27

CONTROL OF THE CHASSIS AND 'BY WIRE' SYSTEMS

27.1 MOTOR VEHICLE CONTROL

As already stated, a road vehicle on pneumatic tires cannot maintain a given trajectory under the effect of external perturbations unless managed by some control device, which is usually a human driver. Its stability solely involves such state variables as the sideslip angle β and the yaw velocity r.

In the case of two-wheeled vehicles the capsize motion is intrinsically unstable forcing the driver not only to control the trajectory but stabilize the vehicle.

A possible scheme of the vehicle-driver system is shown in Fig. 27.1. The driver is assumed to be able to detect the yaw angle ψ, the angular and linear accelerations $\dot{\beta}$, $\dot{r}$, dV/dt, V^2/R and to be able to assess his position on the road (X and Y). Moreover, the driver receives other information from the vehicle, such as forces, moments, noise, vibrations, etc. that allow him to assess, largely unconsciously, the conditions of the vehicle and the road-wheel interactions.

27.1.1 Conventional vehicles

In all classical vehicles of the second half of the twentieth century up to the 1990s, the driver had to perform all control and monitoring tasks. The only assistance came from devices like power steering or power brakes that amplified the force the driver exerted on the controls. In this situation, the human controller is fully inserted in the control loop or, as usually said, the systems include a *human in the loop*.

G. Genta, L. Morello, *The Automotive Chassis, Volume 2: System Design*, Mechanical Engineering Series,

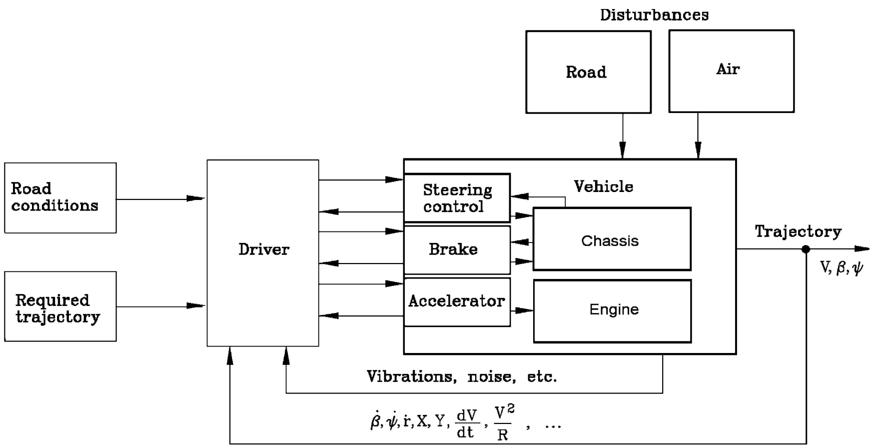

FIGURE 27.1. Simplified scheme of the vehicle-driver system.

Actually the driver must control high level functions (choice of the trajectory, decisions about speed and driving style, about manoeuvres like overtaking, etc.) and intermediate level functions (reacting to perturbations coming from the air and the road, following the chosen trajectory, etc.). Only stability at the lowest level, involving the sideslip angle and the yaw velocity, is provided by the dynamic behavior of the vehicle. As already stated, in motorbikes the driver must also act as a stabilizer against capsizing.

In particular:

- Direction control is implemented by applying a torque to the steering wheel that is then transmitted through a mechanical system (steering box, steering arms, various linkages) to the steering wheels, which are always the front wheels. The torque exerted by the driver may be increased by an hydro-pneumatic or electromechanical system (power steering) that nonetheless never replaces the driver by exerting the whole moment. The required sensitivity is provided by the torque the steering system exerts on the driver through the aligning torque and the contact forces at the wheel-road interface. These, in turn, depend upon the geometry of the steering system (caster angle, toe in, offsets, etc.).

- The control of the power supplied by the engine is managed through the accelerator pedal, operating directly through a mechanical leverage. Sensitivity is supplied by the elastic reaction of a spring that reacts to the motion of the pedal. The driver must control the power accurately enough so that the maximum force the wheel can exert on the ground is not exceeded.

- Engine control is accompanied by control of the gearbox and the clutch, which operate through the clutch pedal and the gear lever. These controls are often automatic.

- Braking control is performed by applying a force on the brake pedal that is then transmitted through a system (usually hydraulic, but pneumatic in industrial vehicles) to the brakes located in all wheels. Here the force exerted by the driver can also be augmented by a hydro-pneumatic device (power braking). In all cases, sensitivity is granted by the fact that the force exerted by the driver is proportional (or at least depends in an almost linear way) to the braking torque and then to the braking force. The driver must control the braking force so that the wheels do not lock.

These basic controls are accompanied by many secondary controls, such as those of the lighting systems, window cleaning and defrosting, parking brake etc. Although not directly used to control the motion of the vehicle, these are extremely important for driving safety. The basic controls are standardized on all vehicles, with some difference in special vehicles, and are subjected to detailed standards. In the case of particular arrangements, to be used by persons with disabilities of various kinds that do not allow them to operate conventional controls directly, a non-conventional *user interface* is provided, designed as needed for each particular installation. The transmission of commands, however, remains the same: for instance, the accelerator control may be brought to the steering wheel with a ring coaxial to the wheel that can be moved axially. This, in turn, operates the conventional accelerator control through levers.

The situation with two-wheeled vehicles is essentially the same, the only difference being that the driver can change the inertial and geometrical characteristics of the vehicle, using these changes as control inputs: for instance, he can move the center of mass sideways or change the aerodynamic characteristics. The controls are obviously different with the front and rear brakes often operating independently.

27.1.2 Automatic and intelligent vehicles

The possibility of introducing automatic control devices in road vehicles has led in recent years to many studies aimed at designing vehicles able to perform automatically a number of those control functions that at present are entrusted to the driver, with the long term goal of building road vehicles able to perform all their functions automatically, essentially transforming the driver into a passenger. This goal is still distant and, as with predictions of so-called strong artificial intelligence, there are doubts as to its achievability, at least with today's technologies and those likely to be developed in the foreseeable future.

However, while the goal of building a fully automatic road vehicle may be a long way off, many partial applications are already available or are about to be realized.

One source of inspiration is what has been done in the field of aeronautics. Since World War II, devices able to keep an aircraft at a given attitude and on a prescribed course, allowing the pilot to leave the controls for a more or less prolonged time, have entered common use. Such devices do not need to sense external conditions and adapt to them; they are simple regulators, that only need to maintain the predetermined motion conditions. Devices of this kind have only a limited use in road vehicles (for instance, *cruise control* devices) because vehicles must continuously adapt their motion to the road and traffic conditions.

Military aircraft are increasingly built on configurations that reduce intrinsic stability or are even unstable, with the goal of improving manoeuvrability; the task of stabilizing the aircraft is given to suitable control devices. Moreover, the senses of the pilot have been enhanced by supplying additional information through the control devices, such as devices that shake the control stick when stall conditions approach. Artificial stability may prove interesting in the vehicular field as well, not so much for improving manoeuvrability as for allowing the use of configurations that are advantageous but reduce stability.

Devices providing an artificial sensibility, often referred to as *haptic*, are those that provide a reaction force through *by wire* controls that is similar to the reaction that conventional mechanical controls would supply. They may also add further information, like the devices that cause the accelerator or the brake pedal to shake when getting close to slip conditions in traction or braking. Such devices are intrinsically necessary when controls are made automatic. They are at present under study and in same cases already on the market.

Nowadays in the aeronautical field commands are no longer transmitted by mechanical (rods, cables, etc.) or hydraulic devices but by electric systems (*fly by wire*), the only exception being small and low cost aircraft. There are two main advantages: first, freedom in architectural and layout is greatly increased (it is much easier to route electric cables than mechanical controls), resulting in a mass reduction. Second, it is much easier to integrate control systems, which are mostly electronic, in *by wire* than in conventional architectures. In the most modern aircraft, the pilot interacts with a computer that in turn actuates the control surfaces through *by wire* devices.

A similar evolution is also underway in the automotive industry. Here the term *steer by wire* is used for the steer control, *brake by wire* for the braking function and *drive by wire* for the accelerator control. The generic term for these systems is *X by wire*, where the generic *X* stands for the various controls. The advantages are similar to those in the aeronautical field, with the added bonus of allowing the use of different user interfaces that can, for example, be designed specifically for disabled persons and even adapted for individual cases.

However, the transfer from the aeronautic *fly by wire* to the automotive *X by wire* is not simple. A first difference between the two fields is linked to cost, or better, to the ratio reliability/cost. The total cost of an aircraft is greater than the cost of a motor vehicle by orders of magnitude, allowing the use of control systems and components much more expensive than those that may be used in vehicles. Something similar can be said for the low cost segment of the aeronautical market: *fly by wire* systems are still not used in light and ultralight aviation.

The scale of production may mitigate this problem: development costs are subdivided, in the automotive market, into a much greater (even by orders of magnitude) number of machines than in the aeronautical market. Reliability is strictly linked to costs: when dealing with functions that are vital for safety, like steering or braking, the need for extremely high reliability leads to high costs, because the required safety is obtained through redundancy of sensors, actuators, control units and communication lines as well as high quality components. Electronic and computer based devices have been available for motor vehicles for several years in non-vital functions and, more often, in gadgets performing tasks that are of little practical use.

However, it is not just a matter of cost: motor vehicles are designed for general use; their mission analysis is less determinate than that of aircraft, and they must be able to work in conditions far from those for which they have been designed, with a less stringent respect for maintenance schedules. This makes technology transfer from aeronautics to automotive industry even more difficult, particularly where complex and even critical technologies are concerned.

One field where technology transfer may be facilitated is that of racing cars, and in particular Formula 1 racers, because these vehicles must be optimized with a limited number of parameters in mind, accrue higher costs and are used in controlled conditions. However their design specifications are strictly linked to racing regulations, which at present (2008) do not allow the use of automatic and control devices.

At present, the fields in which control devices are more common or are at least being actively studied are:

- Engine control systems. All modern automotive internal combustion engines are provided with one or more electronic control units (ECU) that control its main functions. The motor control may be conventional or *by wire*, but in the latter case there is no problem in supplying the driver with adequate sensory inputs. Because these systems are studied in conjunction with the engine and not with the chassis, they will not be dealt with here.

- Longitudinal slip control in traction, (ASR, Anti Spin Regulator[1]). These are systems that detect the beginning of driving wheel skid and reduce the power supplied by the engine. Theoretically, they should measure the longitudinal slip of the tires, but in practice they measure the acceleration of the driving wheels.

- Longitudinal slip control in braking, (ABS, Antilock Braking System). They are systems that detect the beginning of wheel skid and reduce the braking torques. They, too, should measure the longitudinal slip of the tires, but actually measure the deceleration of the wheels.

[1]The acronyms here mentioned are often trade marks of a particular manufacturer, even if many of them have entered technical jargon to designate a variety of similar devices.

- Vehicle dynamics control systems, (VDC, Vehicle Dynamic Control, ESP, Enhanced Stability Program, DSC, Dynamics Stability Control). The goal of these systems is to improve the dynamic response of the vehicle. They often act by differentially braking (and sometimes differentially driving) the wheels of the same axle to produce a yaw torque. The driver controls the trajectory normally through the steering wheel, while the control device tries to counteract the difference between the behavior required and that actually obtained by applying yaw torques.

- Suspension control systems. Many different types of controlled, semi-active and active suspensions have been and are being developed. These can simply adapt the suspension characteristics to the type and conditions of the road or, in the most advanced cases, completely substitute an active system for the conventional suspension.

- Electric Power Steering (EPS). Strictly speaking, EPS should not be considered a control system any more than conventional power steering, but electric actuation allows steering control functions to be added. EPS, then, may be considered as a first step towards *steer by wire*.

- Electric braking. A wide span of functions are available through electric braking, from simple electric power braking with an electric actuator on the master cylinder of a conventional hydraulic system (which should not be listed here) to a true *brake by wire* system, with the electric actuators at the wheels.

- Servo controlled gearbox and clutch. These systems provide automatic gearbox functions by controlling a more or less conventional manual transmission using suitable actuators, with all the advantages of classic automatic transmissions but with a much more efficient mechanical transmission without a torque converter.

- Finally, the parking brake must be counted among the secondary controls that may be made automatic. The advantages are that it is possible to ensure that the brake is applied every time the driver leaves the vehicle, without the possibility of forgetting it, while reducing the effort needed to engage and disengage the brake. Electric parking brake yields a larger freedom to the designer of the interior of the vehicle.

The components of some of these systems and the main control strategies have already been described in Part I.

All the mentioned systems allow the tasks of the driver to be simplified and safety increased, assuming that they meet reliability standards. The driver is still in the control loop, but his work is made simpler by avoiding low level control tasks so that he can concentrate on high level decisions.

Strong research activity is now devoted to going beyond this approach by making it possible to perform higher level functions automatically, as in systems able to recognize and follow the road automatically using video cameras that identify the outer edges of the road and the lines delimiting the lanes. Other examples include systems able to regulate the speed, keeping a constant distance from the preceding vehicle, and anti-collision systems based on obstacle recognition.

There is no doubt that systems of this kind are feasible once the critical technologies have been developed at acceptable costs and with the required reliability. However, systems that can do completely without the presence of a human in the loop are beyond present and predictable technology.

27.2 MODELS FOR THE VEHICLE-DRIVER SYSTEM

Before embarking on the study of automatic systems aimed at controlling the vehicle, it is advisable to study the vehicle driver system in conventional vehicles, where the human driver is fully integrated in the control loop. Such a study has two primary goals:

- To build a mathematical model of a human driver that can be integrated into the mathematical model of the vehicle in simulations. It is not necessarily true that a system made of two subsystems that are both stable is itself stable. The study of stability, therefore, should take into account the behavior of the driver even if the vehicle is intrinsically stable. Moreover, in the case of motorcycles, the intrinsic instability of the system makes it necessary to introduce a driver model, at least as a roll stabilizer, to allow the dynamic behavior of the system to be numerically simulated.
- To supply guidelines for the design of automatic control systems. Automatic controllers are often inspired by the behavior of the human controller, if for no other reasons than that it is the only available model. Moreover, the performance of human controllers is better than that expected from automatic devices. Automatic control systems must interact with a human controller and supply the latter with information and sensory inputs that are not much different from those he is used to.

It is clear that stability of the vehicle-driver system is mandatory, but it is not sufficient to assess the required handling and comfort characteristics of the vehicle. The greater the stability with free and locked controls of the vehicle itself, the fewer the corrections the driver has to introduce to obtain the required trajectory. A vehicle that is stable in β and r requires from the driver only those inputs needed to follow the required trajectory, but not those needed to stabilize the motion on it.

On the other hand, a vehicle that is too stable may lack the manoeuvrability needed to cope with emergency conditions or simply to allow sport driving.

The amount of stability must be assessed in each case, taking into account the type of vehicle, the market target, the traditions and image of the manufacturer.

Usually stability, handling and comfort characteristics of a vehicle are assessed on the basis of prolonged road testing performed by skilled test drivers. This approach has the drawback of being in a way subjective, and above all of focusing on the global characteristics of the vehicle, without giving detailed suggestions on causal relationships between the construction parameters of the vehicle and its behavior. It also demands that long and costly road tests be performed and, above all, forces evaluation of the performance of the vehicle to be postponed to a stage in which prototypes are available.

The availability of mathematical models for the driver-vehicle interaction has a number of advantages that are too obvious for a detailed discussion. The difficulty of translating concepts like comfort and user friendliness into mathematical functions is a serious obstacle in this study, making experimental and numerical approaches likely to remain complementary.

A model able to simulate the behavior of the driver must be built for the study of man-machine interactions. The difficulties encountered in such a task are so large that many different approaches have been attempted. Up to now there is no standard driver model that can be applied.

The first systematic studies were performed in the aeronautical field[2], but beginning in the 1970s, a large number of models specialized for the vehicular field have been published. A quick bibliographic scan identifies more than sixty models published in less than 25 years. These span from simple constant-parameter single-input single-output linear models to multi-variable, nonlinear, adaptive models or models based on fuzzy logic and/or neural networks.

As always, the complexity of the model must be chosen in a way that is consistent with the aims of the study and the availability of significant input data.

27.2.1 Simple linearized driver model for handling

As previously stated, the driver may be thought as a controller receiving a number of inputs from the vehicle and the environment and outputting a few control signals to the vehicle. Under manual control, the driver performs the tasks of the sensors, the controller, the actuators and the source of control power, even if his control actions may be assisted by devices such as power steering or braking.

In building a simple driver model, a small number of the inputs the driver receives is selected and simple control algorithms are chosen to link them with the outputs. The latter are usually only the steering angle δ and the position of the accelerator/brake pedals. Only the former is considered if the driver model is used in connection with a constant speed handling model.

[2]See, for instance, D.T. McRuer, E.S.Crendel, *Dynamic response of human operators*, WADC T.R. 56-524, Oct. 1957.

The simplest driver model is a proportional linear tracking system reacting to the error $\psi - \psi_0$, where ψ_0 is the desired yaw angle, with a control action in terms of steering angle δ proportional to the error. Because the controller has a delay τ, this means

$$\delta(t+\tau) = -K_g[\psi(t) - \psi_0(t)] \,, \tag{27.1}$$

where K_g is the proportional gain of the controller. By developing function $\delta(t+\tau)$ in Taylor series about time t and truncating the series after the linear term, it follows that

$$\tau\dot{\delta}(t) + \delta(t) = -K_g[\psi(t) - \psi_0(t)] \,, \tag{27.2}$$

Equation (27.2) is only an approximation, yielding results that are increasingly inadequate with increasing delay τ. In the present case, it is possible to assume that the values of the delay range between 0,08 s for a professional driver to more than 0,25 s for an occasional driver. Consequently, Eq. (27.2) may lead to non-negligible errors.

The transmission ratio of the steering system must be introduced into the gain K_g, because δ is the steering angle at the wheels and not at the steering wheels.

The simplest handling model that may be coupled to the driver model is a rigid body model that, assuming that the vehicle is neutral steer, reduces to a first order system (Eq.(25.108a)):[3]

$$J_z\dot{r} = N_r r + N_\delta \delta + M_{z_e} . \tag{27.3}$$

Remembering that

$$r = \dot{\psi} \,,$$

the dynamic equation of the controlled system in the state space is

$$\left\{ \begin{array}{c} \dot{r} \\ \dot{\delta} \\ \dot{\psi} \end{array} \right\} = \mathbf{A} \left\{ \begin{array}{c} r \\ \delta \\ \psi \end{array} \right\} + \mathbf{B}_c \psi_0 + \mathbf{B}_d M_{z_e}, \tag{27.4}$$

where the dynamic matrix and the control and disturbances input gain matrices are

$$\mathbf{A} = \begin{bmatrix} \dfrac{N_r}{J_z} & \dfrac{N_\delta}{J_z} & 0 \\ 0 & -\dfrac{1}{\tau} & -\dfrac{K_g}{\tau} \\ 1 & 0 & 0 \end{bmatrix} , \mathbf{B}_c = \begin{bmatrix} 0 \\ \dfrac{K_g}{\tau} \\ 0 \end{bmatrix} , \mathbf{B}_d = \begin{bmatrix} \dfrac{1}{J_z} \\ 0 \\ 0 \end{bmatrix} .$$

If the delay vanishes, the vehicle-driver system reduces to a second order system

$$J_z\ddot{\psi} - N_r\dot{\psi} + N_\delta K_g \psi = N_\delta K_g \psi_0(t) + M_{z_e} . \tag{27.5}$$

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[3] P.G. Perotto, *Sistemi di automazione, Vol.I, Servosistemi*, UTET, Torino, 1970.

Because N_r is always negative, while product $N_\delta K_g$ is always positive, the system is always stable, both statically and dynamically. Its behavior is not oscillatory if

$$|N_r| > 2\sqrt{J_z N_\delta K_g}\ , \tag{27.6}$$

i.e.

$$K_g < \frac{N_r^2}{4J_z N_\delta}\ . \tag{27.7}$$

If the derivatives of stability are computed considering the cornering forces of the tires alone, such a condition becomes

$$K_g < \frac{al^2 C_1}{4J_z}\frac{1}{V^2}\ . \tag{27.8}$$

If the delay τ of the driver is accounted for, the stability of the system can be studied by searching for the eigenvalues of the dynamic matrix. The characteristic equation is

$$s^3 + \left(\frac{1}{\tau} - \frac{N_r}{J_z}\right)s^2 - \frac{N_r}{\tau J_z}s + \frac{N_\delta K_g}{\tau J_z} = 0\ . \tag{27.9}$$

From the Routh-Hurwitz criterion, it follows that the real parts of the solutions of the cubic equation

$$as^3 + bs^2 + cs + d = 0$$

are all negative if

$$a > 0\ , \qquad b > 0\ , \quad \det\begin{bmatrix} b & a \\ d & c \end{bmatrix} = bc - ad > 0\ ,$$

$$\det\begin{bmatrix} b & a & 0 \\ d & c & b \\ 0 & 0 & d \end{bmatrix} = d\,(bc - ad) > 0\ .$$

In the present case, the first two conditions are always satisfied (N_r is always negative) The last condition is always satisfied provided that the third one is, because $d > 0$. The condition for stability is then the third condition

$$\tau\left(1 - \frac{J_z N_\delta K_g}{N_r^2}\right) > \frac{J_z}{N_r}\ . \tag{27.10}$$

Because the term at the right side is negative, the system is always stable if the term in brackets is positive, i.e. when

$$K_g < \frac{N_r^2}{J_z N_\delta}\ . \tag{27.11}$$

If such a condition is not met, the system is stable if

$$\tau < \frac{J_z N_r}{N_r^2 - J_z N_\delta K_g} \; . \tag{27.12}$$

To explicitly express the dependence of the stability of the vehicle-driver system on speed, and remembering that N_r is proportional (at least as a first approximation) to $1/V$, it is possible to introduce parameter VN_r, the reduced delay τ' and the reduced gain K'_g defined as

$$K'_g = K_g \frac{J_z N_\delta}{V^2 N_r^2}, \qquad \tau' = \tau \frac{V\,|N_r|}{J_z}.$$

The first condition for stability then becomes

$$K'_g < \frac{1}{V^2} \; . \tag{27.13}$$

If such a condition is not met, the system is stable if

$$\tau' < \frac{V}{K'_g V^2 - 1} \; .$$

For the system to be stable, the driver must react quickly (with minimal delay) and gradually (small value of the gain). These requests become more demanding with increasing speed. If the delay τ (or better τ') is given, a single condition for stability in terms of K_g (or better K'_g) can be written

$$K'_g < \frac{\tau' + V}{\tau' V^2} \; . \tag{27.14}$$

Condition (27.14) is plotted in Fig. 27.2.

If only the cornering forces of the tire are considered in computing the derivatives of stability, the expressions of K'_g and τ' reduce to

$$K'_g = K_g \frac{J_z}{al^2 C_1}, \qquad \tau' = \tau \frac{alC_1}{J_z} \; . \tag{27.15}$$

To avoid the approximations of Eq. (27.2), it is still possible to obtain the response of the controlled system by numerically integrating the equations of motion. By introducing the steering angle at time t

$$\delta(t) = -K_g[\psi(t-\tau) - \psi_0(t-\tau)] \tag{27.16}$$

into the equation of motion (25.108) of a neutral steer vehicle, it follows that

$$\left\{ \begin{array}{c} \dot{r} \\ \dot{\psi} \end{array} \right\} = \left[\begin{array}{cc} \frac{N_r}{J_z} & 0 \\ 1 & 0 \end{array} \right] \left\{ \begin{array}{c} r \\ \psi \end{array} \right\} + \tag{27.17}$$

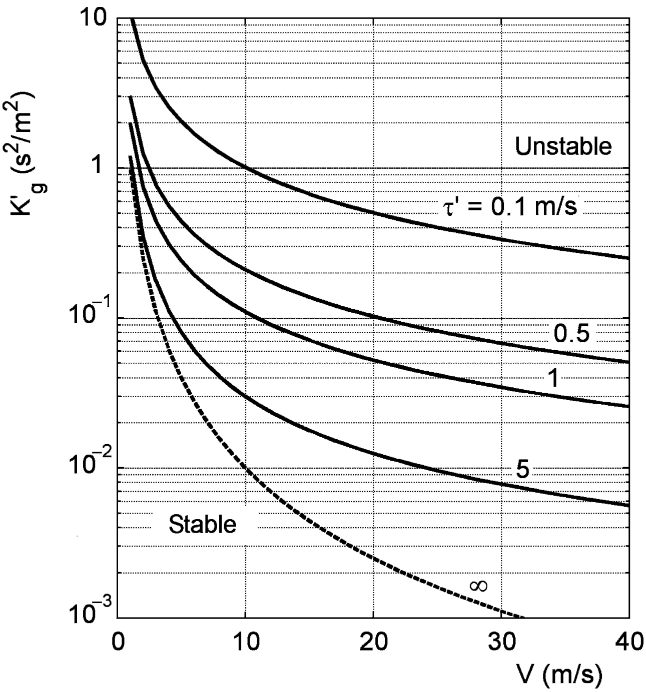

FIGURE 27.2. Values of the gain K'_g allowing a stable working of the vehicle-driver system versus the speed, for different values of the delay τ'.

$$+\begin{bmatrix} \dfrac{K_g N_\delta}{J} & \dfrac{1}{J_z} \\ & \\ 0 & 0 \end{bmatrix} \left\{ \begin{array}{c} -[\psi(t-\tau) - \psi_0(t-\tau)] \\ M_{z_e}(t) \end{array} \right\} .$$

Example 27.1 *Consider a neutral vehicle with the following characteristics:* $J_z = 1,428$ *kg Nm*^{2}, $a = 1.3$ *m*, $b = 1.35$ *m*, $C_1 = 50$ *kN/rad. A linearized drive modelled by Eq. (27.2) steers it along a standard ISO lane change manoeuvre at 80 km/h. Assume a delay* $\tau = 0.1$ *s and a value of the gain such that the system is stable but not overly oscillatory.*

Plot the root locus of the vehicle-driver system for various values of the speed and compute the trajectory obtained through numerical integration of Equations (27.2) and (27.17). Repeat the computations for a delay of 0.3 s.

The ISO lane change manoeuvre, intended to simulate overtaking, was described in Part III. It requires the vehicle to travel for 15 m in the original lane, to change lane with a lateral displacement of 3.5 m in 30 m, to stay in this lane for 25 m and to return to the original lane in 25 m. The manoeuvre must be performed at 80 km/h. The lane changes may be performed using any trajectory, provided that none of the cones delimiting the three straight lanes are touched. The width of these lanes are, in meters, 1.1 B + 0.25 for the first lane, 1.2 B + 0.25 for the second and 1.3 B + 0.25 for the *third, where B is the width of the vehicle. If the vehicle is 1.56 m wide, the three lanes are then 1.966, 2.122 and 2.278 m wide, leaving a margin of 0.203, 0.281 and 0.359 m on both sides of the theoretical trajectory.*

The actual lane changes are left to the driver. In the present simulation, a cosine function is used, which has the advantage of being simple and the drawback of yielding a discontinuity of curvature at each transition with a straight path.

The trajectory and angle ψ_0 *are then*

$$\begin{cases} Y = 0 & \textit{for}\ \ X < 15 \\ Y = \dfrac{3,5}{2}\left\{1 - \cos\left[\dfrac{\pi}{30}(X-15)\right]\right\} & \textit{for}\ \ 15 \le X < 45 \\ Y = 3,5 & \textit{for}\ \ 45 \le X < 70 \\ Y = \dfrac{3,5}{2}\left\{1 + \cos\left[\dfrac{\pi}{25}(X-70)\right]\right\} & \textit{for}\ \ 70 \le X < 95 \\ Y = 0 & \textit{for}\ \ 95 \le X < 125 \end{cases}$$

$$\begin{cases} \psi_0 = 0 & \textit{for}\ \ X < 15 \\ \psi_0 = \arctan\left\{\dfrac{3,5\pi}{60}\sin\left[\dfrac{\pi}{30}(X-15)\right]\right\} & \textit{for}\ \ 15 \le X < 45 \\ \psi_0 = 0 & \textit{for}\ \ 45 \le X < 70 \\ \psi_0 = -\arctan\left\{\dfrac{3,5\pi}{50}\sin\left[\dfrac{\pi}{25}(X-70)\right]\right\} & \textit{for}\ \ 70 \le X < 95 \\ \psi_0 = 0 & \textit{for}\ \ 95 \le X < 125 \end{cases}$$

The value of τ' *corresponding to* $\tau = 0.1$ *s, computed using Eq. (27.15) is* $\tau' = 12.06$ *m/s.The maximum value of* K'_g *to obtain stability at 80 km/h is 0.0058* s^2/m^2*, corresponding to a gain of* $K_g = 1.84$*. To guarantee stability and minimal oscillations, a value equal to 20% of the maximum allowable is assumed:* $K_g = 0.368$*.*

The roots locus (at varying speed) obtained for those values of the gain is plotted in Fig. 27.3a. The results of the numerical simulation are reported in Fig. 27.3b.

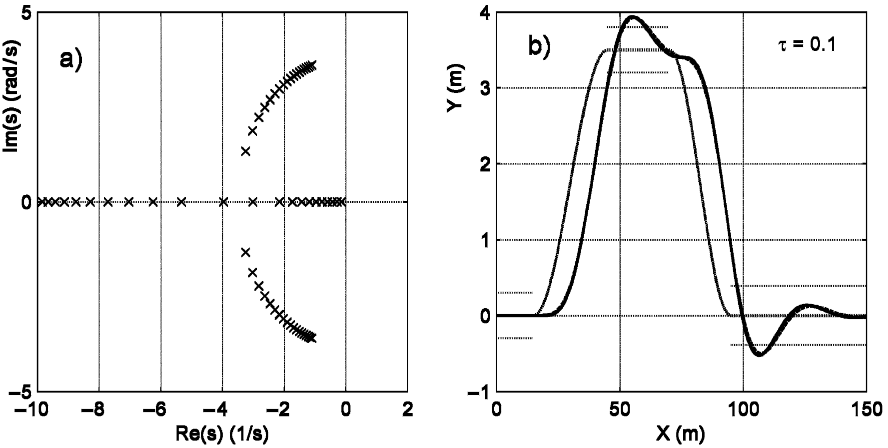

FIGURE 27.3. (a): Root locus of the vehicle-driver system with a delay of 0.1 s. (b): Trajectory obtained during an ISO lane change test computed while taking into account the delay without approximations (full line) and using the approximation of Eq. (27.2).

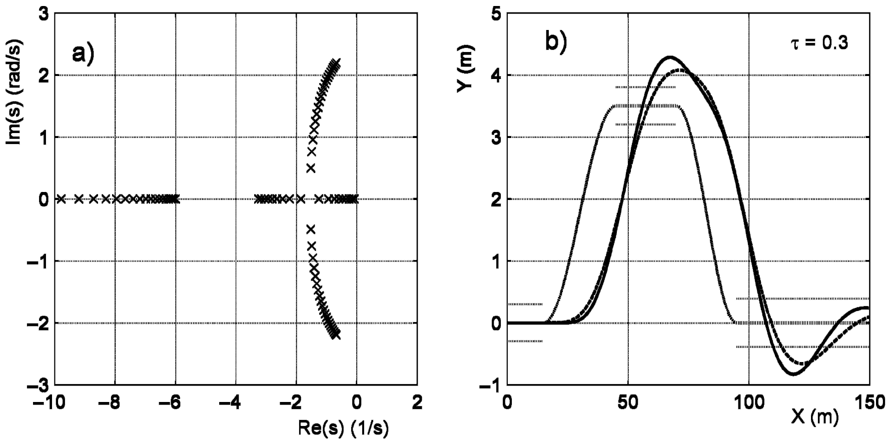

FIGURE 27.4. (a): Root locus of the vehicle-driver system with a delay of 0.3 s. (b): Trajectory obtained during an ISO lane change test computed while taking into account the delay without approximations (full line) and using the approximation of Eq. (27.2).

The computation was repeated with a delay $\tau = 0.3$ s, corresponding to $\tau' = 36.18$ m/s. The maximum value of K'_g to obtain stability at 80 km/h is 0.0033 s^2/m^2, corresponding to a gain of $K_g = 1.04$. Again, to have stability and minimal oscillations, a value equal to 20% of the maximum is assumed: $K_g = 0.21$. The trajectory is shown in Fig. 27.4b. The trajectory follows that required with much delay and strong oscillations. Note that the approximation of Eq. (27.2) leads in this case to non-negligible errors.

In both cases, the driver is unable to perform the lane change manoeuvre without hitting the cones.

To consider a more realistic vehicle behavior, it is possible to remove the assumption that it is neutral steer by inserting Eq. (25.108) instead of Eq. (25.108a) into the vehicle driver model. The equation of the controlled system is then

$$\left\{ \begin{array}{c} \dot{\beta} \\ \dot{r} \\ \dot{\delta} \\ \dot{\psi} \end{array} \right\} = \mathbf{A} \left\{ \begin{array}{c} \beta \\ r \\ \delta \\ \psi \end{array} \right\} + \mathbf{B} \left\{ \begin{array}{c} \psi_0 \\ F_{y_e} \\ M_{z_e} \end{array} \right\}, \tag{27.18}$$

where

$$\mathbf{A} = \begin{bmatrix} \frac{Y_\beta}{mV} & \frac{Y_r}{mV} - 1 & \frac{Y_\delta}{mV} & 0 \\ \frac{N_\beta}{J_z} & \frac{N_r}{J_z} & \frac{N_\delta}{J_z} & 0 \\ 0 & 0 & -\frac{1}{\tau} & -\frac{K_g}{\tau} \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & \frac{1}{mV} & 0 \\ 0 & 0 & \frac{1}{J_z} \\ \frac{K_g}{\tau} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Remark 27.1 *It is, however, uncertain whether it is worthwhile to introduce a more realistic vehicle model when the driver is modelled in such a rough way. An approach of this type is too simple to produce realistic results. The lack of a predictive action and the assumption that the driver reacts only to the yaw angle leads to a reduced overall stability.*

Remark 27.2 *The only interesting result of this model, although quite obvious, is that, to avoid instability, driving must be quick, i.e. with a limited delay, and smooth, i.e. with a low gain.*

27.2.2 More realistic models of linearized driver

The delay is usually assumed to be the sum of three different delays: the first due to the reaction time, i.e. the time needed for the driver to elaborate the information coming from the vehicle and the environment; a neuromuscular delay, the time needed for the command to reach the muscles involved in the control action; and an actuation delay, due to the time needed to actually perform the control action. Some models consider the three delays in distinct ways, and also factor in any predictive action the human operator can perform. Experience shows that such predictive action, which can be much improved with training, is of paramount importance in actual driving conditions. A simple open loop transfer function of a linearized driver is

$$\frac{y(s)}{u(s)} = K_g \frac{(1 + T_L s)e^{-\tau s}}{1 + T_D s} , \tag{27.19}$$

where y, u, K_g, T_L, τ, T_D are respectively the output and the input of the driver, the gain, the prediction time, the reaction time and the neuromuscular delay.

In many cases, the prediction time is neglected and all the delays are added together in a single delay τ, yielding a simpler open loop transfer function

$$\frac{y(s)}{u(s)} = K_g e^{-\tau s} . \tag{27.20}$$

By expressing the exponential as a power series and truncating it after three or two terms, its expression reduces to

$$\frac{y(s)}{u(s)} \approx K_g \frac{1}{1 + \tau s + \frac{1}{2}\tau^2 s^2} \approx K_g \frac{1}{1 + \tau s} . \tag{27.21}$$

The last of these expressions corresponds, in the time domain, to the already seen expression

$$\tau \dot{y}(t) + y(t) = K_g u(t) . \tag{27.22}$$

The choice of which inputs to consider is a delicate one. The results are different if a quantity linked to the position, such as coordinates X and Y or, better, the deviation from the required trajectory, or the yaw angle ψ is used.

It is a common experience that a snaking trajectory is obtained when the driver uses a reference a point close to the front end of the vehicle, as when driving in the fog looking at the curb, while the oscillations disappear when the reference point is far in front of the vehicle.

A simple way to incorporate a kind of predictive behavior into the model is that of using as error not the difference between the desired and the actual value of the yaw angle, but the distance d between a point on the vehicle x-axis at a given distance L in front of the vehicle and the required trajectory (distance d in Fig. 27.5a).

With simple computations and assuming that angle $\psi - \psi_1$ is small, such a distance can be approximated as

$$d = L\left(\psi - \psi_1 + \frac{y}{L}\right) , \tag{27.23}$$

where y is the lateral displacement of the vehicle, i.e. the integral of the lateral velocity v. If the speed of the vehicle is constant, with the usual linearization, it coincides with the integral of β, multiplied by V. Angle ψ_1 is the angle between the X-axis and a line passing through two points of the trajectory at a distance L; it may be easily computed from the shape of the trajectory.

By using the linearized expression for the delay seen above, the equation expressing the time domain model of the driver is then

$$\tau\dot{\delta}(t) + \delta(t) = -K_g\left[\psi(t) - \psi_1(t) + \frac{y(t)}{L}\right] . \tag{27.24}$$

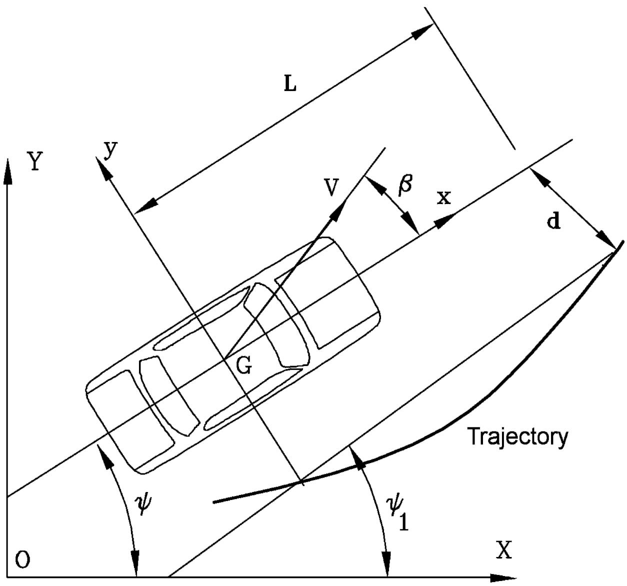

FIGURE 27.5. Definition of distance d.

By introducing Eq. ((27.24) into the simplest mathematical open-loop model of linearized vehicle, remembering that

$$\dot{y} = V\beta \ ,$$

and operating as for the previous model, the state equation for the vehicle-driver system is

$$\left\{ \begin{array}{c} \dot{\beta} \\ \dot{r} \\ \dot{\delta} \\ \dot{\psi} \\ \dot{y} \end{array} \right\} = \mathbf{A} \left\{ \begin{array}{c} \beta \\ r \\ \delta \\ \psi \\ y \end{array} \right\} + \mathbf{B}_c \psi_1 + \mathbf{B}_c \left\{ \begin{array}{c} F_{y_e} \\ M_{z_e} \end{array} \right\}, \tag{27.25}$$

where

$$\mathbf{A} = \begin{bmatrix} \dfrac{Y_\beta}{mV} & \dfrac{Y_r}{mV} - 1 & \dfrac{Y_\delta}{mV} & 0 & 0 \\ \dfrac{N_\beta}{J_z} & \dfrac{N_r}{J_z} & \dfrac{N_\delta}{J_z} & 0 & 0 \\ 0 & 0 & -\dfrac{1}{\tau} & -\dfrac{K_g}{\tau} & -\dfrac{K_g}{L\tau} \\ 0 & 1 & 0 & 0 & 0 \\ V & 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{B}_d = \begin{bmatrix} \dfrac{1}{mV} & 0 \\ 0 & \dfrac{1}{J_z} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$\mathbf{B}_c = \begin{bmatrix} 0 & 0 & \dfrac{K_g}{\tau} & 0 & 0 \end{bmatrix}^T .$$

The errors linked with Eq. (27.24) can be avoided by numerically integrating the equations of motion. Neglecting external disturbances and writing the steering angle at time t

$$\delta(t) = -K_g \left[\psi(t-\tau) - \psi_1(t-\tau) + \frac{y(t-\tau)}{L} \right], \tag{27.26}$$

the equation of motion of the vehicle-driver system is

$$\left\{ \begin{array}{c} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \\ \dot{y} \end{array} \right\} = \begin{bmatrix} \dfrac{Y_\beta}{mV} & \dfrac{Y_r}{mV} - 1 & 0 & 0 \\ \dfrac{N_\beta}{J_z} & \dfrac{N_r}{J_z} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ V & 0 & 0 & 0 \end{bmatrix} \left\{ \begin{array}{c} \beta \\ r \\ \psi \\ y \end{array} \right\} + \tag{27.27}$$

$$
-K_g \left\{ \begin{array}{c} \dfrac{Y_\delta}{mV} \\ \\ \dfrac{N_\delta}{J_z} \\ \\ 0 \\ \\ 0 \end{array} \right\} \left[\psi(t-\tau) - \psi_1(t-\tau) + \frac{y(t-\tau)}{L} \right] .
$$

Example 27.2 *Repeat the simulation of the previous example using the driver model of Eq. (27.24).*

The values of the time delay, the gain and the prediction distance L are assumed to be, respectively, 0.20 s, 0.25 and 30 m.

The trajectory is shown in Fig. 27.6b. The driver model is now successful in performing the required manoeuvre. The tendency to oscillate about the required trajectory is much reduced, and the driver is successful in anticipating the required correction.

Remark 27.3 *Both models here described are a drastic oversimplification of actual human behavior, but the second performs satisfactorily in many instances. In particular, the results obtained in the previous examples were based on a trial and error definition of the characteristics of the driver that satisfied the requirements of the particular case studied. To obtain acceptable results in different manoeuvres, a further adaptation of driver parameters would be needed. To perform simulations of more practical value, more sophisticated models are needed.*

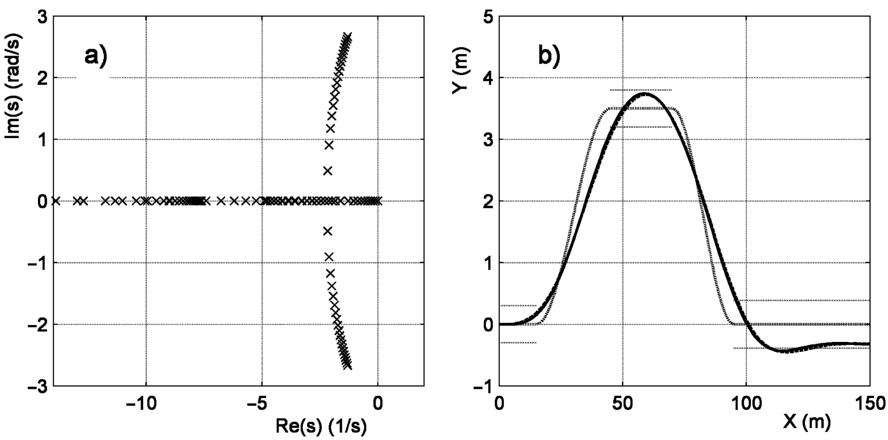

FIGURE 27.6. (a): Root locus of the vehicle-driver system with a delay of 0.2 s. (b): Trajectory obtained during an ISO lane change test computed while taking into account the delay without approximations (full line) and using the approximation of Eq. (27.2).

27.2.3 Longitudinal control

Apart from acting on the steering wheel to maintain course, the driver needs to regulate the vehicle speed by acting on the accelerator pedal and, occasionally, the brakes. If traffic is not intense and the road is largely straight or contains bends with a large radius, it is possible to use a simple regulator maintaining a constant speed at a value set by the driver. These regulators are usually referred to as *cruise control systems*. They are now found in most cars, at least as an option, but they are practically useful in particular conditions: they were first applied in the United States, where they are usable primarily on interstate highways, while in Europe they can be used on highways only when traffic is particularly low.

When traffic is intense, and in particular when there is a line of cars, the driver has to control the accelerator pedal, and occasionally the brakes as well, to maintain a constant distance from the vehicle travelling in the same lane in front of him. In these conditions, long lines of cars can form on highways, with each driver trying to regulate the speed of his vehicle so that the distance from the previous vehicle is constant. Many *anti-collision* systems, imitating this behavior, have been developed.

The simplest model of driver, or of anti-collision system, that follows this approach is a linear controller based on measuring the distance from the previous vehicle. Let V_i and d_i be the speed of the ith vehicle and the distance between the ith and the $(i-1)$th vehicle, whose speed is V_{i-1}.

The derivative of the distance with respect to time is obviously linked to the speed by the relationship

$$\frac{d}{dt}(d_i) = V_{i-1} - V_i \ . \tag{27.28}$$

The driver of the ith vehicle tries to maintain the distance at a fixed value by accelerating when the distance increases and decelerating while it decreases, but this action is applied with a certain delay. A linear model for this action is the following

$$\frac{dV_i(t+\tau)}{dt} = K\frac{d}{dt}(d_i) = K(V_{i-1} - V_i) \ , \tag{27.29}$$

where K is the gain and τ the delay time.

By using the series for the function $V_i(t+\tau)$ truncated after the second term, Eq. (27.29) reduces to

$$\tau\ddot{V}_i + \dot{V}_i + KV_i = KV_{i-1} \ , \tag{27.30}$$

which is formally identical to the equation of motion of a second-order system, with mass τ, unit damping and stiffness K. If

$$K > \frac{1}{4\tau} \tag{27.31}$$

the system has an oscillatory behavior with natural frequency[4] and damping ratio

$$\omega_n = \sqrt{\frac{K}{\tau}} \quad , \quad \zeta = \frac{1}{2\sqrt{K\tau}}. \tag{27.32}$$

Equation (27.30) can be used to predict the behavior of the vehicle-driver system that follows another vehicle travelling at constant speed or, at least, at a speed having a known time history (the law $V_{i-1}(t)$ is known). The behavior must be quick and smooth (both τ and K must be small), so that no oscillations are induced.

The delay here is likely larger than in the previous cases linked with handling, because it is much more difficult to perceive when the distance from the vehicle in front of us changes than to detect changes of trajectory.

The same equation may be used to study the behavior of a line of vehicles. Consider n vehicles ($i = 1, ..., n$) in a line behind a first vehicle ($i = 0$). Let τ_i and K_i be the delay and the gain of the ith vehicle; the n equations of the type of Eq. (27.30) are

$$\begin{bmatrix} \tau_1 & 0 & ... & 0 \\ 0 & \tau_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \tau_n \end{bmatrix} \begin{Bmatrix} \ddot{V}_1 \\ \ddot{V}_2 \\ ... \\ \ddot{V}_n \end{Bmatrix} + \mathbf{I} \begin{Bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ ... \\ \dot{V}_n \end{Bmatrix} + \tag{27.33}$$

$$+ \begin{bmatrix} K_1 & 0 & ... & 0 \\ -K_2 & K_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & K_n \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ ... \\ V_n \end{Bmatrix} = \begin{Bmatrix} K_1 V_0 \\ 0 \\ ... \\ 0 \end{Bmatrix}.$$

As can be seen, the equation describing the behavior of a line of vehicles is then similar, although not identical, to that governing a system made of a set of masses linked to each other by springs and dampers. The stiffness matrix is not symmetrical because the behavior of each vehicle depends on those preceding it, but not those following it.

The eigenvalues can be immediately computed, because the characteristic equation is

$$\prod_{i=1,n} \left(\tau_i s^2 + s + K_i\right) = 0, \tag{27.34}$$

whose $2n$ solutions are

$$s = -\frac{1}{2\tau_i} \pm i\frac{\sqrt{1 - 4\tau_i K_i}}{2\tau_i} \qquad \text{per} \quad i = 1, \ ..., n \tag{27.35}$$

and thus coincide with those of n independent systems described by Eq. (27.30), with the various values of the parameters.

[4]The natural frequency is referred, as usual, to the undamped system. If the damping ratio ζ is larger than one, the damped system will not have free oscillations.

The frequency response of the ith vehicle with respect to the first one is

$$\frac{(V_0)_i}{(V_0)_0} = \frac{\prod_{k=1,i} K_k}{\prod_{k=1,i} \sqrt{(K_k - \tau_k \omega^2)^2 + \omega^2}} \ . \tag{27.36}$$

Remark 27.4 *The present model does not consider the characteristics of the vehicle, and in particular the possibility that its acceleration is not sufficient to get close to the preceding one in the predicted way. If a limitation to the acceleration of the vehicle is introduced, the model is no longer linear, and the approach followed here no longer holds.*

Example 27.3 *Consider a line of vehicles controlled by identical drivers, with a gain* $K = 1.6$ *1/s and delay* $\tau = 0.6$ *s. Plot the frequency response of the various vehicles to an harmonic variation of the speed of the first one.*

The frequency response is plotted in Fig. 27.7 for some values of i*. The line of vehicles has a resonant frequency and may be expected to oscillate.*

This linearized model is obviously too rough to give quantitative indications, but qualitatively explains the oscillatory behavior of long lines of vehicles and also the fact that in the case of dense highway traffic periodic stops can be experienced without any obvious explanation. Clearly, when the oscillations become too large the linearized model loses validity: The acceleration required for some vehicles may be too large or the distance too small, forcing some vehicles to stop.

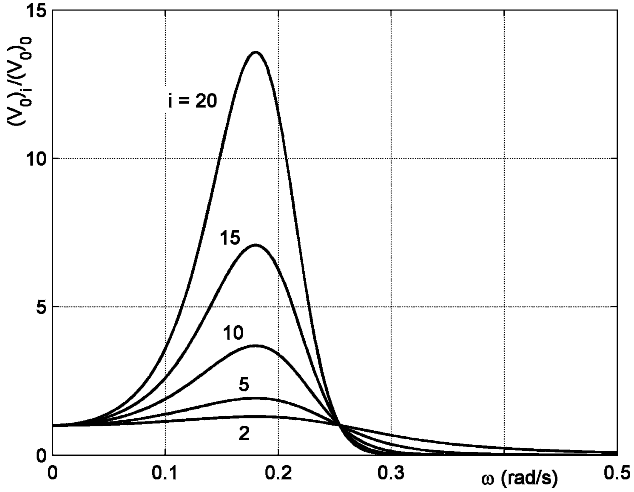

FIGURE 27.7. Frequency response of the velocity of the i-th vehicle in a line using as input the velocity variations of the first one. $K = 1.6$ 1/s; $\tau = 0.6$ s.

27.3 ANTILOCK (ABS) AND ANTISPIN (ASR) SYSTEMS

27.3.1 Basic principles

The force a tire can exert in the longitudinal direction at the wheel-road contact is limited by the available traction. As previously stated when dealing with the tire, to exert a longitudinal force the tire must work with a longitudinal slip, i.e. its angular velocity must be different (smaller when braking, larger when driving) from that characterizing pure rolling. Longitudinal slip was defined as

$$\sigma = \frac{v}{V} \ ,$$

where v is the average slip velocity of the tire on the ground. The longitudinal force, or better the longitudinal force coefficient

$$\mu_x = \frac{F_x}{F_z}$$

is linked to the slip by a nonlinear relationship of the type shown in Fig. 27.8, where both force and slip are assumed to be negative in braking and positive in driving.

The dashed lines indicate unstable working of the tire: once the traction has reached its peak value, not only does the force decrease, but the wheel tends to lock if braking, or to accelerate until a full slip is reached if driving. The equation of motion of a free wheeling wheel in braking is

$$J_r \frac{d\Omega}{dt} = M_m - M_b = |\mu_x| \, F_z r_l - M_b \ , \tag{27.37}$$

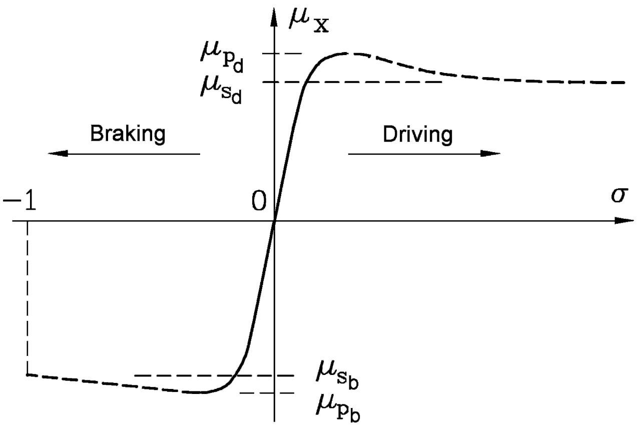

FIGURE 27.8. Longitudinal force coefficient of a tire as a function of the longitudinal slip.

where the driving torque acting on the wheel M_m is due to the longitudinal force at the wheel-road contact $|\mu_x| F_z$ multiplied by the loaded radius r_l. If $|\mu_x|$ decreases at increasing slip, any slowing down of the wheel, with an accompanying increase in slip, will cause a decrease of the longitudinal force and then a further slowing down of the wheel and a further increase of the slip. If the driver does not reduce the braking torque by releasing the pressure on the brakes, the wheels lock. In a similar way, a driving wheel accelerates until it spins freely.

Note that the peak, and the following decrease of traction, are more pronounced in case of high performance tires: racing tires can show values of μ_{xp} much higher than one (even up to 1,8 - 2), while the value at complete slip μ_{xs} remains not much greater than 1. The tendency for the wheel to lock or go into uncontrolled spin is then much greater.

The problems linked with the slipping of a pneumatic tire are, however, not linked solely to the ensuing decrease of the longitudinal force, but deeply affect handling and the very stability of the vehicle. When the traction used in the longitudinal direction increases, the ability of the tire to supply cornering forces decreases and, when the limit traction conditions are reached, the force the tire exerts on the ground has the same direction as the relative velocity between the thread band and the ground. In these conditions, the cornering stiffness of the tire practically vanishes and the wheel loses its ability to supply cornering forces. If the rear wheels lock, the vehicle becomes unstable, while locking of the front wheels causes the vehicle to be uncontrollable, in the sense that it can move only on a straight trajectory.

In traditional vehicles it is the driver who, with his ability to understand the conditions of the vehicle, controls braking forces so that maximum traction conditions are never exceeded. The usefulness of devices that can help the driver in this task is obvious. They can keep the slip of the wheels under control and prevent maximum traction conditions from being exceeded. Actually, it is difficult to measure the longitudinal slip of the tires, because it would require the angular velocity of the wheels to be measured (a simple matter) as well as the speed of the vehicle, a complicated task, and one that requires instrumentation of a kind not usually provided on vehicles.[5]

The first devices of this kind to be used were the anti-lock systems (ABS) used on aircraft for braking during landings and aborted takeoffs. Such devices only measured the speed of the wheel and computed its derivative: a quick decrease of speed shows that locking is about to occur. Subsequently, the device directly reduces the pressure in the hydraulic braking system and then reduces the braking torque. The wheel then returns to a speed close to that of pure rolling and the system ceases to intervene. Braking then recurs, making it likely that the locking conditions repeat, with an ensuing new intervention of the ABS

[5]Normal speedometers, always present on motor vehicles, estimate the velocity of the vehicle from that of an element connected with the output shaft of the gearbox, assuming that the wheels are in pure rolling conditions. Regulations state that speedometers may have only positive errors, i.e. the speed they show must never be less than the actual.

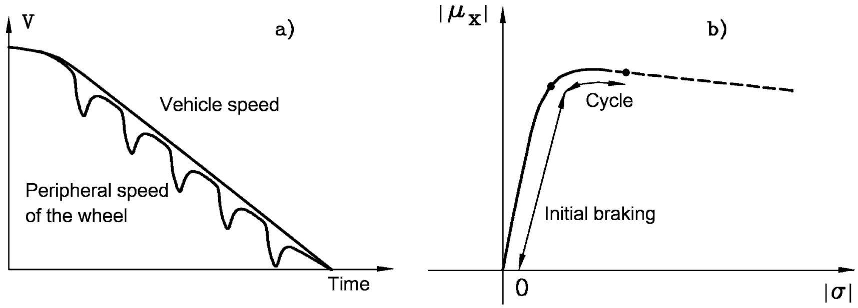

FIGURE 27.9. Working of an anti-lock system (ABS). (a) Time history of the speed of the vehicle and of the peripheral velocity of the wheel during braking with ABS. (b) Zone of the curve $\mu_x(\sigma)$ where the ABS keeps the longitudinal force coefficient.

device. Brakes then act as before, with frequent intervention by the ABS, and the longitudinal force coefficient remains close to its maximum value (Fig. 27.9).

Controlling braking in this way does not allow a traction as good as that at the maximum of the curve $\mu_x(\sigma)$ to be obtained, but it does prevent the wheel from locking and allows a fair lateral force and cornering stiffness to be maintained.

In the automotive field, ABS were initially applied to trailers, owing to the difficulties the driver has in understanding trailer braking conditions. They were then applied to industrial vehicles and luxury cars. Most cars sold today are equipped with ABS.

The control logic of anti-lock systems evolved, even if it is still based mostly on the measurement of the speed of the wheels and on the computation of their deceleration. However, while this method is adequate for free wheels, things are more complex for driving wheels. Here, the inertia of the engine and the transmission must be accounted for along with the inertia of the wheel, and the value of J_r in Eq. (27.37), which depends on the transmission ratio, is much higher. While in the highest gear the apparent increase of the inertia of the wheel may be of 200% or 300%, in the lowest gear the inertia may increase by one or two orders of magnitude. The deceleration of the wheel in these conditions may also depend on both the braking torque of the engine and the position of the accelerator. It may be low enough to prevent the increase of the longitudinal slip from being detected in wheel acceleration measurements.

It is important to evaluate the slip but, as already stated, a measurement of the speed of the vehicle that does not depend on the speed of the wheels is needed, requiring complex and costly equipment. Different strategies are possible, and sometimes more than one can apply. A reference velocity can be defined by elaborating the speeds of all wheels and possibly the longitudinal deceleration of the vehicle as well. By averaging the speed of the wheels, it is possible to obtain a reliable value for the reference velocity, until a quick deceleration of the wheels shows that the longitudinal slip has begun to increase.

By integrating the acceleration of the vehicle in time it is possible to update the values of the speed to obtain a better estimate of the slip. At this point it is possible to use different definitions of the reference speed and different algorithms to compute it, possibly using different definitions for the various axles as well, so that the required performance can be obtained. Suitable corrections may be introduced to take different factors into account, such as the roughness of the road, which is detected from oscillations in the wheel deceleration. It is possible that in the near future devices allowing the vehicle speed to be obtained directly will be available, devices based, for instance, on GPS. By estimating, or even better, by measuring the vehicle speed and then the longitudinal slip, the control system can not only prevent the wheels from locking, but may also try to maintain the slip in the zone where the braking force is highest.

27.3.2 Control strategies for ABS

How the anti-lock system acts on the various wheels has a large effect on stability. Older systems were simple *two channel* devices, i.e. they jointly controlled the two wheels of the same axle. The control of an axle may be performed following two distinct strategies, usually referred to as *select high* and *select low*. In the former case, if the wheels are in different conditions, the wheel governing the behavior of the system is the one in the best condition. In other words, the ABS device allows the wheel in the worst condition to slip, reducing the pressure in the braking system only when the wheel in the best situation begins to slip. The second strategy, on the other hand, begins to reduce the braking pressure when the wheel in the worst condition encounters a critical situation.

The latter strategy guarantees that the two wheels exert the same longitudinal force, thus preventing yawing moments. But it decreases the total amount of braking force to what the axle could exert when all its wheels are in the condition of the wheel that initially slips. Another advantage is that it guarantees a high value for the cornering stiffness of the axle.

Select high strategy, on the other hand, allows the ability of the wheel in the best condition to exert a high force to be exploited, while the other wheel works in conditions close to slipping. The braking force is much higher, but the axle produces a yawing moment that may be quite strong. The ability of the axle to produce cornering forces is compromised, because the cornering stiffness of the wheel in the worst condition vanishes, while that of the other wheel is reduced due to the strong longitudinal force.

A reasonable global strategy is to use *select low* at the rear wheels, which do not, in any case, exert large braking forces, and *select high* at the front wheels which produce most of the braking force. The total braking force is thus close to the highest possible, the rear axle has good cornering stiffness, while that of the front axle decreases, with the result that the vehicle is more understeer and thus mode stable. The drawback is the generation of a yawing moment that compels the driver to control the trajectory by acting on the steering wheel, with a steering angle that, owing to the decrease of the ability of the front wheels to generate lateral forces, may be large.

Performance is increased by using two separate ABS systems at the front axle, so that the wheel in worst conditions does not lock, with an increase of the overall braking force, decrease of the yawing moment and, above all, lower reduction of the corner stiffness of the front wheels. The vehicle remains more manoeuvrable, and it is easier for the driver to counteract the yawing moment.

The rear axle may remain controlled by a single ABS device with a *select low* logic. The system then has three channels, or it may have two distinct ABS devices. In the latter case (four channel ABS), the *select low* strategy may be implemented on the ECU, keeping open the possibility of modifying the strategy depending on the values of a number of parameters.

The ground is usually not uniform, so the traction may change from wheel to wheel. However, the average traction (in time) is the same usually for all wheels. In some cases, the road under the wheels on one side has characteristics that are not the same as those at the other side, such as when the wheels near the curb are on wet road or, even worse, on snow or ice, while those toward the center of the road are on a clean and dry surface. These conditions are usually referred to as *μ-split*. In this case, a *select low* strategy at the front axle would lead to very low values of the braking force, while a *select high* strategy may lead to very high yawing moments. The use of an accelerometer measuring yawing accelerations or an instrument that can measure the yaw angular velocity allows intermediate strategies to be implemented. The braking force on the front wheel that is in better conditions can be limited when the yaw angular acceleration increases, so that the stability of the vehicle can be maintained without overly penalizing the braking force. A strategy of this type depends largely on the characteristics of the vehicle, and particularly on its moment of inertia about the yaw axis and its geometrical characteristics (wheelbase, track). The larger the yaw moment of inertia, the larger the allowable yaw moments.

The devices used to implement an ABS system were described in Part I. Here we must just remember that although an ABS device is conceptually simple (to reduce the braking torque it is sufficient to use an electrovalve discharging a quantity of high pressure fluid from the hydraulic system, thus reducing the pressure in the cylinders of the brakes), in practice a simple ABS of this kind cannot be used, because after a number of interventions the brake pedal would sink owing to the discharge of fluid. This problem may be solved at the cost of greater system complexity, using a pump actuated by an electric motor that takes the discharged fluid and reintroduces it into the high pressure part of the circuit. There are alternatives that avoid adding an electric motor with its control devices to a system that is already complex, but a pump allowing the system to be put under pressure with no intervention by the driver on the brake pedal is required for other functions like traction control.

The ABS system interferes with other devices usually included in the braking system, such as the pressure proportioning valve. The function of the latter can actually be integrated into the ABS system, obtaining what is often referred to as EBD (Electronic Brake Distributor). However, the strategy of an ABS system and a pressure proportioning valve are radically different. The first must step in when locking conditions are approached, while the second must always function, so that the braking torques at the rear axle are reduced when the weight acting on it is reduced because of longitudinal load shift. Conceptually, the slip of the front and rear wheels must be continuously monitored so that the longitudinal slip at the front is larger than that at the rear. All the difficulties seen for measurement of the slip while dealing with ABS systems are present, with the simplification that what matters here is not an absolute measurement but simply the measurement in the difference of slip between the axles. The measurement of the longitudinal acceleration, and then of the load shift, may be very useful.

27.3.3 Traction control systems (TCS, ASR)

The problem of preventing driving wheels from slipping is similar to that of braking wheels, even if it is usually less severe and occurs only in the case of powerful vehicles or in conditions of poor traction. Wheel slipping in this case has two effects: a decrease of the force exerted in the longitudinal direction and a loss of the ability to exert transversal forces, with effects on stability and driveability. The latter are considered less severe in front wheel drive vehicles, which become less controllable, than in rear wheels drive vehicles, which become less stable.

The systems that control the slipping of driving wheels are usually referred to as Traction Control Systems (TCS) or Anti Spin Regulators (ASR). The sensors are the same as for the anti-lock system: they measure the angular velocity of the wheels so that their acceleration can be computed. When a wheel begins to spin, it is possible to react in two ways: either by reducing the power supplied by the engine or operating the brakes. The second strategy is usually quicker, but the actual implementation follows a mixed strategy: The brakes are first used to slow the wheel that has begun to spin, after which the power of the engine is reduced.

The two strategies have different effects and are used to solve different problems. If the vehicle is in symmetrical conditions, i.e., if the right and left wheels are in the same conditions, it is useless to use brakes and not advisable, because both wheels of the same axles should be braked. The result is that the transmission is much more stressed than usual, at least until the power from the engine is reduced. In this case, the reduction of the driving torque prevents both wheels from slipping.

The advantage is not so much improved performance, because the driving force the wheels can exert when not slipping is not much greater than they would exert when slipping (except in the case of high performance tires), but the possibility of exerting side forces. In particular, in the case of rear wheel drive vehicles, the vehicle remains stable, while in that of front wheel drive, the vehicle remains manoeuvrable.

The reduction of power may then be realized by acting on the motor control system.

In case of asymmetric conditions (μ-*split*), this strategy would penalize performances excessively, because it would apply a sort of *select low* strategy: both wheels would exert a force equal to what the wheel in the worst conditions can exert.

On the other hand, by braking the slipping wheel, the differential gear subdividing the driving torque between the driving wheels allows the wheel in better condition to transfer a torque equal to the sum of the braking torque applied on the other wheel plus the torque the latter is able to transfer. By acting on the brakes an increase in performance is obtained, but has a small effect on stability or manoeuvrability. TCS systems can thus be used as an alternative to controlled slip differential because, by applying a braking torque on the wheel that would slip, the system allows a certain driving torque to be transferred to the other wheel even in case where the differential is of the simplest type.

While ABS systems act on the braking system to reduce the pressure exerted by the driver, possibly assisted by the power brake, the TCS must use a pump to put the hydraulic system under pressure, independent of the force exerted by the driver on the brake pedal. However, in many cases such a pump is already included in the ABS system, so the complexity introduced is not great.

To combine the requirements on performance with those on stability, TCS systems must act on both the engine and the brakes, even at the expense of added complexity. The two strategies can be mixed following a logic that is based on many parameters, apart from the wheel slip and the acceleration of the vehicle: for instance, at low speed it is possible to give priority to acceleration performance through a strategy based on the use of brakes, while at high speed it is possible to give priority to stability by acting on the engine.

TCS systems allow drivers with limited ability to drive difficult vehicles, such as rear wheel drive high powered cars, even in critical road conditions.

27.3.4 Electric braking

Electric braking may be performed in two radically different ways: by electrically actuating the pump of a conventional hydraulic (pneumatic in industrial vehicles) system, or by substituting an electric braking system for the hydraulic one. This second strategy may in turn be implemented in two different ways, by replacing the hydraulic system with an electromechanical one (including, for instance, an electric motor in each wheel that operates the calipers using a ball screw) or by putting a pump operated by an electric motor in each wheel, sending pressurized liquid to a more or less conventional caliper.

Even if all these approaches allow ABS and possibly TCS functions to be integrated more easily into the braking system, only the total replacement of the hydraulic system with an electromechanical one where the electric control reaches each wheel directly (possibly using a fully electromechanical actuator) allows the full performance predicted for *by wire* devices to be attained, but at the cost of many reliability problems.

27.4 HANDLING CONTROL

27.4.1 General considerations

As seen in Part I, the lateral dynamics of the vehicle is controlled by the driver, who creates a yawing moment by setting the steering wheels at a certain steering angle; this torque causes a yaw rotation that puts all wheels at a certain yaw angle and consequently produces a lateral force that alters the trajectory. The yawing torque may be produced in many ways: acting on the front steering wheels (setting them at a sideslip angle and then creating a side force that, being applied in front of the center of mass, produces a yawing moment), acting on all wheels that are steering (setting them at a sideslip angle in the opposite direction and then producing a yawing moment), and also by creating a yawing torque directly through differential braking or traction on the right and left wheels. It is also possible to act on all the wheels, setting them at a steering angle in the same direction and thus producing directly the side forces needed to alter the trajectory without any yaw rotation or sideslip angle of the vehicle.

Traditional vehicles are controlled using only the first of these strategies: manual direction control through front wheel steering. If the road conditions are good and the required manoeuvre is not too severe, the simplified models seen in Chapter 25 allow the response of the vehicle to be predicted fairly well. The average driver is able to maintain control without difficulty.

In particular, the dynamic analogy of the vehicle with a mass, spring, damper system (equations (25.110) and (25.111)) is fairly accurate and the response to a steering command $\delta(t)$ is that of a second order system excited by a linear combination of laws $\delta(t)$ and $\dot{\delta}(t)$. The response in terms of yaw velocity and lateral acceleration is expressed by Equations (25.117) and (25.115), and that in term of sideslip angle $\beta(t)$ is expressed by Eq. (25.116).

Even if the response is that of a non-minimum phase system, in which the sideslip angle may initially take values of opposite sign with respect to the steady state value, the sideslip angle can be felt by the driver only to a limited extent and does not create confusion or dangerous situations.

It is interesting to study the response of the vehicle if the control strategy is based on rear wheel steering. The non-stationary response can be computed from previously seen equations by stating $K_1 = 0$ and $K_2 = -1$ into Equations (25.183) and (25.184). By using the simplified expressions of the derivatives of stability, it follows that

$$\frac{r_0}{\delta_0} = C_2 \frac{mbVs + C_1 l}{\Delta} \ , \tag{27.38}$$

$$\frac{a_{y_0}}{\delta_0} = C_2 \frac{-VJ_z s^2 - C_1 las + VC_1 l}{\Delta} \ , \tag{27.39}$$

$$\frac{\beta_0}{\delta_0} = C_2 \frac{-\left(mbV^2 + C_1 la\right) s + J_z V}{V\Delta} \ , \tag{27.40}$$

where Δ is still expressed by Eq. (25.168).

Note that the steady state response in terms of yaw velocity r and lateral acceleration is that already seen for front wheel steering, while that in terms of β is similar except for the term C_2 substituting for C_1. The steady state response is little changed if the vehicle is steered by operating the rear instead of the front wheels.

In non-stationary conditions, things are quite different. By setting the numerator of the first and the third transfer functions to zero, one can see that the responses in terms of r and β have only one real positive zero and thus cause no problem. The zeros of the transfer function a_{y_0}/δ_0 are

$$s = C_2 \frac{-C_1 la \pm \sqrt{C_1^2 l^2 a^2 + 4J_z V^2 C_1 l}}{2VJ_z} \ . \tag{27.41}$$

The two zeros are real, one positive and one negative. The response is then that of a non-minimum phase system at all speeds. The vehicle initially accelerates laterally in a direction opposite to that in which it will accelerate in steady state, disorienting the driver.

It has been proven that it is still possible to drive the vehicle under these conditions, provided that the driver is suitably trained, but driving using the rear wheels is much more difficult. This solution is used only on very slow vehicles (earth-moving machines, dumpers, etc.), because the zero in the right half plane of the roots locus tends to move towards the origin when the speed tends to zero.

The above mentioned considerations are based on linearized models. The experience of the average driver is based on driving conditions in which the behavior of the vehicle is essentially linear. If the limit conditions are approached, either because road conditions are poor or because high performance is required, the nonlinearity of the system, due both to the tires (and possibly to aerodynamic actions), and the geometry of the system, starts to become important. The vehicle may start to behave differently from what the driver expects.

It is difficult for the driver to assess the traction available due to road conditions and the sideslip angle of the tires, so when the wheels start slipping they do so abruptly. The feeling the driver has in normal conditions is that of kinematic driving (the wheels seem not to be at a sideslip angle and the trajectory seems to be defined by the position of the steering wheels in a geometrical way). When the sideslip angles increase to values that can no longer be neglected, the driver feel he has lost control of the vehicle. And indeed he has, for the average driver is unable to control the vehicle when the sideslip angles are large.

Actually, the behavior of the vehicle may change considerably when the sideslip angles take values beyond the linearity range. From the viewpoint of theoretical study, it is possible to compute the steady state working conditions using nonlinear models and then to linearize the equations about those conditions. This allows us to study, for instance, the stability or manoeuvrability under these conditions (Chapter 25). The aim of a study of this kind should be to reduce the difference between handling in limit and in linearized conditions so the driver is able to control the vehicle even outside the linearity range.

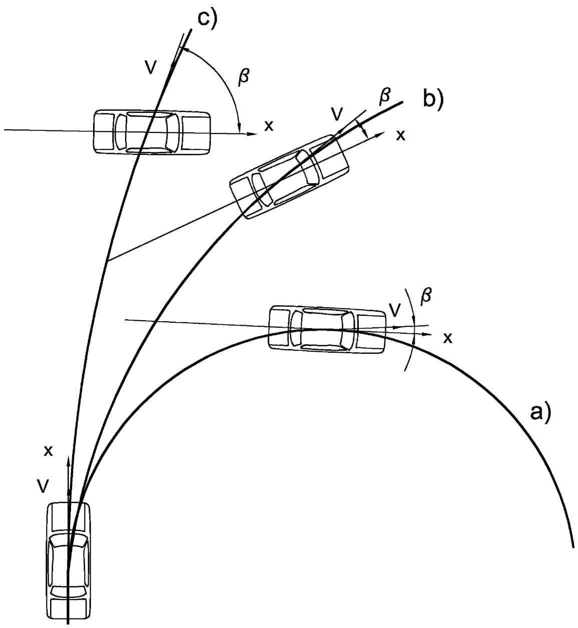

FIGURE 27.10. Response to a steering input. Curve a): conditions far from the limit. Curve b): conditions close to the limit. Curve c): conditions beyond the limit.

Consider, for instance, a steering input, aimed at putting the vehicle on a curved trajectory (Fig. 27.10). If road conditions are good, the vehicle moves on its trajectory in conditions that are far from limit conditions. The sideslip angles are small and the behavior is essentially that of a linear system (curve a). If road conditions are worse (or if the driver steers the tires so as to develop side forces that approach the limit) the sideslip angles become larger and the tires work in nonlinear conditions (curve b). Control is still possible, but the driver must have a proficiency beyond that of an average driver. Finally, if the road conditions are worse still, the vehicle may rotate about the yaw axis, but the wheels slip laterally, so that the vehicle can no longer follow the required trajectory (curve c). The sideslip angles increase in an unbounded way and the yaw rotation becomes uncontrolled. An alternative outcome is that the vehicle cannot rotate due to lateral slipping of the steering wheels. It may then go out of its trajectory with limited, or even small, values of the sideslip angles.

27.4.2 Control using a reference model

It is impossible for the vehicle to maintain a behavior corresponding to what the driver is used to based on his experience with the linearized response when he approaches conditions close to the limit. However, it is possible to introduce control strategies giving the vehicle an apparent behavior the user can find predictable. This can be done by building a reference model allowing the response to the control inputs supplied by the driver to be computed in real time, and by implementing it on a control system that forces the vehicle to behave as close as possible to the computed results. This strategy is not new, and is widely used in the aerospace field, particularly for flying aircraft that are particularly difficult to control. The most typical example is the Space Shuttle[6]: its aerodynamic configuration is such that controlling it during landing, the most critical phase of the flight, is extremely difficult. A mathematical model based on a standard civil aircraft is used as a reference model; it runs on a control system that corrects the behavior of the spacecraft so that it simulates that of the model, making maneuvering much easier.

To transfer this strategy to the automotive field is not simple, but the advantage of providing the vehicle with a response the driver is used to is clear. It is also clear that the model cannot control the vehicle when this is physically impossible, such as when the driver demands a lateral acceleration higher than that made possible by the maximum forces the road-wheels contact can exert. The control system must then realize when the limit conditions are approaching and warn the driver, or manage the high slip conditions in the best possible way.

A possible sketch of a control strategy of this type is shown in Fig 27.11.

If the vehicle used to build the reference model is the actual vehicle used under the linearized conditions, the control device will perform little work in conditions far from the limit, and only when the behavior deviates from the linearized behavior will it be asked to correct the response. If, on the other hand, this strategy is used to induce a vehicle behavior different from standard operations, the control device will have to act in all driving conditions.

Only the steering angle δ is sensed in the sketch of Fig 27.11, but it is possible to use devices that are much more complex, measuring the driving or braking forces (at minimum, sensing the position of the accelerator pedal and the pressure in the braking system), because directional control is much influenced

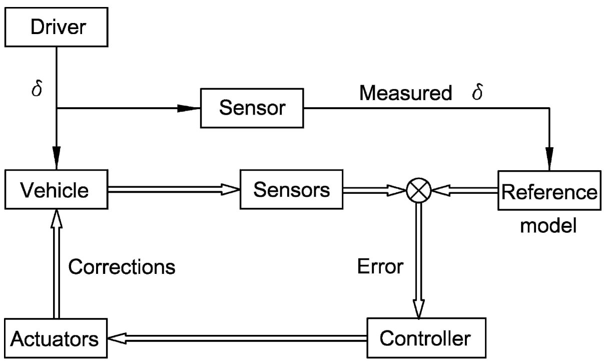

FIGURE 27.11. Sketch of a possible strategy based on a reference model to implement lateral control.

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[6] D. Karnopp, *Vehicle Stability*, Dekker, New York, 2004

by longitudinal forces exerted by the tires. The sensors for the actual behavior of the vehicle may be of a different type, including not only a yaw velocity and a lateral acceleration sensor, but also other sensors allowing the slip of the various wheels to be computed as in ABS systems. Actuators may also be of a different type and corrections may be exerted using different strategies.

27.4.3 VDC systems

Acronyms for the devices implementing lateral dynamics control are many, and they are often trademarks of the various manufacturers, designating systems working with different strategies. In this chapter they will be referred to under the general name of Vehicle Dynamics Control (VDC).

The simplest way to control vehicle dynamics is to leave full control of the steering to the driver, performing the control action by differentially braking the wheels using the existing braking and traction control system, assuming the vehicle already has ABS and TCS devices.

Assume, for instance, that the control system detects a yaw speed that is higher than that computed by the reference mathematical model for the measured steering angle and speed. A situation of this kind, similar to that shown in Fig 27.10, curves b or c, may be defined as an excess of oversteer and corresponds to an excessive sideslip angle of the rear wheels. It may occur for various reasons, such as an underinflated rear tire, low traction at the rear wheels, a center of mass located far to the rear, driving forces created by a rear wheel drive vehicle, or many other possibilities. To reduce the yaw angular velocity it is possible to brake the wheels outside the curved trajectory. In this case, it is expedient to brake the front wheel, both because it is likely that the front axle still has traction available and because by doing so the vehicle becomes more understeer. In a similar way, if the yaw velocity is lower than the computed value, it is possible to brake the rear wheel inside the turn (Fig 27.12).

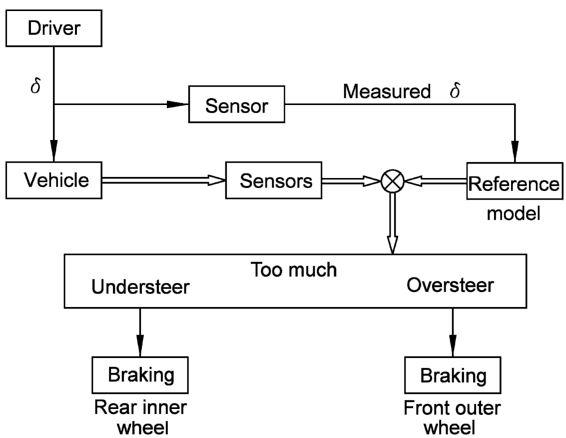

FIGURE 27.12. Differential braking used to correct directional behavior.

Instead of using differential braking, it is possible to have a steering front axle controlled by the VDC system. This strategy is different from that assumed in the section on 4WS where the rear steering was controlled directly by the driver along with the front steering. In the present case, the driver controls the vehicle using the front steering only, while the rear steering is used to perform corrections aimed at forcing the vehicle to follow the behavior of the reference model.

Differential braking has the advantage of acting more quickly than rear steering, because the latter requires not only that the wheels be set at a sideslip angle, something that cannot occur instantly, but also that the wheels start exerting cornering forces, something that requires the vehicle to travel a distance equal to the relaxation length. Nor is the action of the brakes is instantaneous, but its characteristic time is lower.

This drawback of controlling by steering particular wheels is not typical of rear steering, but is present whenever tires exert cornering forces. Another drawback of this strategy is that any steering control performed on the rear wheels involves a non-minimum phase system. The control action initially produces a lateral acceleration in a direction that is opposite to that occurring in steady state. Although this may in itself be an advantage (non-minimum phase systems are often quicker than standard systems), it may be felt as a drawback by the driver, who may prefer a vehicle without VDC.

Instead of using rear steering, or in addition to it, the VDC system may control front steering, removing it in part or totally from the direct control of the driver. While the strategies seen above are additional to the usual ways of controlling the vehicle’s course, this method does offer an alternative. If the control system uses a reference model to perform driver commands, the latter may not even realize that the control system is overriding his command. He may have the feeling that the vehicle behaves as expected, which is true: The goal of the driver is to keep the vehicle on the required trajectory, and it does not matter whether to do so the steering wheels are set to the angle the driver sets on the steering wheel or the control system produces a different steering angle, possibly while the brakes act differentially and the rear wheels steer.

If the vehicle is provided with a *steer by wire* system, there is no difficulty in removing, in part or totally, steering from the direct control of the driver: It is enough that the steer actuator not only receives the signal from the sensor measuring the angle of the steering wheel, but is driven by the controller that acts according to a more complex logic. However, as it will be seen later, at present (2008) there are still difficulties, included those linked to standards, in uncoupling the steering system completely from the steering wheel. It is possible to use a partially mechanical solution, where the steering mechanism is connected to a differential gear, receiving an input from the steering wheel through the shaft in a conventional way, along with the input coming from an electric motor controlled by the VDC system. The steering angle of the wheels is then a combination of the angle of the steering wheel and that imposed by the actuator.

Apart from operating the brakes, the rear steering and the front steering, the VDC system may also operate through the suspensions. If an active anti-roll bar is used, it is possible to shift the load transfer from the front to the rear axle, or vice versa, making the vehicle more or less oversteering. This strategy will be dealt with later when we discuss active suspensions.

Devices of the kind seen above not only affect the driveability of a vehicle and the feeling the driver gets from it, but may also change its intrinsic stability. While the steering on all wheels, implemented by controlling the steering boxes of the two axles using the steering control, has no effect on the stability of the vehicle (although it may affect the stability of the vehicle driver system), a device in which a closed loop controller oversees the lateral behavior (like that sketched in Fig 27.11 or in Fig 27.12) may alter it to some extent.

In general, a VDC system may make the vehicle more stable, by making it less subject to external disturbances, as is predictable for a closed-loop device. If a strategy based on a reference model is applied, the stability of the vehicle should be brought to coincide with that of the model, but this must be accurately studied at the design stage. It must be noted that if the VDC system is designed to operate in conditions in which the vehicle has a nonlinear behavior (in conditions far from the limit, where the vehicle is essentially linear, the usefulness of VDC is debatable) both the system and the controller are nonlinear, complicating the study of stability. This also applies to cases where the goal of the control device is to make the global behavior of the controlled system (as seen by the driver) as linear as possible.

Finally, any system controlling the handling of a vehicle has limitations. The vehicle may remain on the trajectory the driver sets, thanks to the control system, only until the maximum cornering forces are reached. From this viewpoint, a device that changes the behavior of the system so that it remains close to the behavior in conditions far from critical may be dangerous, because it prevents the driver from realizing that he is dangerously approaching critical conditions.

If the control system must warn the driver of the approaching of limit conditions, the controller must be designed so that it changes its control logic when the limit conditions are reached. If, notwithstanding the intervention of the controller, the vehicle cannot be kept on the desired trajectory, the device must perform an emergency manoeuvre, quite a difficult thing because also the driver may be trying to do something similar.

It is unlikely that the control system should have authority to override the driver, because the latter has information the control system ignores (if nothing else, information on the road conditions) and, if the driver is proficient, he may be able to manage the emergency situation better than any control device. This consideration may not apply to average drivers, who may panic and react in a completely wrong way.

Another reason to limit the authority of the controller is to give the driver some margin to react to a malfunctioning of the device. This consideration may be important for present applications, while in the future the authority of the system may be increased as more experience on these systems is gained. However, it is perhaps unrealistic to believe that in the future stability control systems that cannot be a cause of accidents a particularly good human driver could prevent will be built. We must probably reason in statistical terms: A system is useful when the number of accidents it may prevent is larger than the number of accidents it may cause. This criterion is, however, difficult to implement, because it is impossible to say how many accidents a vehicle dynamics control system may prevent.

A further point is that the authority of the VDC system may need to be limited to prevent the driver from feeling that he cannot master his vehicle. An expert driver may appreciate the feeling of being in a complete control and may even enjoy the nonlinear behavior in conditions close to the limit. For this reason the driver can be given the option to switch off the VDC system, as he chooses. The vehicle may have completely different behavior in the two cases, satisfying customers who might otherwise buy two different vehicles, one for everyday use and one for sport driving.

Finally it would be a mistake to think that modifying the handling of a vehicle using a control system makes efforts aimed at designing well-handling vehicles useless. As already stated, it is better to avoid giving too much authority to the control system for safety reasons, and a failure of the control system that makes the vehicle outright unstable must be avoided. Moreover, it must be added that the control system is unable to generate arbitrarily high control forces, leading to limitations to its capabilities.

27.4.4 Simplified VDC with yaw velocity control.

Consider a device controlling the yaw velocity of the vehicle, implemented by steering the front wheels. The steering angle of the front wheels is then a linear combination, one that can be implemented using a differential gear that has as inputs the steering wheel and the controlled actuator, of angles δ_g expressed by the driver and δ_c imposed by the control system. A possible implementation of this system is shown in Section 6.1.2 of Part I.

Assuming that the coefficients of the linear combination have a unit value, it follows that

$$\delta = \delta_g + \delta_c \ . \tag{27.42}$$

Let the reference model be the two degrees of freedom model studied in Chapter 25:

$$\left\{ \begin{array}{c} \dot{v}_r \\ \dot{r}_r \end{array} \right\} = \mathbf{A}_r \left\{ \begin{array}{c} v_r \\ r_r \end{array} \right\} + \mathbf{B}_r \delta_g \ , \tag{27.43}$$

where the dynamic matrix at constant speed and the input gain matrix are

$$\mathbf{A}_r = \begin{bmatrix} \dfrac{Y_{\beta r}}{mV} & \dfrac{Y_{rr}}{m} - V \\ \\ \dfrac{N_{\beta r}}{J_z V} & \dfrac{N_{rr}}{J_z} \end{bmatrix} , \tag{27.44}$$

$$\mathbf{B}_r = \left\{ \begin{array}{c} \dfrac{Y_{\delta r}}{m} \\ \\ \dfrac{N_{\delta r}}{J_z} \end{array} \right\} . \tag{27.45}$$

The parameters of the reference model may be chosen with different criteria. For instance, $Y_{\beta r}$, Y_{rr}, ... may be chosen to be equal to the values Y_β, Y_r, ... as computed for the actual vehicle. In this way the control system will be almost inactive when the vehicle behavior is similar to the linearized model, and the driver will have the impression that the behavior of the vehicle is what he is used to in conditions of smooth driving.

Another strategy may be to modify the behavior of the vehicle in all conditions, giving it, for instance, a neutral steer response. In this case

$$N_{\beta r} = 0, \qquad Y_{rr} = 0 \tag{27.46}$$

while the derivatives of stability may be

$$\begin{array}{ll} Y_{\beta r} = -\dfrac{l}{b} C_1, & \qquad N_{rr} = -\dfrac{al}{V} C_1, \\ \\ Y_{\delta r} = C_1, & \qquad N_{\delta r} = aC_1. \end{array} \tag{27.47}$$

If only the yaw velocity r is to be controlled, it is possible to define an error

$$e = r - r_r \ . \tag{27.48}$$

As an example, a purely proportional controller may be used: It generates a steering angle δ_c proportional to the error through a gain k of the controller, yielding

$$\delta_c = -ke = -k\left(r - r_r\right). \tag{27.49}$$

Although a control of this kind is easy to implement (only r must be measured and r_r computed), its performance would be poor, particularly in terms of control robustness.

An example based on the *sliding mode* technique is reported in *Vehicle Stability* by Dean Karnopp.[7]

Example 27.4 *Consider the vehicle studied in Example 26-4 and apply to it a VDC system of the type shown above with the aim of making it neutral steer, using a simple proportional control. Compute the gains of the vehicle in steady state conditions and simulate a steering step manoeuvre to put it on a circular trajectory with a radius of 200 m at a speed of 100 km/h = 27.78 m/s.*

The behavior of the system may be modelled using an equation of the type

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u} + \mathbf{d}, \tag{27.50}$$

[7] D. Karnopp, *Vehicle Stability*, Dekker, New York, 2004.

where $\mathbf{z}$ *is the state vector, the input* $\mathbf{u}$ *can be assumed as*

$$\delta = \delta_g + \delta_c$$

and $\mathbf{d}$ *is a vector containing the external disturbances and those due to the deviation from the linear behavior implicit in matrices* $\mathbf{A}$ *and* $\mathbf{B}$*.*

The order n *of the model used for the vehicle may be different from the order of the reference model, which is equal to 2, but in the present case also the former is assumed to be a model with two degrees of freedom. Thus assume* $n = 2$*. Neglecting disturbances and nonlinearities,* $\mathbf{d} = 0$

If the output of the system is only the yaw velocity, the output equation reduces to

$$r = \mathbf{C}\mathbf{z}, \tag{27.51}$$

where the output gain matrix $\mathbf{C}$ *is a row matrix which, if* r *is one of the states of the system, has all elements equal to zero and one equal to 1. In the present case,* $\mathbf{C}$ *has just 2 elements.*

The state equation for the system made by the vehicle and the reference model is

$$\left\{ \begin{array}{l} \dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\left(\delta_g + \delta_c\right) + \mathbf{d} \\ \dot{\mathbf{z}}_r = \mathbf{A}_r\mathbf{z}_r + \mathbf{B}_r\delta_g \end{array} \right. \tag{27.52}$$

Remembering equations (27.49) and (27.51) and neglecting $\mathbf{d}$*, Eq. (27.52) becomes*

$$\left\{ \begin{array}{c} \dot{\mathbf{z}} \\ \dot{\mathbf{z}}_r \end{array} \right\} = \left[\begin{array}{cc} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_r \end{array} \right] \left\{ \begin{array}{c} \mathbf{z} \\ \mathbf{z}_r \end{array} \right\} + \left[\begin{array}{c} \mathbf{B} \\ \mathbf{B}_r \end{array} \right] \delta_g - k \left[\begin{array}{c} \mathbf{B} \\ \mathbf{0} \end{array} \right] \left(\mathbf{C}\mathbf{z} - \mathbf{C}_r\mathbf{z}_r\right), \tag{27.53}$$

where $\mathbf{C}_r$ *is a matrix similar to* $\mathbf{C}$*, but for the reference model. In the present case, the two matrices are the same.*

With simple computations, it follows that

$$\left\{ \begin{array}{c} \dot{\mathbf{z}} \\ \dot{\mathbf{z}}_r \end{array} \right\} = \left[\begin{array}{cc} \mathbf{A} - k\mathbf{B}\mathbf{C} & k\mathbf{B}\mathbf{C}_r \\ \mathbf{0} & \mathbf{A}_r \end{array} \right] \left\{ \begin{array}{c} \mathbf{z} \\ \mathbf{z}_r \end{array} \right\} + \left[\begin{array}{c} \mathbf{B} \\ \mathbf{B}_r \end{array} \right] \delta_g \ . \tag{27.54}$$

Assume a unit value for k *(note that* k *in this case is not non-dimensional, but has the dimension of a time. It must then be said that* $k = 1s$*). Using symbols* $\mathbf{A}_{cl}$ *and* $\mathbf{B}_{cl}$ *for the matrices in Eq. (27.54), the steady state response is*

$$\left\{ \begin{array}{c} \dot{\mathbf{z}} \\ \dot{\mathbf{z}}_r \end{array} \right\}_{ss} = -\mathbf{A}_{cl}^{-1}\mathbf{B}_{cl}\delta_g. \tag{27.55}$$

The trajectory curvature gain and the sideslip angle gain are

$$\frac{1}{R\delta_g} = -\frac{1}{V} \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array} \right] \mathbf{A}_{cl}^{-1}\mathbf{B}_{cl}. \tag{27.56}$$

$$\frac{\beta}{\delta_g} = -\frac{1}{V} \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array} \right] \mathbf{A}_{cl}^{-1}\mathbf{B}_{cl}.$$

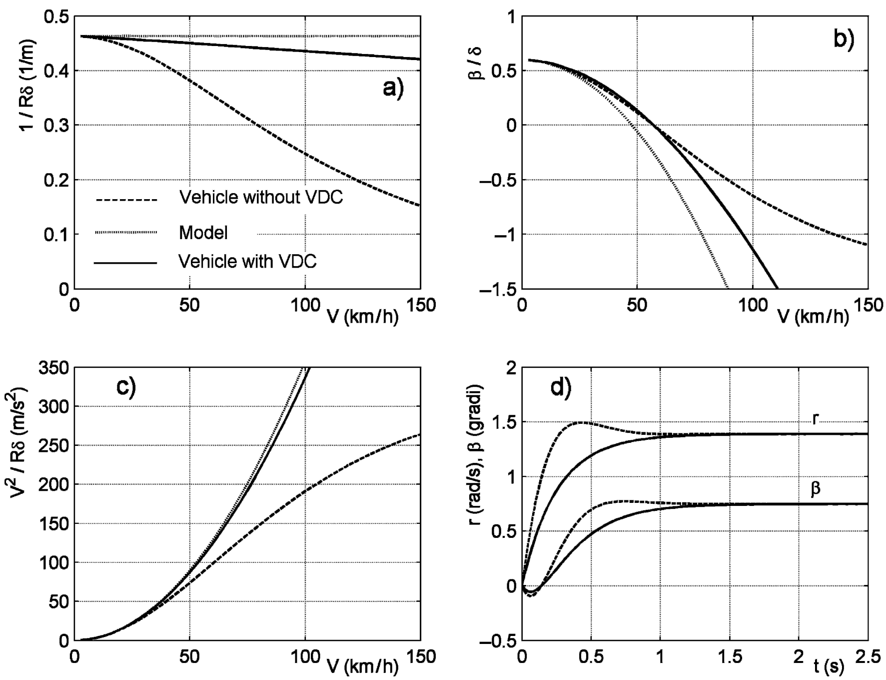

FIGURE 27.13. Trajectory curvature gain (a), sideslip angle gain (b) and lateral acceleration gain (c) as functions of the speed for the vehicle of example 25.4 for the reference model (neutral steer) and for the vehicle with VDC. In (d) the response to a step steering input to insert the vehicle on a trajectory with 200 m radius is shown.

The trajectory curvature gain, the sideslip angle gain and the lateral acceleration gain are plotted as functions of the speed in Fig 27.13. The reference model is that of a neutral steer vehicle with the same geometry and front tires.

From the trajectory curvature gain it is clear that the vehicle has an almost neutral steer behavior, but a proportional control cannot completely compensate for stationary conditions, as expected. To produce an exactly neutral behavior at least a proportional integrative (PI) control must be used.

At a speed of 100 km/h the original vehicle, strongly understeer, has complex conjugate poles. From the model with two degrees of freedom it follows that

$$s_{1,2} = -5.1915 \pm 4.5730i.$$

The poles of the vehicle with VDC are obviously 4, because the model has 4 states, and their values are

$$s_{1,..4} = \left[\begin{array}{cccc} -3.7585 & -4.8822 & -7.7146 & -48.8114 \end{array} \right].$$

Even if a much simplified control has been used, and no optimization of the gain has been attempted, the vehicle is stable.

To obtain a trajectory with a radius of 200 m the vehicle without VDC must have a steering angle $\delta_g = 0.0202$ *rad* $= 1.159°$*, values that coincide with those of Example 25.4. To obtain the same curvature with the VDC system, the steering angle must be* $\delta_g = 0.0115$ *rad* $= 0.658°$*. The time history of the response is shown in Fig 27.13d). The behavior of the understeer vehicle is oscillatory, while that of the controlled vehicle is not.*

27.5 SUSPENSIONS CONTROL

Performance of passive suspensions (i.e. suspensions made of springs and dampers) both in terms of comfort and handling is limited and, as stated in Chapter 26, any improvement of the first is often accompanied by a worsening of the latter and vice versa. The role of damping in the optimization of the handling and comfort characteristics is shown in Fig. 26.17. The figure is here shown in a qualitative way, with its scales starting at zero to show the range in which the parameters span (Fig. 27.14).

The curve is the lower envelope of the performance that may be obtained with passive suspensions, while the zone between the horizontal and vertical tangents shown in the figure is the locus of the points defining optimal performance.

Moreover, as stated earlier, the kinematic errors that are present in all types of suspensions make it impossible to use very soft springs, because they will lead to larger suspension movements and thus to larger unwanted motions of the unsprung mass (changes of track, steering and camber angles, etc., with heave and roll motion).

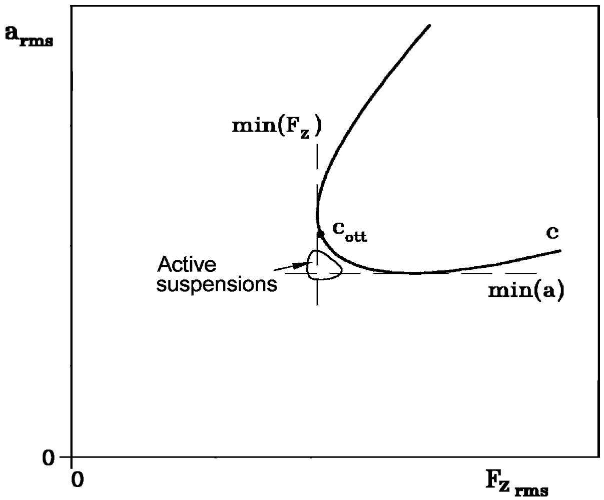

FIGURE 27.14. r.m.s. value of the acceleration of the sprung mass versus the r.m.s. value of the oscillating component of the force on the ground for a quarter car excited by a white noise in velocity. Lower envelope for passive suspensions and zone for active suspensions.

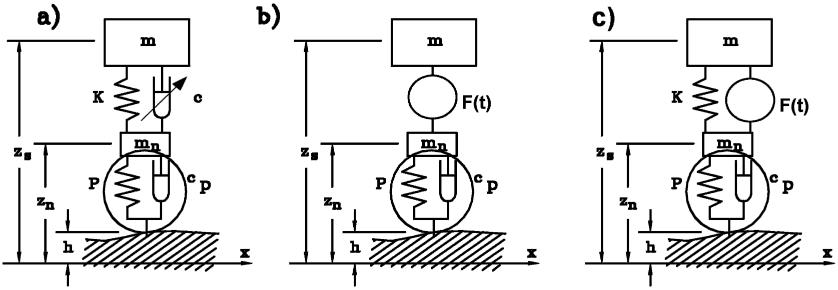

FIGURE 27.15. Quarter car with semiactive (a) suspension (the damping coefficient c is variable in a controlled way) and active suspensions with a controlled actuator supplying all the force exerted between the sprung and unsprung masses with no other mechanical device (b) or with the help of a spring (c).

To improve the situation, suspensions controlled by suitable devices have been studied and sometimes applied in mass produced vehicles. As shown in Section 5.3 in Part I, it is possible to distinguish between *semi-active* suspensions (Fig. 27.15a) and *active suspensions* (Fig. 27.15 b and c) depending on whether the control system simply controls the parameters of the system (usually damping) with limited energy requirements, or directly controls the force exerted by the suspension on the sprung mass, using actuators of a different type that involve a relatively large power consumption.

The performance of semi-active suspensions lies in Fig. 27.14 on the same curve shown for passive suspensions, with the difference that while the latter are represented by a single point, the former may move on the curve when the value of the damping coefficient is changed. It is also possible to imagine semi-active suspensions where the stiffness of the spring can also be changed, as in the system in which the attitude of the vehicle is controlled, shown in the referenced section of Part I. However, devices of this kind usually require much more power than that needed to control damping.

In the case of active suspensions, it is possible to have performance represented by points lying below the envelope, in the zone shown in the plot. In the figure such a zone is very small, because it has been assumed that the stiffness of the suspension is the same as that of the passive suspension, but it may be much larger if a fully active suspension is used.

Another important parameter is the maximum frequency at which the active system operates. The practical difficulties and the power needed in an active system increase with increasing frequency. If the suspension must control the attitude of the vehicle and the frequency at which it works is low, on the order of the frequency of the sprung mass, the system is relatively straightforward. If the control must operate at frequencies on the order of the unsprung masses, or even higher, the systems becomes more complicated.

27.5.1 Active roll control (ARC)

As stated in the chapter on comfort, roll stiffness is in general too low, and roll motions are above all too little damped. Adding anti-roll bars solves the problem only partially, because while limiting the roll angle in static conditions, it makes roll motions even more underdamped. anti-roll bars are actually used on one axle only, to increase load shift on that axle and to decrease it correspondingly on the other. In this way, it is possible to modify the directional behavior, particularly in conditions close to the limit.

Active Roll Control (ARC) has the advantage of reducing roll on a curved path, while increasing roll damping or modifying handling characteristics, depending on the control strategies used, without the drawbacks involved in an increase of the roll stiffness of the vehicle.

First consider roll in quasi-static conditions. If the center of mass is above the roll center, as is almost always the case, the vehicle rolls toward the outside of the path. This is inadvisable both for stability (because the center of mass moves outwards, reducing the rollover factor – making rollover easier – and increasing load shift) and for comfort. Moreover, because of the rolling condition, the wheels take a camber angle that results in camber forces usually directed against the sideslip forces that bend the trajectory. The last effect depends strongly on the type of suspensions used and particularly on their camber recovery.

Even if suspensions were infinitely stiff, the subjective feeling of the passengers when rounding a turn would be that their bodies were inclined outward, because the local vertical direction is inclined with respect to the true vertical. The actual inclination of the vehicle body adds to this effect with results that may be detrimental to comfort, particularly if the lateral acceleration is high.

Roll may be controlled by an actuator exerting a torque between the body and one or more axles. The simplest scheme is that of an active anti-roll bar. Conceptually, imagine cutting the anti-roll bar of an axle, for instance, in the middle, and inserting a rotational actuator exerting a torque. An example of a device of this kind is discussed in section 6.3.4 of Part I.

With the torsional stiffness of the axle unchanged, the vehicle behaves passively if the actuator does not work or is locked. If the actuator exerts a torque to rotate the body back to a position perpendicular to the ground, the effect is an apparent increase of the roll stiffness because the angle is reduced. But the actual stiffness does not change nor does underdamping of the suspension increase, as it would if the stiffness actually had increased.

A first strategy is to cancel the roll angle, obtaining an apparently infinite stiffness. The feeling of lateral tilting of the vehicle is reduced but not cancelled, while above all the changes of track, steering, camber, etc. due to roll vanish. Because kinematic errors related to suspensions linked with roll (or better with the static component of roll) are completely cancelled, handling is improved.

From the viewpoint of mathematical modelling, rigid-body models become more precise, because the assumption that roll angle is vanishingly small holds exactly. Often, while applying this strategy, roll is compensated only up to a certain lateral acceleration. Some rolling toward the outside of the path, although smaller than usual, is accepted. This may be necessary, because the torque the actuator must exert increases with the lateral acceleration and may become quite large. Operating in this way, the roll angle can warn the driver that critical conditions are approaching, but it is doubtful whether the geometric roll angle may be separated from the apparent roll angle due to the inclination of the local vertical.

Another strategy is to overcompensate for rolling by inclining the vehicle toward the inside of the trajectory. The limit condition is defined by aligning axis z with the local vertical. This option will be discussed later in detail, in relation to tilting vehicles.

As seen when dealing with two-wheeled vehicles, the roll angle is

$$\phi = \arctan\left(\frac{V^2}{Rg}\right) . \tag{27.57}$$

There are advantages in both comfort and handling. If the control is precise and quick enough, passengers do not feel any centrifugal acceleration. In rail transportation the diffusion of tilting trains owes mostly to comfort, even if the larger radii of railways when compared to roads cause lower lateral accelerations than in motor vehicles. On the other hand, the great sensitivity of the organs of human equilibrium in detecting lateral acceleration allows us to detect a tilt of less than one degree, and an angle of just a few degrees causes a strong discomfort.

In terms of handling, if roll compensates exactly for lateral acceleration, load shift is exactly zero and all its effects are cancelled. On the other hand, the effects of kinematic suspensions errors do not vanish and may be larger, although of opposite sign, than those typical of conventional vehicles. If the suspensions keep the midplanes of the wheels parallel to the xz plane, i.e. if

$$\frac{\partial \gamma}{\partial \phi} = 1 ,$$

the position of the wheels on the ground is similar to that of the wheels of motorcycles. The strong camber forces then add to the side forces due to sideslip.

The roll angle needed to compensate for the lateral acceleration may be large. For instance, at an acceleration of 0.2 g the angle is 11° while if the acceleration is 0.5 g the angle is 27°. While angles larger than 40° are needed to compensate for the lateral acceleration in high performance cars, it is impossible to use this strategy in racing cars because angles of 70° or more would be required. The steady state torque the actuator must exert may be small even in the case of large lateral accelerations. The actuator must maintain an unstable equilibrium position If it is maintained with precision, reacting quickly to deviations, the moment the actuator must exert is theoretically very small. It is, however, impossible that the equilibrium condition be followed instant by instant with a precision sufficient to cancel all rollover moments, and the actuator may be called to exert a high torque.

Many studies on tilting vehicles based on active roll control have recently been undertaken. These were followed by prototypes and even some small scale production. The vehicles are mostly narrow, often designed for city use, and can be considered a synthesis of motorcycle and car. If the vehicle is very narrow it is possible to reach inclination angles large enough to compensate for lateral acceleration without the body of the vehicle touching the ground at the inside of the path. Many of these *tilting body vehicles* have three wheels.

The simplest model for roll control is the model with a single degree of freedom described in Section 26.6.1 (Fig.26.36). It is quite a rough model because it neglects the compliance of the tires and above all because it is impossible to study roll dynamics separately from lateral dynamics. The equation of motion of a passive vehicle is Eq. (26.119), repeated here

$$J_x\ddot{\phi} + (\Gamma_1 + \Gamma_2)\,\dot{\phi} + (\chi_1 + \chi_2)\,\phi - m_s g h_G \sin(\phi) =$$

$$= \Gamma_1\dot{\alpha}_{t_1} + \Gamma_2\dot{\alpha}_{t_2} + \chi_1\alpha_{t_1} + \chi_2\alpha_{t_2}\ ,$$

where the forcing functions are due to the lateral slope of the road α_{t_i} at the i-th suspension. J_x is the moment of inertia about the roll axis of the sprung mass alone.

If two active anti-roll bars exerting a torque M_{a_i} $(i = 1, 2)$ are added and if a generic moment M_e is included into the model, the equation of motion becomes

$$J_x\ddot{\phi} + (\Gamma_1 + \Gamma_2)\,\dot{\phi} + (\chi_1 + \chi_2)\,\phi - m_s g h_G \sin(\phi) = \tag{27.58}$$

$$= M_{a_1} + M_{a_2} + M_e + \Gamma_1\dot{\alpha}_{t_1} + \Gamma_2\dot{\alpha}_{t_2} + \chi_1\alpha_{t_1} + \chi_2\alpha_{t_2}\ .$$

If moment M_e is due to the centrifugal force on a path with radius R, its value is

$$M_e = \frac{m_s h_G V^2}{R}\cos(\phi)\ . \tag{27.59}$$

Assuming that the moment exerted by the anti-roll bar coincides with that due to the actuator, reduced to the bar through suitable transmission ratios, the stiffnesses χ_i and damping coefficients Γ_i are those of the passive suspensions. If the actuator is controlled by an ideal proportional-derivative (PD) system that uses the roll angle as error, the moments are

$$M_{a_i} = -k_{pi}\phi - k_{di}\dot{\phi}\ ,$$

where k_{pi} and k_{di} are the i-th proportional and derivative gains.

The system behaves as a passive system, with stiffness and damping increased by the gains

$$J_x\ddot{\phi} + (\Gamma_1 + \Gamma_2 + k_{d1} + k_{d2})\,\dot{\phi} + (\chi_1 + \chi_2 + k_{p1} + k_{p2})\,\phi + \tag{27.60}$$

$$-m_s g h_G \sin(\phi) = M_e + \Gamma_1\dot{\alpha}_{t_1} + \Gamma_2\dot{\alpha}_{t_2} + \chi_1\alpha_{t_1} + \chi_2\alpha_{t_2}\ .$$

Assuming that the roll angle is small, the steady state roll angle in a bend is

$$\phi = \frac{m_s h_G V^2}{R\left(\chi_1 + \chi_2 + k_{p1} + k_{p2} - m_s g h_G\right)} \ . \tag{27.61}$$

As expected, a PI control is unable to compensate for steady state roll, even if it is possible to reduce it by increasing the proportional gains. There are, however, limitations on the values of the gains, mainly for stability reasons.

The steady state torque the actuators must exert is

$$M_{a_i} = -k_{pi}\phi \ ,$$

and it grows with V^2.

The load shift on the i-th axle, with track t_i, is

$$\Delta F_{z_i} = \frac{m_s h_G V^2\left(\chi_i + k_{pi}\right)}{R t_i\left(\chi_1 + \chi_2 + k_{p1} + k_{p2} - m_s g h_G\right)} \ . \tag{27.62}$$

To compensate for the steady state roll it is possible to use a proportional-integrative-derivative (PID) control

$$M_{a_i} = -k_{pi}\phi - k_{di}\dot{\phi} - k_{ii}\int \phi dt \ .$$

By introducing the auxiliary states v_ϕ and i_ϕ, respectively the derivative and the integral of ϕ, the state-space model of the system is

$$\left\{\begin{array}{c}\dot{v}_\phi\\ \dot{\phi}\\ \dot{i}_\phi\end{array}\right\} = \left[\begin{array}{ccc}-\frac{K}{J_x} & -\frac{C}{J_x} & -\frac{D}{J_x}\\ 1 & 0 & 0\\ 0 & 1 & 0\end{array}\right]\left\{\begin{array}{c}v_\phi\\ \phi\\ i_\phi\end{array}\right\} + \tag{27.63}$$

$$+\frac{1}{J_x}\left\{\begin{array}{c}m_s g h_G \sin(\phi) + M_e + \Gamma_1\dot{\alpha}_{t_1} + \Gamma_2\dot{\alpha}_{t_2} + \chi_1\alpha_{t_1} + \chi_2\alpha_{t_2}\\ 0\\ 0\end{array}\right\},$$

where

$$\begin{array}{l}K = \chi_1 + \chi_2 + k_{p1} + k_{p2},\\ C = \Gamma_1 + \Gamma_2 + k_{d1} + k_{d2},\\ D = k_{i1} + k_{i2}.\end{array} \tag{27.64}$$

Example 27.5 *Consider the vehicle of example 26.11. The data entering the simplified roll model are J_x = 388.8 kg m^2, m_s = 1,080 kg, $\chi_1 = 11.25$ kNm/rad, $\chi_2 = 9.5$ kNm/rad, $\Gamma_1 = 955$ Nms/rad, $\Gamma_2 = 716$ Nms/rad, h_G = 0.5 m. Compute the time history of the roll angle and of the load shift of the vehicle without active systems after a step steering input to insert it on a curve with a radius of 200 m at a speed of 100 km/h = 27.7 m/s.*

Repeat the computation for a vehicle with an active anti-roll bar at the rear axle, with a PD controller with $k_{pi} = 10$ kNm/rad, $k_{di} = 3$ kNms/rad.

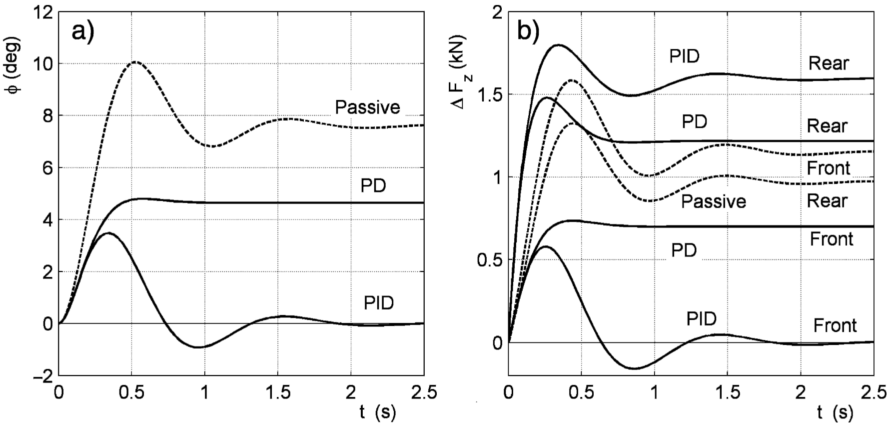

FIGURE 27.16. Time history of the roll angle (a) and of the load shift on both axles (b) for the three cases of a passive vehicle and a vehicle with an active anti-roll bar at the rear axle with PD and PID controller.

Add then an integral control with $k_{ii} = 100$ *kNm/srad. For computing load shift, assume a track of 1.3 m on both axles.*

Because the system is nonlinear, the equation of motion was integrated numerically. The results are reported in Fig.27.16(a) and (b) for the roll angle and the load shift.

Note that the load shift is larger at the front angle in the case of the passive vehicle, while the active anti-roll bar displaces it at the rear axle. The high value of the derivative gain leads to an almost non-oscillatory behavior of the PD system, while oscillations are caused by the integrative control in the case of PID.

The example is only an indication, because the model is quite rough (even a step steering input causes the lateral acceleration to grow more gradually) and because the values of the gains were assumed arbitrarily. In an actual case it would not be advisable to move the load shift to the rear axle (except for correcting understeer), because in this way the oversteer characteristics of the vehicle would increase.

Remark 27.5 *Up to now it has been assumed that the controller has used the absolute roll angle as a reference, something that requires the use of an artificial horizon like those used on aircraft. If the roll angle is measured with reference to the position of the axle, the vehicle body tends to follow the transversal slope of the road, while if the roll angle is measured with reference to the local vertical, a measurement easily performed using an accelerometer, the vehicle tends to tilt toward the inside of the bend, as already shown for tilting vehicles.*

Example 27.6 *Repeat the study of the previous example, using as a reference the roll angle measured from the local vertical. Owing to the type of input assumed, during the entire manoeuvre the local vertical makes an angle*

$$\phi_r = -artg\left(\frac{V^2}{Rg}\right) = -21.36^\circ$$

with the perpendicular to the ground.

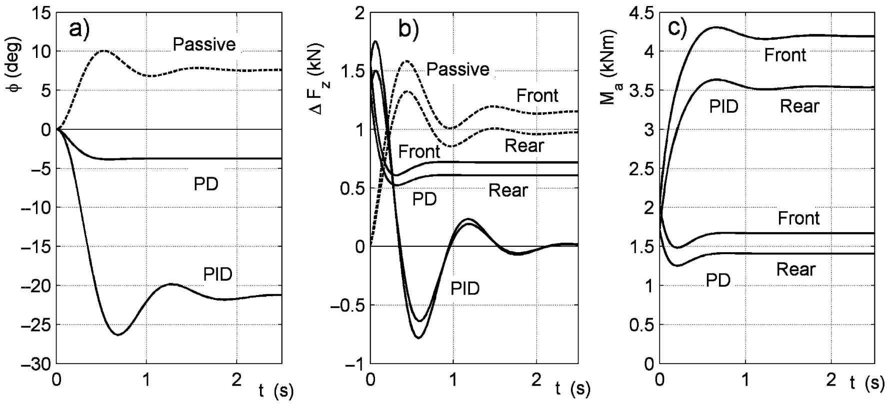

FIGURE 27.17. Time history of the roll angle (a), of the load shift on both axles (b) and of the control torques (c) for the three cases of a passive vehicle and a vehicle with an active anti-roll bar at both the rear and front axles with PD and PID controller.

Because ϕ_r is constant, the moment exerted by the actuators is

$$M_{a_i} = -k_{pi}\left(\phi - \phi_r\right) - k_{di}\dot{\phi} - k_{ii}\int\left(\phi - \phi_r\right)dt =$$

$$= -k_{pi}\phi - k_{di}\dot{\phi} - k_{ii}\int\phi dt + k_{pi}\phi_r + k_{ii}\phi_r t\ .$$

The results of the numerical integration are shown in Fig.27.17(a), (b) and (c) for the roll angle, the load transfer and the torque exerted by the actuators. To avoid too large a load shift on a single axle, two active anti-roll bars are used, with gains distributed between the axles in the same ratio as the stiffness of the suspensions. The sum of the gains on the two axles is, however, the same.

Note that the device with a PID controller succeeds in keeping the vehicle inclined toward the inside of the path so as to compensate for load transfer, even if the actuators must exert large torques. This is due to the fact that they must balance the torques due to the suspension springs, which try to keep the vehicle upright: if the control system is used to keep the vehicle inclined in the curve like a motorcycle, no passive stiffness must be put in series to the actuator. If the suspension must be passively stable with non-working actuators, the suspension springs must be put in series and not in parallel to the actuators.

27.5.2 Heave control

Quarter car with ideal *skyhook*

Consider a quarter car with two degrees of freedom, like that shown in Fig. 26.7b. A damper located not between the two masses but between the sprung mass and

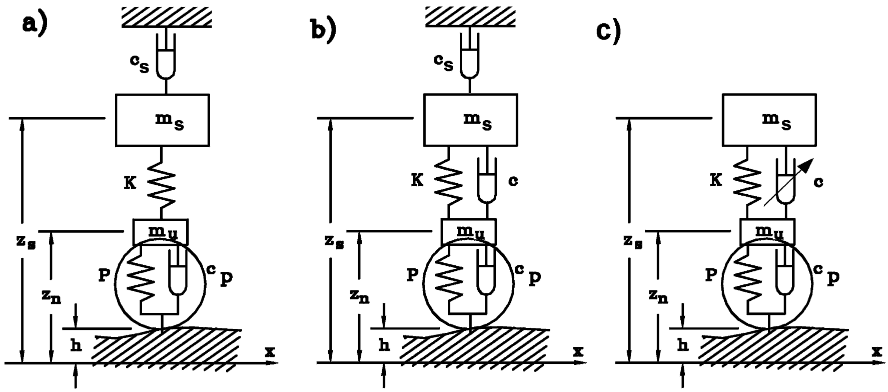

FIGURE 27.18. Quarter car model with two degrees of freedom with skyhook. a) and b): Ideal skyhook, on a quarter car without and with shock absorber between the two masses. c): Practical implementation, in which the ideal skyhook is approximated by a controlled damper located between the two masses (semi-active solution). If the controlled damper is substituted by an actuator, an active solution is obtained.

a fixed point would be needed to damp the motion of the vehicle body in an optimal way. A model with this configuration, usually referred to as *skyhook*[8], is shown in Fig. 27.18 a) or b). The point to which the damper is attached is fixed in an inertial frame; the damper substitutes for the conventional shock absorber in the first scheme, while it is added to it in the second.

With reference to Fig. 27.18 a), the equation of motion is

$$\begin{aligned}&\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix}\left\{\begin{array}{c} \ddot{z}_s \\ \ddot{z}_u \end{array}\right\}+\begin{bmatrix} c_s & 0 \\ 0 & c_p \end{bmatrix}\left\{\begin{array}{c} \dot{z}_s \\ \dot{z}_u \end{array}\right\}+ \\ &+\begin{bmatrix} K & -K \\ -K & K+P \end{bmatrix}\left\{\begin{array}{c} z_s \\ z_u \end{array}\right\}=\left\{\begin{array}{c} 0 \\ c_p\dot{h}+Ph \end{array}\right\},\end{aligned} \tag{27.65}$$

where z_s and z_u are the displacements with respect to the static equilibrium position and are measured in an inertial reference frame.

By introducing the non-dimensional ratios

$$a=\frac{m_u}{m_s}\ ,\quad b=\frac{P}{K}\ ,\quad \zeta_s=\frac{c_s}{2\sqrt{m_sK}}\ ,\quad \zeta_p=\frac{c_p}{2\sqrt{m_sK}}\ , \tag{27.66}$$

[8]The term *skyhook* as used in automotive technology must not be confused with the same term used in aerospace, indicating a long (and hypothetical) cable system attached to a planet (e.g. the Earth), extending beyond synchronous orbit and rotating at the same speed as the planet.

the equation of motion may be written as

$$m_s \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{Bmatrix} + 2\sqrt{m_s K} \begin{bmatrix} \zeta_s & 0 \\ 0 & \zeta_p \end{bmatrix} \begin{Bmatrix} \dot{z}_s \\ \dot{z}_u \end{Bmatrix} + \\ + K \begin{bmatrix} 1 & -1 \\ -1 & 1+b \end{bmatrix} \begin{Bmatrix} z_s \\ z_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ c_p \dot{h} + Ph \end{Bmatrix}. \tag{27.67}$$

By neglecting the damping of the tire c_p, which is usually quite small, the amplification factor of the sprung and unsprung masses are

$$\frac{|z_{s_0}|}{|h_0|} = PK \sqrt{\frac{1}{f^2(\omega) + c_s^2 \omega^2 g^2(\omega)}} \tag{27.68}$$

where

$$\begin{cases} f(\omega) = m_s m_u \omega^4 - [P m_s + K(m_s + m_u)] \omega^2 + KP \\ g(\omega) = m_u \omega^2 - K - P \, . \end{cases}$$

If the shock absorber between the two masses is still present (Fig. 27.18 b), the equation of motion is

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{Bmatrix} + \begin{bmatrix} c + c_s & -c \\ -c & c + c_p \end{bmatrix} \begin{Bmatrix} \dot{z}_s \\ \dot{z}_u \end{Bmatrix} + \\ + \begin{bmatrix} K & -K \\ -K & K+P \end{bmatrix} \begin{Bmatrix} z_s \\ z_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ c_p \dot{h} + Ph \end{Bmatrix}, \tag{27.69}$$

i.e.

$$m_s \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{Bmatrix} + 2\sqrt{m_s K} \begin{bmatrix} \zeta_s + \zeta & -\zeta \\ -\zeta & \zeta_p + \zeta \end{bmatrix} \begin{Bmatrix} \dot{z}_s \\ \dot{z}_u \end{Bmatrix} + \\ + K \begin{bmatrix} 1 & -1 \\ -1 & 1+b \end{bmatrix} \begin{Bmatrix} z_s \\ z_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ c_p \dot{h} + Ph \end{Bmatrix}. \tag{27.70}$$

The frequency response of a quarter car with skyhook is plotted in non-dimensional form in Fig. 27.19. The figure is similar to Fig. 26.12a and c, and was obtaining using the same values of the non-dimensional parameters $b = P/K = 4$, $a = m_u/m_s = 0.1$. The response of the quarter car with skyhook with $\zeta_s = 1$ is compared with that of a conventional quarter car with optimum damping (Eq. (26.29)) and to a quarter car with skyhook plus a damper between the two masses with damping equal to 1/3 of the optimum value.

The amplification factor of the unsprung mass is shown in Fig. 27.19a, while the inertance is reported in Fig. 27.19b. The presence of the skyhook greatly reduces the displacement and the acceleration at low frequency, while causing a very high, although not infinite, resonance peak at the resonant frequency of the

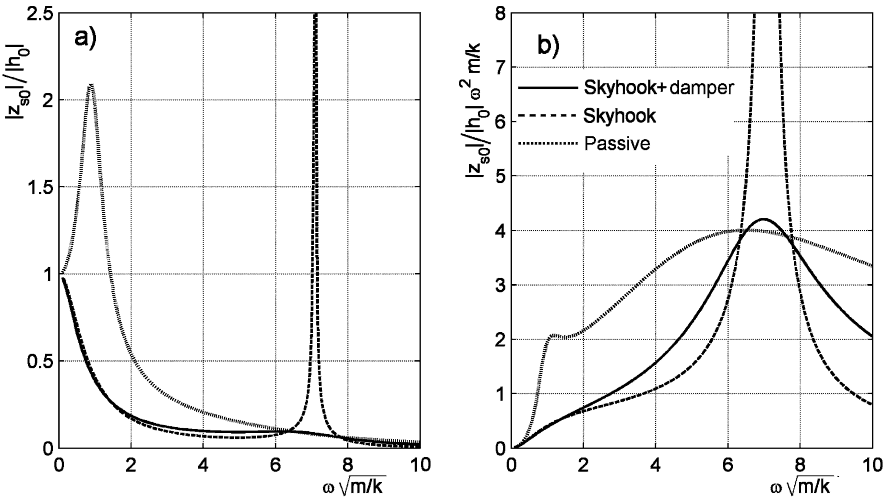

FIGURE 27.19. Non-dimensional frequency response of the sprung mass of a quarter car with two degrees of freedom. Non-dimensional parameters: $b = P/K = 4$ and $a = m_u/m_s = 0.1$.The response of a passive system with optimum damping ($\zeta = 0.433$) is compared with that of a system with skyhook ($\zeta_s = 1$) and with skyhook with the same value of ζ_s plus a damper between the two masses with damping equal to 1/3 of the optimum value, ($\zeta = 0.1443$).

unsprung mass. The peak is fairly narrow and in theory can easily be reduced, because when c_s tends to infinity the response tends to 0. It is, however, impossible to increase the damping of the skyhook indefinitely, with the result that the peak cannot be eliminated with this configuration. If a damper, even a small one, is added between the two masses, the resonance peak in the medium frequency range disappears without greatly changing the response at low frequency.

A skyhook damper of this kind is quite effective in controlling the motion of the sprung mass, but the control of the unsprung mass is unsatisfactory. The amplification factors of the displacement and the acceleration of the unsprung mass are shown in Fig. 27.20 a and b: The skyhook has practically no effect on the motion of the unsprung mass at frequencies close to its resonance. However, the presence of a damper between the two masses strongly reduces the displacement and the acceleration of the unsprung mass. In the figure, the damping of the conventional damper is quite low, being about one third of the optimum; if it is increased the height of the resonance peak decreases, disappearing when the optimum value is reached. In these conditions there is a small increase of the response at low frequency.

The ideal skyhook is, then, an ideal solution to control the low frequency motions of the sprung mass, but it is only a reference solution, because it cannot be implemented in practice.

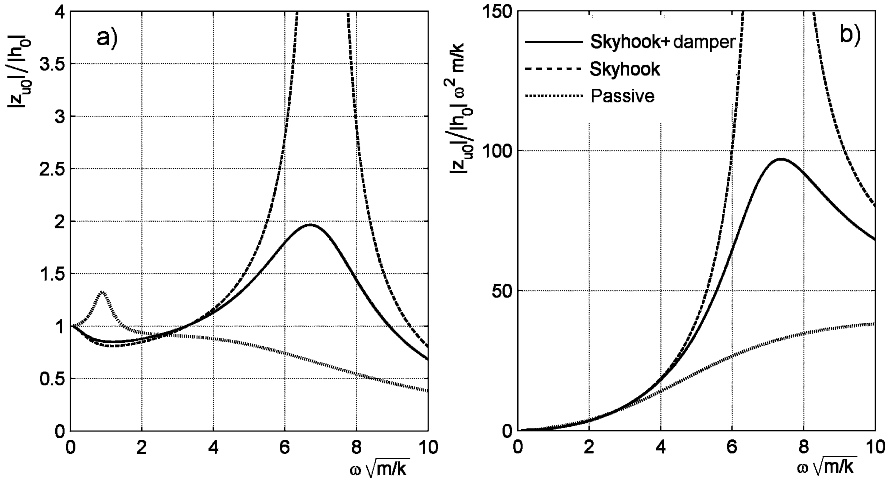

FIGURE 27.20. Non-dimensional frequency response of the unsprung mass of the same quarter car with two degrees of freedom studied in Fig. 27.19.

Semi-active quarter car with 'real world' skyhook

The fixed point where the skyhook damper is attached does not exist in the real world. This strategy must therefore be implemented using a device located between the sprung and the unsprung masses. The semi-active solution, based on a damper with controllable damping coefficient, is shown in Fig. 27.18c and is described in Section 6.3.2. of Part I.

The controlled damper must supply a force

$$F = -c_s\dot{z}_s - c\left(\dot{z}_s - \dot{z}_u\right), \tag{27.71}$$

that is

$$F = -\left(c_s\frac{\dot{z}_s}{\dot{z}_s - \dot{z}_u} + c\right)\left(\dot{z}_s - \dot{z}_u\right). \tag{27.72}$$

Theoretically, it should be possible to implement a device able to simulate the skyhook simply by modulating the damping coefficient of the damper so that it is, in each instant, equal to

$$c_{eq} = c_s\frac{\dot{z}_s}{\dot{z}_s - \dot{z}_u} + c\ . \tag{27.73}$$

Actually, even operating in this way, only an approximation of the ideal skyhook can be obtained because, even if the forces it exerts on the sprung mass are those of the ideal device, the forces exerted on the unsprung mass are different.

Remark 27.6 *Equation (27.73) cannot be implemented by a passive device. When the equivalent damping coefficient is positive, the device dissipates energy,* *something a passive system can do, but when c_{eq} is negative the damper should introduce energy into the system, requiring an active device to be used.*

An approximated but simple solution is to use two different values of the damping coefficient. One is high, and is used when the equivalent damper simulating the skyhook must dissipate energy, i.e. when $\dot{z}_s$ and $\dot{z}_s - \dot{z}_u$ have the same sign (their product is positive). The other is very low, even approaching zero, and is used when the damper should introduce energy into the system ($\dot{z}_s$ and $\dot{z}_s - \dot{z}_u$ have opposite sign, i.e. their product is negative). This method is simple because it only requires a damper with two different values of the damping coefficient. It may be obtained, for instance, with a standard shock absorber with suitable valves to control the motion of the fluid, or by using electrorheological or magnetorheological fluids. The control is also simple, because it requires only an on-off system, while greater difficulties can arise from the measurement of the absolute velocity of the sprung mass.

Other solutions are based on a linear variation of the damping coefficient with ratio $\dot{z}_s/\left(\dot{z}_s - \dot{z}_u\right)$. The damping coefficient is nonetheless set to zero or to a very small positive value when it should be negative.

An approach of this kind leads to a nonlinear behavior of the system, making it impossible to make a general comparison between the behavior of this device and that of other suspensions.

Active quarter car with ‘real world’ skyhook

An active system, able to transfer energy to the system, is needed to follow the law (27.73). As usually stated, a device operating on four quadrants must be used. This expression comes from the force-velocity plot of the damper: All the conditions in which a passive system can operate lie in the second and fourth quadrants, i.e. in the quadrants where force and velocity have opposite signs (if the force is that exerted by the damper to one of its end points and the velocity is that of the same point while the other is constrained). An active system may also exert forces with the same sign of the velocity, in which case it works in all quadrants.

Consider for instance the quarter car of Fig. 26.7c, where the damper with controllable damping is substituted by an actuator operating on four quadrants. By neglecting the damping of the tires, the equation of motion is

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \left\{ \begin{array}{c} \ddot{z}_s \\ \ddot{z}_u \end{array} \right\} + \begin{bmatrix} K & -K \\ -K & K+P \end{bmatrix} \left\{ \begin{array}{c} z_s \\ z_u \end{array} \right\} = \left\{ \begin{array}{c} F \\ -F \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ Ph \end{array} \right\}, \tag{27.74}$$

where F is the force exerted by the actuator on the sprung mass, that is

$$m_s \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \left\{ \begin{array}{c} \ddot{z}_s \\ \ddot{z}_u \end{array} \right\} + K \begin{bmatrix} 1 & -1 \\ -1 & 1+b \end{bmatrix} \left\{ \begin{array}{c} z_s \\ z_u \end{array} \right\} = \left\{ \begin{array}{c} F \\ -F \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ Ph \end{array} \right\}. \tag{27.75}$$

To simulate the skyhook, such a force must be (Eq. (27.73)):

$$F = -c_s \dot{z}_s - c\left(\dot{z}_s - \dot{z}_u\right) = -\left(c_s + c\right)\dot{z}_s + c\dot{z}_u \ . \tag{27.76}$$

The system is equivalent to an ideal proportional control on the states (velocities $\dot{z}_s$ and $\dot{z}_u$ are two of the states of the system). Clearly, this is an idealized system, both because the gains are considered as constants and because in practice it is impossible to measure the absolute velocity of the sprung mass directly. A system of this kind may be approximated using a state observer.

By introducing Eq. 27.76 into Eq. 27.75, it follows

$$m_s \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \left\{ \begin{array}{c} \ddot{z}_s \\ \ddot{z}_u \end{array} \right\} + 2\sqrt{K m_s} \begin{bmatrix} \zeta_s + \zeta & -\zeta \\ -\left(\zeta_s + \zeta\right) & \zeta \end{bmatrix} \left\{ \begin{array}{c} z_s \\ z_u \end{array} \right\} + \tag{27.77}$$
$$+K \begin{bmatrix} 1 & -1 \\ -1 & 1+b \end{bmatrix} \left\{ \begin{array}{c} z_s \\ z_u \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ Ph \end{array} \right\}.$$

As predicted, the damping matrix is not symmetrical, because there is no Raleigh dissipation function able to express the damping of the system.

Consider the quarter car studied in figures 27.19 and 27.20. Because the system is active, its stability must first be checked. The poles of the system are

$$s = \left\{ \begin{array}{l} -1.5732 \pm 6.4207\ i \\ -1.3519, \\ -0.6771. \end{array} \right.$$

All poles have a negative real part, hence the system is stable. One of the poles has a non-vanishing imaginary part, and the behavior of the system is oscillatory.

The frequency response of the sprung mass (in terms of displacement and acceleration) is shown in Fig. 27.21. The response of the quarter car with a skyhook of this kind is not essentially different from that with an ideal skyhook.

The frequency response of the unsprung mass is shown in Fig. 27.22. The response of the quarter car is in this case much different from that with an ideal skyhook and, more significantly, is unsatisfactory. To solve this problem a larger value of ζ is needed. Note that damping between the two masses may be supplied by a passive damper, leaving the active system with only the task of simulating the skyhook.

Quarter car with *groundhook*

The very concept of skyhook was introduced to minimize the vertical accelerations of the sprung mass, with no consideration about the motion of the unsprung mass. It is not surprising, then, that the motion of the unsprung mass is too large and the corresponding variations of the force on the ground are unacceptable. As said in Section 7.4.2, the variable component of the vertical force on the ground F_z may be approximated by neglecting the damping of the tire

$$F_z = -P(z_u - h) \ .$$

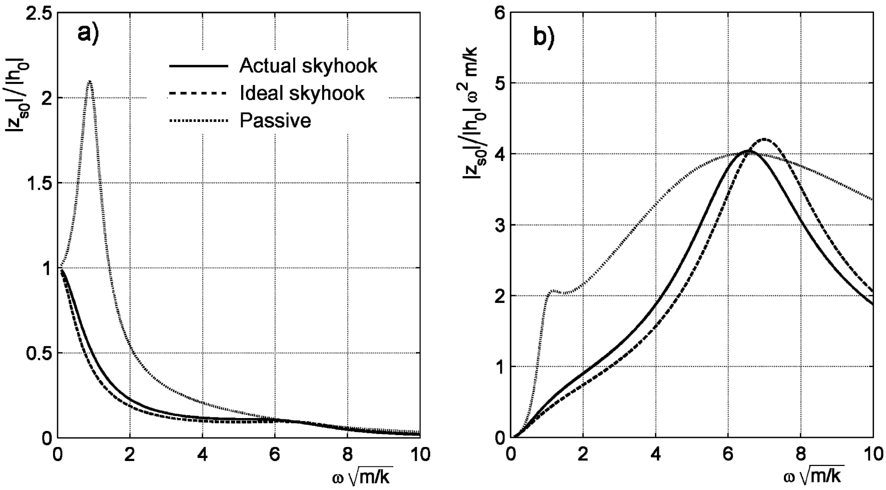

FIGURE 27.21. Non-dimensional frequency response of the sprung mass of a quarter car with two degrees of freedom with $b = P/K = 4$ and $a = m_u/m_s = 0.1$. The response of a passive quarter car with optimum damping ($\zeta = 0.433$) is compared with one with an ideal skyhook ($\zeta_s = 1$, ζ equal to 1/3 of the optimum value, $\zeta = 0.1443$) and with an actual skyhook with the same values of the parameters.

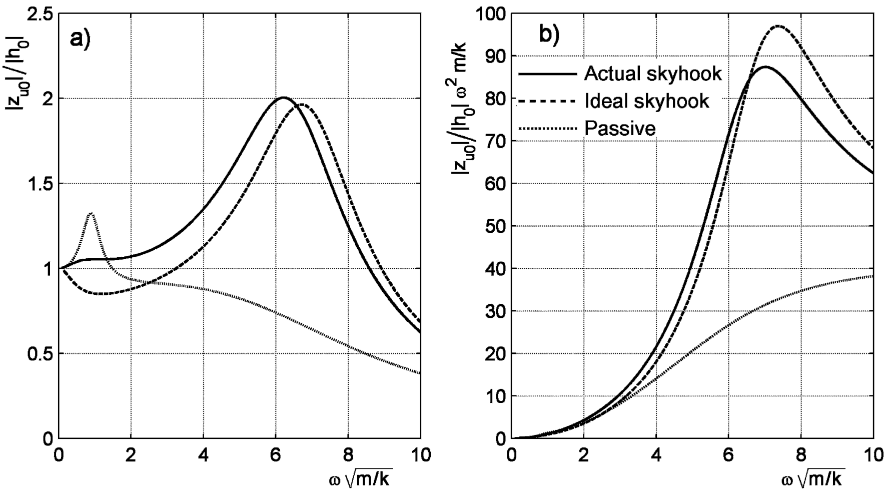

FIGURE 27.22. Frequency response of the unsprung mass of the same quarter car of Fig. 27.21.

The frequency response in terms of tire-ground force of the quarter car of Fig. 27.21 is shown in Fig. 27.23. As can be clearly seen, there is a strong improvement at low frequency, but at high frequency things are much worse.

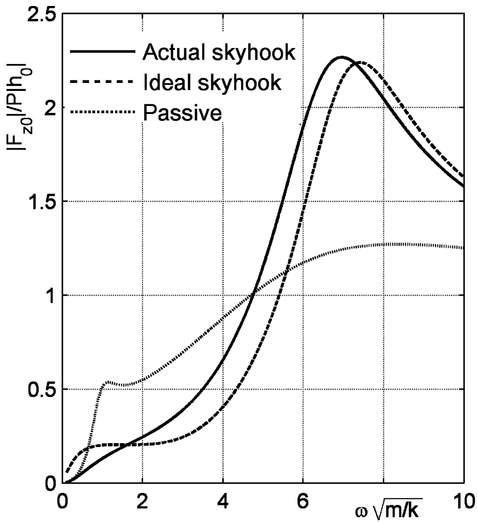

FIGURE 27.23. Frequency response in terms of force on the ground of the same quarter car of Fig. 27.21.

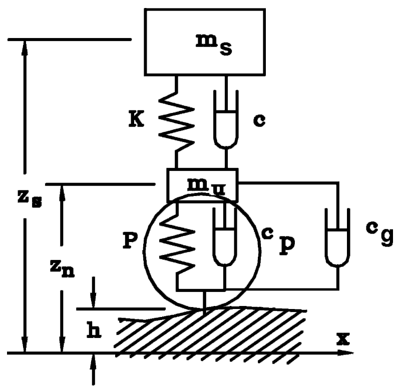

FIGURE 27.24. Quarter car with two degrees of freedom with a damper ideally located between the unsprung mass and the contact point with the ground (*groundhook*).

To minimize the variable component of the force on the ground either the stiffness of the tire or the deformation (z_u-h) must be reduced. To reduce the latter a damper between the unsprung mass and the contact point with the ground may be ideally introduced (Fig. 27.24). Using a designation similar to the skyhook, this approach is usually referred to as *groundhook*.

It is clear from the figure that introducing a groundhook is equivalent to increasing the damping of the tire. Actually, it impossible to do so in an actual situation, and not only for practical reasons. Because the tire rotates, an increase of c_p would cause an unacceptable increase of rolling resistance, accompanied by a strong heating of the tire.

The equation of motion of the system of Fig. 27.24 is

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c + c_p + c_g \end{bmatrix} \begin{Bmatrix} \dot{z}_s \\ \dot{z}_u \end{Bmatrix} + \\ + \begin{bmatrix} K & -K \\ -K & K + P \end{bmatrix} \begin{Bmatrix} z_s \\ z_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ (c_p + c_g)\,\dot{h} + Ph \end{Bmatrix}. \tag{27.78}$$

Here it is not only the homogeneous part of the equation that is affected: The forcing function is also changed. Again, this approach is ideal and in practice may be realized using only the scheme of Fig. 27.18 c), where an active or semi-active device is located between the two masses. If an active system is used, the force exerted on the unsprung mass is

$$F = -c_g \dot{z}_u \ . \tag{27.79}$$

In this way, not only is an equal and opposite force exerted on the sprung mass, but the effect linked with the vertical motion of the contact point on the ground is lost. The equation of motion is not Eq. (27.78), but becomes

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{Bmatrix} + \begin{bmatrix} c & -c - c_g \\ -c & c + c_p + c_g \end{bmatrix} \begin{Bmatrix} \dot{z}_s \\ \dot{z}_u \end{Bmatrix} + \\ + \begin{bmatrix} K & -K \\ -K & K + P \end{bmatrix} \begin{Bmatrix} z_s \\ z_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ c_p\dot{h} + Ph \end{Bmatrix}. \tag{27.80}$$

Note as well that the damping matrix is not symmetrical.

A combination of the two strategies may be implemented introducing the following damping matrix

$$\mathbf{C} = 2\sqrt{Km_s} \begin{bmatrix} \zeta_s + \zeta & -\left(\zeta_g + \zeta\right) \\ -\left(\zeta_s + \zeta\right) & \zeta_g + \zeta + \zeta_p \end{bmatrix}. \tag{27.81}$$

Because $\mathbf{C}$ is not symmetrical, it can be subdivided into a symmetric and a skew-symmetric matrix or, a more expedient procedure, in a matrix corresponding to a passive damper with constant damping coefficient and a matrix corresponding to an active system.

By introducing the mean and the deviatoric values of the damping of the skyhook and the groundhook

$$\zeta_m = \frac{\zeta_s + \zeta_g}{2} \ , \quad \zeta_d = \frac{\zeta_s - \zeta_g}{2} \ , \tag{27.82}$$

it follows that

$$\mathbf{C} = 2\sqrt{Km_s} \begin{bmatrix} \zeta_0 & -\zeta_0 \\ -\zeta_0 & \zeta_0 + \zeta_p \end{bmatrix} + 2\sqrt{Km_s} \begin{bmatrix} \zeta_d & \zeta_d \\ -\zeta_d & -\zeta_d \end{bmatrix}, \tag{27.83}$$

where the passive damping is

$$\zeta_0 = \zeta + \zeta_m = \zeta + \frac{\zeta_s + \zeta_g}{2}. \tag{27.84}$$

The dynamic behavior of the quarter car is then determined by a single parameter, ζ_d, that states the entity of force

$$F = -2\sqrt{Km_s}\zeta_d\left(\dot{z}_s + \dot{z}_u\right) \tag{27.85}$$

the actuator exerts on the sprung mass. A positive value of ζ_d shows that the skyhook effect dominates, while a negative value shows that the system is primarily a groundhook. For instance, the suspension with skyhook of Fig. 27.21 is equivalent to a suspension with $\zeta_0 = 0.6443$ and $\zeta_d = 0.5$.

To compare the two control strategies, consider the same quarter car of Fig. 27.21, choosing a value $\zeta_0 = 0.433$, corresponding to the optimal value of the passive suspension, and two values of ζ_d, equal to 0.4 and -0.4 (Fig. 27.25). In the first case, corresponding to a skyhook, the performance of the active suspension is better than that of a passive system for the sprung mass, in a low frequency range. In the second case (groundhook), on the other hand, performance is improved at high frequencies, particularly if the unsprung mass is considered.

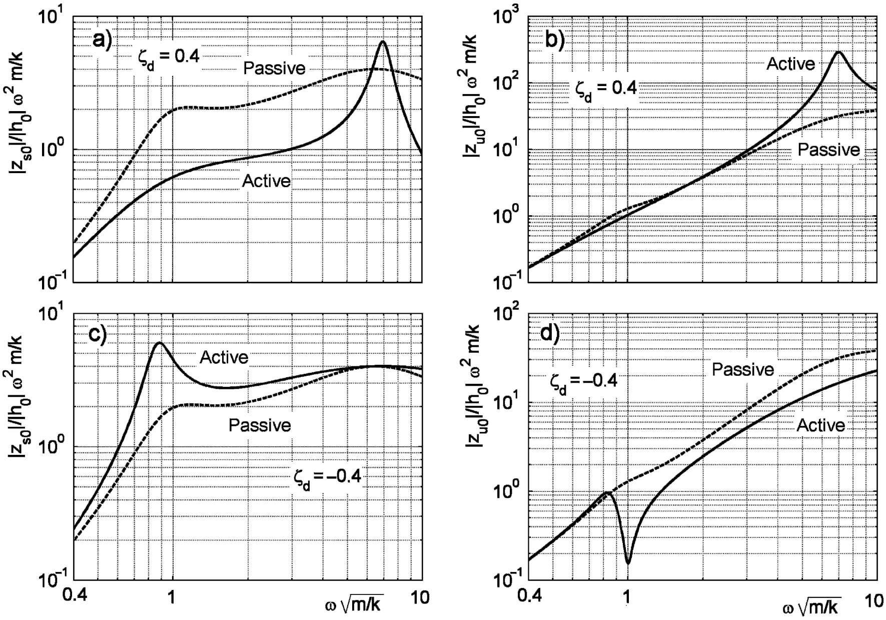

FIGURE 27.25. Non-dimensional frequency response of the sprung (a,c) and unsprung (b, d) mass of a quarter car with two degrees of freedom with $b = P/K = 4$ and $a = m_u/m_s = 0.1$. The response of the passive quarter car with optimum damping ($\zeta = 0.433$) is compared with that with skyhook ($\zeta_d = 0.4$, a,b) and groundhook ($\zeta_d = -0.4$, c,d).

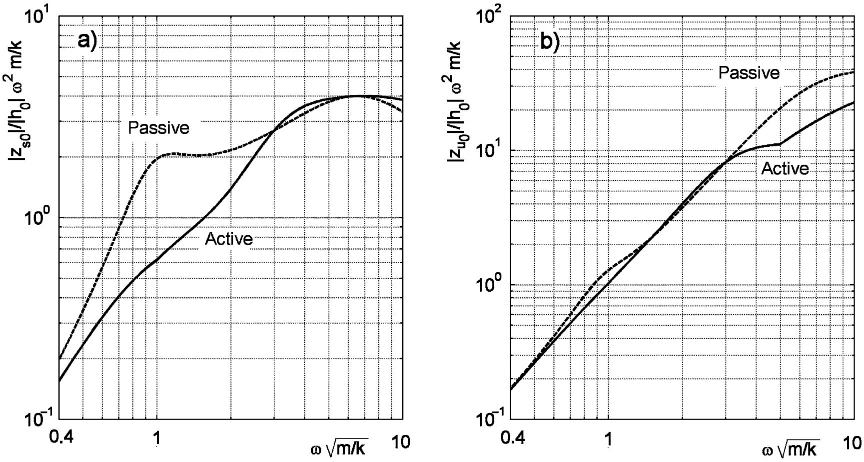

FIGURE 27.26. Non-dimensional frequency response of the sprung (a) and unsprung (b) masses of a quarter car with two degrees of freedom with $b = P/K = 4$ and $a = m_u/m_s = 0.1$. The response of a passive suspension with optimum damping ($\zeta = 0.433$) is compared with that of a quarter car with a value of ζ_d variable with the frequency between 0.4 and -0.4.

An active suspension should thus have characteristics similar to a skyhook at low frequency and to a groundhook at frequencies close to that of the unsprung mass, something that may be actually implemented if ζ_d is a function of frequency. For the quarter car of Fig. 27.25 it is possible to assume a value equal to 0.4 for non-dimensional frequencies lower than $\omega\sqrt{m/K} = 1$ and to -0.4 for frequencies higher than $\omega\sqrt{m/K} = 5$. Between these values a linear variation of ζ_d with the frequency may be assumed, so that the suspension behaves passively at $\omega\sqrt{m/K} = 3$ (Fig. 27.26).

Performance is in this case quite good in the whole frequency range both for the sprung and the unsprung masses. The variable component of the force on the ground is shown in Fig. 27.27 (the figure is similar to Fig. 26.13 plotted for passive suspensions). The strategy seen in Fig. 27.26 is also optimal from this viewpoint.

Example 27.7 *Consider the quarter car studied in Example 26.2 (m_s= 250 kg; K= 25 kN/m; c = 2,150 Ns/m m_u = 25 kg; P = 100 kN/m) and add an actuator realizing the control strategy seen above.*

Assume that the vehicle travels at a speed of 30 m/s on a road whose surface is at the limit between zones B and C following ISO standards, and compute the power spectral density of the acceleration of the sprung mass and the related r.m.s. value.

Compare the results in terms of comfort with those seen for the passive suspension.

The natural frequency of the sprung mass $\sqrt{K/m}$ is equal to 10 rad/s = 1.59 Hz, in which case the deviatoric damping of the active suspensions takes a value $\zeta_d = 0.4$ up to 1.59 Hz and $\zeta_d = -0.4$ above 7.96 Hz. Between these values it varies linearly.

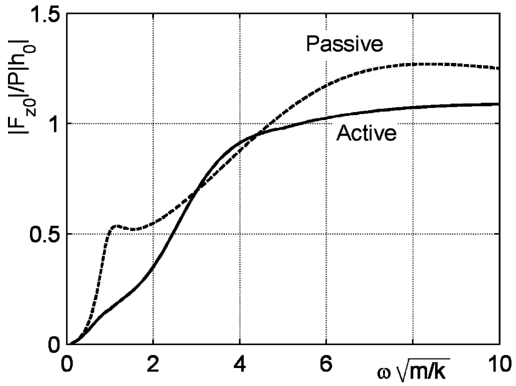

FIGURE 27.27. Variable component of the force in z direction on the ground as a function of frequency for the quarter car of Fig. 27.26.

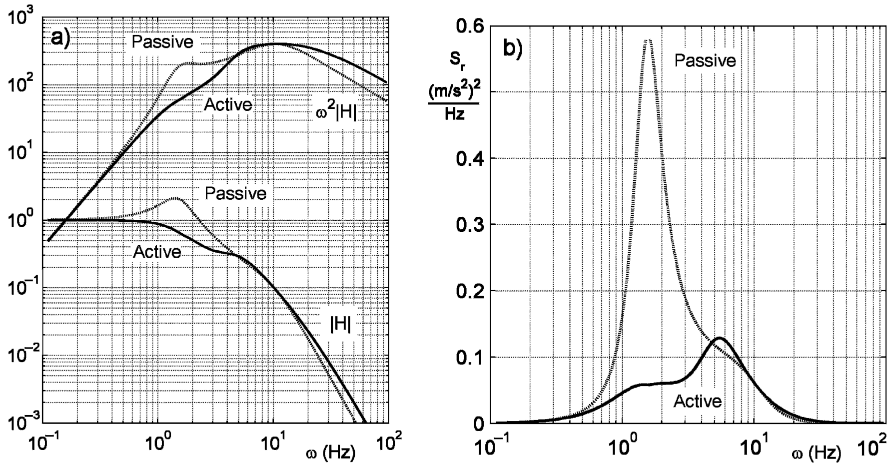

FIGURE 27.28. Dynamic compliance and inertance of the passive and active quarter car with two degrees of freedom (a). Power spectral density of the acceleration of the sprung mass while travelling on a road between zones B and C of ISO standards at a speed of 30 m/s (b).

Frequency response. *The dynamic compliance $H(\omega)$, i.e. the ratio between the displacement of the sprung mass and that of the supporting point on the ground, and the inertance, that is the ratio between the acceleration of the sprung mass and the displacement of the same point, $\omega^2 H$, are plotted in Fig. 27.28a) for both the passive and active suspension. The ability of the active suspension to filter out the perturbations reaching the sprung mass at its natural frequency is clear.*

Response to road excitation. *The power spectral density of the acceleration of the sprung mass may be computed as seen in the previous examples. The result is shown in Fig. 27.28 b). The resonance peak is completely cancelled, while at frequencies close* *to the resonance of the unsprung mass the active suspension is slightly worse than the passive. This is due to the groundhook effect, which has been introduced to reduce the variable component of the force on the ground and thus to improve handling.*

Comfort is, however, much improved, because the r.m.s. value of the acceleration is

$$a_{rms} = 1.11\ m/s^2 \ = \ 0.113\ g$$

about 20% less than in the case of the passive suspension

Quarter car controlled following the acceleration of the sprung mass

Consider a quarter car with two degrees of freedom and put an actuator in parallel to the spring and the shock absorber. By neglecting, as usual, the damping of the tire, the state space equation of the controlled system is

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}_d\mathbf{u}_d + \mathbf{B}_c\mathbf{u}_c, \tag{27.86}$$

where various vectors, the dynamic matrix and the input and gain matrices for inputs due to disturbances and control are

$$\mathbf{z} = \begin{Bmatrix} v_s \\ v_u \\ z_s \\ z_u \end{Bmatrix}, \mathbf{A} = \begin{bmatrix} -\frac{c}{m_s} & \frac{c}{m_s} & -\frac{K}{m_s} & \frac{K}{m_s} \\ \frac{c}{m_u} & -\frac{c}{m_u} & \frac{K}{m_u} & -\frac{K+P}{m_u} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_d = \begin{Bmatrix} 0 \\ \frac{P}{m_u} \\ 0 \\ 0 \end{Bmatrix}, \quad \mathbf{u}_d = h\ , \quad \mathbf{B}_c = \begin{Bmatrix} \frac{1}{m_s} \\ -\frac{1}{m_u} \\ 0 \\ 0 \end{Bmatrix}, \quad \mathbf{u}_c = F\ .$$

Consider the acceleration measured by an accelerometer on the sprung mass as output of the system. Assuming that the sensor is ideal, its output is

$$y = \mathbf{C}\mathbf{z}\ , \tag{27.87}$$

where the output gain matrix is

$$\mathbf{C} = \begin{bmatrix} -\frac{c}{m_s} & \frac{c}{m_s} & -\frac{K}{m_s} & \frac{K}{m_s} \end{bmatrix}.$$

Assuming a simple proportional control law, the control loop can be closed simply by stating that the force exerted by the actuator is proportional to the acceleration of the sprung mass through the gain G:

$$F = -Gy. \tag{27.88}$$

The dynamic equation of the system is

$$\dot{\mathbf{z}} = (\mathbf{A} - \mathbf{B}_c G\mathbf{C})\,\mathbf{z} + \mathbf{B}_d h. \tag{27.89}$$

The transfer function yielding the four components of the state vector is

$$\frac{1}{h}\mathbf{z} = (s\mathbf{I} - \mathbf{A} - \mathbf{B}_c G\mathbf{C})^{-1}\,\mathbf{B}_d. \tag{27.90}$$

The transfer function for the control force is

$$\frac{F}{h} = G\mathbf{C}\,(s\mathbf{I} - \mathbf{A} - \mathbf{B}_c G\mathbf{C})^{-1}\,\mathbf{B}_d. \tag{27.91}$$

If

$$G = m_s,$$

the first row of the closed loop dynamic matrix $\mathbf{A} - \mathbf{B}_c G\mathbf{C}$ vanishes, allowing the suspension to filter out the road irregularities completely. Such a solution, apart from problems linked with its practical feasibility, is also impossible from a theoretical viewpoint, because it would optimize the acceleration of the sprung mass but would lead to a strong increase of the dynamic component of the force on the ground.

Example 27.8 *Repeat the previous example, using a quarter car controlled on the acceleration of the sprung mass. Look for a gain of the control system that reduces the r.m.s. value of the acceleration of the sprung mass with reference to the conditions seen in the previous example (speed of 30 m/s on a road of the same type) without overly penalizing performance in term of forces on the ground or leading to high control forces.*

First, the r.m.s. values of the acceleration, the force on the ground and the control force are computed for different values of the gain. The computation is fairly long but not complicated: The transfer functions can be computed for each value of the gain through Equations (27.90) and (27.91).

Remembering that $s = i\omega$, the frequency responses for the power spectral density of the response can be computed. The required r.m.s. values are obtained by integrating the latter.

The values so computed are plotted in Fig. 27.29a. A range for the gain between 0 (passive system) and 300 Ns^2/m, was chosen. It includes the value $G = 250$ Ns^2/m, at which the response in terms of acceleration of the sprung mass vanishes.

The r.m.s. values of the force on the ground and the acceleration were made non-dimensional by dividing them by the values of the passive system, while those of the control force were divided by the value obtained for a gain $G = 300$ Ns^2/m. As predictable,

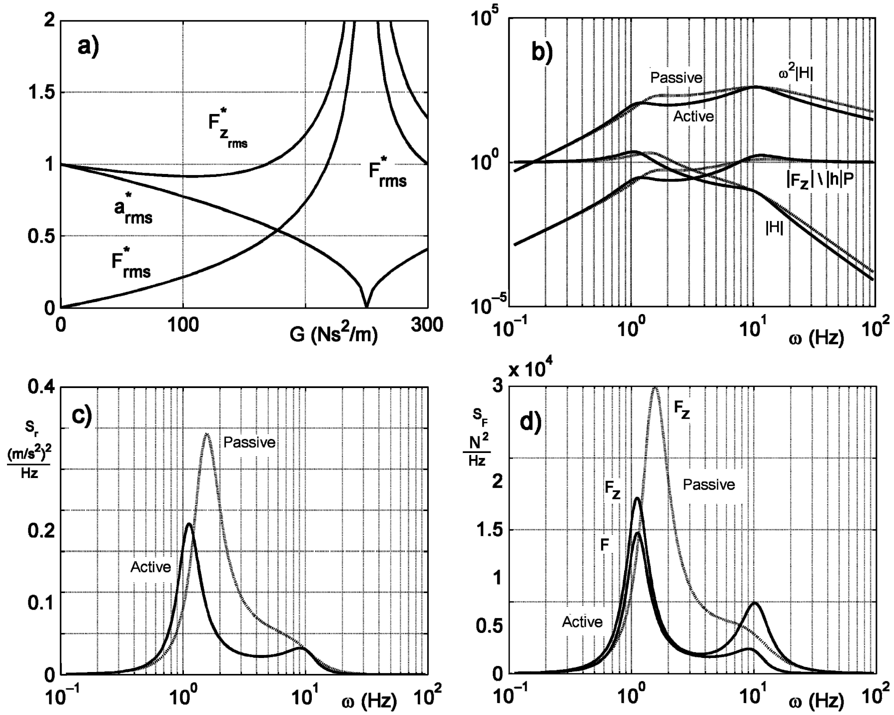

FIGURE 27.29. (a): Non-dimensional r.m.s. values of the acceleration of the sprung mass, the force on the ground and the control force at varying gain. (b): Frequency responses of the displacement and acceleration of the sprung mass and of the variable component of the force on the ground. (c) e (d): Power spectral density of the acceleration of the sprung mass and of the forces.

the acceleration decreases with increasing G, but the force has a shallow minimum to increase again, with a strong increase of the control force. A good compromise value may be

$$G = 120\ Ns^2/m.$$

With this gain

$$a_{rms} = 0.964\ m/s^2 \ = \ 0.098\ g, \ \ F_{z_{rms}} = 356\ N\ , \ \ F_{rms} = 223\ N.$$

With respect to the passive solution, the acceleration is reduced by 28% and the force on the ground by little more than 8%.

The frequency responses of the displacement and the acceleration of the sprung mass and of the variable component of the force on the ground are plotted in Fig. 27.29b, while the power spectral density of the acceleration of the sprung mass and of the forces are reported in Fig. 27.29c and d.

The results show a strong improvement of comfort performance, even if of a different kind than that plotted in the previous example. In that case, the improvement *was entirely at low frequency where the peak disappeared, while at high frequency performance was similar, if not worse, than that of the passive suspension.*

In this case, the improvement is distributed over the entire frequency range. The handling performance seems to be essentially unchanged, at least in terms of the force on the ground. This could be expected, owing to the type of control used.

Final considerations

Some control strategies for active suspensions were considered in the previous sections. It must be first stressed that this is only a theoretical study, because the strategies that may actually be implemented depend on the actual characteristics of the actuators, sensors and power systems used, and in particular on their limitations in terms of manageable power and frequency response. Moreover, all solutions considered are based on a suspension including a passive spring, while the active or semi-active systems introduce a damping force only.

This solution, often considered because it allows relatively small forces to be controlled, clearly has a basic limitation: The elastic characteristics of the suspension are those of the passive device from which it derives. The zone labelled as active suspensions in Fig. 27.14 refers to solutions of this kind. If the stiffness of the suspension can also be controlled or, even better, if the force exerted between the sprung and the unsprung mass is completely controllable, performance can be optimized in a much wider range.

The techniques used are often based on optimal control, with observers used to estimate the quantities that cannot be measured directly, but the greater difficulties are found not so much in the definition and implementation of the control algorithms, but in the implementation and in the optimization of the actuation and power systems.

The models shown above are based on the quarter car model and assume a decentralized control in which each wheel (or as is often said, each corner of the vehicle) acts in an independent way. The roll and pitch characteristics of the vehicle come from those of single suspensions, as was also seen for the passive suspensions. A decentralized control is often implemented, and the interactions between the suspensions are considered as non-modelled dynamics, as well as other phenomena that are neglected. As seen in Section 27.5.1, roll motions may be controlled independently from heave motions by using active anti-roll bars. In the same way it is possible to control pitch motions by connecting the control of the front and rear axis suspensions in what may be defined as a centralized control, even if solutions of this kind are in general not yet applied.

27.6 *By wire* systems

The active systems used on motor vehicles are usually based on hydraulic components or, particularly on industrial vehicles, on pneumatic devices. Control of the actuators is performed by electrovalves and the actuation power is supplied by pumps, which are usually powered by the engine or by electric motors. Traditionally, the same holds for systems helping the driver, such as power steering or power brakes.

The development of electromechanical components and systems has led to the consideration of alternative solutions in which electric actuators are controlled directly without the need for electrovalves, pumps and other hydraulic or pneumatic devices. The tendency to replace hydraulic and pneumatic devices with electric systems is widespread and in many fields *more-electric* or even *all-electric* systems are considered. The advantages are many, among them the following:

- interfacing electric devices to control systems is easier than hydraulic devices;
- the transmission of power and command signals requires electric cables, which allow much greater freedom of layout than hydraulic pipes or mechanical transmissions as used with mechanical controls;
- electric devices are less affected by environmental conditions, above all temperature, than hydraulic systems. In particular, in the latter the viscosity of hydraulic fluids is strongly dependent on temperature;
- the fluids used in hydraulic devices require anti-pollution measures during construction, maintenance and ultimately disposal of the vehicle that are not required in electric devices;

The drawbacks of electric devices that have up to now hampered their diffusion are:

- electric actuators are in general heavier and often more bulky than the corresponding hydraulic actuators;
- the cost of high performance magnetic materials (in particular rare earth magnets) is still high for automotive applications;
- electric systems are in general 'stiffer' than hydraulic systems requiring a more precise control. They may cause more noise and vibration.

Progress in the fields of magnetic materials (above all permanent magnets), control systems and power amplifiers is gradually reducing these drawbacks allowing us to predict that the implementation of hydraulic and pneumatic systems will diminish. The cost of permanent magnets with high energy density is also decreasing, thanks to a liberalization of the markets due to the expiry of many patents.

An apparently marginal problem that is nonetheless hampering the introduction of *by wire* devices is the voltage of the on-board electric system. The increase in the power of the electric devices on motor vehicles makes it convenient to increase the voltage from the traditional (on cars) 12 V to at least

24 V, as on industrial vehicles, or even to 36 or 48 V. In this way, the current needed by the various devices would be reduced, with advantages in cost and weight. This is not a marginal change, as it might seem, because it would require the simultaneous redesign, production and marketing of a large number of new electric components (batteries, bulbs, switches, electric motors, etc.).

Another problem under intense study is electromagnetic compatibility. The electromagnetic environment in which automotive electromechanical and electronic devices must operate is very dirty, which may induce malfunctionings of different kinds. These must be by all means prevented when electric systems are entrusted functions that are vital for the safety of the vehicle.

By wire systems are now one of the most actively researched fields in the automotive industry, with different solutions under study. They will probably enter mass production in the relatively near future. The following sections will deal briefly with some applications already mentioned in this text. Note that the problems to be solved before vehicles completely controlled *by wire* may be marketed are not only technical (design, production, marketing, etc.) but also legal, regulatory and standards-based.

27.6.1 Steer by wire

Electric steering systems like Electric Power Steering (EPS) are in common use, primarily in cars in the low or medium market segment. Their application does not imply substitution for the mechanical steering system, but simply the presence of an electric actuator in parallel with the manual steering system. The electric motor may act directly, exerting a torque on the steering wheel shaft, or it may exert a force on the rack. The torque exerted by the power steering is proportional to that applied by the driver, following strategies that may depend on many parameters. The steering control may be made 'harder' with increasing speed, to compensate for the decrease of steering torque typical of many cars and to induce the driver to act on the wheel with more care.

As already stated in our discussion of handling control devices, the steering actuator may act independently of the command given by the driver, as when using a differential gear having as inputs the steering wheel shaft and an actuator controlled independently. In this case, direct control remains, but the authority of the control system is greater.

In a true *by wire* system, there is no direct control link and the steering wheel is connected solely to a rotation sensor (potentiometer, encoder...) or a torque sensor supplying the value of the angle or the moment to the system that controls the steering actuator. A system of this kind is much more flexible, allowing different control strategies to be used, such as a variable ratio between the rotation of the steering wheel and the steering rotation of the wheels. This allows the command from the driver to interact in more complex way with the command from the control system. Because there is no direct link between the wheels and the steering wheel, there must be an actuator exerting a torque on the steering wheel to supply the driver with information on the working conditions of the wheels (haptic controls).

27.6.2 Brake by wire

Electric power brakes may be simple devices in which an electric motor actuates a pump amplifying the command given by the driver through the master cylinder connected with the brake pedal. In more complicated systems no pump is connected with the brake pedal, with the latter simply supplying a position or force signal that, through a control system, acts on the actuators at the brakes.

The actuator may be a single pump supplying high pressure fluid to a more or less conventional braking system, or an electric actuator located in each wheel. In the latter case, the actuator may pressurize a fluid acting on the pistons of the caliper or may directly actuate the caliper through a ball screw or a mechanical system of other kinds. The choice among the various solutions must take into account the mass, the cost, the reliability of the system, the need for maintenance and adjustments, and the possibility of self-adjusting.

Clearly, an actuator in each wheel allows functions like ABS, TCS, VDC, etc. to be performed without the need of valves discharging the pressure from the branch of the system in each wheel or of pumps that recover the fluid. The brake in each wheel is controlled independently following the commands from the driver and the various control devices.

27.6.3 Electromechanical suspensions

Semi-active suspensions already use systems that are at least partially electromechanical, such as the shock absorbers based on electrorheological of magnetorheological fluids. Electric actuators may replace hydraulic actuators in active suspensions (for instance in ARC systems) and above all, electromechanical eddy current dampers may replace classical shock absorbers. In particular, eddy current dampers may work as passive, uncontrolled components (in this case they may behave as almost perfect viscous dampers, without the drawback of the presence of the fluid and with a greater stability in changing environmental conditions), they may be inserted in an electric circuit containing controlled elements or they may work in a fully active way. At present, their mass is comparable to that of similar hydraulic devices and their cost, although still higher, is quickly decreasing.

27.6.4 Other by wire controls

As already stated, all functions of the vehicle may be controlled by electromechanical actuators. Because control of the engine is the simplest, conversion, *by wire* accelerators are now common. The clutch and the gearbox may also be controlled by electric devices. Apart from the advantage of avoiding mechanical linkages between the gearbox and the passenger compartment, which may transmit vibration, this solution allows for the building of fully automated gearboxes. Secondary controls, such as the parking brake, may also be replaced by electric devices, with the advantages of automation (in the case of the parking brake, it is possible to guarantee that the brake is applied when the vehicle is stopped and released when it moves), thus allowing a greater freedom in command placement and design of the user interface, as well as a much simpler design of the control transmission.