Investigation on the Baumgarte Stabilization Method for Dynamic Analysis of Constrained Multibody Systems

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Abstract This paper presents an investigation on the Baumgarte stabilization method for dynamic analysis of constrained multibody systems. The purpose of this work is to study the influence of the main variables that affect the constraints violation, namely, the values of the Baumgarte parameters. In the process, the formulation of the dynamic equations of motion of constrained multibody systems and the main issues of the Baumgarte stabilization method are revised. Attention is given to the techniques to help in the Baumgarte parameters selection. A demonstrative example is presented and the results of some simulations are discussed.

Keywords Multibody dynamics · Constrained systems · Baumgarte method · Baumgarte parameters

Introduction

Over the last decades the importance of the dynamic simulation of constrained multibody systems (MBS) has been recognized as playing a crucial role in a broad variety of engineering fields, such as robots, biomechanics, automobile systems and railway vehicles [1]. The equations of motion of constrained MBS are composed by a set of differential and algebraic equations (DAE) of index three. The numerical solution of the set of DAE is not straightforward problem. One of the most popular and used methods to solve this problem consists of converting the system of DAE into a set of ordinary differential equations (ODE) by appending the second derivative with respect to time of the constraint equations. However, with this approach, the state of variables, i.e., the generalized position and velocity constraint equations do not satisfy due to numerical error, being the global results not acceptable in practical applications.

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In recent years, a lot of attempts have been made dealing with the constrained multibody systems. Due to its simplicity and easiness for computational implementation, the Baumgarte stabilization method (BSM) is the most attractive technique to overcome the drawbacks of the standard integration of the equations of motion. Baumgarte's method can be looked upon as an extension of feedback control theory [2]. The principle of this method is to damp out the acceleration constraint violations by feeding back the violations of the position and velocity constraints. The choice of the feedback parameters depends on several factors, namely, the integrator used and the model of the MBS. This method does not solve all possible numerical instabilities as, for instance, those that arise near kinematic singularities. The major drawback of Baumgarte's method is the ambiguity in choosing feedback parameters. The coordinate partitioning method and the augmented Lagrangian formulation are alternative methods to deal with the constraints violation. In addition to these approaches, many research papers have been published on the stabilization methods for the numerical integration the equations of motion of multibody systems [3–5].

In this paper, an investigation on the Baumgarte's method for dynamic simulation of constrained MBS is presented. The formulation of the equations of motion of general MBS is also reviewed. The equations of motion are solved by using the Baumgarte stabilization technique with the intent of keeping the constraint violations under control. Finally, an eccentric slider crank mechanism is used as an application example.

Equations of Motion for Constrained MBS

When the configuration of a constrained MBS, with *f* degrees of freedom, is modeled through a set of *n* dependent coordinates, then a set of *m* algebraic constraints can be written as [1],

$$\Phi(\mathbf{q}, t) = \mathbf{0} \tag{1}$$

where $\mathbf{q}$ is the vector of generalized coordinates and t is the time variable. The equations of motion for a constrained MBS can be represented by [1],

$$\mathbf{M}\ddot{\mathbf{q}} + \Phi_{\mathbf{q}}^{T}\boldsymbol{\lambda} = \mathbf{g} \tag{2}$$

where $\mathbf{M}$ is the generalized mass matrix, $\ddot{\mathbf{q}}$ is the system accelerations, $\Phi_{\mathbf{q}}$ is the Jacobian matrix, $\boldsymbol{\lambda}$ is the Lagrange multipliers vector and $\mathbf{g}$ is the generalized force vector that includes the gravitational, centrifugal and Coriolis force terms. In order to progress with solution, the constraint velocity and acceleration equations are required. Thus, differentiating Eq. (1) with respect to time yields the velocity constraint equations,

$$\Phi_{\mathbf{q}}\dot{\mathbf{q}} = -\Phi_t \equiv \boldsymbol{\upsilon} \tag{3}$$

in which $\dot{\mathbf{q}}$ is the vector of generalized velocities and $\boldsymbol{\upsilon}$ is the right hand side of velocity equations, which contains the partial derivates of Φ with respect to time. A second differentiation of Eq. (1) with respect to time leads to the acceleration constraint equations,

$$\Phi_{\mathbf{q}}\ddot{\mathbf{q}} = -(\Phi_{\mathbf{q}}\dot{\mathbf{q}})_{\mathbf{q}}\dot{\mathbf{q}} - 2\Phi_{\mathbf{q}t}\dot{\mathbf{q}} - \Phi_{tt} \equiv \boldsymbol{\gamma} \tag{4}$$

where $\ddot{\mathbf{q}}$ is the acceleration vector and $\boldsymbol{\gamma}$ is the right hand side of acceleration equations. Equations (1), (3) and (4) must be satisfied during the simulation. Equation (4) can be appended to Eq. (2) and rewritten in matrix form as,

$$\begin{bmatrix} \mathbf{M} & \Phi_{\mathbf{q}}^T \\ \Phi_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{Bmatrix} \tag{5}$$

This system of equations is solved for $\ddot{\mathbf{q}}$ and $\boldsymbol{\lambda}$. Then, in each integration time step, the accelerations vector, $\ddot{\mathbf{q}}$, together with velocities vector, $\dot{\mathbf{q}}$, are integrated in order to obtain the system velocities and positions for the next time step. This procedure is repeated up to final time analysis is reached.

In order to keep the constraint violations under control, the Baumgarte stabilization method is used [2]. The BSM allows constraints to be slightly violated before corrective actions can take place, in order to force the violation to vanish. The goal of Baumgarte's method is to replace the differential Eq. (4) by the following equation,

$$\ddot{\Phi} + 2\alpha\dot{\Phi} + \beta^2\Phi = \mathbf{0} \tag{6}$$

Equation (6) is the differential equation for a closed loop system in terms of kinematic constraint equations. The terms $2\alpha\dot{\Phi}$ and $\beta^2\Phi$ in Equation (6) play the role of a control terms. Thus, utilizing the Baumgarte's approach, the equations of motion for a system subjected to constraints are stated in the form,

$$\begin{bmatrix} \mathbf{M} & \Phi_{\mathbf{q}}^T \\ \Phi_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} - 2\alpha\dot{\Phi} - \beta^2\Phi \end{Bmatrix} \tag{7}$$

In general, if α and β are chosen as positive constants, the stability of the general solution of Eq. (7) is guaranteed. When α is equal to β, critical damping is achieved, which usually stabilizes the system response more quickly. Baumgarte [2] highlighted that the suitable choice of the parameters α and β is performed by numerical experiments. It should be highlighted that the improper choice of these coefficients can lead to unacceptable results in the dynamic simulation of multibody systems.

Techniques to Select the Baumgarte Parameters

The first and simplest way to evaluate the Baumgarte parameters consists of expanding in Taylor's series the constraint equation and neglecting the terms of order higher than two. Thus, it is possible to write,

$$\Phi(t+h) = \Phi(t) + \dot{\Phi}(t)h + \ddot{\Phi}(t)\frac{h^2}{2} \tag{8}$$

where h represents the time step. Considering that function Φ is null at instant $t+h$, then Eq. (8) can be written as,

$$\ddot{\Phi}(t) + \frac{2}{h}\dot{\Phi}(t) + \frac{2}{h^2}\Phi(t) = 0 \tag{9}$$

By comparing and analyzing Eqs. (6) and (9) the mathematical relation for Baumgarte parameters and time step can be expressed by,

$$\alpha = \frac{1}{h} \tag{10}$$

$$\beta = \frac{\sqrt{2}}{h} \tag{11}$$

From Eqs. (10) and (11) it can be observed that with this technique the Baumgarte parameters are inversely proportional to the time step. This approach is quite simple, very easy to implement in any general code and works reasonably well from the computational view point. However, this procedure can lead to some numerical instability which ultimately produces incorrect results when the time step is too small, because the damping terms dominate the numerical value of Eq. (6) and make the system to become stiff. Thus, a more sophisticated methodology should be considered, being the Euler's integration method used to show how to select an appropriate set of α and β parameters. Herein, the proposed methodology is based on the stability analysis procedure in digital control theory. Applying the Laplace transform technique to a first order differential yields,

$$sY(s) = F(s) \tag{12}$$

where s is the operator of Laplace domain. Moreover, when Euler's integration method is used, the numerical solution of a first order differential yields,

$$y_{n+1} = y_n + hf_n \tag{13}$$

in which the subscript represents the numerical solution at the corresponding time step and h is the integration time step.

Since Eq. (13) is a difference equation, that is, a discrete data function, the Z transform technique must be used to study it. Thereby, the Z transform of Eq. (13) results in

$$zY(z) = Y(z) + hF(z) \tag{14}$$

where z is the Z transform variable. Re-arranging Eq. (14) yields,

$$\frac{F(z)}{Y(z)} = \frac{z-1}{h} \tag{15}$$

Analyzing Eqs. (12) and (15), striking resemblances between Laplace and Z transform techniques results in,

$$s = \frac{z-1}{h} \tag{16}$$

This means that the substitution of Eq. (16) in any $F(s)/Y(s)$ yields a $F(z)/Y(z)$ based on the Euler's integration method.

Considering now Eq. (6), the corresponding characteristic equation is,

$$s^2 + 2\alpha s + \beta^2 = 0 \tag{17}$$

Equation (17) suggests that if α and β are greater than zero, the system will be stable. However, Eq. (17) is not adequate to select α and β parameters. In order to select the appropriate values of the parameters α and β, the response of the second order characteristic equation (17) for different locations of its roots in the z-plane must be known first. Letting $s = \sigma + j\omega$, it is possible to write,

$$z = e^{t(\sigma + j\omega)} = e^{\sigma t}\angle\,\omega t \tag{18}$$

since $1\angle\omega t = \cos\omega t + j\sin\omega t$.

A system is stable if all roots of the characteristic equation are inside the unit circle on the z-plane, that is, $|z| < 1$. Conversely, a system is said to be unstable when the roots are outside the unit circle, that is, $|z| > 1$. When a system has its roots on the unit circle, $|z| = 1$, is called as marginally stable. In order to study the stability of characteristic Eq. (17), let substitute Eq. (16) yielding the characteristic equation in terms of z-plane as,

$$z^2 + (2\alpha h - 2)z + (\beta^2 h^2 - 2\alpha h + 1) = 0 \tag{19}$$

Equation (19) shows that α, β and h influences the location of the roots and, consequently, the dynamic system response. In order to have a criterion to help in the selection of the α and β parameters independently of the time step h, let

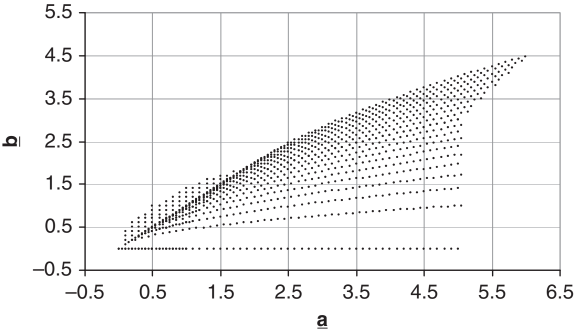

Fig. 1 Stability region in the $\underline{\alpha}$-$\underline{\beta}$ plane for the Euler integration method

consider two additional coefficients $\underline{\alpha}$ and $\underline{\beta}$ defined by, $\underline{\alpha} = \alpha$h and $\underline{\beta} = \beta^2 h^2$. The relationship between the $\underline{\alpha}$ and $\underline{\beta}$ coefficients for the Euler integration method is illustrated in Fig. 1, being easier to identify the stability region as function of the Baumgarte parameters.

Demonstrative Example

The purpose of this section is to demonstrate the computational effectiveness of the presented techniques to an eccentric slider crank mechanism, which is illustrated in Fig. 2. The system is driven by the crank which rotates with a constant angular velocity. Several representative simulations are performed in order to study and compare the efficiency of different values for α and β parameters on the stabilization of the constraint violations. A measure of their efficiency can be drawn from the error of the third constraint equation Φ_3, that are representative of the rest of the constraint equations and their derivatives, which present similar results. Eight sets of

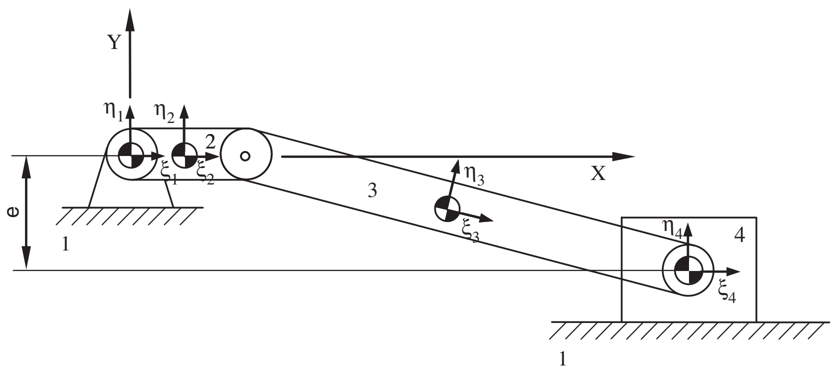

Fig. 2 Initial configuration of the eccentric slider crank mechanism

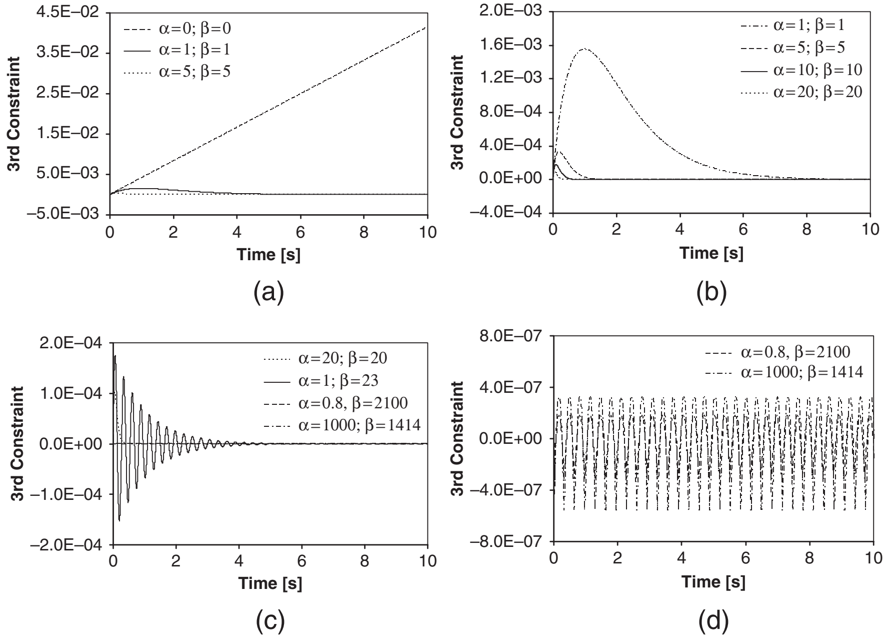

Fig. 3 Error of the 3rd constraint equation Φ_3

Baumgarte parameters were used, which were selected based on the methodologies presented in the previous section. Other variables were studied, chiefly, the values of the integration method, the size of the time step and the quality of the initial system guess for positions, being the results not presented due to lack of space.

Figure 3(a) shows that when $\alpha = \beta = 0$ is used the violation of the constraints grows indefinitely with the time. For nonzero values for parameters α and β, the behaviour of the system is slightly different, as it is illustrated in Fig. 3(b–d). Moreover, when the parameters α and β are equal, the critical damping is reached, which stabilizes the system response more quickly, that is, after a transient phase the first and second derivatives converge to zero. Thus, the constraint equations, and not only their second derivatives, are satisfied at any give time. Figure 3 (d) illustrates a stiff system, which occurs when the values of α and β are high.

Concluding Comments

An investigation on the Baumgarte stabilization method for dynamic analysis of constrained multibody systems was discussed in this work. A demonstrative example is presented and the results of some simulations were discussed. Several numerical simulations were performed in order to study the influence of the main variables that affect the constraints violation. In the process, the formulation of the dynamic equations of motion of constrained multibody systems and the Baumgarte stabilization method are revised. Special attention is also given to the techniques to help in the selection of the Baumgarte parameters. From the main results obtained, it can be concluded that the selection of the Baumgarte parameters play a key role on the dynamic systems' response.

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