SH WAVE PROPAGATION IN LATERALLY HETEROGENEOUS MEDIUM

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Abstract. The present work reports some of the recent investigations in the field of Investigations of our group in wave propagation in laterally heterogeneous media. Such studies have practical applications in developing building code in earth prone area or in wave-guide problem. We present a new analytical approach to the title problem. This is an improved version of our earlier work with M. Mitra and R.K. Bhattacharya[1].

We consider the SH wave propagation through two different elastic media in vertical contact with each other while the medium on the right or left is horizontally layered. One can assume the contact surface to be inclined to each other. Unlike the usual approach of finding the approximate form of the reflection transmission coefficient by the use of the representation theorem, we use the exact solution of the wave equation in each medium. Various boundary conditions at the horizontal and vertical/inclined contact surface are satisfied exactly. Boundary conditions give rise to a set of coupled integral equation connecting the unknown constants of the solution. The source may be a line source or simply a plane incident wave. Iterative solutions of the integral equations provide various wave arrivals which include the scattered waves unlike previous works. Exact computations of various wave arrivals may help in developing accurate building codes in earthquake prone area.

Keywords: lateral heterogeneity, vertical/inclined surface

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1. Introduction

During the past forty years various theoretical methods have been developed to compute the reflection/transmission coefficients of surface waves across vertical discontinuities. These methods are basically based on i) the use of the representation theorem[2-5], ii) some approximation to the boundary conditions at the vertical contact surface[6], iii) representation of the surface wave field as a superposition of plane homogeneous and inhomogeneous waves[7-8]. Scattered waves from the step like irregularities have been studied in [9], [10]. In all the above works except the last three the vertical contact surface conditions are approximately satisfied neglecting the scattering effect.

2. Mathematical formulation

We consider the SH wave propagation in two layered media (Fig. 1) vertical contact with each other.

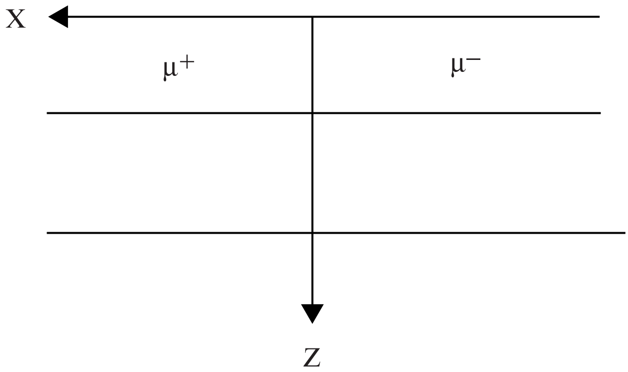

Fig. 1 Two layered media.

In all layers two dimensional SH displacement v satisfies the wave equation and the solution in the i^{th} layer s taken as

$$v_i^{\pm}(x,z) = e^{-i\omega t}\left[\int_0^{\infty}\left[A_i^{\pm}(\zeta)e^{-iz\sqrt{\zeta^2-\omega^2/\beta_i^2}} + B_i^{\pm}(\zeta)e^{iz\sqrt{\zeta^2-\omega^2/\beta_i^2}}\right]\cos(\zeta x)\right]d\zeta$$
$$+\left[C_i^{\pm}(\zeta)e^{-i|x|\sqrt{\zeta^2-\omega^2/\beta_i^2}}\cos(\zeta z)\right]d\zeta + v_0(x,z)$$

where β is the SH wave velocity, ω is the angular frequency and the constants $A_i^{\pm}(\zeta)$ represent waves in the i^{th} medium in $x > 0$ and $x < 0$ moving away from the horizontal surface, $C_i^{\pm}(\zeta)$ corresponding waves moving away from the vertical contact surface and $v_0(x,z)$ is the source effect.

Boundary conditions are:

a) Continuity of displacement and stress across horizontal boundary $z = z_i$

$$v_i(x,z_i-0) = v_{i+1}(x,z_i+0)$$

$$\mu_i \frac{\partial v_i(x,z_i-0)}{\partial z} = \mu_{i+1} \frac{\partial v_{i+1}(x,z_{i+1}+0)}{\partial z}$$

b) Continuity of displacement and stress across vertical contact surface $x = 0$

$$v_i^+(0+,z) = v_i^-(0-,z)$$

$$\mu_i^+ \frac{\partial v_i^+(0+,z)}{\partial x} = \mu_i^- \frac{\partial v_i^-(0-,z)}{\partial x}$$

On satisfying the boundary conditions we derive a set of coupled integral equations connecting $A_i^{\pm}(\zeta)$ with $B_i^{\pm}(\zeta)$ which are solved iteratively. We specifically give the detailed calculations in the case of two layered model.

3. Conclusion

Use of the exact solution of the wave equation and the exact validation of the boundary conditions give not only the reflected and transmitted surface waves as all kind of scattered waves in the model considered. The systematic evaluation of various wave arrivals by the new formulation may be of practical use.

Though we have only considered SH waves our formulation can be easily extended to the case of P and SV wave. Our analysis can be easily modified for the inclined contact surface. Such inclined contact surface is present in the ocean-continental regions.

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