SMAA-III

A Simulation-Based Approach for Sensitivity Analysis of ELECTRE III

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Abstract: ELECTRE III is a well established multiple-criteria decision-making method with a solid track record of real-world applications. It requires precise values to be specified for the parameters and criteria measurements, which in some cases might not be available. In this paper we present a method, SMAA-III, which allows ELECTRE III to be applied with imprecise parameter values. By allowing imprecise values, the method also allows an easily applicable robustness analysis. In SMAA-III, simulation is used and descriptive measures are computed to characterize stability of the results.

1. Introduction

ELECTRE III is a well established multiple-criteria decision-making (MCDM) method for ranking a discrete set of alternatives. It belongs to the ELECTRE family of methods, which are based on constructing and

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exploiting an outranking relation [3]. ELECTRE III has a long history of successful real-world applications in different areas. The inputs for ELECTRE III consist of criteria evaluations on a set of alternatives and preference information expressed as weights and thresholds.

ELECTRE III is a pseudocriteria-based model, and as such it uses a threshold to model indifference between pairs of alternatives. Although this threshold might be an easy concept for a typical decision maker (DM) to understand, simulation studies have shown that it causes the model be quite unstable with respect to changes in the indifference threshold value [8]. Because of this instability, robustness should always be analyzed by considering different values for the threshold.

Real-world decision-making problems in general include various types of uncertainties inherent in problem structuring and analysis [1]. Eliciting the DMs' preferences in terms of relative criteria importance coefficients or weights is usually difficult. Such weights should always be considered imprecise, because humans usually do not think about preferences as exact numerical values, but as more vague concepts [14]. In some cases, weight information may be entirely missing, which corresponds to extremely imprecise weights.

This work presents a tool for dealing with imperfect knowledge within the ELECTRE III method. It can be used either when information is poor or when a robustness analysis needs to be done. The way robustness analysis is conducted comprises intensity of exploration in the parameter space. This is achieved by applying simulation in such a way that the parameter space is explored with a high concentration of discrete values. In addition to this, the exploration is coherent with the model. This means that, for example, when exploring the weight space, the meaning of weight is taken into account. In ELECTRE III weights represent the number of "votes" criteria have.

Capability to derive robust conclusions when applying MCDM methods is nowadays of utmost importance. The main sources of imperfect knowledge that are present in complex and multifaceted decision-making situations require careful observation of the results, and make them dependent on an exploration of the neighborhood of the parameters used mainly to represent preferences or technical aspects of the problem. If an alternative almost always occupies the first position when changing simultaneously all the parameters in a certain neighborhood, it means that it can be a good choice for future implementation; these are the kind of robust conclusions we are interested in.

The method presented in this paper is based on Stochastic Multicriteria Acceptability Analysis (SMAA) [7], a family of decision support methods for aiding DMs in discrete decision-making problems. For a survey of SMAA methods, refer to Tervonen and Figueira [15]. The proposed method, SMAA-III, explores weight, criteria measurement, and threshold

spaces, in order to describe which values result in certain ranks for the alternatives. It allows ELECTRE III to be used with different kinds of imprecise or partially missing information. This brings numerous advantages. Firstly, SMAA-III allows performing an initial analysis without preference information in order to eliminate "inferior" alternatives. Secondly, it allows DMs to express their preferences imprecisely, which can lower the DMs' cognitive effort compared to specifying precise weights. Thirdly, imprecise criteria measurements can be represented with arbitrary joint probability distributions, modeling imprecision in a coherent way not possible with ELECTRE III. Fourthly, it allows representation of the preferences of a group of DMs. Fifthly, the method can be used for analyzing the robustness of the results by representing the imprecision of the elicited weights as constraints or as suitable probability distributions.

In SMAA-III, robustness is analyzed with respect to weights, criteria measurements, and thresholds. Traditionally, robustness with ELECTRE methods is analyzed by considering discrete points in the weight space (see, e.g., [12]). But in the case of ELECTRE III this is not enough: weights between these points that might give contradictory results are missed. There are also simulation techniques for robustness analysis outside the SMAA methodology [2], but to the best of our knowledge, they have never before been applied to ELECTRE III.

This paper is organized as follows: ELECTRE III is briefly introduced in Section 2. SMAA-III is presented in Section 3. We skim rapidly through some computational aspects in Section 4 before proceeding to conclusions in Section 5.

2. ELECTRE III

ELECTRE III is designed for solving a discrete ranking problem. It consists of m alternatives $a_1,\ldots, a_i,\ldots, a_m$, which are evaluated in terms of n criteria $g_1,\ldots, g_j,\ldots, g_n$. We denote by J the set of criterion indices. $g_j(a_i)$ is the evaluation of criterion g_j for alternative a_i. Without loss of generality, we assume that all criteria are to be maximized.

Similarly to the other ELECTRE family methods, ELECTRE III is based on two phases. In the first phase, an outranking relation between pairs of alternatives is formed. The second phase consists of exploiting this relation, producing a final partial pre-order and a median pre-order.

S denotes the *outranking* relation, that is, aSb denotes that "alternative a is at least as good as alternative b."

ELECTRE III applies pseudocriteria in constructing the outranking relation. A pseudocriterion is defined with two thresholds for modeling

preference: an indifference threshold $q_j(g_j(\cdot))$ for defining the difference in criterion g_j that the DM deems insignificant, and a preference threshold $p_j(g_j(\cdot))$ for the smallest difference that is considered absolutely preferred. Between these two is a zone of "hesitation" between indifference and strict preference. ELECTRE III also defines a third threshold: the veto threshold $v_j(g_j(\cdot))$. It is the smallest (negative) difference that completely nullifies (raises a "veto" against) the outranking relation. In addition to the thresholds, preferences are quantified through a weight vector $w = (w_1, \ldots, w_j, \ldots, w_n)$. Without loss of generality, we assume that $\sum_{j \in J} w_j = 1$.

Exploitation of the outranking relation produces a partial pre-order, in which every pair of alternatives is connected with indifference (I), incomparability (R), or preference ($\succ$) relation.

2.1. CONSTRUCTING THE OUTRANKING RELATION

The outranking relation between every pair of alternatives is constructed based on a comprehensive concordance index and partial discordance indices. The concordance index is computed by considering individually for each criterion g_j the support it provides for the assertion aS_jb, "a outranks b with respect to criterion g_j". The partial concordance index is computed as follows, for all $j \in J$:

$$c_j(a,b) = \begin{cases} 1, & \text{if } g_j(b) - g_j(a) \le q_j(g_j(a)), \\ \dfrac{g_j(a) + p_j(g_j(a)) - g_j(b)}{p_j(g_j(a)) - q_j(g_j(a))}, & \text{if } q_j(g_j(a)) < g_j(b) - g_j(a) \le p_j(g_j(a)), \\ 0, & \text{if } g_j(b) - g_j(a) > p_j(g_j(a)). \end{cases}$$

After computing the partial concordance indices, the comprehensive concordance index is computed as follows:

$$c(a,b) = \sum_{j \in J} w_j c_j(a,b).$$

The discordance of criterion g_j describes the veto effect this criterion imposes against the assertion aSb. The partial discordance indices are computed separately for each criterion $j \in J$:

$$d_j(a,b) = \begin{cases} 1, & \text{if } g_j(b) - g_j(a) \geq v_j(g_j(a)), \\ \dfrac{g_j(b) - g_j(a) - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}, & \text{if } p_j(g_j(a)) \leq g_j(b) - g_j(a) < v_j(g_j(a)), \\ 0, & \text{if } g_j(b) - g_j(a) < p_j(g_j(a)). \end{cases}$$

By applying the previously mentioned indices, the degree of credibility of the outranking assertion aSb is defined as:

$$\rho(a,b) = \begin{cases} c(a,b) \prod_{j \in V} \dfrac{1 - d_j(a,b)}{1 - c(a,b)}, & \text{if } V \neq \varnothing \\ c(a,b), & \text{otherwise,} \end{cases}$$

with

$$V = \{ j \in J : d_j(a,b) > c(a,b) \}.$$

Notice that when $d_j(a,b) = 1$ for any $j \in J$, this implies that $\rho(a,b) = 0$.

2.2. THE EXPLOITATION PROCEDURE

The exploitation of the outranking relation consists of two phases. In the first phase, two complete pre-orders, Z_1 (descending) and Z_2 (ascending) are constructed with the so-called distillation procedures. In the second phase, a final partial pre-order or a complete median pre-order is computed based on these two pre-orders.

The distillation procedures work by iteratively cutting the fuzzy outranking relations with descending λ-cutting levels. With a given cutting level λ_*, alternative a outranks alternative b ($aS_{\lambda^*}b$) if the following holds:

$$aS^{\lambda_*}b \Leftrightarrow \begin{cases} \rho(a,b) > \lambda_*, \text{ and} \\ \rho(a,b) > \rho(b,a) + s(\rho(a,b)), \end{cases}$$

where $s(\cdot)$ is the distillation threshold, usually defined as [1]:

$$s(x) = 0.3 - 0.15x.$$

The pre-orders are constructed in an iterative manner. In each step the alternatives with the highest or lowest qualification scores are distilled, depending on whether the distillation is descending or ascending. The qualification score is computed as a difference between the number of alternatives that the selected alternative outranks and the number of alternatives that outrank it for a given cutting level. The procedure is presented in Algorithm 1.

In the original ELECTRE III, a median pre-order is computed based on the two complete pre-orders, Z_1 and Z_2, and the final partial pre-order. The final partial pre-order is computed as the intersection of the two complete pre-orders in such a way that the following relations hold:

$$a \succ b \Leftrightarrow (a \succ^{Z_1} b \wedge a \succ^{Z_2} b) \vee (aI^{Z_1}b \wedge a \succ^{Z_2} b) \vee (a \succ^{Z_1} b \wedge aI^{Z_2}b),$$

$$aIb \Leftrightarrow (aI^{Z_1}b \wedge aI^{Z_2}b),$$

$$aRb \Leftrightarrow (a \succ^{Z_1} b \wedge b \succ^{Z_2} a) \vee (b \succ^{Z_1} a \wedge a \succ^{Z_2} b).$$

Algorithm 1: Distillations

1. Determine the maximum value of the credibility indices in the set under consideration. Assign this to λ.
2. Determine $\lambda_4 = \max_{d(a,b)<\lambda-z(\lambda)} \{d(a,b)\}$, where (a,b) belong to the set under consideration.
3. If $\lambda_4 = 0$, end this distillation.
4. Determine for each alternative its *qualification* score; that is, the difference between the number of alternatives it outranks and the number of alternatives that outrank it. Outranking is determined according to λ_*.
5. The set of alternatives having the largest (or smallest, if the distillation is ascending) qualification is the current distillate.
6. If the number of alternatives in the current distillate is larger than 1, repeat the process from step 2 inside the distillate.
7. Form a new set under consideration by removing the distilled alternatives from the current one. If this set is not empty, repeat the process on the new set from step 1.
8. The final pre-orders are ranked so that the alternatives in the first distillate are given rank 1, in the second rank 2, etc.

After this, the median pre-order can be computed by removing the incomparabilities and calculating the differences of ranks of an alternative in the two complete pre-orders.

2.3. ROBUSTNESS ANALYSIS FOR WEIGHTS

There are numerous weight elicitation techniques proposed for ELECTRE methods; the following are among the most recent and popular:

1. DIVAPIME by Mousseau [10] produces intervals for weights.
2. Hokkanen and Salminen [5] used two different weight elicitation proc edures and found that the normalized sets of weights had minor differences.
3. SRF by Figueira and Roy [4] allows weight elicitation in a user-friendly manner by using a technique based on a pack of "playing cards" to determine the relative importance of criteria coefficients. It can produce interval weights and was also designed to support multiple DMs.
4. The approach proposed by Rogers and Bruen [11] uses pairwise comparisons to elicit the weights.

The first three techniques, which produce intervals or two-weight sets that may be used to define intervals, can be used directly in robustness analysis. With the fourth weight elicitation technique, intervals (such as ± 10%) could be defined around the original weights.

Traditionally the robustness analysis for ELECTRE methods has been an ad hoc investigation into the effect of changing values [1]. This type of investigation typically considers only discrete points (for example, extreme points) of the feasible weight space (e.g., weight intervals). The procedure of building the pre-orders is based on exploiting the fuzzy outranking relation, which is nonlinear and discontinuous by nature. Therefore, instead of just a few discrete points, it is important to analyze the entire continuum of the weight space.

3. SMAA-III

In order to overcome the limitations of ELECTRE III, SMAA-III applies simulation and studies the effect of changing parameter values and criterion evaluations on the results. The imprecision is quantified through joint density functions in the corresponding spaces.

The weights are represented by a weight distribution with joint density function $f_W(w)$ in the feasible weight space W. The weights are non-negative and normalized. The weight space is an $n - 1$ dimensional simplex:

$$W = \left\{ w \in R^n : w \geq 0 \text{ and } \sum_{j \in J} w_j = 1 \right\}.$$

Completely missing preference information is represented by a uniform (constant) weight distribution in W; that is:

$$f_W(w) = 1/\text{vol}(W).$$

If some kind of preference information is available, different weight distributions can be applied [7]. In practice, the preferences can usually be elicited as interval constraints for weights. In this case, a uniform distribution in the space bounded by the constraints is used. Figure 1 illustrates the restricted feasible weight space of a three-criteria problem with lower and upper bounds for w_1. In this paper the focus is on weight information provided as intervals, because:

1. If there are multiple DMs whose preferences need to be taken into account, the weight intervals in general can be determined to contain the preferences of all DMs [7].
2. Weight intervals allow simple robustness analysis even when only deterministic weights are available, by specifying, for example, a ± 10% interval for each weight.

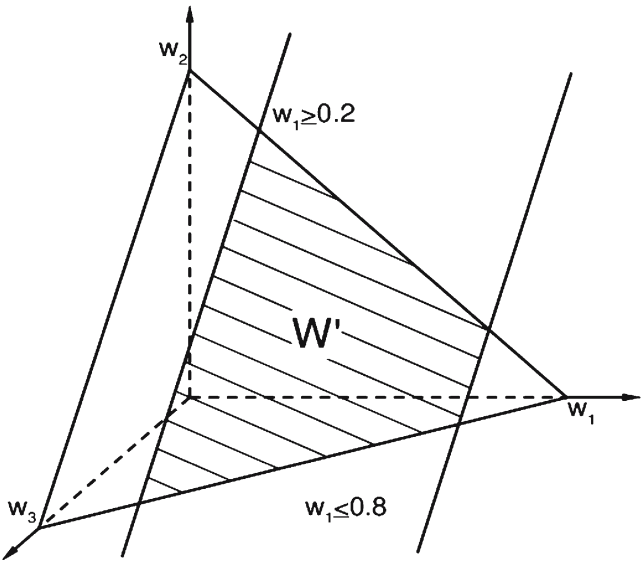

Figure 1. Feasible Weight Space of a Three-criteria Problem with Lower and Upper Bounds for w_1.

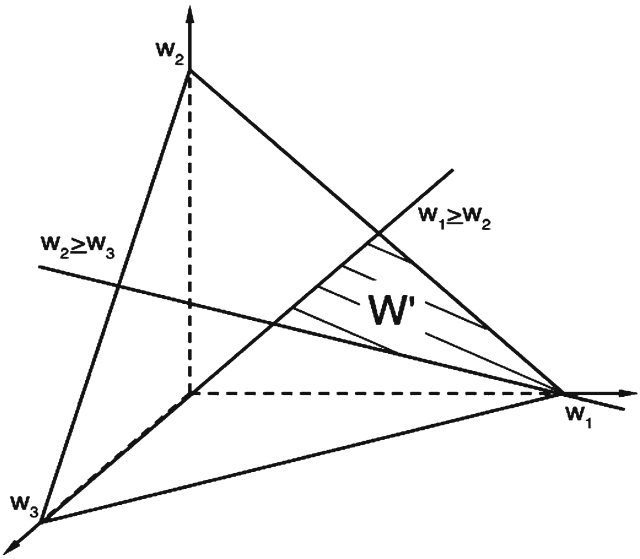

Figure 2. Feasible Weight Space of a Three-criteria Problem with Ranking of the Criteria.

It should be observed that other forms of easily elicitable preference information can be used as well, such as ranking of the criteria. A ranking can be obtained by asking the DMs to identify their most important and second most important criterion etc. Figure 2 illustrates the feasible weight space for a three-criteria problem with the ranking $w_1 \geq w_2 \geq w_3$.

Imprecise thresholds are represented by stochastic functions $\alpha_j(\cdot)$, $\beta_j(\cdot)$, and $\gamma_j(\cdot)$, corresponding to the deterministic thresholds $p_j(\cdot)$, $q_j(\cdot)$, and $v_j(\cdot)$, respectively. To simplify the notation, we define a 3-tuple of thresholds $\tau = (\alpha, \beta, \gamma)$. It has a joint density function f_τ in the space of possible values defining the functions. It should be noted that all feasible combinations of thresholds must satisfy $q_j(a_i) < p_j(a_i) < v_j(a_i)$.

Traditionally the thresholds in ELECTRE models have been used to model preferences of the DMs (e.g., differences deemed significant) as well as data imprecision. But it has been shown that the indifference threshold does not correspond to a linear imprecision interval [8]. Therefore, in SMAA-III thresholds are used only to model preferences (together with weights). Imprecision in the criteria measurements is modeled with stochastic variables.

These stochastic variables are denoted with ξ_{ij} corresponding to the deterministic evaluations $g_j(a_i)$. They have a density function $f_X(\xi)$ defined in the space $X \subseteq R^{m \times n}$. In principle, arbitrary distributions can be used, but in practice a uniform distribution in a certain interval or a Gaussian distribution is used.

Incomparabilities between alternatives can be present in the final results of ELECTRE III. This is one of the main features of ELECTRE methods in comparison with the methods applying classical multi-attribute utility theory [6]. In the late seventies, it was considered a very important theoretical advance. But, in reality when dealing with practical situations, incomparabilities in the final result are inconvenient. This aspect was soon observed [13] and partial pre-orders were replaced by complete pre-orders or median pre-orders. We apply median pre-orders in computing rank acceptability indices. The only information lost in using the median pre-order as the primary measure of the ranking is the incomparability. As our method is also aimed to help analysts accustomed to ELECTRE III, we will later present another index to measure incomparability.

$$a \succ b \Leftrightarrow \begin{cases} (a \succ^{Z_1} b \wedge a \succ^{Z_2} b) \vee (aI^{Z_1} b \wedge a \succ^{Z_b} b) \vee (a \succ^{Z_1} b \wedge aI^{Z_2} b) \\ (a \succ^{Z_1} b \wedge b \succ^{Z_2} a) \wedge (|r^{Z_1}(a) - r^{Z_2}(a)| \prec |r^{Z_1}(b) - r^{Z_2}(b)|), \\ (b \succ^{Z_1} a \wedge a \succ^{Z_2} b) \wedge (|r^{Z_1}(a) - r^{Z_2}(a)| \prec |r^{Z_1}(b) - r^{Z_2}(b)|) \end{cases} \quad (1)$$

$$aIb \Leftrightarrow \neg(b \succ a) \wedge \neg(a \succ b),$$

where $r(\cdot)$ is the ranking of an alternative in the superscripted pre-order. Monte Carlo simulation is used in SMAA-III to compute three types of descriptive measures: rank acceptability indices, pairwise winning indices, and incomparability indices. In order to compute these indices, let us define a *ranking function* that evaluates the rank r of the alternative a_i with the corresponding parameter values:

$$\text{rank}(i, w, \xi, \tau).$$

The evaluation of this function corresponds to executing ELECTRE III and returning the rank of the corresponding alternative in the resulting median pre-order. We will next introduce the three indices. Interpretation of their values is presented in Section 4 through various re-analyses.

3.1. RANK ACCEPTABILITY INDEX

The rank acceptability index, b_i^r, measures the share of feasible weights that grant alternative a_i rank r in the median pre-order by simultaneously taking into account imprecision in all parameters and criterion evaluations. It represents the share of all feasible parameter combinations that make the alternative acceptable for a particular rank, and it is most conveniently expressed as a percentage.

The rank acceptability index b_i^r is computed numerically as a multidimensional integral over the spaces of feasible parameter values as:

$$b_i^r = \int_{W:\text{rank}(i,w,\xi,\tau)=r} f_W(w) \int_X f_X(\xi) \int_T f_T(\tau) dT dw d\xi.$$

The most acceptable ("best") alternatives are those with high acceptability for the best ranks. Evidently, the rank acceptability indices are within the range [0,1], where 0 indicates that the alternative will never obtain a given rank and 1 indicates that it will always obtain the given rank with any feasible choice of parameters.

Using the rank acceptability indices as measures of robustness is quite straightforward: when the index is near 1, the conclusion is robust. Nevertheless, caution should be used when interpreting the results in cases where these indices are computed without weight information to characterize the problem. If an alternative obtains a low score for first-rank acceptability, it does not necessarily mean that it is "inferior". The DMs' actual preferences may well lie within the corresponding (small) set of favorable first-rank weights.

3.2. PAIRWISE WINNING INDEX

The pairwise winning index o_{ik} [9] describes the share of weights that place alternative a_i on a better rank than alternative a_k. An alternative a_i that has $o_{ik} = 1$ for some k always obtains a better rank than alternative a_k, and can thus be said to *dominate* it.

The pairwise winning index o_{ik} is computed numerically as a multidimensional integral over the space of weights that gives a lower rank for one alternative than for another:

$$o_{ik} = \int_{w \in W:\mathrm{rank}(i,w,\xi,\tau) < \mathrm{rank}(k,w,\xi,\tau)} f_W(w) \int_X f_X(\xi) \int_T f_T(\tau) dT dw d\xi.$$

The pairwise winning indices are especially useful when trying to distinguish between the ranking differences of two alternatives. Because the number of ranks in the median pre-order varies among different simulation runs, two alternatives might obtain similar rank acceptabilities although one is in fact inferior. In these cases looking at the pairwise winning indices between this pair of alternatives can help to determine whether one of the alternatives is superior to the other or if they are equal in "goodness."

3.3. INCOMPARABILITY INDEX

Because median pre-orders are used in computing the rank acceptability indices, it is no longer possible to model incomparability. As some DMs might be accustomed to make decisions that take incomparabilities into account, another index is introduced. Incomparability index σ_{ik} measures

the share of feasible parameter values that cause alternatives a_i and a_k to be incomparable. For this reason, we define the incomparability function:

$$R(i,k,\xi,\tau) = \begin{cases} 1, \text{ if the alternatives } a_i \text{ and } a_k \text{ are incomparable,} \\ 0, \text{ if not.} \end{cases}$$

This function corresponds to running ELECTRE III with the given parameter values and checking if the alternatives are judged incomparable in the final partial pre-order. In practice we do not compute the final partial pre-order, because this information can be extracted from the two partial pre-orders Z_1 and Z_2 as shown (1). By using the incomparability function, the incomparability index is computed numerically as a multidimensional integral over the feasible parameter spaces as:

$$\sigma_{ik} = \int_W f_W(w) \int_X f_X(\xi) \int_T f_T(\tau) R(i,j,\xi,\tau) dT dw d\xi.$$

4. Computation

All of the indices mentioned above are computed with Monte Carlo simulation. The procedure is similar to that presented and analyzed by Tervonen and Lahdelma [16]. SMAA-III differs in the sense that it applies the ELECTRE III procedure to derive the descriptive values instead of a utility function.

In each simulation iteration, sample parameter values are generated from their corresponding distributions, and ELECTRE III is executed with these values. Then the corresponding hit counters are updated as with the original SMAA. If standard distributions are used for defining the imprecise parameter values, then all sampling operations except weight generation are computationally very light. In the case of weight generation, if tight upper bounds are used, we can have very high weight rejection ratios (up to 99.9%). Nevertheless, even with 99.9% weight rejection, the method is fast enough to use in an interactive decision-making process with problems of reasonable size.

To obtain sufficient accuracy for the indices, we suggest using at least 10,000 simulation iterations. This gives error limits of less than 0.01 with 95% confidence [16].

5. Conclusions

In this paper we introduced a new method, SMAA-III, which allows the parameters and criteria measurements of ELECTRE III to be imprecise and to be defined with various types of constraints: no deterministic values are

required. This has numerous advantages, especially in the context of MCDM with multiple DMs, because the parameters can be determined as intervals that contain the preferences of all DMs. It also allows an easily applicable robustness analysis to be performed.

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