23

DRIVING DYNAMIC PERFORMANCE

When computing the performance of a vehicle in longitudinal motion (maximum speed, gradeability, fuel consumption, braking, etc.), the vehicle is modelled as a rigid body, or in an even simpler way, as a point mass.

The presence of suspensions and the compliance of tires are then neglected and motion is described by a single equation, the equilibrium equation in the longitudinal direction. If the x-axis is assumed to be parallel to the ground, the longitudinal equilibrium equation reduces to

$$m\ddot{x} = \sum_{\forall i} F_{x_i}, \tag{23.1}$$

where F_{x_i} are the various forces acting on the vehicle in the longitudinal direction (aerodynamic drag, rolling resistance, traction, braking forces, etc.).

As will be seen later, Eq. (23.1) is quite a rough model for various reasons. For one thing, when the vehicle is accelerated, a number of rotating masses must be accelerated as well; this, however, can be accounted for easily. Other approximations come from the fact that the vehicle does not travel under symmetrical conditions, particularly when the trajectory is not straight and the direction of the x-axis does not coincide with the direction of the velocity or, in other words, the sideslip angle β is in general different from zero.

23.1 LOAD DISTRIBUTION ON THE GROUND

Longitudinal dynamics is influenced by the distribution of normal forces at the wheels-ground contact. A vehicle with more than three wheels is statically indeterminate, and the load distribution is determined by characteristics of the

G. Genta, L. Morello, *The Automotive Chassis, Volume 2: System Design*, Mechanical Engineering Series,

suspensions which, as seen in Part I, also have the task of distributing the load on the ground in proper way. However, if the system is symmetrical with respect to the xz plane, all loads are equally symmetrical, and the velocity is contained in the symmetry plane, then the two wheels of any axle are equally loaded. In this case, it is possible to think in terms of axles rather than wheels, and a two-axle vehicle may be considered as a beam on two supports which is, then, a statically determined system. In this case, the forces on the ground do not depend on the characteristics of the suspensions and the vehicle can be modelled as a rigid body.

23.1.1 Vehicles with two axles

Consider the vehicle as a rigid body and neglect the compliance of the suspensions and of the body. As previously stated, if the vehicle is symmetrical with respect to the xy-plane[1], it can be modelled as a beam on two supports, and normal forces F_{z_1} and F_{z_2} acting on the axles can be computed easily.

With the vehicle at a standstill on level road the normal forces are

$$\begin{cases} F_{z_1} = mg\epsilon_{0_1} \\ F_{z_2} = mg\epsilon_{0_2} \end{cases} \quad \text{where} \quad \begin{cases} \epsilon_{0_1} = b/l \\ \epsilon_{0_2} = a/l \; . \end{cases} \tag{23.2}$$

The forces acting on a two-axle vehicle moving on straight road with longitudinal grade angle α (positive when moving uphill) are sketched in Fig. 23.1. Note that the x-axis is assumed to be parallel to the road surface.

Taking into account the inertia force $-m\dot{V}$ acting in x direction on the centre of mass, the dynamic equilibrium equations for translations in the x and z direction and rotations about point O are

$$\begin{cases} F_{x_1} + F_{x_2} + F_{x_{aer}} - mg\sin(\alpha) = m\dot{V} \\ F_{z_1} + F_{z_2} + F_{z_{aer}} - mg\cos(\alpha) = 0 \\ F_{z_1}(a + \Delta x_1) - F_{z_2}(b - \Delta x_2) + mgh_G\sin(\alpha) - M_{aer} + |F_{x_{aer}}|h_G = -mh_G\dot{V} \; . \end{cases} \tag{23.3}$$

If the rolling resistance is ascribed completely to the forward displacement of the resultant F_{z_i} of contact pressures σ_z, distances Δx_i can be easily computed as

$$\Delta x_i = R_{l_i} f = R_{l_i}(f_0 + KV^2) \; . \tag{23.4}$$

Except in the case of vehicles with different wheels on the various axles, such as F-1 racers, the values of Δx_i are all equal.

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[1] In the present section on longitudinal dynamics, a complete symmetry with respect to the xz plane is assumed: The loads on each wheel are respectively $F_{z_1}/2$ and $F_{z_2}/2$ for the front and the rear wheels. To simplify the equations, the x-axis is assumed to be parallel to the road surface.

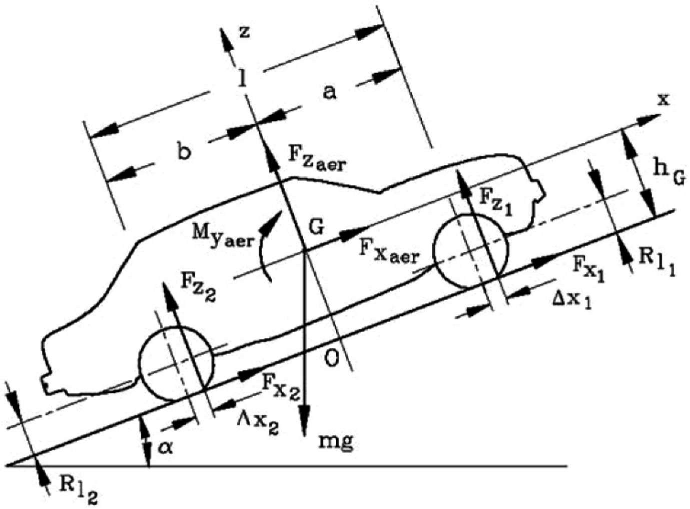

FIGURE 23.1. Forces acting on a vehicle moving on an inclined road.

The second and third equation (23.3) can be solved in the normal forces acting on the axles, yielding

$$\begin{cases} F_{z_1} = mg\dfrac{(b-\Delta x_2)\cos(\alpha) - h_G\sin(\alpha) - K_1V^2 - \dfrac{h_G}{g}\dot{V}}{l+\Delta x_1 - \Delta x_2} \\ \\ F_{z_2} = mg\dfrac{(a+\Delta x_1)\cos(\alpha) + h_G\sin(\alpha) - K_2V^2 + \dfrac{h_G}{g}\dot{V}}{l+\Delta x_1 - \Delta x_2} \end{cases} , \tag{23.5}$$

where

$$\begin{cases} K_1 = \dfrac{\rho S}{2mg}\left[C_x h_G - lC_{M_y} + (b-\Delta x_2)C_z\right] \\ \\ K_2 = \dfrac{\rho S}{2mg}\left[-C_x h_G + lC_{M_y} + (a+\Delta x_1)C_z\right] . \end{cases}$$

The values of Δx_i are usually quite small (in particular, their difference is usually equal to zero) and can be neglected. If considered, they introduce a further weak dependence of the vertical loads on the square of the speed, owing to the term KV^2 in the rolling resistance.

Example 23.1 *Compute the force distribution on the ground of the small car of Appendix E.1 at sea level, with standard pressure and temperature, in the following conditions:*
a) at standstill on level road;
b) driving at 100 km/h on level road;

c) driving at 70 km/h on a 10% grade;
d) braking with a deceleration of 0.4 g on level road at a speed of 100 km/h.

The air density in the mentioned conditions is 1.2258 kg/m^3*.*

a) Using Eq. (23.2), the static load distribution between the axles is

$$\epsilon_{0_1} = 0.597, \qquad \epsilon_{0_2} = 0.403.$$

The forces acting on the axles are then

$$F_{z_1} = 4863\ N, \qquad F_{z_2} = 3280\ N.$$

b) From Eq. (23.4), at 100 km/h = 27.78 m/s the value of Δx *is 4.6 mm for all tires. This value is so small that it could be neglected; it will, however, be considered in the following computations.*

Constants K_1 *and* K_2 *are easily computed*

$$K_1 = 8.505 \times 10^{-6}\ s^2/m, \qquad K_2 = -5.869 \times 10^{-5}\ s^2/m.$$

The forces acting on the axles are then

$$F_{z_1} = 4820\ N, \qquad F_{z_2} = 3491\ N.$$

c) A 10% grade corresponds to a grade angle $\alpha = 5.7^\circ$*. Operating in the same way, at 70 km/h = 19.44 m/s the value of* Δx *is 4.0 mm for all tires. The other results are*

$$K_1 = 8.490 \times 10^{-6}\ s^2/m, \qquad K_2 = -5.867 \times 10^{-5}\ s^2/m,$$

$$F_{z_1} = 4643\ N, \qquad F_{z_2} = 3542\ N.$$

d) The acceleration is $\dot{V} = -3.924\ m/s^2$*. As the speed is the same as in case b), the same values for* Δx*,* K_1 *and* K_2 *hold. The forces are*

$$F_{z_1} = 5498\ N, \qquad F_{z_2} = 2813\ N.$$

23.1.2 Vehicles with more than two axles

If more than two axles are present, even in symmetrical conditions the system remains statically indeterminate and it is necessary to take into account the compliance of the suspensions (Fig. 23.2a). The equilibrium equations (23.3) still hold, provided that the terms

$$F_{x_1} + F_{x_2}\ , \quad F_{z_1} + F_{z_2}\ , \quad F_{z_1}(a + \Delta x_1) - F_{z_2}(b - \Delta x_2)$$

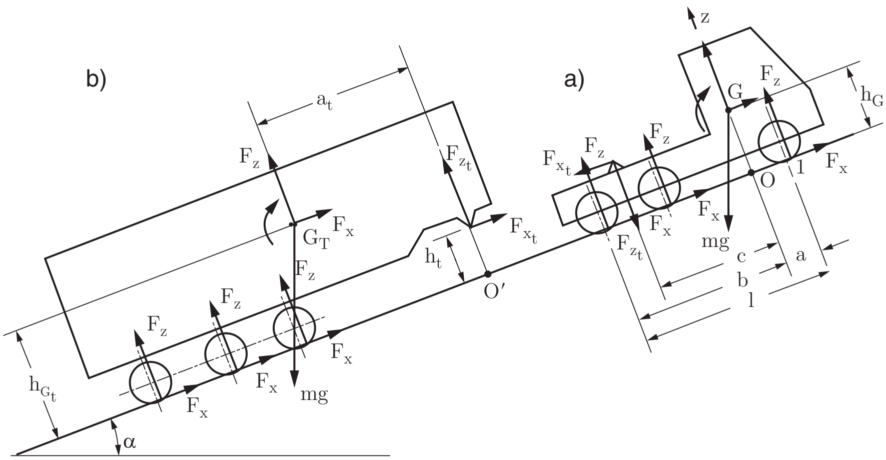

FIGURE 23.2. Forces acting on an articulated vehicle moving on an inclined road. (a) Tractor or vehicle with more than two axles; (b) trailer.

are substituted by

$$\sum_{\forall i} F_{x_i} \ , \quad \sum_{\forall i} F_{z_i} \ , \quad \sum_{\forall i} F_{z_1}(x_i + \Delta x_i) \ ,$$

where distances x_i are positive for axles located forward of the centre of mass and negative otherwise.

For computation of normal loads on the ground a number $(n-2)$ of equations, where n is the total number of axles, must be added. Each one of them simply expresses the condition that the vertical displacement of the point where each intermediate suspension is attached to the body is compatible with the displacement of the first and the last.

To account for possible nonlinearities of the force-displacement curves of the suspension, it is advisable to compute a reference position in which each suspension exerts a force $(F_{z_i})_0$. The linearized stiffness of the ith suspension, possibly taking into account the compliance of the tires as well, is K_i. The vertical displacement of the point where the ith suspension is attached is

$$\Delta z_i = -\frac{1}{K_i} \left[F_{z_i} - (F_{z_i})_0 \right] \ . \tag{23.6}$$

With reference to Fig. 23.3, the vertical displacement of the vehicle body in the point where the ith suspension is attached can be expressed as a function of the displacement of the first and nth suspension by the equation

$$\frac{1}{l} \left(\Delta z_n - \Delta z_1 \right) = \frac{1}{a - x_i} \left(\Delta z_i - \Delta z_1 \right) \ . \tag{23.7}$$

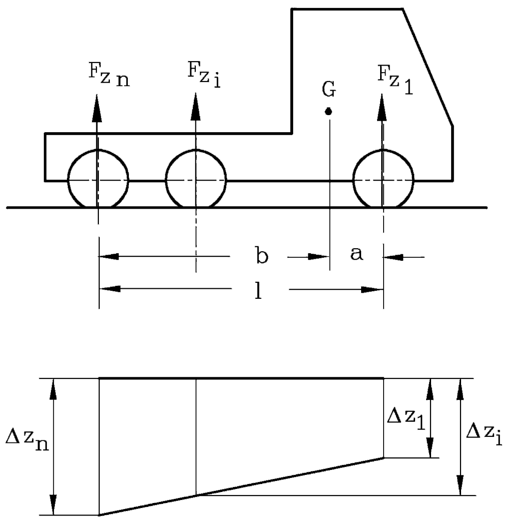

FIGURE 23.3. Compatibility condition for vertical displacements of the points where the suspensions are attached. In the figure, the ith axle is behind the center of mass and its coordinate x_i is negative.

By eliminating displacements Δz_i between equations (23.6) and (23.7), the required equation is obtained,

$$\frac{b + x_i}{K_1}\left[F_{z_1} - (F_{z_1})_0\right] + \frac{a - x_i}{K_n}\left[F_{z_n} - (F_{z_n})_0\right] - \frac{l}{K_i}\left[F_{z_i} - (F_{z_i})_0\right] = 0 \;, \quad (23.8)$$

$$\text{for } i = 2, \ldots, n-1 \;.$$

The mentioned reference condition can be referred to any value of the load or any position of the centre of gravity, provided that the values of the linearized stiffnesses are the same as those in the actual condition. Forces $(F_{z_i})_0$ can all be set to zero if the springs are linear and the suspensions are such that a position (i.e. a vertical and a pitch displacement) exists in which all wheels just touch the ground, exerting on it vanishing forces (neglecting the weight of the axles).

Equations (23.8), together with the second and third equation (23.3), form a set of n equations that can be solved to yield the n normal forces acting on the axles.

Remark 23.1 *Forces F_{z_i} can never become negative: If a negative value is obtained, it means that the relevant axle loses contact with the ground and the computation must be repeated after setting the force to zero due to the relevant axle. The procedure is repeated until no negative force is present.*

23.1.3 Articulated vehicles

In the case of articulated vehicles with a tractor with two axles and one or more trailers with no more than a single axle each (Fig. 23.2), the computation is straightforward. In this case, the equilibrium equations of the tractor are

$$\left\{\begin{array}{l}\displaystyle\sum_{i=1}^{n} F_{x_i} - F_{x_t} + F_{x_{aer}} - mg\sin(\alpha) = m\dot{V}\\ \displaystyle\sum_{i=1}^{n} F_{z_i} - F_{z_t} + F_{z_{aer}} - mg\cos(\alpha) = 0\\ \displaystyle\sum_{i=1}^{n} F_{z_i}(x_i + \Delta x_i) + F_{z_t}c + F_{x_t}h_t + mgh_G\sin(\alpha) - M_{aer}+\\ \qquad +|F_{x_{aer}}|h_G = -mh_G\dot{V}\ ,\end{array}\right. \tag{23.9}$$

where forces F_{x_t} and F_{z_t} are those the tractor exerts on the trailer, as in the figure, the number of axles of the tractor is assumed to be n (in the present case $n = 2$), the moments are computed with reference to point O, and the aerodynamic forces and moments are those exerted on the tractor only.

Similarly, the equilibrium equation of the trailer are

$$\left\{\begin{array}{l}\displaystyle\sum_{i=1}^{m} F_{x_i} + F_{x_t} + F_{x_{R_{aer}}} - m_R g\sin(\alpha) = m_R\dot{V}\\ \displaystyle\sum_{i=1}^{m} F_{z_i} + F_{z_t} + F_{z_{R_{aer}}} - m_R g\cos(\alpha) = 0\\ \displaystyle\sum_{i=1}^{m} F_{z_i}(x_i + \Delta x_i) - F_{x_t}h_t + m_R g h_{G_R}\sin(\alpha) + m_R g a_R\cos(\alpha) - M_{R_{aer}}+\\ \qquad +|F_{x_{R_{aer}}}|h_{G_R} = -m_R h_{G_R}\dot{V}\ ,\end{array}\right. \tag{23.10}$$

where the number of axles of the trailer is assumed to be m (in the present case $m = 1$), the moments are computed with reference to point O′, the aerodynamic forces and moments are those exerted on the trailer only and x_i are the coordinates of the axle in the reference frame centred in O′. Note that all x_i are usually negative.

The last two equations (23.9), together with the last two equations (23.10) are sufficient only on level road at a standstill, when force F_{x_t} vanishes. If it is other than zero the first equation (23.9) must also be used. However, the forces F_{x_i} it contains are not known since they depend on the normal forces F_{z_i}. A simple iterative scheme can be used, to compute the normal forces with $F_{x_t} = 0$, repeating the computation until a stable solution is found. If the wheels of the trailer exert driving or braking forces, these forces must also be introduced into the computation.

If the tractor has more than two axles or the trailer has more than one, additional equations must be introduced. The additional $(n - 2)$ equations of the tractor (n is the number of axles of the tractor), are equations (23.8) while the additional $(m - 1)$ equations for the trailer, where m is the number of its axles, are

$$\begin{aligned}&\frac{(a+c)(x_m-x_i)}{lK_1}\left[F_{z_1}-(F_{z_1})_0\right]+\frac{(b-c)(x_m-x_i)}{lK_n}\left[F_{z_n}-(F_{z_n})_0\right]+\\&+\frac{x_i}{K_{R_m}}\left[F_{zR_m}-(F_{zR_m})_0\right]-\frac{x_m}{K_{R_i}}\left[F_{zR_i}-(F_{zR_i})_0\right]=0\,,\end{aligned}\tag{23.11}$$

$$\text{for } i=1,\dots,m-1\,,$$

where K_{t_i} and $F_{z_{t_i}}$ are the linearized stiffness of the ith suspension of the trailer and the force acting on it and $(F_{z_{t_i}})_0$ is the normal force in the same axle in any reference condition.

The first two terms of Eq. (23.11) are linked to the vertical displacement of the hitch, and the equation expresses the displacements of the hitch, the last axle and the relevant axle.

The number of unknowns and equations is then equal to the total number of axles plus one, since the normal force the two parts of the vehicle exchange is also unknown. When force F_{x_t} does not vanish, it must be computed iteratively, as seen above.

Example 23.2 *Compute the force distribution on the ground of the five-axle articulated truck of Appendix E.9 at sea level, with standard pressure and temperature, in the following conditions:*
a) at standstill on level road;
b) at standstill on a 10% grade;
c) driving at 70 km/h on a 10% grade;

The air density in the mentioned conditions is 1.2258 kg/m^3.
a) The static load distribution on level road can be computed directly, as the horizontal force exchanged between the two parts of the vehicle vanishes. The unknowns are six, the loads of the five axles and the vertical force exchanged between tractor and trailer. These can be computed from the set of linear equations

$$\begin{bmatrix}1.000 & 1.000 & 0 & 0 & 0 & -1.000\\1.175 & -2.310 & 0 & 0 & 0 & 1.860\\0 & 0 & 1.000 & 1.000 & 1.000 & 1.000\\0 & 0 & -6135 & -7.395 & -8.715 & 0\\-1,070 & -0,1109 & 4,054 & 0 & -4,446 & 0\\-0,5474 & -0,06087 & 0 & 4,054 & -5,359 & 0\end{bmatrix}\times 10^{-3}$$

$$\times\begin{Bmatrix}F_{z_1}\\F_{z_2}\\F_{zR_1}\\F_{zR_2}\\F_{zR_3}\\F_{z_t}\end{Bmatrix}=\begin{Bmatrix}70.100\\0\\313.900\\-1.597.900\\0\\0\end{Bmatrix}.$$

The forces acting on the axles are then 58.660 kN, 105.700 kN, 80.060 kN, 83.600 kN and 56.050 kN. The force at the tractor-trailer connection is 94.210 kN.

b) A 10% grade corresponds to a grade angle $\alpha = 5.7°$. *In this case the load distribution can also be computed directly, since the horizontal force exchanged between the two parts of the vehicle does not depend on the normal forces. Operating as in the previous case, the forces acting on the axles are 45.050 kN, 115.720 kN, 78.900 kN, 84.490 kN and 58.000 kN. The forces at the tractor-trailer connection are 90.980 kN in the vertical direction and 31.236 kN in the horizontal direction.*
c) At 70 km/h = 19.44 m/s the value of Δx *is 3.7 mm for all tires. In this case, owing to rolling resistance, an iterative solution must be obtained. However, the convergence is very fast, as only five iterations are needed to reach a difference between the results at the* i *-th and at the* $(i-1)$*-th iteration smaller than* 10^{-6} *in relative terms. The other results are not dissimilar to those obtained in the previous case: The forces on the axles are 43.980 kN, 116.970 kN, 78.710 kN, 84.440 kN and 58.060 kN; those at the tractor-trailer connection are 91.150 kN in the vertical direction and 33.440 kN in the horizontal direction.*

Note that the matrix of the coefficients of the relevant set of equations is the same in all cases.

23.2 TOTAL RESISTANCE TO MOTION

Consider a vehicle moving at constant speed on a straight and level road. The forces that must be overcome to maintain a constant speed are aerodynamic drag and rolling resistance.

By using the simplified formula seen in Part I to express the dependence of rolling resistance on speed, the modulus of the first is

$$R_r = \sum_{\forall i} F_{z_i} \left(f_0 + KV^2 \right) , \tag{23.12}$$

where F_{z_i} is the force acting in a direction perpendicular to the ground on the ith wheel.

Assuming that the rolling coefficient f is the same for all wheels[2], the sum of all normal forces can be brought out from the sum and, taking into account aerodynamic lift as well, it follows that

$$R_r = \left(f_0 + KV^2 \right) \sum_{\forall i} F_{z_i} = \left[mg\cos(\alpha) - \frac{1}{2}\rho V_r^2 SC_z \right] \left(f_0 + KV^2 \right) . \tag{23.13}$$

Aerodynamic drag (or, better, the aerodynamic force in the x direction, Eq.(21.11)) has a value (always as an absolute value) of

$$R_a = \frac{1}{2}\rho {V_r}^2 SC_x . \tag{23.14}$$

[2]This assumption holds only as a first approximation, since it does not take into account the dependence of f on the driving or braking conditions or other variables.

With increasing speed, the importance of the former grows; at a given value of the speed aerodynamic drag becomes more important than rolling resistance. This speed is lower for small cars while for larger vehicles, particularly for trucks at full load, rolling resistance is the primary form of drag. Another factor is that usually the mass of the vehicle grows with its size more rapidly than the area of its cross section.

If the road is not level, the component of weight acting in a direction parallel to the velocity V, i.e. the grade force

$$R_p = mg\sin(\alpha) \tag{23.15}$$

must be added to the resistance to motion.

The grade force becomes far more important than all other forms of drag even for moderate values of grade (Fig. 23.1). Since the force acting in a direction perpendicular to the ground on a sloping road is only the component of weight perpendicular to the road, the total resistance to motion, or road load, as it is commonly referred to, can be written in the form

$$R = \left[mg\cos(\alpha) - \frac{1}{2}\rho V^2 SC_z\right](f_0 + KV^2) + \frac{1}{2}\rho V^2 SC_x + mg\sin(\alpha)\ , \tag{23.16}$$

where, assuming that the air is still, the velocity with respect to air V_r becomes conflated with velocity V.

To highlight its dependence on speed, the road load can be written as

$$R = A + BV^2 + CV^4\ , \tag{23.17}$$

where

$$A = mg\left[f_0\cos(\alpha) + \sin(\alpha)\right]\ ,$$

$$B = mgK\cos(\alpha) + \frac{1}{2}\rho S[C_x - C_z f_0]\ ,$$

$$C = -\frac{1}{2}\rho SKC_z\ .$$

The last term in Eq. (23.17) becomes important only at very high speed in the case of vehicles with strong negative lift: It is usually neglected except in racing cars.

Since the grade angle of roads open to vehicular traffic is usually not very large, it is possible to assume that

$$\cos(\alpha) \approx 1\ , \qquad \sin(\alpha) \approx \tan(\alpha) \approx i\ ,$$

where i is the grade of the road. In this case coefficient B is independent of the grade of the road and

$$A \approx mg(f_0 + i)$$

depends linearly on it. C never depends on grade.

23.3 POWER NEEDED FOR MOTION

The power needed to move at constant speed V is obtained simply by multiplying the road load given by Eq. (23.17) by the value of the velocity

$$P_n = VR = AV + BV^3 + CV^5 \,. \tag{23.18}$$

Example 23.3 *Plot the curves of the road load of the car of Appendix E.1 on level road and on a 10% grade. Plot the curve of the power needed for constant speed driving on level road.*

The results obtained through Eq. (23.16) are shown in Fig. 23.4.

Example 23.4 *Plot the curves of the road load of the articulated truck of Appendix E.9 on level road and on a 10% grade.*

The results obtained through Eq. (23.16) are shown in Fig. 23.5. Note that in this case aerodynamic drag amounts to a relatively small part of the road load and that on a 10% grade the grade force is very high.

Motion at constant speed is possible only if the power available at the wheels at least equals the required power given by Eq. (23.18). This means that the

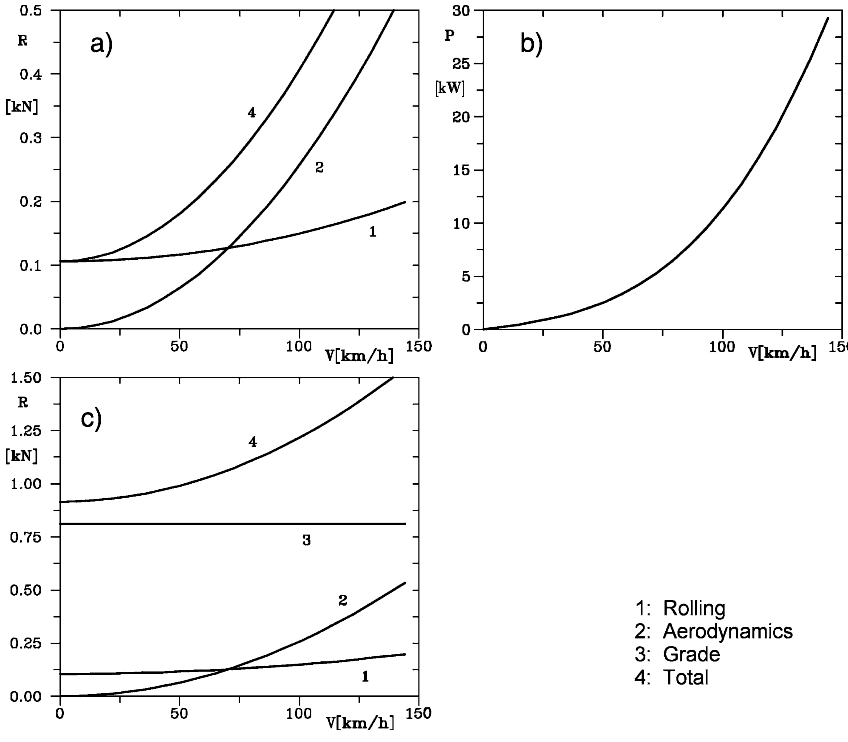

FIGURE 23.4. Resistance (a), and power (b) needed for motion at constant speed for the small car of Appendix E.1. Road load on the same car driving on a 10% slope (c).

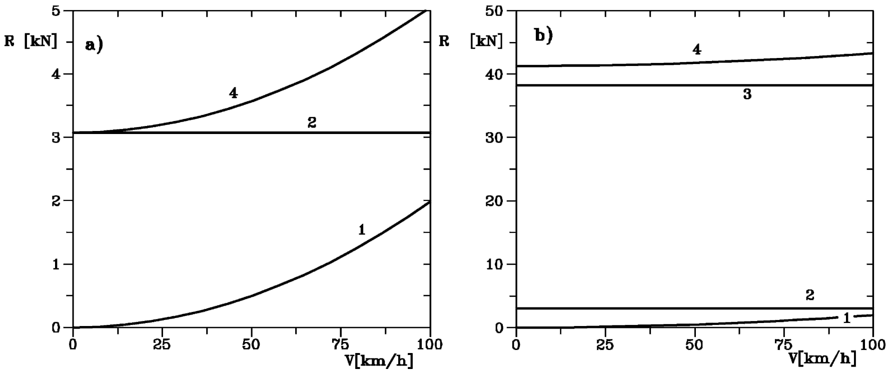

FIGURE 23.5. Aerodynamic drag (curve 1), rolling resistance (2), grade force (3) and total road load (4) for the articulated truck of Appendix E.9 on level road (a) and on a 10% grade (b).

engine must supply sufficient power, taking into account losses in the transmission as well, and that the road-wheel contact must be able to transmit this power.

When assessing the longitudinal performance of motor vehicles, it is often expedient to plot the power required for motion as a function of speed on a logarithmic plot. If the term CV^5 is neglected, the two remaining terms of Eq. (23.18) are represented by straight lines with slopes respectively equal to 1 and 3. The two straight lines cross in a point whose x-coordinate is the so called characteristic speed, easily obtained from Eq. (23.18)

$$V_{car} = \sqrt{\frac{A}{B}} \; . \tag{23.19}$$

Its y-coordinate is the logarithm of half the corresponding characteristic power (Fig. 23.6), whose value is:

$$P_{car} = AV_{car} + BV_{car}^3 = 2A\sqrt{\frac{A}{B}} \; . \tag{23.20}$$

The plot of the power required for motion cannot be obtained directly by adding the ordinates of the two straight lines of Fig. 23.6: the power must be computed from its logarithm, the values of the powers given by the straight lines added to each other, and the logarithm computed once again.

How to plot the curve $P_n(V)$ on a logarithmic plot is described in detail by M. Bencini[3]. Once the curve for $\alpha = 0$ has been obtained, all other curves for any value of α can be obtained by moving the curve on the $P(V)$ plane. The

[3] M. Bencini, *Dinamica del veicolo considerato come punto*, Tamburini, Milano, 1956.

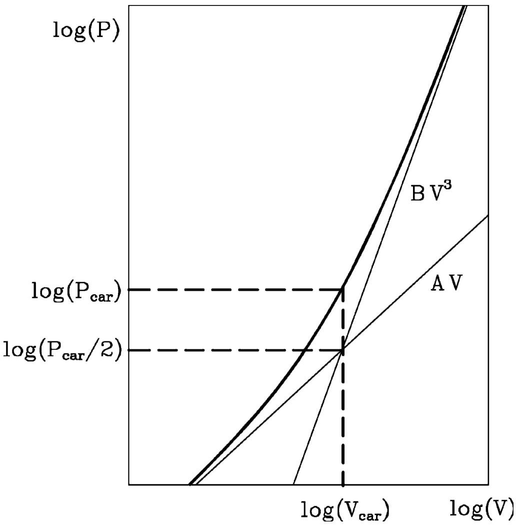

FIGURE 23.6. Power needed for motion. Logarithmic plot.

curve must be moved by

$$\Delta_1 = \frac{1}{2}\log\left[\frac{f_0+\tan(\alpha)}{f_0}\right] + \frac{1}{2}\log\left[\cos(\alpha)\frac{2gK+\lambda}{2gK\cos(\alpha)+\lambda}\right] , \tag{23.21}$$

where

$$\lambda = \rho S C_x / m ,$$

in the horizontal direction, and by

$$\Delta_2 = \frac{3}{2}\log\left[\frac{f_0+\tan(\alpha)}{f_0}\right] + \frac{1}{2}\log\left[\cos^3(\alpha)\frac{2gK+\lambda}{2gK\cos(\alpha)+\lambda}\right], \tag{23.22}$$

in the vertical direction. For values of the road slope small enough to accept the approximation $\cos(\alpha) \approx 1$, equations (23.21) and (23.22) simplify as

$$\Delta_1 = \frac{1}{2}\log\left(\frac{f_0+i}{f_0}\right) , \qquad \Delta_2 = \frac{3}{2}\log\left(\frac{f_0+i}{f_0}\right) , \tag{23.23}$$

where $i = \tan(\alpha)$. The curves move along a straight line with slope 3 and the displacement depends only on the value of f_0, and obviously on the slope i of the road.

It is then possible to obtain a single logarithmic plot containing a set of curves $P_n(V)$ for different values of α that can be used for any vehicle in the range where $\cos(\alpha) \approx 1$. Such a plot must be adapted to any particular vehicle by computing the values of characteristic speed and power (V_{car}, P_{car}) on level road. If the value of the rolling coefficient f_0 coincides with the reference value

f_{0_r} used to plot the diagram, the reference value of the slope i_r shown on the plot is the actual one, otherwise

$$i = i_r f_0 / f_{0_r} \ . \tag{23.24}$$

The plot $P_n(V)$ obtained for the values reported in the caption is shown in Fig. 23.7. Such a plot holds for any vehicle in the zone characterized by low values of α. The error made using Fig. 23.7 for other vehicles increases with increasing α, but remains negligible for values of i up to $0.3 \div 0.4$.

23.4 AVAILABLE POWER AT THE WHEELS

The engine drives the wheels through a mechanical transmission whose task is essentially that of reducing the angular velocity of the engine to that required at the wheels. If a reciprocating internal combustion engine is used, the transmission also has the task of uncoupling the engine from the wheels at a stop or at low speed, for which reason the driveline includes a clutch or a torque converter as well.

It is possible, at least as a first approximation, to state a value of the efficiency η_t for any type of driveline. The power available at the wheels is then

$$P_a = P_e \eta_t \ . \tag{23.25}$$

Depending on the type of transmission, the efficiency can be considered as a constant (obviously only as a first approximation), or may be computed, but only after assessing the working conditions of the driveline and above all of the torque converter, if present.

To compute the efficiency of the driveline, or the power it dissipates, see the relevant section of Part II.

The equation linking the speed of the engine to that of the wheels is simply

$$V = \Omega_e R_e \tau_t \ , \tag{23.26}$$

where τ_t is the overall gear ratio, defined as the ratio between the speed of the wheels and that of the engine shaft, and is usually smaller than 1. Once the gear ratios of all parts of the transmission are known, the power available at the wheels can be plotted as a function of the speed of the vehicle on the same plot where the power needed for motion at constant speed is reported.

If the curves of the required and available power are plotted on a logarithmic plot, any change of the gear ratio causes a translation of the curve related to the available power along the V-axis, while a change in the efficiency of the driveline causes a translation of the same curve along the P-axis (Fig. 23.8). If a continuous transmission (CVT) is present, the position of the curve is a function of the gear ratio, and then it is possible to define a zone on the VP-plane where all possible working conditions are included.

See Part II for situations where a torque converter is present.

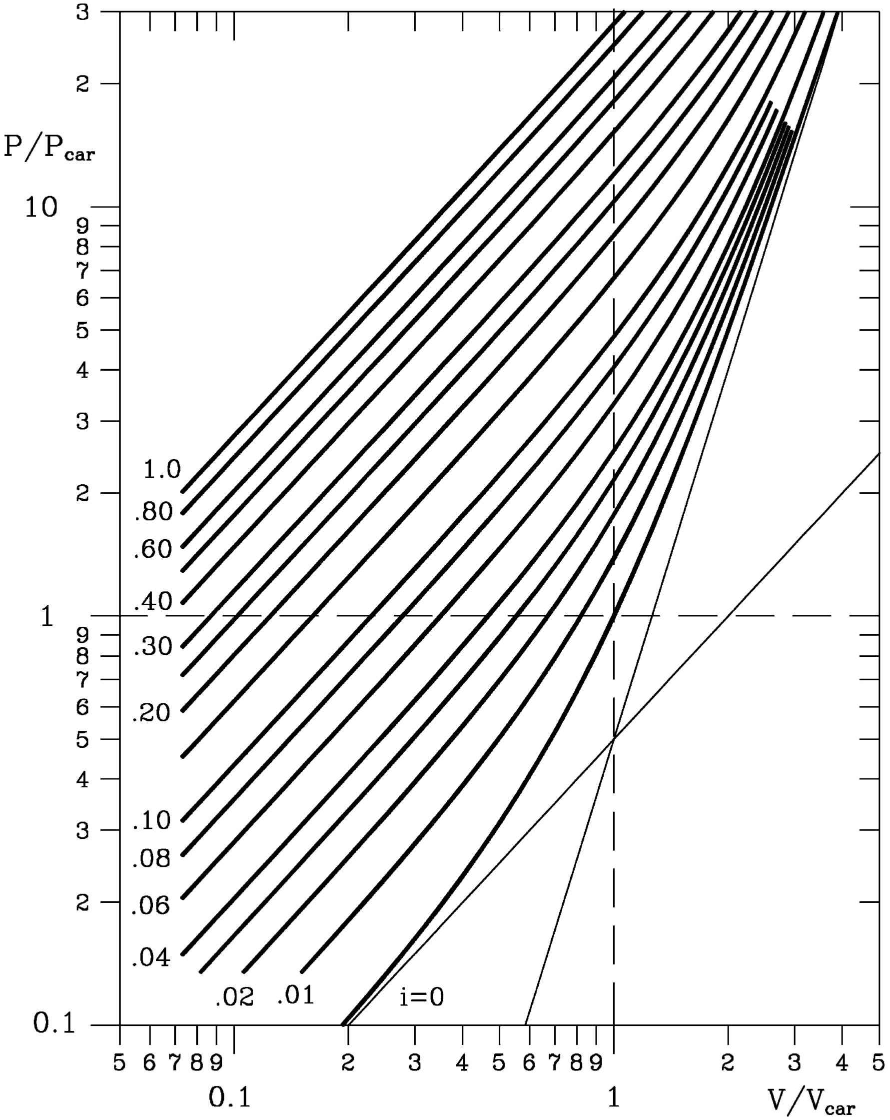

FIGURE 23.7. Logarithmic plot of the power needed for motion. The characteristic speed and power refer to a level road with $f_0 = 0.013$. For high values of the slope the plot holds only if $m = 1000$ kg; $S = 1.7$ m^2; $C_x = 0.42$; $K = 6.5 \times 10^{-6}$ s^2/m^2; $g = 9.81$ m/s^2; $\rho = 1.22$ kg/m^3.

23.5 MAXIMUM POWER THAT CAN BE TRANSFERRED TO THE ROAD

The power needed to overcome the road load must be transferred through the road-wheel contact. As it increases with both increasing speed and the grade of

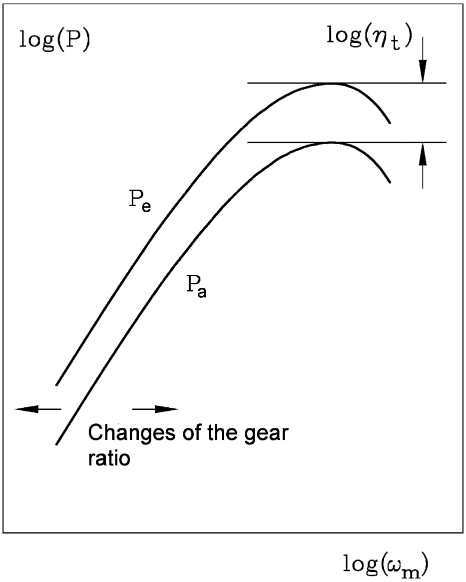

FIGURE 23.8. Curves of maximum engine power and power available at the wheels plotted with logarithmic scales. Changing the efficiency of the transmission and the overall transmission ratio.

the road, there is a limit on the maximum speed that can be reached and the maximum grade that can be managed because of this limit on the driving force the vehicle can exert, even if no limit to the power supplied by the engine exists.

The maximum power that can be transferred by the vehicle is

$$P_{max} = V \sum_{\forall i} F_{z_i} \mu_{i_p} \, , \tag{23.27}$$

where the sum is extended to all the driving wheels.

Remark 23.2 *If the maximum longitudinal force coefficient μ_{i_p} and the load acting on the driving wheels were independent of speed, the maximum power would increase linearly with V. The optimum engine characteristic $P_e(\Omega_e)$ for a vehicle with a transmission with fixed ratio would be a linear characteristic. This is not the case, however, as the situation is far more complicated.*

To begin with, consider the case of a vehicle with two axles, all of which are driving and assume that all wheels work with the same longitudinal slip, i.e. that the values of μ_i are equal. This situation will be referred here to as "ideal driving force".

The maximum power that can be transferred to the road is then

$$P_{max} = V \mu_{i_p} \left(mg \cos(\alpha) - \frac{1}{2} \rho V^2 S C_z \right) \, . \tag{23.28}$$

23.5.1 Vehicles with all wheels driving

When all wheels are driving wheels, it is possible to assume that the rolling resistance of the wheels is due only to the forward displacement of force F_z at the road-wheel contact, and hence is overcome directly by the driving torque exerted by the engine.

To compute the maximum grade that can be managed at low speed, it is possible to assume that the only road load that must be overcome at the wheel-road contact at such speeds is the grade load. By equating the power required for motion (Eq. 23.18) to the maximum power that can be transferred (Eq. 23.28), the maximum grade is readily obtained,

$$\tan(\alpha_{max}) = i_{max} = \mu_{x_p} \ . \tag{23.29}$$

To compute the maximum speed that can be reached, the decrease of traction at the road-wheel contact occurring with increasing speed must be modelled. A very simple way is to assume a linear law

$$\mu_{i_p} = c_1 - c_2 V \ . \tag{23.30}$$

By equating the power required for motion at constant speed (except that related to rolling resistance) to Eq. (23.28) and using Eq. (23.30) for expressing the decrease of the available driving force with the speed, the maximum speed can be obtained from the cubic equation

$$C_z c_2 V^3 + \left(C_z c_1 + C_x\right) V^2 - \frac{2mg}{\rho S}\left[(c_1 - c_2 V)\cos(\alpha) - \sin(\alpha)\right] = 0 \ . \tag{23.31}$$

The values of the maximum grade and of the maximum speed can only be achieved in ideal conditions, since the longitudinal slip of all wheels has been assumed to be the same. The forces acting on the driving axles can be computed by using the procedure already seen: They generally depend not only on the static load distribution but also on the speed and the acceleration.

In the case of a vehicle with two axles, both driving, the ratio

$$K_T = \frac{F_{x_1}}{F_{x_2}}$$

between the driving force at the front wheels and that at the rear wheels is usually a constant. If the wheels all have the same diameter, it coincides with the ratio between the driving torques supplied to the two axles. Assume that the two axles have tires of the same type and operate on patches of road with the same characteristics. If

$$K_T \frac{F_{z_2}}{F_{z_1}} > 1 \ ,$$

the limit conditions occur at the front wheels. At the onset of slipping, the power that can be transferred to the road is then

$$P_{max} = V \mu_{x_p} F_{z_1} \frac{1 + K_T}{K_T} \ . \tag{23.32}$$

By plotting the maximum power obtained by Eq. (23.32) versus the speed together with the power required given by Eq. (23.17) multiplied by V, the maximum speed at which the vehicle is able to transfer enough power to maintain its speed is readily obtained. It must be remembered that rolling resistance must be neglected in the computation of the required power.

If, on the contrary,

$$K_T \frac{F_{z_2}}{F_{z_1}} < 1 \ ,$$

the limit conditions occur at the rear wheels and the maximum power that can be transferred to the road is

$$P_{max} = V \mu_{x_p} F_{z_2} (1 + K_T) \ . \tag{23.33}$$

The maximum grade that can be managed is also easily obtained. Since in this case the speed can be set to zero, it follows that, for an all-wheel drive vehicle,

$$\tan(\alpha_{max}) = i_{max} = \frac{b c_1 \mu_{x_p} \left(1 + \frac{1}{K_T}\right)}{l + z_G \mu_{x_p} \left(1 + \frac{1}{K_T}\right)} \ ,$$

$$\tan(\alpha_{max}) = i_{max} = \frac{a \mu_{x_p} (1 + K_T)}{l + z_G \mu_{x_p} (1 + K_T)} \ , \tag{23.34}$$

respectively if the rear wheels slip first, i.e. if

$$K_T < \frac{b - h_G \tan(\alpha_{max})}{a + h_G \tan(\alpha_{max})} \ ,$$

or if this condition does not hold. It is, however, unlikely that the rear wheels are in a critical condition on a very steep grade, since this would require an abnormally low value of K_T. The value of μ_{x_p} is that for a vanishingly small speed.

23.5.2 Vehicles with a single driving axle

If not all axles are driving, the power that can be transferred to the ground is smaller. Aerodynamic drag increases the load on the rear wheels, as does a positive grade of the road: The power that can be transferred by a rear-wheels drive vehicle thus increases with speed, due to drag, and with the slope. Aerodynamic moment and lift have different effects depending on the sign of the moments and the position of the centre of mass. The maximum power is then

$$P_{max} = V \mu_{x_p} F_{z_1} \ , \qquad P_{max} = V \mu_{x_p} F_{z_2} \ , \tag{23.35}$$

respectively for the cases of front and rear wheel drive. Only the rolling resistance of the free wheels must be accounted for in the computation of the power needed for constant speed driving ; this is easily done by introducing F_{z_2} or F_{z_1}, respectively for front- and rear-wheels drive, in the expression of the road load instead of the total load on the ground.

Equations (23.34) could still be used for the computation of the maximum grade that can be managed. The first equation holds for front-wheel drive ($1/K_T = 0$) and the second for rear-wheel drive, ($K_T = 0$) but they do not include the rolling resistance of the free wheels.

If this effect is accounted for, the equations are modified as

$$\tan(\alpha_{max}) = i_{max} = \frac{b\mu_{x_p} - af_0}{l + z_G\left(\mu_{x_p} + f_0\right)} ,$$
$$\tan(\alpha_{max}) = i_{max} = \frac{a\mu_{x_p} - bf_0}{l - z_G\left(\mu_{x_p} + f_0\right)} . \tag{23.36}$$

Example 23.5 *Plot the curves of maximum transmissible and required power for the vehicle of Appendix E.2 on dry and wet road. Compute the maximum speed and the maximum grade that can be managed.*

Repeat the computations assuming that the same vehicle has rear-wheel drive, without changing the static load distribution on the ground.

Assume that $c_1 = 1.1$ *and* $c_2 = 6 \times 10^{-3}$ *s/m on dry road and* $c_1 = 0.8$ *and* $c_2 = 8 \times 10^{-3}$ *s/m on wet road.*

The curves of the maximum transmissible power are shown in Fig. 23.9, together with those of the required power. The vehicle of the example can thus reach a maximum speed of 225 km/h (wet road) or 308 km/h (dry road) for reasons linked only to the wheel driving force.

The computations were repeated assuming that the driving wheels are the rear ones. In this case, the maximum power that can be transferred to the ground

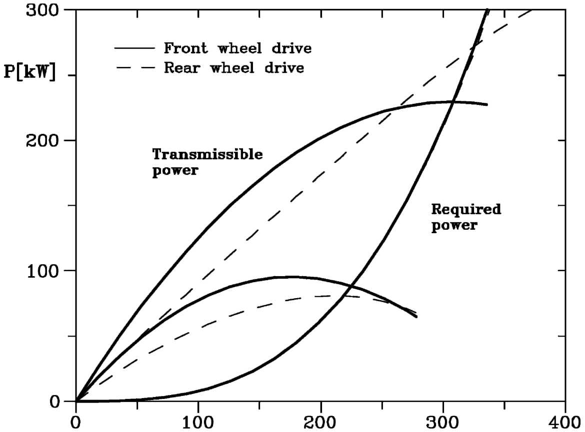

FIGURE 23.9. Maximum transmissible power and required power on level road in the case of Example 23.5.

at low speed is lower than in the previous case, since the static load distribution was specified in order to obtain good performance with front-wheel drive.

The load on the rear wheels increases with increasing speed and eventually gets larger than that on the front wheels. On dry road, the maximum speed is then higher for the vehicle with rear-wheel drive, despite the fact that at standstill the front wheels are loaded by about 60% of the weight.

The values of the maximum speed for the rear-wheel drive vehicle are of 218 km/h on wet road or 328 km/h on dry road.

Note that the required power curve includes only the rolling resistance of the non-driving wheels, and is slightly different in the two cases.

Also note that the curves do not take into consideration the load shift due to acceleration, so for speeds lower than the maximum speed, where the vehicle would accelerate if the maximum power is applied, they are not realistic.

The maximum grade angle that can be managed when only the wheel driving force is considered may be computed using Eq. (23.36), obtaining 28.0° on dry road and 22.1° on wet road, corresponding to grades of 53.1% and 40.6% respectively. If the driving wheels were the rear ones the values would have been 29.4° (56.3%) and 20.6° (37.6%).

In the case of rigid axles in which the final gear is directly mounted on the axle and the propeller shaft is in the longitudinal direction, the drive torque M_d applied to the axle causes a transversal load shift between the driving wheels of the same axle.

With reference to Fig. 23.10 the load shift ΔF_z could be determined easily as

$$\Delta F_z = \frac{M_d}{t_i} \qquad \text{where} \qquad M_d = F_x R_l \tau_f \ , \tag{23.37}$$

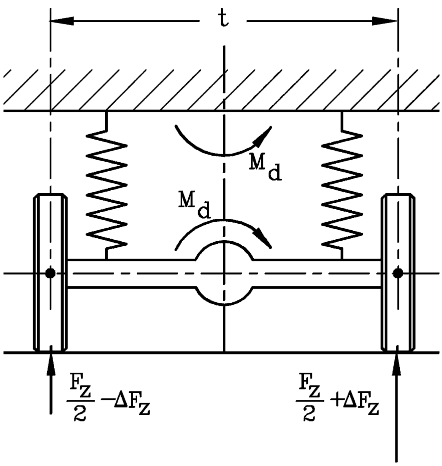

FIGURE 23.10. Transversal load shift due to the driving torque T_d.

F_x is the longitudinal force exerted by the axle on the ground and τ_f is the gear ratio of the final drive, defined as the ratio between the speed of the wheels and that of the propeller shaft (it is usually smaller than unity).

Equation (23.37) is not, however, usually correct as under the action of the driving torque the vehicle body is subject to a roll rotation, which in turn produces an added torque on the axle through the suspension system. If the roll stiffness of the ith suspension is K_{t_i}, the roll angle is

$$\phi = -\frac{M_d}{\sum_{\forall i} K_{t_i}} .$$

The torque exerted on the axle is then equal to

$$\phi K_t = -\frac{M_d K_t}{\sum_{\forall i} K_{t_i}} ,$$

where K_t is the roll stiffness of the relevant suspension.

The load shift is thus

$$\Delta F_z = \frac{F_x R_l \tau_f}{t_i} \left(1 - \frac{K_t}{\sum_{\forall i} K_{t_i}}\right) . \tag{23.38}$$

If the vehicle has a standard differential gear, the maximum driving force which can be exerted by the driving axle is equal to twice that which can be exerted by the less loaded wheel, i.e.

$$F_{x_{max}} = \mu_p (F_z - 2\Delta F_z) . \tag{23.39}$$

If, on the contrary, a locking differential is used, within the limits of the assumption that the force coefficient μ_p is independent of the load, the transversal load shift does not affect the maximum driving force.

Example 23.6 *Consider the articulated truck of Appendix E9. Compute*
a) the maximum driving force at a constant speed of 70 km/h on level road;
b) the same as in a), but on a 10% grade;
c) the maximum grade that can be managed at 10 km/h.

All the above computations must be performed taking into account the transversal load shift and repeated for the case of a locking differential. Assume that the maximum longitudinal force coefficient is $\mu_p = 1$.
a) At 70 km/h = 19.44 m/s the load on the driving axle is 106.940 kN while the required driving force is 3.187 kN. Taking into account the gear ratio of the final drive, the driving torque on the axle is 344 Nm, yielding a roll angle of 2.67°. The transversal load shift is $\Delta F_z = 96.4N$, *and the maximum longitudinal force is 106.75 kN. This value compares with the 106.94 kN that could be exerted if a locking differential were used, showing that the latter would improve the ability to exert longitudinal forces only marginally in this case.*
b) At 70 km/h on a 10% grade the load on the driving axle is 116.97 kN and the required driving force is 91.15 kN, corresponding to a driving torque on the

axle of 4453 Nm. A very large roll angle, namely 34.6°, results from the values of the stiffness of the axles, but this is an unrealistic result as for large torques the nonlinear nature of the suspensions would limit rotations. Assuming that the stiffness distribution between the suspensions in the nonlinear range is the same as in the linear range, the transversal load shift is $\Delta F_z = 1249N$*, yielding a maximum longitudinal force of 114.47 kN; if a locking differential were used, a force of 116.97 kN would have been exerted.*
c) By computing the force required for motion and the maximum force that can be exerted by the driving wheels at 10 km/h = 2.78 m/s for different values of the grade, it is possible to find the value of the latter at which the two are equal. This procedure allows one to find the maximum value of the grade as 34.9%, i.e. a grade angle of 19.2°.

Note that the driving torque is very large on that grade and the suspensions operate clearly outside their linear range: The load shift can thus be far smaller than that computed. If no load shift was accounted for, a value of the grade of 37.8%, i.e. a grade angle of 20.7°, would have been found.

23.6 MAXIMUM SPEED

The maximum speed that can be reached on level road with a given transmission ratio can be found by intersecting the curve of the available power at the wheels with that of the required power on level road. The transmission ratio causing this intersection to occur at the maximum available power allows the highest speed that can be attained by a given vehicle-engine combination (curve 1 in Fig. 23.11) to be reached.

The computation of the maximum speed and of the overall gear ratio τ_t necessary to reach it is straightforward. By intersecting the required power curve with the horizontal straight line

$$P = P_{a_{max}} = P_{e_{max}}\eta_t \ ,$$

a fifth degree equation is obtained

$$AV + BV^3 + CV^5 = P_{e_{max}}\eta_t \ , \tag{23.40}$$

whose solution directly yields the maximum value of the speed.

If aerodynamic lift is neglected (actually it is sufficient to neglect the contribution to rolling resistance proportional to the square of the speed due to lift), C vanishes and the equation is cubic. Its solution can be obtained in closed form

$$V_{max} = A^* \left(\sqrt[3]{B^*+1} - \sqrt[3]{B^*-1}\right) \ , \tag{23.41}$$

where

$$A^* = \sqrt[3]{\frac{P_{e_{max}}\eta_t}{2mgK + \rho S C_X}} = \sqrt[3]{\frac{P_{e_{max}}\eta_t}{2B}} \ ,$$

$$B^* = \sqrt{1 + \frac{8m^3g^3f_0^3}{27P_{m_{max}}^2\eta_t^2(2mgK + \rho SC_X)}} = \sqrt{1 + \frac{4A^3}{27P_{m_{max}}^2\eta_t^2 B}} \ .$$

Once the maximum speed has been obtained, the gear ratio allowing the vehicle to reach it is

$$\tau_t = \frac{V_{max}}{R_e(\Omega_e)_{P_{max}}} \ , \tag{23.42}$$

where $(\Omega_e)_{P_{max}}$ is the engine speed at which the peak power is obtained.

If the transmission is of the mechanical type, the overall gear ratio is the product of the gear ratio at the gearbox (in the relevant gear) and that of the final drive

$$\tau_t = \tau_g \tau_f \ .$$

The transmission ratio of the gearbox, which in top gear is usually close to 1, can be stated and consequently the gear ratio τ_f at the final drive can be computed.

Note that this procedure is based on the assumption that the intersection in Fig. 23.11 occurs at the peak of the engine power curve. This can, however, occur in only one given condition, since not only the load, but also the rolling resistance coefficient and even the air density, affect the road load curve. Air density also affects the engine power curve. If the intersection occurs in the descending branch of the curve (situation 2 in Fig. 23.11) the vehicle is said to be "undergeared", i.e., the overall transmission ratio is "too short". Conversely, if the intersection occurs in the ascending branch of the curve (situation 3 in Fig. 23.11), the vehicle is "overgeared" and the overall transmission ratio is "too long".

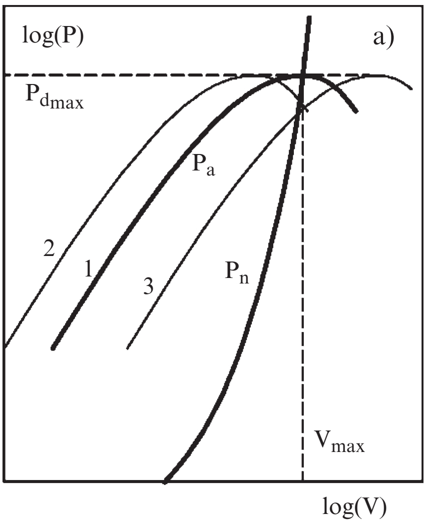

FIGURE 23.11. Maximum speed for a vehicle with internal combustion engine.

The first situation can be purposely obtained to improve the acceleration and grade performance of the vehicle, while the second allows fuel consumption to be reduced. The degree of undergearing λ_u can be defined as

$$\lambda_u = \frac{(\Omega)_{V_{max}}}{(\Omega)_{P_{max}}} . \tag{23.43}$$

It is greater than unity if the vehicle is undergeared and smaller than 1 in case of overgearing.

There are thus two ways of choosing the top gear ratio: One has already been stated, namely a "fast" gear ratio, with a degree of undergearing equal to about unity, i.e., chosen in order to reach the maximum speed. A different approach is to use a longer overgeared ratio, with the goal of reducing fuel consumption (see below). In practical terms, this trade-off is typical of five or six speed transmissions: Either the maximum speed is reached in fifth (sixth) gear or in fourth (fifth) gear, the longest one being an overdrive gear.

Remark 23.3 *This strategy works only in the case of vehicles with high power/ weight ratio: In low powered vehicles, this "economy" gear would be very difficult to use since any increase of the required power, e.g., due to a slight grade, headwind, etc., would compel a shift to a shorter gear. Undergearing may be a necessity in this case.*

23.7 GRADEABILITY AND INITIAL CHOICE OF THE TRANSMISSION RATIOS

The maximum grade that can be managed with a given gear ratio may be obtained by plotting the curves of the required power at various values of the slope and looking for the curve that is tangent to the curve of the available power (Fig. 23.12). The slope so obtained is, however, only a theoretical result, since it can be managed only at a single value of the speed: If the vehicle travels at a higher speed, it slows down because the power is not sufficient, but if its speed is reduced the power is insufficient and the vehicle slows down further: The condition is therefore unstable and the vehicle stops.

To be able to manage a specified slope safely, the curve of the available power must be above that of the required power in a whole range of speeds, starting from a value low enough to assure that starting on that slope is possible. To choose a value of the gear ratio of the bottom gear allowing the vehicle to start on a given grade, it is possible to state a reference speed and to compute the gear ratio in such a way that at that speed the P_a and P_n curves intersect.

As the vehicle is moving at low speed, only the first term of the required power curve needs to be accounted for. As the power developed by the engine can be written in the form

$$P_e = M_e \Omega_e = M_e \frac{V}{R_e \tau_g \tau_f} , \tag{23.44}$$

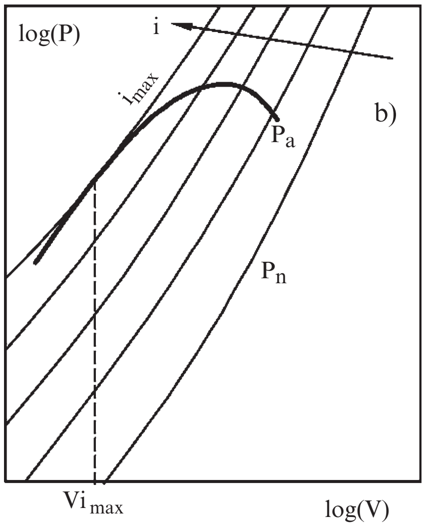

FIGURE 23.12. Maximum slope for a vehicle with internal combustion engine.

where M_e is the engine torque, the equilibrium condition allows the overall gear ratio to be computed as

$$\tau_t = \frac{M_e \eta_t}{R_e m g [f_0 \cos(\alpha) + \sin(\alpha)]} \; . \tag{23.45}$$

The value of the engine torque to be introduced into Eq. (23.45) can be the maximum torque available at the minimum engine speed, possibly multiplied by a number smaller than 1 for safety. The mass of the vehicle must be that at full load, including the maximum trailer mass the vehicle is allowed to tow. For the grade, values of 25% or even 33% for road vehicles can be considered, but it must be kept in mind that in some cases, as in ferry ramps or private garage ramps, very steep grades may be encountered. For off-road vehicles values up to 100% can be considered.

Another consideration in the choice of the gear ratio for the bottom gear is to assure a regular working of the engine at a speed chosen so as to avoid a prolonged use of the clutch in very low speed driving. A reference value may be 6 or 8 km/h. Both criteria must be satisfied.

Once the ratios of the bottom and top gears have been chosen, the intermediate ones can be stated using different criteria. The simplest is to set them in geometric sequence, i.e., stating that the ratios between two subsequent gear ratios are all equal. Operating in this way, the available power curves on the $P(V)$ logarithmic plot are all equispaced.

There may be some advantages in having the curves a bit closer to each other in the high speed range, so that the third gear (in a four speed gearbox) is closer to the fourth. If this is required, it is possible to set in a geometric sequence not the gear ratios τ_i but the ratios between them τ_i/τ_{i+1}. This can give a feeling of sport driving, since the gear ratios are more crowded in the zone of most common use.

The choice of the transmission ratios is much influenced by considerations that are beyond the scope of the present section, being mostly linked to the acceleration performance of the vehicle. This aspect was introduced in Part III and will be dealt with in Section 23.10.

Remark 23.4 *Since the values of the gear ratios have a large influence on the performance of the vehicle and above all on the driver's perception of them, the trade-off dominating their choice is also a matter of subjective impressions and the traditions of various manufacturers. The market sector a manufacturer aims at may have more influence in deciding the matter than technical considerations alone*

Example 23.7 *Choose the overall top gear ratio for the car of Appendix E.2 to reach the maximum speed in the load condition indicated. Choose the bottom gear ratio to start on a 33% grade with a safety margin of 1.1 with respect to the maximum engine torque. Compare the ratio obtained with those listed in the Appendix.*

Equation (23.40), solved numerically, yields a maximum speed of 42.6 m/s = 153.4 km/h.

The overall transmission ratio $\tau_g\tau_f$ *allowing the intersection between the two curves on the* $P(V)$ *plane to occur at the peak power is 0.3044. If a value of 22/21 = 1.048 is accepted for the top gear ratio, the transmission ratio of the final drive is 0.2906, which can be approximated as 18/62 with an error of about 0.08%.*

The actual ratio of the final drive is 0.284. By computing the maximum speed with this value of the transmission ratio, a value of 41.2 m/s = 148.36 km/h is found. The top speed is reached at 5147 rpm, yielding a degree of undergearing $\lambda_u = 0.99$.

The overall transmission ratio of the bottom gear can be found using Eq. (23.45). By dividing the maximum engine torque by a factor of 1.1, a value of 0.1056 is obtained, corresponding to a value for the gearbox ratio of 0.3639. This value is far longer than the actual one (0.2154), since the computation was performed with the vehicle unloaded.

23.8 FUEL CONSUMPTION AT CONSTANT SPEED

The energy needed to travel at constant speed can be immediately computed by multiplying the power required for constant speed driving by the time

$$e = P_n t = \frac{P_n d}{V} , \tag{23.46}$$

where d is the distance travelled. Note that Eq. (23.46) gives the energy required at the wheels: To obtain the energy actually required, it must be divided by the various efficiencies (transmission, engine, etc.).

If the efficiency of the engine η_e and the thermal value H of the fuel are known, the fuel consumption can be computed. Introducing the expression derived from Eq. (23.16) for the total road load into the expression for the power, the fuel consumption per unit distance Q is

$$Q = \frac{A + BV^2 + CV^4}{\eta_t \eta_e H \rho_f} , \tag{23.47}$$

where ρ_f is the density of the fuel, introduced to obtain the consumption in terms of volume of fuel per unit of distance. In S.I. units it is measured in m^3/m, while liters per 100 km is a more practical, although not consistent, unit. Often the reciprocal of Q, expressed in km per liter or miles per gallon, is used.

From Eq. (23.47), if the aerodynamic lift is neglected, the fuel consumption would be a quadratic function of the speed if the efficiency of the engine could be considered as a constant. The plot $Q(V)$ for a car with a mass of 1,000 kg, with $H = 4.4 \times 10^7$ J/kg, $\rho = 730$ kg/m^3 and $\eta_e = 0.25$ is shown in Fig. 23.13.

This is not the case, however, as the efficiency of the engine is strongly influenced by its rotational speed and above all by the power the engine is required to supply.

To compute the consumption Q, the simplest procedure is to obtain the power required at the wheels as a function of the speed and hence to compute the power the engine must supply to travel at constant speed

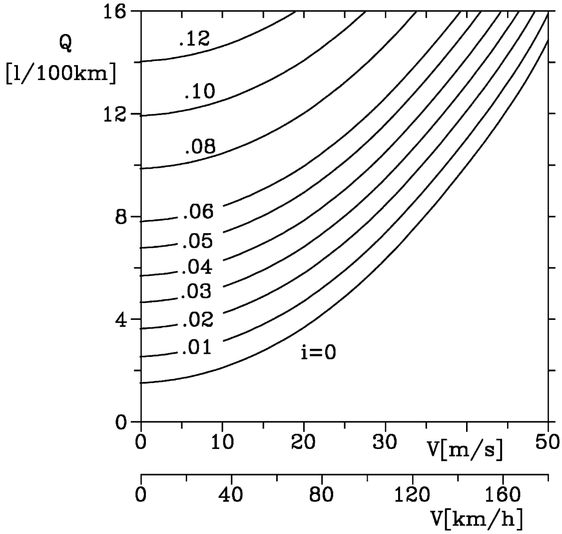

FIGURE 23.13. Fuel consumption at constant speed as a function of the speed, assuming that the efficiency of the engine is constant.

$$P_e = \frac{P_n}{\eta_t} \ .$$

Once the transmission ratio has been stated, the rotational speed of the engine is known and thus the working point on the map of the engine is located. From it the efficiency η_e or, which is the same, the specific fuel consumption

$$q = \frac{H}{\eta_e}$$

is obtained and the fuel consumption can be computed as

$$Q = \frac{qP_n}{\eta_t V \rho_f} \ . \tag{23.48}$$

The curves $Q(V)$ are of the type shown in Fig. 23.14. They usually have a minimum at low speed, obtained in conditions in which the engine works at low power with low efficiency.

Since the conditions in which the engine works depend on the overall transmission ratio, the fuel consumption is also largely influenced by the value of the gear ratio. Usually the longer the ratio, the lower the consumption, as a "long" ratio allows the engine to be used at low speed in conditions which are close to the maximum power, where the specific fuel consumption is low.

As already stated, a transmission ratio longer than that needed to reach the maximum speed can be used. It is possible to choose it in such a way that the

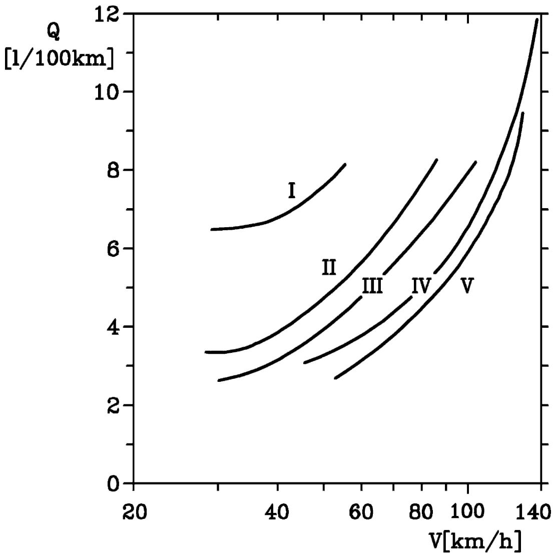

FIGURE 23.14. Fuel consumption with different gear ratios at constant speed on level road. Passenger vehicle with five-speed gearbox.

curve of the required power crosses that of the maximum efficiency at a given cruise speed, e.g. at a speed equal to 3/4 of the top speed. The fuel consumption at that speed is consequently the minimum possible value, with the added advantages of a reduction in noise and engine wear due to the reduced engine speed. Obviously, the performance in terms of maximum speed, acceleration and gradeability is reduced with respect to that available with a shorter gear ratio.

If a CVT is used, it is possible to control it in such a way that the engine works at conditions of maximum efficiency at all speeds, i.e. at all speeds the working point on the map lies on the curve of the maximum efficiency. This is really expedient, however, only if the increase in efficiency so obtained is greater than the loss of efficiency, with respect to that of a simpler transmission, due to the use of the CVT. Moreover, the control law for the transmission ratio of the CVT is a trade-off among different requirements, which also take into account acceleration and gradeability.

Example 23.8 *Plot the fuel consumption curve in top gear for the car of Appendix E.2.*

The map of the engine is shown in Fig. 23.15a. The curves of the power required at the engine, i.e. of the power required at the wheels divided by the transmission efficiency, are plotted for the different gear ratio in the same figure.

The curves identify the working conditions of the engine.

The points at which the curve of the power required in top gear intersects the curves at constant specific fuel consumption are reported in the first two columns of the following table

Ω *[rpm]*	*P [kW]*	*q [g/HPh]*	*V [km/h]*	*Q [l/100km]*	*1/Q [km/l]*
2083	*3.819*	*400*	*60.05*	*4.65*	*21.52*
3157	*9.711*	*300*	*91.02*	*5.85*	*17.10*
4135	*19.610*	*250*	*119.20*	*7.51*	*13.31*
4664	*27.152*	*240*	*134.46*	*8.85*	*11.30*
5320	*37.876*	*250*	*153.36*	*11.28*	*8.87*

The other columns list the specific fuel consumption, the speed and the fuel consumption (in l/100 km) and its reciprocal (in km/l). A value of 730 kg/m^3 *has been used for the density of the fuel. The fuel consumption is also reported in Fig. 23.15b.*

The experimental data do not allow the fuel consumption to be computed directly in the other gears, since the required power curves do not cross the curves of the map. Although there is no difficulty in repeating the tests and plotting the specific fuel consumption in the relevant zone of the map, not having other experimental data available it is still possible to interpolate linearily the values of the efficiency between the lowest curve and the Ω*-axis where the efficiency is zero.*

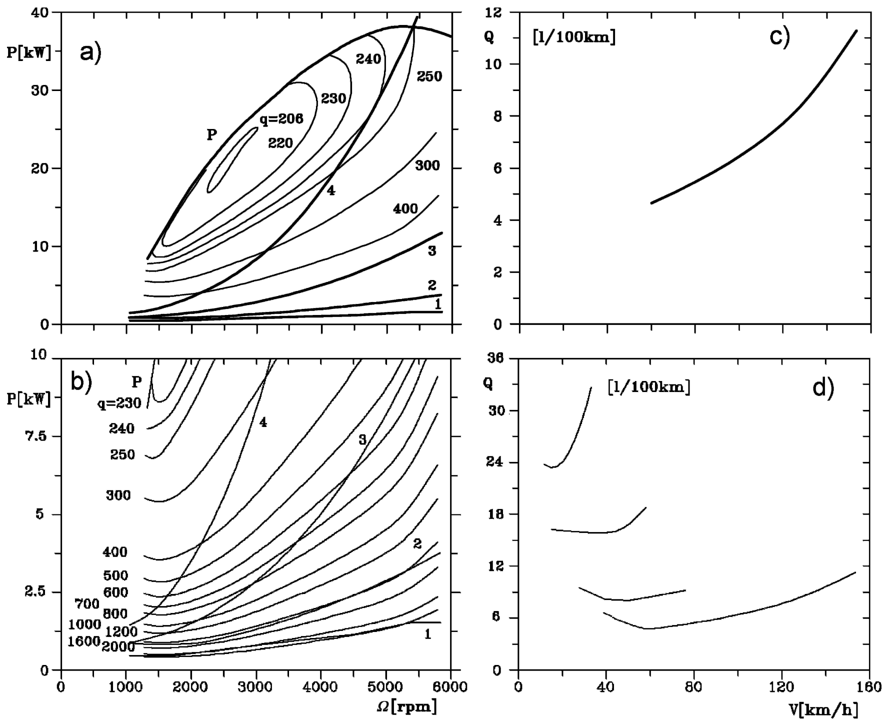

FIGURE 23.15. Fuel consumption for the car of Appendix E.2. (a) Map of the engine with superimposed curves of power required at the engine in various gears (1: bottom gear; 2, 3: intermediate gears; 4: top gear). The specific fuel consumption is reported in g/CVh. (b) Fuel consumption in l/100 km as a function of the speed (top gear). (c) Zone of the engine map for low-power operation, with curves of power required at the engine in various gears. (d) Fuel consumption in l/100 km as a function of the speed in the various gears.

To interpolate the efficiency means to interpolate the reciprocal of the specific fuel consumption[4]*; consequently, the curve midway between the curve at 400 gCV/h and the* Ω*-axis is that related to a doubling of the fuel consumption, 800 gCV/h, and so on. The lower part of the plot of Fig. 23.15a, obtained in this way, is shown in Fig. 23.15c.*

The fuel consumption curves (Fig. 23.15d) were then obtained in the same way seen above for the top gear. The results are only a rough approximation, but at any rate their pattern is realistic.

[4] To interpolate directly on the specific fuel consumption has little meaning, since the latter tends to infinity on the Ω-axis.

23.9 VEHICLE TAKE-OFF FROM REST

Since internal combustion engines cannot operate below a minimum speed Ω_{min}, the vehicle cannot slow down below the speed

$$V_{min} = \Omega_{min} R_e \tau_f \tau_g$$

with the engine connected to the driving wheels. Either a torque converter or a friction clutch must be used, both for starting and stopping the vehicle and to facilitate the shifting of gears.

The starting manoeuvre may be easily simulated in an approximate way by accepting the following assumptions:

1. The manoeuvre is started with the engine running at a speed Ω_{e_0} and the clutch control is released gradually from time $t = 0$ to time $t = t_i$ in such a way that the torque M_c it transmits increases linearly in time from 0 to the maximum value it can handle in slipping conditions M_c^*, and then remains constant until time t_s when no more slipping occurs;
2. the engine torque is maintained constant at the value M_e;
3. if the vehicle starts on a sloping road, it is kept stationary by some external means until the clutch torque is sufficient to produce motion;
4. the longitudinal slip of the wheels is small;
5. the terms in V^2 and V^4 of the road load are neglected owing to the low speed at which the manoeuvre is performed.

The vehicle can be modelled in terms of two moments of inertia, one to model the engine J_e and one to model the vehicle J_v (Fig. 23.16a). The first includes

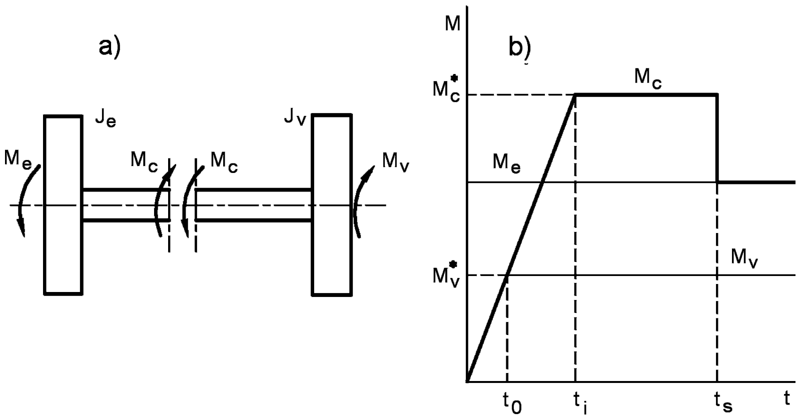

FIGURE 23.16. (a) Model of the vehicle for the starting manoeuvre. (b) Time history of the torques acting on the vehicle.

the moment of inertia of the engine, up to the flywheel, while the moments of inertia of the clutch disks, of the shaft entering the gearbox, of all the rotating parts (reduced to the engine shaft), and the mass of the vehicle as "seen" from the engine are included in the second. For the computational details, see Section 23.10.

Torque M_e, which has been assumed to be constant, acts on the moment of inertia J_e. On J_v a drag torque M_v is acting, whose value is simply

$$M_v^* = mg\left[f_0\cos(\alpha) + \sin(\alpha)\right]R_e\frac{\tau_f\tau_g}{\eta_t} \ , \tag{23.49}$$

when the vehicle is moving. When the vehicle is stationary, at the beginning of the starting manoeuvre, the drag torque is simply equal to the torque the clutch is supplying, if it is smaller than M_v^*,

$$M_v = \min(M_v^*,\ M_c) \ . \tag{23.50}$$

The maximum torque the clutch can transfer to the vehicle M_c^* is usually slightly larger, by 10% to 20%, than the maximum engine torque.

The torques acting on the system are plotted versus time in Fig. 23.16b. The manoeuvre can thus be subdivided into three phases:

1. From $t = 0$ to
$$t = t_0 = t_i\frac{M_v^*}{M_c^*} \ ,$$
in which the vehicle is at a standstill, since the torque transferred by the clutch is not yet sufficient to overcome the drag. The engine speeds up.

2. From $t = t_0$ to $t = t_i$, the clutch slips, the vehicle accelerates and the engine initially continues to speed up, but when M_c^* becomes greater than M_e, it starts to slow down.

3. From $t = t_i$ to $t = t_s$, the clutch continues to slip until time t_s, when the transmission starts to behave as a rigid system and the acceleration continues, as will be seen in Section 23.10.

The equation of motion of the system is simply

$$\begin{cases} \dot{\Omega}_e = \dfrac{M_e - M_c}{J_e} \\[2ex] \dot{\Omega}_v = \dfrac{M_c - M_v}{J_v} \end{cases} \ . \tag{23.51}$$

23.9.1 First phase

In the first phase the moments are

$$\begin{cases} M_e = M_e^* \\ \\ M_c = M_v = M_c^* \dfrac{t}{t_i} \ , \end{cases} \tag{23.52}$$

and then the equations of motion are

$$\begin{cases} \dot{\Omega}_e = \dfrac{M_e^*}{J_e} - \dfrac{M_c^*}{J_e} \dfrac{t}{t_i} \\ \\ \dot{\Omega}_v = 0 \ , \end{cases} \tag{23.53}$$

with the initial conditions

$$\begin{cases} \Omega_e = \Omega_{e_0} \\ \\ \Omega_v = 0 \end{cases} \qquad \text{for } t = 0. \tag{23.54}$$

23.9.2 Second phase

In the second phase the moments are

$$\begin{cases} M_e = M_e^* \\ \\ M_v = M_v^* \\ M_c = M_c^* \dfrac{t}{t_i} \ . \end{cases} \tag{23.55}$$

The equations of motion are then

$$\begin{cases} \dot{\Omega}_e = \dfrac{M_e^*}{J_e} - \dfrac{M_i^*}{J_e} \dfrac{t}{t_i} \\ \\ \dot{\Omega}_v = \dfrac{M_c^*}{J_v} \dfrac{t}{t_i} - \dfrac{M_v^*}{J_v} \ , \end{cases} \tag{23.56}$$

with the initial conditions that can be obtained at the end of the first phase.

23.9.3 Third phase

In the third phase the moments are all constant and their values are M_e^*, M_c^* and M_v^*. The equations of motion are

$$\begin{cases} \dot{\Omega}_e = \dfrac{M_e^* - M_c^*}{J_e} \\ \\ \dot{\Omega}_v = \dfrac{M_c^* - M_v^*}{J_v} \ , \end{cases} \tag{23.57}$$

while the initial conditions can be obtained from those at the end of the second phase.

The manoeuvre ends when the condition $\Omega_e = \Omega_v$ holds, i.e., when the clutch does not slip any more.

By integrating Eq. (23.51) separately for the three phases, the following time histories for the engine and for the vehicle are obtained:

$$\begin{cases} \Omega_e = \Omega_{e_0} + \dfrac{1}{J_e}\left(M_e t - \dfrac{M_c^*}{2t_i}t^2\right) & \text{for } 0 < t < t_i \\ \\ \Omega_e = \Omega_{e_0} + \dfrac{1}{J_e}\left[t\left(M_e - M_c^*\right) - \dfrac{M_c^*}{2}t_i\right] & \text{for } t_i < t < t_s \ , \end{cases} \tag{23.58}$$

$$\begin{cases} V = 0 & \text{for } 0 < t < t_0 \\ \\ V = \dfrac{R_e}{J_v}\left(\dfrac{M_c^*}{2t_i}t^2 - M_v^* t + \dfrac{M_v^{*2} t_i}{2M_c^*}\right) & \text{for } t_0 < t < t_i \\ \\ V = \dfrac{R_e}{J_v}\left[t\left(M_c^* - M_v^*\right) + \dfrac{M_v^{*2} t_i}{2M_c^*}\left(M_v^{*2} - M_c^{*2}\right)\right] & \text{for } t_i < t < t_s \ . \end{cases} \tag{23.59}$$

The starting time t_s can be defined as the time at which the clutch stops slipping: $\Omega_v = \Omega_e$. By equating the two angular velocities it follows that

$$t_s = \frac{2J_e J_v M_c^* \Omega_{e_0} + {M_c^*}^2 t_i (J_v - J_e) - t_i {M_v^*}^2 J_e}{2M_c^*\left[J_e(M_c^* - M_v^*) + J_v(M_c^* - M_e)\right]} \ . \tag{23.60}$$

To make the subsequent acceleration of the vehicle possible, the angular velocity of the engine at time t_s must be in excess of the minimum velocity at which it can work regularly; otherwise, it stops. This can occur if the values of Ω_{e_0} or of M_e are too low or if the clutch engages too quickly (t_i too low).

If $t_s < t_i$ the vehicle completes the starting manoeuvre before the clutch is fully engaged: This poses no problem, but Eq. (23.60) fails to yield a correct value of t_s.

During the manoeuvre, the engine delivers an energy equal to the difference between its kinetic energy at times 0 and t_s added to the energy it produces in the time interval

$$e_e = \int_0^{t_s} M_e \Omega_e dt + \frac{1}{2} J_e \left(\Omega_{e_0}^2 - \Omega_{e_s}^2\right) \ . \tag{23.61}$$

Similarly, the vehicle receives the energy

$$e_v = \int_0^{t_s} M_v \Omega_v dt + \frac{1}{2} J_v \Omega_{v_s}^2 \ . \tag{23.62}$$

The difference

$$e_c = e_e - e_v$$

yields the energy which is dissipated by the clutch during the starting manoeuvre. It is strictly linked to the quantity of friction material removed from the disc of the clutch, i.e. with the wear of that element.

The overall efficiency of the clutch is

$$\eta_c = \frac{e_v}{e_e} \; . \tag{23.63}$$

The space travelled during the take-off manoeuvre may be computed by integrating the speed in time. As the vehicle speed follows a pattern that is roughly quadratic, it may be approximated as

$$\frac{V_s t_s}{3} \; .$$

Example 23.9 *Simulate a starting manoeuvre for the car of Appendix E.2. Assume that the manoeuvre is started at 2000 rpm with the engine supplying a torque equal to 60% of the maximum torque while the clutch can transfer a torque equal to 120% of the maximum torque. Assume that* $t_i = 0.5$ *s, but repeat the computations for* $t_i = 0.2$ *s and* $t_i = 0.8$ *s.*

With simple computations it follows that the moment of inertia simulating the vehicle is $J_v = 0.2113$ *kg m*^{2} *and that* $M_v^* = 1.829$ *Nm,* $M_e = 52.2$ *Nm,* $M_c^* = 104.4$ *Nm and* $\Omega_{e_0} = 209.4$ *rad/s. The results are shown in Fig. 23.17.*

The angular velocity of the flywheel simulating the vehicle at the end of the manoeuvre is 160.4 rad/s, corresponding to a vehicle speed $V = 2.561$ *m/s = 9.22 km/h.*

The engine speed, 1532 rpm, is low but sufficient for accelerating the vehicle.

The results obtained for the three cases are

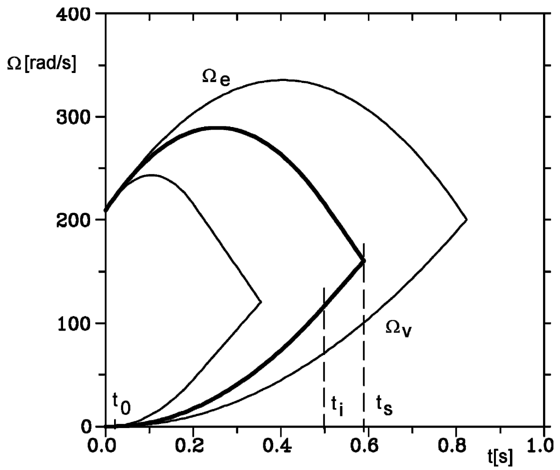

FIGURE 23.17. Angular velocities of the engine and flywheel, simulating the vehicle during a starting manoeuvre. Results for $t_i = 0.5$ s, 0.2 s and 0.8 s.

t_i [s]	t_s [s]	Ω_{e_s} [rpm]	Ω_{v_s} [rad/s]	V_s [m/s]	V_s [km/h]	e_e [kJ]	e_v [kJ]	e_c [kJ]	η	s_s [m]
0.2	0.36	1147	120.1	1.918	6.90	5.08	1.63	3.45	36%	0.257
0.5	0.59	1532	160.4	2.561	9.22	8.52	2.78	5.74	33%	0.494
0.8	0.83	1910	200.0	3.193	11.5	12.8	4.45	8.37	35%	0.863

Remark 23.5 *The efficiency of the clutch is lower than the value 0.5 which is often assumed. Actually, it would be 0.5 if the engine rotates at constant speed with no drag acting on the inertia that has to be accelerated.*

The assumptions made are quite rough, particularly those on the laws $M_e(t)$ and $M_c(t)$. However, the results do allow one to obtain reference values that are independent of the actual behavior of the driver.

In cases where the transmission has a torque converter instead of a clutch, the torque entering the gearbox may be computed using the equations discussed in Part II. It is then possible to integrate the equations of motion numerically and to obtain the time history of the speed.

In the case of a servo-controlled clutch, a procedure similar to the one shown can be followed by introducing the relevant control laws.

23.10 ACCELERATION

If the curve of the required power lies, at a certain speed, below that of the power available at the wheels, the difference $P_a - P_n$ between the two is the power available to accelerate the vehicle.

Remark 23.6 *Note that the engine power P_e is usually measured in steady-state running, in which case using it for acceleration is arbitrary; however, the time scales of the acceleration of the crankshaft and of the thermodynamic cycle are different by orders of magnitude, and thus the error introduced by using the values obtained from the steady-state map is negligible. The driving torque is then almost the same in steady-state conditions and in acceleration, but in the latter case part of the engine torque is used to accelerate the engine itself.*

Consider a vehicle with a mechanical transmission with a number of different gear ratios. During acceleration a number of rotating elements (wheels, transmission, the engine itself) must increase their angular velocity, and it is expedient to write an equation linking the engine power with the kinetic energy $\mathcal{T}$ of the vehicle

$$\eta_t P_e - P_n = \frac{d\mathcal{T}}{dt} \; . \tag{23.64}$$

The transmission efficiency should not be included for the part or the engine power needed to accelerate the engine, but the error created as a result is usually negligible.

Once the transmission ratio has been chosen, Eq. (23.26) gives the relationship between the speed of the vehicle and the rotational speed of the engine. Similar relationships may be used for the other rotating elements that must be accelerated when the vehicle speeds up.

The kinetic energy of the vehicle can then be expressed as

$$\mathcal{T} = \frac{1}{2}mV^2 + \frac{1}{2}\sum_{\forall i} J_i\Omega_i^2 = \frac{1}{2}m_eV^2 \ , \tag{23.65}$$

where the sum extends to all rotating elements which must be accelerated when the vehicle speeds up. The term m_e is the equivalent or apparent mass of the vehicle, i.e., the mass of an object that, when moving at the same speed as the vehicle, has the same total kinetic energy. Usually it is written in the form

$$m_e = m + \frac{J_w}{R_e^2} + \frac{J_t}{R_e^2\tau_f^2} + \frac{J_e}{R_e^2\tau_f^2\tau_g^2} \ , \tag{23.66}$$

where J_w is the total moment of inertia of the wheels, which are assumed to have the same radius and hence to rotate at the same speed, and of all elements rotating at their speed, J_t is the moment of inertia of the propeller shaft and of all elements of the transmission, and J_e is the moment of inertia of the engine, the clutch and all the elements rotating at speed Ω_e.

To account for the fact that the engine is accelerated directly, at least in an approximate way, the last term is sometimes multiplied by η_t. The modifications to Eq. (23.66) to take the presence of different wheels on different axles into account are obvious.

Of the three last terms the first is usually small, the second negligible, while the third may become very important, particularly in low gear. As only the last term depends on the transmission ratio at the gearbox, the equivalent mass can be written in the form

$$m_e = F + \frac{G}{\tau_g^2} \ , \tag{23.67}$$

where

$$F = m + \frac{J_w}{R_e^2} + \frac{J_t}{R_e^2\tau_f^2} \ , \qquad G = \frac{J_e}{R_e^2\tau_f^2}$$

or, possibly

$$G = \frac{J_e\eta_t}{R_e^2\tau_f^2} \ .$$

As the equivalent mass is a constant, once the gear ratio has been chosen, Eq. (23.64) yields

$$\eta_t P_e - P_n = m_e V\frac{dV}{dt} \ . \tag{23.68}$$

Equation (23.68) holds only in the case of constant equivalent mass. If a CVT or a torque converter is used, the overall transmission ratio, and hence the equivalent mass, changes in time and the equation should be modified as

$$\eta_t P_e - P_n = m_e V \frac{dV}{dt} + \frac{1}{2} V^2 \frac{dm_e}{dt} , \tag{23.69}$$

and then

$$\eta_t P_e - P_n = \left(m_e + \frac{1}{2} V \frac{dm_e}{dV} \right) V \frac{dV}{dt} . \tag{23.70}$$

The correction present in Eq. (23.69) is, however, usually very small, since the equivalent mass does not change very quickly.

From Eq. (23.68), the maximum acceleration the vehicle is capable of at various speeds is immediately obtained

$$\left(\frac{dV}{dt} \right)_{max} = \frac{\eta_t P_e - P_n}{m_e V} , \tag{23.71}$$

where the engine power P_e is the maximum power the engine can deliver at the speed Ω_e, corresponding to speed V.

The plot of maximum acceleration versus speed for a passenger vehicle with a four speed gearbox is shown in Fig. 23.18.

The minimum time needed to accelerate from speed V_1 to speed V_2 can be computed by separating the variables in Eq. (23.71)

$$dt = \frac{m_e V dV}{\eta_t P_e - P_n} \tag{23.72}$$

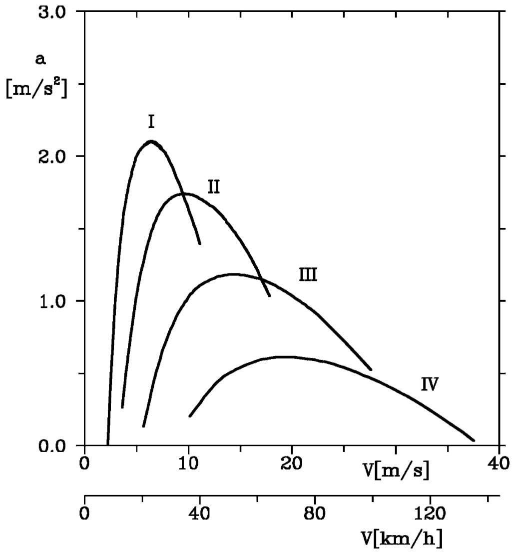

FIGURE 23.18. Maximum acceleration as a function of speed. Vehicle with a 4-speed gearbox.

and integrating

$$T_{V_1 \to V_2} = \int_{V_1}^{V_2} \frac{m_e}{\eta_t P_e - P_n} V dV \ . \tag{23.73}$$

The integral must be performed separately for each velocity range in which the equivalent mass is constant, i.e. the gearbox works with a fixed transmission ratio. Although it is possible to integrate Eq. (23.73) analytically if the maximum power curve is a polynomial, numerical integration is usually performed.

A graphical interpretation of the integration is shown in Fig. 23.19: The area under the curve

$$\frac{V m_e}{\eta_t P_e - P_n} = \frac{1}{a}$$

versus V is the time required for the acceleration.

The speeds at which gear shifting must occur to minimize acceleration time are readily identified on the plot $1/a(V)$. Since the area under the curve is the acceleration time or the time to speed, the area must be minimized and gears must be shifted at the intersection of the various curves. If they do not intersect, the shorter gear must be used up to the maximum engine speed.

A criterion for choosing the gear ratios can also be evolved. The lower envelope of the curves (dashed line in the figure) does not depend on the transmission ratios and may be thought of as the curve that can be followed using a CVT

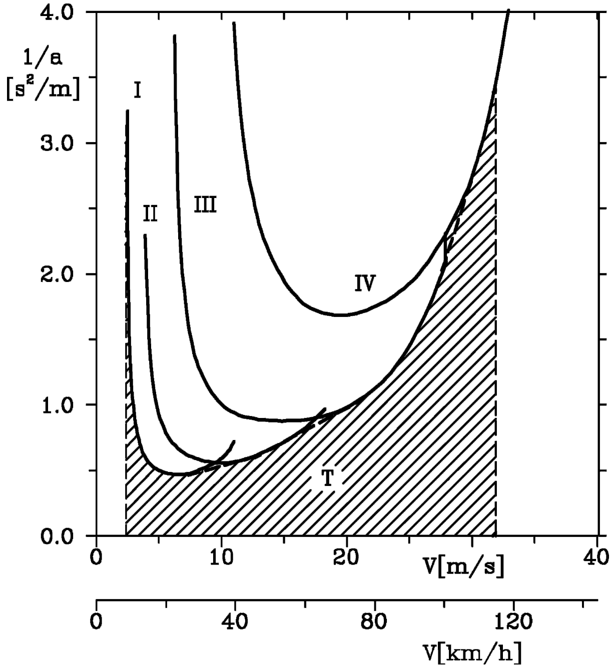

FIGURE 23.19. Function $1/a(V)$ showing the optimum speeds for gear shifting. The hatched area is the time to speed.

having the same efficiency as the gearbox and optimized to obtain the maximum acceleration. The area under the dashed curve is the minimum time to speed under ideal conditions.

The areas between the dashed and the continuous lines account for the time which must be added due to the presence of a finite number of speeds: The transmission ratios can be chosen in such a way as to minimize this area.

By increasing the number of speeds the acceleration time is reduced, since the actual curve gets closer to the ideal dashed line. However, at each gear shifting there is a time in which the clutch is disengaged and, consequently, the vehicle does not accelerate: Increasing the number of speeds leads to an increase in the number of gear shifts and thus of the time wasted without acceleration. This restricts the use of a high number of gear ratios.

The speed-time curve at maximum power can be easily obtained by integrating Eq. (23.73). An example is shown in Fig. 23.20. The actual curve, obtained by adding the time needed for gear shifting, is also reported. The speed is assumed to be constant during gear shift.

By further integration it is possible to obtain the distance needed to accelerate to any value of the speed

$$s_{V_1 \to V_2} = \int_{t_1}^{t_2} V dt \,. \tag{23.74}$$

It is, however, possible to obtain the acceleration space directly, by writing the acceleration as

$$a = \frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = V\frac{dV}{dx} \,. \tag{23.75}$$

By separating the variables and integrating it follows that

$$s_{V_1 \to V_2} = \int_{V_1}^{V_2} \frac{V}{a} dV = \int_{V_1}^{V_2} \frac{m_e}{\eta_t P_e - P_n} V^2 dV \,. \tag{23.76}$$

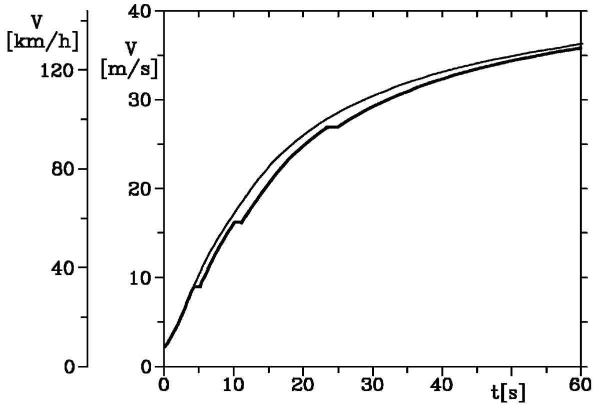

FIGURE 23.20. Speed versus time curve for the vehicle studied in the previous figures.

Instead of modelling the vehicle as an equivalent mass accelerated along the road, it is sometimes modelled as an equivalent moment of inertia attached to the flywheel of the engine, as seen in the previous section. Its value is

$$J_e = F'\tau_g^2 + G' \ , \tag{23.77}$$

where

$$F' = FR_e^2\tau_f^2 \ , \qquad G' = J_e \ .$$

The acceleration curves can thus be obtained in terms of acceleration of the engine instead of acceleration of the vehicle.

It is possible to choose the gear ratio of the bottom gear to optimize the acceleration at low speed. When the transmission ratio is shortened, the torque available at the wheels increases; however, the equivalent mass also increases and it is not convenient, from the viewpoint of acceleration, to use transmission ratios that are too short.

Assuming that the engine torque M_e is constant and discarding the terms in V^3 and V^5 in the required power since at low speed their contribution is negligible, Eq. (23.71), written for the case of level road, yields

$$\left(\frac{dV}{dt}\right)_{max} = \frac{\eta_t M_e \Omega_e - AV}{m_e V} = \frac{\eta_t M_e - AR_e\tau_f\tau_g}{R_e\tau_f\tau_g\left[F + \frac{G}{\tau_g^2}\right]} \ . \tag{23.78}$$

By differentiating Eq. (23.78) with respect to τ_g and equating the derivative to zero, a quadratic equation in τ_g, yielding the value of the gear ratio which maximizes the acceleration, is obtained. If the road load is neglected, which is reasonable on level road when dealing with strong accelerations, the value of the optimum gear ratio is

$$(\tau_g)_{opt} = \sqrt{\frac{G}{F}} \approx \sqrt{\frac{J_e}{mR_e^2}} \ . \tag{23.79}$$

The last value has been obtained by neglecting the terms representing the inertia of the wheels and transmission in the expression of the equivalent mass. Note that the value so obtained leads to equal contributions for the mass of the vehicle and the inertia of the engine in the equivalent mass.

The value of the transmission ratio obtained with this criterion is, however, too short: It usually yields driving torques exceeding the maximum torque that may be transmitted by the driving wheels without slipping.

Example 23.10 *Plot the acceleration curve for the vehicle in Appendix E.2 and compute the time needed to reach 100 km/h. Compute also the time needed to travel for 1 km from standstill.*

Assume that the time needed for gear shifting is 0.5 s and that the takeoff manoeuvre follows the results obtained in the previous example.

Constants F and G are $F = 855.2$ kg and $G = 15.96$ kg, leading to the following values of the equivalent mass and moment of inertia:

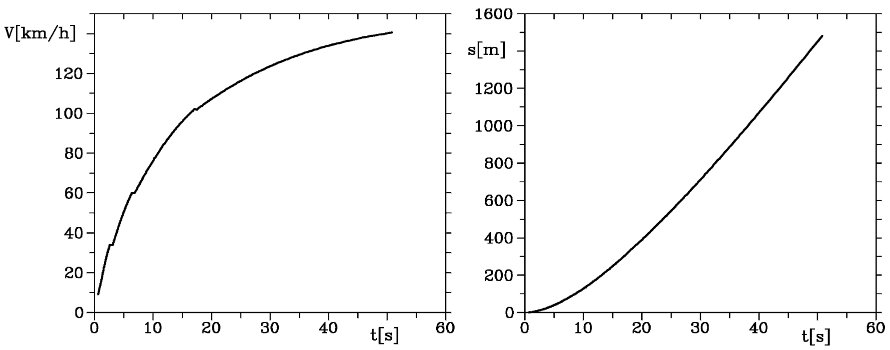

FIGURE 23.21. Speed and distance travelled as functions of time during a full power acceleration. The initial take-off manoeuvre has also been considered.

$m_e = 1199\ kg = 1.45\ m$	$J_e = 0.296\ kg\ m^2$	*in first gear,*
$m_e = 975\ kg = 1.18\ m$	$J_e = 0.692\ kg\ m^2$	*in second gear ,*
$m_e = 897\ kg = 1.08\ m$	$J_e = 1.823\ kg\ m^2$	*in third gear,*
$m_e = 870\ kg = 1.05\ m$	$J_e = 5.085\ kg\ m^2$	*in fourth gear.*

The results of the numerical integration yielding the speed and the distance travelled as functions of time during an acceleration are shown in Fig. 23.21. They were computed based on the results obtained in the previous example with a time $t_i = 0.5$ *s, namely a time of 0.59 s, a speed of 9.22 km/h and a distance of 0.494 m.*

The engine power was introduced in the computation through the best-fit third degree polynomial found in Example 4.5, and the speeds at which gear shifting occurs were determined as the minimum value between that corresponding to the maximum speed of the engine (6000 rpm) and the speed at which the acceleration obtainable in the following gear equals that obtainable with the gear under consideration. They are

5784 rpm (V=34.3 km/h)	*for the first gear,*
6000 rpm (V=60.6 km/h)	*for the second gear,*
6000 rpm (V=102.3 km/h)	*for the third gear.*

The time to reach a speed of 100 km/h is 16.3 s and that needed to reach the 1 km mark is 38.1 s.

23.11 FUEL CONSUMPTION IN ACTUAL DRIVING CONDITIONS

Fuel consumption at variable speed gives the customer a more reliable estimate of the actual fuel consumption, but its determination is much more difficult. For this reason, several simplifications are usually accepted.

The first describes the actual use of the vehicle through a cycle, i.e. a time history of the speed of the vehicle, that takes into account neither the behavior of an actual driver nor the actual road and traffic conditions. This time history is used for all vehicles.

This simplification is implicitly accepted by European standards, which establish an urban and a suburban cycle to evaluate fuel consumption. These cycles were described in Part I. Fuel consumption measured in these cycles is the only value that may be supplied to the customer, and must be expressly stated on the data sheet of the vehicle.

Different cycles, obtained directly by manufacturers on similar vehicles in actual operating conditions may give more realistic values, but they are useful only to designers; this subject has also been covered in Part I.

A second simplification is that of computing the fuel consumption in a cycle as the sum of the partial consumption obtained in the various parts that approximate the chosen cycle assuming quasi-steady-state operation.

It is clear that the error so introduced decreases with decreased duration of the parts of the cycle that are assumed to be steady state; the error, however, is also due to the fact that fuel consumption in variable conditions is different from the fuel consumption obtained by approximating them with a sequence of steady state operations. This is due to the following reasons:

- In non-stationary operation, the thermal conditions of the engine are variable, so that the thermal energy losses are different from those occurring when the temperature has reached its steady-state value;
- in non-stationary conditions, part of the fuel burns with a lower efficiency due to condensation of the vapor on the intake manifold in indirect injection engines, or to a different evaporation rate of the fuel droplets in direct injection engines.

The difference is never very large, particularly if the instant power required by the cycle is much lower than the maximum engine power; this occurs often in statistically relevant cycles, since traffic conditions are always such as to prevent the engine from obtaining maximum performance. The comparison between measured and computed data always shows differences between 2 and 5%, with the computed consumption always lower than the actual one, due to the mentioned causes.

Following the above mentioned simplifications, the reference cycle is subdivided into a series of short time intervals; experience shows that a duration of about 1 s is acceptable for the intervals.

If V_i is the speed at instant t_i of the cycle, the fuel consumption in the time interval from t_i to t_{i+1} will be

$$\Delta e_i = \frac{1}{\eta_t}\left(A + BV_{mi} + CV_{mi}^4 + m_e V_{mi}\frac{V_{i+1} - V_i}{t_{i+1} - t_i})\right)(t_{i+1} - t_i)\,\frac{1}{\eta_e H \rho_f}\ , \tag{23.80}$$

where:

$$V_{mi} = \frac{V_{i+1} + V_i}{2} . \tag{23.81}$$

This equation holds if the value of the first term in brackets is positive or vanishes, that is if the vehicle accelerates or decelerates with a rate low enough to compensate for the road load with inertia forces. If its value is negative, the contribution must be set to zero, since controllers on all modern engines cut off the fuel supply as the vehicle slows.

The contribution to fuel consumption when $V_i = 0$ is

$$\Delta e_i = Q_i(t_{i+1} - t_i) , \tag{23.82}$$

where Q_i is the fuel consumption at idle in liters/s.

The total fuel consumption in the cycle is

$$Q = \sum_{i=1}^{n} \Delta e_i . \tag{23.83}$$

An idea of the relative importance of the various forms of resistance to motion on fuel consumption in actual conditions is given by Fig. 23.22, where two different driving conditions are considered. Although the figure was obtained for a particular car (a medium sized saloon car), the results are typical. While in motorway driving aerodynamic drag is important, most of the energy in urban driving is expended to accelerate the vehicle.

The average was computed by using statistical data on average European conditions. From the average it is clear that reducing the mass of the vehicle, which affects both rolling resistance and the power needed to accelerate, is more important than reducing aerodynamic drag, and that the possibility of recovering

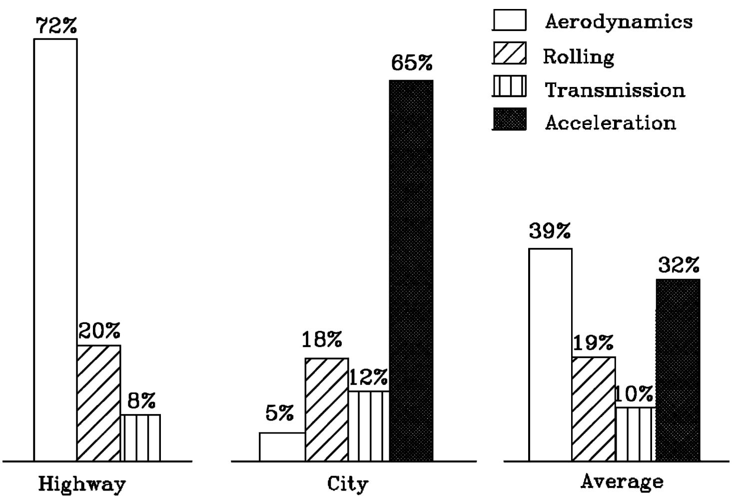

FIGURE 23.22. Energy required for motion in two different driving conditions.

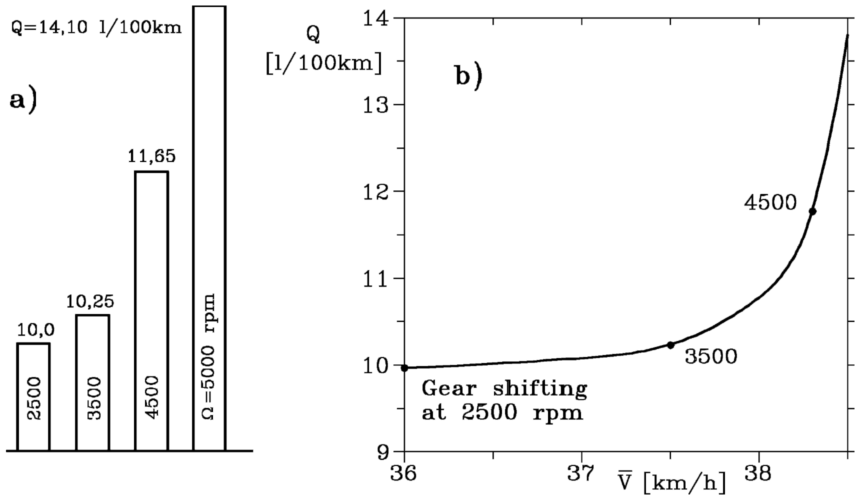

FIGURE 23.23. Effect of the engine speed at which gear shifting occurs (a) and of the average speed (b) on the fuel consumption in city driving.

braking energy, which allows a part of the energy used to accelerate the vehicle to be recovered, allows important energy savings to be obtained.

Numerical simulations can be used to study the effects of driving style on fuel consumption. In city driving, it is expedient to use the engine at the lowest speed consistent with its regular operation and particularly to maintain it near the speed of maximum torque and maximum efficiency. Prolonged use of low gears increases consumption without increasing the average speed appreciably (Fig. 23.23).