UNDERWATER ROCKFALL KINEMATICS: A PRELIMINARY ANALYSIS

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Abstract

The marine environment presents various settings in which talus slopes are formed via a rock fall process similar to what exists on land. This is the case along fjords and submarine canyons in particular. Although many studies have been carried out on land, surprisingly very little is known for the submarine environment. We propose here the first kinematics analysis of underwater rockfall. It is postulated that the block have a diameter of more than one meter. As it can be expected, the main addition to the subaerial case is the consideration of the role of water (e.g. density or current). For submarine rockfall analysis, the effect of the ambient fluid cannot be neglected. Hydrodynamic constraints are controlled by the speed, shape, and size of the moving mass. Wind does not have a significant role in subaerial rockfall analysis, but currents must be considered in the subaqueous environment. In addition, coefficients of restitution are not only controlled by the elastic properties of the material, but also by impact Stokes number. This paper provides a summary of underwater rockfall kinematics in order to formulate underwater rockfall governing equations.

1. Introduction

Kinematics of falling rocks have been studied analytically and experimentally for several decades (Ritchie, 1963; Pfeiffer and Bowen, 1989; Azzoni *et al.,* 1995). Almost all of these works have been done for subaerial environment, but rockfalls also happen in underwater environment (Edmunds *et al*., 2006). In order to protect submarine installations such as pipelines or to be able to protect the environment, for example coral reef (Edmunds *et al*., 2006), underwater rockfall analysis must be undertaken. To the authors' knowledge, the only work on submarine rockfalls is from Beranger *et al.* (1998): they have introduced two constants to account for water effects (drag and added mass forces). In its preliminary form, we propose here the first kinematics analysis of underwater rockfall. As a first step, geometry of the problem and forces will be defined. Rock shape will be taken as a disk for simplification reasons. Effect of hydrodynamic forces on every part of the movement will then be presented in order to develop governing equations. A discussion on how coupling these equations will then be done. Further work will be to develop a mathematical model and laboratory or in-situ validation.

2. Physics

It can be expected that for underwater rockfalls, general modes of motion will be the same as for subaerial rockfalls: rolling or sliding, bouncing and freefall (Figure 1). Since the values of water density and viscosity are higher than those of the air, hydrodynamic forces cannot be neglected. They will influence modes of motion. Among all hydrodynamic forces, lift, drag and added mass will be discussed.

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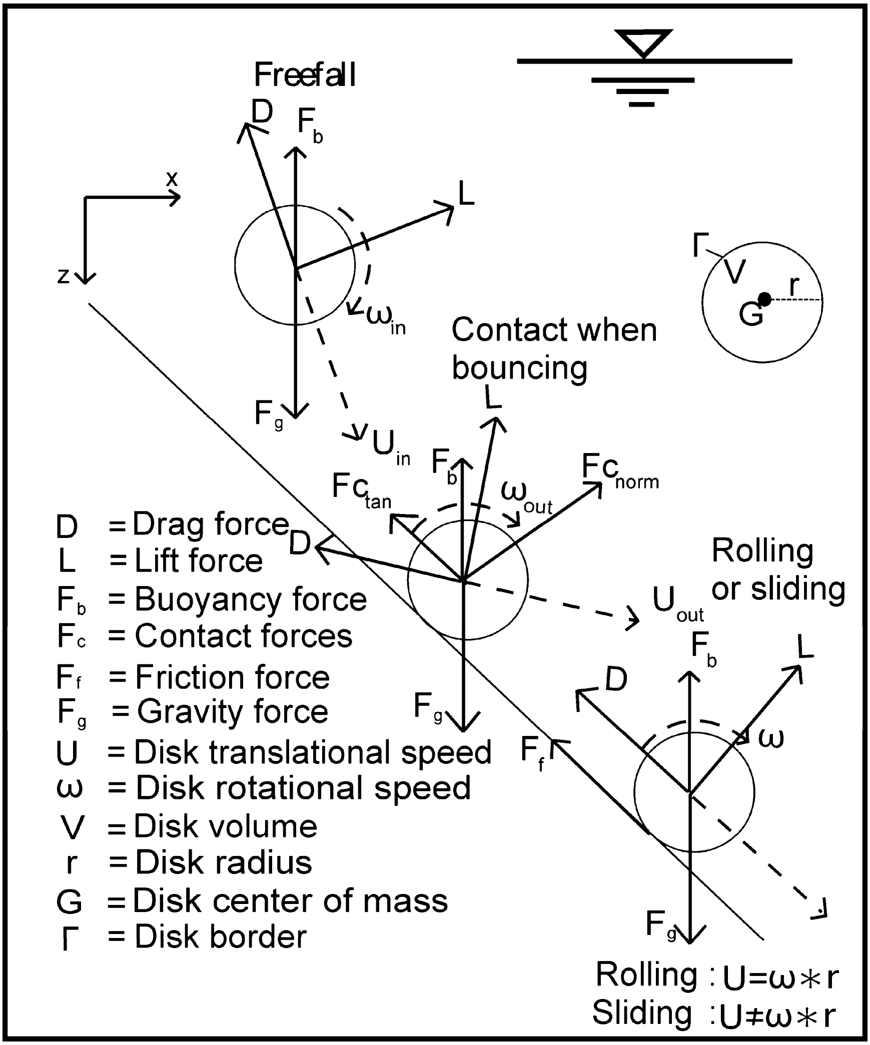

Figure 1 General modes of motion and corresponding forces.

These forces are shown in Figure 1. Basset history force, force due to the instationarity of the flow, can be neglected because of large particle diameter and density of the rock mass (Thomas, 1992).

2.1 HYDRODYNAMIC FORCES

2.1.1 Buoyancy forces

Buoyancy force is the force that obeys Archimedes principle which states that a body immersed in a fluid is buoyed up by a force equivalent to the mass of displaced fluid. This force is directed upward, and is defined by: $F_b = -\rho_w V g$, where ρ_w is water density, V is the body volume, and g is the gravitational acceleration directed downward.

2.1.2 Lift and drag forces

Lift and drag forces are related forces acting in perpendicular directions. When a body moves through a fluid, an interaction between the body and the fluid occurs. This interaction can be described in terms of shear stress due to viscous effects and normal stress due to pressure. (Munson *et al.*, 1998). Lift force (L) is oriented perpendicular to the direction of motion and drag force (D) is oriented in a direction opposite to that of motion.

It is not always possible to directly calculate these two forces, without complex numerical analysis. Two adimensional terms have been introduced to be able to evaluate lift and drag forces for different object shapes, lift coefficient (C_l) and drag coefficient (C_D):

$$C_l = \frac{L}{0.5\rho U^2 A} \qquad C_D = \frac{D}{0.5\rho U^2 A} \qquad \text{(1) and (2)}$$

Where U is the relative velocity of the object in the fluid and A is a characteristic area of the body. Numerical values of these coefficients have been found after several laboratory experiments (Allen, 1900, Wieselsberger, 1922, Liebster, 1927, Achenbach, 1965). Both coefficients are function of Reynolds number (Re) which represents the ratio between inertial and viscous forces: $Re = \frac{\rho U l}{\mu}$, roughness of the object and rotation of the object (Magnus effect). One can show that for a moving disk without rotation, lift force is zero so $C_l = 0$.

A standard drag curve (SDC) has been developed by Lapple and Shepherd (1940) (Figure 2). When it exceed $Re = 10^5$, the drag coefficient shows a sudden drop, called drag crisis, due to the transition from laminar to turbulent boundary layer. In our case, because of the large radius of the disk and its speed, Re will vary from around 10^5 to much higher Re numbers (Fig. 2). With such high Re numbers, C_D may vary from 0.08 to 0.7.

2.1.3 Robins effect

Robins effect is the lift force observed when a spinning sphere moves through a fluid. The effect of spin is to delay separation on the retreating side and enhance it on the advancing side (Mehta, 1985). Little work has been done on smooth spheres, and they have only been done for a very narrow range of Re numbers ($Re \leq 2000$).

Work of Kurose and Komori (1999) shows that both C_l and C_D varies with Re and a rotational parameter defined as $\Omega = \omega r / U$ where ω is the rotation speed of the sphere, r is the radius of the sphere, and U is the relative velocity of the sphere in the fluid. Most work has been done on Magnus effect for rotating cylinders or for sports balls such as golf or cricket ball. Up to now, no correlation seems to exist to link rotation rate, Reynolds number and lift or drag coefficient in viscous flow when Re number is high.

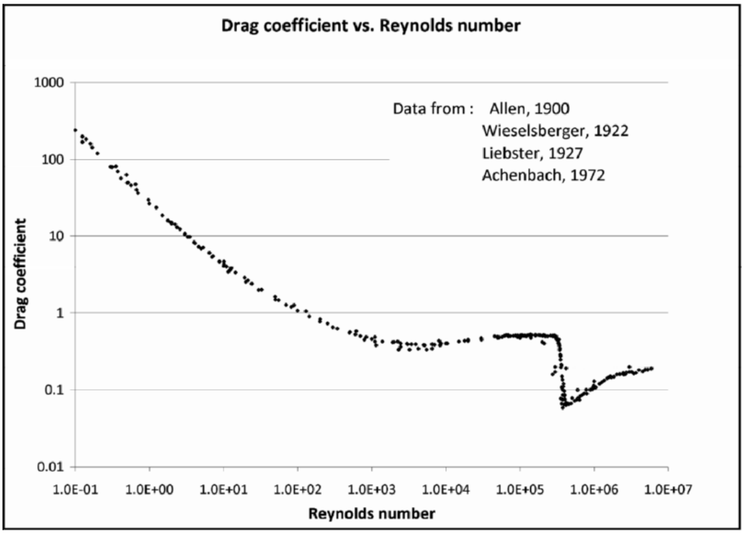

Figure 2. Standard drag curve.

2.1.4 Added mass force

When a body accelerates in a fluid, it must also accelerate part of the fluid. The kinetic energy associated with the moving fluid will be changed. Kinetic energy can be defined as (Brennen, 1982) (with Einstein notation):

$$T = \frac{\rho}{2}\int_V u_i u_i dV \tag{3}$$

Where u_i *(i=1,2,3)* represents components of fluid velocity. If the object speed is constant, this kinetic energy will be constant and dependant of the square of the object translation velocity (U^2). If the flow is a potential flow, it can be said that when U is altered, the velocity u_i at each point in the fluid varies proportionally with U (Brennen, 1982) then:

$$T = \rho \frac{I}{2} U^2 \quad \text{where } I = \int_V \frac{u_i}{U}\frac{u_i}{U} dV \tag{4}$$

If the body accelerates, then an additional work will have to be done to increase the kinetic energy of the fluid and can be expressed as dT/dt. This extra work result in an additional drag experienced by the object, $-FU$ and is equal to dT/dt, so (Brennen, 1982):

$$F = -\frac{1}{U}\frac{dT}{dt} = -\rho I \frac{dU}{dt} \tag{5}$$

The resultant form of the added mass force is expressed as (modified from Gondret *et al.*, 2002):

$$F_{am} = -\frac{4}{3}\pi r^3 C_a \rho_f \frac{dU}{dt} \tag{6}$$

where C_a is the added mass force coefficient, varying from 0.5 to 1.05 dependant of the ratio ρ_s / ρ_f (Odar and Hamilton, 1964).

2.2 CONTACT FORCES

In most of the modeling software for rockfall analysis and because in any non-perfectly elastic collision some kinetic energy is lost (Pfeiffer and Bowen, 1989), contacts are modeled by use of two adimensional restitution coefficients: normal and tangential. Normal coefficient of restitution is defined as the ratio between rebound to impact velocity normal to the slope. Normal coefficient of restitution is function of material properties, angle of contact (α) and impact (Chau *et al.,* 2002). Tangential restitution coefficient is also function of slope inclination, impact angle, impact velocity, and material properties.

CRSP formulation (Pfeiffer and Bowen, 1989) takes most of these parameters into account and reproduces well experimental data (Heidenreich, 2004).

2.2.1 Effect of ambient fluid on restitution coefficients

Few studies have been done on particle-collision in fluids, but all of them show the same results. The viscous liquid dissipates the energy and may weaken the restitution process in collision (Yang, 2006). McLaughlin (1968), Joseph *et al.* (2001), and Gondret *et al.* (2002) investigated normal (*i.e.* without tangential velocity) particle-wall collision in liquid and found that normal restitution coefficient is also function of Stokes number. Stokes number is defined by the ratio of viscous to inertial forces of an object in a fluid:

$$St = (\rho_s / p_f) Re / 9 \tag{7}$$

For $St < St_{critical} = 10$, there is no rebound and for very high St (>1000), viscous forces are negligible and wet coefficient of restitution equals dry coefficient of restitution (Figure 3). When there is a tangential component, Joseph and Hunt (2004) showed that normal coefficient have the same behaviour as the one with normal collision and is not affected by tangential speed component. Stokes number must be modified to take into account normal component of impact velocity. For the tangential component, two cases can occur: (1) there is solid-solid contact or (2) there is no solid-solid contact. Joseph and Hunt (2004) showed that, when mean surface roughness is larger than elastohydrodynamic (EHD) lubrication minimum distance of approach, there will be solid-solid contact and the collision will exhibit the same behaviour as subaerial collision. Otherwise, when the surface roughness is smaller than EHD lubrication minimum distance of approach, there is a substantial decrease in the rotational impulse, when compared to collisions in air. This will also cause a reduction of friction coefficient by almost an order of magnitude (Joseph and Hunt, 2004).

2.3 WALL EFFECTS

When an object moves close to a wall, the classical lubrication theory predicts a force that increase as the inverse of the gap width (Yang, 2006). This force is a lift force and is directed normal to the wall.

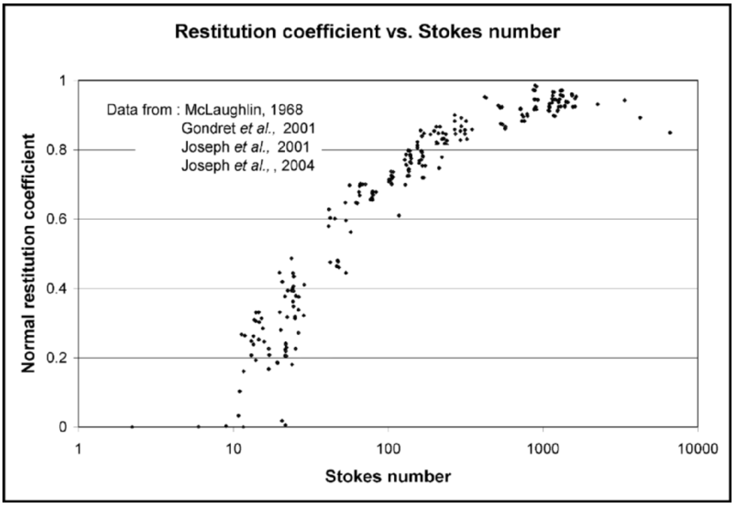

Figure 3. Effect of fluid on restitution coefficient.

It has been shown by Jan and Chens (1997) and Chhabra and Ferreira (1999) that for a sphere rolling down a smooth plane, drag coefficient in the supercritical regime (C_D) is about 0.74. For the SDC, C_D is about 0.45. Unfortunately, no data seems to exist for critical and transcritical regimes. Drag coefficient is much larger near a wall than in the freefall case.

Jan and Chens (1997) have also studied the effect of wall proximity for added mass. They have shown that for a sphere rolling down an incline added mass coefficient is larger than that in the freefall. $C_a = 2$ have better agreement with their experimental data.

3. Governing equations

Underwater rockfalls is a multi-physic problem involving a moving rigid body in a Newtonian fluid. Body motion is due to a constant force: gravity and hydrodynamic forces. Contact forces also have to be taken into account. We have shown that no hydrodynamic forces are constant throughout the same problem. They are function of moving body speed, rotation, proximity to a wall, etc.

3.1 FREEFALL MOVEMENT

Governing equation for the spherical body will be expressed as (forces balance and moment balance) (Glowinski *et al.*, 1999):

$$M\frac{dV}{dt} = Mg + F_h + F_c \qquad I\frac{d\omega}{dt} = T_h \qquad \frac{dG}{dt} = V \qquad \text{(8), (9) and (10)}$$

Where V is the rigid body speed, M the body mass, T the hydrodynamic moment, G the center of mass and ω the angular velocity. F_h is hydrodynamic forces (lift force, drag force and added mass force) and F_c is contact force.

For the surrounding fluid, governing equations are Navier-Stokes equations and continuity equation:

$$\rho_f\left(\frac{\partial u}{\partial t}+u\cdot\nabla u\right)=\rho_f g+\nabla\cdot\sigma \qquad \nabla\cdot u=0 \qquad \text{(11) and (12)}$$

Where σ is the stress tensor given by:

$$\sigma=-pI+\mu_f\left[\nabla u+\left(\nabla u\right)^T\right] \qquad (13)$$

Where p is pressure, I is identity matrix and μ_f is fluid viscosity. To model the problem, we must set boundary conditions on the moving block and on the wall. This condition is called "no-slip condition". Fluid is moving at the same speed than the solid on each boundary.

Hydrodynamic forces and moment are defined as (Glowinski *et al.,* 1999):

$$F_h=\int_\Gamma \sigma n \mathrm{d}\gamma \qquad T_h=\int_\Gamma (x-G)\times(\sigma n)\mathrm{d}\gamma \qquad \text{(14) and (15)}$$

n is a unit vector normal to Γ. Presence of water current can be added to this model by imposing an initial water speed.

By coupling Navier-Stokes and motion equations, it is possible, at each instant, to numerically calculate hydrodynamic forces acting on the moving rigid body. It also allows using any kind of block geometry (while modifying *eq.* 9). Coupling these equations have already been done for simulating particle-fluid system such as fluid-induced erosive failures or debris flows, but have never been done for underwater rockfall analysis. Such an analysis is complicated by the fact that fluid motion neither block motion is known a priori, because block motion influence fluid motion and *vice-versa*.

3.2 BOUNCING AND ROLLING MOTION

Contact forces have to be seen as independent of the freefall case. When distance between the moving body and a wall approach block radius, new velocities will be computed to take into account contact forces. CRSP formulation will be modified to consider Stokes number. After collision, if normal speed after the collision is small compared to tangential speed, the block will be rolling (or sliding). Otherwise, a new freefall trajectory will be calculated.

When the block is rolling or sliding on or near a wall, another set of equations will have to be developed to take into consideration friction coefficient of the wall and EHD theory. If a rebound occurs because of a change in the shape of the wall, then a new freefall part of motion will be computed.

4. Discussion

In their work, Beranger *et al.* (1998) have assumed constant drag and mass coefficients: 0.4 and 0.5, respectively. This seems to be a good approximation for boulders in freefall subcritical regime. In our paper, it can be seen that for underwater rockfall, where expected *Re* number will be over 10^5, drag coefficient will be less than 0.5 and will

increase with speed. If rotation is not neglected, Magnus effect should play an important role in the motion. When the block is near a wall two different forces will interact. Added mass force will be greater, so block acceleration or deceleration will decrease because of fluid momentum. In addition, drag coefficient will increase as well as lift coefficient.
Equations 8 to 13 can be joined via *eq*. 14 and 15. To solve this system, with specified boundary conditions, two main different methods can be used: Arbitrary Lagrange-Euler method or Distributed Lagrange multiplier. The last one seems to be the best one to use in our case, if we want to optimize computation time (Carlson *et al*., 2004). Carlson *et al.* (2004) used finite differences to solve these equations, which is easier to implement than finite elements. Distributed Lagrange Multiplier method takes the solid as a rigid fluid (Patankar *et al.,* 2000; Patankar, 2001). In our case, a turbulent model for the fluid will have to be added to account for high Re number flow.

5. Concluding remarks

It has been shown that for underwater rockfalls effect of ambient fluid cannot be neglected. The shape of the block, speed, and presence of a wall control hydrodynamic constraints. Presence of the fluid also controls bouncing motion and friction coefficient. Lift force, drag force, and added mass force are all non-constant forces. It seems necessary, in order to model behaviour of underwater rockfall, to join block equations of motion (*eq*. 8 and *eq*. 9) with Navier-Stokes (*eq*. 11) and continuity equations (*eq*. 12), and to modify collision equations with Stokes number dependence. A turbulence model will have to be added to Navier-Stokes equations. It has not been done so far for submarine rockfall analysis, but it will be the next step of this work.

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