Logical Fragments in Ibn Khaldūn's *Muqaddimah*

Hans P. van Ditmarsch

Abstract In this short contribution we briefly present life and times of Ibn Khaldūn, his magistral accomplishment in the *Muqaddimah*, and present *Muqaddimah* fragments related to logic and epistemology from the perspective of modern modal logic.

1 Life of Ibn Khaldūn

Ibn Khaldūn was a fourteenth century historiographer and author of the well-known *Muqaddimah*, equally well-known by its Latin title *Prolegomena*. He lived from 1332 to 1406. Though born in Tunis, his family originated in Seville, where they lived prior to its conquest by the king of Castille, the king of Spain so to speak. This conquest was part of the grander scheme that became later known as the Reconquista. His life is rather well-documented, as he wrote an autobiography (the autobiography is included in the French edition by de Slane (de Slane 1934–38)). This autobiography already makes for absolutely fascinating reading. Ibn Khaldūn lived an itinerant life serving as a magistrate for—in modern geographic terms—Spanish, Moroccan, Tunisian and Egyptian Islamic courts. In that function in Granada, Spain, he negotiated treaties with the Christian Spanish crown (with Pedro the Cruel, which does not sound too encouraging). The autobiography follows a stupefying cyclic pattern: Ibn Khaldūn goes to state X to serve ruler A; then, unfortunately, ruler A dies/is murdered/is deposed, due to intervention of his son/his prime minister/other family or court official B. Ibn Khaldūn then: flees from state X to state Y in case he remained loyal to the former ruler A, or, alternatively, remains in state X in case he had switched allegiance to the new ruler B in time. This suggests, rather improperly as it is the undersigned suggesting it, a somewhat flighty character, but the picture in fact emerging from these repetitive sequences of events is that of a steady mind living in troubled times, who chooses according to principles of justice and fairness, with the greater good of the population and

H.P. van Ditmarsch
University of Otago, Dunedin, New Zealand & IRIT, Toulouse, France
e-mail: hans@cs.otago.ac.nz

S. Rahman et al. (eds.), *The Unity of Science in the Arabic Tradition*,

the desirability of a stable society very much in mind. He writes utterly matter-of-factly about the continuous change of power and focuses on his achievements to the administration: his mutterings about immorality in Cairo, where he deposed corrupt judges (irregularities at trials and inheritances, such as appropriation of religious bequests, were a great illegitimate source of income), could be equally found in today's Watergates and the like.

A well-known exploit during the later period of his life in the politically more stable environment of Cairo, where he also taught at the renowned al-Azhar University, is his meeting with the Turkish conqueror Tamerlane (a.k.a. Timur) during the siege of Damascus. The story goes that Ibn Khaldūn dared outside the city walls to propose parley with the attacking army—by no means a safe pursuit that might already cost one one's life. But—according to his own and contemporary documentation—he succeeded to contact the army's leader Tamerlane and had a discussion on history, philosophy and very practical matters such as rules and customs of peoples still to conquer further West. Whether this contributed to the delivery of Damascus on more favourable terms is not really known, but is of course suggested. Part of the historical evidence is that he interacted with Tamerlane by way of an interpreter ʿAbd Al-Jabbār Al-Khwārizmī.

2 The *Muqaddimah*

Ibn Khaldūn's major heritage to civilization is his encyclopedic overview of science and philosophy, and of as well—and mainly so—the history of North-African and Andalucian Islamic culture and politics at the time. The encyclopedic approach was in the Arabic tradition of the general philosophical project of the ninth century known as the translation project, which was implemented by the House of Wisdom in Baghdad and directed by Al-Kindī (Tahiri et al. 2007, fn. 6). According to the author himself he wrote his voluminous compendium (mainly) in a period of five months in 'the year 779' (AH, i.e. 1377 AD). He continued to expand this for the remainder of his life. An obvious bibliographic source for the history of the *Muqaddimah* is (Rosenthal 2005). Ibn Khaldūn's entry in Wikipedia (http://en.wikipedia.org/wiki/Ibn_Khaldun) also gives fairly precise references concerning the genesis of the *Muqaddimah*. The first western language edition was the French translation by M. de Slane from 1863, that was reprinted in the 1930s (de Slane 1934–38). The German and English translations are from the twentieth century, much later. One has to be careful about one's wording of *first* here: western, European, Modern? There is a Turkish translation from 1745, published in Istanbul, in Europe... And of course Granada, whose rulers Ibn Khaldūn served, lies very Westerly in Europe anyway—which makes Arabic a western European language at that time. My apologies for the digression.... A wonderfully concise—for the present-day itinerant scholar—English edition is the 2005 Princeton University Press reprinted abridgement (Rosenthal 2005) of the 1958 Rosenthal translation. This source was used for the quotations involving logic below—although we performed that search in the unabridged French translation by de Slane.

This brings us closer to our research question: what evidence does the *Muqaddimah* provide for epistemic logical concepts, and theoretical or otherwise precise treatment of knowledge and related concepts? We further focussed this question as follows: is there any evidence in the *Muqaddimah* of the three postulates of epistemic modal knowledge—truthfulness, awareness of knowledge and awareness of ignorance, a.k.a., respectively, *the postulate of truth*, *the postulate of positive introspection*, and *the postulate of negative introspection*? Now this concerns *modern* epistemic logic, in which these postulates make sense given a Tarskian distinction between syntax and semantics and a Kripke semantics for modal logic. And apart from that, this concerns epistemic logic in its rather contested appearance that became popular in areas as computer science and artificial intelligence, which makes rather encompassing simplifications about the nature of knowledge and truth. There is no reason a priori to assume that this perspective makes sense in a medieval setting that is much more concerned with truly epistemological investigations, that question the nature of knowledge rather than its formal or structural behaviour given some simplifying assumptions. On the other hand, the interest for not necessarily epistemic modal logic but for the more purely modal logic of necessary and possible throughout early modern times, with roots back in Aristotle and well-known from later medieval authors as Thomas Aquinas, suggests that some such pursuit might not be totally in vain.

And apart from looking ahead, we might as well look further back in time, closer to the roots of the Translation Project. The 8th century Arabic (Iraq/Oman area) philologist Al-Khalīl Ibn Aḥmad composed the first Arabic dictionary and is credited with the following famous epigraph that as well adorns the introduction (Tahiri et al. 2007) to this volume:

الرِّجالُ أربعة: رَجُلٌ يَدْري و يَدْري أنه يَدْري فسلوه،
و رَجُلٌ يَدْري و لا يَدْري أنه يَدْري فذاكَ ناس فذكِّروه،
و رَجُلٌ لا يَدْري و يَدْري أنه لا يَدْري فذلِكَ مُسْتَرْشِدٌ فعلموه،
و رَجُلٌ لا يَدْري ولا يَدْري أنه لا يَدْري فذلِكَ جاهِلٌ فارفضوه.

There are four kinds of men: men who know and know that they know; ask them.
Men who know and do not know that they know, they are forgetful; remind them.
Men who do not know and know that they do not know, they search for guidance; teach them.
And men who do not know and do not know that they do not know, they are ignorant; shun them.

(Al-Khalīl ibn Aḥmad al-Farāhīdī, in Ibn Qutaybah *'Uyūn al-akhbār* 1986, II, p. 142)

It is therefore clear that the epistemological enterprise is at the very heart of Islamic philosophy, and as this is so obviously related to the postulates of introspection we can expect to find some relation to them in the *Muqaddimah* or in contemporary early medieval writings.

Section 3 provides some essential formal background to understand the three postulates of knowledge. Section 4 reports on the fragments found. Section 5 discusses these results in relation to known other work from the era relating to the knowledge postulates and to reasoning about knowledge in general.

1. Autograph of Ibn Khaldûn (upper left corner)
From MS. C (Atif Effendi 1936)

Muqaddimah manuscript fragment (http://www.muslimphilosophy.com/ik/klf.htm)

3 Modern Epistemic Logic

Modern epistemic logic starts with Hintikka's *Knowledge and Belief—An introduction to the logic of the two notions* (Hintikka 1962). The postulates of knowledge, and the names under which they are commonly known, are that:

- what you know is true (truthfulness),
- you are aware of your knowledge (positive introspection),
- you are aware of your ignorance (negative introspection).

Such linguistic utterances are, firstly, formalised and, secondly, interpreted in their formal logical appearance on a relational structure representing 'the information'. This structure is also known as a *Kripke model*. It consists of *possible worlds*. A feature of these worlds is that, unlike the real world, they can be completely described by enumerating factual truths. Assume a very simple world in which only two facts are relevant: whether it rains in Bonn, and whether it rains in Cairns. Given two such facts we can only base four different world descriptions on them: it rains in Bonn and in Cairns, it rains in Bonn but not in Cairns, it does not rain in Bonn but rains in Cairns, and it neither rains in Bonn nor in Cairns. I am currently not in Bonn, so I have no idea whether it rains there. I am

currently in Cairns, so I know that it rains here: I am getting wet. We are therefore concerned with only two of these four different worlds: one where it rains in Bonn and in Cairns, abbreviated as (rainBonn, rainCairns) and another one where it does not in Bonn but rains in Cairns, abbreviated as (norainBonn, rainCairns). Only one of these can be the case, assume (surely...) that this is (rainBonn, rainCairns). Now what?

The idea is that we continue to reason from the perspective of the rational agent about what is possible and what is not possible. In fact, we can think of the rational agent as ourselves. With 'possible' in this epistemic context is meant: what facts that we know to be relevant are conceivably false or true given our observations about the world, our background knowledge, and our deductive abilities. As an observation counts that we are wet in Cairns—where it rains. In this case we assume that there is no background knowledge at all, except our 'realisation' that the only other relevant fact that we care to be uncertain about is: whether it rains in Bonn. In this setting surely the actual world (rainBonn, rainCairns) is considered possible. But also the world (norainBonn, rainCairns) is possible: even though it rains in Bonn, we cannot observe it. Now as a rational observer we do not actually know from which world we reason. Therefore, also if (norainBonn, rainCairns) had been the actual world, we would have considered that possible and also, in that case: (rainBonn, rainCairns). On an abstract structure with a domain of two objects we have so described a binary relation between worlds consisting of four pairs. This is called the accessibility relation. For example, given that in (norainBonn, rainCairns) we consider it possible that (rainBonn, rainCairns), this means in other words that the (ordered) pair [(norainBonn, rainCairns), (rainBonn, rainCairns)] is in this accessibility relation.

Now we proceed to modal logic. In the actual world you are said to *know* a proposition if and only if it holds in *all* worlds that are *possible* given that actual world. For example, you know that it rains in Cairns, because it rains in Cairns in world (rainBonn, rainCairns) and in world (norainBonn, rainCairns) and those are the *only* worlds you consider possible in actual world (rainBonn, rainCairns). You don't know something if and only if it is not the case that you know it. That one's easy. And you consider something possible (the diamond form of the modal operator) if and only if it you don't know that it is not the case. In relational terms this means that you consider something possible if and only if there is (at least) an accessible/possible world where it holds. For example, in the actual world (rainBonn, rainCairns), where it rains in Bonn, you consider it possible that it does not rain in Bonn, because the world (norainBonn, rainCairns) is accessible from the actual world.

An interesting aspect of this interpretation schema is that it can be applied iteratively—and that will be where the three postulates of knowledge also come in. We have already computed that in the actual world (rainBonn, rainCairns) you know that it rains in Cairns. Now if (norainBonn, rainCairns) had been the actual world we could have similarly computed that you know that it rains in Cairns. Consider (rainBonn, rainCairns) again... This world is considered possible—and you know there that it rains in Cairns. The other world is also considered possible—and you know there as well that it rains in Cairns. Therefore, in the actual world you know

that (you know that it rains in Cairns), because the proposition 'you know that it rains in Cairns' is possible in both accessible worlds! In other words, in the actual world you are aware of your knowledge that it rains in Cairns. (We will use 'knowing that you know' and 'being aware of your knowledge' as interchangeable.) If this holds regardless of which is the actual world, and regardless of the proposition known, the *postulate of positive introspection* is satisfied.

The first postulate of knowledge prescribes that what you know should be true. And this is also the case for our rainy example. We have just computed that the modal proposition 'you know that it rains in Cairns' is true in actual world (rainBonn, rainCairns). But this is also really the case: the proposition 'it rains in Cairns' is evidently true in this actual world. If this holds regardless of the actual world and regardless of the proposition, the *postulate of truth* is satisfied. Note that, as for the positive introspection example, the true propositions can also be modal. For example, you know that (you don't know whether it is raining in Bonn), and this is also true: it is indeed the case that you don't know whether it is raining in Bonn.

This also brings us to the third postulate, awareness of ignorance: consider the example argument in the previous sentence in reverse: you don't know whether it is raining in Bonn, and indeed it is also true that you *know* that. In other words, you are aware of your ignorance. If this is always the case, and for every proposition, the *postulate of negative introspection* is satisfied.

When the three knowledge postulates are satisfied the accessibility relations between worlds are always equivalence relations. This means that we can think of the domain of possible worlds as partitioned into non-overlapping subsets called equivalence classes. Your equivalence class consists of all the worlds that are indistinguishable from your point of view—where 'your point of view' is the real world: one of those in that class.

A difference between knowledge and belief is that beliefs may be false. Clearly, the truth postulate can in that case not be satisfied. There is a wealth of alternatives to this simplifying setting for the analysis of knowledge and belief and it has been contested from various sides. The original (Hintikka 1962) is still an excellent reference for that. In particular, negative introspection is unrealistic, as it requires us to be aware of all our ignorance: there are many things that we don't know of which we are unaware. In the words of a soon forgotten American government official: there are *unknown unknowns*. The technical reason for this discrepancy is that our assumption that *we are aware of all the relevant facts but just not their truth value*, is incredibly unrealistic for human reasoning.

Let us not proceed into this direction, but finish by mentioning a puzzling phenomenon of this logic of knowledge, often called a paradox (one of many epistemic paradoxes). Consider the actual world (rainBonn, rainCairns) again. If I tell you that (it rains in Bonn and you don't know that), then (a) this is *true*, and (b) after having told you that, it is *false*: you now know that it rains in Bonn, so it's no longer true that you don't know it! A somewhat different way to address this matter, is to say that the following proposition is inconsistent, or incoherent: *you know that* (*it rains in Bonn and you don't know that*). Already in the Middle Ages this was known as the *Knower Paradox*, e.g. in the works of Thomas Bradwardine

(Read 2007a, Read 2007c). Or at least it relates to the complexities, normally explained as truth-functional, involved in this paradox. This should at least make us hopeful to find similar phenomena of epistemic interest in Ibn Khaldūn's work.

4 *Muqaddimah* Text Fragments

I consulted Ibn Khaldūn's *Prolegomena* from A to Z searching for references to logic or knowledge. My source was the authorative French translation by de Slane from the 1860s (de Slane 1934–38), the first complete edition of the Prolegomena in a western language. In particular I was interested to find out whether Ibn Khaldūn considered the three postulates of knowledge as formalised in the logic *S5*: truthfulness, positive introspection, and negative introspection. My recent publication not accidentally entitled *Prolegomena to dynamic logic for belief revision* refers in a long footnote (the main text is on purely modern epistemic matters) to such text fragments and suggests that the answer to that tripartite question is: yes, yes, no. I am now even less certain of the two 'yes's. I will here present and discuss these fragments—they were only afterwards matched with their English counterparts in the Rosenthal translation (Rosenthal 2005).

I found four relevant fragments. They are all in Chapter 6 of the *Muqaddimah*, entitled: *The various kinds of sciences. The methods of instruction*. As said, the main part of the *Muqaddimah*, the content of most other chapters, is a history of North-African and Andalucian peoples of the era, including various sociological ramifications that are praised by others loudly (and justifiedly) enough already. But his overview of the academic accomplishments of his era—or rather metaphysics and natural philosophy—is certainly also very much worth reading. In Chapter 6, the relevant fragments are in Section 1—*Man's ability to think*, Section 2—*The world of things that come into being as the result of action ...*, Section 3—*The knowledge of human beings and the knowledge of angels*, and Section 22—*The science of logic*. It turns out that for our purposes the last is not the most interesting of the four! Our observations in the quotations are between [and].

Chapter 6, Section 1: Man's ability to think

> God distinguished man from all the other animals by an ability to think (...). This comes about as follows. Perception — that is, consciousness on the part of the person who perceives — is something peculiar to living beings to the exclusion of all other possible and existent things. (...) Man has this advantage over other beings: he can perceive things outside his essence through his ability to think, which is something beyond his sense. (...) The ability to think is the occupation with pictures that are beyond sense perception, and the application of the mind to them for analysis and synthesis. The ability to think has several degrees. The first degree (...) mostly consists of perceptions. (...) The second degree (...) mostly conveys apperceptions. (...) This is called the experimental intellect. The third degree (...) is the speculative intellect (Rosenthal 2005, p. 333–334).

This serves as an introduction to our further observations on positive introspection ad fragment three, below. We should point out that the corresponding French terms in (de Slane 1934–38) for 'perception', 'ability to think', and 'apperception'

are: *perception*, *réflexion*, and *affirmation*. The French terminology appears to lend itself more to an interpretation suggesting a link to epistemic logic and positive introspection. (See http://www.cnrtl.fr/lexicographie/reflexion: Réflexion: Faculté qu'a la pensée de faire retour sur elle-même pour examiner une idée, une question, un problème. Liberally translated: Faculty of thought that introspectively considers an idea, question, or problem.) It seems peculiar (to a non-francophone) not to find the terms *pensée* and *apercevoir* instead. My apologies for not being familiar with the distinctions intended in the originally used Arabic terminology. The transliteration of the Arabic term corresponding to 'ability to think' or 'réflexion' is *fu'ād*. This is the singular of *af'ida*, for heart. See (de Slane 1863, v.2, p. 427).

Chapter 6, Section 2: The world of things that come into being as the result of action.

> The ability to think is the quality of man by which human beings are distinguished from other living beings. The degree to which a human being is able to establish an orderly causal chain determines his degree of humanity. Some people are able to establish a causal nexus for two or three levels. Some are not able to go beyond that. Others may reach five or six. Their humanity, consequently, is higher. For instance, some chess players are able to perceive (in advance) three or five moves (...) (Rosenthal 2005, p. 335).

This is the citation I like best, although it is not really related to epistemic logic. Again, it is tempting to compare causal chains of reasoning to iterations of knowledge operators, where you know that you know that you know that... But this relation only exists to the extent that in either case a chain of reasoning is necessary to make an argument. A present-day philosopher or cognitive scientist immediately thinks of Turing tests and intelligent computers when reading this! Given that computers now exceed humans in computational power, computational power ceased to be seen as a sign of intelligence per se. And present-day philosophers prefer to see the *creativity* of humans as what makes them human, and not their *rationality*... I do find the observation above uncannily accurate though: this is not just some wild guess but an experimental observation; one could easily link it to the often suggested limit of six of seven items that can be concurrently processed in short-term working memory. Ibn Khaldūn makes good reading!

Chapter 6, Section 4: The knowledge of human beings and the knowledge of angels.

> We observe in ourselves through sound intuition the existence of three worlds. The first of them is the world of sensual perception. We become aware of it by means of the perception of the senses, which the animals share with us. Then, we *become aware of the ability to think* [our emphasis] which is a special quality of human beings. We learn from it that the human soul exists. This knowledge is necessitated by the fact that we have in us scientific perceptions which are above the perceptions of the senses. They must thus be considered as another world, above the world of the senses [The third world is the world of spirits and angels.] (Rosenthal 2005, pp. 337–338).

This fragment is the most pertinent to our quest. From 'become aware of the ability to think' it may seem a big step to 'awareness of knowledge' in the

epistemic sense, but a small case can be made. This step seemed on the whole a lot smaller in the French translation where 'réflexion' is used to denote ability to think, although for this particular passage the French version is less striking than the English version: la réflexion, faculté spéciale à l'homme, nous enseigne de la manière la plus positive l'existence de l'âme humaine; (elle nous le fait savoir) au moyen des connaissances acquises et enfermées dans notre intérieur; connaissances bien au-dessus de celles qui proviennent des sens (de Slane 1934–38, v.2, p. 433). It is surely comforting to a modal logician that awareness of knowledge provides proof of the existence of the soul.

Chapter 6, Section 22: The science of logic.

> (Logic concerns) the norms enabling a person to distinguish between right and wrong, both in definitions that give information about the essence of things, and in arguments that assure apperception. (...) Eventually, Aristotle appeared among the Greeks. He improved the methods of logic and systematized its problems and details (...) (Rosenthal 2005, pp. 382–383).

This concerns a roughly 2000 word overview of Aristotle's Categories, and how they found their way into the Arab world by way of translations and commentaries, such as by—using their Latinised names—Averroes and Avicenna. (Ibn Khaldūn was of course familiar with the works of these philosophers—he received a classical Arabic education in Tunis at an early age and his teacher was a Al-Abili (http://en.wikipedia.org/wiki/Ibn_Khaldun).) For our epistemic logical concerns this part is of no interest.

There seems to be a missing logic treatise by Ibn Khaldūn (de Slane 1934–38) on which contents it would be unwise to conjecture. So, the above is all. What can we conclude? Concerning the *postulate of truth*: that real knowledge is true seems easily read into various phrases, as some kind of reliability or certainty corresponds to the connotation of the word knowledge anyway. For example, we give a final corroborating quote: "Their knowledge [of prophets] is one of direct observation and vision. No mistake or slip attaches itself to it, and it is not affected by errors or unfounded assumptions." (Rosenthal 2005, p. 339). Concerning the *postulate of positive introspection*: It is tempting to see reflection on acquired knowledge as a form of introspection in the modern epistemic logical sense, but it is not found in some general form that involves arbitrary iteration or reflection on knowledge. Concerning the *postulate of negative introspection*: I did not find a reference to negative introspection. In fact, the main context of the knowledge postulates (at least of the ones on introspection) is in (i) *derivations* of factual from epistemic knowledge, or vice versa, and (ii) *higher-order* settings where you know that you know that you know that something is the case (or derive something else from that), or where such a setting is necessary to explain or analyse seemingly paradoxical (the Knower Paradox) or otherwise too complex phenomena of reasoning. None of this I found in Ibn Khaldūn's writings. Is this therefore a failed enterprise? Not really, I presume to suggest. In the first place this small investigation might keep others from also fruitlessly repeating it, and it provides at least a record of all the logical fragments in the *Muqaddimah* (insofar as such a record is needed given the splendidly accessible translations for this masterpiece). Apart

from that, there is the more general question whether the three postulates can be found as such in medieval logical arena. I close with a section referring to such matters. A surprising observation there will be that the works of Avicenna—a main source for Ibn Khaldūn's schooling, and to which he refers in the Section *The Science of Logic* when discussing Aristotle—certainly contain such epistemic modal content.

5 Related Sources and Discussion

In this section we discuss some sources related to epistemic logic of Ibn Khaldūn's contemporaries and predecessors. This overview is not exhaustive and may not even be typical for the era. First, we need to point out that the study of modalities as such was widely pursued in the early modern period, see e.g. the various sources mentioned in (Kneale & Kneale 1962), or in this volume Paul Thom's *Logic and Metaphysics in Avicenna's Modal Syllogistic* (Thom 2007). It should then be pointed out that this mainly concerns the general logic of reasoning about the necessary and the possible, where the typical understanding of 'something is necessarily the case' is that 'something will be the case in all future developments of the world'. In other words, the modality is temporal, as in Aristotle's sea-battle argument. For the epistemic modality, a main source appears to be Avicenna and his Arab predecessors (Black 2007)—this we will present in some detail. We also report on an obvious relation apparent from the presentation of the *Knower Paradox* in Thomas Bradwardine's fourteenth century writings, and a stipulated relation conjectured from the Scholastic medieval notion of *Obligatio*. (For further sources, see, again (Kneale & Kneale 1962), and (Boh 1993)). In our explorations we focus on positive and negative introspection, as identified with awareness of knowledge and awareness of ignorance.

5.1 Avicenna, Al-Fārābī, and Positive Introspection

Deborah Black's *Avicenna on Self-Awareness and Knowing that One Knows* (Black 2007) addresses epistemic aspects in Avicenna's (Ibn Sīnā, 980–1037) work, in relation to relevant work of his predecessor Al-Fārābī (870–950). Avicenna's experiment of the Flying Man is worth recounting: imagine yourself in a state where you have no sensory perception to distract you, you are, as it were, floating in the air in a suspended state. Are you then still aware of yourself? The answer to that is clearly: yes, you are. This proves awareness of the self as a, so to speak, semantic object. One's interpretation of this phenomenon depends on whether we see this as an observation about sensory perception or as an observation about intellectual reflection. In the second case we are more clearly not just talking about awareness of an (semantic) object but about awareness of knowledge of factual information. Given our modern identification of awareness with knowledge (an identification that seems also questioned, in principle, in the original

source as reported by (Black 2007)) this clearly amounts to second-degree knowledge of factual information. Awareness of knowledge is then (somewhat surprisingly, to the modern mind) identified with certainty of knowledge (Black 2007). Positive introspection goes beyond that: it is awareness of any knowledge, also of epistemic propositions. This amounts to arbitrarily higher-order knowledge: to know that to know that to know that ... This problem of infinite regress 'seems not to worry Avicenna although his justification—in our interpretation—is that otherwise certainty about knowledge would not be possible, which is undesirable (Black 2007). The modern justification arguing away infinite regress problems is that knowledge is interpreted on Kripke models where the accessibility relation satisfies the structural property of *transitivity*: this corresponds in an exact formal way to the postulate of positive introspection—we refrain from further details, see (Hintikka 1962). Black further mentions Avicenna's predecessor Al-Fārābī who also wrote on infinite regress of knowing that—it would be of clear interest to investigate that aspect of the work of Al-Fārābī. Black's *Knowledge and Certitude in Al-Fārābī's epistemology* (Black 2006) mentions six conditions for certain knowledge. The first three clearly (as she observes) seem to define knowledge as justified true belief, and they relate to the postulates of truth and positive introspection.

5.2 Bradwardine and Epistemic Paradox

In *Bradwardine's Revenge* (Read 2007a) Stephen Read discusses the Knower Paradox (our source was his GPMR workshop presentation delivered in Bonn). We quote Read:

> Thomas Bradwardine, writing in the early 1320s, developed a solution to the semantic paradoxes (insolubilia) based on a closure principle for signification: every proposition signifies whatever is implied by what it signifies. In ch. 9 of his treatise, he extends his account to deal with various epistemic paradoxes. Comparison of Fitch's paradox with one of these paradoxes, the Knower paradox ('You do not know this proposition') explains the puzzlement caused by Fitch's paradox. Bradwardine's argument shows that the Knower paradox signifies its own truth, and is false. (Read 2007b)

In epistemic logic, one way to model the Knower Paradox is to see 'You do not know this proposition' as the announcement of 'The proposition is true and you do not know that', where the proposition may as well—but does not have to—be a factual proposition. The example we already gave in Section 3 was 'it is raining in Bonn and you do not know that'. In dynamic epistemic logic (as mentioned in van Ditmarsch 2005) an announcement as 'it is raining in Bonn and you do not know that' is proposed to be processed as a Kripke model transforming operation: in case of this announcement, it restricts the current information state consisting of the two worlds (rainBonn, rainCairns) and (norainBonn, rainCairns) to a single state (rainBonn, rainCairns). This is then the only remaining possible state, in which you therefore know that it rains in Bonn. This explanation resolves the paradoxical character of the Knower Paradox. Now, not surprisingly of course, this 'modern' dynamic explanation is not found in Bradwardine's work (we consulted the translation in progress by Stephen Read (Read 2007c) with his kind

permission)—it is not even found in G.E. Moore's work, one of the much more recent sources (early 1940s) that addresses this matter (for very detailed references to Moore's work on this paradox, see, yet again, (Hintikka 1962)). Still, it is quite a surprise to see Bradwardine explain the paradoxical character of the 'Knower' in rather similar terms as the modern epistemic logician would do. His starting point is the phrase 'This proposition is not known by you' (or, in another manuscript, 'this proposition is not known by Socrates'—as observed by Read in work in progress) and the derivation towards a contradiction uses distribution of knowledge over conjunction (if I know A and B, then I know A and I know B), and, indeed, the truth postulate (what I know is true) applied to a proposition of ignorance. Here, we come fairly close to negative introspection again. Unlike his method to derive the paradox, Bradwardine's method to resolve the paradox seems quite different from our above dynamic approach. Bradwardine uses a certain treatment of self-reference that can in a different, unrelated, context (personal communication, Stephen Read), also be used to address the Liar's Paradox (see Read 2007a, Rahman et al. 2007).

5.3 Obligatio and Negative Introspection

The postulate of negative introspection concerns awareness of ignorance. It was suggested by Catarina Dutilh Novaes at the GPMR Workshop Medieval Logic in Bonn that there is a link between negative introspection and the medieval concept of '*obligatio*'. *Obligatio* is a philosophical method of dialogue (a game) between opponents, with the object of confirming or rejecting agreement, or to resolve inconsistencies (Dutilh Novaes 2007). An important medieval source are the *Obligationes Parisiensis*, translated by a group headed by Sara Uckelman (Uckelman et al. 2008a, Uckelman et al. 2008b). We quote the abstract of their GPMR presentation:

> Oxford MS Canon misc 281 contains a tract on *obligatio* which can be tentatively dated and placed in early 13th century France (the text was edited by de Rijk in *Vivarium*). The tract is divided into three sections, *positio*, *dubitatio*, and *depositio*.

The link with negative introspection is, that in the case of the *dubitatio* obligation, the uncertainty about information brings the obligation to question it, followed by a process of attempted justification. In as far as this *obligation* can be identified with *awareness* and *uncertainty* with *ignorance*, what takes place here is indeed a step from 'I do not know this proposition' to 'I am aware that I do not know this proposition', in other words: I know that I do not know this proposition—negative introspection. On the other hand, it seems to us that one might as well interpret the doubt or uncertainty here as the absence of knowing that or knowing that not. That would be a somewhat stronger interpretation: in that case, *doubting* a proposition would mean 'I am aware that (I do not know this proposition and I do not know the negation of this proposition)'. Sara Uckelman (personal communication) also suggests that apart from 'true' and 'false', 'doubtful' may well function as a third truth value. A *multi-valued* approach to reasoning would be

fairly different from an *epistemic modal* approach, that we are trying to read into the obligation of doubt. We are uncertain at this stage which of the two is a more suitable modern re-interpretation.

Finally, one might observe, as Shahid Rahman with reason does, that negative introspection as a method is part of the general epistemogical approach to logic, and in this form this brings us back to the Arabic tradition where the realisation of ignorance is a condition to learn, and where the desire to learn was the original motivation for the Translation Project.

Acknowledgements I am acknowledged to Shahid Rahman and Hassan Tahiri for encouraging me to write this contribution, and for their persistence in doing so, overcoming my never ending doubts: negative introspection can be exhausting when applied to oneself. I am further acknowledged to the GPMR workshop on Medieval Logic that took place in Bonn in June 2007, and to the comments of and interaction with the participants of that event. For their comments on this investigation I thank Catarina Dutilh Novaes, Shahid Rahman, Stephen Read, Hassan Tahiri, and Sara Uckelman. I acknowledge Nicola Bidwell, of James Cook University in Cairns, Australia, for her hospitality: this contribution was mainly written during a one-week stay at the IT department of that university, in July 2007; and I also thank John Haynes, of Charles Darwin University in Darwin, Australia, for his hospitality; the contribution was revised while staying at his department, in August 2007.

Last but not least I acknowledge the hospitality of the Netherlands Institute of Advanced Study in the Humanities and the Social Sciences (NIAS), and in particular I thank NIAS Rector Wim Blockmans for his confidence in me and in the outcome of my Lorentz Fellowship 2007/2008 project, which I estimate beyond measure.

References

Black, D. L.: 2007, "Avicenna on Self-Awareness and Knowing that One Knows", in S. Rahman, T. Street and H. Tahiri (eds.), *The Unity of Science in the Arabic Tradition: Science, Logic, Epistemology and their Interactions*, Dordrecht/Boston/London, Springer/ Kluwer (this volume).

Black, D.L.: 2006, "Knowledge (*'Ilm*) and Certitude (*Yaqīn*) in Al-Farabi's Epistemology", *Arabic Sciences and Philosophy*, **16** (1), 11–46.

Boh, I.: 1993, *Epistemic Logic in the Later Middle Ages*, Routledge.

de Slane, M.: 1934-38, *Les prolegomènes d'Ibn Khaldoun*, translated and commented upon by M. de Slane, 3 vol., originally published in 1863, integrally reprinted in 1934, 1936, and 1938, Paris, Librairie Orientaliste Paul Geuthner.

Dutilh Novaes, C.: 2007, *Formalizing Medieval Logical Theories*, series Logic, Epistemology and the Unity of Science edited by S. Rahman and J. Symons, volume 7, Dordrecht/Boston/London, Springer.

Hintikka, J.: 1962, *Knowledge and Belief: An Introduction to the Logic of the Two Notions*, Cornell University Press.

Kneale W. and Kneale, M.: 1962, *The Development of Logic*, Oxford, Clarendon Press.

Read, S.: 2007a, "Bradwardine's Revenge", in J.C. Beall, *Revenge of the Liar*, Oxford University Press, To appear.

Read, S: 2007b, *Thomas Bradwardine and Epistemic Paradox*, (Electronic) Proceedings of GMPR workshop on Medieval Logic, Bonn http://www.illc.uva.nl/GPMR-LS1/abstracts.html

Read, S. (translator and editor): 2007c, *Insolubles*, translation in progress of Bradwardine's Insolubilia.

Rahman S., T. Tulenheimo & E. Genot (eds.): 2007, *Truth, Unity and the Liar: The Modern Relevance of Medieval Solutions to Semantic Paradoxes*, series Logic, Epistemology and the Unity of Science edited by S. Rahman and J. Symons, volume 8, Springer/Kluwer.

Rosenthal, F.: 2005, *The Muqaddimah: An Introduction to History*, (Abridged Edition), translated and introduced by Franz Rosenthal, edited and abridged by N.J. Dawood, with a new introduction by Bruce B. Lawrence, Princeton University Press.

Tahiri H., S. Rahman and T. Street: 2007, "Introduction I: The Major Breakthrough in Scientific Practice", in S. Rahman, T. Street and H. Tahiri (eds.), *The Unity of Science in the Arabic Tradition: Science, Logic, Epistemology and their Interactions*, Dordrecht/Boston/London, Springer/Kluwer (this volume).

Thom, P.: 2007, "Logic and Metaphysics in Avicenna's Modal Syllogistic" in S. Rahman, T. Street and H. Tahiri (eds.), *The Unity of Science in the Arabic Tradition: Science, Logic, Epistemology and their Interactions*, Dordrecht/Boston/London, Springer/Kluwer (this volume).

Uckelman S.L., J. Maat & K. Rybalko: 2008a, The Art of Doubting in *Obligationes Parisienses*, to appear in C. Kann, B. Loewe, C. Rode, and S.L. Uckelman, eds., *Modern Views of Medieval Logic*, Recherches de Theologie et Philosophie Medievales – Bibliotheca.

Uckelman S.L. et al. (translators): 2008b, *Parisian Obligations*, ILLC technical note X-2008-3, see http://www./illc.uva.nl/Publications/ResearchReports/X-2008-03.text.pdf.

van Ditmarsch, H.P.: 2005, "Prolegomena to Dynamic Logic for Belief Revision", *Synthese*, **147**, 229–275.