Searching for Factors that Limit Observed Extreme Maximum Wave Height Distributions in the North Sea

Gerrit Burgers, Frits Koek, Hans de Vries, and Martin Stam

Abstract The probability that individual waves are much larger than the significant wave height is studied in a large set of observations. It is investigated whether steepness and shallow water effects are limiting factors for extreme wave heights. The relation between observations and a model freak wave index is examined.

Measurements from two locations in the North Sea are used, one with a depth of 80 m, and another with a depth of 20 m. The data consist of the significant wave height, wave period and maximum wave height of 20-min records. The total amount of the records covers several years. The freak wave model index from the European Centre for Medium-Range Weather Forecasts (ECMWF) wave model is collocated with the observations.

The instrumental data show Rayleigh like distributions for the ratio of maximum wave height to significant wave height. Our analysis is limited by uncertainties in the instrumental response in measuring maximum wave height. The data indicate that steepness is a limiting factor for extreme wave height. At the shallow water location, extreme waves are not more frequently observed than at the deep water location. The relation between the freak wave index of the ECMWF wave model and enhanced extreme wave probability is studied.

1 Introduction

During the All Saints Day storm of November 2006, a waverider buoy recorded extreme individual waves of 17 and 20 m, around twice the significant wave height at the time (Fig. 1). For the same storm, the ECMWF (European Centre

G. Burgers
Royal Netherlands Meteorological Institute, P.O. Box 201, NL-3730 AE De Bilt, The Netherlands
burgers@knmi.nl

F. Koek, H. de Vries, and M. Stam
Royal Netherlands Meteorological Institute, De Bilt, The Netherlands
koek@knmi.nl, jwdv@knmi.nl, and stam@knmi.nl

E. Pelinovsky, C. Kharif (eds.) *Extreme Ocean Waves*, 127–138.

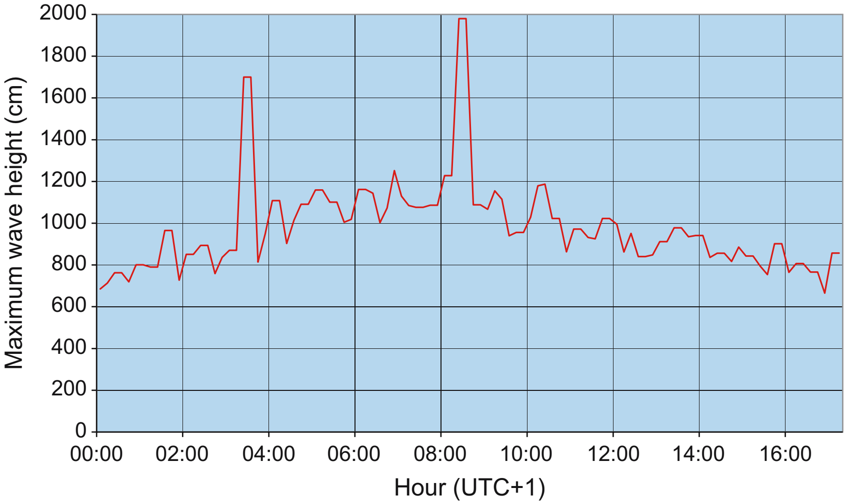

Fig. 1 Registration of maximum wave height at Schiermonnikoog Island (SMN) at (53°35N, 6°10E) during the night of 1 November 2006

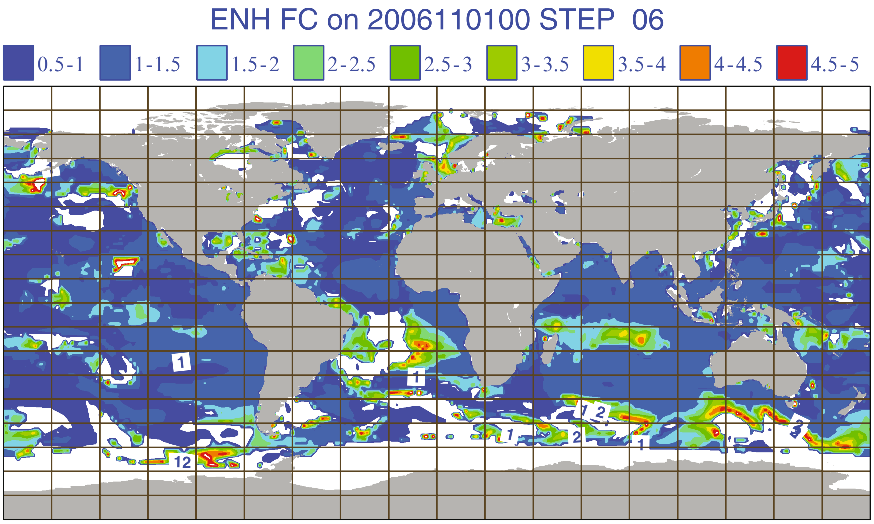

Fig. 2 WAM enhancement factor. This factor gives the ratio of the WAM model estimate to the standard linear model probability for waves with a height more than twice the significant wave height

for Medium-Range Weather Forecasts) wave model WAM indicated an enhanced probability of extreme waves in the North Sea, see Fig. 2. For a water depth of 20 m, a 20-m wave would be quite exceptional. Whether the record is correct remains unclear: analysis of the buoy by the manufacturer showed that the measurement is not reliable in these conditions (Datawell 2006), and in the same storm, damage was reported at a platform more than 15 above sea level (Bojanowski 2007). But the buoy has been in place for years, so in principle we can determine multi-year return times for wave extremes.

In this chapter, we study the following questions: how exceptional are extreme waves in long records? Can the WAM model identify conditions with enhanced extreme wave probability? More precisely, we focus on the ratio $r = H_{max}/H_s$, where H_s is the significant wave height and H_{max} the maximum wave height in a 20-min record, and from the WAM model we use the BFI index (Janssen 2003). Our study differs from the one by Holliday et al. (2006) in that we do use the 20-min record summary information instead of individual wave records, and that we have used data that accumulate to a much longer time. However, because various instruments and algorithms have been used, the interpretation as one single dataset is problematic.

Standard linear wave theory (see e.g. the textbook of Holthuijsen (2007) or the introduction by Berg and Rhome (2005)) gives rise to a Rayleigh distribution for wave height (Appendix). Freak waves are sometimes defined as waves that are higher than twice the significant wave height. According to the Rayleigh distribution, about one in 3,000 waves is a freak wave. For waves with a period of about 10 s, this is of the order of once every 8 h. In practice, of course, usage of the term freak wave is often restricted to cases where the absolute value is exceptionally high. Over the last few years, several mechanisms have been proposed, that give rise to enhanced extreme wave distribution compared with standard Rayleigh theory (Janssen 2003; Mori and Janssen 2006). In Fig. 3, which is discussed in Appendix A, such an enhanced extreme wave distribution is compared with the standard Rayleigh distribution. On the one hand, the WAM model index is based on a non-linear effect that enhances the extreme wave height distribution. On the other hand, for very large r, when

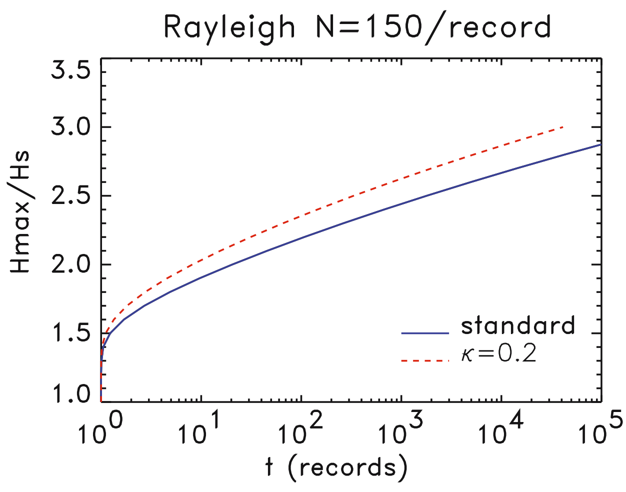

Fig. 3 Rayleigh and Janssen distribution of H_{max}/H_s of the return time of the ratio of H_{max}/H_s of maximum wave height over significant wave height. Time is measured in 150-wave records. The *thick line* is the Rayleigh distribution, the *dashed line* distribution according to Janssen theory for the case that the kurtosis of the sea surface $\kappa = 0.2$, corresponding to BFI = 0.33

steepness becomes a limiting factor, non-linear effects will lead to a suppression with respect to the Rayleigh distribution. Moreover, in shallow water, the behaviour might be quite different. So it is interesting to examine the behaviour of the observed distribution.

In Sect. 2, we discuss the observational dataset as well as the WAM model index. In Sect. 3, we present and analyse maximum wave height distributions, including a comparison between model results and observations. It is clear from the results that the maximum wave height as measured by an instrument depends on the sensor used. In Sect. 4, we give a discussion of the results, and in Sect. 5, we present the conclusions.

2 Data Sources

2.1 North Sea Data

The data consist of reports of the Meetnet Noordzee (MNZ), a network of measuring instruments at a number of platforms and buoys in the North Sea set up by the Dutch authorities in cooperation with platform operating companies. In this chapter, we use data from the AUK platform located in the central North Sea at (56°24N, 0°02E) and a depth of 80 m, and from the wave buoy near the coast of Schiermonnnikoog Island (SMN) in the north of the Netherlands at (53°35N,6°10E) and a depth of 20 m.

A report consists of a set of wave parameters extracted from a 20-min record, the frequency of the reports is 3 h. In this chapter, in addition to significant wave height and wave period, also the maximum wave height of the record is used. The MNZ data for wave height and wave period have been monitored for years by the Royal Netherlands Meteorological Institute (KNMI), and we know them to be reliable.

The measurements have been made by several types of instruments (Table 1). The radars operate from fixed platforms and measure surface elevation with a frequency

Table 1 Overview of instruments used in this study at the locations of Auk at 56°24N, 0°02E and Schiermonnikoog Island (Schier) at 53°35N, 6°10E

Label	Location	Type	Period
1	Auk	Waverider	19840326–19861010
2	Auk	Saab radar	19860413–19870714
A2	Auk	Saab radar	19930616–19990625
A3	Auk	Wavec	19901024–19991109
AUK1	Auk	Saab radar	20000204–20050928
AUK2	Auk	Wavec	20010718–20010925
AUK2	Auk	Directional waverider	20020621–20030731
4	Schier	Wavec	19901024–19930331
W4	Schier	Wavec	19931101–19991109
SMN1	Schier	Wavec	19931109–20031216
SMN1	Schier	Directional waverider	20020220–20070831

of 5.12 Hz. Wave buoys calculate wave data on the basis of acceleration measurements. The sample frequency is 1.28 Hz. A linear time-domain filter is used to reconstruct a wave height record, and to estimate the maximum wave height from these accelerations. The number of waves (from the estimated zero-upcrossing period) in a record falls typically in the range 150–200. The results of Sect. 3 show that wave buoys give systematically lower estimates for the maximum wave height than the radar estimates, and that there are significant differences between the various combinations of buoys and filters that have been used over the years.

Quality control included rejection of duplicates and the rejection of gross errors. For example, for the radar altimeter, some short periods with on average unrealistically high values were skipped. There are some features of the dataset we cannot explain. For example, there seems to be a preference for 'nice' values of the ratio $H_{\max}/H_{\mathrm{s}}$ such as 1.5 or 2. We have not been able to trace what part of the processing is responsible for this feature.

2.2 WAM Model BFI

The Benjamin Feir index *BFI* proposed by Janssen (2003) is a measure of the strength of the effect of non-linear interactions on wave height distribution. Non-linear effects are stronger if (1) waves are steeper, and (2) the wave spectrum is more narrow allowing for waves to travel longer together. For a narrow-band spectrum the definition of the BFI is

$$\mathrm{BFI} = \frac{\sqrt{2}km_o^{1/2}}{(\sigma/\omega)}, \tag{1}$$

where k denotes the dominant wave number, ω the frequency of the spectrum, σ the spectral width, and $m_o^{1/2} = 0.25H_{\mathrm{s}}$ the amplitude of the spectrum. In the numerator, $km_0^{1/2}$ is the steepness, and the denominator is the narrowness of the spectrum.

For general spectra, the above expression for the spectral width is rather ambiguous, and Janssen (2003) uses the following expression for the BFI:

$$\mathrm{BFI} = \sqrt{2\pi}km_o^{1/2}Q_p, \tag{2}$$

with

$$Q_{\mathrm{p}} = \frac{2}{m_0^2}\int \mathrm{d}\omega\omega E^2(\omega) \tag{3}$$

where $E(\omega)$ is the spectral density.

The BFI was added as an output parameter to ECMWF's wave model in the fall of 2003. Since then all model forecasts have been archived and from these archives BFI and significant wave height have been extracted from 6 October 2003 until 31 December 2006 in 6-hourly steps for model grid points near platform AUK and Schiermonnikoog.

3 Results

The first quantity we consider is the return period of $r = H_{max}/H_s$ for the deep-water station AUK. In Fig. 4, this quantity is plotted as a function of return time (in units of 20-min records), for various instruments and periods. It appears that the differences between the instruments and periods are large. The radar altimeter values are close to those which one would expect from a Rayleigh, the wave-buoy data are generally lower.

For situations where the average steepness (H_s/λ) is large, steepness may be a limiting factor. First, we check in Fig. 5, which gives a plot of H_{max} vs. T if wave heights do not exceed the limiting steepness line of $H = \lambda/7$. This figure shows that it is not uncommon that the steepness approaches the limiting steepness.

If steepness is a limiting factor, then one would expect that for a given significant wave height, longer periods that go with less steep waves would lead to an enhanced probability of high values of r. Figure 6, where the average value of r for records with $H_s \approx 4m$ is plotted as a function of T, gives some evidence for this fact.

Now we turn to the shallow water results. In Fig. 7, the return periods for the shallow-water station Schiermonnikoog (SMN) are shown. Comparing these results to the deep water-data of AUK is hampered by the fact that different stations and periods are hard to compare. Even when matching periods and instruments, there remain problems: the SMN1 Wavec is much lower than the AUK2 Wavec, while the SMN W4 is only slightly lower than the AUK A3. We checked that this also holds when we restricted the comparison to periods that both instruments yielded data, (not shown). We note that period of the W4 vs. A3 comparison is much

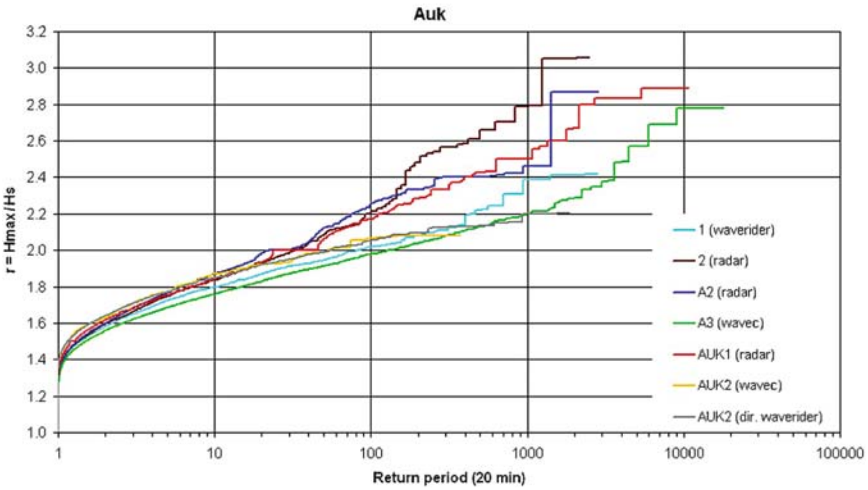

Fig. 4 Return times at the deep water station AUK. Time is measured in 20-min records. The labels refer to different instruments and periods, see Table 1

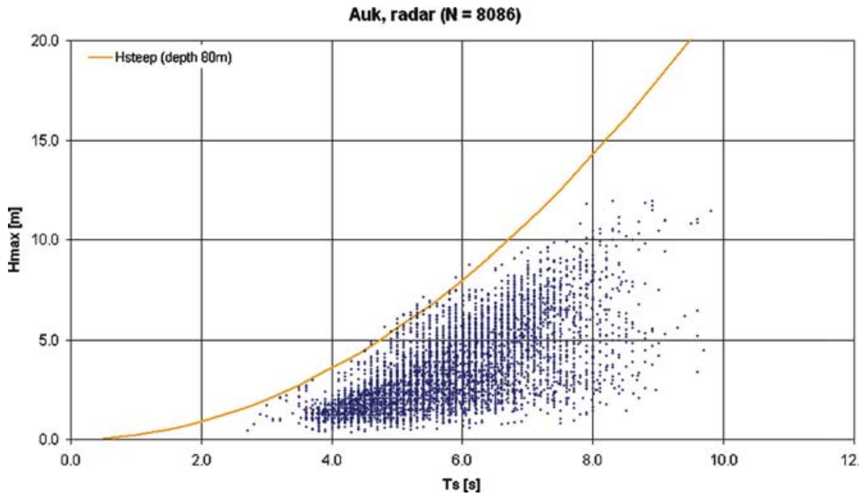

Fig. 5 Scatter plot of maximum wave height $H_{\max}$ vs. mean period T in deep water for the radar measurements at AUK

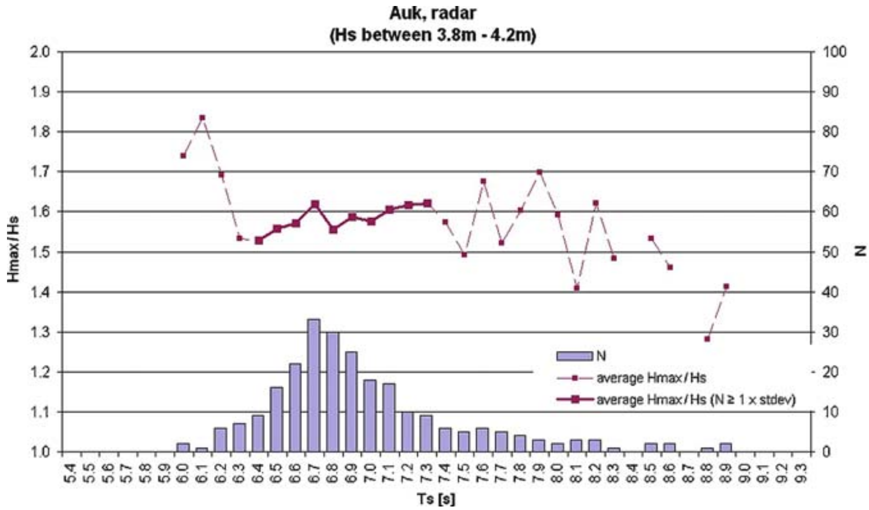

Fig. 6 The ratio $r = H_{\max}/H_s$ as function of the mean period T, for radar measurements at AUK. The *histogram* indicates the number of data in each bin. The *solid line* connects the average value of r for bins with 9 or more entries

longer (19931101–19991109) than the period of the AUK2 vs. SMN1 comparison (20010718–20010925). What we can conclude is that there is no indication that outside the surf zone, values of r are higher in shallow water than in deep water

For this shallow water location, the constant steepness line (yellow) in a H-T diagram has a different shape than for deep water. Figure 8, which gives a plot of

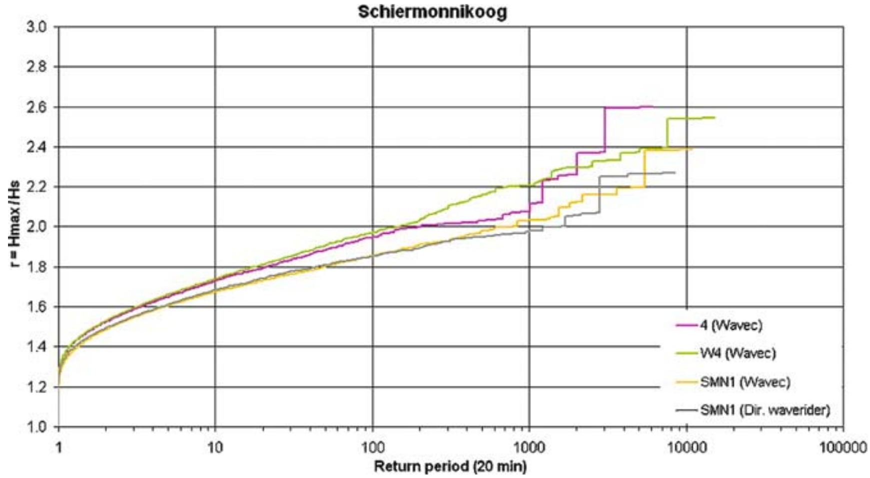

Fig. 7 Return times at the shallow water station Schiermonnikoog Island (SMN). Time is measured in 20-min records. The labels refer to different instruments and periods, see Table 1

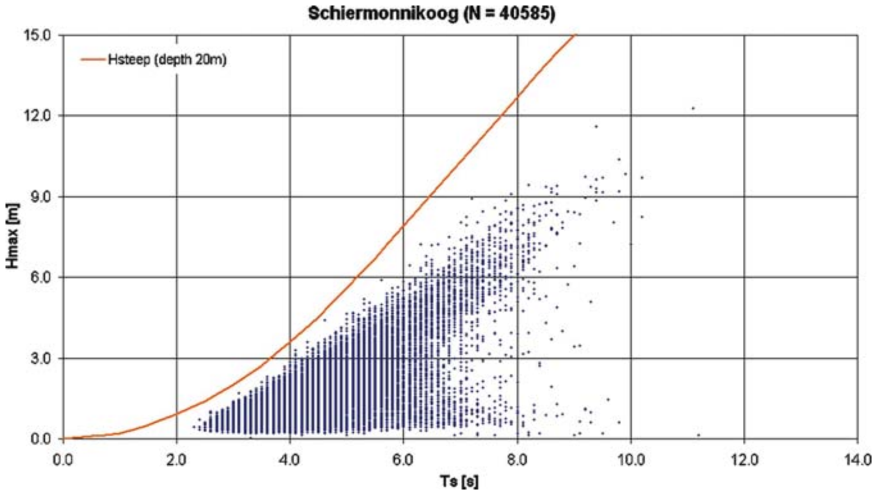

Fig. 8 Scatter plot of maximum wave height H_{max} vs. mean period T at the shallow water station Schiermonnikoog Island (SMN)

H_{max} vs. T for SMN, shows that for this station maximum wave heights are not as close to yellow line as for deep water waves. We checked in plots of H_s against steepness, see Figs. 9 and 10, that although the general picture that high waves are more often steep than low waves remains valid, for waves with $H_S > 2m$, the limiting steepness at the shallow water location decreases with significant wave height, while it stays roughly constant at the deep-water location.

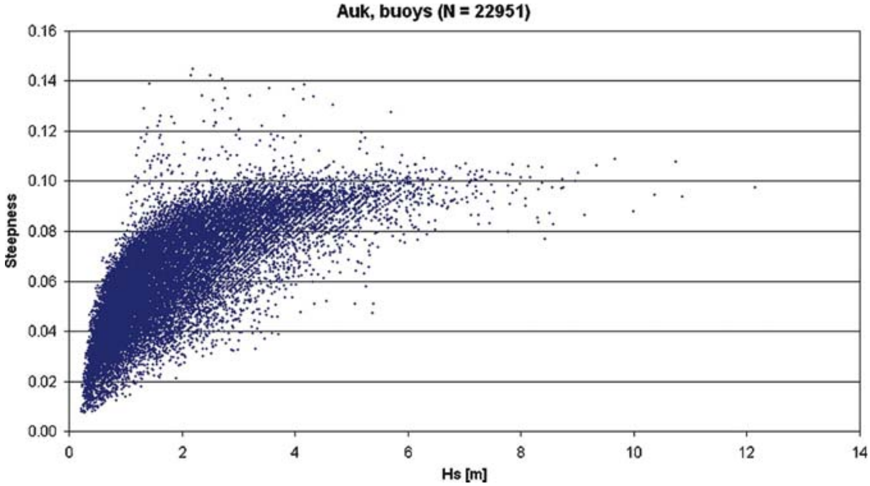

Fig. 9 Scatterplot of wave steepness vs. H_s at the deep water station AUK

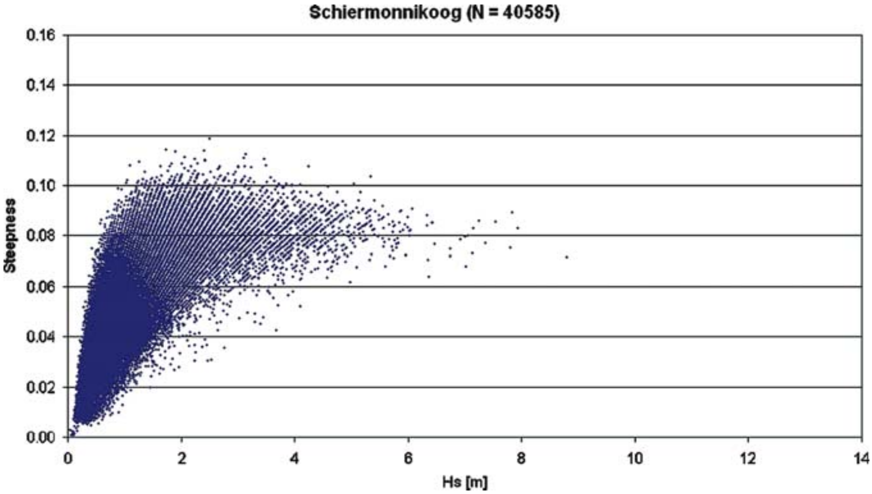

Fig. 10 Scatterplot of wave steepness vs. H_s at the shallow water station Schiermonnikoog Island

Finally, we checked whether there is a relation between the BFI index of WAM of the ECMWF and the probability of high *r* values. To this end, average values of *r* have been collocated with the model BFI. The results are shown in Fig. 11 for AUK and Fig. 12 for SMN. During the period for which the model BFI was available, there were many more observations at the shallow water station than at the deep water station. The data for the deep water station AUK do not exclude a relation between high *r* observations and high WAM model BFI indices. For the shallow water location, there is hardly a correlation between high *r* observations and high WAM model BFI indices.

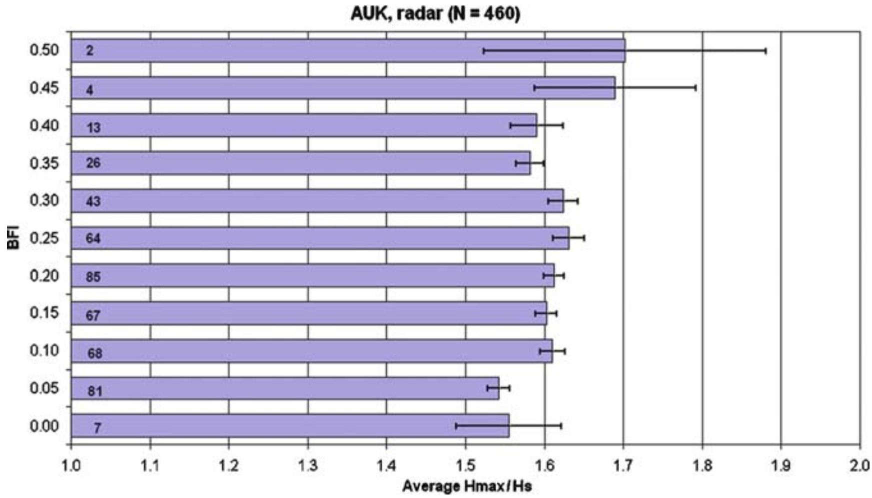

Fig. 11 Model BFI vs. measured H_{max}/H_s in deep water at AUK. Observations have been binned according to model BFI, the number of observations is indicated by the *black number* in each bar. The length of the bar gives the average value of r for that bin, and the *black line* indicates the standard deviation

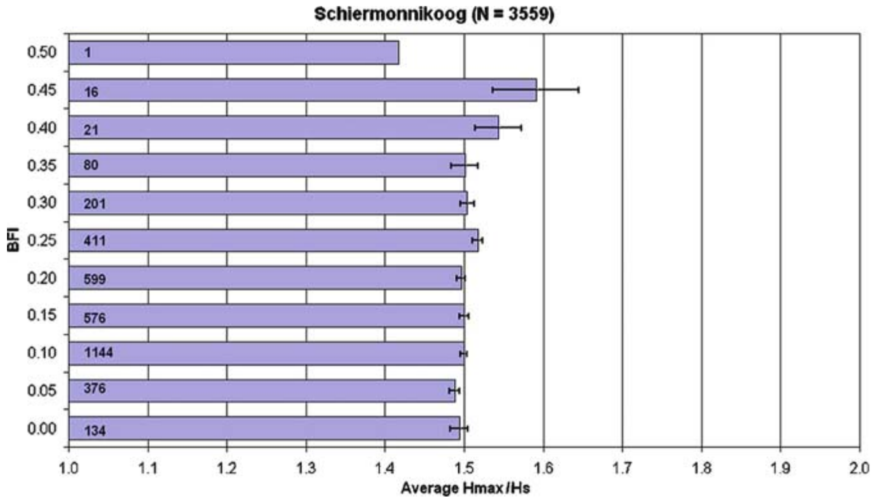

Fig. 12 Model BFI vs. measured H_{max}/H_s in shallow water at SMN. Observations have been binned according to model BFI, the number of observations is indicated by the *black number* in each bar. The length of the bar gives the average value of r for that bin, and the *black line* indicates the standard deviation

4 Discussion

From the measurements it appears that there are systematic differences between the various instruments. Because the radar has a higher sampling frequency, and makes a more direct measurement than the accelerations measured by the buoys, we consider the radar measurements to be more reliable. Additional confidence in the radar results comes from the fact that they are close to those predicted by standard Rayleigh theory. Apparently, the frequency of around 1 Hz of accelerations by the buoys is not sufficient for capturing maximum wave heights well, making that the buoy results are systematically below the radar results. That differences between various periods are so large indicates that not only the instrument but also the processing algorithm has a large impact on the distribution of the measurements. This makes a comparison of the deep water station with the shallow water station far from straightforward. What remains is a number of series of several months of data. The most obvious result is that there is no change in behaviour for timescales ranging from hours to months. There is neither evidence for an enhanced tail because of non-linear enhancement effects nor for a damped tail because of limiting steepness effects. Considering subclasses, steepness may have an effect: there is some indication that given the significant wave height and the period, extreme waves are more likely in case long periods. In shallow water, wave energy can converge and give rise to high waves. But our results do not indicate a higher probability of extreme waves in shallow water than in deep water. If any, we observe the opposite effect: extreme waves are less likely in shallow water. We do find a clear difference in plots of steepness vs. wave height between deep water and shallow water: in deep water, there is a limiting steepness that does not depend on wave height, in shallow water, this limit decreases with wave height, probably because of bottom friction effects.

For deep water, Janssen theory expects an increase of the mean value of r of the order of 0.1 if the BFI is varied from 0 to 0.5 (Peter Janssen, personal communication). Such an increase is compatible with our results for the deep water station AUK in Fig. 11. Janssen theory does not expect a correlation between BFI and r in shallow water, because in shallow water conditions for four-wave interactions differ from deep water. This is confirmed by our shallow water results in Fig. 12.

5 Conclusion

The analysis has been hampered by the fact that the instruments report approximations for maximum wave height, and that those approximations differ between different instruments and observing periods. Our main result is that the instrumental data are consistent with a Rayleigh like extreme wave distributions up to return periods of many months. There is slight indication that for long waves steepness can be a limiting factor for maximum wave height. In shallow water, there is some evidence that extreme waves are less common than in deep water. The distributions of wave height vs. steepness in shallow water and deep water are different, which may be related to the overall damping effect of bottom friction that causes a reduction of significant wave height in shallow water. As expected, in shallow water, there is no relation between the WAM model BFI and the probability of extreme waves. For deep water, such a relation cannot be ruled out.

Acknowledgement We thank Peter Janssen for valuable and stimulating discussions.

APPENDIX: Rayleigh Distribution

According to linear wave theory, see e.g. Holthuijsen (2007), the probability $p(r)$ that an individual wave has a height $H = rH_s$ is given by

$$p(r) = \exp(-r^2). \tag{4}$$

From this expression it follows the probability $P(\bar{r})$ that in a series of N waves all waves have $r < \bar{r}$ is

$$P(\bar{r}) = (1 - p(\bar{r}))^N. \tag{5}$$

The exact number of waves in a 20-minute record is not determined. In the North Sea, periods are shorter than in the open ocean and vary from 5 to 10 s, in very severe storms the dominant period can be higher. So a typical number of waves in a 20-min record is about 150. In Fig. 3, return maximum wave height as a function of the number of records is plotted as a solid line. The maximum wave height ratio r reaches a value of 2 for about 20 records (about 6 h) and increases slowly with the number of records. Even for 10^5 records, that is about 4 years, r is below 3.

For comparison, a distribution that follows from the theory of Mori and Janssen (2006) is shown as well (dotted line). The case shown corresponds to a BFI of 0.33. In Janssen theory, the BFI is directly related to κ, the kurtosis of the sea surface, by $\kappa = (\pi\ \sqrt{3})BFI^2$, so the case of Fig. 3 corresponds to $\kappa = 0.2$.

References

Berg R, Rhome J (2005) Expecting the unexpected wave: How the national weather service marine forecasts compare to observed seas. Mar. Weath. Log 49:4–7

Bojanowski A (2007) Monsterwellen bedrohen Schiffe im Nordatlantik http:\\www.spiegel.de\wissenschaft\natur\0,1518,470 359,00.html

Datawell (2006) Versnellingswaarden: Detectie en reparatie van extreme versnellingswaarden. Datawell internal report. Datawell BV, Haarlem, The Netherlands

Holliday N, Yelland M, Pascal R, Swail V, Taylor P, Griffitchs C, Kent E (2006) Were extreme waves in the Rockall trough the largest ever recorded? Geophys Res Lett 33:L05613 doi:10.1029/2005GL025238

Holthuijsen L (2007) Waves in oceanic and coastal waters. Cambridge University Press, Cambridge, UK, pp 404

Janssen P (2003) Nonlinear four-wave interactions and freak waves. J Phys Ocean 33:863–884

Mori N, Janssen P (2006) On kurtosis and occurence probability of freak waves. J Phys Ocean 36:1471–1483