9 Upscaling of Slug Test Hydraulic Conductivity Using Discrete Fracture Network Modelling in Granitic Aquifers

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INTRODUCTION

The Indian sub-continent is underlain by hard rocks aquifers that are devoid of primary porosity and occupy more than two thirds of the landmass. These aquifers are highly vulnerable to pollution and resource depletion because they are the most heavily exploited by the population for agriculture, industry and domestic needs. These hard rocks aquifers, in general, consist of three layers: the weathered zone that can be considered as a porous media, the fissured/fractured zone, which is heterogeneous, and the fresh basement which is generally devoid of any openings unless and until some deep seated tectonic fractures are present. Aquifers in such geological settings are, therefore, of very variable quality, inherently to the heterogeneous nature of the fracture networks. Due to the adaptation of the latest drilling technology in India during the last few decades, ground water has been exploited on a large scale. But this uncontrolled exploitation of the resource has resulted in an alarming decline in water levels. The characterization of the flow and storage of ground water in these systems is a challenging task since flow and transport processes are very different from those occurring in the porous matrix. Furthermore there is an extremely high degree of contrast between the hydraulic conductivity of the fractures within short distances. An estimation of the groundwater resources is only possible through an estimation of flow and storage parameters in the fracture systems but it is anticipated that the responses of pumping tests performed in well field cannot be analyzed through classical aquifer-testing methods that assume a homogeneous aquifer.

STUDY AREA AND AVAILABLE DATA

For the above mentioned reasons, a joint Indo-French Collaborative Project on groundwater research was launched in 1999 (Ahmed & Ledoux, 1999) and a number of field investigations were conducted in a watershed in Maheswaram mandal, about 30 km south of Hyderabad, Andhra Pradesh, India. Some historical data of rainfall, water levels in wells and land-use were available from the Groundwater Department but much of the field investigations were jointly performed by the scientists involved in the project. The studies included the base-line data generation of well inventory, preparation of geomorphological maps based on aerial photographs and satellite imageries, geophysical investigations for the delineation of the extension of weathering in dykes, and across lineaments, mapping of the weathering profiles, drilling of wells at specific locations for regular monitoring of water levels, conducting hydraulic tests of short (30 minutes) duration to mid and long duration (4 to 6 hours, 18 hours), and the monitoring of a hydrometeorological station. Under the present project, about 25 piezometers have been drilled in the entire watershed to carry out the hydrological tests and for monitoring water levels and quality parameters. This considerable but necessary amount of data will form the basis for the preparation of a model so that future scenarios of water balance could be established for the management of the limited resource. This paper mainly deals with the results of slug tests and the related subjects. Figure 1 shows the set of normalized slug test responses which will be used later.

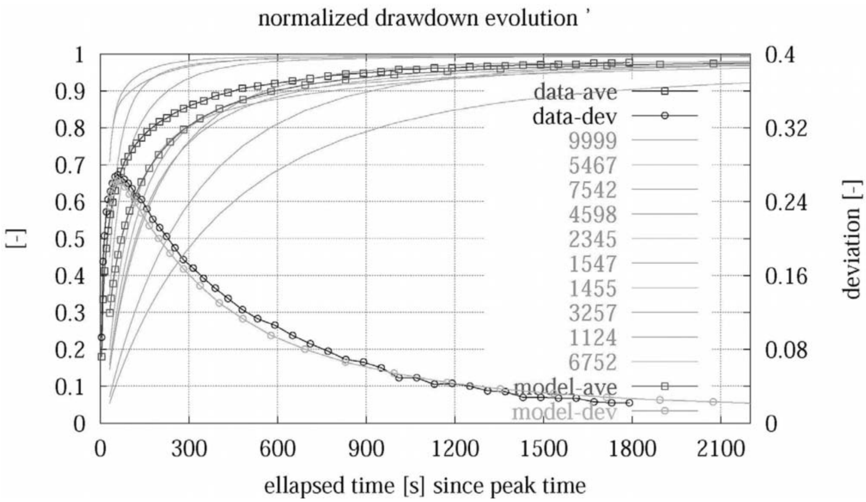

Figure 1. Slug tests responses, normalized versus the initial water level change. Superimposed is the average value observed at a given time and the standard deviation measuring the spreading of the responses at that particular time (three poor responses have been discarded).

THE DISCRETE FRACTURE NETWORK APPROACH

Conceptual Model

Discrete fracture network approaches have been developed in the past few years to handle the question of fluid and mass transport in fractured heterogeneous systems where discontinuities are likely to exist at many scales. Various conceptual approaches were proposed to describe the geometry of network of discontinuities, and capture the uneven nature of flow and solute transfer within a single fracture. 2D flow as well as channel models in random or structured network of planar fractures were investigated and tested against a variety of in situ experiments. Different softwares now exist with most of these capabilities (i.e. NAPSAC developed by AEA Tech in UK, FRACMAN package, developed by Goldberg ass.). The FRACAS software used hereafter is a similar product, gradually developed at the Paris School of Mines by Cacas et al. (1990) and Bruel et al. (1994), in the framework of National and European research programmes dedicated to nuclear waste insulation, to geothermal projects in hard rocks (Bruel, 2002), and to the estimation of hydrogeological properties in the vicinity of mined areas. Some of the capabilities of the code were recently tested in an international benchmark exercise (Rejeb and Bruel, 2001).

Geometry

The FRACAS modelling approach is based on the assumption that fluid moves through a rock mass within a system of interconnected fractures and that flow in the rock matrix is negligible by comparison. To alleviate the problems faced in the interpretation of fractured rock geometry the following are adapted for a better representation and interpretation of the aquifer characteristics.

The three-dimensional hydraulically conductive network of planar, disc-shaped fractures is generated within a rectangular block of rock based on stochastic descriptions of fracture density (Poisson distribution), fracture orientation (Fisher von Mises distribution), and fracture diameter (log-normal distribution) for specific fracture sets. Fractures may arrange into five types of fracture systems for modelling purposes.

1. Pure random network of disk shaped fractures.
2. Fisher type set of disk shaped fractures, that is a directional set.
3. Sub-vertical, with no preferential strike, set of disk shaped fractures.
4. Planar structures of deterministic location, can be infinite or with a finite elliptical extension, partly glued/non-persistent.
5. For an array of discs, with a fully deterministic description.

Flow Rule

Time-dependant analysis requires assumptions to be made concerning the form of fluid flow within the fracture network. The general form of fluid flow assumed in each fracture is based on an analytical solution, known as the "cubic law", for fluid flow between approximately parallel surfaces (Witherspoon et al., 1980). For ground water purposes at shallow depth, in unconfined or semi-confined situations, a linear form is used. Transmissivity is proportional to the permeability and to the fracture thickness. It is also assumed that fractures are filled with a porous material of storativity *S*. However transmissivities and storativities are modified to account for the effects of pressure changes between fracture surfaces when the fracture becomes desaturated. Thus, in FRACAS, the volumetric flux (m^3s^{-1}) in the x-direction through a length *l* (m) of a fracture has the form:

$$Q = \frac{-a^3 glF}{12\upsilon}\frac{dh}{dx} \tag{1}$$

where *a* is the hydraulic aperture (m) of the fracture, *g* is acceleration due to gravity (ms^{-2}), υ is the kinematic viscosity (m^2s^{-1}), dh/dx is the hydraulic head gradient driving flow through the fracture, and *F* is a dimensionless function dependent on effective pressure.

Analytical and empirical expressions for *F*, in which *F* decreases as the effective pressure becomes negative ($F = 1$ under saturated conditions) are presented in the literature. As an example we adopt the Van Genuchten formalism: the water content θ and the fluid pressure ψ are linked by equation (2), where the residual water content θ_r, α and *k* are adjustable parameters to be calibrated. θ_s is the water content at the saturation ($\psi = 0$) and $1 = 1-1/k$. In this case the *F* factor is derived as a power function of the water content, according to the Brooks and Corey formulation (3).

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1 + (\alpha\psi)^k\right]} \tag{2}$$

$$F(\psi) = \theta_e^v \tag{3}$$

The FRACAS geometry underlying the fluid flow law is illustrated here below. For this geometry, the volumetric flux from the centre of fracture *i* to the centre of fracture *j* may be approximated by equation (4), where the geometric mean fracture hydraulic conductivity k_{ij} (m^3s^{-1}) is defined as:

$$Q_{ij} = k_{ij}\frac{h_i - h_j}{L_i + L_j},\quad k_{ij} = \frac{k_i k_j (L_i + L_j)}{k_j L_i + L_j k_i} \tag{4}$$

in which, based on equation (1), the fracture hydraulic conductivities are defined as:

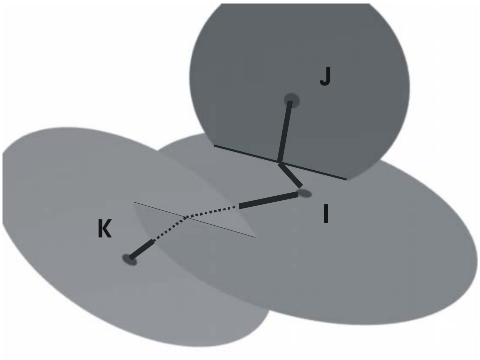

Figure 2. Geometry of the fracture network underlying the fluid flow law.

$$k_j = \frac{a^3 gly F_j}{12\upsilon}, \quad k_i = \frac{a^3 gly F_i}{12\upsilon},$$

If n_i fractures intersect fracture i, then the time dependent mass conservation equation for fracture i is given by:

$$\sum_j^{n_i} Q_y = V_i S_i \frac{dh_i}{dt}$$

The fracture volume term V_i (m^3) is:

$$V_i = \pi R_i^2 \alpha$$

and the specific storage of the S_i (m^{-1}) is a constant or, under unsaturated condition, a function of the water content:

$$S_i = \rho g \frac{1}{a} \frac{\partial\theta}{\partial\psi} \tag{5}$$

where R_i is the fracture radius (m) and ρ is the fluid density (kgm^{-3}).

In deriving the specific storage term, the rock stress has been assumed constant, such that changes are caused only by fluid pressure changes. Also, fluid compressibility is generally much smaller than fracture compressibility and, thus, has been neglected. The form of the term $d\theta/d\psi$, calculated at the centre of the fracture disc, depends on the assumed form of the relationship between the pore pressure and the water content that we use to describe the material infilling the fractures.

Modelling the Slug Tests Using the FRACAS DFN Approach

The objective of this section is to identify and calibrate a set of parameters suitable for modelling the observed spatial variability of the slug test responses, described in Fig. 1. Our working assumption is that the variability of the numerical responses resulting from statistically equi-consistent alternatives of the fracture network should be similar to the variability observed on the field. Therefore, our strategy will be (i) to built a set of fracture networks, using the same geometrical characteristics, (ii) simulate a transient hydraulic test, similar to a slug test, in each one of these networks and (iii) analyse the set of responses in terms of mean behaviour and deviation between the tested alternatives. This should be comparable with the in-situ corresponding curves in Fig. 1.

Determination of the Geometrical Fracture Network Parameters

Identification of Fracture Sets

Assuming that the rose diagram for the fracture orientations reflects the strike of sub-vertical fractures only, we define two fracture sets. First one representing sub-vertical fractures, striking to North +/– 30° and a second set, for the rest of the population. A third set is introduced for sub-horizontal fractures. There is much less constraint on this third set, since we only have visual indications in dry dug-wells and indirect indications of fluid occurrence in producing wells.

Densities of the Fracture Sets and Fracture Sizes

The determination of these parameters is highly empirical. At the scale of the visited excavated areas, there is no evidence of single sub-horizontal fractures that crosscut the domain. However, the sub-horizontal fractures may arrange into relay structures, therefore resulting at a larger scale in more continuous horizontal features. We assume that horizontal single elements have extensions ranging in between 5 and 10 m that is the size of a dug-well's side, with a mean value of 75 m. This in turn, indicates a mean fracture area of about 45 m^2. With a frequency of about 0.4 (i.e. one fracture in 2.5 m) we deduce the density, expressed in fracture centre/unit volume of rock, in the range of 0.4/45 = 0.0088. With a frequency of about 0.1 we end with a fracture density of about 0.0022. As a starting value for modelling purposes, the density of sub-horizontal fractures has been set to 0.005.

Concerning the sub-vertical structures, we arbitrarily assume a total density of about 0.01 and smaller sizes, ranging from 2 to 5 m. As the sub-vertical north striking fractures do not form the majority, we assume a density of 0.004 for this set. These numbers are starting numbers and will be subject to numerical tests to evaluate how sensible the model is to any significant variation.

The FRACAS model assumes a lognormal distribution for the fracture radius. Two parameters are required, a mean value μ and a deviation σ. The mean size in the real space, expressed in [m] is given by the expression $r = \exp(\mu + 0.5\,\sigma^2)$. The parameters used in the next sections are tabulated hereafter.

Table 1: Fracture Set Parameters

Fracture set	*Mean value*	*Deviation*	*Mean radius (m)*
Set 1: Sub-horizontal	1.20	0.5	3.75
Set 2: Sub-vertical, North-South	0.79	0.5	2.50
Set 3: Sub-vertical	0.79	0.5	2.50

Hydraulic Properties of the Different Sets of Fractures

Fracture Aperture, Fracture Permeability and Fracture Storativity

These parameters refer to single fracture and have to be calibrated. There is poor site specific information about realistic bracketing values, since no tests between packers are available, and since even the slug tests we are looking at refer to open hole sections that may intersect more than one fracture (2 to 5 intersections are frequently detected in the present numerical models). Fracture aperture are therefore set to 10^{-2} m, and we assume that the infilling materials, some weathering by products trapped in between the natural rough fracture walls, have a porosity of about 30%. We investigate fracture permeabilities ranging in between 10^{-2} ms^{-1} and 10^{-4} ms^{-1}. Dealing with storativity, we started with values close to that of confined aquifers, about 10^{-4} to 10^{-5}, and move towards values more appropriate to unconfined situations, i.e. some per cent.

Calibration Procedure and Model Outputs

Size and Shape of the Modelled Volume

The modelled volume of rock is a vertical cylinder, 50 m in diameter and 30 m in elevation. A vertical bore-hole is simulated along the vertical cylinder axis. The open hole section is 22 m, centered at the mid-height of the block. This value reflects the average aquifer thickness, which was observed on the 25 tested IFP wells.

Inner and Outer Prescribed Conditions at the Model Boundaries

The outer surface represents an open boundary where hydrostatic conditions prevail all the time. These conditions are applied at all the fractures (i.e. at the fracture centre in the numerics) that intersect this outer boundary. Along the central bore-hole is the inner boundary, where a transient chart is prescribed. Two phases are described. During the first two seconds, the hydraulic head at the well linearly increases from 0 to 0.49 m, the theoretical maximum value of the change in water level. Then from 2 to 2100 seconds, the well is subject to a ‘no flow’ condition and the head perturbation dissipates in the fracture network. The initial hydraulic head distribution is uniform and set to 0 m.

Calibration Strategy and Results

The scenario is applied and we present the period during which the pressure perturbation vanishes in the fracture network, in a normalized way, starting from 0 at the peak time and increasing to 1 as time increases. Series of 10 equi-consistent network alternatives are generated and we produce a set of

10 hydraulic drawdown curves. These curves are summed up into two curves, an average and a deviation curve respectively that can be compared to those derived from the *in situ* measurements. The first point is try to simulate the spatial variability of the responses, and then try to match the average behaviour. Hereafter (Figs 3 and 4), we show some of satisfactory results obtained using the parameters listed in Table 2. Obviously there is no unique solution to the problem and we only suggest set of values that produce an acceptable fit to the data, qualitatively and quantitatively.

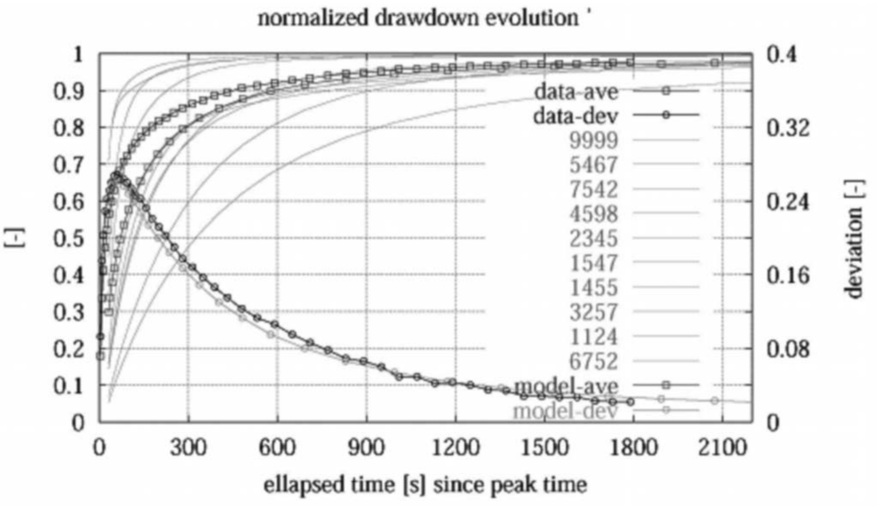

Figure 3. Calibration test, case 4.

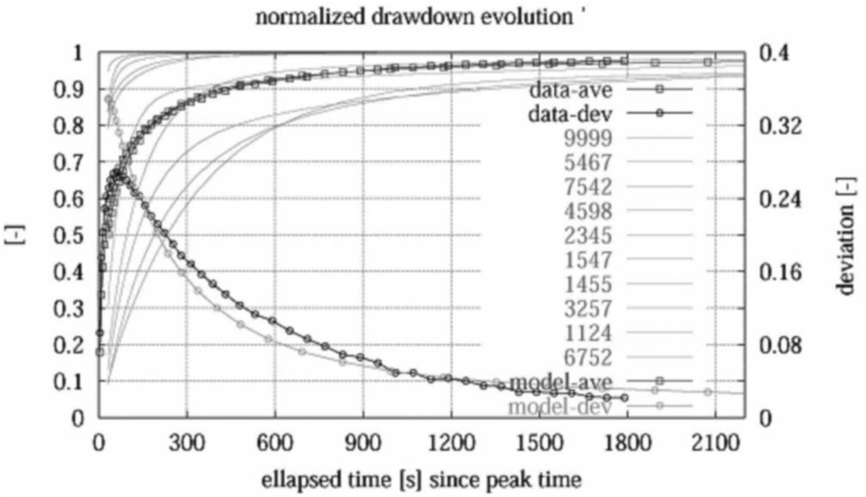

Figure 4. Calibration test, case 5.

Table 2: Parameters used to calibrate the slug test model, according to Figs 3 and 4

Calibration test	*Case 4*	*Case 5*
Density set 1 (frac/m^3)	0.004	0.0033
Density set 2 (frac/m^3)	0.004	0.0033
Density set 3 (frac/m^3)	0.006	0.005
K-1 (ms^{-1})	1.2×10^{-2}	1.0×10^{-2}
K-2 (ms^{-1})	1.2×10^{-2}	1.0×10^{-2}
K-3 (ms^{-1})	1.2×10^{-2}	1.0×10^{-2}
S-1(–)	1.0×10^{-3}	1.0×10^{-4}
S-2(–)	2.0×10^{-2}	8.0×10^{-3}
S-3(–)	2.0×10^{-2}	8.0×10^{-3}

UPSCALING THE TRANSMISSIVITY AND STORATIVITY PARAMETERS

Objectives

The upscaling phase of this study consists in a numerical derivation of equivalent hydraulic parameters at a spatial scale greater than the one investigated by the slug tests. The aim is to evaluate properties at the scale of an elementary cell of a global hydrogeological model based on standard porous media theory. The set of calibrated parameters, at the local scale, will be directly used for this purpose.

Method

The method consists in simulating parallel flow through a square cell of varying size. The size of this elementary cell is set successively to 50 m × 50 m × 30 m and 100 m × 100 m × 30 m, the sides of which are parallel to east-west and north south directions. Such a cell is filled with fracture elements, with geometrical properties directly inferred from the previous calibration phase. The flow is established across the cell, by assigning a head gradient between two opposite faces, the two others remaining closed to flow. The head gradient can be applied along X axis and Y axis in two successive runs.

Because the hydraulic regime is calculated in transient conditions, the numerical experience has to last until a quasi steady state situation is reached. Practical numerical prescriptions are as follows:

Initial head distribution: 0 m
Initial head value at the inlet face: + 5 m
Initial head value at the outlet face: – 5 m
Duration of the simulation: two days

Equivalent Transmissivity

The results of the tests are obtained for the two directions. They are obtained as the averaged value of the results produced by five equi-consistent networks, in a statistic sense. The five independent responses for a cell of 50 m × 50 m × 30 m for case 4 are listed hereafter:

Table 3: Flow in X and Y directions based on the parameters used for calibrating the slug test model in Table 2

Alternative	*Seed number*	*Flow in X direction (l/s)*	*Flow in Y direction (l/s)*
1	7571	2.76	3.49
2	3317	2.25	2.66
3	4531	2.31	3.04
4	9999	2.35	2.59
5	5703	1.92	2.91
Mean Value	-	2.32	2.93

From the calculated mean flux value combined with the geometry of the block and the applied head gradient, we derive an equivalent horizontal transmissivity, for both directions.

Because the cell has a square shape, the transmissivity is directly given when dividing the flux, expressed in m^3s^{-1} by the head difference, expressed in m. In the East-West direction (resp. along X axis), we obtain $T_{XX} = 2.3 \times 10^{-4}\ m^2s^{-1}$ and in the North-South direction (resp. along Y axis), we obtain $T_{YY} = 2.94 \times 10^{-4}\ m^2s^{-1}$. Therefore the equivalent porous media is likely to be slightly anisotropic, with a factor T_{XX}/T_{YY} close to 0.8. An equivalent isotropic horizontal permeability would be about $8.7 \times 10^{-5}\ ms^{-1}$. For case 5, there is no connection at a big scale, $T = 0$.

Equivalent Storativity

To derive an equivalent storativity factor suitable for hydraulic calculations at the regional scale, the calibrated fracture storativities are combined, using the densities of the fracture sets as weighting factors.

Table 2 shows that, to fit the data, the storativities values for the horizontal fracture set are smaller than the one for the sub-vertical and vertical fracture set. This can be due to the fact that because of the geometry, the vertical fractures are more in unconfined condition than the horizontal ones. In that case, the storativity calculated for the vertical and horizontal fracture set is more representative of confined aquifer and more near from a storativity value. The storavities calculated for the vertical and sub-vertical fracture sets are a 'mix' between a storativity and a specific yield coefficient. That is why the resulting storativity value (no unit) is calculated as following according to the set of horizontal fracture parameters:

$$S = e.\pi.S_{set1}.(d_{set1}.R^2_{set1} + d_{set2}.\mathrm{R}^2_{set2} + d_{set3}.R^2_{set3}) \quad (6)$$

where d_{seti} = density, set i (centres/m^2), e = fracture thickness (m), R_{seti} = fracture radius, set i (m^2) and S_{set1} = horizontal fracture storavity (1/m) and falls within the range of 3.89×10^{-6}, case 4 (Table 2) and 3.22×10^{-7}, case 5 (Table 2).

CONCLUSIONS

Numerical calibration was performed following a trial and error process. Although the solution may not be unique, we end with a combination of parameters that provides the set of responses shown in Fig. 1 when simulating a slug-test in a random fracture network. Total fracture density is about 0.02 m^{-3}, 30% of the fractures being sub-horizontal. Sizes are ranging in between 1 and 10 m, fracture thickness is about 0.01 m and porosity of the infilling material is set to 30%. Calibrated fracture permeability is close to 10^{-2} m/s, while the fracture storativity lies in between 10^{-3} and 10^{-4}m^{-1}.

Using these numbers to evaluate the permeability tensor by simulating parallel flow in a 100 m × 100 m × 30 m cell, in two perpendicular directions successively, leads to equivalent permeabilities ranging from 5.0×10^{-6} to 7.3×10^{-6} m/s, with a mean value of 6.2×10^{-6}m/s. An anisotropy factor of 1.25 is found in favour of the North/South direction, as a result of the existence of a set of north-south sub-vertical fractures.

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