Singularity-Consistent Torque Control of a Redundant Flexible-Base Manipulator

Naoyuki Hara, Yusuke Fukazu, Yoshikazu Kanamiya and Daisuke Sato

Abstract A path tracking control method for a kinematically redundant manipulator on a flexible base is proposed. The method is based on dynamic redundancy resolution through a vibration suppression constraint. It is shown that the end-effector path can pass via an algorithmic singularity without destabilizing the system. Simulation data from a planar system is presented, confirming that stable path tracking can be achieved within large portions of the manipulator workspace.

1 Introduction

Manipulators mounted on a flexible base have been studied widely in the past in view of two fields of applications mainly: nuclear waste cleanup [1,2] and space robotics [3,4]. In the former application, a manipulator is mounted on a long beam to ensure access to a remote site. In the latter application, the manipulator is mounted at the end of a large arm that allows for relocation of the manipulator base. Such systems are known as "macro-micro" manipulators. Examples include the Canadian SSRMS/Dextre and the Japanese JEMRMS/SFA manipulator systems on the International Space Station.

Flexible base mounted manipulators induce base vibrations via the reaction force. A few control methods have been proposed in the past that can ensure base vibration suppression control [5–8], design of control inputs that induce minimum vibrations [9], and end-point control in the presence of vibrations [10, 11].

Appropriate control methods depend very much on the structure of the manipulator, e.g. dual-arm or single-arm and the presence of kinematic and/or dynamic redundancy. In this work, we focus on a kinematically redundant flexible base manipulator. End-effector control in the presence of base vibrations becomes possible

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with such a manipulator. In addition, there is also a possibility for vibration suppression control via manipulator self-motion.

We should note, however, that redundancy resolution techniques usually suffer from the presence of algorithmic singularities [12]. In the case of a flexible base manipulator, algorithmic singularities are due to the imposed vibration suppression constraint, and are located inside the workspace. As noted in [13], it is physically impossible to realize vibration suppression at such manipulator configurations. The work of Hanson and Tolson demonstrates this fact [14]. Unfortunately, the importance of this problem has been usually underestimated in literature, even in recent studies [15].

We have addressed the problem of flexible base manipulator teleoperation control in the presence of both algorithmic and kinematic singularities in a recent work [16]. A velocity control framework has been designed, based on the Reaction Null-Space [17] and the Singularity-Consistent [18] methods, named Singularity-Consistent Vibration Suppression (SCVS) control. The aim was to achieve stable teleoperation control throughout the entire workspace.

The aim of the present work is twofold. First, we highlight a problem with the SCVS velocity controller related to the presence of algorithmic singularities due to the Reaction Null-Space constraint. Second, we develop a dynamic torque control framework and show how the algorithmic singularity problem can be tackled within such framework.

2 Background and Notation

The equation of motion of a manipulator mounted on a flexible base can be written in the following form [17]:

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\nu}}_b \\ \ddot{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_b \\ \boldsymbol{c}_m \end{bmatrix} + \begin{bmatrix} \boldsymbol{D}_b \boldsymbol{\nu}_b \\ \boldsymbol{D}_m \dot{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_b \Delta\boldsymbol{\xi} \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\tau} \end{bmatrix} \tag{1}$$

where $\Delta\boldsymbol{\xi} \in \Re^k$ denotes the positional and orientational deflection of the base from its equilibrium, $\boldsymbol{\nu}_b$ is the twist (velocity/angular velocity) of the base, $\boldsymbol{q} \in \Re^n$ stands for the generalized coordinates of the arm, $\boldsymbol{H}_b(\boldsymbol{q}, \Delta\boldsymbol{\xi})$, $\boldsymbol{D}_b$, and $\boldsymbol{K}_b \in \Re^{k\times k}$ denote base inertia, damping and stiffness, respectively. $\boldsymbol{H}_m(\boldsymbol{q}) \in \Re^{n\times n}$ is the inertia matrix of the arm, $\boldsymbol{D}_m$ stands for joint damping and $\boldsymbol{H}_{bm}(\boldsymbol{q}, \Delta\boldsymbol{\xi}) \in \Re^{k\times n}$ denotes the so-called *inertia coupling matrix*. $\boldsymbol{c}_b(\boldsymbol{q}, \dot{\boldsymbol{q}}, \Delta\boldsymbol{\xi}, \boldsymbol{\nu}_b)$ and $\boldsymbol{c}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}, \Delta\boldsymbol{\xi}, \boldsymbol{\nu}_b)$ are velocity-dependent nonlinear terms, and $\boldsymbol{\tau} \in \Re^n$ is the joint torque. No external forces are acting neither on the base nor on the manipulator.

Under the simplifying assumptions, described in [17], the equation of motion can be linearized around the equilibrium of the base, as follows:

$$\boldsymbol{H}_b \dot{\boldsymbol{\nu}}_b + \boldsymbol{D}_b \boldsymbol{\nu}_b + \boldsymbol{K}_b \Delta\boldsymbol{\xi} = -\boldsymbol{H}_{bm} \ddot{\boldsymbol{q}}. \tag{2}$$

Then, choose the control acceleration as

$$\ddot{\boldsymbol{q}}_c = \boldsymbol{H}_{bm}^{+} \boldsymbol{G}_b \boldsymbol{v}_b + (\boldsymbol{U} - \boldsymbol{H}_{bm}^{+} \boldsymbol{H}_{bm})\boldsymbol{\zeta}, \tag{3}$$

where $\boldsymbol{G}_b$ is a positive definite matrix, and $\boldsymbol{H}_{bm}^{+} \in \Re^{n \times k}$ denotes the Moore-Penrose generalized inverse of the inertia coupling matrix, $\boldsymbol{U}$ denotes the unit matrix of proper dimension, and $\boldsymbol{\zeta}$ is an arbitrary vector. Since $\boldsymbol{H}_{bm}\boldsymbol{H}_{bm}^{+} = \boldsymbol{U}$ and $\boldsymbol{H}_{bm}(\boldsymbol{U} - \boldsymbol{H}_{bm}^{+}\boldsymbol{H}_{bm}) = \boldsymbol{0}$, it becomes apparent that controlled damping can be achieved by a proper choice of matrix $\boldsymbol{G}_b$.

Note that the second term on the RHS of the above equation stands for the Reaction Null-Space. In [17], the term was used to ensure the desired end-effector motion constraint. In [16] it was shown that the desired end-effector motion can be realized without the Reaction Null-Space term.

We aim to control both end-tip motion and flexible base vibrations. Denote by $\boldsymbol{v}_e \in \Re^m$ the manipulator end-effector twist. Then we have:

$$\dot{\boldsymbol{v}}_e = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \dot{\boldsymbol{v}}_b, \tag{4}$$

where $\boldsymbol{J}(\boldsymbol{q}) \in \Re^{m \times n}$ is the manipulator Jacobian.

3 Singularity-Consistent Redundancy Resolution with Vibration Suppression Capability

3.1 Redundancy Resolution via Additional Constraint

A well known method for resolving kinematic redundancy is to impose an additional constraint [12]. We derive such an additional constraint in terms of joint acceleration from the vibration suppression control acceleration (3):

$$\boldsymbol{H}_{bm}\ddot{\boldsymbol{q}} = \boldsymbol{G}_b \boldsymbol{v}_b. \tag{5}$$

Note that the Reaction Null-Space term has been thereby ignored.

Let us assume now that the dimension k of base deflection space equals the *degree of redundancy* of the manipulator, that is $k = n - m$. Combining the imposed end-effector acceleration constraint from (4) with the above additional constraint, we obtain:

$$\begin{bmatrix} \dot{\boldsymbol{v}}'_e \\ \boldsymbol{G}_b \boldsymbol{v}_b \end{bmatrix} = \boldsymbol{J}_{vs}\ddot{\boldsymbol{q}}, \tag{6}$$

where $\dot{\boldsymbol{v}}'_e = \dot{\boldsymbol{v}}_e - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \dot{\boldsymbol{v}}_b$, $\boldsymbol{J}_{vs} = \begin{bmatrix} \boldsymbol{J}^T & \boldsymbol{H}_{bm}^T \end{bmatrix}^T \in \Re^{n \times n}$. The joint acceleration can then be written as:

$$\ddot{\boldsymbol{q}} = \boldsymbol{J}_{vs}^{-1} \begin{bmatrix} \dot{\boldsymbol{v}}'_e \\ \boldsymbol{G}_b \boldsymbol{v}_b \end{bmatrix}. \tag{7}$$

Though the above solution was obtained in a straightforward manner, we must note that performance will inevitably degrade when matrix $\boldsymbol{J}_{vs}$ becomes singular. The condition $\det \boldsymbol{J}_{vs} = 0$ means that the linear system (6) becomes singular. When displayed in workspace, the singularities are mapped to both isolated points and continua. A well-known subclass of singularities are the kinematic singularities, defined by the condition $\det \boldsymbol{J}\boldsymbol{J}^T = 0$. For articulated manipulators, these appear mainly at the workspace boundaries. The rest of the singularities, referred to as "algorithmic singularities," are located within the workspace, though. Since the additional constraint used here is the vibration suppression constraint, we can expect that the capability to suppress vibrations will deteriorate around these algorithmic singularities [13]. In addition, the system may destabilize. This hinders the task planning problem significantly.

3.2 Singularity-Consistent Solution for the Acceleration

To cope with the singularity problem, we will rewrite the above joint acceleration (7) according to the Singularity-Consistent method [18]. First, we compose the column-augmented Jacobian and the respective homogeneous equation:

$$\bar{\boldsymbol{J}}_{vs}\ddot{\bar{\boldsymbol{q}}} = \boldsymbol{0}, \tag{8}$$

where

$$\bar{\boldsymbol{J}}_{vs} = \begin{bmatrix} \boldsymbol{J} & -\dot{\boldsymbol{v}}'_e & \boldsymbol{0} \\ \boldsymbol{H}_{bm} & \boldsymbol{0} & -\boldsymbol{G}_b\boldsymbol{v}_b \end{bmatrix} \in \Re^{n\times(n+2)} \tag{9}$$

and

$$\ddot{\bar{\boldsymbol{q}}} = \left[\ddot{\boldsymbol{q}}^T\ 1\ 1\right]^T. \tag{10}$$

Next, we write the set of solutions to the above homogeneous equation as follows:

$$\ddot{\bar{\boldsymbol{q}}} = \bar{\boldsymbol{N}}_{vs}\boldsymbol{b}_{vs}, \tag{11}$$

where $\bar{\boldsymbol{N}}_{vs} = \left[\bar{\boldsymbol{n}}_m\ \bar{\boldsymbol{n}}_b\right] \in \Re^{(n+2)\times 2}$. The two column vectors of $\bar{\boldsymbol{N}}_{vs}$ are: $\bar{\boldsymbol{n}}_m = \left[\boldsymbol{n}_m^T\ \det \boldsymbol{J}_{vs}\ 0\right]^T$ and $\bar{\boldsymbol{n}}_b = \left[\boldsymbol{n}_b^T\ 0\ \det \boldsymbol{J}_{vs}\right]^T$, and $\boldsymbol{b}_{vs} = \left[b_m\ b_b\right]^T$ is a vector with arbitrary components. The last equation can be expanded as:

$$\ddot{\boldsymbol{q}} = b_m\boldsymbol{n}_m(\boldsymbol{q}, \dot{\boldsymbol{v}}'_e) + b_b\boldsymbol{n}_b(\boldsymbol{q}, \boldsymbol{v}_b) \tag{12}$$

$$-1 = b_m \det \boldsymbol{J}_{vs} \tag{13}$$

$$1 = b_b \det \boldsymbol{J}_{vs}. \tag{14}$$

$\boldsymbol{n}_m(\boldsymbol{q}, \dot{\boldsymbol{v}}'_e)$ denotes a vector field component that ensures *reactionless motion* along the desired end-effector trajectory. The $\boldsymbol{n}_b(\boldsymbol{q}, \boldsymbol{v}_b)$ vector field component, on the other hand, ensures vibration suppression.

It is easy to show that if the two arbitrary scalars b_m and b_b are determined from the last two equations, respectively, and substituted into (12), then the joint acceleration obtained will be the same as that in (7), and hence, the system may destabilize around singularities.

One possible way to deal with such problem is by proper choice of b_m and b_b. This is the essence of the Singularity-Consistent method. We sacrifice thereby performance in terms of end-effector acceleration along the desired path and in terms of vibration suppression capability, but gain overall stability.

We should note also an important property of the above solution: the $b_m \boldsymbol{n}_m$ component restricts the manipulator motion in a conservative way due to the Reaction Null-Space constraint $\boldsymbol{H}_{bm}\ddot{\boldsymbol{q}} = \boldsymbol{0}$.[1] The algorithmic singularities appear as a consequence of this constraint. The CoM should not be restricted to move in such conservative way, because inevitably an algorithmic singularity will be reached [16].

4 Pseudoinverse-Based Solution

To relax the constraint on the CoM motion, we will employ a Moore–Penrose generalized inverse (pseudoinverse)-based acceleration component for the end-effector motion. Recall that the general solution for the joint acceleration can be written as [12]:

$$\ddot{\boldsymbol{q}} = \boldsymbol{J}^{+}\left(\dot{\boldsymbol{v}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}\right) + (\boldsymbol{U} - \boldsymbol{J}^{+}\boldsymbol{J})\boldsymbol{\zeta}_a, \tag{15}$$

where $\boldsymbol{\zeta}_a$ is an arbitrary $n-$vector. We can then replace $b_m \boldsymbol{n}_m$ in (12), to obtain:

$$\ddot{\boldsymbol{q}} = \boldsymbol{J}^{+}\left(\dot{\boldsymbol{v}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \dot{\boldsymbol{v}}_b\right) + b_b \boldsymbol{n}_b(\boldsymbol{q}, \boldsymbol{v}_b). \tag{16}$$

When analyzing the above equation, recall that the set of joint accelerations $\ddot{\boldsymbol{q}}_n = b_b \boldsymbol{n}_b$ satisfies the two constraints: $\boldsymbol{J}\ddot{\boldsymbol{q}}_n = \boldsymbol{0}$ and $\boldsymbol{H}_{bm}\ddot{\boldsymbol{q}}_n = \boldsymbol{G}_b \boldsymbol{v}_b$. The former constraint means that vector $\boldsymbol{n}_b$ belongs to the null space of the Jacobian: $\boldsymbol{n}_b \in \mathcal{N}(\boldsymbol{J}_{vs})$. Hence, from a well known property of the pseudoinverse-based inverse kinematics solution for kinematically redundant manipulators, it can be concluded that the two components of the above joint acceleration are orthogonal [12]. Thus, their mutual interference will be minimized, and we can expect that the vibration suppression constraint will be enforced constantly during end-effector motion, without disturbing it.

[1] We should note that $\boldsymbol{n}_m$ is derived as the null-space vector of a matrix obtained from $\bar{\boldsymbol{J}}_{vs}$ by removing its last column.

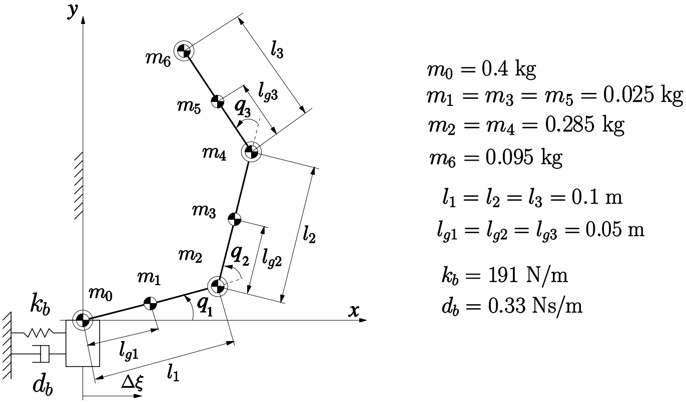

Fig. 1 A planar 3R manipulator on a flexible base.

5 Implementation of the Method

We will demonstrate the method with the help of the planar 3R manipulator shown in Figure 1. The base deflects along the x axis. Hence, the reaction moment and the reaction force component along the y axis can be neglected as a disturbance. We have: $n = 3,\ k = 1$. The parameters of the manipulator are shown in the figure.

End-tip path tracking control (meaning that $m = 2$) is envisioned according to the following control law:

$$\dot{\boldsymbol{v}}_{ref} = \ddot{\boldsymbol{p}}_d + \boldsymbol{K}_v(\dot{\boldsymbol{p}}_d - \dot{\boldsymbol{p}}) + \boldsymbol{K}_p(\boldsymbol{p}_d - \boldsymbol{p}). \tag{17}$$

$\boldsymbol{p}$ and $\boldsymbol{v} \equiv \dot{\boldsymbol{p}}$ denote end-tip position and velocity, respectively. The subscript $(\circ)_d$ denotes a desired quantity, $\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are positive definite feedback gain matrices.

Using (16), the reference joint acceleration is written as

$$\ddot{\boldsymbol{q}}_{ref} = \boldsymbol{J}^+ \left(\dot{\boldsymbol{v}}_{ref} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \dot{\boldsymbol{v}}_b\right) + b_b\tilde{\boldsymbol{n}}_b g_b v_{bx}, \tag{18}$$

where $\boldsymbol{v}_b = [v_{bx},\ 0]^T$ is the base velocity vector, g_b is the vibration suppression gain and $\boldsymbol{n}_b = \tilde{\boldsymbol{n}}_b g_b v_{bx}$.

Further on, the joint torque vector can be written as

$$\boldsymbol{\tau} = \boldsymbol{H}_m(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}_{bm}^T(\boldsymbol{q})\dot{v}_{bx} + \boldsymbol{D}_m\dot{\boldsymbol{q}} + \boldsymbol{c}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}), \tag{19}$$

according to the equation of motion. The joint damping term plays the important role of damping out the momentum, conserved during vibration suppression [17].

Next, insert the reference joint acceleration (18) into the last equation, to obtain the control torque as:

$$\boldsymbol{\tau}_c = \boldsymbol{H}_m \boldsymbol{J}^+ \left(\dot{\boldsymbol{v}}_{ref} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \dot{\boldsymbol{v}}_b\right) + \boldsymbol{D}_m \dot{\boldsymbol{q}} + \boldsymbol{c}_m + \boldsymbol{h}_{bm}^T \dot{v}_{bx} + b_b \boldsymbol{H}_m \tilde{\boldsymbol{n}}_b g_b v_{bx}. \quad (20)$$

6 Simulation Results

Starting from initial configuration $q_1 = 0.0$, $q_2 = q_3 = 0.5$ rad (nonsingular), the manipulator end-tip is required to track a straight-line path parallel to the x axis. The desired current end-tip position, speed and acceleration along the straight-line are calculated from a fifth-order spline function. During this motion, the CoM accelerates/decelerates along the low-stiffness (x axis) direction. Hence, vibrations are induced, that are then to be suppressed by the vibration suppression component (the last term on the r.h.s. in (20)). We should also note that an algorithmic singularity will be encountered along the path.

In the first simulation, the final time for the spline function is set to 9 s. The vibration suppression gain is $g_b = 30$ kgs^{-1}, the feedback gain matrices are $K_p =$ diag $[20000,\ 20000]$ s^{-2} and $K_v =$ diag $[200,\ 200]$ s^{-1}. Joint damping is set to $\boldsymbol{D}_m =$ diag $[0.05,\ 0.05,\ 0.05]$ kgms^{-1}. The vibration suppression scalar b_b is determined from $b_b = 1/\det \boldsymbol{J}_{vs}$. In the neighborhood of the algorithmic singularity, vibration suppression is turned off (by setting $b_b = 0$) to avoid destabilization. The neighborhood is determined by a threshold, selected as $|b_b| = 1.0 \times 10^4$ m^{-2}s^{-2}.

The results from the simulation are shown in Figure 2. It becomes apparent that vibration is successfully suppressed during the motion. At around 5 s, the algorithmic singularity is crossed. From Figure 2(c) it can be observed that CoM acceleration increases around the singularity. Nevertheless, no significant base deflection is observed, and the end-tip error remains within acceptable limits.

In the next simulation, we shortened the execution time of the same path, from 9 s to 3 s, reading to higher overall velocities/accelerations (see Figure 3). The base deflects significantly around the algorithmic singularity. In addition, large peak velocities are observed and the system tends to destabilize. After passing the singularity, vibration suppression is invoked again, further vibrations are suppressed and the system stabilizes.

7 Conclusions

We have developed a path tracking control method for a kinematically redundant flexible base manipulator, capable of simultaneous vibration suppression, based on dynamic redundancy resolution. The effect achieved is similar to that of reactionless path motion control. In addition, we have shown that it is possible to cross an algorithmic singularity without destabilizing the system, despite using high PD-feedback gains.

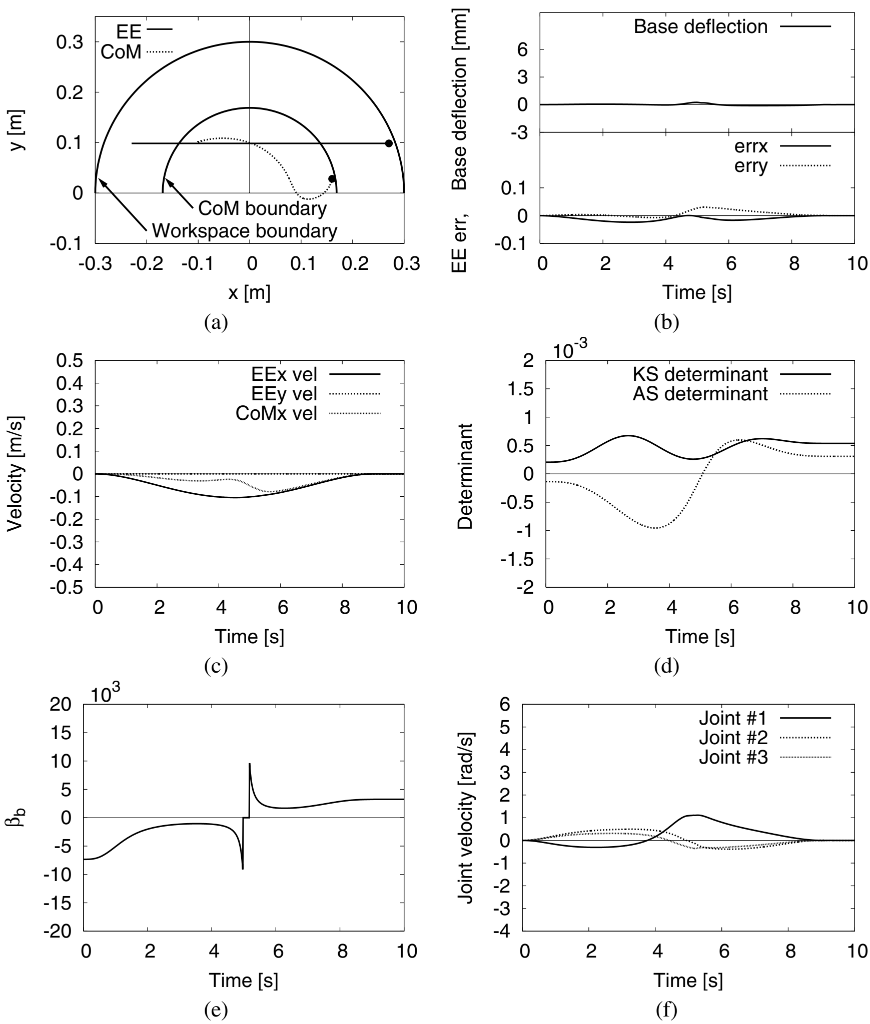

Fig. 2 Straight-line tracking and vibration suppression in case of a relatively slow movement.

Unfortunately, with faster movements, the base may deflect locally, around the singularity, since vibration suppression is switched off for a short time to avoid destabilization. We intend to tackle this problem in a future work by proper end-tip speed/acceleration replanting.

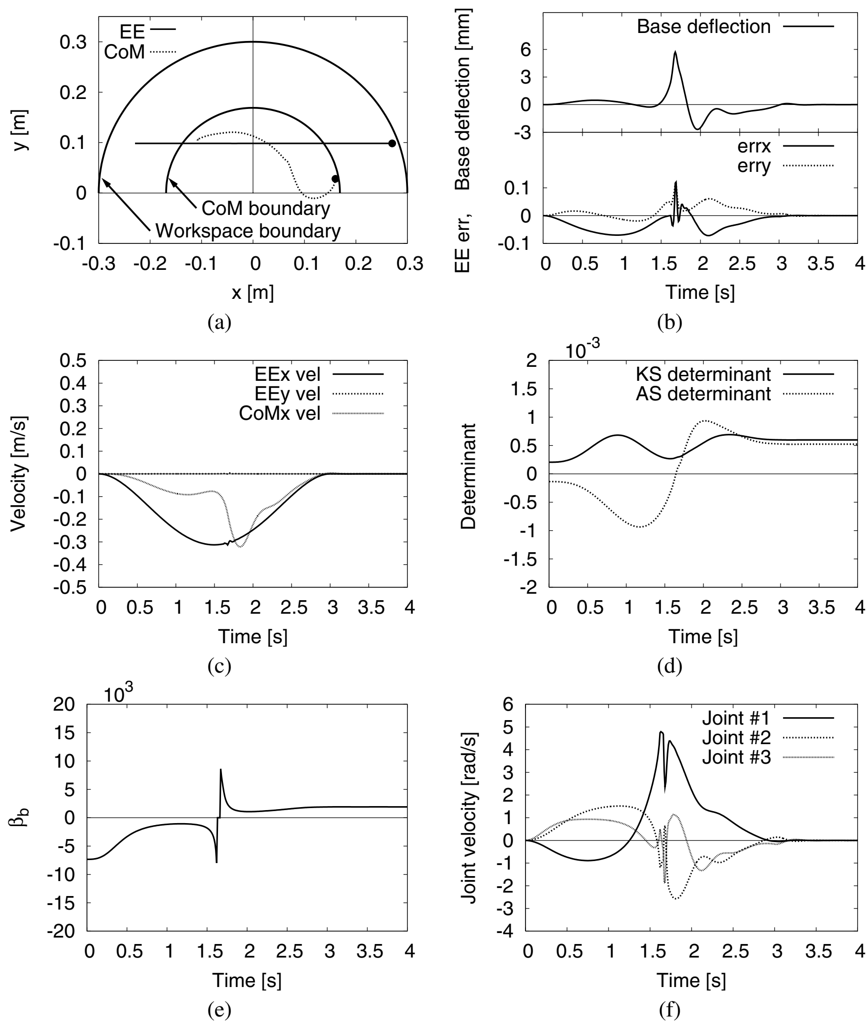

Fig. 3 Straight-line tracking and vibration suppression in case of a relatively fast movement.

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