GPS and Low Cost Sensors in Navigation

Nasrin Bourghani Farahani and Nader Pouryaie

Abstract In this paper, we present our work on integrated navigation for low speed vehicles at low altitudes, using Global Positioning Satellites (GPS).

The commercial GPS systems are used for determination of speed and location of low speed objects on Earth or airborne objects, with acceptable accuracy. On the other hand, low cost Inertial Navigation System (INS) is suited for similar applications. These systems are self-contained, non-interrupted and do not need external signals. However, GPS errors result in constant drifts in position. These errors in position grow with time, or are unlimited.

The INS and GPS systems have complementary characteristics, and hence, can be selected for integrated navigation.

To determine the flight position, the INS system operates uninterrupted in an independent mode. Whenever, the GPS system determines the position (usually at lower rate than the INS system), the INS system corrects the speed and position.

The commercial GPS systems can not produce the required altitude estimations independently. Therefore, pressure sensor has been used to determine the errors in vertical channel. Kalman filter has been used to integrate all the measurements.

1 Introduction

We have used off the shelf low cost sensors, and obtained the required integrity and reliability using an efficient algorithm. Integrity means these can judge the health of input data.

N.B. Farahani
Iranian Research Organization for Science & Technology (IROST), Faculty Member of Electrical & Computer Department of IROST
e-mail: farahani@irost.org

N. Pouryaie
Iranian Research Organization for Science & Technology (IROST), Electrical Expert of Electrical & Computer Department of IROST

R. Sandau et al. (eds.), *Small Satellites for Earth Observation*,

2 INS & Magnetometers

Inertial navigation system is composed of three accelerometers & three gyros each on its three measuring axes. Before mission INS needs initial alignment & calibration to find out deterministic errors of sensors & initial attitude of vehicle. INS has also some stochastic or random erros. Random errors of gyros can be compensated by making an AHRS. If vehicle is stationary (no thrust acceleration & centripetal acceleration) then by using the direction of local gravity vector measured by accelerometer, INS can measure two angles *roll & pitch*. Gravitational acceleration is always downwards & can be transformed in body frame (as sensed by accelerometers) according to following relationship.

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} -g \sin\theta \\ g \cos\theta \sin\varphi \\ g \cos\theta \sin\varphi \end{bmatrix} \tag{1}$$

where θ is the pitch angle & φ is the roll angle with these relations a rough estimate of *roll & pitch* can be made as under:

$$\begin{aligned} \varphi &= \tan^{-1}\left(\frac{a_y}{a_z}\right) \\ \theta &= \sin^{-1}\left(\frac{a_x}{g}\right) \end{aligned} \tag{2}$$

The third angle or heading angle Ψ comes from magnetometer, with three magnetometers, we calculate magnetic field horizontal & vertical component in x & y direction of inertial frame using frame transformation (body to inertial) as M_x, M_y & ultimately our heading angle is:

$$\psi = -\tan^{-1}\left(\frac{M_y}{M_x}\right) \tag{3}$$

The difference between geographic north & magnetic north is compensated each time during navigation Gyros also give three angles, combining this information with the above estimated angles we can estimate the day to day random biases of gyros, which can be compensated later during the mission. Random errors in accelerometers (which accumulate errors in position & velocity) are corrected using GPS data.

3 GPS & Errors in GPS Data

Four satellites are used to position the object on earth, three satellites determine the position & fourth one is used to synchronize the clock of receiver with atomic clock of satellite. The GPS gives the position in geographic frame Latitude, Longitude, Altitude & velocities in North, East, down (local geographic) frame. Every satellite emits a signal which has following information:

Information about status of satellite, Data Acquisition, precision of satellite, ionosphere delay, Ephemeris of satellite.

The precision of system depends upon six classes of errors:

Ephemeris data, Satellite clock, Ionosphere, Troposphere, Multipath
Receiver. noise, software accuracy, and inter-channel biases
Geometry of satellite with respect to user (Geometric Dilution of precision)
Error of Position $=$ GDOP.UERE

Table 1 Standard error model - L1 C/A (no SA), (One-sigma error, m)

Error source	Bias	Random	Total	DGPS
Ephemeris data	2.1	0.0	2.1	0.0
Satellite clock	2.0	0.7	2.1	0.0
Ionosphere	4.0	0.5	4.0	0.4
Troposphere	0.5	0.5	0.7	0.2
Multipath	1.0	1.0	1.4	1.4
Receiver measurement	0.5	0.2	0.5	0.5
User equivalent range				
Error (UERE), rms	5.1	1.4	5.3	1.6
Filtered UERE, rms	5.1	0.4	5.1	1.5
Vertical one-sigma errors – VDOP = 2.5			**12.8**	**3.9**
Horizontal one-sigma errors – HDOP = 2.0			**10.2**	**3.1**

Taking account of all these errors PPS service (with out selective availability) contribute to following errors in position: 22 meter of incertitude in horizontal position, 23 meter of incertitude in vertical position.

4 Navigation Algorithm & Quaternion

In navigation different reference frames are used, for applications close to the surface of earth, earth centered inertial frame is used. The velocity of vehicle in inertial frame can be given as

$$v_i = v_e + \omega_e^i \times r \tag{4}$$

Differentiating above équation gives:

$$a_{i/i} = \underbrace{a_{e/i}}_{\textit{measured acceleration}} + \underbrace{\omega_e^i \times v_e}_{\textit{coriolis acceleration}} + \underbrace{\omega_e^i \times \omega_e^i \times r}_{\textit{centripetal acceleration}} \tag{5}$$

v_e $=$ $velocity$ of vehicle w.r.t earth frame
ω_e^i $=$ $rotation$ rate of earth w.r.t inertial frame
r $=$ position vector of vehicle
a $=$ acceleration in respective frame

As the accelerations are acquired in body frame, a transformation is required between body to inertial frame. Usually this is done using quaternion approach for less computation & no singularity.

The transformation or direction cosine matrix in form of quaternion can be represented as:

$$T_b^i = \begin{bmatrix} q_3^2 + q_0^2 - q_1^2 - q_2^2 & 2(q_0q_1 - q_2q_3) & 2(q_0q_2 + q_1q_3) \\ 2(q_0q_1 + q_2q_3) & q_3^2 + q_1^2 - q_0^2 - q_2^2 & 2(q_1q_2 - q_0q_3) \\ 2(q_0q_2 - q_1q_3) & 2(q_1q_2 + q_0q_3) & q_3^2 + q_2^2 - q_0^2 - q_1^2 \end{bmatrix} \tag{6}$$

The rate of change of quaternion can be represented as follows with out using trigonometric identities & avoiding singularities. This relation is used at a very fast rate for updating attitude matrix at the acquisition rate of INS

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \cdot \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{7}$$

$$\dot{q} = \frac{1}{2}\Omega \cdot q$$

Ultimately we always need Euler angles as output because they are more realizable &, it is also possible to represent Euler angles in the form of quaternion & vice versa:

$$eul(q) = \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\dfrac{2(q_2q_3 + q_0q_1)}{1 - 2(q_1^2 + q_2^2)}\right) \\ \sin^{-1}\left(-2\left(q_1q_3 - q_0q_2\right)\right) \\ \tan^{-1}\left(\dfrac{2(q_1q_2 + q_0q_3)}{1 - 2(q_2^2 + q_3^2)}\right) \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right) + \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right) \\ \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right) - \cos\left(\frac{\varphi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right) \\ \cos\left(\frac{\varphi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right) + \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right) \\ \cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right) - \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right) \end{bmatrix} \tag{9}$$

Accelerations are then g-compensated (accelerometer measure only specific force)

$$a_I = \mathbf{T}_b^I a_b + \mathbf{T}_l^I g_l \tag{10}$$

Where gravitational force is assumed to be in the direction of Z_1 (spherical earth), and is given in local geographic frame as

$$g_l = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{11}$$

$g \;=\; \Gamma / r^2$
$\Gamma \;=\;$ Gravitational Constant
$\tau \;=\;$ position vector

These accelerations are integrated twice using trapezoidal rule to achieve velocity & position. Vehicle position is transformed first from inertial coordinate system to geocentric one:

$$\begin{gathered} \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \mathbf{T}_I^G \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\ \mathrm{T}_I^G = \begin{bmatrix} \cos(\Omega t) & \sin(\Omega t) & 0 \\ -\sin(\Omega t) & \cos(\Omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{gathered} \tag{12}$$

And finally to geographic coordinate system

$$\begin{aligned} h &= R - R_E \\ \alpha_G &= \tan^{-1}(Y_G / X_G) \\ d_G &= \sin^{-1}\left(\sqrt{X_G^2 + Y_G^2}/R\right) \\ R &= \sqrt{X_G^2 + Y_G^2 + Z_G^2} \end{aligned} \tag{13}$$

Here b *i, l* & *G* denote body, Inertial, local geographic & geographic frame respectively & T shows transformation (from subscript to superscript frame). h is mean sea level height, α_G is longitude, δ_G is latitude, R is position vector & Ω is earth rotation rate. GPS positions are transformed from geographic to inertial frame & velocities are transformed from local geographic to inertial frame before integration with INS calculated results.

5 Error Model of Inertial Sensors

Accelerometers & gyros all posses some deterministic as well as stochastic errors. The error model of accelerometers & gyros is given as (IEEE standard)

$$d_{\omega x} = b + s_f \omega_x + m_y \omega_y + m_z \omega_z + \eta$$
$$d_{fx} = b + s_f a_x + m_y a_y + m_z a_z + \eta \quad (14)$$

b = bias component
s_f = scale factor
η = random noise
m_y, m_z = misalignment coefficient ω_{xyz} = Gyro rates
a_{xyz} = Acceleration

The deterministic errors in model can be compensated in Navigation algorithim. The stochastic components of errors in sensors give unbounded errors, when integrated & are called random walk. These random errors need to be estimated & compensated in real time.

6 Integration Techniques

GPS & INS in our particular case are integrated in loosely coupled mode. In this mode Integration Kalman filter treats both GPS & INS as separate navigation systems. It takes their outputs as measurements in navigation solution & only INS errors are estimated explicitly. It means dynamic errors of gyros are ignored. There are other integration techniques like, tightly coupled & ultra tightly coupled integrations but for our application loosely coupled mode was found to be suitable

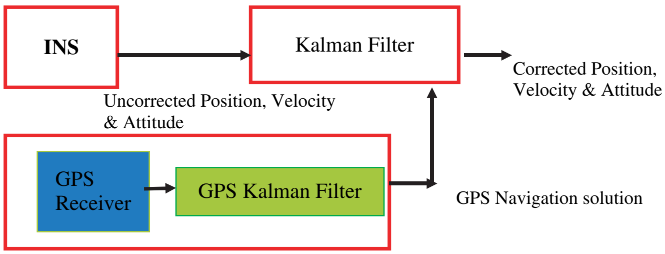


Fig. 1 Loosely coupled INS/GPS intégration

6.1 *Kalman Filtering*

In navigation Kalman filtering is used to optimally combine information from different sensors. Kalman filter uses Kinematics equations of vehicle which are fed by INS measurements. To converge the output of this filter absolute sensor (GPS) correction is used. Since the equations of system are highly non-linear we use Kalman filter in extended mode. Extended Kalman filter line arises the equation at small instant of time taking partial derivatives. General formulization of extended Kalman filter can be given as:

$$\begin{cases} \hat{x}_{k+1} = f(\hat{x}_k, u_k, 0) \\ \hat{z}_k = h(\hat{x}_k, 0) \end{cases} \tag{15}$$

($\hat{x}_k$ *is* an estimation states calculated using k iterations)

The linearization is realized by introducing the Jacobean matrix

$$A = \left[\frac{\partial f}{\partial x}\right] H = \left[\frac{\partial h}{\partial x}\right] \tag{16}$$

There are two stages of Kalman filter algorithm, Prediction & Correction

6.1.1 Prediction

This step is evaluated at the frequency of quantization of filter (1/Ts). In this step matrix of covariance & states are predicted as follows

$$\begin{gathered} \hat{x}_{k+1}^{-} = f_k(\hat{x}_k, u_k, 0) \\ P_{k+1}^{-} = P_k + T_s\left(A_k P_k + P_k A_k^t + Q\right) \end{gathered} \tag{17}$$

6.1.2 Correction

This step is evaluated when new measurements are available. Measurements are used to correct the state vector & the matrix of covariance as follows:

$$\begin{gathered} K_k = P_k^{-} H_k^t \left[H_k P_k^{-} H_k^t + R\right]^{-1} \\ \hat{x}_k = \hat{x}_k^{-} + K_k\left(z_k - H_k \hat{x}_k^{-}\right) \\ P_k = (I - K_k H_k) P_k^{-} \end{gathered} \tag{18}$$

Where:

P_k = covariance matrix
K_k = Kalman gain
H_k = Observation matrix
R = covariance of measurement noise
Q = covariance of system noise
$\hat{x}_k$ = Estimated state vector
A = state transition matrix

For our particular case the state vector & output vector are

$$\begin{gathered} x^{(14)} = \left(xyz^{(3)}, uvw^{(3)}, q^{(4)}, g^{(1)}, \delta_{pqr}^{(3)}\right)^t \\ z^{(9)} = \left(xyz^{(3)}, uvw^{(3)}, \varphi, \theta, \psi\right)^t \end{gathered} \tag{19}$$

Where:

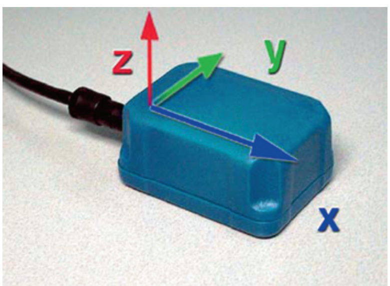
xyz : Positions local NED frame
uvw : Velocity in body frame
q : Quaternion
g : Acceleration of gravity
δ_{pqr} : Bias of gyros
$\phi\theta\psi$: Euler angles

We estimate gravity to correct the bias of vertical accelerations. For proper tuning of filters several test were made in order to reach a suitable value of R, Q & P matrices

7 Experimental Setup

The components of hybrid INS used for this experiment are as follows

- MEM type INS
  - Three Accelerometer (a triad orthogonal, measure acceleration vector in body frame)
  - Three gyros (a triad orthogonal, measure attitude of system in Body frame)
  - Three Magnetometer (measure magnetic heading of system)
  - Thermometer
- GPS receiver

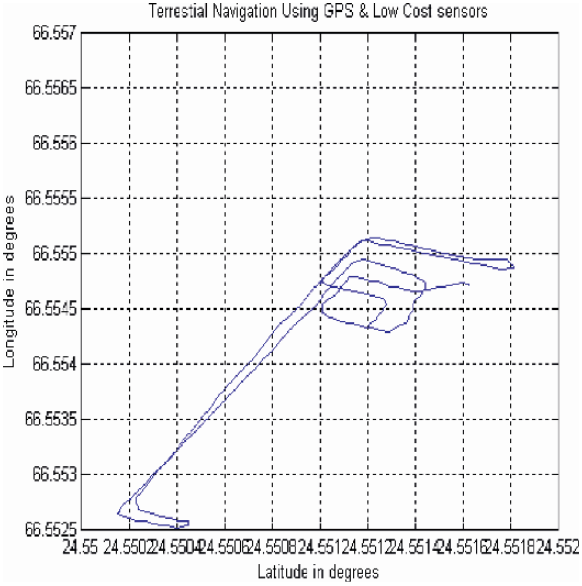


MEM Type Ins

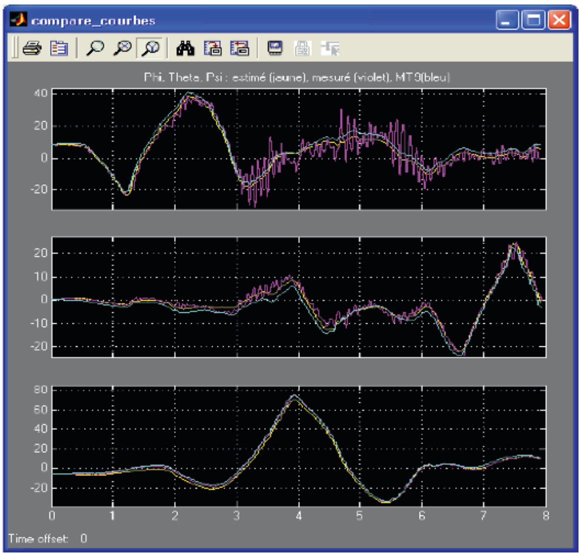
Hand held GPS receiver

8 Results & Verification

A terrestrial navigation system is built & tested using different trajectories & velocities. An accuracy of 10–20 m in position & 2–5° in attitude is achieved.



Estimation of Position



Estimation of Attitude

References

1. R.M. Rogers, Applied Mathematics in Integrated Navigation System, AIAA Education series, 2nd ed., 2003
2. J. Tithertton & C. Weston, Strap down inertial navigation, AIAA Education series, 2nd Rev. Ed., 2004
3. M.S. Grewal et al., GPS Inertial Navigation & Integration, John Wiley & Sons, 2001
4. S.C. Bose, Multisensor Navigation & Fault Isolation, Technalytics, 2004
5. http://edu-observatory.org/gps/gps_accuracy.html