13 The Shadow of Light: Lorentzian Violation of Electrodynamics in Photon Systems

The experiments on superluminal group propagation, analyzed in terms of the tools of DSR, provided evidence for a breakdown of local Lorentz invariance with a threshold both in energy and space (at least for the electromagnetic interaction). In order to confirm these results, we carried out new experiments explicitly designed to test them. Let us briefly discuss these experiments, together with their implications.

13.1 Double-Slit-Like Experiments

The experiments we performed were optical ones, in the infrared range, of the double-slit type. We were essentially aimed at searching for a possible anomalous photon behavior, at variance with the predictions of classical and/or quantum electrodynamics, and therefore related to Lorentz invariance violation. Let us briefly report the main features and results of these three experiments, carried out at L'Aquila University [63–66].

The employed apparatus (schematically depicted in Fig. 13.1) consisted of a Plexiglas box with wooden base and lid. The box (thoroughly screened from those frequencies susceptible of affecting the measurements) contained two identical infrared (IR) LEDs, as (incoherent) sources of light, and three identical photodiodes, as detectors (A, B, C). The two sources S_1 , S_2 were placed in front of a screen with three circular apertures F_1 , F_2 , F_3 on it. The apertures F_1 and F_3 were lined up with the two LEDs A and C respectively, so that each IR beam propagated perpendicularly through each of them.



FIGURE 13.1. Above view of the experimental apparatus used in the first double-slit experiment

The geometry of this equipment was designed so that no photon could pass through aperture F_2 on the screen.¹ Let us stress that the dimensions of the apparatus were inferred from the geometrical size of the Florence microwave experiment [35], namely the horizontal distance between the planes of the antennas (see Fig. 12.2).

The wavelength of the two photon sources was $\lambda = 8.5 \times 10^{-5}$ cm. The apertures were circular, with a diameter of 0.5 cm, much larger than λ . We worked therefore in absence of single-slit (Fresnel) diffraction. However, the Fraunhofer diffraction was still present, and its effects have been taken into account in the background measurement.

Detector C was fixed in front of the source S_2 ; detectors A and B were placed on a common vertical, movable panel (see Fig. 13.1). This latter feature allowed us to study the space dependence of the anomalous effect, predicted by DSR.

Let us highlight the role played by the three detectors. Detector C destroyed the eigenstates of the photons emitted by S_2 . Detector B ensured that no photon passed through the aperture F_2 . Finally, detector A measured the photon signal from the source S_1 .

In summary, detectors B and C played a controlling role and ensured that no spurious and instrumental effects could be mistaken for the anomalous effect which had to be revealed on detector A. The design of the

¹In this connection, let us notice that the dotted line S in Fig. 13.1 is a mere geometrical one, and does not represent any physical trajectory of photons emitted by the source S_2 , since the aperture F_2 was well outside the emission cone of S_2 [63]. It is only to mean that the distance between S_2 and the detector A is the same as the distance S in Fig. 12.2.

box and the measurement procedure were conceived so that detector A was not influenced by the source S_2 according to the known and officially accepted laws of physics governing electromagnetic phenomena: classical and/or quantum electrodynamics. In other words, with regards to detector A, all went as if the source S_2 was not there at all or as if it was always kept turned off.

In essence, the experiment just consisted in the measurement of the signal of detector A (aligned with the source S_1) in two different states of source lighting. Precisely, a single measurement on detector A consisted of two steps:

- 1. Sampling measurement of the signal on A with source S_1 switched on and source S_2 off
- 2. Sampling measurement of the signal on A with both sources S_1 and S_2 on

As already stressed, due to the geometry of the apparatus, no difference in signal on A between these two source states ought to be observed, according to either classical or quantum electrodynamics. If $A(S_1i \ S_2k)$ (i, k = on, off) denotes the value of the signal on A when source S_1 is in the lighting state *i* and S_2 in the state *k*, a possible nonzero difference $\Delta A = A(S_1 \text{ "on" } S_2 \text{ "off"}) - A(S_1 \text{ "on" } S_2 \text{ "on"})$ in the signal measured by A when source S_2 was off or on (and the signal in B was strictly null) has to be considered evidence for the searched anomalous effect.

The outcomes of the first experiment were positive, namely the differences ΔA between the measured signals on detector A in the two conditions were different from zero and below the threshold value of energy for the breakdown of local Lorentz invariance as predicted by DSR. In particular, ΔA ranged from (2.2 ± 0.4) to $(2.3\pm0.5) \mu V$, values well below the threshold $E_{0,e.m.}=4.5 \mu V$. Moreover, such an anomalous effect was observed within a distance of at most 4 cm from the sources [63], thus confirming the spatial threshold obtained from the analysis of the Cologne and Florence experiments (see Chap. 12). We can consider such an effect as the consequence of an "hidden" (Lorentzian) interference.

The purpose of the second experiment was to corroborate the results of the previous one [64,65]. The experimental set-up was essentially the same (for instance, the dimensions of the apparatus, and the relevant quantities, like photon wavelength and aperture diameter, were identical to those of the first experiment). The main difference with respect to the equipment of the first experiment was in a right-to-left inversion along the bigger side of the box, and in the three used detectors, which were not photodiodes but phototransistors (of the type with a convergent lens). In this way, it was possible to study how the phenomenon changes under a spatial parity inversion and for a different type of detector. We want to point out that in this second experiment the time procedure to sample the signals on the

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detectors was different from that used in the first experiment. We indeed realized that the sampling time procedure was apparently crucial in order to observe the anomalous interference effect.

The results of this second experiment confirmed those of the first one. The value of the difference measured on detector A was $(0.008 \pm 0.003) \mu V$, which is consistent, within the error, with the difference $\Delta A \simeq 2.3 \,\mu V$ measured in the first experiment, provided that the unlike efficiencies of the phototransistors with respect to those of the photodiodes are taken into account [64].² The consistency between the results of the first two experiments shows apparently that the effect is not affected by the parity of the equipment and by the type of detector used (at least for photodiodes and phototransistors). Let us notice that one was compelled to use two different sampling time procedures for the two different types of detecting devices in order to make the effect evident. It turned out that there was apparently a sort of unavoidable bond between detector and sampling-time procedure, to be taken into account in order to reveal the effect.

The third experiment was planned and carried out in order to shed some light on this issue and to obtain a further evidence of the searched effect [66]. In order to test the apparent bond between detectors and sampling time procedures, the experiment was carried out by means of the box with photodiodes but using the sampling-time procedure adopted with phototransistors. The results of this third experiment were consistent with those of the two previous ones. By this statement we mean that the average value of the differences on detector A in the two lighting situations of the sources was below the threshold energy for the breakdown of LLI for the electromagnetic interaction, as required by the theory. In particular the maxima of $|\Delta A|$ accumulate around the value of $2.3 \,\mu V$ (see Fig. 13.2), in agreement with the results of the other two experiments.³ One can conclude that the sampling time procedure, which permitted the effect to be evidenced on phototransistors, could reveal it on photodiodes as well.

²One can define the relative geometrical efficiency $\eta_{\rm g}$ of the phototransistor (with respect to the photodiode) as the ratio of their respective sensitive areas, and their relative time efficiency η_t as the ratio of their respective detection times. Then, one can define the relative total efficiency η_T of the phototransistor with respect to the photodiode as the product $\eta_{\rm T} = \eta_{\rm g} \eta_t$. From the values of $\eta_{\rm g}$ and η_t in this case, one gets $\eta_{\rm T} = 0.0015$ [64].

Therefore, it was reasonable to foresee that the value of the expected phenomenon in the second experiment to be given by the product of the total relative efficiency times the value measured in the first experiment, i.e., $\eta_{\rm T}[(2.3\pm0.5)\mu V] = (0.004\pm0.001)\,\mu V$, in agreement with the experimental result.

³Let us note that the photodiodes used as detectors in the first and third experiment were integrated to a transimpedance amplifier, transducing the photocurrent signal into a voltage signal. Such a voltage, measured by means of a multimeter, does not depend therefore on the value of the circuit resistances of the voltage measuring system.



FIGURE 13.2. Value of the differences ΔA of signal sampled on detector A for the two lighting states of the sources S_1 on, S_2 off, and S_1 on, S_2 on (third experiment). The differences are clearly incompatible with zero

Therefore, it is possible to state that, although there is not such a tight bond between detector and time procedure, the latter plays a very important role in giving evidence to the effect. More explicitly, one could say that the phenomenon one tried to detect and to study possesses very complex features, which make it hard to be grasped both in literal and figurative sense. We already know that the anomalous Lorentzian interference manifests itself only under precise conditions, namely below an energy threshold and within some spatial threshold as well. In this sense, it is endowed with a peculiar structure both in energy and in space. The global view of these three experiments teaches us that there exists also some sort of threshold for the sampling time interval. Because of this time structure, the effect looks quite different depending on the time procedure adopted to sample the signals on the detectors, as it is apparent from the two responses we got from the first and the third experiments. In order to evidence this anomalous photon behavior, which is the consequence of a very complex physical phenomenon, i.e., the breakdown of LLI, one has to adapt the physical inquiry to it and be aware of the existence of these thresholds in energy, space and time.

We want to add that the third experiment was repeated several times over a whole period of four months in order to collect a fairly large amount of samples and hence have a significant statistical reproducibility of the results. Thanks to this large quantity of data, it was possible to study the distribution of the differences of signals on detector A, which is shown in Fig. 13.2. For clarity' sake, we reported only the differences ΔA outside the interval [-1,1], which is the interval of compatibility with zero of the values of ΔA .

The circumstance that the majority of the differences ΔA is negative (namely $A(S_1 on S_2 of f) < A(S_1 on S_2 on)$) might wrongly induce to deem that, when source S_2 was turned on, the signal detected on A increased. One might then be incorrectly tempted to account for this by stating that some photons of S_2 passed through aperture F_2 (see Fig. 13.1). Conversely, if one takes into account the mode of operation of the photodiodes chosen as detectors,⁴ it becomes immediately apparent that the above inequality means exactly the opposite situation. Namely, when S_2 was turned on, detector A recorded a lower signal and hence received less photons, although there was a larger number of photons in the box because both sources were on.⁵ On the other hand, it is impossible to account for this reduction of the signal on A when S_2 got turned on and S_1 was already on as a destructive interference between photons from the two sources, because the LEDs are incoherent sources of light.

13.2 Crossing Photon Beam Experiments

The results of the double-slit experiments suggest that similar anomalous effects can be observed also in different experimental situations involving photon systems, like e.g., in interference experiments. Further evidence for the anomalous photon system behavior (and for the related anomalous photon-photon cross section) was observed indeed in orthogonal crossing photon beams.

These interference experiments were carried out after our first one, one with microwaves emitted by horn antennas (see Fig. 13.2), at IFAC-CNR (Ranfagni and coworkers) [67–69], and the other with infrared CO₂ laser beams (Fig. 13.3), at INOA-CNR (Meucci and coworkers) [69]. Let us summarize the results obtained.

⁴In order to understand this point, let us give some brief details about the mode operation of the type of photodiode (OPT301 Burr Brown) used in the third experiment. First of all, we have to say that its pins were connected to the input pins of a transimpedance operational amplifier which was integrated along with the photodiode on the same chip. The photodiode was not inversely polarized and the dark current was always greater than the photocurrent. As is well known, the two currents flow in opposite directions, and the total current flowing in the photodiode is given by their subtraction. When the total current increases, the op-amp output voltage increases too. However, a rise of the total current (and hence a rise of the output voltage) means a decrease of the photocurrent (the dark current cannot change) and this means a drop of the number of photons received by the photodiode. Thus, when both sources were on, the increase of the output voltage means that the photodiode A was receiving less photons.

⁵Needless to say, the stability of power supplies was constantly checked.



FIGURE 13.3. Schematic view of the crossed-beam experiment in the infrared range, exploiting a CO_2 laser emitting at 10.6 μm on the fundamental TEM₀₀ Gaussian mode. The laser beam is split in two orthogonal beams (beam 1 and beam 2) by means of a beam splitter. By using two flat mirrors the two beams are directed to the crossing area within the near field of the Gaussian mode, estimated at 1.5 m from the out-coupler mirror of the laser cavity. Beam 2 is periodically interrupted by means of a chopper whose frequency is the reference frequency in a lock-in amplifier connected to the detector

The main result of the IFAC experiment consists in an unexpected transfer of modulation from one beam to the other, which cannot be accounted for by a simple interference effect. This confirms the presence of an anomalous behavior in photon systems, in the microwave range too.

In the optical experiment carried out at INOA-CNR [69], the wavelength of the used infrared laser beams was 10,600 nm, namely one order of magnitude higher than the wavelength of the sources (LEDs) used in our experiments (850 nm). Let us also remark that the energy of the photons of the three double-slit experiments was 10^4 times higher than that of the photons in the Cologne and Florence experiments [32, 33, 35], and 10 times higher than that of the INOA-CNR experiment [69].

The optimum alignment which can be achieved with lasers and the laser beam confinement make this optical set-up especially suitable for investigating the anomalous behavior of the photon systems. This allowed one to perform a statistical test on the averaged results [64, 69]. The signal statistics provided a significant variation in the mean values obtained with or without beam crossing. Hence the chance to have two identical statistics was rejected with a sufficient level of confidence. Moreover, it was estimated [64] that the actual shift of the crossed beam signal with respect to the single beam signal is $(2.08 \pm 0.13) \,\mu\text{V}$. This value agrees excellently with that obtained in our first experiment $\Delta A \simeq 2.3 \,\mu\text{V}$. Notice that the laser experiment shows that the observed phenomenon does depend neither on the infrared wavelength, nor on the coherence properties of the light.

Although further checks are needed, one can conclude that the crossing photon-beam experiments do preliminarily support the evidence for an anomalous interference effect under the space and energy constraints obtained by the DSR formalism. 206 13. Lorentzian Violation of Electrodynamics in Photon Systems

13.3 The Shadow of Light: Hollow Wave, LLI Breakdown and Violation of Electrodynamics

We want now to provide an interpretation, and discuss the implications, of the observed anomalous photon behavior.

Needless to say, the results obtained in different photon systems in different experiments are consistent with LLI breakdown. The signature of violation of LLI is provided by the marked threshold behavior the phenomenon exhibited. In fact, the anomalous effect was observed within a distance of at most 4 cm from the sources (1 cm in the second experiment), and the measured signal difference on detector A ranged from $\Delta A \simeq 2.3 \,\mu V$ (first and third experiment) to $\Delta A \simeq 0.008 \,\mu V$ (second experiment) [63–66]. These values are consistent with the space and energy threshold behavior for the electromagnetic breakdown of LLI, obtained in the framework of DSR (see Chapt. 4 and 12).

Moreover, in our opinion, the results of the photon experiments described earlier cannot be explained in the framework of the Copenhagen interpretation of quantum wave [65], or in its implementation in terms of path integrals in Feynman's approach.

Indeed, let us consider the difference ΔA in the signal measured by detector A according to whether only S₁ is turned on or both sources are on, and recall the role played by the three detectors in our experiments (see Sect. 13.1). On one side, detector C measures – and hence destroys – the superposition of states belonging to the photons emitted by S₂ (thus manifesting their corpuscle nature); on the other hand, detector B is always underneath the dark voltage threshold, thus ensuring no transit of photons through aperture F₂. Therefore in no way – according to the Copenhagen interpretation – photons from S₂ can interact with those from S₁, thus accounting for the signal difference on detector A.

On the contrary, such a result can be understood by interpreting – following Einstein, de Broglie and Bohm [70–72] – the quantum wave as a pilot (or hollow) wave.

In such a framework, pilot waves can interact with quantum objects (as assumed by de Broglie and Andrade y Silva [73]). Then, the region outside aperture F_2 is optically forbidden to the photons emitted by the source S_2 , but not to the hollow waves associated to them. Thence, the photons emitted by the source S_1 can interact with the hollow waves of photons from the source S_2 , which have gone through the aperture F_2 . Consequently, the change ΔA in the A signal – in absence of any change in the response signal of detectors B and C – finds a natural explanation, in the Einstein–de Broglie–Bohm interpretation of quantum wave, in terms of the interaction of the S_1 photons (and their hollow waves) with the hollow waves (of S_2 photons) passed through F_2 . The role played by the aperture F_2 is fundamental, since, although hollow waves can penetrate in optically forbidden regions, nonetheless the mass distribution and density are expected to affect their propagation. Hence, they can pass only through space regions with a lower mass density.

Since, according to DSR, the breakdown of LLI is connected to a deformation of the Minkowski metrics, it is possible to put forward the hypothesis [63] that the hollow wave (at least for photons) is nothing but a deformation of space-time geometry, intimately bound to the quantum entity ("shadow of light").

This can be depicted as follows. Most of the energy of the photon is concentrated in a tiny extent; the remaining part is employed to deform the space-time surrounding it and, hence, it is stored in this deformation. It is just the deformations ("shadows") of the photons from S_2 that expand, go through F_2 and interact with the shadows of the photons emitted from S_1 .

Therefore, in this view, the difference of signal measured by the detector A in all the double-slit experiments can be interpreted as the energy absorbed by the space-time deformation itself, which cannot be detected by the central detector $B^{.6}$ In other words, the experimental device, used in these experiments, "weighed" the energy corresponding to the space-time deformation by the measured difference on the first detector.

If the interpretation we have given here is correct, the double-slit experiments do provide for the first time, among the others, direct evidence for the Einstein-de Broglie-Bohm waves and yield a measurement of the energy associated to them.

The hypothesis of the hollow wave as space–time deformation is able to explain also the anomalous behavior observed in crossed photon-beam experiments (see Sect. 13.2). In fact, the shadow of the photon spreads beyond the border of the space and time sizes corresponding to the photon wavelength and period, respectively. This changes the photon-photon cross section (strongly depressed both in classical and in quantum electrodynamics),⁷ and gives rise to the anomalous effects observed in the photon–photon interactions in crossing beams.

The earlier interpretation is of course incompatible with standard electrodynamics (either classical or quantum). This is also easily seen by the ensuing violation of LLI, on account of the strict connection between Lorentz invariance and electrodynamics (as is well known, the standard Lorentz group is the covariance group of Maxwell equations). We want now to show that a more detailed analysis of the measurements of the third experiment

⁶One might think to detect such an "energy of deformation field" (corresponding to the hollow waves of photons) by a detector operating by the gravitational interaction, rather than the electromagnetic one. However, this would still be impossible, because the deformation value lies within the energy interval for a flat (Minkowski) gravitational space–time, according to DSR (see Sect. 4.1).

⁷In fact it goes as α^4 (with α being the fine structure constant).

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FIGURE 13.4. Values of the differences $\Delta A'$ of signal sampled on detector A with both sources on (third experiment). The differences are clearly compatible with zero

are just in favor of the anomalous (Lorentzian) interference observed as signature of a possible violation of electrodynamics.

This is easy to realize, by noting that the distribution of the results of the third experiment (reported in Fig. 13.2) is unmistakably different from that expected from the theoretical predictions of both quantum and classical electrodynamics.

With reference to Sect. 13.1, we recall that Fig. 13.2 shows the signal differences measured on A in correspondence to the two different states of lighting of the sources, $\Delta A = A(S_1 \text{ on } S_2 \text{ off}) - A(S_1 \text{ on } S_2 \text{ on})$. For comparison, we report in Fig. 13.4 the differences of the two values sampled on A in the same lighting condition of the sources, i.e., with both sources turned on: $\Delta A' = A(S_1 \text{ on } S_2 \text{ on}) - A(S_1 \text{ on } S_2 \text{ on}).$ Again, for clarity' sake, we show only the differences outside the interval [-1, 1]. There is no surprise in observing that the differences are almost evenly distributed around zero, since the subtracted values belong to the same population. However, by the very design of the experimental box, according to either classical or quantum electrodynamics detector A was not to be affected by the state of lighting of the source S_2 . Hence, one would expect that the mean value of these differences was zero and that the differences ΔA were uniformly distributed around it. In other words, one would expect to find roughly the same number of positive and negative differences, and therefore that Figs.13.2 and 13.4 displayed two compatible distributions of differences evenly scattered across zero. On the contrary, the differences in Fig. 13.2 are not uniformly distributed around zero but are markedly shifted downward (as compared to



FIGURE 13.5. Oscillations of the percentage of differences ΔA in the five measurement sessions of the third experiment

those in Fig. 13.4), and hence the number of negative differences is larger than the positive ones.

One can go into this point in more depth by considering the oscillations of the percentage of negative differences ΔA (Fig. 13.5). The five points in such a figure represent the five percentages of negative differences attained in the five different sessions of the third experiment. It is quite evident that they do oscillate around a mean value as expected, but this mean value is approximately 85% and not 50% as predicted by electrodynamics. Then, it follows that the downward displacement of the differences in going from Fig. 13.4 to Fig. 13.2 is not a mere chance, but is a systematic result obtained every time the experiment was performed. Let us notice that each of the five sessions reported in Fig. 13.5 has actually to be counted as if it were four sessions, due to the particular procedure adopted to sample the signal on detector A [66]. Then one has 20 sessions of the experiment in which the percentage of negative differences is always much greater than 50%.

In order to further enforce the evidence for the difference of the two physical situations corresponding to Figs.13.2 and 13.4, we carried out a statistical analysis of the results found in the two cases (only the differences outside the interval [-1, 1] have been considered), by taking also into account the instrumental drift. The Gaussian curves obtained are shown in Fig. 13.6. The dashed curve refers to the signal differences $\Delta A =$ $A(S_1 \text{ on } S_2 \text{ off}) - A(S_1 \text{ on } S_2 \text{ on})$, whereas the solid one to $\Delta A' = A(S_1 \text{ on } S_2 \text{ on}) - A(S_1 \text{ on } S_2 \text{ on})$. The two curves differ by 2.5 σ , clearly showing that



FIGURE 13.6. Gaussian curves (normal frequency vs. signal difference in μV) for the signal differences ΔA and $\Delta A'$ on detector A for the two cases of source S₂ off and on (*dashed* and *solid curve*, respectively). The instrumental drift has been taken into account. It is $\overline{\Delta A} = -3.15$ ($\sigma = 0.45$); $\overline{\Delta A'} = -2.56$ ($\sigma = 0.24$)

the two cases are statistically distinct, the latter one representing a mere fluctuation (unlike the former).

We can therefore conclude that the results obtained on the anomalous behavior of photon systems brings to light a more complex physics of the electromagnetic interaction, which again calls for giving up the local Lorentz invariance in order to be accounted for. They are apparently at variance with both standard quantum mechanics (in the Copenhagen interpretation) and usual (classical and quantum) electrodynamics.

The interpretation in terms of DSR is quite straightforward. Under the energy threshold $E_{0,e.m.}=4.5 \,\mu\text{eV}$, the metric of the electromagnetic interaction is no longer Minkowskian. The corresponding space-time is deformed. Such a space-time deformation shows up as the hollow wave accompanying the photon, and is able to affect the motion of other photons. This is the origin of the anomalous interference observed. It was noted at the beginning of this section that the difference of signal measured by the detector A in all the double-slit experiments can be regarded as the energy spent to deforme space-time. In space regions where the external electromagnetic field is present (regions of "standard" photon behavior), we can associate such energy to the difference $\Delta \mathcal{E}$, (3.124), between the energy density corresponding to the external e.m. field $F_{\mu\nu}$ and that of the deformed one $\tilde{F}_{\mu\nu}$ given by (3.119).

But it is known from the experimental results that the anomalous interference effects observed can be explained in terms of the shadow of light, namely in terms of the hollow waves present in space regions where no external e.m. field occurs. How to account for this anomalous photon behavior within DSR? The answer is provided by the internal structure of the deformed Minkowski space discussed in Sect. 9.4. In fact, we have seen that the structure of the deformed Minkowski space M as Generalized Lagrange Space implies the presence of two internal e.m. fields, the horizontal field $\mathcal{F}_{\mu\nu}$ and the vertical one, $f_{\mu\nu}$. Whereas $\mathcal{F}_{\mu\nu}$ is strictly related to the presence of the external electromagnetic field $F_{\mu\nu}$, vanishing if $F_{\mu\nu} = 0$, the vertical field $f_{\mu\nu}$ is geometrical in nature, depending only on the deformed metric tensor $g_{\text{DSR},\mu\nu}(E)$ of $\text{GL}^4 = \widetilde{M}$ and on E. Therefore, it is present also in space-time regions where no external electromagnetic field occurs. In our opinion, the arising of the internal electromagnetic fields associated to the deformed metric of M as Generalized Lagrange space is at the very physical, dynamic interpretation of the experimental results on the anomalous photon behavior. Namely, the dynamic effects of the hollow wave of the photon, associated to the deformation of space-time – which manifest themselves in the photon behavior contradicting both classical and quantum electrodynamics -, arise from the presence of the internal v-electromagnetic field $f_{\mu\nu}$ (in turn strictly connected to the geometrical structure of M).

Moreover, as is well known, in relativistic theories, the vacuum is nothing but Minkowski geometry. An LLI breaking connected to a deformation of the Minkowski space is therefore associated to a lack of Lorentz invariance of the vacuum. Then, the view by Kostelecky [55] that the breakdown of LLI is related to the lack of Lorentz symmetry of the vacuum accords with our results in the framework of DSR, provided that the quantum vacuum is replaced by the geometric vacuum. Notice also that in the Kostelecky formalism it is impossible to recover local Lorentz invariance. On the contrary, DSR recovers it in a generalized sense, in the form of deformed Lorentz invariance (see Sects. 3.3.5, 3.3.7). Let us also recall (as we shall see in Part IV) that, as already said, DSR admits a natural immersion in a 5D-space, and that the vacuum solutions of the Einstein equations in such a space reproduce the phenomenological metrics discussed in Sect. 4.1. In this connection, it was proved [74] that waves and particles admit a common geometrical interpretation as isometries of a 5D space. One can therefore hazard the view that local Lorentz invariance, apparently violated, is actually recovered in the 5D version of DSR as an exact symmetry, intimately related to the propagation of quantum waves in the 4D space-time.