Chapter 9
Mathematical Models for Reservoir Operation in Tunisia

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Abstract A genetic algorithm model has been developed and applied to solve a planning problem of optimum allocation of water resources within a complex reservoir system. The specific conditions of the surface water resource utilization in Tunisia, exemplified in a 10-reservoir case study system (Louati 2005 thèse de doctorat en sciences agronomiques "Spécialité: Génie rural eau et forets", Inat, Tunis, Tunisie), have required that the allocation of the available resources be analyzed considering both the quantity as well as salinity of supply. Therefore, the analyses included resource allocation optimization under the assumption of five different objective functions reflecting the relationship between the two supply criteria. In addition, the obtained solutions under the five objective assumptions have further been assessed across a range of system performance indicators. This step has proven essential in obtaining a more comprehensive insight into the operation of the system under the different objectives.

Introduction

The availability of, and the demand for water form one of the most complex relationships the mankind is facing. Under "availability of water" one should primarily underline the limiting amount and acceptable quality of water in our hydrological cycle in arid and semi arid zones, and the uneven distribution of its quantities in space and time. By "demand for water" one should consider drinking and agricultural water consumption as the essential preconditions for human life sustenance, as well as the areas of water use, which could be considered as contributing factors to the improvement of the quality of life (i.e. non consumptive household, industrial and tourism, energy production, recreation water demands etc.).

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Quality factor is the required to reconcile water availability and minimum required quality demand, as drinking water or some crop irrigation requirements. On the one hand, the quality of available water resources determines, to a varying degree, their suitability for different purposes. The quality of water released back to the environment after its use, on the other, influences the extent of environmental pollution and, in turn, prospects for the maintenance of the sustainable use of the water resources in the future. Furthermore, both the use of the available water resources and the release of the used effluents back to the environment have an impact on the environmental balance in the affected areas.

The aforementioned quantitative and qualitative aspects of the balance of water resources have been recognized as crucial in the strive to maintain the necessary environmental quality, ensuring at the same time that everyone gets a just share of water of good quality. The water resources management aims to improve the water use efficiency, equity of distribution and sustainability of the water system.

The objective criterion is to optimize the water management rules, with the quality as salinity and quantity objectives, for dams' network. The case study system consists of 15 large reservoirs in the Northern part of Tunisia. The reservoirs are mutually interconnected in either serial or parallel fashion, both through natural river reaches as well as man-made water transfers.

The system encompasses 36 individual demand centers grouped into three principal water user types: urban (five demands), irrigation (30 demands) and environmental (one demand). The demands have been described by two parameters: demand volume and the maximum acceptable supply salinity.

System topology studied indicates that the analyses are to address a rather difficult operations research problem. On the one hand, the system itself can contain multiple reservoirs and demand centers, which can be linked together in an intricate network. On the other hand, the consideration of salinity of reservoir inflows and releases, and thereby allocations to individual demands, adds additional complexity to the operation problem. It is obvious therefore that the optimization problem must apply criteria that will be able to address both the quantity and salinity of reservoir allocations to individual demands. Furthermore, reservoir operating storage targets (rule curves) are considered as an additional objective criterion.

The primary goal of the analyses is to identify the preferable water resource allocation strategies within a complex water supply reservoir system and, at the same time, to derive the respective optimum operating policies of system reservoirs. To achieve this goal, three objective criteria have been defined and adopted for the analyses:

- To minimize the supply quantity deficit;
- To minimize the violation (surpassing) of supply salinity thresholds set for individual demands; and
- To minimize the deviation of the operating final storage of reservoirs from the predefined final storage targets.

Structure of the Optimization Problem

The main goal of this work is to assess the applicability of a combination of several operations research approaches to a strategic operational problem of complex reservoir supply systems. System topology requires that the adopted approach for the analyses be able to tackle rather complex system configurations. With regard to such a system topology, the focus of the work is limited to the optimization of the long-term operating strategy of a multiple reservoir water supply system. In principle, an operating strategy of such a complex system may be understood as a composition of two main parts:

- Reservoir-demand allocation patterns; and
- Reservoir operating policies reflecting the aforementioned allocation patterns.

Such a decomposition of the operating strategy is justified by the fact that the original problem is rather complex and mathematically none polynomial.

Reservoir-demand allocation patterns are introduced to resolve the problem of demand sharing among groups of reservoirs. The task of optimization is therefore to identify those demand sharing patterns that would lead to the best allocation of water resources within a system.

Once reservoir-demand allocation patterns have been derived, the optimization of individual reservoir operating policies can be carried out. This process is therefore based on the assumption that the derived allocation patterns have to be complied with in policy optimization. As a consequence, the obtained operating policies will preserve the imposed reservoir-demand allocation patterns.

The stochasticity of reservoir inflows is considered where uncertainty of the inflow processes is sufficient for the case being analyzed. With regard to the temporal discretization, the analyses are limited to monthly time steps assuming the stationarity of the stochastic properties of monthly river flows (i.e. the probability distribution of a stochastic process is not changing over time). Monthly water demands, on the other hand, are assumed to be deterministic and considered to be recurring in annual cycles. Since the chosen monthly time base is long enough the required time for the released water to travel between any two serially linked reservoirs and any reservoir and the respective demand centers can safely be neglected.

Since the size of such a problem can be prohibitively large (i.e. number of reservoirs and demand centers, the complexity of reservoir-reservoir and reservoir-demand interconnections, consideration of flow stochasticity, and multiple objectives), it is inevitable to employ an iterative derivation procedure to arrive at the respective solution. One common characteristic of almost all the approaches of this kind is, however, that the global optimality of the obtained solution cannot be guaranteed. It is, therefore, necessary to emphasize that the starting point of this work was not to pursue a methodology which would guarantee the derivation of the global optimum operating strategy at any cost, but rather to try and identify a relatively simple and transparent, however yet efficient and effective approach for the analysis of the operation of complex reservoir systems. With this notion in view, the decomposition applied in this study is done at two levels:

- Problem decomposition may be understood as a coupling of reservoir system (topology) decomposition and reservoir-demand allocation patterns; and
- Reservoir operating policies reflecting the aforementioned allocation patterns.
  The main features of the applied system decomposition approach as iterative approach are:
- A multiple reservoir system is decomposed into single-reservoir sub-systems;
- Appropriate optimization/simulation techniques are applied to single-reservoir sub-systems;
- Single reservoirs are entering an iterative cycle of analyses in a predefined sequence;
- The interaction between the reservoirs is modeled by an auxiliary model, which is selected on the basis of the type of problem being solved (i.e. reservoir-demand allocation patterns or reservoir operating policies).

Based on the aforementioned description of the problem and its decomposition, optimization and general structure of adopted approach to derive long-term operating strategy of a complex reservoir system can be formulated as follows:

- Decompose the problem into resource allocation and policy optimization;
- Decompose the reservoir system into individual reservoir sub-systems;
- Solve the resource allocation sub-problem applying the appropriate optimization method combined with the reservoir system decomposition principles;
- Solve the policy optimization sub-problem applying the appropriate optimization method combined with the reservoir system decomposition principles;
- Simulate the operation of the system according to the derived resource allocation patterns and operating policies;
- Evaluate the performance of the system.

Namely, the resource allocation sub-problem is solved by a genetic algorithm (GA) based search model. The principal idea of a GA search is to sweep the objective function space looking for solutions that bring improvement to the objective function. In this specific case, the GA model assumes that a solution is a collection of reservoir-demand allocation targets for the entire system and uses reservoir system simulation to estimate the objective function value for each potential solution to the allocation problem.

The adopted methodology for the optimization of the long-term operating policies for individual reservoirs combines a physical decomposition of the system into individual reservoir subsystems, stochastic dynamic programming (SDP) optimization of a single reservoir operation, simulation and release allocation among each reservoir's water users. Since the SDP model derives the operating policy for a single reservoir (as opposed to the GA model which derives the allocation pattern for the entire system) its application has to be combined with system decomposition, simulation and release allocation. In addition, the developed SDP model utilizes the reservoir-demand allocation patterns derived by the preceding run of the GA.

Finally, simulation of the system operation according to the derived policies is essential due to three reasons:

- It is necessary for the evaluation of potential solutions in the genetic algorithm;
- It is an integral component of the stochastic dynamic optimization model; and
- System performance evaluation could be done using simulation.

To transform the multi-objective decision making problem into a single objective optimization, the obvious choice is to opt for a composite objective function, which would include all three objectives. The composite objective has been made to combine two objective criteria in deriving reservoir-demand allocation patterns, and different pairs of criteria for the optimization of reservoir operating policies:

- Reservoir-demand allocation patterns: supply quantity and supply salinity objectives; and
- Reservoir operating policies reflecting the aforementioned allocation patterns: supply quantity and storage target objectives.

To solve this, a genetic algorithm search is used to derive reservoir-demand allocation patterns. A GA search is based on objective function estimation using simulation of system operation and, therefore, it is no problem to develop a simulation model for a single reservoir that is able to simulate both the volumetric and salt balance of water in a reservoir during a time step. Hence, supply salinity objective can be applied to the first problem without difficulty. On the other hand, stochastic dynamic programming is applied to derive reservoir-operating policies and considers reservoir inflows as a stochastic process. Thus, SDP describes reservoir inflows as a Markov process through estimation of monthly inflow transitional probabilities. Consideration of salinity would therefore also require that inflow salinity time series is also described as a Markov process, which would impose that joint probability distributions of flow volumes and salinities are estimated. This would however, render a discrete SDP formulation rather complicated. Furthermore, salinity data available for the research show very little variability over the years of record, thus justifying the assumption that the consideration of supply salinity objective only in reservoir-demand allocation sub-problem. That is, the derived allocation patterns would then sufficiently reflect the objective to minimize the violation of supply salinity threshold and would thereafter implicitly incorporate the salinity consideration into the SDP-based operating policies derived within the second sub-problem.

Genetic algorithm search for the best reservoir-demand allocation patterns is also used to derive the storage targets of individual reservoirs.

Finally, the combination of supply quantity and storage target objectives in SDP optimization of reservoir operating policies completes the combination of the three objectives. In addition, the derived SDP operating policies would reconcile, in a single policy, the aim to maintain the optimum level of supply quantity and salinity, and the desired storage target curve.

Since there are three objective criteria adopted, the selection of performance indicators must also reflect the criteria themselves. Therefore, three distinctive sets of performance indicators are defined to provide additional information on the analyzed system performance:

- Performance indicators for the supply quantity objective;
- Performance indicators for the supply salinity objective; and
- Performance indicators for the storage target objective.

Reliability Criteria Assessment in Evaluation of Reservoir Performance

Within stochastic optimization concepts the most frequently used objective criteria include either the maximization of the expected system output or benefit function, or the minimization of the expectation of some form of loss function. Utilization of this type of criteria provides the estimate of the expected performance of the system in the long run. However, they cannot shed any light on the frequency of the system's failing to provide the required service, the duration and severity of potential failures, nor the ability of the system to return to a satisfactory operating state once a failure has occurred. These important facets of a system's performance are widely known as reliability indicators. Consequently, substantial effort has been put into the explicit consideration of reliability in the optimization of the operation of reservoir systems. It could be said that the most significant in the field started with the work on chance-constrained programming by ReVelle et al. (1969), which was further extended by, to name just a few, ReVelle and Kirby (1970), Eastman and ReVelle (1973), ReVelle and Gundelach (1975), Gundelach and ReVelle (1975), Lebdi et al. (1997, 2003), Loucks and Dorfman (1975), Houck (1979), Houck and Datta (1981), and many others, including the works on reliability programming by Simonovic and Mariño (1980, 1981, 1982).

Recognizing that the simulated estimates of the mean and the variance of the selected performance measure (e.g. output, operating cost) could not provide accurate information about the frequency and magnitude of operational failures, Hashimoto et al. (1982) used three additional performance indicators (PI) to compare a number of different operating policies of a single irrigation water supply reservoir. They introduced *reliability* to describe how often the system failed to meet the target; *resiliency* to assess how quickly the system managed to return to a satisfactory state once a failure had occurred; *vulnerability* to estimate how significant the likely consequences of a failure might be. Based on simulation of the reservoir's operation over a long synthetic inflow time series, a set of operating strategies was evaluated by deriving trade-offs among the expected loss, reliability, resiliency and vulnerability. For instance, one conclusion that could be drawn from the analyses was that, for the given case study, high system reliability was always accompanied by high vulnerability (i.e. the fewer failures the reservoir had, the higher the deficits encountered in the failure periods). The authors also pointed out that each problem bears its own unique features and, therefore, the selection of appropriate performance indicators should always reflect upon those unique characteristics of the problem.

Similar conclusions were also drawn by Moy et al. (1986) in their study of the operation of a single water supply reservoir. They used mixed-integer linear programming to derive trade-off curves among the virtually same three performance indicators presented by Hashimoto et al. (1982). Namely, they defined *reliability* as the probability of failing to meet the desired target; *resilience* as the maximum number of consecutive failures prior to the reservoirs return to the full supply state of operation; and *vulnerability* as the maximum supply deficit observed during simulation. The major finding described the relationship between vulnerability and the other two PIs. In general, the results showed that a reservoir would likely exhibit higher vulnerability (i.e. larger magnitude of failures) if it were more reliable (i.e. had fewer operating failures), or if it were more resilient (i.e. had short sequences of repeated failures).

The extensive study of Bogardi and Verhoef (1995) presented a more detailed analysis of the sensitivity of the operation of the same three-reservoir Mahaweli river development scheme in Sri Lanka. Using a range of different objective criteria, they optimized the operation of the system by means of SDP and subsequently appraised the derived operating strategies by simulation. In addition to the simulated objective criterion estimates, the comparisons were carried out on the basis of an array of both energy and irrigation related PIs (n.b. for each PI, separate estimates were derived for energy and irrigation).

Nandalal and Bogardi (1996) used an array of quantity-related PIs to evaluate the performance of a single water supply reservoir whose operating strategies were derived by optimization considering both the quantity and quality of reservoir releases. Specifically, they adopted seven PIs to investigate the impact of different salinity reduction measures of reservoir releases on the quantitative aspects of the reservoir's performance.

A number of PIs is selected to compare different operating strategies of the case study system in this work. The defined PIs do not depict the operating details of individual reservoirs. They rather describe the performance of the entire multiple-reservoir system with respect to the quantitative fulfillment of the water demand imposed upon the system (n.b. a similar approach has also been adopted in Milutin and Bogardi, 1995, 1996a and 1996b). The set of PIs used in this case study includes a number of criteria defined to evaluate various facets of reliability, resilience and vulnerability of the system's operation. A detailed definition of the adopted PIs is given in "Performance Indicators".

Objective Criteria

This section provides the detailed description of the three objective criteria used. Each of the three objective functions (i.e. supply quantity achievement, salinity threshold non-breach and reservoir storage target achievement) is presented in its full mathematical formulation. In addition, an introduction and an argumentation about the combined use of the objective functions in different optimization steps are given here as well.

Supply Quantity Objective

The supply quantity objective aims at minimizing the deviation of supply from the respective demand targets. The objective function is defined as an aggregate of the squared supply deviations from the respective demand targets over all individual demands and over the entire time span of the analyses:

$$Z_1 = \sum_{t=1}^{T} \sum_{i=1}^{N} (R_{ti} - D_{ti})^2 \tag{9.1}$$

Where:

- Z_1 supply quantity objective criterion achievement
- T number of time steps in the objective criterion assessment
- N number of demands
- R_{ti} allocation of supply to demand i in time step t
- D_{ti} demand i in time step t

To force the optimization procedure to seek the solution that is reducing the risk from extreme supply shortages, this objective is penalizing the supply deviation from its respective target as the square of the resulting deviation. If the objective function were linear, the optimization procedure would not make any distinction between, for example, a single large deficit and a number of smaller deficits amounting to the same total volume.

By adopting such an objective function form, it is ensured that the optimization procedure will disregard, to the maximum extent possible, solutions that result in excessive supply shortages or surpluses. This approach therefore strives to reduce the vulnerability of the system performance.

Supply Salinity Objective

In essence, the initial assumptions used to define this objective function have been very similar to the ones used in the definition of the other two objectives. That is, given a certain salinity threshold beyond which the salinity of supply to a demand center should not occur, this objective function should represent a penalty if such a case does happen. There are two principal differences between the supply salinity threshold objective and the other two objective functions:

- Supply salinity objective penalizes only the surplus of salt concentration beyond the specified threshold value, whereas the other two penalize the deviation from their respective target; and
- The units and the magnitude of surplus of salinity differ significantly from those in the other two objectives.

The first difference is no obstacle for the definition of the objective function. However, the second one does require careful consideration when defining the objective function. This is due to the fact that the intrinsic multiobjective decision making problem is to be transformed into a single (composite) objective optimization, thus requiring that different objective function components be additive (i.e. supply quantity achievement and salinity threshold non-breach objectives).

Since the objective functions should be used jointly in optimization, the second obstacle is overcome by redefining the supply salinity surplus formulation into a volumetric equivalent (volume of water) describing the relationship between the supplied volume and salinity, and the imposed supply salinity threshold. Namely, let the following be the variables and relations describing the aforementioned quantities:

- Salt concentration of the allocated supply to a demand center (C_{ti}):

$$C_{ti} = \sum_{j=1}^{M} r_{tij} c_{tj} \Bigg/ \sum_{j=1}^{M} r_{tij} \tag{9.2}$$

- The total amount of water allocated (R_{ti}) to meet the demanded volume D_{ti}

$$R_{ti} = \sum_{j=1}^{M} r_{tij} \tag{9.3}$$

where the newly introduced symbols so far are:

r_{tij} volume released from reservoir j for demand i in time step t
c_{tj} salinity of release from reservoir j in time step t

If the salinity of the supply C_{ti} is beyond the maximum threshold salinity $C_{i\max}$ for that particular demand, one can assume that the supplied volume will have to be additionally treated or partially replaced by some fresh water amount (volume A_{ti} of salinity c_{ext}) which would then reduce the salinity of the originally supplied water to the threshold level, or lower. This amount of additional fresh water can be estimated from the salt balance inequality:

$$R_{ti} \cdot C_{i\,\max} \geq (R_{ti} - A_{ti}) \cdot C_{ti} + A_{ti} \cdot c_{ext} \tag{9.4}$$

or, expressed as the equality for estimating the minimum value of the volume A_{ti}:

$$A_{ti} = \begin{cases} R_{ti} \dfrac{C_{i\,\max} - C_{ti}}{c_{ext} - C_{ti}} & , \quad C_{ti} > C_{i\,\max} \\ 0 & , \quad otherwise \end{cases} \tag{9.5}$$

It need not be mentioned that the assumed salinity c_{ext} of this "external" source of fresh water must be lower than the supply salinity threshold $C_{i\max}$ of the demand in question.

Given the estimates of the required external source supply A_{ti} to dilute the allocated volumes in each time step when the supply salinity threshold breach occurs, the objective function value can be estimated as:

$$Z_2 = \sum_{t=1}^{T} \sum_{i=1}^{N} A_{ti}^2 \tag{9.6}$$

The objective is penalizing the volumetric equivalent of the supply salinity surplus beyond its respective threshold as the square of the equivalent volume of fresh water needed to dilute the allocated salinity to the respective threshold value. Again, the choice of a squared rather than linear form of the penalty is forcing the optimization procedure to opt for more failures of lesser magnitude rather than just a few high ones.

Reservoir Storage Target Objective

The reservoir storage target objective function is very similar in its form to the supply quantity objective described before. Namely, it penalizes the deviation of the final storage volume of a reservoir observed in optimization/simulation from the respective target storage volume. The function itself is defined as an aggregate of the squared final storage volume deviations from their respective targets over all individual reservoirs and over the entire time span of the analyses:

$$Z_3 = \sum_{t=1}^{T} \sum_{j=1}^{M} \left(SF_{tj} - ST_{tj}\right)^2 \tag{9.7}$$

where the newly introduced symbols so far are:

Z_3 reservoir storage target objective criterion achievement
M number of reservoirs
SF_{tj} observed final storage volume of reservoir j in time step t
ST_{ti} target final storage volume of reservoir j in time step t

Similarly to the discussion on the other two objective functions presented in "Supply Quantity Objective and Supply Salinity Objective", the storage target objective function is also defined as an aggregate of squared deviations to force the optimization procedure to avoid solutions with fewer high deviations as opposed to those with numerous lower deviations from the target.

Composite Objective Within Resource Allocation Optimization

A genetic algorithm search for the best resource allocation pattern is based on the objective that minimizes the value of a so-called fitness function. In essence, a genetic algorithm fitness function is the equivalent of an objective function in an optimization procedure. The adopted fitness function is defined as an aggregate of two distinct components:

- Quantity-related squared deviation of supply from the target demand, multiplied by the respective weight factor; and
- Salinity related squared penalty of a volumetric equivalent of the violation of the maximum acceptable supply salinity, multiplied by the respective weight factor.

Given the definition of the two individual objective functions in "Supply Quantity Objective and Supply Salinity Objective", it is necessary to adjust their estimation for the purpose of their combined use in the aforementioned fitness evaluation. It should also be noted here that in the definition of the genetic algorithm's fitness evaluation model the allocated consumptive release cannot exceed the respective demand. Therefore supply shortage is the only possible quantitative supply failure, and surplus can never occur.

The penalty associated with a failure of meeting the quantity and/or quality requirement is derived under the assumption that either of the two is to be compensated for from an imaginary external source with water of a constant (low and known) salt concentration. The joint penalty for utilization of such a source is proportional to the square of the amount of water withdrawn regardless of the purpose of such a withdrawal (i.e. to compensate for quantity shortage or to improve the quality of delivered water or both). The penalty is thus estimated in four steps described below:

1. Based on the observed quantitative supply deficit associated with a demand during a certain time step, the imaginary external source provides full compensation for the incurred shortage. The external compensation for the supply deficit affects the salt concentration of the water delivered to the demand center. The estimation of the resulting salinity of the assumed "full supply" is computed from the following equations:

   - Salinity of the original supply from the associated reservoirs:

$$C_{ti} = \sum_{j=1}^{M} r_{tij} c_{tj} \Big/ \sum_{j=1}^{M} r_{tij} \tag{9.8}$$

   - Total volume supplied by the associated reservoirs:

$$R_{ti} = \sum_{j=1}^{M} r_{tij} \tag{9.9}$$

- Salinity of "full supply" (including the volume provided by the external source):

$$C'_{ti} = \frac{R_{ti} \cdot C_{ti} - (D_{ti} - R_{ti}) \cdot c_{ext}}{D_{ti}} \tag{9.10}$$

- Salinity of "full supply" (in a slightly different form):

$$C'_{ti} = \frac{R_{ti}}{D_{ti}} \cdot C_{ti} + \left(1 - \frac{R_{ti}}{D_{ti}}\right) \cdot c_{ext} \tag{9.11}$$

2. Having estimated the salinity of the "full supply" after the initial compensation from the external source for the quantitative shortage, it is necessary to assess whether the newly obtained supply salinity is below the supply salinity threshold associated with this demand:

- The "full supply" salinity is below the threshold value,

$$C'_{ti} \leq C_{i\max} \tag{9.12}$$

  and there is no need for additional fresh water supply, i.e. $A_{ti} = 0$.
- The "full supply" salinity is still higher than the threshold value,

$$C'_{ti} > C_{i\max} \tag{9.13}$$

  and the additional fresh water volume (A_{ti}) is estimated from the salt balance equation for this demand (it needs no mention that $c_{ext} < C_{i\,\max}$)

$$D_{ti} \cdot C_{i\max} = (D_{ti} - A_{ti}) \cdot C'_{ti} + A_{ti} \cdot c_{ext} \tag{9.14}$$

  which leads to

$$A_{ti} = D_{ti} \cdot \frac{C'_{ti} - C_{i\,\max}}{C'_{ti} - c_{ext}} \tag{9.15}$$

3. The total penalty f_{ti} (both quantity and salinity related) associated with the supply to this demand center during one time step then becomes (w_q and w_s are penalty weights associated with the quantity and quality penalty components respectively):

$$f_{ti} = w_q \cdot (R_{ti} - D_{ti})^2 + w_s \cdot A_{ti}^2 \tag{9.16}$$

where

$$w_q \geq 0 \tag{9.17}$$

$$w_s \geq 0 \tag{9.18}$$

$$w_q - w_s = 1.0 \tag{9.19}$$

4. Summing up these individual penalties over all demand centers and over the entire period under consideration gives the total penalty associated with the system for the chosen release distribution pattern:

$$f = w_q \sum_{t=1}^{T} \sum_{i=1}^{N} (R_{ti} - D_{ti})^2 + w_s \sum_{t=1}^{T} \sum_{i=1}^{N} A_{ti}^2 \tag{9.20}$$

The volume $(R_{ti} - D_{ti})$ in the above equation is the penalty base associated with the quantitative supply shortage whereas the amount of water A_{ti} represents the penalty base for the inadequate salinity of the delivered water.

Since genetic algorithms are essentially maximization search procedures, the presented penalty function must be transformed into an equivalent whose maximum will refer to the optimum solution of the allocation problem. In this case, the choice of transformation is rather simple. Namely, the actual fitness (objective) function f^* used is computed as the difference between the maximum possible penalty f_{max} estimated on the basis of equation (9.20) and the actual penalty f for a particular alternative solution (equation (9.20)):

$$f^* = f_{\max} - f \tag{9.21}$$

where f_{max} is estimated assuming the following:

- Weight factors w_q and w_s are set to 1.0 and 0.0, respectively.
- Demands supplied by a single reservoir only encounter 100% deficit (no supply).
- Demands supplied by multiple reservoirs receive full demand supply from each of the reservoirs (maximum surplus.) It should be noted here that such a case is actually not possible within the settings of the genetic algorithm model. Nevertheless, it does ensure that the maximum possible fitness be certainly beyond any penalty value that can be encountered in the search.

Composite Objective Within Operating Policy Optimization

The operating policy optimization is carried out using stochastic dynamic programming (SDP). The SDP model applies reservoir system decomposition and optimizes the operating policies of individual reservoirs in an iterative fashion. Therefore, the objective function does not reflect the objective achievement of the entire system like the allocation optimization model (Composite Objective Within Resource

Allocation Optimization), but only a contribution of a single reservoir operation to the overall objective function value. The adopted objective function is the sum of two components:

- The annual aggregate of the squared monthly deviation of release from the respective demand, multiplied by a given weight factor; and
- The annual aggregate of the squared deviation of monthly final storage volume from the respective target storage volume, multiplied by a given weight factor.

Since this model applies stochastic dynamic programming, the objective function value represents the expectation of the objective achievement covering the span of one annual cycle.

Unlike the combination of supply deficit and supply salinity objectives (Composite Objective Within Resource Allocation Optimization), this compound objective function does not require transformation of either of its components since both represent volumetric quantities of the same type:

$$G = w_d \cdot \sum_{t=1}^{T} \left(R_{tj} - D_{tj}\right)^2 + w_v \cdot \sum_{t=1}^{T} \left(SF_{tj} - ST_{tj}\right)^2 \qquad (9.22)$$

where the newly introduced symbols so far are:

w_d weight factor for supply deviation component ($w_d \geq 0$)
w_v weight factor for storage target deviation component ($w_v \geq 0$)
R_{tj} total consumptive release of reservoir *j* in time step *t*
D_{ti} total demand imposed upon reservoir *j* in time step *t*

Suffice it to say at this stage that both weight factors are predefined positive real numbers and must meet the condition:

$$w_d - w_v = 1.0 \qquad (9.23)$$

Performance Indicators

This section gives a full description of the risk and reliability indicators, hereafter referred to as performance indicators (PI), used in the present work. Performance Indicators (PIs) provide specific information about the performance of a system with regard to, for instance, the likelihood of the occurrence of insufficient supply, the probable severity of such a failure and the estimate of the likely duration of periods of full and insufficient supply, respectively. Since there are three objective criteria, the description distinguishes which indicators are appropriate for use in which of the objective cases. Furthermore, and due to the complexity of the system being analyzed, the estimation of performance indicators can be applied either to the system as a whole, to individual reservoirs or groups thereof, or to individual/groups of demand centers. The ultimate choice among the aforementioned alternatives is made during the analyses and is addressed accordingly.

Definitions

Since there are three distinct objective criteria considered it is deemed appropriate to introduce a few important terms at this stage to ensure that consistent terminology is used throughout the text:

- **Level of service.** The term "level of service" describes the extent to which a "service provider" (i.e. reservoir, reservoir system) fulfils its obligations towards meeting the agreed requirements of its "client(s)" (i.e. demand centers) during a single time step.
- **Failure vs success.** Contrary to a "success" event, a "failure" event indicates that a "service provider" has not managed to provide the full service to meet the requirement of its "client(s)" during a certain time step (e.g. supply shortage occurred, maximum acceptable salinity of supply surpassed, storage target not achieved).
- **Quantity-based performance indicators.** This set of PIs evaluates the performance of the selected system (i.e. single reservoir, system of reservoirs, single or group of demands) from the level of service point of view (i.e. supply quantity, supply salinity, storage target). Thus, the performance is assessed reflecting the magnitude of failure events and not their temporal distribution.
- **Time-based performance indicators.** Contrary to quantity-based PIs, time-based indicators describe the temporal facets of failure and success event occurrence related to the level of service of the selected system (i.e. single reservoir, system of reservoirs, single or group of demands).

Quantity-Based Performance Indicators

1. *Quantity-based reliability* (PI_1), is a simulation-based estimate of the mean level of service delivery over the entire period under consideration:

$$PI_1 = \frac{\sum_{i=1}^{N_t} \max(0, T_i - S_i)}{\sum_{i=1}^{N_t} T_i} \qquad \text{(failure: shortage)} \tag{9.24}$$

2. *Average magnitude of failure* (PI_3) is the simulation-based estimate of the mean magnitude of failure:

$$PI_3 = \frac{\sum_{i=1}^{N_t} \max(0, T_i - S_i)}{N_t} \qquad \text{(failure: shortage)} \tag{9.25}$$

$$PI_3 = \frac{\sum_{i=1}^{N_t} \max(0, S_i - T_i)}{N_t} \qquad \text{(failure: surplus)} \tag{9.26}$$

$$PI_3 = \frac{\sum_{i=1}^{N_t} (T_i - S_i)}{N_t} \qquad \text{(failure: deviation)} \tag{9.27}$$

3. *(Undershooting) vulnerability* (PI_5) indicates the magnitude of the most severe failure, i.e. shortage failure type, observed over the entire simulation period:

$$PI_5 = \max_i[\max(0, T_i - S_i)] \qquad \text{(failure: shortage)} \tag{9.28}$$

4. *(Overshooting) vulnerability* (PI_6) indicates the magnitude of the most severe failure, i.e. surplus failure type, observed over the entire simulation period:

$$PI_6 = \max_i[\max(0, S_i - T_i)] \qquad \text{(failure: surplus)} \tag{9.29}$$

Time-Based Performance Indicators

5. *Time-based reliability* (PI_7) is the simulation-based estimate of the long-term probability that the system service will be able to meet the target (consequently, the likelihood that the system will fail to provide the targeted service is 1 – PI_7):

$$PI_7 = 1 - \frac{1}{N_t} \sum_{i=1}^{N_t} u_i \tag{9.30}$$

6. *Average (success) recovery time* (PI_8) is defined as the average number of successive time steps the system continuously fails to meet the target, thus stating the expected time required by the system to switch to an operating mode characterized by full service delivery once it has encountered an operating service failure during one time step (this PI can thus be described as the *average duration of failure*):

$$PI_8 = \frac{\sum_{i=1}^{N_t} u_i}{\sum_{i=1}^{N_t} v_i} \tag{9.31}$$

7. *Average (failure) recurrence time* (PI_9) is defined as the average number of successive time steps the system sustains full service delivery before switching to a failure operating mode. In other words, it gives the estimate on how long the system may be expected to provide full service once it has recovered from an operating failure (this PI can thus be described as the *average duration of success, or full service*):

$$PI_9 = \frac{N_t - \sum_{i=1}^{N_t} u_i}{\sum_{i=1}^{N_t} w_i} \tag{9.32}$$

8. *Resilience (or failure persistence)* (PI_{10}) is the longest interval Δi (in number of time steps) of consecutive operating failure events:

$$\begin{aligned} PI_{10} = \max_i \big(\Delta i | v_i = 1 \wedge w_{i+\Delta i} = 1,\ \Delta i \geq 0 \\ \wedge\, u_j = 1 \forall j \in \{i-1, \ldots, i-\Delta i - 1\}\big) \end{aligned} \tag{9.33}$$

9. *Resistance (or success persistence)* (PI_{11}) is the longest interval Δi (in number of time steps) of consecutive full operating service:

$$\begin{aligned} PI_{11} = \max_i (\Delta i | w_i = 1 \wedge v_{i+\Delta i} = 1,\ \Delta i \geq 0 \\ \wedge u_j = 0 \forall j \in \{i-1, \ldots, i-\Delta i - 1\}\big) \end{aligned} \tag{9.34}$$

The notation used in equations above is described in the following:

i	the index depicting a time step (i.e. month);
N_t	the length, in time steps (i.e. months), of the simulation time period;
N_y	the length, in years, of the simulation time period;
T_i	the target that the system service is expected to reach in time step i;
S_i	the service that the system is expected to provide in time step i;
$\sum_{i=1}^{12} T_{ij}$	the annual target that the system service is expected to reach in year j;
$\sum_{i=1}^{12} S_{ij}$	the annual service that the system is expected to provide in year j;
u_i	the *success/failure* ($u_i = 0/u_i = 1$) descriptor which indicates whether the system has managed to provide the expected service during time step i:

$$u_i = \begin{cases} 1, & T_i > S_i \\ 0, & T_i \leq S_i \end{cases}, \quad \forall i \qquad \text{(failure: shortage)} \tag{9.35}$$

$$u_i = \begin{cases} 0, & T_i \geq S_i \\ 1, & T_i < S_i \end{cases}, \quad \forall i \qquad \text{(failure: surplus)} \tag{9.36}$$

$$u_i = \begin{cases} 0, & T_i = S_i \\ 1, & T_i \neq S_i \end{cases}, \quad \forall i \qquad \text{(failure: deviation)} \tag{9.37}$$

v_i the descriptor indicating a *success-to-failure* operating transition:

$$v_i = \begin{cases} 1, & u_{i-1} = 0 \wedge u_i = 1 \\ 0, & otherwise \end{cases}, \quad \forall i > 1, \quad v_1 = u_1 \tag{9.38}$$

w_i the descriptor indicating a *failure-to-success* operating transition:

$$w_i = \begin{cases} 1, & u_{i-1} = 1 \wedge u_i = 0 \\ 0, & otherwise \end{cases}, \quad \forall i > 1, \quad w_1 = 1 - u_1 \tag{9.39}$$

It should be noted here that the definitions and functional relationships of all the PIs have been presented assuming that the system's operation is characterized by both success and failure events thus excluding a possibility of a division by zero in the estimation of any of the PIs. Similarly, it is assumed that the target service imposed upon the system over the whole simulation span, as well as the length of the simulation period, are not zero.

To conclude, Table 9.1 summarizes the applicability of individual PIs to the assessment of system performance with regard to each of the three objective criteria.

Table 9.1 Summary on performance indicators applicability

Performance indicator	Objective		
	Supply quantity	Supply quality	Storage target
Quantity-based			
1 Reliability	✓		
2 Shortage index	✓		
3 Average magnitude of failure	✓	✓	✓
4 Average absolute magnitude of failure			✓
5 (Undershooting) vulnerability	✓		✓
6 (Overshooting) vulnerability		✓	✓
Time-based			
7 Reliability	✓	✓	✓
8 Average (failure) recurrence time	✓	✓	✓
9 Average (success) recovery time	✓	✓	✓
10 Resilience (or failure persistence)	✓	✓	✓
11 Resistance (or success persistence)	✓	✓	✓

Case Study and Results

The methodology is summarized in Fig. 9.1.

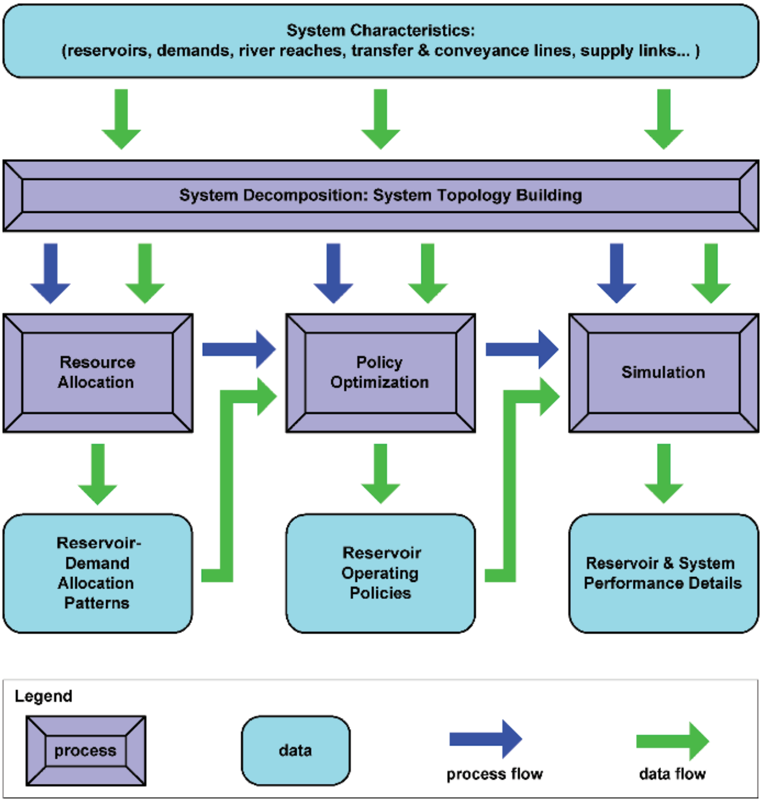

Fig. 9.1 The adopted approach for system operation optimization

The approaches developed and applied in this work have been thoroughly tested on the 15-reservoir case study system (Louati, 2005). It is therefore of primary importance to seek an opportunity for further research to appraise the applicability of these methods to different reservoir systems.

This study has been restricted to several long-term operational aspects associated with a multiple-reservoir-multiple-demand water supply system. Two particular optimization problems have been identified in this regard:

- Optimum allocation of available resources within such a system; and
- Optimization of the individual reservoir operating policies.

The two aforementioned optimization problems have been formulated and solved so as to reflect the desire of a decision maker to reconcile two primary objectives and one secondary goal. The primary objectives have been defined as:

- Quantitative satisfaction of water demand imposed upon the system; and
- Maintenance of supply salinity below the salt concentration limits predefined for each of the individual demands.

The work has focused on the assessment of applicability of a technique combining system and optimization problem decomposition, resource allocation, operating policy optimization and simulation to solving a strategic operational problem of a "multiple-reservoir-multiple-demand" water resource system. The complexity of the operational problem has brought about an assumption that the problem itself could be split into two main components. Namely, an operating strategy of such a complex system may be understood as a composition of two main parts:

- Resource (reservoir-demand) allocation patterns; and
- Reservoir operating policies reflecting the aforementioned allocation patterns.

The effectiveness of the proposed optimization and search methods have been appraised and compared not only on the basis of the applied objective criterion but rather over an array of simulated performance indicator estimates describing different aspects of system operation.

Given the findings of this research, genetic algorithms seem to be a good choice for this type of water resource management problems. The main advantage is their robustness and insensitivity to the size of the problem. Secondly, genetic algorithms rely on the objective function estimate derived by simulation, thus allowing the use of detailed simulation models. Finally, genetic algorithms can easily identify a number of equally good alternative solutions, which is frequently the case in water resources management problems.

The selected genetic algorithm-based resource allocation strategy has further been used to estimate the individual reservoir storage targets. The storage targets have been computed upon simulation of the entire system operation over 20 sets of 250 years of synthetic monthly inflows to individual reservoirs. The inflows to individual reservoirs have been generated using the autoregressive lag-one Thomas-Fiering model with seasonally varying coefficients, however without modeling the stream flow cross-correlation among the different streamflow processes.

It should be noted that the reservoir storage targets are derived assuming equal importance of supply towards all demand types, i.e. drinking water, irrigation and environmental needs. The simplification of the approach in this regard is made because this issue is extending beyond the scope, main objectives and resources of this research and should be treated to a greater detail elsewhere.

Since flood control is an integral part of any reservoir operation, additional analyses are required to assess the effects of flood control rules on the system operation. The consideration of flood control may also prove important to the assessment of the scope and magnitude of policy violation simulation used in this work. Namely, seasonal flood control related storage limitations would certainly influence the extent of the applied violations of stochastic dynamic programming policies. However, such an approach would also require deeper consideration of issues like river flow forecasting and/or rainfall-runoff modeling.

Ultimately, the findings of this research have shown that there are multiple aspects of system operation affecting the final decision on the preferred planning option. Namely, the use of performance indicators depicting the reliability, risk, resilience and vulnerability of different aspects of system performance have proven invaluable in making the final assessment of the efficiency and effectiveness of the proposed resource allocation and reservoir operating policy options. It is therefore sensible to assume that further research considering objective criteria like reliability, risk and/or vulnerability in devising water resource allocation plans may offer additional valuable insight into the available planning alternatives for such a reservoir system.

References

Bogardi J J, Verhoef A (1995) Reliability analysis of reservoir operation. In Kundzewicz Z W (ed.), New Uncertainty Concepts in Hydrology and Water Resources. Proceedings of the International Workshop on new Uncertainty Concepts in Hydrology and Water Resources (Madralin, 1990):306–315. Cambridge University Press.

Eastman J, ReVelle C (1973) Linear decision rule in reservoir management and design.3. Direct capacity determination and intraseasonal constraints. Water Resources Research, 9(1):29–42.

Gundelach J, ReVelle C (1975) Linear decision rule in reservoir management and design. 5. A general algorithm. Water Resources Research, 11(2):204–207.

Hashimoto T, Stedinger J R, Loucks D P (1982) Reliability, resiliency, and vulnerability criteria for water resource system performance evaluation. Water Resources Research, 18(1):14–20.

Houck M H (1979) A chance constrained optimization model for reservoir design and operation. Water Resources Bulletin, 15(5):1011–1016.

Houck M H, Datta B (1981) Performance evaluation of a stochastic optimization model for reservoir design and management with explicit reliability criteria. Water Resources Bulletin, 17(4):827–832.

Lebdi F, Slimani M, Parent E (1997) Empirical strategy for water resources system management : The example of the semi arid irrigated perimeter. Revue des sciences de l'eau, 10(1):121–134.

Lebdi F, Bergaoui M, Bouslimi M A (2003) Optimisation of reservoir management in semi arid countires: Case of lakhmess in tunisia. In Rossi G, et al. (eds.). Tools for drought mitigation in mediterranean regions, pp. 293–304.

Louati M H (2005) Optimisation des règles de gestion des réservoirs multiples avec considération du risque, thèse de doctorat en sciences agronomiques "Spécialité: Génie rural eau et forets", Inat, Tunis, Tunisie.

Loucks D P and Dorfman P J (1975) An evaluation of some linear decision rules in chance-constrained models for reservoir planning and operation. Water Resources Bulletin, 11(6):777–782.

Milutin D and Bogardi J J (1995) Reliability criteria in the assessment of a multiple reservoir operational strategy under Mediterranean conditions. Proceedings of the European Symposium on Water Resources Management in the Mediterranean Under Drought or Water Shortage Conditions: Economic, Technical, Environmental and Social Issues (Nicosia, 1995), pp. 265–271. Rotterdam: Balkema.

Milutin D, Bogardi J J (1996a) Hierarchical versus distributed release allocation within optimization of a multiple reservoir system operation. In Rao K (ed.), Proceedings of the International Conference on Aspects of Conflicts in Reservoir Development and Management (London, 1996), pp. 485–494. London City University.

Milutin D, Bogardi J J (1996b) Application of genetic algorithms to derive the release distribution within a complex reservoir system. In Muller A (ed.), Hydroinformatics '96, Proceedings of the Second International Conference on Hydroinformatics (Zurich, 1996), pp. 109–116. Rotterdam: Balkema.

Moy W-S, Cohon J L, ReVelle C S (1986) A programming model for analysis of the reliability, resilience, and vulnerability of a water supply reservoir. Water Resources Research, 22(4):489–498

Nandalal K D W, Bogardi J J (1996) Reliability analysis of a reservoir for salinity control. Proceedings of the International Conference on Water Resources and Environment Research: Towards the 21th Century (Kyoto, 1996), pp. 263–269. Kyoto University.

ReVelle C, Joeres E, Kirby W (1969) The linear decision rule in reservoir management and design 1. Development of the stochastic model. Water Resources Research, 5(4):767–777.

ReVelle C, Kirby W (1970) Linear decision rule in reservoir management and design 2. Performance optimisation. Water Resources Research, 6(4):1033–1044.

ReVelle C, Gundelach J (1975) Linear decision rule in reservoir management and design. 4. A rule that minimizes output variance. Water Resources Research, 11(2):197–203.

Simonovic S P, Mariño M A (1980) Reliability programming in reservoir management: 1. Single multipurpose reservoir. Water Resources Research, 16(5):844–848.

Simonovic S P, Mariño M A (1981) Reliability programming in reservoir management: 2. Risk-loss Functions. Water Resources Research, 17(4):822–826.

Simonovic S P and Mariño M A (1982) Reliability programming in reservoir management: 3. System of multipurpose reservoirs. Water Resources Research, 18(4):735–743.