IMPACT SYSTEMS WITH UNCERTAINTY

W. Schiehlen, R. Seifried
Institute of Engineering and Computational Mechanics, University of Stuttgart, 70569 Stuttgart, Germany

Abstract: The coefficient of restitution is mostly required for impact analysis in multibody dynamics. Using a multiscale simulation approach the coefficient can be computed on a fast time scale. Thereby modal models with local contact models proof to be efficient and accurate models for the simulations on the fast time scale. For many impact systems the coefficient of restitution is assumed to be deterministic, depending on essential parameters such as material, shape and initial collision velocity. In this paper impacts on beams are investigated numerically and experimentally. The investigated beam impacts feature multiple impacts, resulting in an uncertainty for the coefficient of restitution.

Key words: Multibody systems, multiscale simulation, multiple impacts, beam, experiments, coefficient of restitution, uncertainty.

1. INTRODUCTION

Impacts occur in passive mechanical systems constraint by bearing with clearance, and in actively controlled mechanical systems like robots with colliding links. Such mechanical systems are often modeled as multibody systems to describe large nonlinear motions, and the impacts are treated by the coefficient of restitution, see e.g. Pfeiffer and Glocker **[7]** and Stronge **[19]**. The coefficient of restitution is considered as deterministic number depending on the material, the shape and the velocity of the colliding bodies see e.g. Goldsmith **[3]**. However, in experiments and simulations it was observed that for a sphere striking a beam the coefficient of restitution is uncertain due to multiple impacts resulting in random behavior.

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2. IMPACTS IN MULTIBODY SYSTEMS

The method of multibody systems allows the dynamical analysis of machines and structures, see References **[8-10]**. More recently contact and impact problems featuring unilateral constraints were considered too, see Pfeiffer and Glocker **[7]**. A multibody system is represented by its equations of motion as

$$\mathbf{M}(\mathbf{y})\ddot{\mathbf{y}} + \mathbf{k}(\mathbf{y},\dot{\mathbf{y}}) = \mathbf{q}(\mathbf{y},\dot{\mathbf{y}}), \tag{1}$$

where $\mathbf{y}(t)$ is the global position vector featuring f generalized coordinates, $\mathbf{M}$ the inertia matrix, $\mathbf{k}$ the vector of Coriolis and gyroscopic forces and $\mathbf{q}$ the vector of the applied forces. The continuous motion of the multibody system might be interrupted by collision. Collisions with non-zero relative velocity result in impacts and impact modeling is required.

Using the instantaneous impact modeling the motion of the multibody system is divided into two periods with different initial conditions, see e.g. Glocker **[2]**, Pfeiffer and Glocker **[7]** or Eberhard **[1]**. During impact the equations of motion (1) have to be extended by the impact force F which is assumed to act in normal direction to the impact points,

$$\mathbf{M}(\mathbf{y})\ddot{\mathbf{y}} + \mathbf{k}(\mathbf{y},\dot{\mathbf{y}}) = \mathbf{q}(\mathbf{y},\dot{\mathbf{y}}) + \mathbf{w}_N\, F. \tag{2}$$

The vector $\mathbf{w}_N$ projects the impact force from the normal direction of the impact on the direction of the generalized coordinates. Due to the assumption of infinitesimal impact duration, the velocity changes in a jump, whereas the position remains unchanged. The equation of motion during impact is then formulated on velocity level,

$$\lim_{t_e \to t_s} \int_{t_s}^{t_e} \left(\mathbf{M}\ddot{\mathbf{y}} + \mathbf{k} - \mathbf{q} - \mathbf{w}_N\, F\right) dt = \mathbf{M}\left(\dot{\mathbf{y}}_e - \dot{\mathbf{y}}_s\right) - \mathbf{w}_N\, \Delta P = \mathbf{0}, \tag{3}$$

where the indices s and e mark the start and end of the impact, respectively. In the limit case $t_e \to t_s$ the quantities $\mathbf{M}$ and $\mathbf{w}_N$ are constant and all but the impact forces vanish due to their limited amplitudes. However, the infinitely large impact force F yields a finite force impulse ΔP which results in the jump of the generalized velocities and the non-smooth behavior. The impact force F and, therefore, the impulse ΔP are still unknown. The coefficient of restitution e provides additional information for the assessment of the impulse. Using the kinetic coefficient of restitution due to Poisson, the impact duration is divided into a compression and a restitution phase. The compression phase starts at time t_s and ends with time t_c, which is marked by the vanishing relative normal velocity. The restitution phase starts at time t_c and ends at t_e. The kinetic coefficient of restitution is defined as the ratio of the impulses ΔP_c and ΔP_r during the compression and restitution of the impact, respectively. An impact with e=1 is called elastic and indicates no energy loss, whereas an impact with e=0 is called plastic or inelastic and indicates maximal energy loss, resulting in a permanent contact. However, it should be noted, that the terms 'elastic' and 'plastic' describe here only the impact behavior and have little to do with the material behavior. As shown in Reference **[13, 17]** the impulse during the compression phase reads as

$$\Delta P_c = \frac{-\dot{g}_{Ns}}{\mathbf{w}_N^T \mathbf{M}^{-1} \mathbf{w}_N} \tag{4}$$

where $\dot{g}_{Ns}$ is the relative normal velocity of the contact points before impact. The total impulse during impact follows as

$$\Delta P = \Delta P_c + \Delta P_r = (1+e)\Delta P_c \tag{5}$$

and using Equation (3) the generalized velocities after impact $\dot{\mathbf{y}}_e$ can be computed. In the case of more than one impact occurring simultaneously or a permanent contact opening due to impact, respectively, the corresponding equations have to be solved simultaneously resulting in linear complementarity problems (LCPs), see Pfeiffer and Glocker **[7]**.

The impact modeling using Poisson's coefficient of restitution is a very efficient method for treating impacts in multibody systems if the coefficient of restitution is known. The coefficient of restitution is usually found by experiments or it is known from experience. However, the coefficient of restitution may be evaluated numerically by additional simulations on a fast time scale, too, see References **[11-13]**. This results in a multiscale simulation approach. The simulation on the slow time scale is interrupted by an impact. Then, for the impact, a detailed simulation with deformable bodies is performed on a fast time scale including elastodynamic wave propagation and elastic-plastic material phenomena. The generalized coordinates and velocities before impact are used as initial conditions for the simulations on the fast time scale. These simulations are limited to the impact duration and from the time-continuous impact force F the resulting impulse ΔP is computed and the kinetic coefficient of restitution follows as

$$e = \frac{\Delta P_r}{\Delta P_c} = \frac{\Delta P - \Delta P_c}{\Delta P_c} = -\frac{\mathbf{w}_N^T \mathbf{M}^{-1} \mathbf{w}_N}{\dot{g}_{Ns}} \Delta P - 1, \tag{6}$$

see References **[13, 17]** for more details. The coefficient of restitution is now fed back to the slow time scale. Then, the generalized velocities $\dot{\mathbf{y}}_e$ after impact are computed using Equation (3-5).

3. NUMERICAL MODELS

The computation on the fast time scale requires numerical models which include wave propagation within the bodies, and elastic or elastic-plastic deformation of the contact region. First of all, a complete Finite Element (FE) model of the impacting bodies is used. A small overall element length is required to comprise the wave propagation in the bodies and an additional refinement is necessary for the modeling of the contact region, see Reference **[14]**. Thus, FE-models for impact analysis are excessively time consuming and not suitable for larger impact systems as found in engineering. Therefore, in a more time efficient numerical approach, impact processes are divided into two parts, a small contact region and the remaining body featuring wave propagation, see Reference **[13, 17]**. This procedure is also called boundary approach. The contact is a nonlinear problem which is limited to a small region, while the wave propagation is a linear problem encompassing the entire body. Thus, combined models are developed in which the elastodynamic behavior of the impacting bodies is represented by a modally reduced model and the deformation of the contact region is presented by a local contact model based on FE-models of the contact region. The local contact model is than either concurrently computed or pre-computed and then coupled with the reduced elastodynamic model of the impacting bodies, see References **[12, 13, 15-17]**.

The efficiency and consistency of the combined models is demonstrated for the impact of a steel sphere (radius=15mm) on aluminum rods (radius=10mm, length=1000mm) with initial velocity of 0.3 m/s. The rods have elastic and elastic-plastic material behavior, respectively. The computed coefficients of restitution and computation times are summarized in Table 1. It turns out clearly that the simulation results obtained from the different models agree very well. It is also obvious that the completely nonlinear FE model is very time consuming, especially when including elastic-plastic material behavior. Using a modal model with concurrently computed FE-contact the computation time is reduced by 40-60%. A tremendous decrease in the computation time is achieved using the modal model with pre-computed FE contact. However, it should be noted that the pre-computation of the force-deformation diagram is time consuming, too, especially for elasto-plastic material behavior. The computation time corresponds to about 15 impact simulations with the nonlinear FE model.

Table 1. Comparison of numerical models for sphere to rod impact

	coeff. of restitution		computation time [s]	
model	elastic	plastic	elastic	plastic
A. complete nonlinear FE-model	0.633	0.481	462	937
B. modal model+concurrently computed FE-contact	0.631	0.477	285	354
C. modal model+pre-computed FE-contact	0.632	0.477	0.04	0.05

Therefore, the benefit of the modal model with pre-computed FE-contact takes place especially when many impacts are investigated.

4. ESSENTIAL PARAMETERS FOR THE COEFFICIENT OF RESTITUTION

The coefficient of restitution depends not only on the material parameters but also strongly on the contact geometry, the body geometry and the initial velocity. Early experimental results for the evaluation of the coefficient of restitution are summarized in Goldsmith **[3]**, more recent numerical and experimental results are presented in Minamoto **[6]**, Sondergard **[18]**, Wu et al. **[20]**, Zhang et al. **[21]** and References **[11-17]**.

In Figure 1 the influence of the material properties and the initial velocity on the coefficient of restitution is presented for the impact of a steel sphere (radius 15mm) on two different aluminum rods (radius=10mm, length=1000mm). Rod 1 has a low yield stress of 205Mpa and rod 2 has a high yield stress of 575Mpa. The sphere has an initial velocity in the range of 0.05-0.5m/s, the rods are initially in rest. For the experimental evaluation a test bench with two Laser-Doppler-Vibrometer is used, see Hu et al. **[4, 5]**.

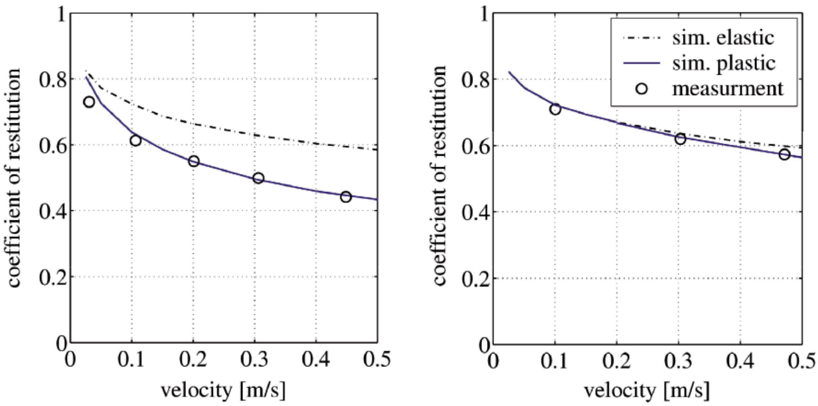

Figure 1. Impact of a hard steel sphere on two aluminum rods (left: low yield stress 205MPa, right: high yield stress 575MPa).

It is clearly seen from simulations and experiments that for both impact systems the coefficient of restitution decreases with increasing initial velocity. For rod 1 the measured coefficients and the ones obtained from simulations with elastic-plastic material behavior agree very well. However they are significantly lower than coefficients obtained from simulations with purely elastic material behavior. For rod 2, which has a high yield stress, simulations with elastic and elastic-plastic material behavior show for the investigated velocity range nearly identical behavior and agree well with experimental results. In References **[13, 15]** the influence of plastification on the coefficient of restitution for repeated impacts is investigated for both rods.

The influence of the shape of the bodies on the coefficient of restitution is investigated in Reference **[11]** for the impact of a steel sphere on four elastic aluminum bodies with equal mass but different shape. These are a compact cylinder, a half-circular plate, a long rod and a slender beam. Figure 2 shows the computed coefficients of restitution of these impact systems for the velocity range 0.025-0.5m/s. The computed coefficient of restitution for the cylinder is close to e=1 for the investigated velocity range. For the impact on the cylinder the transformation of initial kinetic energy into waves and vibrations can be neglected. From the simulations for the rod and half-circular plate it is seen that the coefficient of restitution decreases steadily with increasing initial velocity. This indicates an increase of energy transformation from the initial rigid body motion into waves and vibrations with increasing velocity. The transverse impact on the beam excited very strong vibration phenomena in the beam resulting in multiple successive impacts within a very short time period. In sharp contrast to the previous impact systems the beam impact shows no clear pattern but a strong uncertainty, see also Reference **[17]**.

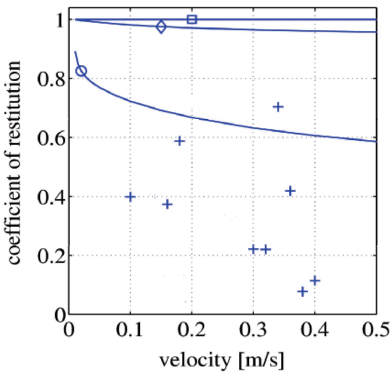

Figure 2. Impact of a hard steel sphere on differently shaped aluminum bodies (□ compact cylinder, ◊ half circular plate, ○ rod, + beam).

5. UNCERTAINTY OF THE COEFFEICIENT OF RESTITUTION

The impact on a beam features multiple impacts which are caused by the strong bending vibrations of the beam, resulting from the first impact. The multiple impacts are the source of the uncertainty of the coefficient of restitution. Since more than one successive impact occur within a short time period efficient numerical methods for impact simulation on the fast time are even more important than for single impacts.

5.1 Comparison of Numerical Models

A comparison of the simulation results using the different numerical models is presented in Figure 3 for the impact of a steel sphere (radius=15mm) with exactly the same initial velocity 0.2m/s on an elastic aluminum beam (radius=10mm, length=1000m). After the first impact the sphere still moves forward in its initial direction until a successive second impact occurs. This overall behavior is consistently observed in all simulations using the three different numerical models and shows the good overall agreement of the models. Moreover, it proves that the uncertainty is not a numerical problem.

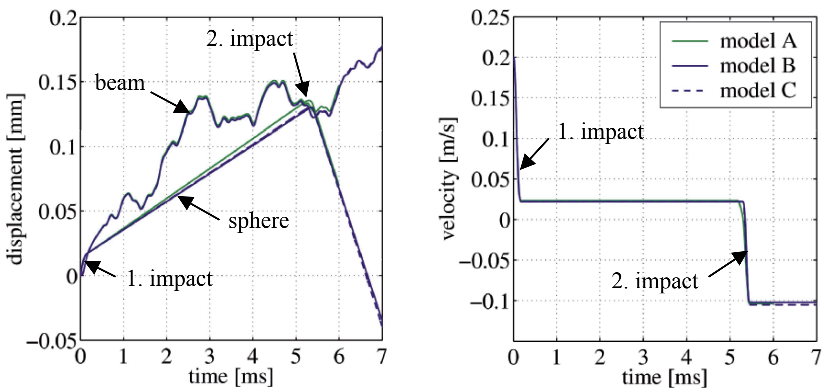

Figure 3. Comparison of numerical models for beam impact (A: complete FEM, B: modal+concurrently computed FE-contact, C: modal+pre-computed FE-contact.

Table 2 summarizes the coefficients of restitution and computation times of the simulations. This shows again the good agreement of the modal models with FE-contact and the complete FE-model. It turns out that the complete FE-model is very time consuming. By using modal models the computation times can be reduced significantly. Using the modal model with concurrently computed FE-contact the computation time can be reduced by 97%.

Using the modal model with pre-computed FE-contact the computation time can be reduced further, however the computation time for the force-displacement diagram has to be considered, which takes in this case about 1000s. This shows clearly, that for a larger and complex impact system, such as the transverse impact on a beam, the modal model with pre-computed FE-contact is the most efficient approach.

Table 2. Comparison of numerical models for sphere to beam impact

model	coeff. of restitution	computation time [s]
A. complete nonlinear FE-model	0.707	80564
B. modal model+concurrently computed FE-contact	0.700	2422
C. modal model+pre-computed FE-contact	0.717	16

5.2 Experimental validation

For the experimental validation of the simulation results an experimental setup, originally developed by Hu et al. **[4, 5]**, was adapted to beam impacts, see Figure 4. The sphere and beam are suspended with thin Kevlar wires in a frame as pendula. The sphere is released by a magnet from a predefine height and it impacts on the beam along its symmetry line. Two Laser-Doppler-Vibrometers are used for displacement and velocity measurement of sphere and beam in the central line of impact.

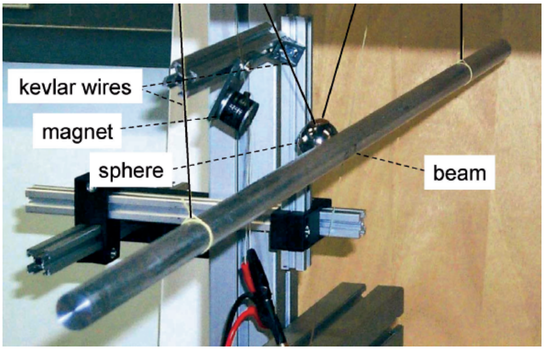

Figure 4. Experimental setup for sphere to beam impact.

Figure 5 shows for the three initial velocities v=0.276m/s, v=0.287m/s and v=0.303m/s the measured and simulated displacement of sphere and beam, as well as the velocity of the sphere. The measurement and simulation show for all three initial velocities, that within a few milliseconds several impacts occur. Although the initial velocities chosen are close together, the impact response is quite different which is due to the multiple impacts.

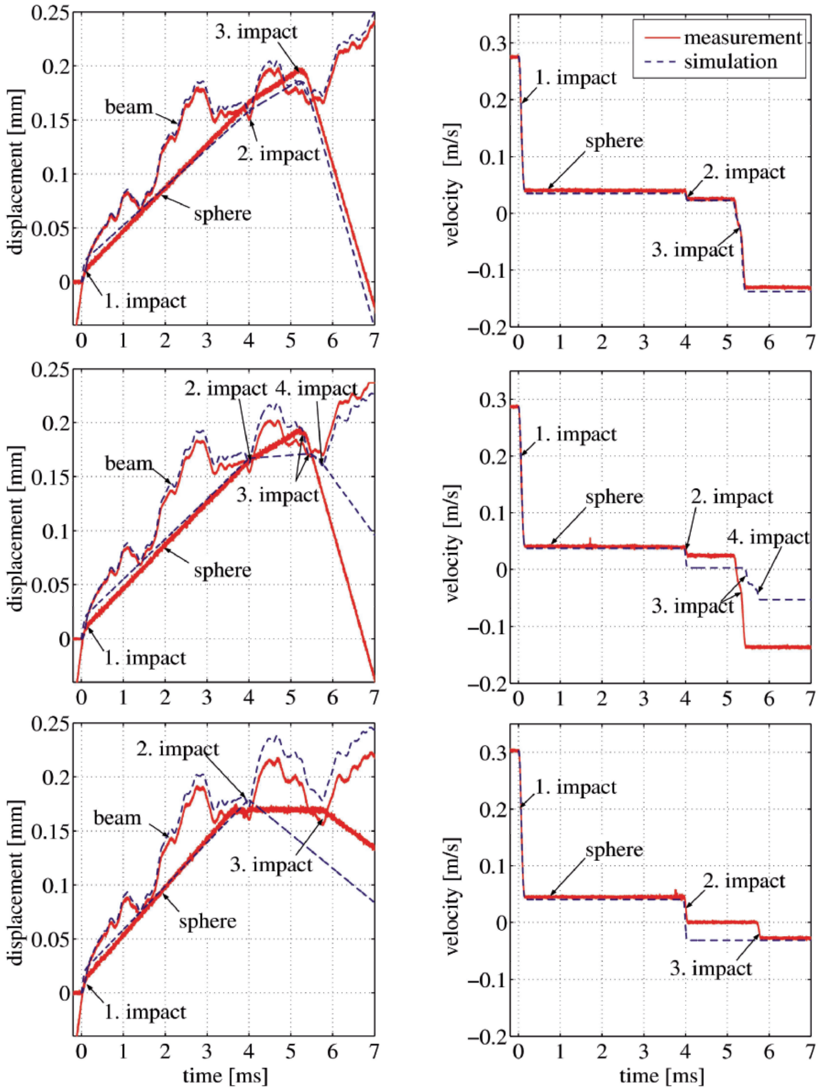

Figure 5. Impact on beam with initial velocity v=0.276m/s (top), v=0.287m/s (middle) and v =0.303 m/s (bottom).

Figure 5 shows for all three velocities a very good agreement for the first impact as well as consistently a second impact after 4 ms. However, for the successive impacts significant differences occur resulting in an overall uncertainty.

For the impact with an initial velocity v = 0.276 m/s the second impact yield only to a small velocity change. Therefore, after 5.2 ms a third impact occurs, which results in a large velocity change of the sphere. In this case experiment and simulation agree very well. This is also reflected by the good agreement of the measured and simulated coefficients of restitution which are $e_m=0.664$ and $e_s=0.687$, respectively.

The impact with the initial velocity v=0.287 m/s shows in the simulation a much stronger second impact than in the experiment. This results in a very different behavior of the following motion. Consequently the coefficient of restitution computed from measurement and simulations differ strongly and are $e_m=0.620$ and $e_s=0.334$.

For the impact with initial velocity v=0.303 m/s the experiment proves that sphere is in rest after the second impact and a third impact occurs after 5.7 ms. In the simulation the second impact is stronger as the one in the experiment. Thereby the sphere rebounds and no further impact occurs in the simulation. Measurement and simulation yield hereby nearly identical coefficients of restitution of $e_m=0.230$ and $e_s=0.243$.

5.3 Analysis of the coefficient of restitution

In Figure 6 simulated and measured coefficients of restitution are presented for 53 different initial velocities of the sphere. Due to the multiple impacts the coefficient of restitution depends strongly on the initial velocity, however, without showing a clear pattern but strong uncertainty, see Reference **[17]**. The coefficients of restitution are in the range e=0.07-0.73. Small differences of the simulated and measured motion of beam and sphere after the first impact result in very different behavior of the successive impacts. As a result, the investigated impacts show significant differences of the measured and simulated coefficients of restitution, for different initial velocities.

For the simulated and measured impacts presented in Figure 6 the mean value of the initial velocity of the sphere is $\bar{v}=0.25$ and the standard deviation is $\sigma_v=0.0929$. The mean value of the simulated coefficients is $\bar{e}_s=0.3981$ and the standard deviation is $\sigma_s=0.2275$. This is in good accordance with the measured coefficients of restitution which have a mean value of $\bar{e}_m=0.3800$ and a standard deviation of $\sigma_m=0.2125$. This statistical analysis shows that although large deviations between measured and simulated impacts may occur, the overall behavior is represented accurately by the numerical models. In Figure 6 the mean value of the measurements and deviation intervals are added. Thereby the areas A-D corresponds to the intervals represented by the mean values and the deviations $0.5\sigma, \sigma, 1.5\sigma, 2\sigma$, respectively. However, it turns out that using this statistical approach the interval D, defined by $e=\bar{e}_m \pm 2\sigma_m$, includes nonphysical negative values for the coefficient of restitution. This shows that mechanical aspects and the simple statistical evaluation of the coefficient of restitution are contradicting for this uncertain mechanical system.

In the right plot of Figure 6 the numbers of multiple impacts are indicated for simulation and measurements. It turns out that only for very low velocities one impact occur. For higher velocities 2, 3 or 4 successive impacts occur, however no relationship between the coefficient of restitution and the number of multiple impacts is obvious.

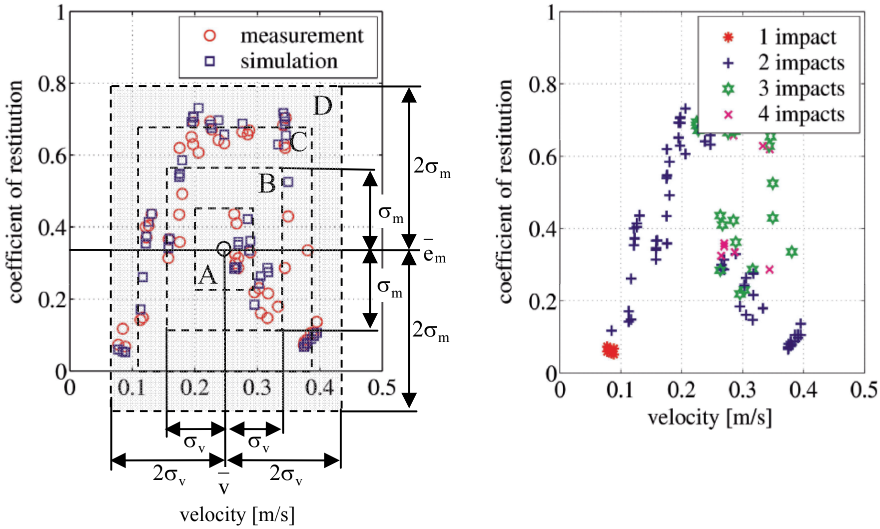

Figure 6. Multiple impacts on an elastic aluminum beam.

6. CONCLUSION

Measurements and simulations for the transverse impact of a steel sphere on an aluminum beam show multiple successive impacts within a very short time period, resulting in an uncertain behavior of the coefficient of restitution. For the evaluation of the numerical and experimental data a probabilistic approach using mean value and variance of the coefficient of restitution shows good overall agreement of simulation and measurement. However a simple statistical approach for describing the coefficient of restitution has its limitations in overcoming its uncertainty.

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