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Embedding Deformed Minkowski Space in a 5D Riemann Space

19.1 From LLI Breakdown to Energy as Fifth Dimension

Both the analysis of the physical processes considered in deriving the phenomenological energy-dependent metrics for the four fundamental interactions, and the experiments discussed in Part III, seem to provide evidence (indirect and direct, respectively) for a breakdown of LLI invariance (at least in its usual, special-relativistic sense). But it is well known that, in general, the breakdown of a symmetry is the signature of the need for a *wider, exact* symmetry. In the case of the breaking of a space–time symmetry – as the Lorentz one – this is often related to the possible occurrence of higher-dimensional schemes. It will be shown that this is indeed the case, and that *energy does in fact represent an extra dimension*.

In the description of interactions by energy-dependent metrics, we saw that energy plays in fact a *dual* role. On one side, as more and more stressed, it constitutes a dynamic variable. On the other hand, it represents a parameter characteristic of the phenomenon considered (and therefore, for a given process, it cannot be changed at will). In other words, when describing a given process, the deformed geometry of space–time (in the interaction region where the process is occurring) is "frozen" at the situation described by those values of the metric coefficients $\left\{b_{\mu}^{2}(E)\right\}_{\mu=0,1,2,3}$ corresponding to the energy value of the process considered. Namely, a fixed value of E determines the space–time structure of the interaction region at that given energy. In this respect, therefore, the energy of the process has to be considered as *a geometrical quantity* intimately related to the very geometrical structure of the physical world. In other words, from a geometrical point of view, all goes on as if were actually working on “slices” (sections) of a 5D space, in which the extra dimension is just represented by the energy. Then, the 4D, deformed, energy-dependent space–time is just a manifestation (or a “shadow,” to use the famous word of Minkowski) of a larger space with energy as fifth dimension.

The simplest way to take account of (and to make explicit) the double role of energy in DSR is assuming that E represents an extra metric dimension – on the same footing of space and time – and therefore embedding the 4D deformed Minkowski space $\widetilde{M}(E)$ of DSR in a 5D (Riemann) space $\Re_5$. This leads to build up a “Kaluza–Klein-like” scheme, with energy as fifth dimension, we shall refer to in the following as *5D Deformed Relativity* (DR5) [6,130,131].

Let us recall that the use of momentum components as metric variables on the same foot of the space–time ones can be traced back to Ingraham [117]. On the contrary, it was just shown by Lee that time (namely, a space–time coordinate) can be used as a (discrete) dynamic variable [132]. Moreover, many authors (starting from Dirac [133]) treated mass as a dynamic variable in the context of scale-invariant theories of gravity [134,135]. Such a point of view has been advocated also in the framework of modern Kaluza–Klein theories by the already quoted “Space–Time–Mass” (STM) theory [123].

It is worth stressing that, apart from the previous considerations, we already ran across some clues of a possible 5D structure underlying DSR. One such an indication is provided e.g., by generalized energy-momentum dispersion law (3.100), which – as already stressed – is typical of multidimensional theories. Another one is provided by the form of the phenomenological metric of strong interaction (see Sect. 4.1.3), in particular expressions (4.12), (4.13) of the space coefficients $b_{2,\text{strong}} = \sqrt{2}/5$ and $b_{3,\text{strong}} = 2/5$. Indeed, the 5 at the denominators are reminiscent of the same factor entering the relation between the Ricci tensor and the scalar curvature in a 5D Riemann space, $R_{AB} = (R/5)g_{AB}$, with g_{AB} being the 5D metric tensor.[1] Another clue is the interpretation of the

[1] In fact, let us consider the vacuum Einstein equations with a cosmological constant Λ in a N-dimensional Riemann space:

$$R_{AB} - \frac{1}{2}Rg_{AB} = \Lambda g_{AB}.$$

By contracting on A, B and using the well-known property $g^{DA}g_{AB} = \delta^D_B$, one gets

$$R = \frac{2n}{2-n}\Lambda.$$

Then

$$R_{AB} = \left(\frac{1}{2}R + \Lambda\right) g_{AB} = \frac{R}{n} g_{AB}$$

(M. Mamone Capria, private communication).

hadronic law of time deformation, (17.3), as a relation of power conservation, $W = const.$ (needed to explain the mechanism of piezonuclear reactions: see Sect. 16.3.5). As already stressed, in a 5D optics this means moving along the extra dimension energy at constant speed (namely it amounts to a principle of inertia for energy). Another possible experimental inkling of the fifth dimension can be found in the double-slit-like experiments (Chap. 13). Indeed, we have seen that, in order to put the anomalous interference effect into evidence, it is necessary to employ a suitable time sampling of the measurements. On account of the fact that the phenomenon has a threshold behavior both in space and in energy, we can state it to occur in a well-defined space–time–energy, 5D region. Moreover, the crucial dependence on the time sampling can be interpreted as follows. As is well known, a way to realize one lives on a curved manifold is by means of the geodesic deviation. For instance, on Earth surface, moving from Equator along two meridians shows that the meridian separation decreases, thus implying Earth surface is curved (Wheeler's "parable of the two travelers"). However, the travelers are able to discern the decrease of their relative separation only if they move an appreciable distance (compared to the Earth radius of curvature). Otherwise, no separation is seen and they remain convinced that Earth is flat. In our opinion, the anomalous interference effect is not only related to the deformation of space–time (and therefore to the breakdown of LLI), but also to the Gaussian curvature of the 5D space–time–energy manifold $\Re_5$. Selecting the suitable time sampling amounts therefore to choose the time magnitude scale necessary to detect the curvature of the 5D region in which the anomalous effect shows up.

19.2 The 5D Space–Time–Energy Manifold $\Re_5$

On the basis of the arguments of Sect. 19.1, we assume therefore that physical phenomena do occur in a world which is actually described by a 5D space–time–energy manifold $\Re_5$ endowed with the energy-dependent metric:[2]

$$g_{AB,\mathrm{DR5}}(E) \equiv \mathrm{diag}(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E), f(E)) \overset{\mathrm{ESC\ off}}{=}$$

$$= \delta_{AB} \left(b_0^2(E)\delta_{A0} - b_1^2(E)\delta_{A1} - b_2^2(E)\delta_{A2} - b_3^2(E)\delta_{A3} + f(E)\delta_{A5} \right) . \tag{19.1}$$

[2]In the following, capital Latin indices take values in the range $\{0, 1, 2, 3, 5\}$, with index 5 labeling the fifth dimension. We choose to label by 5 the extra coordinate, instead of using 4, in order to avoid confusion with the notation often adopted for the (imaginary) time coordinate in a (formally) Euclidean Minkowski space.

It follows from (19.1) that E, which is an independent *nonmetric* variable in DSR, becomes a *metric* coordinate in $\Re_5$. Then, whereas $g_{\mu\nu,\mathrm{DSR}}(E)$ (given by (2.17)) is a deformed, Minkowskian metric tensor, $g_{AB,\mathrm{DR5}}(E)$ is a genuine Riemannian metric tensor.

Therefore, the infinitesimal interval of $\Re_5$ is given by

$$\begin{aligned} &\mathrm{d}s^2_{\mathrm{DR5}}(E) \equiv \mathrm{d}S^2(E) \equiv g_{AB,\mathrm{DR5}}(E)\mathrm{d}x^A\mathrm{d}x^B \\ &= b_0^2(E)\left(\mathrm{d}x^0\right)^2 - b_1^2(E)\left(\mathrm{d}x^1\right)^2 - b_2^2(E)\left(\mathrm{d}x^2\right)^2 - b_3^2(E)\left(\mathrm{d}x^3\right)^2 + f(E)\left(\mathrm{d}x^5\right)^2 \\ &= b_0^2(E)c^2\left(\mathrm{d}t\right)^2 - b_1^2(E)\left(\mathrm{d}x^1\right)^2 - b_2^2(E)\left(\mathrm{d}x^2\right)^2 - b_3^2(E)\left(\mathrm{d}x^3\right)^2 + f(E)l_0^2\left(\mathrm{d}E\right)^2, \end{aligned} \tag{19.2}$$

where we have put

$$x^5 \equiv l_0 E \ , \ l_0 > 0. \tag{19.3}$$

The constant l_0 provides the dimensional conversion energy→ length, and it has therefore the dimensions of the inverse of a force. On physical grounds, it is expected to be a fundamental constant of DR5, so it is worth trying to guess a possible identification of l_0. Let us recall that in Sect. 4.2 we already came across a quantity built up by fundamental constants with dimensions of a force: the Kostro constant or Planck force $K = c^4/G$ (see (4.27), which can be interpreted as the greatest possible force in Nature [40]). Then, it is natural to assume

$$l_0 = \frac{1}{K} = \frac{G}{c^4} = \frac{1}{8\pi}\kappa, \tag{19.4}$$

where κ is the gravitational coupling constant of the usual, four dimensional Einstein equations $G_{\mu\nu} = \kappa T_{\mu\nu}$ (with $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $T_{\mu\nu}$ being the Einstein curvature tensor and the energy–momentum tensor, respectively). Therefore, identifying l_0 with the inverse of the Kostro constant has as consequence that it coincides with the gravitational constant κ apart from the numerical factor 8π (which however is essentially due to the choice of the unit system). As is well known, in General Relativity κ determines the effectiveness of the energy density of the source in deforming space–time and can be interpreted as the force per unit area required to give space–time a unit curvature.[3] If the identification (19.4) is correct, then l_0, and consequently κ, plays an analogous role in DR5, namely it is related in an essential way to the curvature of $\Re_5$ – which in turn reflects itself in the deformation of the 4D space–time $\widetilde{M}$ – *whatever the interaction involved.* In the framework of DR5, therefore, the gravitational constant rises, from mere coupling constant for the gravity only, to the role of *universal constant of deformation, valid for all interactions.*

Since the space–time metric coefficients are dimensionless, it can be assumed that they are functions of the ratio E/E_0, where E_0 is an energy

[3] Remember that curvature has dimensions l^{-2}.

scale characteristic of the interaction (and the process) considered (for instance, the energy threshold in the phenomenological metrics of Sect. 4.1). The coefficients $\{b_\mu^2(E)\}$ of the metric of $\widetilde{M}(E)$ can be therefore expressed as

$$\left\{b_\mu\left(\frac{E}{E_0}\right)\right\} \equiv \left\{b_\mu\left(\frac{x^5}{x_0^5}\right)\right\} = \{b_\mu(x^5)\} \quad \forall\mu = 0,1,2,3, \tag{19.5}$$

where we put

$$x_0^5 \equiv l_0 E_0 \ . \tag{19.6}$$

As to the fifth metric coefficient, one assumes that it too is a function of the energy only: $f = f(E) \equiv f(x^5)$ (although, in principle, nothing prevents from assuming that, in general, f may depend also on space–time coordinates $\{x^\mu\}$, $f = f(\{x^\mu\}, x^5)$). Unlike the other metric coefficients, it may be $f(E) \lessgtr 0$. Therefore, a priori, the energy dimension may have either a time-like or a space-like signature in $\Re_5$, depending on $\operatorname{sgn}(f(E)) = \pm 1$. In the following, it will be sometimes convenient assuming $f(E) \in R_0^+$ and explicitly introducing the double sign in front of the fifth coefficient.

In terms of x^5, the (*covariant*) metric tensor can be written as

$$\begin{aligned} g_{AB,\mathrm{DR5}}(x^5) &= \operatorname{diag}(b_0^2(x^5), -b_1^2(x^5), -b_2^2(x^5), -b_3^2(x^5), \pm f(x^5)) \\ &\stackrel{ESC\ off}{=} \delta_{AB}\left[b_0^2(x^5)\delta_{A0} - b_1^2(x^5)\delta_{A1} - b_2^2(x^5)\delta_{A2} - b_3^2(x^5)\delta_{A3} \pm f(x^5)\delta_{A5}\right]. \end{aligned} \tag{19.7}$$

On account of the relation

$$g_{\mathrm{DR5}}^{AB}(x^5) g_{BC,\mathrm{DR5}}(x^5) = \delta_C^A \ , \tag{19.8}$$

the *contravariant* metric tensor reads

$$\begin{aligned} g_{\mathrm{DR5}}^{AB}(x^5) &= \operatorname{diag}(b_0^{-2}(x^5), -b_1^{-2}(x^5), -b_2^{-2}(x^5), -b_3^{-2}(x^5), \pm\left(f(x^5)\right)^{-1}) \\ &\stackrel{ESC\ off}{=} \delta_{AB}\left[\begin{array}{c} b_0^{-2}(x^5)\delta_{A0} - b_1^{-2}(x^5)\delta_{A1} - b_2^{-2}(x^5)\delta_{A2} - b_3^{-2}(x^5)\delta_{A3} \\ \pm\left(f(x^5)\right)^{-1}\delta_{A5} \end{array}\right]. \end{aligned} \tag{19.9}$$

The space $\Re_5$ has the following “slicing property”

$$\Re_5\big|_{\mathrm{d}x^5=0 \Leftrightarrow x^5=\overline{x^5}} = \widetilde{M}(\overline{x^5}) = \left\{\widetilde{M}(x^5)\right\}_{x^5=\overline{x^5}} \tag{19.10}$$

(where $\overline{x^5}$ is a fixed value of the fifth coordinate) or, at the level of the metric tensor:

$$\begin{aligned} & g_{AB,\mathrm{DR5}}(x^5)\big|_{\mathrm{d}x^5=0 \Leftrightarrow x^5=\overline{x^5}\in R_0^+} \\ &= \operatorname{diag}\left(b_0^2(\overline{x^5}), -b_1^2(\overline{x^5}), -b_2^2(\overline{x^5}), -b_3^2(\overline{x^5}), \pm f(\overline{x^5})\right) = g_{AB,\mathrm{DSR}}(\overline{x^5}). \end{aligned} \tag{19.11}$$

We recall that in general, in the framework of 5D Kaluza–Klein (KK) theories, the fifth dimension must be necessarily space-like, since, in order to avoid the occurrence of causal (loop) anomalies, the number of time-like dimensions cannot be greater than one. But it is worth to stress that the present theory is not a Kaluza–Klein one. In "true" KK theories, due to the lack of observability of the extra dimensions, it is necessary to impose to them the cylindricity condition. This is not required in the framework of DR5, since the fifth dimension (energy) is a physically observable quantity (think to the Minkowski space of standard SR: There is no need to hide the fourth dimension, since time is an observable quantity). Actually, in DR5 not only the cylindricity condition is not implemented, but it is even reversed. In fact, the metric tensor $g_{AB,\mathrm{DR5}}(x^5)$ depends only on the fifth coordinate x^5. Therefore, one does not assume the compactification of the extra coordinate (one of the main methods of implementing the cylindricity condition in modern hyperdimensional KK theories, as discussed in Chap. 18), which remains therefore extended (i.e., with infinite compactification radius). The problem of the possible occurrence of causal anomalies in presence of more time-like dimensions is then left open in the "pseudo-Kaluza–Klein" context of DR5. This is reflected in the uncertainty in the sign of the energy metric coefficient $f(x^5)$. In particular, it cannot be excluded a priori that the signature of x^5 can change. This occurs whenever the function $f(x^5)$ does vanish for some energy values. As a consequence, in correspondence to the energy values which are zeros of $f(x^5)$, the metric $g_{AB,\mathrm{DR5}}(x^5)$ is *degenerate*.

DR5 belongs therefore to the class of noncompactified KK theories . Moreover, it has some connection with Wesson's STM theory [123]. Both in the DR5 formalism and in the STM theory (at least in its more recent developments) it is assumed that all metric coefficients do in general depend on the fifth coordinate. Such a feature distinguishes either models from true Kaluza–Klein theories. However, DR5 differs from the STM model – as well as from similar ones, like e.g., the Fukui STMC [126] – at least in the following main respects:

(1) Its physical motivations are based on the phenomenological analysis of Part I and on the experimental results of Part III, and therefore are not merely speculative.

(2) The fact of assuming *energy* (which is a true variable), and not rest mass (which instead is an invariant), as fifth dimension.[4]

(3) The *local* (and not *global*) nature of the 5D space $\Re_5$, whereby the energy-dependent deformation of the 4D space–time is assumed to provide a geometrical description of the interactions.

[4]In this respect, therefore, the DR5 formalism rensembles more the one due to Ingraham [117].

We want to stress that, in embedding the deformed Minkowski space $\widetilde{M}(x^5)$ in $\Re_5$, *energy does lose its character of dynamic parameter* (the role it plays in DSR), *by taking instead that of a true metrical coordinate,* on the same footing of the space–time ones. This has a number of basic implications. The first one is of geometrical nature, and is just the passage from a (flat) pseudoeuclidean metric to a genuine (curved) Riemannian one. The others consequences pertain to both symmetries and dynamics, as we shall see in this Part and in the next one. In such a change of role of energy, with the consequent passage from $\widetilde{M}(x^5)$ to $\Re_5$, some of the geometrical and dynamic features of DSR are lost, whereas others are still present and new properties appear. Among the former, we recall the basic one – valid at the slicing level $x^5 = const.$ ($dx^5 = 0$) – related to the Generalized Lagrange Space structure of $\widetilde{M}(x^5)$, which implies *the natural arising of gauge fields*, intimately related to the inner geometry of the deformed Minkowski space (see Part II). Let us also stress that, in the framework of $\Re_5$, the dependence of the metric coefficients on a true metric coordinate make them fully analogous to the gauge functions of non-abelian gauge theories, thus implementing DR5 as a metric gauge theory (in the sense specified in Sect. 4.4).

19.3 Phenomenological 5D Metrics of Fundamental Interactions

Let us now consider the 4D metrics of the deformed Minkowski spaces $\widetilde{M}(x^5)$ for the four fundamental interactions (electromagnetic, weak, strong, and gravitational) (see Sect. 4.1). In passing from the deformed, special-relativistic 4D framework of DSR to the general-relativistic 5D one of DR5 – geometrically corresponding to the embedding of the deformed 4D Minkowski spaces $\left\{\widetilde{M}(x^5)\right\}_{x^5 \in R_0^+}$ (where x^5 is a parameter) in the 5D Riemann space $\Re_5$ (where x^5 is a metric coordinate), in general the phenomenological metrics (4.2)–(4.3), (4.7)–(4.8), (4.10)–(4.13), and (4.17)–(4.18) take the following 5D form ($f(x^5) \in R_0^+ \ \forall x^5 \in R_0^+$):

$$
\begin{aligned}
&g_{AB,\mathrm{DR5,e.m.}}(x^5)\\
&= \mathrm{diag}\left(1, -\left\{1 + \widehat{\Theta}(x^5_{0,\mathrm{e.m.}} - x^5)\left[\left(\frac{x^5}{x^5_{0,\mathrm{e.m.}}}\right)^{1/3} - 1\right]\right\},\right.\\
&\quad -\left\{1 + \widehat{\Theta}(x^5_{0,\mathrm{e.m.}} - x^5)\left[\left(\frac{x^5}{x^5_{0,\mathrm{e.m.}}}\right)^{1/3} - 1\right]\right\},\\
&\left. -\left\{1 + \widehat{\Theta}(x^5_{0,\mathrm{e.m.}} - x^5)\left[\left(\frac{x^5}{x^5_{0,\mathrm{e.m.}}}\right)^{1/3} - 1\right]\right\}, \pm f(x^5)\right); \qquad (19.12)
\end{aligned}
$$

$$
\begin{aligned}
&g_{AB,\mathrm{DR5,weak}}(x^5)\\
&= \mathrm{diag}\left(1, -\left\{1+\widehat{\Theta}(x^5_{0,\mathrm{weak}}-x^5)\left[\left(\frac{x^5}{x^5_{0,\mathrm{weak}}}\right)^{1/3}-1\right]\right\},\right.\\
&\quad -\left\{1+\widehat{\Theta}(x^5_{0,\mathrm{weak}}-x^5)\left[\left(\frac{x^5}{x^5_{0,\mathrm{weak}}}\right)^{1/3}-1\right]\right\},\\
&\quad \left. -\left\{1+\widehat{\Theta}(x^5_{0,\mathrm{weak}}-x^5)\left[\left(\frac{x^5}{x^5_{0,\mathrm{weak}}}\right)^{1/3}-1\right]\right\}, \pm f(x^5)\right); \qquad (19.13)
\end{aligned}
$$

$$
\begin{aligned}
&g_{AB,\mathrm{DR5,strong}}(x^5)\\
&= \mathrm{diag}\left(1+\widehat{\Theta}(x^5-x^5_{0,\mathrm{strong}})\left[\left(\frac{x^5}{x^5_{0,\mathrm{strong}}}\right)^{2}-1\right], -\left(\frac{\sqrt{2}}{5}\right)^2,\right.\\
&\quad \left. -\left(\frac{2}{5}\right)^2, -\left\{1+\widehat{\Theta}(x^5-x^5_{0,\mathrm{strong}})\left[\left(\frac{x^5}{x^5_{0,\mathrm{strong}}}\right)^{2}-1\right]\right\}, \pm f(x^5)\right); \quad (19.14)
\end{aligned}
$$

$$
\begin{aligned}
&g_{AB,\mathrm{DR5,grav.}}(x^5)\\
&= \mathrm{diag}\left(1+\widehat{\Theta}(x^5-x^5_{0,\mathrm{grav.}})\left[\frac{1}{4}\left(1+\frac{x^5}{x^5_{0,\mathrm{grav.}}}\right)^{2}-1\right], -b^2_{1,\mathrm{grav.}}(x^5),\right.\\
&\quad \left. -b^2_{2,\mathrm{grav.}}(x^5), -\left\{1+\widehat{\Theta}(x^5-x^5_{0,\mathrm{grav.}})\left[\frac{1}{4}\left(1+\frac{x^5}{x^5_{0,\mathrm{grav.}}}\right)^{2}-1\right]\right\}, \pm f(x^5)\right). \\
&\qquad (19.15)
\end{aligned}
$$

As we are going to show, all the earlier metrics – derived on a mere phenomenological basis, from the experimental data on some physical phenomena ruled by the four fundamental interactions, at least as far as their space–time part is concerned – can be recovered as solutions of the vacuum Einstein equations in the 5D space $\Re_5$, natural covering of the deformed Minkowski space $\widetilde{M}(x^5)$.