Chapter 3
Multi-Criteria Decision Problems

3.1 Theoretical Aspects

The basis for decision making is that given two objects, say A and B, people can meaningfully say whether they prefer A to B, B to A or whether they are indifferent (von Winterfeldt and Edwards 1986). Usually, it is assumed that people can also state the strength of this preference. The strength could be expressed either in ordinal terms, or in cardinal terms. If the decision maker can say that change from A to B is preferable to change from B to C, then the judgment is ordinal. If the decision maker can say by how much, the judgment is cardinal.

Multi-Attribute Utility Theory, where the utility of the decision maker is considered to consist of several attributes, is usually shortened with MAUT. In different contexts, concepts 'objectives' and 'criteria' are used instead of 'attributes'. Malczewski (1999, p. 85) defines multi-criteria decision making as a broader class, which includes both multi-attribute decision making for discrete decision problems and multi-objective decision making for continuous problems. He defines attributes as measures of performance of an object, and objects as statements about the desired state of a system. In this book, the attributes and criteria are used as synonyms, and multi-objective is only used in the context of optimization.

In many cases also the term MAVT, Multi-Attribute Value Theory, is used. The difference between MAUT and MAVT is that the value functions do not include the risk preferences of the decision maker but the utility function does. von Winterfeldt and Edwards (1986, p. 215) think this difference is spurious, however. A utility function is also a value function, while a value function is not necessarily a utility function.

The problem is formulated with a set of distinct alternatives $d_i,\ i = 1,\ldots,n$ and a set of decision criteria $c_j,\ j = 1,\ldots,m$ so that c_{ij} represents the performance of alternative i with respect to criterion j. It is simply not possible to independently maximize or minimize several criteria at the same time: you cannot maximize the gross incomes while at the same time minimizing the costs, or maximize the yield and minimize the risks (Keeney and Raiffa 1976, p. 66).

Therefore, the big issue in multi-criteria decision making is that of tradeoffs: how much is the decision maker willing to give up in one criterion, in order to improve the performance with respect to another criterion by some fixed amount. For instance, the decision maker needs to decide, how much more risk of beetle outbreak she/he will tolerate in order to improve net incomes by 100€, or how much incomes is she/he willing to give up in order to reduce the risk by 10%.

The tradeoffs decisions are about personal values, and thus, they require subjective judgment of the decision maker. This means that there are no correct or wrong answers to the value questions; people may have very different preference structures. The tradeoffs problem can be solved in two ways: (1) the decision maker can informally weigh the tradeoffs is his/her mind or (2) the decision maker can formalize his/her preferences to a multi-criteria utility function and use it to solve the problem (Keeney and Raiffa 1976). Either way, the tradeoffs are inevitable in any multiple-criteria decision.

There are a few choice procedures that do not require utility functions in order to make choices. One of these is dominance. We can say that alternative i dominates alternative i', if

$$\begin{aligned} &c_{ij} \geq c_{i'j}, \ \forall j = 1, \ldots, m \\ &c_{ij} > c_{i'j}, \ \text{for some } j = 1, \ldots, m \end{aligned}$$

It means that alternative i is at least as good as alternative i' for all criteria j, and, in addition, alternative i is strictly better than alternative i' with respect to at least one criterion j. If an alternative is dominated by some other alternative, it can be left out from the analysis: it can never be chosen as the best alternative, whatever the preference structure. If there is one alternative among the n alternatives that dominates all other alternatives, the choice would then be easy. However, such is the case only rarely. It may, however, be that some of the alternatives can be eliminated before further analysis.

What is important in the non-dominated alternatives, is that they form the efficient frontier or the Pareto optimal set among the alternatives (Fig. 3.1).

One possible way of choosing among a set of alternatives, without considering the tradeoffs, is to utilize a lexicographic ordering (Keeney and Raiffa 1976). In this method, the decision maker first ranks the criteria with respect to their importance. Then, the most important criterion is c_1 and the least important criterion is c_m. Then, alternative i is preferred to alternative i', denoted by $i \succ i'$, if and only if

(a) $c_{i1} > c_{i'1}$ or

(b) $c_{ij} = c_{i'j}, \ j = 1, \ldots, k$

$c_{i(k+1)} > c_{i'(k+1)}$, for some $k = 1, \ldots, m-1$

It means that the choice is made only based on the most important criterion. If there are several alternatives that are equal with respect to that criterion, then the second most important criterion is considered and so on.

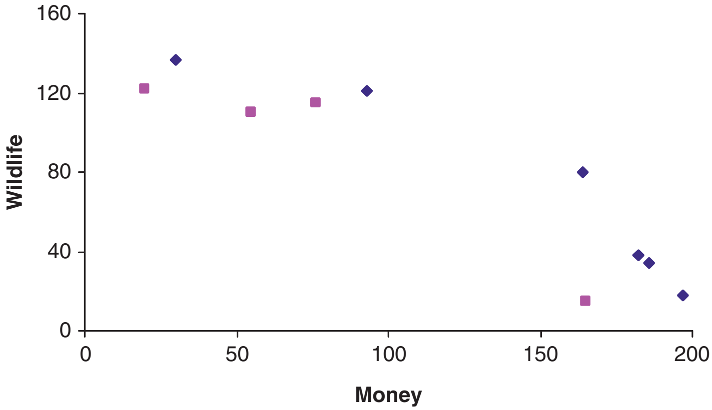

Fig. 3.1 Dominated alternatives are presented with squares, and non-dominated with diamonds. The non-dominated alternatives form the efficient frontier of the problem

3.2 Multi-Attribute Utility Functions

3.2.1 Function Forms

In a case of multi-attribute utility function, it is assumed that there are m criteria, and a unidimensional utility (or value) function is evaluated or can be evaluated for each of these criteria. The task is now to aggregate these utility functions to describe the overall utility of the alternatives. This aggregation is done by weighting the different criteria in the utility function with respect to their importances. The relations between the weights of different criteria describe the tradeoffs between the criteria.

The most applied multi-attribute utility function is the linear additive utility function

$$U_i = \sum_{j=1}^{m} a_j c_{ji} \tag{3.1}$$

where U_i describes the overall utility of alternative i (or priority of alternative i) and c_{ij} is the performance of the alternative i with respect to criterion j and a_j is the importance weight of criterion j. In this equation, it is assumed that the criteria values c_{ij} are already in utility scale or are scaled to a utility scale with Formula (2.4) or (2.5), for instance.

Typically, it is required that

$$\sum_{j=1}^{m} a_j = 1, \tag{3.2}$$

otherwise the utility could always be increased by increasing the weights. The trade-offs between criterion k and k' can be calculated from the ratio of the weights $a_k/a_{k'}$.

In general, the marginal rate of substitution between criteria k and k' can be calculated as a ratio of partial derivatives of the utility function as

$$\lambda = \frac{U'_k}{U'_{k'}} = \frac{a_k}{a_{k'}} \tag{3.3}$$

This means that the decision maker is willing to give up λ units of criterion k' in order to increase the value of criterion k by one.

Example 3.1. Assume a utility function with $U_k = 0.67c_{1k} + 0.33c_{2k}$, where c_1 denotes units of money and c_2 the number of animals in some wildlife species. Then, the decision maker is willing to give up one animal in order to increase the incomes with $0.67/0.33 = 2$ units of money.

In the linear additive utility function, the tradeoff between the criteria is constant. It means, that the willingness of the decision maker to change one unit of criterion k' to λ units of criterion k does not change, even if there is only a few units left of criterion k', or even if there is already plenty of criterion k. If a decreasing marginal utility is assumed, this does not hold. In this case, a more general function, namely the additive utility function needs to be used. It is of the form

$$U_i = \sum_{j=1}^{m} a_j u_j(c_{ij}) \tag{3.4}$$

where $u_j(c_{ij})$ is the partial utility due to criterion j. It is described with the unidimensional utility function for this criterion. In this function, the marginal rate of substitution is a function of the current values of criteria j.

The additive utility models are compensatory at nature. In this respect, this utility function differs from other forms of utility functions. It means that even if some criteria assume their lowest possible levels, this can be compensated if some other criterion or criteria assume very good values. The compensation is dictated by the marginal rate of substitution.

Example 3.2. Let us consider six non-dominant alternatives from Fig. 3.1. The overall utility function is assumed to be

$$U_i = a_1 \cdot 0.00506 \cdot c_{i1} + a_2 \cdot 1.55251 \cdot \exp(-60.2556/c_{i2})$$

and the resulting utilities are presented in Table 3.1. The weights of these two criteria are assumed to be equal, i.e. 0.5, and the greatest overall utility is observed for alternative 2, with 164 units of money and 80 animals. If the number of wildlife were 1, this could be fully compensated with 309 units of money, i.e. such an alternative would get the largest overall utility, 0.782. On the other hand, if the amount of money were 10 units, this could be compensated with 2,100 animals, giving overall utility 0.780. The marginal utility of additional animals is very small, when the number of animals is high.

The partial derivatives of these functions are $a_1 \cdot 0.00506$ with respect to criterion c_1, money, and a_2 $1.55251 \cdot \exp(-60.2556/c_{i2})(60.2556/c_{i2}^2)$ with respect to criterion c_2, number of wildlife animals. Thus, the marginal rate of substitution does not

Table 3.1 Overall utilities of the alternatives

Alternative	Money	Wildlife	Overall utility
1	197	18	0.526
2	164	80	**0.780**
5	30	137	0.576
8	186	34	0.603
9	182	38	0.619
10	93	121	0.707

depend on money, but it depends on the amount of animals. When the number of wildlife animals varies from 30 to 530, the marginal rate of substitution varies from 0.36 to 17.02 animals: the more animals there are, the more the decision maker is willing to give up for additional unit of money. The partial derivatives and the resulting λ are presented in the Table 3.2.

In addition to these two utility functions, there exist a number of different functions. One example is the multiplicative model (von Winterfeldt and Edwards 1986, p. 291)

$$1 + aU_i = \prod_{j=1}^{m} [1 + aa_j u_j(c_{ij})] \tag{3.5}$$

which can also be presented in a form of an additive utility function with interaction terms

$$U_i = \sum_{j=1}^{m} a_j u_j(c_{ij}) + a \sum_{j=1}^{m} \sum_{k>j}^{m} a_i a_j u_i(c_{ij}) u_k(c_{ik}) + \cdots + a^{m-1} \prod_{j=1}^{m} a_j u_j(c_{ij}). \tag{3.6}$$

In this function, the interaction between the partial utility of different criteria is presented using products of each pair of alternatives, products of each group of three criteria, and finally the product of all criteria. The interactions are dealt with

Table 3.2 Ratio of partial derivatives

Wildlife	Money	Wild	λ
30	0.00253	0.00697	0.3628
80	0.00253	0.00344	0.7352
130	0.00253	0.00174	1.4531
180	0.00253	0.00103	2.4493
230	0.00253	0.00068	3.7184
280	0.00253	0.00048	5.2589
330	0.00253	0.00036	7.0704
380	0.00253	0.00028	9.1527
430	0.00253	0.00022	11.5057
480	0.00253	0.00018	14.1293
530	0.00253	0.00015	17.0234

using one parameter, a, with a power of $p-1$, where p is the number of terms in the product. It means, that for two-term interactions $p=1$, and for m-term interactions $p=m-1$. This parameter a must lie between -1 and ∞. As the value of a increases, the amount of interaction increases, for $a=0$ this formula simplifies to additive utility function. In a two-dimensional case a can be calculated from (von Winterfeldt and Edwards 1986)

$$a=\frac{1-a_1-a_2}{a_1a_2} \tag{3.7}$$

It means that if $a_1+a_2=1,\ a$ is 0, and if $a_1=a_2=0,\ a$ is infinite.

Using this function, the degree of compensation cannot be calculated as neatly as with the previous additive models. Furthermore, also the interpretation of the model is more complicated, as the level of any one criterion affects to the utility obtained from a fixed level of another criterion.

Example 3.3. Assume the criteria in the example 3.2 have an interaction, i.e. the level of money affects to the utility obtained from wildlife. Both criteria have equal weight, 0.4, and thus the interaction term $a=\frac{1-0.4-0.4}{0.4\cdot 0.4}=1.25$. The partial utilities are calculated with the same functions as in example 3.2 and the resulting utilities are presented in Table 3.3. In this case, as the alternative 2 has the most even distribution of the criteria, it is the best alternative also with this utility model.

Since all the interactions are dealt with the same parameter, this function is fairly inflexible. In principle, it would be possible to estimate separate interaction term for each interaction pair of criteria, but this would require a lot of data from decision makers: each parameter requires at least one observation in order to be estimated, and the number of observations increases very fast as m increases.

Another common utility function form is the conjunctive function

$$U_i=\prod_{j=1}^{m}(u_j(c_{ij}))^{a_j} \tag{3.8}$$

This model is non-compensatory at nature. If the utility due to one of the criteria assumes zero value, the overall utility is also zero. It favours alternatives having fairly similar partial utility values for all criteria (e.g. Tell 1976).

Table 3.3 Overall utilities with interaction term

Alternative	Money	Wildlife	Overall utility
1	197	18	0.431
2	164	80	**0.746**
5	30	137	0.491
8	186	34	0.532
9	182	38	0.554
10	93	121	0.654

One example of utility functions is also the distance function (Tell 1976; Tell and Wallenius 1979)

$$U_i = \sqrt{\sum_{j=1}^{m} a_j^2 \left(c_j^{opt} - c_{ij}\right)^2} \quad (3.9)$$

where c_j^{opt} defines the optimal state of each criterion. This model needs to be scaled so that the optimal value of criterion is given value 1 and the least preferred value of that criterion is given zero value.

The weights a_j have throughout the chapter been interpreted as importances of the different criteria. However, this approach has also been criticized. For instance, Keeney and Raiffa (1976) consider the weights to be simple rescaling method which is necessary to match the units of one unidimensional utility function with another. Since it is possible to obtain very different value functions from the same data for any one criterion (e.g. Formulas 2.4 and 2.5) this needs to be kept in mind. For instance, when Formula 2.4 is used, the weights can be interpreted to describe the importance of change from 0 level in natural scale to the maximum level, and when Formula 2.5. is used, the same weights should describe the importance of change from the minimum value at natural scale, i.e. 0 at utility scale, to the maximum value at both natural and utility scale.

Example 3.4. In the data of the example 3.2, also a conjunctive utility function can be used. The partial utilities are calculated with the same functions as in example 3.2. The criteria are assumed to be equally important also in this example, and the resulting utilities are presented in Table 3.4. In this case, as the alternative 2 has the most even distribution of the criteria, it is the best alternative also with this utility model.

3.2.2 Basis for Estimating the Weights

There exist a large amount of methods that can be used for estimating the weights in the utility function. Generally, they can be divided to two main categories: direct and indirect methods. In indirect methods, the estimation may be based on the earlier, true decisions. These revealed preferences are commonly utilised for evaluating

Table 3.4 Overall utility assuming a conjunctive utility function

Alternative	Money	Wildlife	Overall utility
1	197	18	0.233
2	164	80	**0.779**
5	30	137	0.390
8	186	34	0.498
9	182	38	0.541
10	93	121	0.666

non-market values. In direct methods, the estimation is based on direct questions concerning the importances of criteria in the decision situation at hand.

It has been claimed that the old decisions are the only reliable way to estimate the true utility function. Such utility function is true only with respect to the old decision, however. The preferences and situation of the decision maker may have totally changed after that old decision was made, and thus, the model may not be at all useful in a new decision situation. The estimated utility model may also be incorrect for the old decision, if it was made based on imperfect information on the alternatives. Therefore, studying the old decisions and the preferences implied by these decisions are mainly useful in descriptive studies. When aiming at decision support, direct methods can be assumed more useful.

Direct estimation methods can be further divided to two groups, statistical and subjective methods (Schoemaker and Waid 1982). In statistical methods, the decision makers are asked to holistically evaluate some decision alternatives, and the utility function is estimated based on these values. In subjective methods, the decision problem is divided to several criteria, and preferences are asked regarding these criteria.

Different estimation method for utility functions, have been tested, for instance by Eckenrode (1965), Fishburn (1967), Tell (1976), Tell and Wallenius (1979), Eliashberg (1980), Jacquet-Lagreze and Siskos (1981), Schoemaker and Waid (1982), Horsky and Rao (1984), and Laskey and Fischer (1987). In this sub-chapter, only one group of subjective direct methods of estimation are presented, namely SMART. Later in the chapter, some additional techniques like the pairwise comparisons of Analytic Hierarchy Process are presented. These can also be used for estimating a utility function. However, these methods do not necessarily belong under the MAUT, and therefore they are dealt with separately.

3.2.3 Smart

SMART (Simple Multi-Attribute Rating Technique) is a decision support method developed at the close of the 1960s and early 1970s in the field of multi-attribute utility theory (von Winterfeldt and Edwards 1986). In fact, several methods based on direct evaluation are involved in the family of SMART methods, of which various researchers have developed new versions over the years. Generally, in SMART, additive models are applied.

Direct rating in SMART means, for example, that criteria are directly assigned numerical values depicting their importance. The least important criterion is first given 10 points and the points of other criteria are related to that. Then, the points are summed, and the final weights are the points of each criterion divided by the sum.

$$a_j = \frac{p_j}{\sum\limits_{i=1}^{m} p_i} \tag{3.10}$$

The same principles can, of course, also be used for estimating the value function with respect to each criterion (Chapter 2).

When the importance of the individual criteria and the priorities of each of the alternatives with respect to each of the criteria have been determined, the overall utility of alternatives can be calculated. SMART methods have been applied in natural resources management by Reynolds (2001) and Kajanus et al. (2004), for instance.

Another version of SMART, namely SMARTS (SMART using Swings) also exists. In SWING weighting, it is first assumed that all criteria are their lowest possible values, e.g. at utility function value 0. Then it is asked, which criteria is most important to change from its minimum level to its maximum level. This criterion is given 100 points. After that, the rest of the criteria are evaluated relative to that criterion. This approach has the advantage that the importance weights explicitly depend on the range of criteria values on the problem at hand, while the former SMART weighting does not necessarily do so.

Example 3.5. The case example presented next is applied throughout the book to illustrate different MCDM tools. The data comes from a real-life case of strategic planning in a 320.9 ha area in northern Finland, owned by state and managed by the Finnish State Forest Enterprise Metsähallitus. It has been used in many planning studies. The data consists of three criteria and six alternatives (see Kangas et al. 1992). The problem was to assess the priority of the forest plans generated for the area. The plans were

Continue natural growth with no cuttings (NAT)
Optimize scenic beauty index (SCEN)
Normal forestry guidelines (NORM)
Optimize game values (GAME)
Modest regeneration (MREG)
Maximize income (INC)

The decision criteria in this problem were timber production, scenic beauty, and game management. Each of the criteria was further decomposed into several sub-criteria. In order to keep the example as simple as possible, only timber production and scenic beauty of the main criteria were used here. The timber production was divided to two sub-criteria, namely net incomes during first 10 years and stumpage value after 20 years. The decision hierarchy is presented in Fig. 3.2.

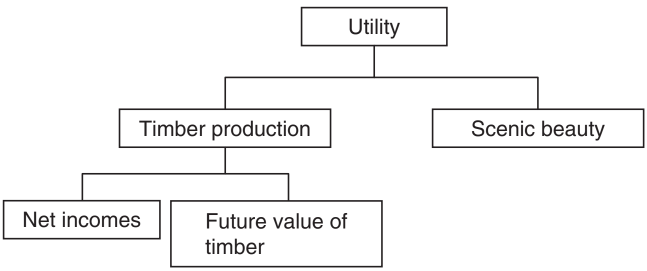

Fig. 3.2 The decision hierarchy in SMART example

Table 3.5 Net incomes, stumpage values and scenic beauty index of the six alternative plans

	Net incomes 1000€	Stumpage value million€	Scenic beauty index
NAT	0.00	0.71	5.5
SCEN	79.6	0.28	5.7
NORM	38.0	0.60	5.4
GAME	33.0	0.61	5.5
MREG	102.3	0.51	5.2
INC	191.7	0.13	4.9

Table 3.6 Cutting schemes of the example alternatives

	Regeneration 1st period (ha)	Per cent Clear-cut	Regeneration 2nd period (ha)	Per cent Clear-cut
NAT	0	0	0	0
SCEN	55.1	0	104.3	0
NORM	14.9	81	15.6	81
GAME	17.0	0	17.2	0
MREG	50.3	96	44.5	11
INC	109.6	68	87.2	29

Table 3.7 Sub-utility function values

	Net incomes	Stumpage value	Scenic beauty
NAT	0.000	1.000	0.750
SCEN	0.631	0.546	1.000
NORM	0.360	0.954	0.625
GAME	0.319	0.959	0.750
MREG	0.740	0.893	0.375
INC	1.000	0.000	0.000

Table 3.8 Global and local utilities

	Global utility	Timber production	Scenic beauty
NAT	0.350	0.200	0.150
SCEN	0.688	0.488	**0.200**
NORM	0.532	0.407	0.125
GAME	0.533	0.383	0.150
MREG	**0.697**	**0.622**	0.075
INC	0.600	0.600	0.000

Table 3.9 Weights based on ranking

	Rank reciprocal	Rank sum	Rank Exponent with $z = 1.6$	ROC
Timber production	0.67	0.67	0.75	0.75
Scenic beauty	0.33	0.33	0.25	0.25

Net incomes describe the timber harvesting income during the first half of the 20-year planning period, and stumpage value describes the monetary value of the remaining trees at the end of the 20-year period. Scenic beauty is an index describing scenic beauty after 20 years, calculated with MONSU software (see Pukkala 2006). The data for the example is presented in Table 3.5. From the alternatives, NAT means that there are no cuttings at all. In SCEN and GAMES alternatives, regeneration is carried out, but no clear-cuts are allowed. In INC alternative, the area of clear-cutting is largest: in MREG the regeneration area is only half of that, although most of the regeneration is carried out with clear-cutting (Table 3.6.).

In SMART analysis, an exponential utility function (of the form $a \cdot \mathrm{e}^{-b/x}$) was assumed for net incomes and stumpage value, and a linear utility function (Formula 2.5) for Scenic beauty. The utility functions were calculated with WebHIPRE program (http://www.hipre.hut.fi). The utility function values for each alternative and each criterion are given in the Table 3.7.

The criteria were next weighted. Since the criteria form a decision hierarchy, the lowest-level criteria were first weighted. The least important criterion, stumpage value, was given 10 points and the most important, net incomes, was given 30 points. Then, the weight of net incomes becomes 0.75 and that of stumpage value 0.25. The higher-level criteria were compared in the same way: the scenic beauty was given 10 points and the timber production was given 40 points, which gave weights 0.2 for scenic beauty and 0.8 for timber production. Then, the global weights of net incomes and stumpage value were calculated as $0.8 \cdot 0.75 = 0.6$ and $0.8 \cdot 0.25 = 0.2$. With these weights, the priorities of the alternatives could be calculated. In Table 3.8. the priorities of each alternative with respect to both higher-level criteria, and the global priorities are shown.

It is also possible to use the importance ranks of the criteria to calculate the weights a_j for the alternatives. This approach is called SMARTER (Simple Multi-Attribute Rating Technique Exploiting Ranks). One possible approach for calculating the weights from ranks are the so-called Rank Order Centroid or ROC weights (Edwards and Barron 1994)

$$a_j = (1/m) \sum_{i=j}^{m} 1/i, \tag{3.11}$$

where the criteria are assumed to be arranged from most important $(j = 1)$ to least important $(j = m)$. von Winterfeldt and Edwards (1986; also Stillwell et al. 1981) presented three other formulas that can be used for calculating weights from

Table 3.10 Global utilities with ROC weighting

	Global utility
NAT	0.375
SCEN	**0.707**
NORM	0.538
GAME	0.547
MREG	0.677
INC	0.563

importance ranks of criteria, namely the rank reciprocal rule,

$$a_j = \frac{1/r_j}{\sum_i 1/r_i} \tag{3.12}$$

the rank sum – rule

$$a_j = (m+1-r_j) \Big/ \sum_{i=1}^{m} r_i \tag{3.13}$$

and the rank exponent rule

$$a_j = (m+1-r_j)^z \Big/ \sum_{i=1}^{m} r_i^z, \tag{3.14}$$

where z is estimated with

$$\frac{a_j}{a_i} = \frac{(m+1-r_j)^z}{(m+1-r_i)^z} \tag{3.15}$$

where r_j is the rank of criterion j.

In the last formula, the decision maker needs to give a preference ratio (3.15) for one pair of weights, in order to calculate the rest of them. This ratio could, for instance, be the weight ratio of the most and least important criteria.

All three of the formulas can be considered ad hoc procedures (Stillwell et al. 1981; Edwards and Barron 1994). Yet, they could be useful if the decision maker does not want to evaluate the magnitude of his/her preferences.

Example 3.6. In the example above, in the higher hierarchy level there are two criteria. In this case, the weights for these two criteria, calculated with the formulas based on ranks are in Table 3.9. Using ROC weighting, the global priorities of the alternatives are presented in Table 3.10., SCEN alternative being the most preferred one.

3.3 Even Swaps

Even swaps, originally developed by Hammond et al. (1998a, b), is a method for making tradeoffs among criteria across a range of alternatives. The method is based on direct comparison of the preferences of each pair of decision elements: one criterion is valued in terms of another criterion. The criteria can be qualitative as well as quantitative. The method goes on in four steps (Hammond et al. 1998a, b).

Step 1. The consequence matrix is created. Each row represents an alternative, and each column a criterion. Each cell contains the consequence of the given alternative with respect to the given criterion.
Step 2. Dominated alternatives are eliminated. For instance, if alternative A is better than alternative B on one or more criteria, and no worse on all other criteria, then alternative B can be eliminated. Also such alternatives that are practically dominated (i.e. have only a slight advantage in one criterion and are dominated in other criteria) can be removed.
Step 3. Criteria, which have equal rating for each alternative, can be ignored in decision making. Therefore, the criteria are made equivalent by making tradeoffs. This is carried out with the following steps (Kajanus et al. 2001):

- Determining the change necessary to eliminate one criterion
- Assessing what change in another objective would compensate for the needed change
- Making the even swap in the consequence table by reducing the one objective while increasing the other
- Cancelling out the now irrelevant objective

Step 4. Steps 2 and 3 are repeated until there is only one objective left. Then, the dominant alternative is selected.

Example 3.7. The original table of consequences is the same as in example 3.5

	Net incomes 1000€	Stumpage value million€	Scenic beauty index
NAT	0.00	0.71	5.5
SCEN	79.6	0.28	5.7
NORM	38.0	0.60	5.4
GAME	33.0	0.61	5.5
MREG	102.3	0.51	5.2
INC	191.7	0.13	4.9

From this table, the swaps are made in order to get either some alternatives dominated, or either some criteria irrelevant. The example was carried out with the SMART-SWAPS program (http://www.smart-swaps.hut.fi, Mustajoki and Hämäläinen 2005; Mustajoki and Hämäläinen 2006). The SMART-SWAP program actively proposes swaps for the decision maker, to make the analysis easier.

The first swap was to compensate a change 5.4 → 5.5 in NORM's scenic beauty with a decrease of incomes 38 → 33. The resulting table shows that NORM is now dominated by GAME, and can be removed.

	Net incomes 1000€	Stumpage value million€	Scenic beauty index
NAT	0.00	0.71	5.5
SCEN	79.6	0.28	5.7
~~NORM~~	~~33.0~~	~~0.60~~	~~5.5~~
GAME	33.0	0.61	5.5
MREG	102.3	0.51	5.2
INC	191.7	0.13	4.9

The second swap was to compensate a change 4.9 → 5.5 in INC's scenic beauty with a decrease of incomes 191.7 → 170

	Net incomes 1000€	Stumpage value million€	Scenic beauty index
NAT	0.00	0.71	5.5
SCEN	79.6	0.28	5.7
GAME	33.0	0.61	5.5
MREG	102.3	0.51	5.2
INC	170.0	0.13	5.5

The next swap was to compensate a change 5.2 → 5.5 in MREG's scenic beauty with a decrease of incomes 102.3 → 90

	Net incomes 1000€	Stumpage value million€	Scenic beauty index
NAT	0.00	0.71	5.5
SCEN	79.6	0.28	5.7
GAME	33.0	0.61	5.5
MREG	90.0	0.51	5.5
INC	170.0	0.13	5.5

and the next swap was to compensate a change 5.7 → 5.5 in SCEN's scenic beauty with a increase of stumpage value 0.28 → 0.38. Now, SCEN alternative is dominated and can be removed from the table. In addition, scenic beauty is irrelevant and can be removed from the table.

	Net incomes 1000€	Stumpage value million€	~~Scenic beauty index~~
NAT	0.00	0.71	~~5.5~~
~~SCEN~~	~~79.6~~	~~0.38~~	~~5.5~~
GAME	33.0	0.61	~~5.5~~
MREG	90.0	0.51	~~5.5~~
INC	170.0	0.13	~~5.5~~

The next swap was to compensate a change 0.61 → 0.71 in GAME's stumpage value with a decrease of incomes 33 → 28, resulting NAT being a dominated alternative that can be removed.

	Net incomes 1000€	Stumpage value million€
~~NAT~~	~~0.0~~	~~0.71~~
GAME	28.0	0.71
MREG	90.0	0.51
INC	170.0	0.13

The next swap was to compensate a change 0.51 → 0.71 in MREG's stumpage value with a decrease of incomes 90 → 80, resulting GAME being a dominated alternative that can be removed

	Net incomes 1000€	Stumpage value million€
~~GAME~~	~~28.0~~	~~0.71~~
MREG	80.0	0.71
INC	170.0	0.13

Then, the final swap was to compensate a change 0.13 → 0.71 in INC's stumpage value with a decrease of incomes 170 → 145, resulting MREG being a dominated alternative, and INC the recommended one

	Net incomes 1000€	Stumpage value million€
~~MREG~~	~~80.0~~	~~0.71~~
INC	145.0	0.71

3.4 Analytic Hierarchy Process

3.4.1 Decision Problem

The Analytic Hierarchy Process (AHP), originally developed by Saaty (1977, 1980), is a widely used MCDS method and perhaps the most popular in many application fields, including natural resource management. Mendoza and Sprouse (1989),

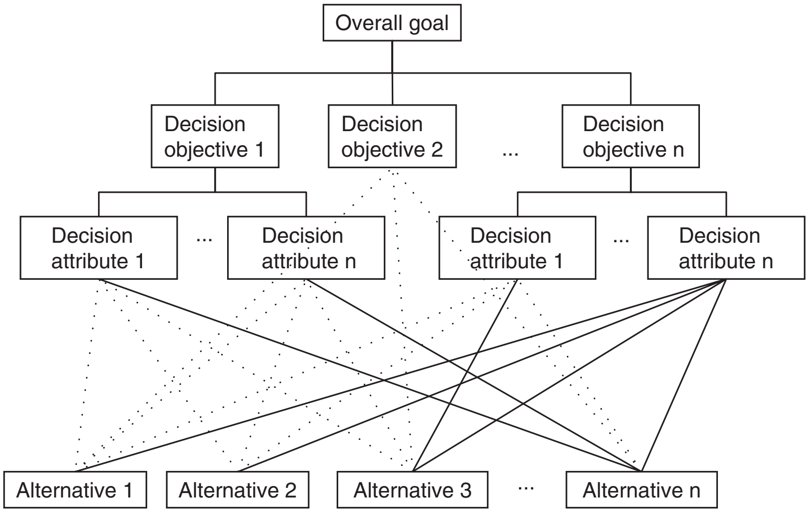

Fig. 3.3 The decision hierarchy

Murray and von Gadow (1991), and Kangas (1992), among others, have used AHP in forestry applications, and the number of applications is continuously increasing (e.g. Rauscher et al. 2000; Reynolds 2001; Vacik and Lexer 2001). AHP has also gained interest among forestry practitioners. The Finnish State Forest Enterprise Metsähallitus, which governs the vast majority of state-owned lands in Finland, has used AHP, or more precisely the HIPRE software, in practical natural resource planning (Pykäläinen et al. 1999). For a review of AHP forestry applications, readers are referred to Kangas (1999), and to Schmoldt et al. (2001) for extensions and for AHP-related development.

Basically the AHP is a general theory of measurement based on some mathematical and psychological principles. In the method, a hierarchical decision schema is constructed by decomposing the decision problem in question into decision elements – goals, objectives, attributes and decision alternatives. The general goal is at the top of a decision hierarchy, and decision alternatives constitute the lowest level (Fig. 3.3).

The branches of the decision hierarchy are assumed to be independent of each other. It means that the decision criteria are not supposed to measure the same values from the decision maker's point of view. For instance, if scenic beauty measures, for instance, the recreational value of the forest estate, but has no intrinsic value besides that, it should not be included as a criterion of its own, but as a sub-criterion for recreational value. Defining the decision hierarchy is important, as splitting the criteria in different ways has been noted to affect their weight in the decision analysis (e.g. Pöyhönen and Hämäläinen 1998; Pöyhönen et al. 2001). This does not mean, however, that the decision criteria should not be correlated. On the contrary, it often happens that, for instance, criteria describing biodiversity and recreation value may be correlated.

On the other hand, the independency means that the criteria should not have interactions. It is possible that the incomes in the different alternatives also affect the importance of aesthetic or ecological values (e.g. Leskinen et al. 2003). In basic AHP this is not allowed, but in regression AHP context (e.g. Leskinen and Kangas 2005a) and in ANP such interactions can be accounted for.

The importances or preferences of decision elements are compared in a pairwise manner with regard to each element above in the hierarchy. Based on these comparisons, an additive model on a ratio scale describing the preferences of the decision maker and priorities of decision alternatives with respect to the objectives or attributes is estimated. The model is called a priority function. The decision alternative which produces the greatest global priority is considered the "best" and most satisfactory.

Differences in measurement scales and units do not present any difficulty when the AHP is used, because the method is based on straight comparison between the significance and preference of each pair of decision elements without using any physical unit. Thus, AHP can deal with qualitative attributes as well as those which are quantitative.

3.4.2 Phases of AHP

The four basic steps involved in using the AHP to address decision problems are:

1. The decision hierarchy is constructed by decomposing the original decision problem into a hierarchy of interrelated decision elements.
2. Pairwise comparisons are made at each level of the hierarchy. In making the comparison, the question is, which of the two factors has a greater weight in decision making, and how much greater, or which of the two decision alternatives is more preferred with regard to a certain decision attribute.
3. Using the pairwise comparisons as input, the relative weights (importance/preference) of elements at each level are computed using the eigenvalue method. The resulting weights or priorities represent the decision maker's perception of the relative importance or preference of the elements at each level of the hierarchy.
4. The ratings for the decision alternatives are calculated based on the relative weights of the decision elements.

Pairwise comparisons give the decision maker a basis on which to reveal his/her preferences by comparing two elements at a time. The importances or preferences of decision elements are compared in a pairwise manner with regard to each element above in the hierarchy. First, each of the alternatives from 1 to n is compared to each other alternative with respect to decision attribute 1. These comparisons, with respect to one decision element, form one comparison set. Then, each of the alternatives is compared to each other alternative with respect to rest of the decision attributes, one by one. After that, the decision attributes are compared pairwisely with respect to each decision objective above them, and finally, the decision objectives are compared in a pairwise manner with respect to the goal.

In the standard method presented by Saaty (1977, 1980), the decision maker has the option of expressing preferences between the two elements as:

(i) Equal importance or preference of both elements
(ii) Weak importance or preference of one element over another
(iii) Essential or strong importance or preference of one element over another
(iv) Demonstrated importance or preference of one element over another
(v) Absolute importance or preference of one element over another

These preferences are then translated into numerical values of 1, 3, 5, 7 and 9, respectively, with 2, 4, 6 and 8 as intermediate values. Many other variations of the scale have been presented (see Leskinen 2001). It is also possible to carry out comparisons by using a continuous scale, e.g. by making use of graphical bars in the computer interface (e.g. Pukkala and Kangas 1993).

For estimating the priorities, the matrix of pairwise comparisons **A** is constructed for each set of comparisons. The elements of the matrix, a_{ij}, describe the comparison of alternative (or attribute or objective) i to j. The matrix is required to be reciprocal, i.e. in the matrix the element $a_{ij} = 1/a_{ji}$. It means that if alternative i is twice as good as j, then j has to be half as good as i. Each alternative is then indifferent with itself, i.e. when $i = j$, $a_{ij} = 1$.

If there were no inconsistencies in judgements, matrix **A** has unit rank since every row is a constant multiple of the first row, and all the eigenvalues of the matrix are zero except one. (Rank of a matrix is the number of mutually independent rows in it). Based on a consistent matrix **A**, relative weights can be determined by solving the equation

$$\mathbf{A}\mathbf{w} = \lambda \mathbf{w}, \tag{3.16}$$

where λ is the only nonzero eigenvalue of a consistent matrix **A**, and **w** is its right eigenvector. The solution **w** of this problem is any column of **A**. These solutions differ only by a multiplicative constant. Thus, the same relative weights are got based on any column of the matrix. In human decision making, some inconsistencies can be expected: people's feelings and preferences are not always consistent. Furthermore, as the largest value used in the comparison matrix is 9, and there are only nine possible answers, in many cases it may be impossible to compare the decision elements consistently using this scale.

Example 3.8. If the comparisons were such that decision maker considers alternative 1 to be twice as good as 2, alternative 2 three times as good as 3, and alternative 1 six $(= 2 \cdot 3)$ times as good as alternative 3, (i.e. $a_{12} = 2$, $a_{23} = 3$ and $a_{13} = 6$), the decision matrix is considered to be consistent. The matrix **A** would then be

$$\begin{bmatrix} 1 & 1/2 & 1/6 \\ 2 & 1 & 1/3 \\ 6 & 3 & 1 \end{bmatrix}$$

In this case, the weights can be obtained simply by dividing each column with the sum of its cell values, giving weights

$$\begin{bmatrix} 0.111 \\ 0.222 \\ 0.667 \end{bmatrix}$$

When **A** contains inconsistencies, the estimated weights can be obtained using the eigenvector equation.

$$(\mathbf{A} - \lambda_{\max}\mathbf{I})\mathbf{q} = 0 \tag{3.17}$$

where λ_{max} is the largest eigenvalue of matrix **A, q** its right eigenvector and **I** the identity matrix. The right eigenvector, **q**, constitutes the estimation of relative weights. It is the first principal component of the matrix of pairwise comparisons. The first principal component of a matrix is a linear combination of the variables (i.e. comparisons with respect to one alternative) that describes the largest part of the variation in the matrix. It gives the relative weights to the compared elements, which best fit to the made comparisons. If the matrix does not include any inconsistencies, i.e. the judgements made by a decision maker have been consistent, **q** is an exact estimate of the priority vector.

Each eigenvector is scaled to sum to one to get the priorities. The form of priority functions is the same as the form of additive linear utility functions without interaction terms.

Global priorities of decision elements are calculated downwards from the top of the hierarchy by multiplying the local priorities by the priority of their corresponding decision element at the level above. Global priority of an element is then used to weight the local priorities of elements at the level below and so on down to the bottom level. Global priorities at each level sum up to one.

It has been shown that λ_{max} of a reciprocal matrix **A** is always greater or equal to n (= number of rows = number of columns) (e.g. Saaty 1977). If the pairwise comparisons do not include any inconsistencies, $\lambda_{max} = n$. The more consistent the comparisons are, the closer the value of computed λ_{max} is to n. Based on this property, a consistency index, *CI*, has been constructed

$$CI = (\lambda_{max} - n)/(n-1). \tag{3.18}$$

CI estimates the level of consistency with respect to the entire comparison process. A consistency ratio, *CR*, also measures the coherence of the pairwise comparisons. To estimate the *CR*, the average consistency index of randomly generated comparisons, *ACI*, has to be calculated ($CR = CI/ACI$). *ACI* varies as a function of the size of matrix (e.g. Saaty 1980). As a rule of thumb, a *CR* value of 0.1 or less is considered to be acceptable. Otherwise, all or some of the comparisons must be repeated in order to resolve the inconsistencies of pairwise comparisons.

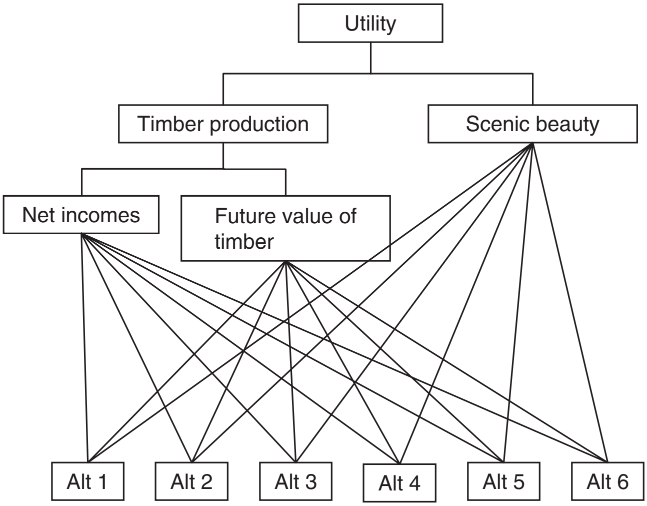

Fig. 3.4 The decision hierarchy of the AHP example

Example 3.9. The problem of example 3.5 is analysed using AHP. There are six decision alternatives and three criteria. They form a decision hierarchy (Fig. 3.4). The alternatives were compared pairwisely with respect to each criterion, namely net incomes, value of timber and scenic value (Table 3.11). The obtained CR results mean that the comparisons were a bit too inconsistent for a good analysis, and it would be recommendable to make some of them again. After comparing the alternatives, the criteria were compared. First, the second-level criteria, net incomes and value of timber, were compared to each others and then the higher level criteria were compared (Table 3.12)

The priorities and the consistency rations were calculated with webHIPRE program (http://www.hut.hipre.fi/). The priorities were first calculated at the level of decision attributes and objectives (Table 3.13). Then, INC is the best alternative with respect to net incomes, NAT with respect to stumpage value and SCEN with respect to scenic beauty. The local weight of net incomes was 0.75 and that of stumpage value 0.25, the local weights of scenic beauty was 0.2 and that of timber production 0.8. Thus, the global weight of net incomes was $0.8 \cdot 0.75 = 0.6$ and that of stumpage value $0.25 \cdot 0.8 = 0.2$. The global priorities of the alternatives with respect to decision attributes were obtained by multiplying the priorities at the decision objective level with the global weights (Table 3.14). Global priorities with respect to timber production were obtained by adding together the global priorities with respect to net incomes and stumpage value, and the overall priorities by adding the priorities of timber production and scenic beauty together (Table 3.14). It can be noted that INC is the best alternative with respect to timber production as a whole, since the weight of stumpage value is quite low, and also its overall priority was the best.

To illustrate the notion of inconsistency, the priorities of the alternatives were calculated from each column in matrix 3.11 (Table 3.15). It means that six

Table 3.11 Pairwise comparisons

	NAT	SCEN	NORM	GAME	MREG	INC
			(a) with respect to net incomes			
NAT	1/1	1/4	1/2	1/2	1/7	1/9
SCEN	4/1	1/1	2/1	2/1	1/3	1/5
NORM	2/1	1/2	1/1	1/1	1/4	1/7
GAME	2/1	1/2	1/1	1/1	1/5	1/7
MREG	7/1	3/1	4/1	5/1	1/1	1/2
INC	9/1	5/1	7/1	7/1	2/1	1/1
CR = 0.099						
			(b) with respect to value of timber			
NAT	1/1	6/1	3/1	3/1	4/1	8/1
SCEN	1/6	1/1	1/4	1/4	1/3	3/1
NORM	1/3	4/1	1/1	1/1	2/1	6/1
GAME	1/3	4/1	1/1	1/1	2/1	6/1
MREG	1/4	3/1	1/2	1/2	1/1	4/1
INC	1/8	1/3	1/6	1/6	1/4	1/1
CR = 0.159						
			(c) with respect to scenic beauty			
NAT	1/1	1/2	1/1	1/1	3/1	5/1
SCEN	2/1	1/1	3/1	5/1	5/1	6/1
NORM	1/1	1/3	1/1	1/1	2/1	4/1
GAME	1/1	1/5	1/1	1/1	2/1	4/1
MREG	1/3	1/5	1/2	1/2	1/1	3/1
INC	1/5	1/6	1/4	1/4	1/3	1/1
CR = 0.146						

Table 3.12 Pairwise comparisons between the criteria

	Net incomes	Value of timber
Net incomes	1/1	3/1
Value of timber	1/3	1/1
CR = 0.000		
	Timber production	Scenic beauty
Timber production	1/1	4/1
Scenic beauty	1/4	1/1
CR = 0.000.		

Table 3.13 Local utilities with respect to different criteria

	Net incomes	Stumpage value	Scenic beauty
NAT	0.036	0.421	0.186
SCEN	0.113	0.057	0.382
NORM	0.064	0.188	0.155
GAME	0.061	0.188	0.155
MREG	0.272	0.114	0.081
INC	0.455	0.031	0.04

Table 3.14 Overall utility and global utility with respect to timber production criterion and scenic beauty

	Overall utility	Timber production	Scenic beauty
NAT	0.143	0.106	0.037
SCEN	0.156	0.079	**0.076**
NORM	0.107	0.076	0.031
GAME	0.105	0.074	0.031
MREG	0.202	0.186	0.016
INC	**0.287**	**0.279**	0.008

different priority estimates were obtained. It can be noted that these priority estimates have a lot of variation among them, for instance, the priority of INC with respect to net incomes varies from 0.360 to 0.509 (0.455 in eigenvalue analysis, Table 3.13), and that of NAT from 0.024 to 0.053 (0.036 in eigenvalue analysis).

3.4.3 Uncertainty in AHP

Many decision scientists have criticized the AHP method. Perhaps the two foremost problems with the application of AHP have been that the original comparison scale does not allow for the expression of any hesitation regarding a single comparison, and that the AHP itself does not provide tools for in-depth analyses of the comparisons, particularly of the uncertainty inherent in the data (e.g. De Jong 1984; Crawford and Williams 1985; Alho et al. 1996). The only means to analyse the uncertainty in AHP is to calculate the inconsistencies. Moreover, part of the inconsistencies in the pairwise analysis may be due to the scale, not the answers given by the people (Leskinen 2001).

In basic AHP the number of comparisons increases rapidly as the number of alternatives and criteria increases. Large numbers of comparisons may be too costly

Table 3.15 Priorities calculated by scaling each column of pairwise comparisons matrix separately

	1	2	3	4	5	6
NAT	0.040	0.024	0.032	0.030	0.036	0.053
SCEN	0.160	0.098	0.129	0.121	0.085	0.095
NORM	0.080	0.049	0.065	0.061	0.064	0.068
GAME	0.080	0.049	0.065	0.061	0.051	0.068
MREG	0.280	0.293	0.258	0.303	0.255	0.238
INC	0.360	0.488	0.452	0.424	0.509	0.477

and tedious, especially for participatory planning problems. However, eigenvalue technique requires a full comparison matrix in order to be carried out.

Finally, rank reversal occurring when using the AHP may cause problems (e.g. Belton and Gear 1983). This means that if new alternative is included in the analysis, it is possible that the rank of the previously considered alternatives changes, although the preferences do not change. For instance, if the preferences originally are so that A is preferred to B, and B to C, after including a new alternative D the situation may change so that B is preferred to A. The rank reversal may be partly due to the arithmetic aggregation rule applied in the basic AHP, and partly due to inconsistencies in the pairwise comparisons (Leskinen and Kangas 2005b). According to, rank reversal is acceptable if it is due to inconsistencies (i.e. new alternatives give new information concerning the preferences), but not acceptable if it is due to the method itself. Using geometric aggregation rule, the rank reversal problem can be avoided (Barzilai and Golany 1994; Leskinen and Kangas 2005b). The problem of rank reversal does not only apply to AHP, but also, for instance, SMART, if the sub-utility functions are calculated using interval scale.

To alleviate these problems, different modifications of AHP have been developed. In these, the concept of decision hierarchy and the pairwise comparisons may be similar to the basic AHP, but the techniques are different. The number of comparisons can be reduced by the use of regression techniques for estimating preferences instead of the eigenvalue technique (Alho et al. 1996, 2001; Leskinen 2001). The pairwise comparisons are denoted with $r_{ij} = v_i / v_j \exp(\varepsilon_{ij})$, where $\exp(\varepsilon_{ij})$ describes the uncertainty in each pairwise comparison. Since all the values of items i, v_i are positive, with no loss of generality, it can be expressed as

$$v_i = \exp(\mu + \alpha_i) \tag{3.19}$$

where μ and α_i are parameters. Then, the ratio can, in turn, be expressed as

$$v_i / v_j = \exp(\alpha_i - \alpha_j) \tag{3.20}$$

and the model can be expressed as

$$\log(r_{ij}) = y_{ij} = \alpha_i - \alpha_j + \varepsilon_{ij} \tag{3.21}$$

Thus, expressing the values v_i as exponents and using a logarithmic transformation enables using a linear model. The parameters α_i, $i = 1, \ldots, n-1$ are then estimated using standard regression tools, for instance SAS program. The parameter α_n related to the item n is assumed to be zero for definiteness, i.e. otherwise there would be an infinite number of solutions to this model. The minimum number of observations in regression analysis is the number of parameters to be estimated, i.e. it would be enough to include only one row or column from the pairwise matrix in the analysis. In that case, however, it would not be possible to estimate the inconsistency involved. For that, additional observations are needed.

In this model, the distribution of the error term, ε_{ij}, describes the uncertainty of the analysis. With this formulation, the error variance describes the inconsistency of the comparisons: if the variance is 0, the comparisons are consistent and the higher the variance, the more inconsistency there is.

The priorities of the alternatives that sum up to one are finally calculated as

$$q_i = \frac{\exp(\alpha_i)}{\sum_{i=1}^{n} \exp(\alpha_i)} \tag{3.22}$$

With this formula, the scale is transformed from logarithmic scale back to original one (value scale). The division by the sum of transformed values scales the sum of priorities to one. Finally, if the weight for each criterion j is denoted by a_j, the utility of each alternative i can be calculated with geometric aggregation rule as

$$U_i = \prod_j q_{ij}^{a_j} / \sum_i \prod_j q_{ij}^{a_j}. \tag{3.23}$$

Example 3.10. The pairwise comparisons of example 3.9 with respect to net incomes were analyzed with regression AHP. In Table 3.16 are the data used in the analysis. The explanatory variables are just zeros and ones. They describe, which alternative has been compared to which, according to model 3.21. Since the parameter for the last alternative is set to 0, INC is not included in the data. The parameters were calculated using SAS, and the priorities of the alternatives with respect to net incomes could be calculated (Table 3.17).

The standard error of the model is 0.19 and R^2 is 0.9863. They indicate that the pairwise comparisons are not quite consistent, but the consistency is fairly high. These comparisons were consistent enough also with respect to CR criterion.

Table 3.16 Pairwise comparison data for a model

NAT	SCEN	NORM	GAME	MREG	r	y
1	−1	0	0	0	0.25	−1.38629
1	0	−1	0	0	0.5	−0.69315
1	0	0	−1	0	0.5	−0.69315
1	0	0	0	−1	0.143	−1.94491
1	0	0	0	0	0.111	−2.19823
0	1	−1	0	0	2.0	0.693147
0	1	0	−1	0	2.0	0.693147
0	1	0	0	−1	0.33	−1.10866
0	1	0	0	0	0.2	−1.60944
0	0	1	−1	0	1.0	0
0	0	1	0	−1	0.25	−1.38629
0	0	1	0	0	0.143	−1.94491
0	0	0	1	−1	0.2	−1.60944
0	0	0	1	0	0.143	−1.94491
0	0	0	0	1	0.5	−0.69315

Table 3.17 Estimated priorities of alternatives

	α	Exp(α)	q
NAT	−2.55106	0.077999	0.035316
SCEN	−1.38936	0.249235	0.112846
NORM	−1.95364	0.141757	0.064183
GAME	−1.99083	0.136582	0.06184
MREG	−0.50575	0.603053	0.273044
INC	0	1	0.45277
	Sum	2.208626	1

Another example of accounting for uncertain preferences in AHP framework is interval AHP (Leskinen and Kangas 1998; see also Arbel 1989; Salo and Hämäläinen 1992). In interval AHP, the decision makers are asked the probability that the preference lies in a certain interval, or the interval the preference lies in with a certain probability.

The regression approach to AHP can also be reformulated to Bayesian framework (e.g. Alho and Kangas 1997; Basak 1998). Bayes theorem is then used to derive the conditional distributions of the parameters. Then, it is easy to calculate the probabilities that one plan is better than all the others, for example.

Yet another example of accounting for uncertainty in AHP context is the fuzzy AHP. In fuzzy AHP, preference ratios of criteria or alternatives are described by membership functions (e.g. Mendoza and Prabhu 2001).

3.4.4 ANP

The Analytic Network Process (ANP) is an extension of the AHP (Saaty 2001) that answers some of the development challenges of the basic AHP methodology. Basically, the ANP is a general theory of ratio scale measurement of influence, with a methodology that deals with dependence and feedback. The comparisons are made using pairwise comparisons like in original AHP, but the relations between the criteria are included in the comparisons. The main idea is to avoid the assumption of independence among criteria of the standard AHP.

ANP model can be designed either using a so-called control hierarchy (i.e. a hierarchy of subsystems with inner dependencies) or as a non-hierarchical network, which includes both decision criteria and alternatives as clusters (e.g. Wolfslehner et al. 2005). The clusters are connected with arrows that describe the flow of influence. Thus, each criterion can have an interaction with other criteria (outer dependence), and each sub-criterion can have interaction with other sub-criteria in the same cluster (inner dependence).

A hypothetical example of ANP network is presented in Fig. 3.5. In the example, decision objective 1 influences to objectives 2, 3 and 6, and objective 3 is also influencing objective 1. In addition, there is a feedback loop back to the objective

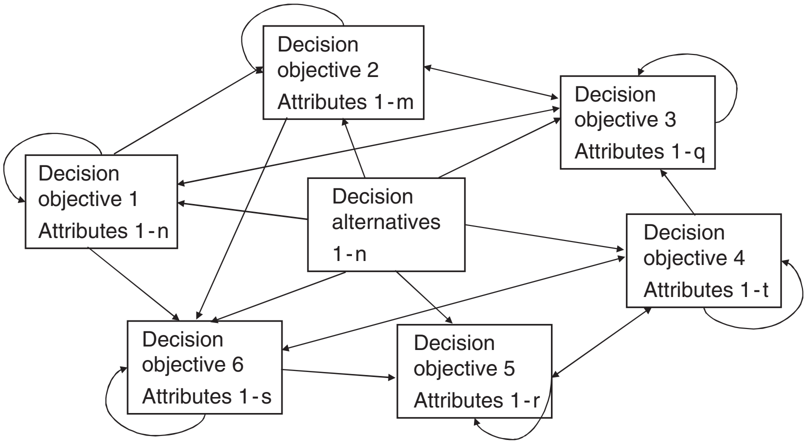

Fig. 3.5 An example of ANP network

itself. The influence means that instead of comparing decision attributes of objective 2 in pairwise manner, the influence is accounted for. Thus it is asked, for instance, "With regard to decision objective 1, decision attribute 1 is how many times more important than decision attribute 2?". For a forestry example concerning criteria and indicators of sustainable management, readers are referred to Wolfslehner et al. (2005).

The ANP utilises a so-called supermatrix calculation in order to deal with interactions among the network of criteria and decision alternatives. Saaty (2001) stated that, generally taken, the ANP is more objective and more likely to capture what happens in the real world than the AHP. However, applying the ANP is much more laborious and time-consuming. Obviously the ANP has potential application in forest management, where different kinds of interdependencies between decision elements are usual.

3.5 A'WOT

In the so called A'WOT method (Kurttila et al. 2000; Pesonen et al. 2001a) the Analytic Hierarchy Process (AHP) and its eigenvalue calculation framework are integrated with SWOT analysis. SWOT is a widely applied tool in strategic decision support. In SWOT, the internal and external factors most important for the enterprise's future are grouped into four categories: Strengths, Weaknesses, Opportunities, and Threats. By applying SWOT in a strategic planning process, the aim usually is to develop and adopt a strategy resulting in a good fit between these internal and external factors. When used properly, SWOT can provide a good basis for strategy formulation. However, SWOT could be used more efficiently than normally has been the case in its applications. The most crucial problem with SWOT is

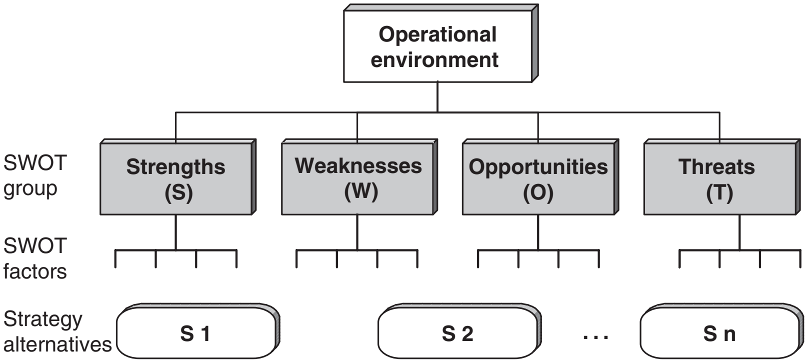

Fig. 3.6 A'WOT framework (Kurttila et al. 2000)

that it includes no means of analytically determining the importance of factors or of assessing the fit between SWOT factors and decision alternatives.

The aim in applying the hybrid method is to improve the quantitative information basis of strategic planning processes. AHP's linking with SWOT yields analytically–determined priorities for the factors included in SWOT analysis and makes them commensurable (Fig. 3.6). In addition, decision alternatives can be evaluated with respect to each SWOT factor by applying the AHP. So, SWOT provides the basic frame within which to perform an analysis of the decision situation, and the AHP assists in carrying out SWOT more analytically and in elaborating the analysis so that alternative strategic decisions can be prioritised.

The main phases of A'WOT are as follows:

1. The SWOT analysis is carried out. The relevant factors of the external and internal environment are identified and included in SWOT analysis.
2. Pairwise comparisons between the SWOT factors are carried out separately within each SWOT group. When making the comparisons, the issue at stake is which of the two factors compared is more important and how much more important. With these comparisons as the input, the mutual priorities of the factors are computed.
3. The mutual importance of the SWOT groups is determined by applying pairwise comparisons. There are several possibilities as how to do this. For instance, it is possible to compare the groups as such or the most important factors in each group pairwisely.
4. The strategy alternatives are evaluated with respect to each SWOT factor by using pairwise comparisons and the eigenvalue technique.
5. Global priorities are calculated for the strategy alternatives.

In the earliest A'WOT applications (Kurttila et al. 2000; Pesonen et al. 2001a), only steps (1)–(3), as listed above, were carried out in an early stage of a strategic planning process. A'WOT strengthens the decision basis also in the case where the result is only the quantification and commensuration of SWOT factors. However, the final goal of any strategic planning process as a whole is to develop and propose a strategy resulting in a good fit between internal and external factors. When steps (4) and (5) are included in the A'WOT process, the initial SWOT analysis might not always be applicable as such (Pesonen et al. 2001b).

The reason for this is that the SWOT factors could have been formulated so that strategy alternatives can not be evaluated with respect to them. This being the case, SWOT factors need some value-focused modification and fine-tuning (e.g. Leskinen et al. 2006). For A'WOT, SWOT factors should be determined by asking, which are the internal and external factors of the operational environment that should be taken into account in choosing the strategy for the enterprise. Then it is possible to compare strategy alternatives with respect to strengths, weaknesses, opportunities, and threats as listed in SWOT. To take an example of the pairwise comparisons: which of the two strategy alternatives compared (when implemented) makes it possible to better exploit a certain opportunity, and how much better? According to the experiences of A'WOT applications and tests, the combined use of the AHP and SWOT analysis is a promising approach in supporting strategic decision-making processes (Kurttila et al. 2000; Pesonen et al. 2001a, b).

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