Thermal Vibrational Convection in a Porous Medium Saturated by a Pure or Binary Fluid

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1 Introduction

1.1 What is Thermal Vibration?

The importance of mechanical vibration, as a source of pattern generating mechanism on the surface of a container filled with liquid, was recognized as early as the beginning of the 19th century by Faraday (1831). However, its importance as a mechanism controlling the convective motion has only been recognized during the 20th century.

Originally, mechanical vibration was used in mathematical modeling aimed at increasing the stability threshold of thermo-fluid system (Gershuni et al. 1970, Gresho and Sani 1970). The space exploration and especially the benefits expected from material production in space stations accelerated its development (Alexander 1994).

Formally, the thermo-vibrational convection studies concern the form of a mean flow in a confined cavity filled with a fluid presenting temperature non-homogeneities. Compared to the gravity-induced convection, this type of convection presents the advantage that it may exist under weightlessness condition.

Under micro-gravity conditions, the gravitational force is reduced drastically. However this situation may cause other forces, which under earth conditions are of minor importance, to be more significant.

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1.2 A Brief History of Thermal Vibration in Porous Media: Suppression of Motion and Generation of Motion

The theory of thermo-vibrational convection in the fluid medium is summarized by Gershuni and Lyubimov (1998) which mainly covers Russian studies in this field. Contrary to the thermo-vibrational problems in fluid media, studies of the vibrational counterpart in porous media are quite recent.

Most of the studies concerning thermo-vibrational convection in porous media are theoretical and are focused on the linear stability analysis. The preferred method is the time-averaged method (see Simonenko and Zenkovskaya (1966) for details). In this method the time dependent acceleration does not appear explicitly in the governing equations. Furthermore, this method provides us with closed form solution by which it is possible to obtain the stability threshold. Given the fact that thermal vibration problems generally depend on many parameters, the existence of some closed form relation are quite beneficial in understanding these problems. For porous media saturated by pure fluid, Zenkovskaya (1992) studies the effect of vertical vibration (parallel to the temperature gradient) on the thermal stability of the conductive solution. The geometry considered is an infinite horizontal porous layer. Zenkovskaya and Rogovenko (1999) consider the same problem with variable direction of vibration. The results of their linear stability analysis show that only the vertical vibration always has a stabilizing effect. These authors find that, for other directions of vibration, depending on the vibrational parameter and the angle of vibration, stabilizing and destabilizing effects are possible.

Malashetty and Padmavathi (1997) consider the same geometry with finite frequency. They use the Brinkman–Forcheimer model in their momentum equation. It has been found that the low frequency g-jitter has a significant effect on the stability of the system and that the effect of gravity modulation can be used to stabilize the conductive solution.

In a confined porous cavity heated from below, Bardan and Mojtabi (2000) consider the effect of vertical vibration. The vibration is in the limiting range of high frequency and small amplitude, which justifies their use of the time averaged method. The transient Darcy model is used in their momentum equation. It is shown that vibration reduces the number of convective rolls. Their results show that, in order to apply the time-averaged formulation effectively, the transient Darcy model should be kept. Further, they find that vibration increases the stability threshold. They also perform a weakly nonlinear stability analysis which indicates that primary bifurcations are of a special type of symmetry-breaking pitch fork bifurcation.

Pedramrazi et al. (2002) and Charrier-Mojtabi et al. (2003) discuss the validity of the time-averaged formulation in the Horton–Rogers–Lapwood problem using two different approaches; the time-averaged and the direct method. They also explain, from a physical point of view, the necessary assumptions for performing the time-averaged method. They further study the stability of the conductive solution via the Mathieu equation. Charrier-Mojtabi et al. (2006) revisited the horizontal layer and confined cavity and found a relation between stability analysis of these two problems via Mathieu equation. Bardan et al. (2004) revisited the effect of vertical vibration on confined cavity and they argued to obtain well-interpreted physical results, vibrational Rayleigh number should be redefined. Pedramrazi (2004) emphasized the restrictions of time-averaged method and showed the importance of sub-harmonic solutions. Govender studied the effect of vertical vibration on a horizontal porous layer (Pedramrazi 2004, Govender 2004, 2005a,b, 2006a,b, Pedramrazi et al. 2005). His results are in agreement with Palm et al. (1972).

2 The Effect of Vibration in Horizontal Porous Layer Saturated by a Pure Fluid

2.1 Infinite Horizontal Porous Layer

2.1.1 Mathematical Formulation

The problem of the onset of thermal instability in an infinite horizontal porous layer heated from below is well suited to illustrate mathematical and physical nature of thermo-vibrational problem.

The geometry of the problem consists of two horizontal parallel plates having lateral infinite extension (Fig. 1). The two rigid and impermeable plates are kept at two constant but different temperatures T_1 and T_2. The two plates are placed in a distant H apart. The porosity and permeability of the porous mesh forming the layer are ε and K respectively.

The porous medium is considered homogenous and isotropic. The porous layer and its boundaries are subjected to a harmonic vibration. As the objective is to study the onset of convection, the Darcy model can be used in the momentum equation. The fluid saturating porous media is assumed to be Newtonian and to satisfy the Oberbeck–Boussinesq approximation. The thermophysical properties are considered constant except for the density of fluid in the buoyancy term which depends linearly on the local temperature:

$$\rho(T) = \rho_0 \left[1 - \beta_T (T - T_2)\right]. \tag{1}$$

where T_2 is taken as the reference state, and the coefficient of volumetric expansion β_T is assumed constant ($\beta_T > 0$). In a coordinate system linked to the layer, the gravitational field is replaced by the sum of the gravitational and vibrational accelerations

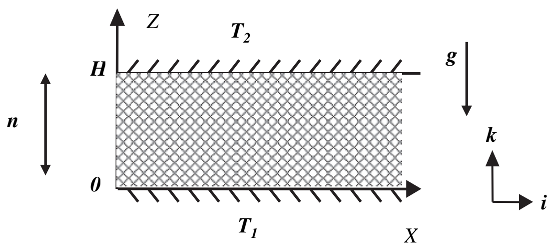

Fig. 1 Problem definition sketch

$\boldsymbol{g} \to -g\boldsymbol{k} + b\omega^2 \sin(\omega t)\boldsymbol{n}$. In this transformation $\boldsymbol{n}$ is the unit vector along the axis of vibration, b is the displacement amplitude and ω is the angular frequency of vibration. After making standard assumptions (local thermal equilibrium, negligible viscous heating dissipations. . .), the governing equations may be written as:

$$\begin{aligned}
&\nabla \cdot \mathbf{V} = 0, \\
&\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} = -\nabla P + \rho_0 \left[\beta_T (T - T_2)\right] \left(-g\,\mathbf{k} + b\omega^2 \sin \omega t\, \mathbf{n}\right) - \frac{\mu_f}{K}\mathbf{V}, \\
&(\rho c)_* \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{V} \cdot \nabla T = \lambda_* \nabla^2 T.
\end{aligned} \tag{2}$$

The boundary conditions corresponding to this system are written as:

$$\begin{aligned}
&\mathrm{V}_z(x, z = 0) = 0, \quad T(x, z = 0) = T_1, \\
&\mathrm{V}_z(x, z = H) = 0, \quad T(x, z = H) = T_2.
\end{aligned} \tag{3}$$

In (2), μ_f is the dynamic viscosity of fluid, $(\rho c)_*$ represents the effective volumic heat capacity, $(\rho c)_f$ is the volumic heat capacity of fluid and λ_* is the effective thermal conductivity of saturated porous media.

2.1.2 Time-Averaged Formulation

In order to study the mean behavior of mathematical system (1–2), the time-averaged method is used. This method is adopted under the condition of high-frequency and small-amplitude of vibration. Under these conditions, it is shown that two different time scales exist, which make it possible to subdivide the fields into two different parts. The first part varies slowly with time (i.e. the characteristic time is large with respect to vibration period) while the second part varies rapidly with time and is periodic with period $\tau = 2\pi/\omega$. Simonenko and Zenkovskaya (1966) used this procedure in thermo-vibrational problem in a horizontal fluid layer under the action of vertical vibration. So we may write:

$$\begin{aligned}
\mathbf{V}(M, t) &= \bar{\mathbf{V}}(M, t) + \mathbf{V}'(M, \omega t), \\
T(M, t) &= \bar{T}(M, t) + T'(M, \omega t), \\
P(M, t) &= \bar{P}(M, t) + P'(M, \omega t).
\end{aligned} \tag{4}$$

In the above transformations $(\bar{\mathbf{V}}, \bar{T}, \bar{P})$ represent the averaged fields (for a given function f(M, t), the average is defined as $\bar{f}(M, t) = \frac{1}{\tau} \int_{t-\tau/2}^{t+\tau/2} f(M, s)\, \mathrm{d}s$).

On replacing (4) in (2), we obtain two coupled systems of equations, one for the mean flow and the other for oscillatory one.

By making some assumption, we may express the oscillating fields in terms of the averaged ones. This procedure is detailed elsewhere (Mc Lachlan 1964).

Governing Time-Averaged Equations

By introducing the reference parameter, $T_1 - T_2$ for temperature, H for height, $\sigma H^2/a_*$ for time, ($a_* = \lambda_*/(\rho c)_f$ is the effective thermal diffusivity), a_*/H for velocity, $\beta_T \Delta T$ for **W** and $\mu a_*/K$ for pressure, we obtain dimensionless governing equation :

$$\begin{aligned}
&\nabla \cdot \bar{\mathbf{V}}^* = 0, \\
&B\frac{\partial \bar{\mathbf{V}}^*}{\partial t} = -\nabla \bar{P}^* + Ra_T \bar{T}^* \mathbf{k} + Ra_v (\mathbf{W} \cdot \nabla) \bar{T}^* \mathbf{n} - \bar{\mathbf{V}}^*, \\
&\frac{\partial \bar{T}^*}{\partial t} + \bar{\mathbf{V}}^* \cdot \nabla \bar{T}^* = \nabla^2 \bar{T}^*, \\
&\nabla \cdot \mathbf{W}^* = 0, \\
&\nabla \times \mathbf{W}^* = \nabla \bar{T}^* \times \mathbf{n}
\end{aligned} \tag{5}$$

The corresponding boundary conditions for the mathematical problem can be written as:

$$\begin{aligned}
&\forall x^*, \quad \text{for } z^* = 0, \quad \bar{\mathrm{V}}_z^* = 0, \quad \bar{T}^* = 1, \quad \mathrm{W}_z^* = 0, \\
&\forall x^*, \quad \text{for } z^* = 1, \quad \bar{\mathrm{V}}_z^* = 0, \quad \bar{T}^* = 0, \quad \mathrm{W}_z^* = 0.
\end{aligned} \tag{6}$$

where:

$$Ra_T = \frac{\mathrm{K} g \beta_T \Delta T H}{\nu a_*}, \quad Ra_v = \frac{(\delta^* Fr_F Ra_T \omega^*)^2}{2B},$$
$$\left(\delta^* = \frac{b}{H}, \; Fr_F = \frac{a_*^2}{g H^3 \sigma^2}, \; \omega^* = \omega \frac{\sigma H^2}{a_*}, B = \frac{a_* \mathrm{K}}{\varepsilon \nu \sigma H^2} = \frac{\tau_{hyd}}{\tau_{ther}}\right)$$

In the above relations Ra_T is the thermal Rayleigh number, Ra_v is the vibrational Rayleigh number ω^* is the dimensionless pulsation, B is the transient coefficient, Fr_F is the filtration Froude number and δ^* is the dimensionless amplitude. It should be added that these parameters (Fr, δ^*) are originally used in a horizontal fluid layer under vertical vibration in the pioneering work of Gresho and Sani (1970).

It should be noted that **W** is solenoidal field resulting from the Helmholtz decomposition ($(\bar{T} - T_2)\mathbf{n} = \mathbf{W} + \nabla\varphi$., **W** and $\nabla\phi$ are soloneidal and irrotational parts).

Linear Stability Analysis of the Time-Averaged System of Equations (Case of Vertical Vibration)

In the presence of vertical vibration ($\mathbf{n} = \mathbf{k}$), mechanical equilibrium is possible. In order to find the necessary condition for thermal stability of the problem, we set the velocity field equal to zero in (4). Then the steady-state distribution of fields are sought.

The equilibrium state corresponds to:

$$\bar{T}_0^* = 1 - z^*, \qquad \mathbf{W}_\mathbf{0}^* = 0. \tag{7}$$

For stability analysis, the fields are perturbed around the equilibrium state (for simplicity bars are omitted):

$$\mathbf{V}^* = 0 + \mathbf{v}',\ T^* = T_0^* + T',\ P^* = P_0^* + p',\ \mathbf{W}^* = \mathbf{W_0^*} + \mathbf{w}'.$$

Replacing the above equations in system (4) and (5), and after eliminating the non-linear terms we obtain:

$$\begin{aligned} \nabla \cdot \mathbf{v}' &= 0, \\ B\frac{\partial \mathbf{v}'}{\partial t^*} &= -\nabla p' + Ra_T T'\mathbf{k} + Ra_v(\mathbf{w}' \cdot \nabla T_0^* + \mathbf{W_0^*} \cdot \nabla T')\mathbf{k} - \mathbf{v}', \\ \frac{\partial T'}{\partial t^*} + \mathbf{v}' \cdot \nabla T_0^* &= \nabla^2 T', \\ \nabla \cdot \mathbf{w}' &= 0, \\ \nabla \times \mathbf{w}' &= \nabla T' \times \mathbf{k}. \end{aligned} \tag{8}$$

with corresponding boundary conditions:

$$\begin{aligned} &\mathrm{v}_z'(x^*, z^* = 0) = 0,\ T'(x^*, z^* = 0) = 0,\ \mathrm{w}_z'(x^*, z^* = 0) = 0, \\ &\mathrm{v}_z'(x^*, z^* = 1) = 0,\ T'(x^*, z^* = 1) = 0,\ \mathrm{w}_z'(x^*, z^* = 1) = 0. \end{aligned} \tag{9}$$

Introducing the stream functions Ψ, F, one can write:

$$\mathrm{v}_x' = \frac{\partial \psi}{\partial z^*},\ \mathrm{v}_z' = -\frac{\partial \psi}{\partial x^*},\ \mathrm{w}_x' = \frac{\partial F}{\partial z^*},\ \mathrm{w}_z' = -\frac{\partial F}{\partial x^*}. \tag{10}$$

On considering the 2D disturbances which are developed in normal modes:

$$(\psi,\ T',\ F) = (\phi(z^*),\ \theta(z^*),\ f(z^*)) \exp(-\lambda t^* + ikx^*) \tag{11}$$

in which k is the wave number in the horizontal direction 0x. Replacing (10) in (8) and (9), and eliminating pressure leads one to:

$$\begin{aligned} &(-\lambda B + 1)\left(\frac{d^2\phi(z^*)}{dz^{*2}} - k^2\phi(z^*)\right) = -ikRa_T\theta(z^*) + k^2 Ra_v f(z^*), \\ &-\lambda\theta(z^*) + ik\phi(z^*) = \frac{d^2\theta(z^*)}{dz^{*2}} - k^2\theta(z^*), \\ &-k^2 f(z^*) + \frac{d^2 f(z^*)}{dz^{*2}} = -ik\theta(z^*). \end{aligned} \tag{12}$$

System (12) is a spectral amplitude problem where λ is the eigenvalue of the system, which depends on:

$$\lambda = \lambda(Ra_T, Ra_v, k, B)$$

Generally, λ is a complex number ($\lambda = \lambda_r + i\lambda_i$).

The mathematical system (21), admits exact solutions of the form:

$$(\phi(z^*),\ \theta(z^*),\ f(z^*)) = (\phi, \theta, f)\sin n\pi\, z^* \tag{13}$$

By substituting (13) in (12), the necessary condition for obtaining marginal stability ($\lambda = 0$):

$$Ra_T = \frac{(\pi^2 + k^2)^2}{k^2} + Ra_v \frac{k^2}{\pi^2 + k^2}. \quad (n = 1) \tag{14}$$

One can understand from the above equation that, under micro-gravity ($Ra_T = 0$), the system is always stable.

Under the condition of vibration in presence of gravity, we can replace Ra_v with $(\delta^* Fr_F \omega^* Ra_T)^2/2B$. From (13), we get:

$$Ra_T = \frac{B}{\delta^{*2} Fr_F^2 \omega^{*2}} \frac{k^2}{k^2 + \pi^2}\left[1 - \sqrt{1 - 2\frac{\delta^{*2} Fr_F^2 \omega^{*2}}{B}(k^2 + \pi^2)}\right]. \tag{15}$$

Another interesting feature of this equation is that it gives additional information:

$$\omega^*_{\max} = \frac{\sqrt{B/2}}{\delta^* Fr_F \pi}. \qquad (k \to 0) \tag{16}$$

Relation (16) gives a possible maximum frequency for achieving absolute stabilization for high-frequency and small-amplitude vibration.

Weakly Non-linear Stability Analysis of the Time-Averaged System of Equations

In order to determine the characteristics of the solutions near the bifurcation point, the normal form of amplitude equation is sought.

The weakly nonlinear stability analysis of the time-averaged equations is expressed in terms of (ψ, θ, F) as follows:

$$\frac{\partial}{\partial t}\begin{bmatrix} B\nabla^2\psi \\ \theta \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -\nabla^2 & -Ra_T \frac{\partial}{\partial x^*} & -Ra_v \frac{\partial^2}{\partial x^{*2}} \\ -\frac{\partial}{\partial x^*} & \nabla^2 & 0 \\ 0 & \frac{\partial}{\partial x^*} & \nabla^2 \end{bmatrix}}_{L} \begin{bmatrix} \psi \\ \theta \\ F \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ 0 \end{bmatrix} \tag{17}$$

in which $\mathbf{L}$ represents a linear operator whereas N_1 and N_2 are nonlinear operators:

$$\begin{aligned} N_1 &= -Ra_v\left[\frac{\partial^2\theta}{\partial x^{*2}}\frac{\partial F}{\partial z^*} + \frac{\partial\theta}{\partial x^*}\frac{\partial^2 F}{\partial x^*\partial z^*} - \frac{\partial^2 F}{\partial x^{*2}}\frac{\partial\theta}{\partial z^*} - \frac{\partial F}{\partial x^*}\frac{\partial^2\theta}{\partial x^*\partial z^*}\right], \\ N_2 &= \frac{\partial\psi}{\partial x^*}\frac{\partial\theta}{\partial z^*} - \frac{\partial\psi}{\partial z^*}\frac{\partial\theta}{\partial x^*}. \end{aligned} \tag{18}$$

In order to study the onset of thermo-vibrational convection near the critical thermal Rayleigh number, the linear operator and the solution are expanded into power series of the positive small parameter η, defined by:

$$Ra_T = Ra_{Tc} + \eta\, Ra_{T1} + \eta^2\, Ra_{T2} + \cdots \tag{19}$$

Thus:

$$\begin{aligned} &[\psi, \theta, F] = \eta\,[\psi_1, \theta_1, F_1] + \eta^2\,[\,\psi_2, \theta_2, F_2] + \cdots \\ &\mathbf{L} = \mathbf{L}_0 + \eta\mathbf{L}_1 + \eta^2\mathbf{L}_2 + \cdots \end{aligned} \tag{20}$$

($\mathbf{L}_0$ is the operator which governs the linear stability). It should be noted that, in the operators, Ra_v is also expanded:

$$Ra_v = \frac{(\delta^* Fr_F \omega^*)^2}{2B}\left[Ra_{Tc}^2 + 2\eta Ra_{T1} Ra_{Tc} + \eta^2(2Ra_{Tc}Ra_{T2} + Ra_{T1}^2) + \cdots\right] \tag{21}$$

By replacing (19–21) in (17), and after introducing time transformation:

$$\frac{\partial}{\partial t^*} = \eta\frac{\partial}{\partial t_1^*} + \eta^2\frac{\partial}{\partial t_2^*} + \cdots$$

By equating the same power of η we obtain a sequential system of equations.

At each order of η, a linear eigenvalue problem is found. At the first order (η) the perturbation is written in the following form:

$$\begin{bmatrix} \psi_1 \\ \theta_1 \\ F_1 \end{bmatrix} = A(t_1^*, t_2^*, \ldots) \begin{bmatrix} (\pi^2 + k^2)/k^2\ \sin\pi z^* \sin kx^* \\ -(\pi^2 + k^2)/k \sin\pi z^* \cos kx^* \\ \sin\pi z^* \sin kx^* \end{bmatrix}. \tag{22}$$

The amplitude A depends on slow time evolutions $(t_1^*, t_2^*, \ldots)$.

At the second order η^2, the existence of a convective solution requires that the solvability lemma be satisfied, in other words there must be a non-zero solution for the adjoint of $\mathbf{L}_0$ associated with identical boundary conditions. From the adjoint operator, we obtain:

$$Ra_{Tc}^* = Ra_{Tc}.$$

Also, we find $Ra_{T1} = 0$ and amplitude A does not depend on time scale $t_1{}^*$.

At the third order η^3 by invoking the solvability condition and the Fredholm alternative we obtain the amplitude equation:

$$\frac{dA}{dt_2^*} = \alpha(A - \beta A^3).$$

in which α and β are defined as:

$$\alpha = \frac{k^2}{(k^2+\pi^2)^2}\left[(k^2+\pi^2) - \frac{(\delta^* Fr_F \omega^*)^2}{B} k^2 Ra_{Tc}\right] Ra_{T2},$$
$$\beta = \frac{(\pi^2+k^2)^2\left[1 - \dfrac{k^4(\delta^* Fr_F \omega^*)^2}{B(\pi^2+k^2)^3} Ra_{Tc}^2\right]}{8Ra_{T2}\left[(\pi^2+k^2) - \dfrac{(\delta^* Fr_F \omega^*)^2}{B} k^2 Ra_{Tc}\right]}. \tag{23}$$

In α and β, Ra_{T2} is defined as $Ra_{T2} = (Ra_T - Ra_{Tc})/\eta^2$ which is the control parameter.

When there is no vibrational effect, we find that the amplitude of thermo-convective flow near the bifurcation point is proportional to:

$$A \approx \sqrt{Ra_T - Ra_{Tc}}$$

which is in agreement with Palm et al. (1972). Under the effect of vibration α and β are both positive, which results in a supercritical pitchfork bifurcation.

2.1.3 Linear Stability Analysis from Direct Formulation

In this section, we study the stability of the solution corresponding to governing equations where the time dependent buoyancy term appears explicitly. As was stated before, when the direction of vibration is parallel to gravitational acceleration, mechanical stability is possible. This equilibrium is characterized by a linear temperature and parabolic pressure distribution. In order to study linear stability, the field variables are infinitesimally perturbed around the motionless equilibrium state. The perturbed system of equation becomes:

$$\nabla \cdot \tilde{\mathbf{v}} = 0,$$
$$\frac{\rho_0}{\varepsilon}\frac{\partial \tilde{\mathbf{v}}}{\partial t} = -\nabla \tilde{p} + \rho_0 \beta_T \tilde{\theta}(g + b\omega^2 \sin\omega t)\,\mathbf{k} - \tfrac{\mu}{\mathrm{K}}\tilde{\mathbf{v}}, \tag{24}$$
$$\sigma\frac{\partial \tilde{\theta}}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla T_0 = a_* \nabla^2 \tilde{\theta}.$$

By eliminating the pressure in the momentum equation and by substituting the normal modes as:

$$\tilde{\mathrm{v}}_z = X(t)\, e^{ik\frac{x}{H}} \sin\frac{z}{H}\pi,\ \tilde{\theta} = h(t) e^{ik\frac{x}{H}} \sin\frac{z}{H}\pi. \tag{25}$$

in the resulting equations, we get:

$$\frac{d^2h}{dt^2} + \left[\frac{a_*}{\sigma H^2}(k^2+\pi^2) + \frac{\varepsilon\nu}{K}\right]\frac{dh}{dt} +$$
$$\left[\frac{\varepsilon \nu a_*}{K H^2 \sigma}(k^2+\pi^2) - \frac{\varepsilon \beta_T \Delta T}{\sigma H}\frac{k^2}{k^2+\pi^2}(g + b\omega^2 \sin\omega t)\right] h = 0. \tag{26}$$

The above equation is similar to a mechanical pendulum with oscillating support:

$$\ddot{Y} + 2\xi\omega_n \dot{Y} \pm ({\omega_n}^2 - \omega^2 \frac{\delta}{\ell} \sin \omega t)\, Y = 0. \tag{27}$$

in which ω_n represents the natural frequency, ξ damping ratio, ω vibrational frequency, ℓ pendulum length and finally δ the amplitude of vibration. The plus sign in (27) corresponds to a normal hanging pendulum while the negative sign corresponds to an inverted pendulum. Equalizing the vibrational effect in the two systems gives:

$$\ell_{eff} \approx \frac{H}{\frac{\varepsilon}{\sigma}\beta_T \Delta T}. \tag{28}$$

which is the effective length of the equivalent system. In addition it is clear that this effective length is quite long ($\beta_T \Delta T << 1$).

Equation (26) can be written in dimensionless form:

$$B\frac{d^2h^*}{dt^{*2}} + \left[B(k^2+\pi^2)+1\right]\frac{dh^*}{dt^*} + \left[(k^2+\pi^2) - Ra_T \frac{k^2}{k^2+\pi^2}(1+R\sin\omega^* t^*)\right]h^* = 0. \tag{29}$$

where B, Ra_T, ω^* are defined as in the previous section. Also we can define R as $\delta^* Fr_F {\omega^*}^2$. For the above equation two different cases are distinguished:

(i) $B\omega^* << 1$

In this case, the governing equation is written as:

$$\frac{dh^*}{dt^*} + \left[(\pi^2+k^2) - Ra_T \frac{k^2}{k^2+\pi^2}(1+\delta^* Fr_F {\omega^*}^2 \sin\omega^* t^*)\right]h^* = 0. \tag{30}$$

The solution of this first order differential equation with periodic coefficient is:

$$h^* = h_0^* \exp - \left[(\pi^2+k^2) - \frac{k^2}{k^2+\pi^2} Ra_T\right] t^* \cdot \exp\left(2\delta^* Fr_F \omega^* \frac{k^2}{k^2+\pi^2} Ra_T \sin^2 \omega^* t^*\right),$$
$$h^*(0) = h_0^*. \tag{31}$$

When there is no vibration ($\delta^* Fr_F \omega^* = 0$), from (30) the classical result of $Ra_{Tc} = 4\pi^2$ for marginal stability may be deduced. In the presence of vibration, if the layer is heated from above the solution is always stable. This is true because, in this situation, the arguments in exponential functions (31) are always positive. When the layer is heated from below, the solution is composed of two parts, see (30) the second part of which can be considered as a positive bounded periodic function. Therefore, for marginal stability, the first part is important and gives $\mathrm{Ra_T} = 4\pi^2$. In other words, vibration has no effect on stability threshold. Physically from the mechanical analogy, this case corresponds to a pendulum in which the viscous damping is much larger than angular acceleration. Strong damping is able to destroy the oscillatory movements.

(ii) $B\omega^* >> 1$

Using transformation $h^*(t^*) = e^{-mt^*} M(t^*)$ equation (29) is cast into Mathieu's equation (m being $(\pi^2 + k^2 + 1/B)/2$):

$$\frac{d^2 M(\tau)}{d\tau^2} + (A - 2Q\cos 2\tau)M(\tau) = 0, \quad \left(\omega^* t^* = 2\tau - \frac{\pi}{2}\right) \tag{32}$$

in which A and Q are:

$$A = \frac{\pi^2 + k^2}{B} - m^2 - \frac{k^2}{B(\pi^2 + k^2)} Ra_T, \qquad Q = \frac{2k^2}{B(\pi^2 + k^2)} \delta^* Fr_F Ra_T. \tag{33}$$

Detailed analysis of the stable regions for this equation can be found elsewhere (Mc Lachlan 1964, Cunningham 1958, Jordan and Smith 1987). They divide the domain into alternate stable and unstable regions. In order to solve (32), the Floquet theory is used, which considers the solution as:

$$M = R(\tau)e^{\mu\tau}.$$

in which $R(\tau)$ is a periodic function having period π or 2π, the parameter μ is the Floquet exponent and the marginal stability condition is $m = \mu\omega^*/2$. The details of this method can be found elsewhere Aniss et al. (2000).

To obtain the critical thermal Rayleigh and wave numbers for marginal stability, all the working parameters ($B, \omega^*, \delta^*, Fr_F$) are fixed except Ra_T and k. Then we search for the minimum Ra_T vs k. It can be concluded that, for given dimensionless amplitude δ^* and dimensionless frequency of vibration ω^*, there are two modes of convection onset, namely harmonic (with dimensionless frequency ω^*) and sub-harmonic (with dimensionless frequency: $\omega^*/2$) (Fig. 2)

For heating from below ($Ra_T > 0$, which corresponds to $A < 0$), two different behaviors for harmonic and sub-harmonic modes are distinguished: for harmonic mode with increasing ω^*, thermal Rayleigh number Ra_T increases. This means that vibration has a stabilizing effect which depends significantly on dimensionless amplitude δ^*. On decreasing δ^*, the stable region with harmonic response widens. If the frequency is increased, the critical wave number for this mode decreases. For the sub-harmonic mode we have a different scenario, the vibration has a destabilizing effect, in other words Ra_{Tc} decreases and ultimately reaches a limiting value. The critical wave number in this mode increases with increasing dimensionless frequency. It should be noted that the intersection of harmonic and sub-harmonic modes corresponds to different values of wave number. For other heating condition, one may consult Cunningham (1958).

2.1.4 Comparison of the Two Methods

The objective of this section is to compare the two approaches of stability analysis in the thermo-vibrational problem, namely the time-averaged and the so-called direct methods. The time-averaged method under high frequency and small amplitude

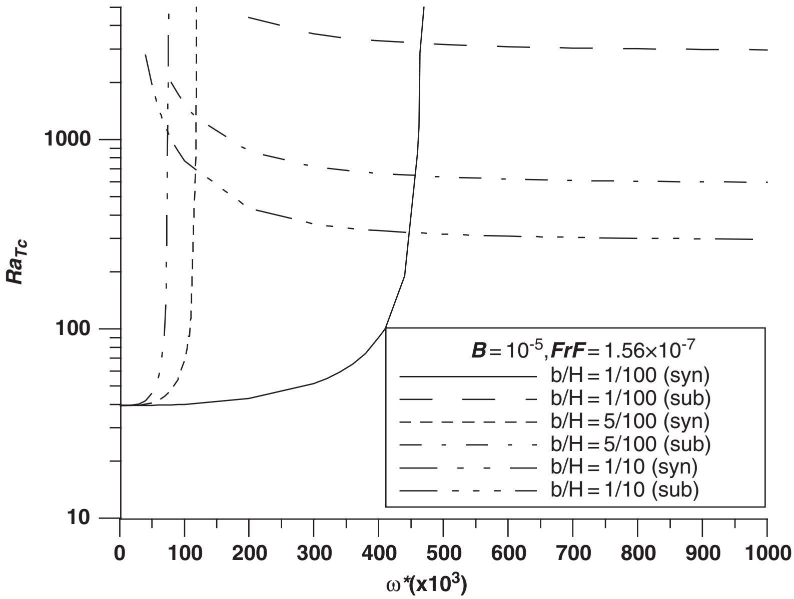

Fig. 2 The effect of vibrational frequency on the critical Rayleigh Number Ra_{Tc}, for different values of b/H: harmonic response (syn) and sub-harmonic response (sub)

vibration is considered in the previous section. As has been stated before, this limiting case permits us to subdivide the fields (temperature, velocity and pressure) into two parts. The question is under which condition we can find this characteristic of solution (subdivision of fields) if we adopt the direct method. Let us examine what will happen if we apply the assumptions needed for finding the criteria of high frequency and small amplitude to the coefficients of Mathieu's equation. We write Mathieu's equation and its coefficients A and Q as:

$$\frac{d^2M(\tau)}{d\tau^2} + (A - 2Q\cos 2\tau)M(\tau) = 0, \tag{34}$$

$$A = -\left[\frac{a_*}{\sigma H^2\omega}\left(k^2+\pi^2\right) + \frac{\varepsilon\nu}{K\omega}\right]^2 + 4\left(\frac{\varepsilon\nu}{K\omega}\right)\left(\frac{a_*}{\omega\sigma H^2}\right)\left(k^2+\pi^2\right)$$
$$- 4\left(\frac{\varepsilon\beta_T\Delta T g}{\sigma H\omega^2}\right)\frac{k^2}{k^2+\pi^2},$$
$$Q = 2\left(\frac{\varepsilon}{\sigma}\beta_T\Delta T\frac{b}{H}\right)\frac{k^2}{k^2+\pi^2}.$$

Let us examine A and Q closely:

The first and second terms in A involve the ratios of vibrational time scale to conductive and viscous time scales. The third term in A involves the ratio of vibrational time scale to pseudo buoyancy time scale.

Q involves a kind of amplitude ratio. Based on hypothesis of high-frequency and small amplitude all these terms are very small (vibration time scale is the smallest time scale and amplitude of temperature oscillation should be also small) so A and Q tend to zero. A regular perturbation method in which Q is considered as a small parameter may be used:

$$M(\tau) = M_0(\tau) + QM_1(\tau) + Q^2 M_2(\tau) + \cdots$$
$$A = A_0 + QA_1 + Q^2 A_2 + \cdots \tag{35}$$

Replacing the above expansions in Mathieu's equation and invoking solvability condition results:

$$A = -\frac{Q^2}{2}, \tag{36}$$
$$M = a_0 - \frac{a_0}{2}\cos 2\tau. \tag{37}$$

(a_0 is an arbitrary constant)

On replacing A and Q in the (36) and using the fact that $\mu = [a_*(k^2 + \pi^2)/\sigma H^2\omega + \varepsilon\nu/K\omega)] = 0$, we find:

$$Ra_{Tc} = \frac{(\pi^2 + k^2)^2}{k^2} + Ra_v \frac{k^2}{k^2 + \pi^2}. \quad (Ra_v = \frac{(\delta^* Fr_F \omega^* Ra_T)^2}{2B}) \tag{38}$$

Which means that imposing the assumptions needed for the averaging method on Mathieu's equation gives identical results to the time-averaged formulation. The most interesting thing about this fact is that the time-averaged method gives only harmonic (with dimensionless frequency ω^*) mode and is not able to give subharmonic mode.

2.1.5 Effect of the Direction of Vibration

Zenkovskaya and Rogovenko (1999) study the effect of the direction of vibration on the onset of convection. They use the time-averaged formulation and discuss several physical situations. When the direction of vibration is not parallel to the temperature gradient, there exists a quasi-equilibrium; i.e. the mean velocity is zero but the oscillating velocity is not zero. The equilibrium solution is characterized by:

$$\mathbf{V_0} = \mathbf{0}, T_0 = 1 - z \quad \text{and} \quad W_{0x} = (\frac{1}{2} - z)\cos(\alpha) \tag{39}$$

where α is the angle between the vibration direction and the horizontal direction.

We study different physical situations:

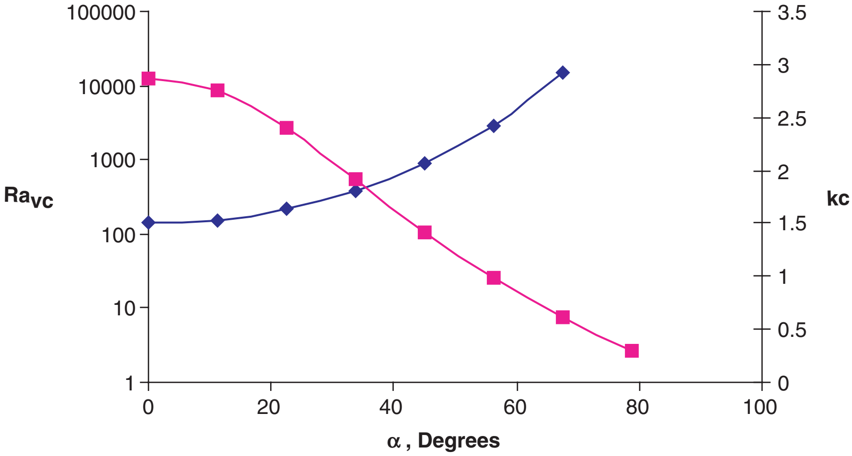

Fig. 3 Influence of the direction of the vibration on the values of the vibrational critical Rayleih number Ra_{vc} (*left*) and the critical wave number kc (*right*)

The Onset of Convection Under Micro-gravity ($Ra_T = 0$)

One of the most interesting results reported in Zenkovskaya and Rogovenko (1999) is that, if the direction of vibration is not parallel to the temperature gradient, there is a possibility of convective motion under micro-gravity conditions. In this situation, only vibrational controlling force is operational. Figure 3 shows the critical values of the vibrational Rayleigh number (Ra_V) and the critical values of wave number (k_c) as a function of α (the direction of vibration with respect to the heated plate). It can be observed that, with increasing direction of vibration α, the domain of stability increases. At the same time, the wave number decreases with increasing direction of vibration. It should be emphasized that for $\alpha = \pi/2$, i.e. the vertical vibration, the equilibrium solution is infinitely linearly stable.

The Onset of Thermo-Vibrational Convection in the Presence of Gravity ($Ra_T \neq 0$ *and* $R \neq 0$)

In this case, the two controlling mechanisms, namely the vibrational and gravitational, are present. For the direction of vibrations $5\pi/16 < \alpha < \pi/2$, there are some values of R for which maximum stability may be obtained. Another interesting feature of the effect of direction of vibration is that for the layer heated from above, we may obtain convective motions. This is in severe contrast to the classical Horton–Rogers–Lapwood problem in which, for the case of the layer heated from above, the layer is infinitely stable. A detailed summary of the effect of direction of vibration can be found elsewhere (Cunningham 1958).

2.2 Confined Cavity

2.2.1 Introduction

From few studies devoted to confined geometries under the effect of vibration, we may mention Bardan and Mojtabi (2000). A numerical and an analytical study of convective motion in a rectangular porous cavity saturated by a pure fluid and subjected to a high-frequency and small-amplitude vibration is presented in Bardan and Mojtabi (2000). The transient Darcy formulation is adopted in the momentum equation. As vibration has high frequency and small amplitude, the relevant equations are solved by the time-averaged method. The same problem under arbitrary frequency of vibration has been studied recently by Charrier-Mojtabi et al. (2006). They conclude that the results found from stability analysis of infinite porous layer can be used to calculate the onset of convection in a confined cavity.

2.2.2 Stability Analysis

Linear Stability Analysis

In the study (Bardan and Mojtabi 2000), the case of vertical vibration, i.e. $\alpha = \pi/2$ has been considered. For this situation mechanical equilibrium is possible. To perform linear stability analysis, the field variables are infinitesimally perturbed around the motionless equilibrium state. By eliminating the pressure in the momentum equation and introducing the perturbed field as:

$$\tilde{v}_z = \sum_{n=1}^{p}\sum_{m=1}^{q} X_{nm}(t)\sin n\pi z \sin\frac{m\pi x}{A_L},\ \tilde{\theta} = \sum_{n=1}^{p}\sum_{m=1}^{q} h_{nm}(t)\sin n\pi z \sin\frac{m\pi x}{A_L}. \quad (40)$$

m and n are integer numbers and represent number of rolls in the x and z directions respectively.

By defining $Y = (X_{mn}(t), h_{mn}(t))$ with components as explained in (40), we obtain the following equation:

$$\frac{dY}{dt} = M_0 Y + N_0 Y \sin\omega t \quad (41)$$

with M_0 and N_0 defined as:

$$M_0 = \begin{bmatrix} -\frac{1}{B} & \frac{Ra_T}{B}\frac{\left(\frac{m\pi}{A_L}\right)^2}{\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2} \\ 1 & -\left(\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2\right) \end{bmatrix},\quad N_0 = \begin{bmatrix} 0 & -\frac{Ra_T}{B}\delta Fr\omega^2\frac{\left(\frac{m\pi}{A_L}\right)^2}{\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2} \\ 0 & 0 \end{bmatrix} \quad (42a)$$

Elimination of $X_{mn}(t)$ in (40) and setting $h_{mn} = h$, leads us to a damped Mathieu equation:

$$B\frac{\mathrm{d^2h}}{\mathrm{dt^2}} + \left[B\left(\left(\frac{m\pi}{\mathrm{A_L}}\right)^2 + (\mathrm{n}\pi)^2\right) + 1\right]\frac{\mathrm{dh}}{\mathrm{dt}}$$
$$+\left[\left(\left(\frac{\mathrm{m}\pi}{\mathrm{A_L}}\right)^2 + (\mathrm{n}\pi^2)\right) - \mathrm{Ra_T}\frac{\left(\frac{\mathrm{m}\pi}{\mathrm{A_L}}\right)^2}{\left(\frac{\mathrm{m}\pi}{\mathrm{A_L}}\right)^2 + (\mathrm{n}\pi)^2}(1 - \delta \mathrm{Fr_F}\omega^2 \sin\omega \mathrm{t})\right]\mathrm{h} = 0. \tag{42b}$$

Dividing by B and using the transformation $h(t) = e^{-\lambda t}M(t)$, the above equation is cast into Mathieu's equation (λ being $[(m\pi/A_L)^2 + (n\pi)^2 + 1/B]/2$:

$$\frac{d^2M}{d\tau^2} + (A - 2Q\cos 2\tau)M = 0 \qquad (\omega t = 2\tau - \pi/2) \tag{43}$$

in which A and Q are defined as:

$$A = \frac{4}{B\omega^2}\left[\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2\right] - \frac{\left(\frac{m\pi}{A_L}\right)^2}{\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2}\frac{Ra_T}{B\omega^2},$$
$$Q = \frac{2\left(\frac{m\pi}{A_L}\right)^2}{\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2}\frac{Ra_T\tilde{R}}{B\omega^2} \tag{44}$$

If we compare (43) and (44) with the coefficients of the Mathieu equation obtained from the linear stability analysis of an infinite horizontal layer of previous section, we observe an interesting analogy between these two equations. It means that one should set: $m\pi/A_L = k$ (k being the wave number in the infinite direction (Ox) of the layer). Thus we obtain:

$$Ra_T = \frac{\left[\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2\right]^2}{\left(\frac{m\pi}{A_L}\right)^2} + \frac{\left(\frac{m\pi}{A_L}\right)^2}{\left[\left(\frac{m\pi}{A_L}\right)^2 + (n\pi)^2\right]}Ra_V \tag{45}$$

with $Ra_v = Ra_T^2 R^2$ and $R_v^2 = \dfrac{(\delta\,\mathrm{Fr_F}\,w)^2}{2\,B}$

The critical values of Ra_{Tc} and $m\pi/A_L = k_c$ are given as:

$$\begin{aligned} Ra_{Tc} &= \frac{\left(2\pi^2 - k_c^2\right)\left(\pi^2 + k_c^2\right)^2}{\pi^2 k_c^2} && \text{(a)} \\ R_v^2 &= \frac{\pi^2\left(\pi^2 - k_c^2\right)}{\left(2\pi^2 - k_c^2\right)^2\left(\pi^2 + k_c^2\right)} && \text{(b)} \end{aligned} \quad (46)$$

In order to find the critical value of the thermal Rayleigh number, we should simultaneously solve (46a) and (46b) for given values of A_L and m.

Weakly Non-linear Stability Analysis

In order to obtain the normal form of the amplitude equation and to determine the characteristics of solutions (stream function and temperature) near the bifurcation point, a weakly non-linear analysis is carried out in Bardan and Mojtabi (2000). This analysis is based on the multi-scale approach. The procedure is the same as in previous section and will not be repeated. The amplitude equation can be written as:

$$a\frac{\partial K}{\partial t} = bK(\mu + cK^2) \quad (47)$$

where $\mu = (Ra - Ra_c)/\eta^2$ is the bifurcation parameter.

The sign of these coefficients (a, b, and c) are functions of Ra_v and A_L. We distinguish two different cases:

$C > 0$ and $-b\mu/a < 0 \Rightarrow$ bifurcation is stable supercritical pitch-fork

$C < 0$ and $-b\mu/a > 0 \Rightarrow$ bifurcation is unstable sub-critical pitch-fork

2.2.3 2D Numerical Simulations

The time-averaged equations are solved using a collocation spectral method.

The time discretization is a second order Adams–Bashforth–Euler backward scheme (Sovran et al. 2002). The influence of the vibrational Rayleigh number Ra_v (B is fixed to 10^{-5}), is investigated for a cavity of aspect ratio $A_L = 3$. The spatial resolution is 63×27 collocation points along the horizontal and vertical axes respectively.

For $A_L = 3$, we deduce from the results of the stability analysis (46) that for $Ra_{Tc} = 4\pi^2$ and $Ra_v = 0, m = 3$ and $n = 1$, the onset of the convection corresponds to a three rolls motion in the horizontal direction. We also obtain, for $Ra_{Tc} = 72.09$ and $Ra_v = 82.94$, the transition from the convective motion characterized by three rolls to the two rolls motion ($m = 2, n = 1$).It should be observed that, as the problem depends on two control parameters , Ra_T and Ra_v, the results are expressed as function of the couple (Ra_T, Ra_v).

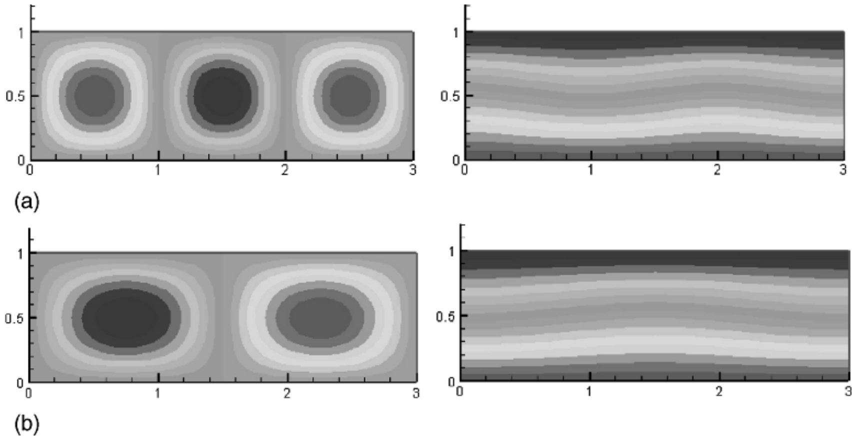

Fig. 4 Streamlines and isotherms for: (**a**) $A_L = 3.$, $Ra_{Tcnum} = 41$ and $Ra_{vnum} = 0$, (**b**) $A_L = 3$, $Ra_{Tcnum} = 73$ and $Ra_{vnum} = 84$

For the onset of stationary convection, the numerical results are presented as the streamlines and the isotherms associated to the mean field for $Ra_{Tcnum} = 41$ and $Ra_{vnum} = 0$ (Fig. 4a).We can observe three rolls as predicted by the linear stability theory. For the set of parameters: $Ra_{Tcnum} = 73$ and $Ra_{vnum} = 84$, the streamlines and isotherms are shown in Fig. 4b. We can observe the existence of two rolls which is in good agreement with our theoretical results.

For the aspect ratio $A_L = 4$, the stability analysis indicates that the onset of convection for $m = 4$ and $n = 1$ corresponding to four rolls is obtained for $Ra_{Tc} = 4\pi^2$ and $Ra_v = 0$ .We also obtain for $Ra_{Tc} = 61.58$ and $Ra_v = 52.06$, the transition from the convective motion characterized by four rolls to the three rolls motion. The numerical results obtained for the onset of convection for the solution with four rolls are $Ra_{Tcnum} = 39.6$ and $Ra_{vnum} = 0$ (global Nusselt number, Nu = 1.0061). For the onset of the solution with three rolls, we obtain: $Ra_{Tcnum} = 62.5$ and $Ra_{vnum} = 54$ (global Nusselt number, Nu = 1.0187).

For all the numerical cases studied, the global Nusselt number, Nu, is close to one which indicates that we are close to the threshold of convection. In conclusion for these two aspect ratios the numerical results corroborate the theoretical ones.

2.3 Some Key Results

The stability analysis of a porous layer under the effect of mechanical vibration is presented. The layer can be heated uniformly from below or from above. It is shown that vibration can change the onset of convective motion in porous media. The change of threshold depends on direction, amplitude and frequency of vibration. For the case of mechanical vibration parallel to the temperature gradient (vertical vibration), mechanical equilibrium is possible. For this case, under different heating conditions (heating from above or below), there is a possibility of convective motion that largely depends on the chosen values of amplitude and frequency of vibration. The response of the system shows harmonic or sub-harmonic behavior. For heating from below, the harmonic mode exhibits a stabilizing behavior whereas the sub-harmonic mode exhibits a destabilizing one. For heating from below the results indicate that, under the condition of high-frequency and small amplitude of vibration, the harmonic part shows a strong stabilizing effect. Under this limiting situation, the time-averaged formulation can be adopted. A weakly nonlinear stability analysis is performed for this averaged system revealing that bifurcation is of supercritical pitch-fork type. For the case of other directions of vibration ($\alpha \neq \pi/2$) under high-frequency and small amplitude, it is shown that, in the presence of gravitational acceleration for the layer heated from below, vibration may produce stabilizing or destabilizing effects. These depend largely on the choice of vibrational parameter and the direction of vibration. For the layer heated from above, decreasing the direction of vibration from $\alpha = \pi/2$ to $\alpha = 0$ reduces the stability domain (Ra_{Tc} decreases). For the case of convection under micro-gravity conditions, it is shown that there is a possibility of thermo-vibrational convection for all directions of vibration except vertical vibration ($\alpha \neq \pi/2$).

A simple procedure for obtaining the critical Rayleigh number in a confined cavity is proposed through an analogy with the linear stability analysis results in an infinite porous layer.

3 Influence of Mechanical Vibration on a Porous Media Saturated by a Binary Mixture

In this section, we study the effect of vibrational mechanism on coupled dissipative phenomena, namely, the Soret driven convective motion in a porous medium saturated by a binary mixture. Under the Soret effect, a concentration gradient is established as a result of the temperature gradient (De Groot and Mazur 1984). This problem in the context of vibration in fluid media was studied in an infinite horizontal layer (Gershuni et al. 1997, 1999). The limiting case of high-frequency and small amplitude vibration was studied which enabled the time-averaged method to be used. It is found that vibration could drastically change the stable zones in the stability diagram. Generally, vertical vibration (parallel to the temperature gradient) increases the stability of the conductive mode. Smorodin et al. (2002) studied the same problem under finite frequency. They also showed that, in synchronous mode, vibration has a stabilizing effect.

On existing vibrational thermosolutal convection, we may mention Jounet and Bardan (2001) and Charrier-Mojtabi et al. (2004). In Jounet and Bardan (2001) the solutal and temperature differences are imposed while in Sovran et al. (2002) and Charrier-Mojtabi et al. (2004) the temperature gradient generates mass flux (Soret effect).

It should be noted that the study of thermosolutal problem provides us with more interesting instability mechanisms and pattern generating phenomena, which are normally absent in single component fluid.

3.1 Problem Description

The geometry consists of a rectangular cavity filled with a porous medium saturated by a binary mixture. The aspect ratio is defined as $A = L/H$ where H is the height and L is the length of the cavity. The boundaries of the cavity are rigid and impermeable; the horizontal ones can be heated from below or above. The lateral boundaries are thermally insulated and impermeable (Fig. 5). The governing equations are written in a reference frame linked to the cavity. The case with high frequency and small amplitude vibration is considered.

The direction of vibrations is defined in this Section 3 by:

$$\mathbf{e} = \cos(\alpha)\mathbf{i} + \sin(\alpha)\mathbf{j} \qquad \text{with} \quad g = -g\mathbf{j}$$

Under the Boussinesq approximation the dimensionless governing equations for the mean flow averaged over the vibration period can be written as:

$$\begin{aligned}
&\nabla \cdot \mathbf{V} = 0\\
&B\tfrac{\partial \mathbf{V}}{\partial t} + \mathbf{V} = -\nabla P + Ra(T + \psi C)\mathbf{j} + Rv(\mathbf{W}_T + \psi \mathbf{W}_c)\cdot\\
&\nabla(T + \tfrac{\psi}{\varepsilon^*}C)(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})\\
&\frac{\partial T}{\partial t} + \vec{\mathbf{V}} \cdot \nabla T = \Delta T,\\
&\varepsilon^*\frac{\partial C}{\partial t} + \vec{\mathbf{V}} \cdot \nabla C = \tfrac{1}{Le}(\Delta C - \Delta T)\\
&\nabla \cdot \mathbf{W}_T = 0, \quad \nabla \cdot \mathbf{W}_c = 0\\
&T\mathbf{e} = \mathbf{W}_T + \nabla\xi_T, \quad C\mathbf{e} = \mathbf{W}_c + \nabla\xi_c
\end{aligned} \tag{48}$$

Where $\mathbf{V}$, T, C are velocity, temperature and mass fraction fields and $\mathbf{W_T}$ and $\mathbf{W_c}$ are solenoidal vectors corresponding to the temperature and concentration fields respectively.

The corresponding boundary conditions are:

$$\begin{aligned}
&\mathbf{W}_T \cdot \mathbf{n} = \mathbf{W}_c \cdot \mathbf{n} = 0\\
&y = 0 : T = T_1, \quad \mathbf{J_m}\cdot\mathbf{n} = 0\\
&y = 1 : T = T_2, \quad \mathbf{J_m}\cdot\mathbf{n} = 0\\
&x = 0,\ A : \frac{\partial T}{\partial x} = \frac{\partial C}{\partial x} = 0
\end{aligned} \tag{49}$$

Mathematical system (48) depends on eight parameters; the thermal Rayleigh number $Ra = Kg\beta\Delta T H/\nu a^*$, the vibrational Rayleigh number

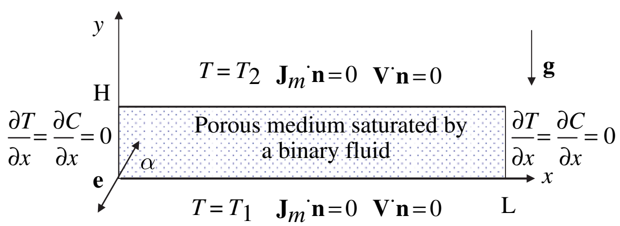

Fig. 5 Geometry and boundary conditions: case of a porous medium saturated by a binary fluid

$Rv = (\tilde{R}^2 Ra^2 B)/2(B^2\omega^2 + 1) = R^2 Ra^2 (\tilde{R} = b\varpi^2/g)$, the separation factor $\psi = -C_i(1 - C_i)(\beta_c/\beta_T)D_T/D^*$, the normalized porosity ε^* ($\varepsilon^* = \varepsilon/\sigma$,), the Lewis number Le ($Le = a^*/D^*$ in which a^* is the effective thermal diffusivity and D^* is the effective mass difusivity), the coefficient of the unsteady Darcy term in the momentum equation $B = Da/(\sigma\varepsilon Pr^*)$ (in porous media $B \approx 10^{-5}$ and Da represents the Darcy number $Da = K/H^2$), and finally α the direction of vibration with respect to the heated boundary.

3.2 Linear Stability Analysis

For the direction of vibration parallel to the temperature gradient ($\alpha = \pi/2$), there exists a mechanical equilibrium (for both an infinite horizontal layer and a confined cavity), which is characterized by:

$$\mathbf{V}_0 = 0, \quad T_0 = 1 - y, \quad C_0 = cst - y, \quad \mathbf{W}_{T0} = 0, \quad \mathbf{W}_{C0} = 0 \tag{50}$$

However, for other directions of vibration, we may obtain quasi-equilibrium solution only for the infinite horizontal layer. This is characterized by:

$$\begin{aligned} &\mathbf{V}_0 = 0, \quad T_0 = 1 - y, \quad C_0 = c_1 - y, \quad \mathbf{W}_{T0_x} = c_2 - y\cos\alpha; \quad \mathbf{W}_{T0_y} = 0, \\ &\mathbf{W}_{C0_x} = c_3 - y\cos\alpha, \quad \mathbf{W}_{C0_y} = 0 \end{aligned} \tag{51}$$

It should be noted that, for a confined cavity, there is no equilibrium solution under the horizontal vibration ($\alpha = 0$).

3.2.1 Infinite Horizontal Porous Layer

In order to investigate the stability of the conductive solution, the fields are perturbed around the equilibrium state. Then after linearization, the disturbances are developed in the form of normal modes. We introduce the stream function perturbation ϕ, the temperature perturbation θ and the mass fraction perturbation c. Also we designate the stream function perturbations ϕ_θ and ϕ_c for corresponding solenoidal fields $\mathbf{W_T}$ and $\mathbf{W_c}$. In order to facilitate our study, we use the transformations $\eta = c - \theta$ and $\varphi_\eta = \varphi_c - \varphi_\theta$. We may write:

$$\begin{aligned} &\varphi = \sum_{i=1}^{N} \mathrm{a_i} \sin(i\pi y)\exp(\sigma t + Ikx); \theta = \sum_{i=1}^{N} \mathrm{b}_i \sin(i\pi y)\exp(\sigma t + Ikx); \\ &\eta = \sum_{i=0}^{N-1} \mathrm{c_i} \cos(i\pi y)\exp(\sigma t + Ikx) \\ &\varphi_\theta = \sum_{i=1}^{N} \mathrm{d}_i \sin(i\pi y)\exp(\sigma t + Ikx); \quad \varphi_\eta = \sum_{i=1}^{N} \mathrm{g}_i \sin(i\pi y)\exp(\sigma t + Ikx) \end{aligned} \tag{52}$$

in which k is the wave number in the infinite horizontal direction Ox and $I^2 = -1$.

The corresponding linear stability problem is solved using the Galerkin method.

Vertical Vibration ($\alpha = \pi/2$)

For different sets of parameters $0 < Rv < 100$, $2 < Le < 100$ and $\varepsilon^* = 0.5, 0.7$, the numerical simulations are carried out. The results of linear stability analysis for $Le = 2$, $\psi = -0.2$ and $\varepsilon^* = 0.5$ are presented in Table 1. It should be noted that, in the range of Lewis numbers studied, the results are qualitatively the same. As can be observed from Table 1, we may distinguish two types of bifurcations; namely stationary and Hopf bifurcations. For the stationary bifurcation, we assume that the principle of the exchange of stability is valid (i.e. $\sigma \in \Re$). From this the marginal state is determined ($\sigma = 0$). For the Hopf bifurcation ($\sigma = \sigma_r + I\omega_0$); the marginal state corresponds to $\sigma_r = 0$. It should be added that the Hopf bifurcation is present only for negative separation factors and, for the layer heated from below, it can be formed before the stationary bifurcation ($Ra_{co} < Ra_{cs}$). The effects of vibration on the Hopf bifurcation for $\psi = -0.2$, $\varepsilon^* = 0.5$ and $Le = 2$ are represented in Table 1. We conclude from Table 1 that vibration has a stabilizing effect; it increases the critical value of thermal Rayleigh number for the onset of convection. This is true for both the stationary and the Hopf bifurcation. It should be mentioned that vibration reduces the critical wave number (k_{cs}, k_{co}) and the Hopf frequency (ω_o).

Here after, we present the evolution of the critical Rayleigh number (Ra_{cs}) (Fig. 6a) and the critical wave-number (k_{cs}) (Fig. 6b) versus the separation factor Ψ for different values of the vibrational Rayleigh number Rv in the case $Le = 10$ and $\epsilon^* = 0.5$ and for stationary bifurcations.

One can observe that for $\Psi > 0$, when Rv increases, the value Ψ_1 of the separation ratio beyond which the critical wave number vanishes (i.e $k_{cs} = 0$) decreases. We show that for $\Psi > \Psi_1$ and for vertical vibrations:

$$Ra_{cs} = \frac{12}{Le\psi}; \forall R_v \tag{53}$$

Horizontal Vibration ($\alpha = 0$)

The stability domain for different vibrational parameters in the (Ra, Ψ) stability diagram has been determined.This diagram is characterized by stationary and

Table 1 Effect of vertical vibrations on stationary and Hopf bifurcations ($Le = 2$, $\Psi = -0.2$ and $\varepsilon^* = 0.5$)

Rv	Ra_{cs}	k_{cs}	Ra_{co}	k_{co}	ω_o
0	153.19	4.75	95.43	2.59	10.78
10	157.53	4.73	97.78	2.56	10.75
50	173.63	4.65	107.1	2.41	10.50
100	193.60	4.54	117.8	2.26	10.26

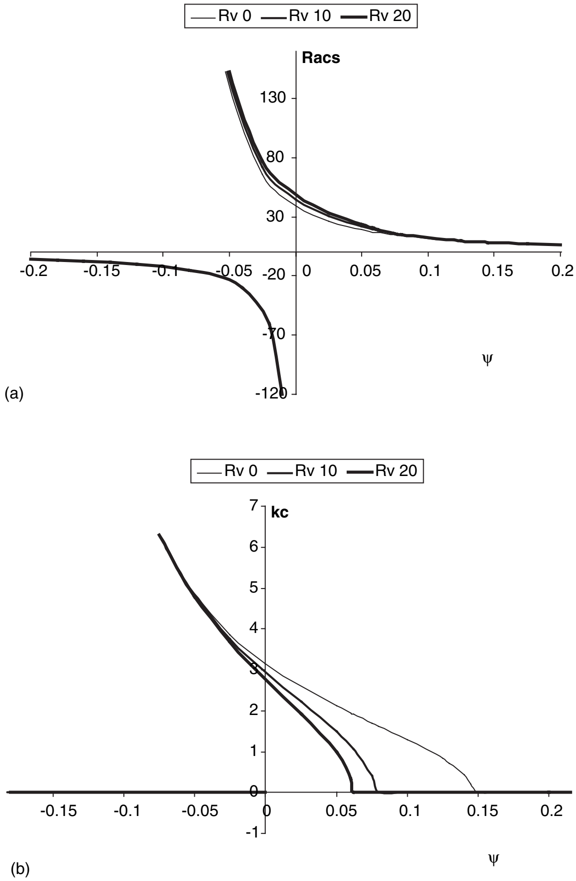

Fig. 6 (**a**) $Ra_{cs} = f(\Psi)$, $Le = 10$, $\varepsilon^* = 0.5$ (stationary bifurcations) and ($\alpha = \pi/2$), (**b**) $k_{cs} = f(\Psi)$, $Le = 10$, $\varepsilon^* = 0.5$ (stationary bifurcations) and ($\alpha = \pi/2$)

oscillatory bifurcations. For $\Psi > 0$ the bifurcation is always of the stationary type while for $\Psi < 0$, we may obtain oscillatory or stationary bifurcations. The computations are performed for $\varepsilon^* = 0.3$, $B = 10^{-6}$ (usual values used in porous media) and $Le = 2, 10, 100$. The results show that horizontal vibrations have a destabilizing effect on both stationary and Hopf bifurcations while vertical vibrations have stabilizing effect.

As for the case of vertical vibrations, we note the existence of long-wave-mode instability (i.e. $k_{cs} = 0$) for $\Psi > 0$. A regular perturbation method with the wave number as small parameter has been performed and the following relation:

$$Ra_{cs} + R_v(1+\psi)\frac{\psi}{\epsilon^*} = \frac{12}{Le\psi} \quad (\alpha = 0) \tag{54}$$

has been obtained. From (54), for fixed values of Rv, Le, Ψ and ϵ^*, we can calculate the value of the critical Rayleigh number $\mathrm{Ra_{cs}}$, corresponding to the onset of a monocellular flow, for $\alpha = 0$.

3.3 Numerical Simulations in a Confined Cavity ($A = 1$ and $A = 10$)

The numerical simulations for a confined cavity are performed for vertical and horizontal vibrations. The calculations are made for different aspect ratios $A = 1$ and $A = 10$. The 27*27 collocation points are used for $A = 1$ while 63*27 collocation points are used for $A = 10$.

3.3.1 Vertical Vibration ($\alpha = \pi/2$)

In this section we provide a qualitative representation of the flow and thermal fields to complete the results of our stability analysis. In order to study the effect of vibrations on the convective pattern, we set $Le = 2$, $\psi = 0.4$, $A = 1$ and $Ra = 30$ and we changed the value of the vibrational Rayleigh number R_v. To study the importance of the normalized porosity, computations have been performed, for $\varepsilon^* = 0.5$ and 0.7, with an increasing Rv, in order to determine the value of Rv, note Rvc, below which the conductive state appears (the global Nusselt number, defined as follows, $Nu = \frac{1}{A}\int_0^A \left.\frac{\partial T}{\partial y}\right)_{y=1} \mathrm{d}x$, is equal to one). The results are presented in Table 2 and Figs. 7a and 7b.

Thus these results illustrate clearly the stabilizing effect of vertical vibrations and the importance of the normalized porosity. When ϵ^* increases, for a fixed value of the thermal Rayleigh number, the value of Rv for which the conductive regime appears increases. In addition, for the combination of Rv, ψ and ε^*, there exists an interesting relation $Rv_c(1 + \psi/\varepsilon^*) = cst$. In our case this constant is equal to 31.5.

On Fig. 8a and b, we illustrate the modification of the convective structure due to vibrations for $A = 10$, $Le = 10$, $\epsilon^* = 0.5$, $\Psi = 0.1$ and $Ra = 12.5$.

For this set of parameters and $Rv = 0$ (Fig. 8a), we obtain five cells and the iso-concentrations are deformed. For $Rv = 10$ (Fig. 8b), a monocellular motion takes place and induces a separation of the binary fluid components.

Table 2 Evolution of the global Nusselt number, Nu, versus Rv for $\varepsilon^* = 0.5$ and 0.7. ($\alpha = \pi/2$)

$\varepsilon^* = 0.5$ Rv	$\varepsilon^* = 0.5$ Nu	$\varepsilon^* = 0.7$ Rv	$\varepsilon^* = 0.7$ Nu
5	1.2047	5	1.2057
10	1.1483	10	1.1528
15	1.0789	15	1.0942
16	1.0604	17	1.0669
17	1.0358	18	1.0514
17.5	1.0124	19	1.0329
17.53	1.0084	19.5	1.0208
		19.7	1.0156
		20.02	1.0036

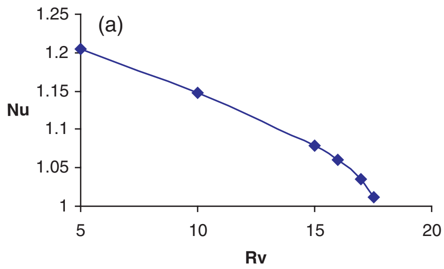

Fig. 7a Effect of mechanical vibration on the global Nusselt number for $A = 1$, $\alpha = \pi/2$, $Le = 2$, $\varepsilon^* = 0.5$, $\psi = 0.4$ and $Ra = 30$

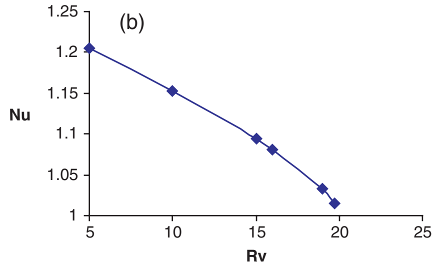

Fig. 7b Effect of mechanical vibration on the global Nusselt number for $A = 1$, $\alpha = \pi/2$, $Le = 2$, $\varepsilon^* = 0.7$, $\psi = 0.4$ and $Ra = 30$

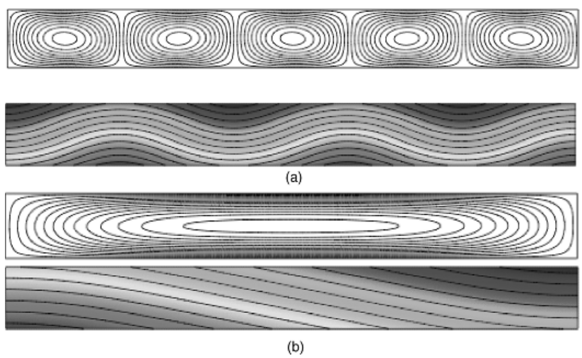

Fig. 8 Streamlines and iso-concentrations for: (**a**) $Le = 10$, $\varepsilon^* = 0.5$, $\Psi = 0.1$ and $Ra = 12.5$ for $Rv = 0(\alpha = \pi/2)$, (**b**): $Le = 10$, $\varepsilon^* = 0.5$, $\Psi = 0.1$ and $Ra = 12.5$ for $Rv = 10(\alpha = \pi/2)$

3.3.2 Horizontal Vibrations ($\alpha = 0$)

To illustrate the influence of horizontal vibrations, we consider the case $A = 1$, $Le = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $R = 0.3$. For a confined cavity, there is no equilibrium solution under horizontal vibrations. However, we study the different convective structures obtained when the thermal Rayleigh number increases for a fixed value of R. Note that for $Le = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $R = 0.3$ and for an infinite layer, the critical value of the Rayleigh number corresponding to the onset of convection is $\mathrm{Ra}_{cs} = 14.06$.

Firstly we set the value of Ra to the previous value so that only the vibrational mechanism is in action. For $Ra = 6$, Figs. 9 and 10 show the corresponding fluid

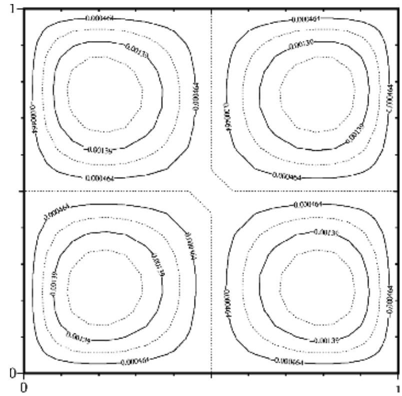

Fig. 9 Stream functions for $A = 1$, $\mathrm{Le} = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $\mathrm{Ra} = 6$, $R = 0.3$ and $\alpha = 0$

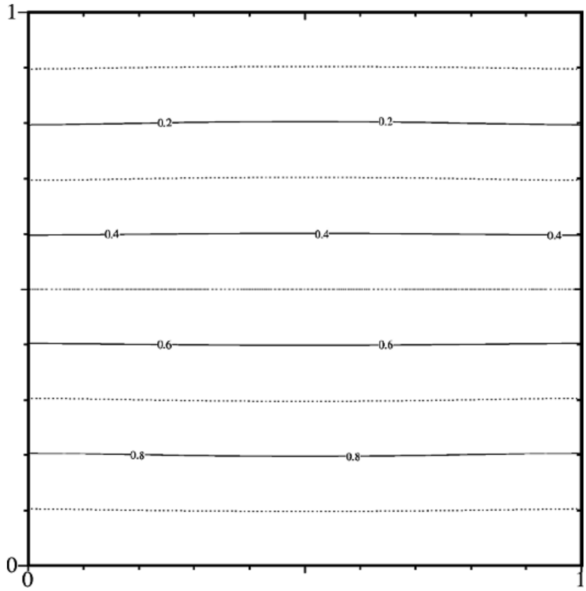

Fig. 10 Isotherms for $A = 1$, $\mathrm{Le} = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $\mathrm{Ra} = 6$, $R = 0.3$ and $\alpha = 0$

flow structure and temperature distribution; the stream functions are characterized by symmetrical four-vortex rolls. This structure is a typical example of an imperfect bifurcation. The sum of stream functions is zero in this case.

Secondly the thermal Rayleigh number is increased further to $Ra = 13.15$, the gravitational acceleration will be in action. The intensity of convective motion will be accordingly increased. The sum of stream functions at all points in the domain is a good criterion for the intensity of convective motion. This case is shown in Figs. 11 and 12 . As can be seen from the figures, we obtain a symmetry breaking structure. This is explained by coalescence of the two rolls with the same sign in the diagonal direction and the existence of two separate off-diagonal rolls with weaker intensity.

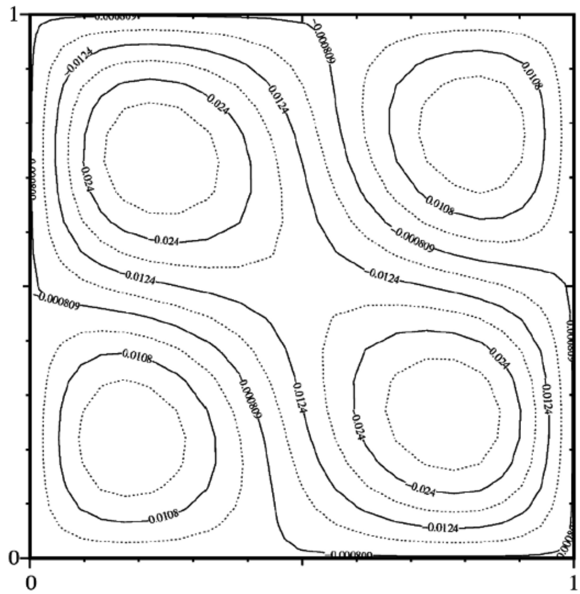

Fig. 11 Stream functions for $A = 1$, $Le = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $Ra = 13.15$, $R = 0.3$ and $\alpha = 0$

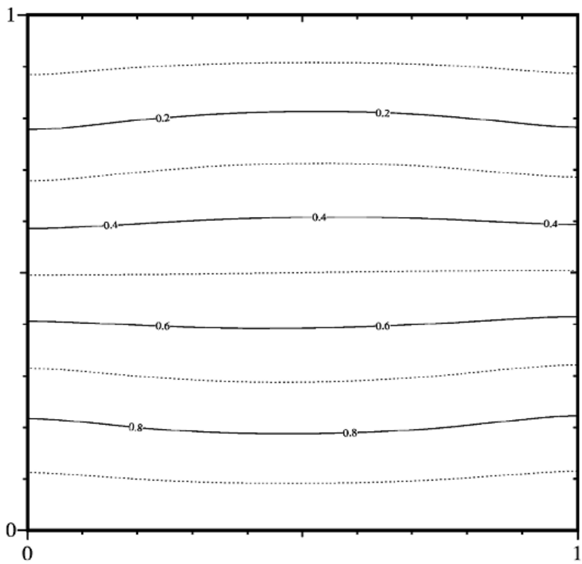

Fig. 12 Isotherms for $A = 1$, $Le = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $Ra = 13.15$, $R = 0.3$ and $\alpha = 0$

Then we increase further the thermal Rayleigh number to $Ra = 15$, we find a single convective roll, which means that the gravitational effect is more important than the vibrational effect, Figs. 13 and 14.

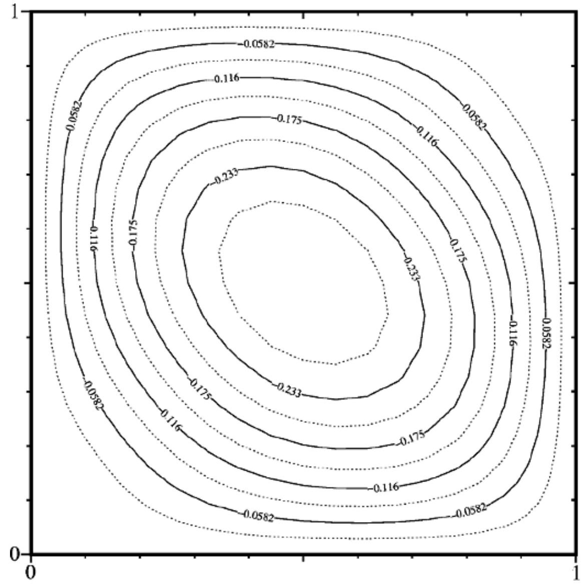

Fig. 13 Stream functions for $A = 1$, $Le = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $Ra = 15$, $R = 0.3$ and $\alpha = 0$

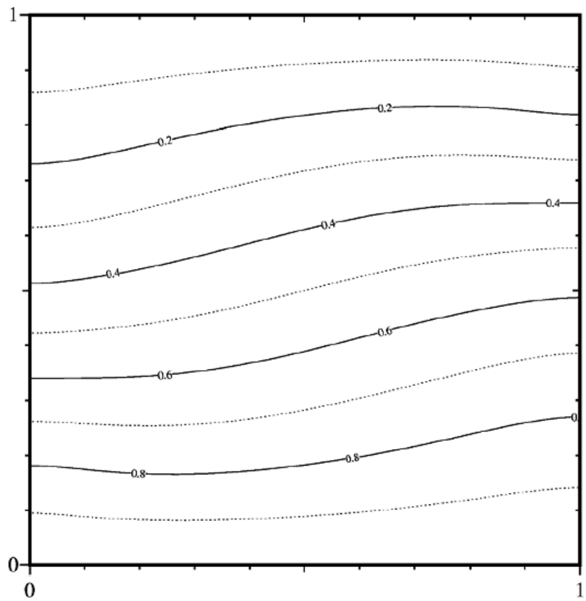

Fig. 14 Isotherms for $A = 1$, $Le = 2$, $\varepsilon^* = 0.5$, $\psi = 0.2$, $Ra = 15$, $R = 0.3$ and $\alpha = 0$

3.4 Conclusions

In this section, we studied two-dimensional thermo-solutal convection under mechanical vibration analytically and numerically. The vibration is in the limiting range of high frequency and small amplitude. Linear stability analyses of equilibrium and quasi-equilibrium states are performed for infinite horizontal layer. It is found that when the direction of vibration is parallel to the temperature gradient, vibration has a stabilizing effect for both the stationary and the Hopf bifurcation. The action of vibration reduces the number of convective rolls and the Hopf frequency. However, when the direction of vibration is perpendicular to the temperature gradient, vibration has a destabilizing effect. The numerical simulation shows that the vertical vibration can reduce the number of convective rolls. The effect of vibration on reducing the Nusselt number is presented. In this respect, the importance of the group $(Rv_c(1 + \psi/\varepsilon^*))$ is emphasized. For the cases in which mechanical equilibrium is impossible the fluid flow structures are sought. For a fixed value of vibrational Rayleigh number, we increase the Rayleigh number from a value much less than the critical value corresponding to the onset of convection in an infinite layer. We observe first a symmetrical four-vortex structure, then a diagonal dominant symmetry breaking structure and finally a mono-cellular structure. These results are similar to the results obtained in a cavity filled with pure fluid under the action of vibration in weightlessness (Bardan et al. 2000). The interesting result of this study is that, by appropriate use of residual acceleration in micro-gravity environment, we may obtain significant enhancement in heat and mass transfer rates.

NOMENCLATURE

Roman Letters

a	effective thermal diffusivity, $m^2.s^{-1}$
b	vibration amplitude, m
C_i	initial mass fraction
C'	dimensional mass fraction
C	perturbation of concentration
D^*	mass diffusion coefficient
D_T	thermodiffusion coefficient
Da	Darcy number
$\boldsymbol{e}$	direction of vibration
$\boldsymbol{g}$	gravitational acceleration, $m.s^{-2}$
H	height, m
$\boldsymbol{j}$	unit vector in y direction
k	wave number
K	permeability, m^2
Le	Lewis number (a/D^*)
P	pressure, $N.m^{-2}$
Pr	Prandtl number (ν/a)
R	acceleration ratio
R_v	vibrational parameter
Ra	Rayleigh number

Ra_v vibrational Rayleigh number
T temperature, K
T' dimensional temperature
t' dimensional time
$\boldsymbol{V}$ velocity, m.s^{-1}
$\boldsymbol{W}$ solenoidal vector

Greek Letters

β_C coefficient of mass expansion
β_T coefficient of thermal expansion
ε porosity
ε^* normalized porosity
θ perturbation of temperature
λ^* effective thermal conductivity
ν kinematic viscosity, m^2.s^{-1}
ρ density, kg.m^{-3}
$(\rho c)^*$ volumic heat capacity of medium
τ vibration period
ψ separation ratio
φ steam function perturbation
ϖ pulsation
ω dimensionless pulsation
α angle of vibration
σ^* dimensionless volumic heat capacity ratio
$\Delta T = T_1 - T_2$

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