MODELING STAKEHOLDER PREFERENCES WITH PROBABILISTIC INVERSION

Application to Prioritizing Marine Ecosystem Vulnerabilities

R. NESLO

Delft University of Technology
Delft, The Netherlands

F. MICHELI

Stanford University
Palo Alto, California, USA

C.V. KAPPEL

National Center for Ecological Analysis and Synthesis
Santa Barbara, California, USA

K.A. SELKOE

University of Hawaii
Manoa, Hawaii, USA

B.S. HALPERN

National Center for Ecological Analysis and Synthesis
Santa Barbara, California, USA

R.M. COOKE

Delft University of Technology and Resources for the Future
Delft, The Netherlands
Cooke@rff.org

I. Linkov et al. (eds.), Real-Time and Deliberative Decision Making.

Abstract: A panel of 64 experts ranked 30 scenarios of human activities according to their impacts on coastal ecosystems. Experts were asked to rank the five scenarios posing the greatest threats and the five scenarios posing the least threats. The goal of this study was to find weights for criteria that adequately model these stakeholders' preferences and can be used to predict the scores of other scenarios. Probabilistic inversion (PI) techniques were used to quantify a model of ecosystem vulnerability based on five criteria. Distinctive features of this approach are:

1. A model of the stakeholder population as a joint distribution over the criteria weights is obtained. This distribution is found by minimizing relative information with respect to a noninformative starting distribution, but makes no further assumptions about the interactions between the weights for different criteria. Criteria distributions with dependence emerge from the fitting procedure.
2. The multicriteria preference model can be empirically validated with expert preferences not used in fitting the model.

1. Introduction

This article presents an analysis of the 64 experts' rankings of 30 scenarios of human activities and their impacts to coastal ecosystems. The elicitation protocols were designed and executed by researchers at the National Center for Ecological Analysis and Synthesis. Experts were asked to rank the five scenarios posing the greatest threats and the five scenarios posing the least threats. The goal of this study was to find weights for criteria that adequately model these stakeholders' preferences and can be used to predict the scores of other scenarios. Probabilistic inversion (PI) techniques were used to quantify a model of ecosystem vulnerability based on five criteria. Stakeholder preference modeling can also serve as a form of expert elicitation when the stakeholders are domain experts, as in the present case. Their preferences are taken to prioritize threats to marine ecosystems, with a view to optimizing mitigation and abatement actions.

Other multicriteria weighting methods [9, 10, 22] require stakeholders to evaluate the criteria directly. Of course, the weights assigned to a criterion cannot be assessed independently of the scale on which *all* criteria scores are measured—a fact that is sometimes overlooked. The present approach asks the stakeholders to rank scenarios rather than evaluate criteria. Criteria weights are then derived to fit the stakeholder preference rankings as well as possible. This has the significant advantage of allowing us to assess the validity of our fitted model of stakeholder preference.

Probabilistic inversion denotes the operation of inverting a function over a probability distribution, rather than at a point. Such problems arise in quantifying uncertainty in physical models [8, 13, 14, 15, 23]. One has uncertainty distributions on observable phenomena, either from data or from expert judgment, and one wishes to find a distribution over the parameters of a predictive model, such that one recovers the observed distributions when the parameter distributions are "pushed through" the model. PI algorithms used in the past were computationally intensive, involving sophisticated interior point optimization techniques and duality theory as well as ad hoc steering [16]. Recent computational advances [26, 34] clarify the mathematical foundations for PI and yield simple algorithms with proven convergence behavior, suitable for use by nonspecialists. The results depend on a variant of the classical Iterative Proportional Fitting algorithm [6–8, 12, 17, 19, 20, 26].

In stakeholder preference modeling, the data is discrete-choice preference data elicited from a set of stakeholders. The distributions to be inverted are those of indicator variables such as:

- Alternative i is better than alternative j.
- Alternative i is ranked third in the given set of alternatives.

We are interested in the probability of such variables, taking the values "yes" or "no" for a set of stakeholders. We can measure these probabilities by querying a large representative set of stakeholders. Existing discrete-choice—or random-utility—techniques construct a value or utility function from discrete-choice data [1, 3, 23, 24, 27, 28, 30–33], and they strongly restrict the form of the utility functions. Using PI, this form can be inferred from choice data.

We first discuss the data, then address model adequacy and model fit. Summary statistics for the 30 scenarios are then given. The conclusion of this analysis is that the data are broadly consistent with a linear model of stakeholder preferences.

2. Data

The 30 threat scenarios were scored on five criteria:

- C1 Spatial scale
- C2 Frequency
- C3 Trophic (functional) impact
- C4 Recovery time
- C5 Resistance

These criteria were developed and tested elsewhere [11, see also 5, 25, 29].

The stakeholders' preference data is represented with a linear model:

$$\text{Score for scenario S} = \Sigma_{i=1...5}(\text{score of S on } C_i \times \text{weight for } C_i) \quad (1)$$

The weights are random variables that are nonnegative and sum to 1. The (joint) distribution for the weights is modeled to represent the distribution of weights in a population of stakeholders, of which the 64 elicited experts are a random sample. Since the weights are normalized, the scores are transformed so that the product *score* × *weight* is positive and falls within the same range. Spatial scale is given in square kilometers, and the values for spatial scale range from 0.1 to 50,000 km^2. These values are transformed to ln(100 m^2), whose values thus range from 2.3 to 15.4. Frequency was scored as ln(360*# /year). Trophic or functional impact is the number of trophic layers affected. Resistance is scored as the percent of species affected per trophic layer. These transformations are chosen for mathematical convenience.

A salient feature of these data is dominance. Scenario A dominates Scenario B from above if A's scores on all five criteria are greater or equal to the scores of B. A dominates B from below if A's scores on all five criteria are less than or equal to those of B. If A dominates B from above, then B can never be ranked above A in any model that computes the scenario score as a monotonic function of the five criteria scores. The presence of dominated scenarios enables us to analyze whether the experts' rankings are broadly consistent with a monotonic model of criteria scores.

3. Model Adequacy

Of the 30 scenarios, only seven were nondominated. This means that none of the 23 scenarios dominated from above could be ranked 1 by a stakeholder whose preferences were consistent with the model. In fact, 22.4% of the top rankings were inconsistent in this sense: 77.6% of the top rankings went to four of the seven nondominated scenarios. A scenario dominated from above by two or more scenarios could not consistently be ranked second; in fact, 23.7% of the second rankings were inconsistent in this sense. Dominance from below was much less prevalent than dominance from above.

In view of the large number of dominated scenarios, we view the percentages of inconsistent rankings as indicating that the stakeholders' preferences were broadly, though not wholly, consistent with a monotonic model.[1] We therefore proceeded to fit the linear model (1).

The 30 scenarios and their criteria scores are shown in Table 1. The nondominated scenarios are shaded.

[1] If the 64 experts had chosen their top-ranked scenario at random, the probability that 14 or fewer would chose one of the 23 dominated scenarios is in the order of 10^{-20}.

TABLE 1. Scenarios and Criteria Scores.

Nr	Code	Scenario	Scale $\ln((km^*10)^2)$	Freq ln (360*#/year)	Func (# trophic layers)	Recov (years)	Resist
1	am	Aquaculture: marine plant	5.30	11.77	1	1	0.2
2	as	Aquaculture: shellfish	6.21	11.77	1	0.1	0.05
3	cl	Climate change: sea level rise	13.82	5.19	2	5	0.2
4	ct	Climate change: sea temp	15.42	5.89	3	50	0.25
5	cu	Climate change: UV	13.82	3.58	1	1	0.05
6	ca	Coastal engineering: habitat alteration	4.61	5.89	4	25	0.75
7	dh	Direct human impact: trampling	9.62	11.77	2	25	0.35
8	fd	Fishing: demersal destructive	6.68	2.89	4	0.5	0.1
9	fn	Fishing: demersal nondestructive low bycatch	2.30	2.89	1	0.5	0.1
10	fa	Fishing: nondestructive artisanal	4.61	2.89	1	1	0.5
11	fp	Fishing: pelagic high bycatch	6.21	1.28	1	0.5	0.05
12	fr	Fishing: recreational	6.68	9.84	2	5	0.2
13	fu	Freshwater input: increase	6.91	4.28	2	1	0.1
14	is	Invasive species	14.51	11.77	1	20	0.25
15	ma	Military activity	6.91	8.37	1	5	0.1
16	nh	Nutrient input: causing harmful algal blooms	9.21	4.28	2	1	0.1
17	nz	Nutrient input: causing hypoxic zones	6.68	4.28	3	1	0.05
18	no	Nutrient input: into oligotrophic waters	8.29	4.97	1	0.5	0.3
19	og	Ocean dumping: lost fishing gear	2.30	5.89	3	3	0.15
20	os	Ocean dumping: ship wrecks	3.91	2.89	4	10	0.5
21	ox	Ocean dumping: toxic materials	6.91	2.89	1	1	0.1
22	po	Ocean pollution	6.91	6.58	1	3	0.2
23	pa	Pollution input: atmospheric	9.62	3.58	1	0.5	0.2
24	pi	Pollution input: inorganic	8.29	4.28	2	3	0.2
25	pr	Pollution input: organic	8.52	5.19	2	5	0.2
26	ps	Power, desalination plants	4.61	11.77	3	10	0.5
27	sr	Scientific research: collecting	2.30	8.37	1	2	0.15
28	sd	Sediment input: decrease	3.91	1.28	1	0.5	0.05
29	si	Sediment input: increase	10.82	5.19	2	10	0.3
30	ts	Tourism: surfing	2.30	10.49	1	1	0.05

4. Model Fitting: Criteria Weights

We fit the linear model by finding a distribution over criteria weights which fit as well as possible the probabilities of rankings given by the stakeholders. The fitting is done by probabilistic inversion. We start with a noninformative distribution over criteria weights (which however are constrained to add to 1). We then adapt this distribution to optimally recover the stakeholders' rankings. That is, if we sample randomly from the adapted distribution, the probability of drawing a set of weights with which Scenario A is ranked first equals, to the extent possible, the percentage of experts who ranked A first, and so on. The fitting based on first ranks applies only to the percentages for the scenarios that were ranked first. Similarly, the fitting based on the first two ranks applies only to the percentages for the scenarios ranked 1 or 2.

We are interested in finding a fitting that can be validated by predicting rankings *not* used in the fitting. Since the goal is to prioritize threats, the top rankings are most important. Satisfactory results were found by fitting the model based on the first four rankings; this model could then and used to predict the fifth rankings. Table 2 and Figure 4 compare the predicted and observed percentages of rankings. The model is first used to "retrodict" or "recover" the first four rankings. These are the data actually used to fit the model, so this comparison is a check of model fit rather than model prediction. Using the model, we can predict the percentages of experts ranking the various scenarios in the fifth position (Figure 5). These percentages were not used in fitting the model and test the ability of the model to predict preferences of the population of stakeholders. Of course, we should hope that the predictions and retrodictions show similar agreement with the observed rankings.

Because we are fitting a linear model, the expected score of any scenario may be computed by using the expected values of the criteria weights in the adapted distribution. A new scenario, not among the original 30, can be scored by multiplying its (transformed) criteria scores by the expected weight of each criterion. This of course is the great advantage of a linear model, and explains the preference for this model above more complex models, even though the latter might yield a better fit. Figure 1 shows the expected criteria weights based on fitting only the first ranks, the first two ranks, the first three ranks, and the first four ranks, and finally, based on fitting all ranks. We observe that these expected weights do not change significantly between the two-, three-, and four-rank options. Using all ranks causes changes, and also causes greater variance in the criteria scores (see Table 4).

TABLE 2. Model Predictions and Stakeholder Probabilities for Top Five Rankings.

Constraint	Prediction I	Prediction I,II	Prediction I,II,III	Prediction I,II,III,IV	Stakeholders
#S3 = 1	0.0000	0.0000	0.0000	0.0000	0.0597
#S4 = 1	0.3424	0.3428	0.3420	0.4359	0.3433
#S6 = 1	0.2695	0.2687	0.4164	0.3008	0.2687
#S7 = 1	0.0296	0.0299	0.0453	0.0329	0.0299
#S8 = 1	0.0000	0.0000	0.0114	0.0000	0.0149
#S9 = 1	0.0000	0.0000	0.0000	0.0000	0.0149
#S11 = 1	0.0000	0.0000	0.0000	0.0000	0.0149
#S12 = 1	0.0000	0.0000	0.0000	0.0000	0.0149
#S14 = 1	0.0744	0.0748	0.0580	0.0800	0.0746
#S16 = 1	0.0000	0.0000	0.0000	0.0000	0.0299
#S19 = 1	0.0000	0.0000	0.0000	0.0000	0.0149
#S22 = 1	0.0000	0.0000	0.0000	0.0000	0.0448
#S25 = 1	0.0000	0.0000	0.0000	0.0000	0.0149
#S28 = 1	0.0000	0.0000	0.0000	0.0000	0.0299
#S29 = 1	0.0000	0.0000	0.0000	0.0000	0.0299
#S2 = 2	0.0000	0.0000	0.0000	0.0000	0.0339
#S3 = 2	0.0001	0.0339	0.0442	0.0392	0.0339
#S4 = 2	0.2295	0.2213	0.1713	0.2218	0.2203
#S5 = 2	0.0000	0.0000	0.0000	0.0000	0.0169
#S6 = 2	0.4753	0.0511	0.0663	0.0661	0.0508
#S7 = 2	0.1557	0.0676	0.0825	0.0681	0.0678
#S8 = 2	0.0000	0.0679	0.0432	0.0725	0.0678
#S9 = 2	0.0000	0.0000	0.0000	0.0000	0.0169
#S11 = 2	0.0000	0.0000	0.0000	0.0000	0.0169
#S14 = 2	0.0275	0.2700	0.1855	0.2829	0.2712
#S16 = 2	0.0000	0.0000	0.0000	0.0000	0.0508
#S18 = 2	0.0000	0.0000	0.0000	0.0000	0.0169
#S20 = 2	0.0238	0.0170	0.0214	0.0174	0.0169
#S22 = 2	0.0000	0.0000	0.0000	0.0000	0.0508
#S23 = 2	0.0000	0.0000	0.0000	0.0000	0.0169
#S24 = 2	0.0000	0.0000	0.0000	0.0000	0.0169
#S29 = 2	0.0000	0.0000	0.0000	0.0000	0.0508
#S2 = 3	0.0000	0.0000	0.0000	0.0000	0.0317

(continued)

TABLE 2. (continued)

Constraint	Prediction I	Prediction I,II	Prediction I,II,III	Prediction I,II,III,IV	Stakeholders
#S3 = 3	0.0015	0.0084	0.1924	0.3305	0.1587
#S4 = 3	0.0798	0.0656	0.0769	0.1486	0.0635
#S6 = 3	0.0707	0.2063	0.0713	0.1131	0.0635
#S7 = 3	0.5732	0.4615	0.0816	0.1401	0.0794
#S8 = 3	0.0005	0.0053	0.0328	0.0514	0.0317
#S9 = 3	0.0000	0.0000	0.0000	0.0000	0.0159
#S12 = 3	0.0000	0.0000	0.0000	0.0000	0.0635
#S14 = 3	0.0730	0.0649	0.1276	0.1616	0.1270
#S16 = 3	0.0000	0.0000	0.0000	0.0000	0.0317
#S17 = 3	0.0000	0.0000	0.0000	0.0000	0.0635
#S18 = 3	0.0000	0.0000	0.0000	0.0000	0.0317
#S20 = 3	0.0968	0.1582	0.0158	0.0189	0.0159
#S21 = 3	0.0000	0.0000	0.0000	0.0000	0.0159
#S22 = 3	0.0000	0.0000	0.0000	0.0000	0.0159
#S24 = 3	0.0000	0.0000	0.0000	0.0000	0.0159
#S25 = 3	0.0000	0.0000	0.0000	0.0000	0.1111
#S26 = 3	0.1044	0.0293	0.0160	0.0181	0.0159
#S29 = 3	0.0000	0.0000	0.0000	0.0000	0.0794
#S3 = 4	0.0137	0.0687	0.0137	0.2174	0.1864
#S4 = 4	0.1508	0.1125	0.0150	0.0392	0.0339
#S5 = 4	0.0001	0.0036	0.0372	0.0392	0.0339
#S6 = 4	0.0889	0.3417	0.2099	0.0958	0.0847
#S7 = 4	0.1196	0.1948	0.2580	0.1091	0.1017
#S8 = 4	0.0031	0.0074	0.0028	0.0737	0.0678
#S11 = 4	0.0000	0.0000	0.0000	0.0000	0.0339
#S12 = 4	0.0017	0.0000	0.0000	0.0000	0.0847
#S14 = 4	0.2851	0.0784	0.0235	0.0910	0.0847
#S16 = 4	0.0000	0.0000	0.0000	0.0000	0.0339
#S17 = 4	0.0000	0.0000	0.0000	0.0000	0.0169
#S18 = 4	0.0000	0.0000	0.0000	0.0000	0.0169
#S20 = 4	0.0990	0.0795	0.3630	0.0176	0.0169
#S22 = 4	0.0000	0.0000	0.0000	0.0000	0.0678
#S24 = 4	0.0000	0.0000	0.0000	0.0000	0.0169

(continued)

TABLE 2. (continued)

Constraint	Prediction I	Prediction I,II	Prediction I,II,III	Prediction I,II,III,IV	Stakeholders
#S25 = 4	0.0000	0.0000	0.0000	0.0000	0.0508
#S29 = 4	0.0000	0.0001	0.0000	0.1024	0.1017
#S2 = 5	0.0122	0.0024	0.0035	0.0705	0.0333
#S3 = 5	0.0140	0.0845	0.0276	0.0000	0.0500
#S4 = 5	0.1189	0.1671	0.0182	0.0270	0.0500
#S6 = 5	0.0544	0.0789	0.0974	0.0969	0.0667
#S7 = 5	0.1121	0.1460	0.4151	0.2279	0.1667
#S8 = 5	0.0034	0.0091	0.0057	0.0000	0.0167
#S12 = 5	0.0094	0.0005	0.0000	0.0004	0.0167
#S13 = 5	0.0000	0.0000	0.0000	0.0000	0.0167
#S14 = 5	0.1848	0.1641	0.1137	0.1036	0.1167
#S16 = 5	0.0000	0.0000	0.0000	0.0000	0.0833
#S17 = 5	0.0000	0.0000	0.0114	0.0000	0.0167
#S19 = 5	0.0000	0.0000	0.0000	0.0000	0.0167
#S20 = 5	0.1714	0.0864	0.0883	0.2331	0.0333
#S22 = 5	0.0000	0.0000	0.0000	0.0000	0.0333
#S24 = 5	0.0000	0.0000	0.0000	0.0000	0.0167
#S25 = 5	0.0000	0.0000	0.0000	0.0000	0.0500
#S26 = 5	0.2552	0.1646	0.1003	0.0493	0.0667

Although the expected weights are most important in using the model, it is also of interest to examine the distributions of weights. Figure 2 shows the cumulative distribution functions of the five weights in the four cases shown in Figure 1. The joint distributions for one rank, four ranks, and all ranks are shown in Figure 3.

The rightmost cumulative distributions indicate greatest importance. The picture from Figure 2 echoes that in Figure 1 for the first two ranks: resistance is most important, followed by trophic impact. Of course, we must bear in mind that these results are relative to the scaling chosen to represent the criteria scores.

Figures 1 and 2 show that the mean values and marginal distributions are somewhat similar in all fitting situations. The joint distributions, however, are quite different. One sample of weights represents one virtual stakeholder. If we plot these five weights on five vertical lines, we get a jagged line representing

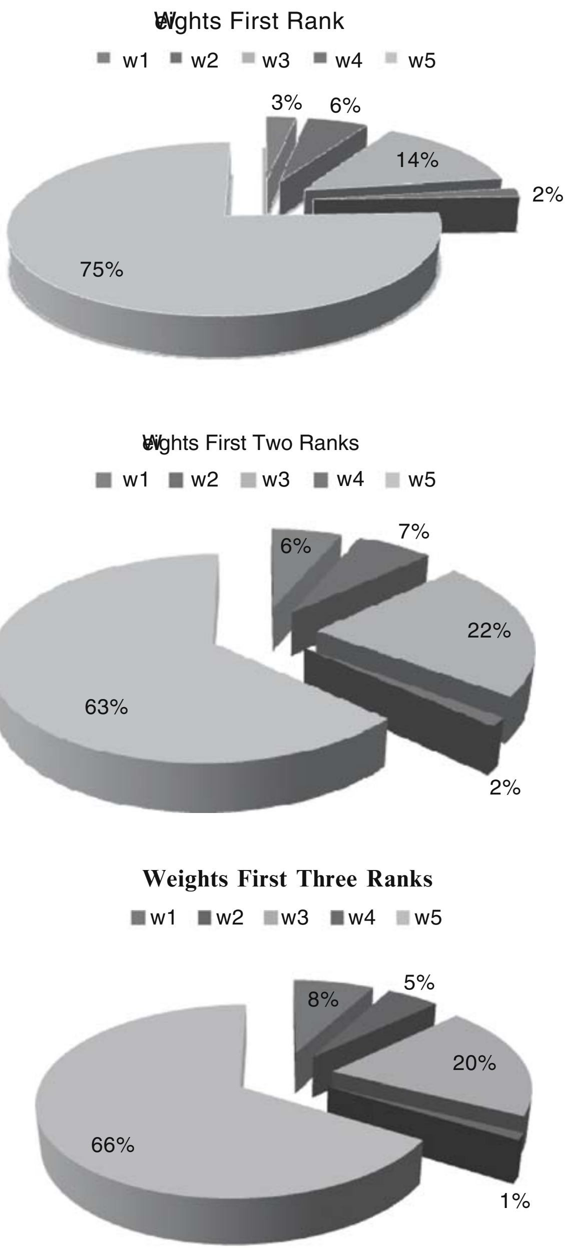


Figure 1. Expected criteria weights based on ranks 1, 1&2, 1&2&3, 1&2&3&4, and all ranks.

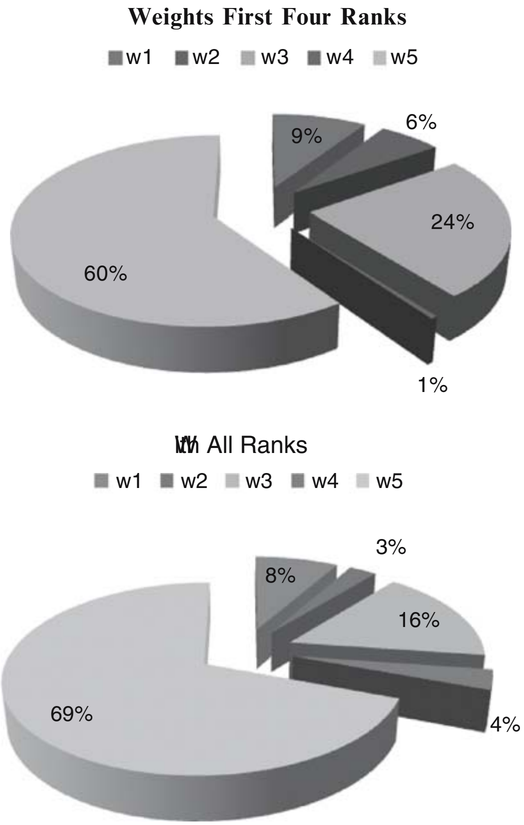
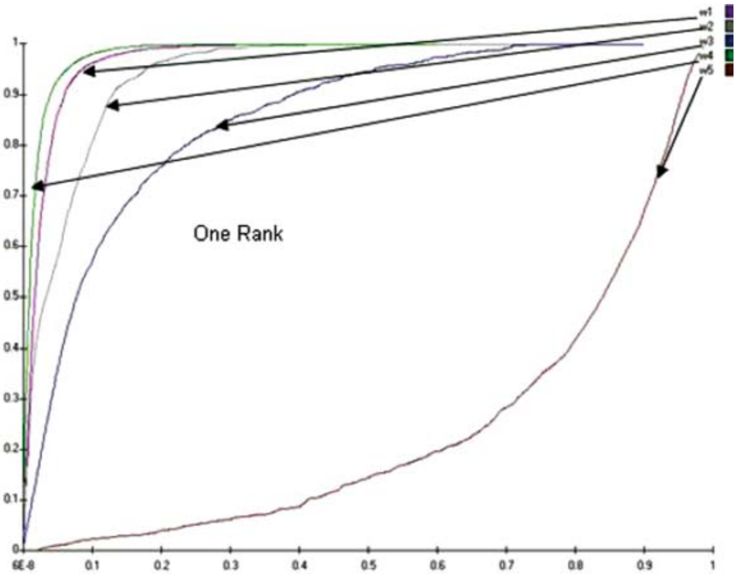


Figure 1. (continued)

one virtual stakeholder. If we plot 16,000 such lines we get a picture of the population of stakeholders. We say that the stakeholder weights have *interactions* if, for example, knowledge that a stakeholder assigns high weight to the "frequency" criterion gives significant information regarding weights for other criteria. A quick visual impression of the joint distributions is given by the "percentile cobweb plots" shown in Figure 3. Instead of the weights themselves, Figure 3 plots the weights' percentiles, as this makes the dependence structure more visible. Evidently the joint distributions are complex, and are different for the different fitting situations. A detailed analysis of interactions is not undertaken here. It is worth noting that the probabilistic inversion infers the dependence structure from the stakeholder data; it does not assume or impose any structure. We note that as we use more ranks in the fitting, the fitting becomes less smooth. The departure from the starting distribution grows more pronounced as the number of constraints that the fitting tries to satisfy increases.



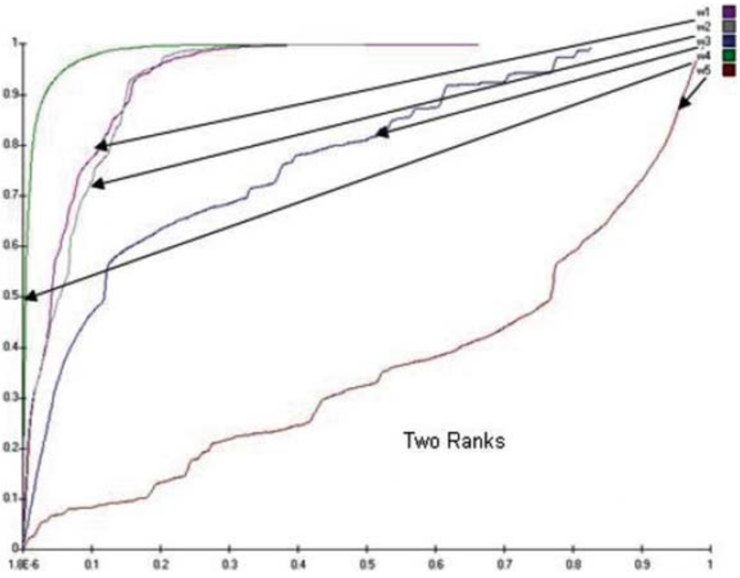
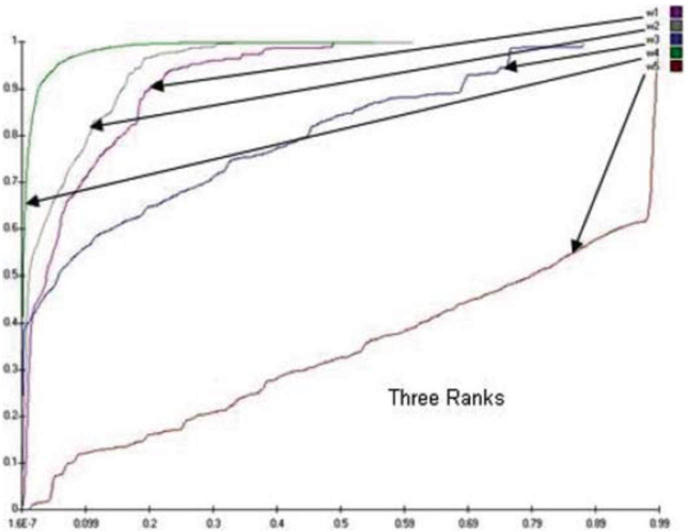


Figure 2. Cumulative weight distributions based on rank 1, 1&2, 1&2&3, 1&2&3&4, and all ranks.



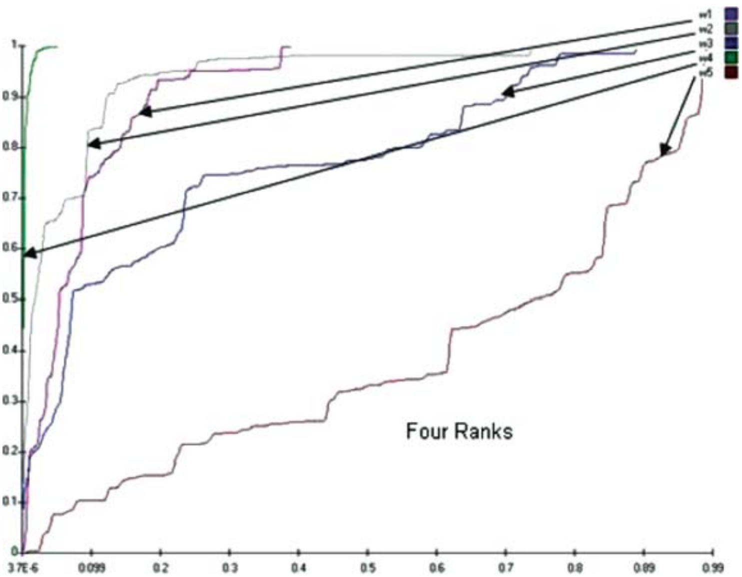


Figure 2. (continued)

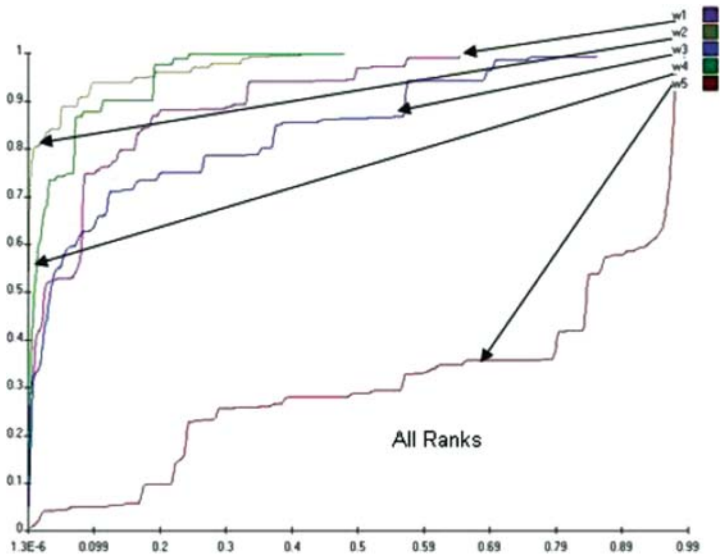


Figure 2. (continued)

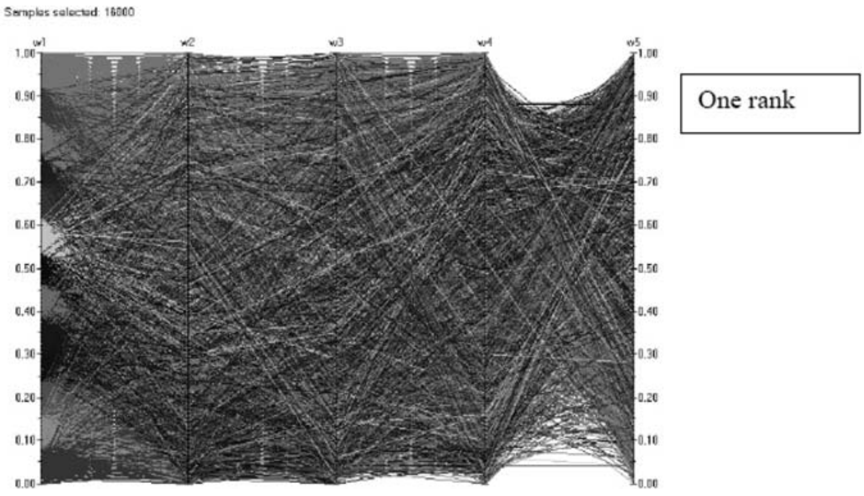


Figure 3. Percentile cobweb plots for criteria weights fitting one rank, four ranks, and all ranks.

Table 2 shows the predicted probabilities of rankings based on the fitting in the four cases discussed above. Thus "prediction I" indicates the prediction based on fitting only the first-ranked scenarios. The first column gives the constraints. "#S4=1" denotes the constraint that Scenario 4 was ranked 1. The last

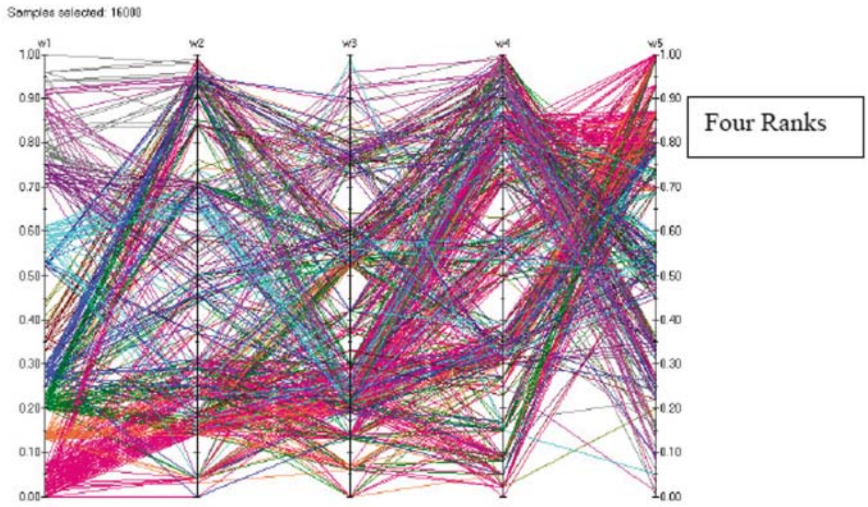


Figure 3. (continued)

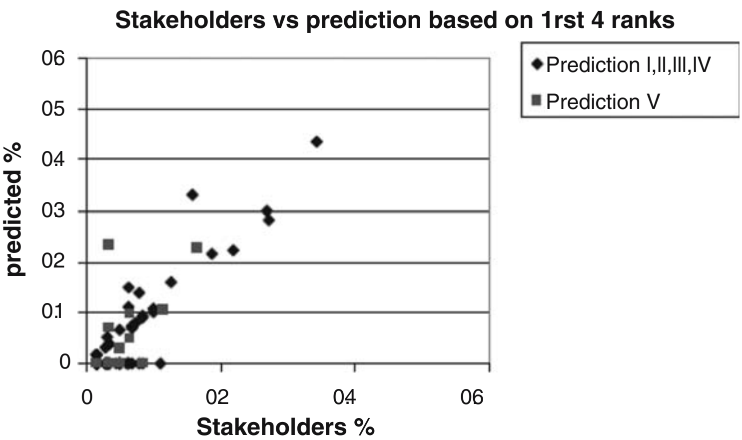


Figure 4. Predictions based on ranks 1– 4 of stakeholder percentages for the first four ranks (diamonds), and for the fifth ranks (squares)

column shows that 34.33% of the stakeholders ranked Scenario 4 as 1. Using the fitting based only on the first ranks predicts that 34.24% of the population of stakeholders would rank Scenario 4 as 1. Similarly, using the fitting based on the first four ranks, 43.59% of the population would rank Scenario 4 first. Of course, owing to the presence of inconsistent rankings, the fitting can never be perfect. Indeed, 22.4% of the first ranks were inconsistent with the model; as we fit 77.6% of the consistent rankings, the remaining probability mass must be distributed over the other feasible rankings. Some of the discrepancies are sizeable, as in the case of #S20 = 5 for the prediction based in the top four ranks. On the whole, however, the predictions do capture the drift of stakeholder preferences. Fitting all ranks is numerically quite burdensome and conflates issues that determine the most serious and least serious threats. The fitting based on the top four rankings presents the best compromise.

Figure 4 shows the information in Table 2 graphically. On the horizontal axis are stakeholders' percentages for rankings of scenarios; on the vertical axis are the predicted percentages based on the fitted model. The diamonds are scenarios which were ranked first, second, third, or fourth. These percentages were used to fit the model. The squares are scenarios that were ranked fifth. We see that these percentages are reasonably well predicted by the model. Scenarios plotted on the horizontal axis correspond to rankings that are inconsistent with the model.

5. Scenario Scores

Figure 5 shows the densities of the scores of the top four scenarios, ranked according to their mean values. These densities are generated by the distribution of criteria weights, which models the distribution of participants. It is interesting to note that the modes of these densities are all

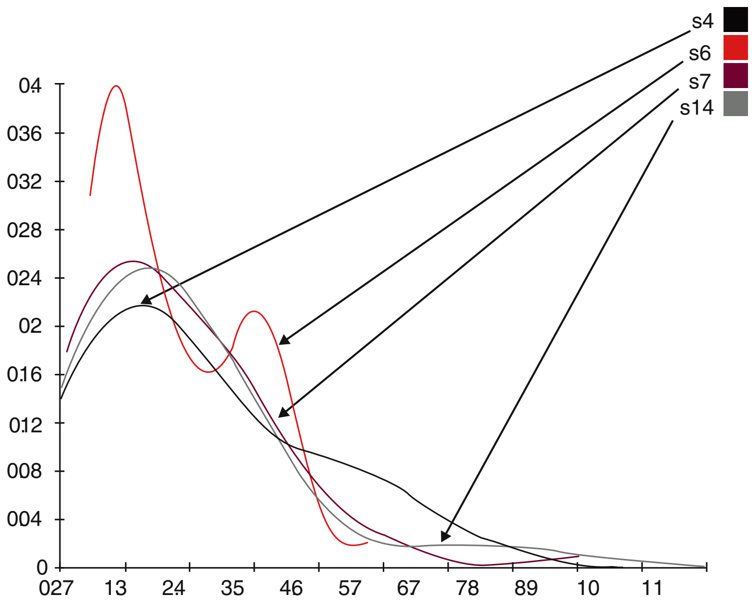


Figure 5. Densities for the top four ranked scenarios.

similar, but the shapes are different. The top-ranked scenario, Scenario 4 (Sea level rise), is distinguished by a large right tail. Scenario 6 (Coastal engineering) shows a bimodal form, suggesting that there are two distinct subgroups of participants. The remaining two scenarios, Scenario 14 (Invasive species) and Scenario 7 (Direct human impact) are quite similar in distribution.

Table 3 shows the mean, variance, and standard deviation of the five criteria weights and the 30 scenarios, based on the first four ranks. Table 4 gives the same information based on all ranks. Note that the variances in Table 4 tend to be larger, sometimes much larger. The top-ranked Scenario 4 has a

TABLE 3. Scenario scores using the first four ranks.

	Using first four ranks		
Variable	Mean	Variance	SD
S1	1.572	1.362	1.167
S2	1.561	1.523	1.234
S3	2.214	2.103	1.450
S4	2.901	3.702	1.924
S5	1.763	1.649	1.284
S6	2.328	1.464	1.210
S7	2.420	2.441	1.562
S8	1.825	1.732	1.316
S9	0.693	0.193	0.439
S10	1.146	0.214	0.462
S11	0.923	0.412	0.642
S12	1.844	1.547	1.244
S13	1.446	0.920	0.959
S14	2.540	2.953	1.719
S15	1.472	1.075	1.037
S16	1.657	1.226	1.107
S17	1.639	1.352	1.163
S18	1.446	0.688	0.829
S19	1.405	1.035	1.017
S20	1.858	1.165	1.079
S21	1.118	0.533	0.730
S22	1.412	0.794	0.891
S23	1.465	0.856	0.925
S24	1.642	1.050	1.025
S25	1.729	1.178	1.085
S26	2.220	1.944	1.394
S27	1.065	0.677	0.823
S28	0.712	0.228	0.477
S29	2.024	1.513	1.230
S30	1.129	1.023	1.012

TABLE 4. Scenario scores using all ranks.

Using all ranks			
Variable	Mean	Variance	SD
S1	1.118	1.725	1.313
S2	1.065	2.115	1.454
S3	2.066	4.997	2.235
S4	4.196	17.187	4.146
S5	1.613	4.405	2.099
S6	2.689	4.173	2.043
S7	2.726	6.270	2.504
S8	1.396	2.081	1.442
S9	0.528	0.255	0.505
S10	1.041	0.482	0.694
S11	0.822	0.932	0.965
S12	1.510	2.294	1.514
S13	1.167	1.585	1.259
S14	2.768	7.954	2.820
S15	1.274	1.975	1.405
S16	1.384	2.433	1.560
S17	1.263	1.874	1.369
S18	1.264	1.688	1.299
S19	1.039	0.961	0.980
S20	1.786	1.714	1.309
S21	0.981	1.232	1.110
S22	1.220	1.596	1.263
S23	1.302	2.185	1.478
S24	1.445	2.066	1.437
S25	1.568	2.362	1.537
S26	1.918	2.361	1.537
S27	0.757	0.651	0.807
S28	0.605	0.431	0.656
S29	2.051	3.663	1.914
S30	0.701	0.902	0.950

variance of 3.7 based on four ranks, and 17.2 based on all ranks. This suggests that trying to fit the top *and* bottom ranks just muddies the water—it does not give more insight into the factors determining high-threat scenarios.

6. Conclusion

By design, this study involved many dominated scenarios. This enabled us to test the extent to which the stakeholder preferences were consistent with a model for scenario scores based on a monotonic function of the five criteria scores. A stakeholder who prefers a dominated to a nondominated scenario is not consistent with any such model. Of course, this does not mean that such a stakeholder is inconsistent, it simply means that his/her preferences are not consistent with this type of model. In view of the large number of dominated scenarios, we may conclude that these stakeholders are broadly, though not wholly, consistent with such a monotonic model. A more complex model—possibly involving other criteria or interactions of criteria—might produce a better fit, but such models would be much more cumbersome in practice.

The linear model (1) is one type of monotonic model. Owing to the inconsistencies noted above it can never yield a perfect fit, but it does seem to capture the main drift of the stakeholder preferences. This means that the expected weights (Figure 1) can be used to score coastal ecosystem threat scenarios, provided their scores on the five criteria are given and scaled appropriately.

References

1. Anderson, S.P., de Palma, A., and Thissen, J-F., 1996. *Discrete Choice Theory of Product Differentiation*. MIT Press, Cambridge.
2. Bradley, R., 1953. Some statistical methods in taste testing and quality evaluation. *Biometrika* 9:22–38.
3. Bradley, R., and Terry, M., 1952. Rank analysis of incomplete block designs. *Biometrika* 39:324–345.
4. Cooke, R. M., and Misiewicz, J., 2007. Discrete choice with probabilistic inversion: application to energy policy choice and wiring failure. Presented at *Mathematical Methods in Reliability*, July.
5. Covich, A. P., Austen, M. C., Barlocher, F., Chauvet, E., Cardinale, B. J., Biles, C. L., Inchausti, P., Dangles, O., Solan, M., Gessner, M. O., Statzner, B., and Moss, B., 2004. The role of biodiversity in the functioning of freshwater and marine benthic ecosystems. *Bioscience* 54:767–775.
6. Csiszar, I., 1975. I-divergence geometry of probability distributions and minimization problems. *Annals of Probability* 3:146–158.
7. Deming, W. E., and Stephan, F. F., 1944. On a least squares adjustment to sample frequency tables when the expected marginal totals are known. *Annals of Mathematical Statistics* 40(11):427–444.
8. Du, C., Kurowicka, D., and Cooke, R. M., 2006. Techniques for generic probabilistic inversion. *Computational Statistics & Data Analysis* 50:1164–1187.
9. Linkov, I., Kiker, G. A., and Wenning, R. J. (Eds.) 2007. *Environmental Security in Harbors and Coastal Areas*. Springer, Dordrecht.
10. French, S., 1988. *Decision Theory; An Introduction to the Mathematics of Rationality*. Ellis Horwood, Chichester.
11. Halpern, B. S., Selkoe, K. A., Micheli, F., and Cappel, C. V. 2007. Evaluating and ranking global and regional threats to marine ecosystems. *Conservation Biology* 21:1301–1315.
12. Ireland, C. T., and Kullback, S., 1968. Contingency tables with given marginals. *Biometrika* 55:179–188.

13. Kraan, B. C. P. and Cooke, R. M. 2000. Processing expert judgments in accident consequence modeling. *Radiation Protection Dosimetry* 90(3):311–315.
14. Kraan, B.C.P. and Cooke, R. M., 2000. Uncertainty in compartmental models for hazardous materials - a case study. *Journal of Hazardous Materials* 71:253–268.
15. Kraan, B.C.P., and Bedford. T. J. 2005. Probabilistic inversion of expert judgments in the quantification of model uncertainty. *Management Science* 51(6):995–1006.
16. Kraan, B. C. P., 2002. *Probabilistic Inversion in Uncertainty Analysis and Related Topics*. Ph.D. dissertation, TU Delft, Dept. Mathematics.
17. Kruithof, J., 1937. Telefoonverkeersrekening. *De Ingenieur* 52(8):E15–E25.
18. Kullback, S., 1959. *Information Theory and Statistics*. Wiley, New York.
19. Kullback, S., 1968. Probability densities with given marginals. *The Annals of Mathematical Statistics* 39(4):1236–1243.
20. Kullback, S., 1971. Marginal homogeneity of multidimensional contingency tables. *The Annals of Mathematical Statistics* 42(2):594–606.
21. Kurowicka, D., and Cooke, R. M., 2006. *Uncertainty Analysis with High Dimensional Dependence Modelling*. Wiley, New York.
22. Linkov, I., Sahay, S., Kiker, G., Bridges, T., Belluck, D., and Meyer, A., 2006. Multicriteria decision analysis; comprehensive decision analysis tool for risk management of contaminated sediments. *Risk Analysis* 26(1):61–78.
23. Luce, R. D., and Suppes, P., 1965. Preference, utility, and subjective probability. In *Handbook of Mathematical Psychology*, vol. 3, eds. Luce, R. D., Bush, R., and Calanter, E. Wiley, New York.
24. Luce, R. D., 1959. *Individual Choice Behavior; A Theoretical Analysis*. Wiley, New York.
25. M.E.A. (Millennium Ecosystem Assessment), 2005. *Ecosystems and Human Well-Being: Synthesis Report*. Island Press, Washington, DC.
26. Matus, F., 2007. On iterated averages of I-projections, *Statistiek und Informatik,* Universität Bielefeld, Bielefeld, Germany.
27. McFadden, D., 1974. Conditional logic analysis of qualitative choice behavior. In *Frontiers in Econometrics*, ed. Zarembka, P., 105–142. New York Academic Press, New York.
28. Siikamäki, J., and Layton, D. F., 2007. Discrete choice survey experiments: a comparison using flexible methods. *Journal of Environmental Economics and Management* 53:127–139.
29. Stringer, L. C., Dougill, A. J., Fraser, E., Hubacek, K., Prell, C., and Reed, M. S., 2006. Unpacking "participation" in the adaptive management of social ecological systems: a critical review. *Ecology and Society* 11(2):39.
30. Thurstone, L., 1927. A law of comparative judgment. *Psychological Review* 34:273–286.
31. Torgerson, W., 1958. *Theory and Methods of Scaling*. Wiley, New York.
32. Train, K. E., 2003. *Discrete Choice Methods with Simulation*. Cambridge University Press, New York.
33. Train, K. E., 1998. Recreation demand models with taste differences over people. *Land Economics* 74: 230–239.
34. Vomlel, J., 1999. *Methods of Probabilistic Knowledge Integration*. Ph.D. thesis, Czech Technical University, Faculty of Electrical Engineering.