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Editors

Rediscovering Phenomenology

Phenomenological Essays on Mathematical Beings, Physical Reality, Perception and Consciousness

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Phenomenological Essays on Mathematical Beings, Physical Reality, Perception and Consciousness

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INTRODUCTION

L. Boi, P. Kerszberg and F. Patras

Scientific thought has undergone a series of major developments in the second half of the twentieth century. Retrospectively, disregarding some special cases such as biology, this evolution, however radical it may have been, was more a deepening of the ideas born at the beginning of the century than a real transvaluation. Within the limits of mathematics and physics, axiomatic method, algebraic geometry and topology, general relativity or quantum mechanics, have been the driving forces of the great breakthroughs accomplished in the twentieth century.

The second half of the twentieth century has witnessed the systematic development of theories, which had been established and methodologically grounded in mathematics and physics (a process that had led to the paradoxes of set theory followed by Gödel's theorems, the change in the ontology of elementary particles and the problems of interpretation of quantum mechanics). Their qualitative and quantitative rise, in addition to their operational scope, have been such that they led to a change of attitude in the scientific community concerning the need for theoretical and epistemological foundations. Thus, after the second world war, the mathematicians of the Bourbaki group have stopped concerning themselves with the architectonic problems that attracted the attention of such great mathematicians as Poincaré, Weyl, von Neumann or Gödel. Likewise, though perhaps in a less significant manner, and as a result of the incredibly efficient power of description and prediction in quantum theories, the physical sciences have put up with a number of ontological aporias.

Nevertheless, in recent times a number of profound changes have emerged. To be sure, they do not have the dramatic character of the non-Euclidean, axiomatic or quantum revolutions that shook the world of Euclid, Newton or Kant; but they modified our perception and understanding of scientific thought and its mode of operation. In symbolic logic and mathematics, Gödel's heritage or category theory have greatly

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modified, if not invalidated, the significance of most of the reflective work and dispute over the foundations of axiomatic thought. In other words, our understanding of the phenomenology of logical and mathematical beings (in the sense in which experimental research uses this word) has been sufficiently renewed and deepened to allow for new roads in epistemological analysis, which until then could not be pursued because of the lack of relevant tools. In physics too, Bell's inequalities and the experimental verification of non-locality as predicted in the Einstein-Podolski-Rosen paradox have had a deep effect over the intelligibility of quantum theories.

The purpose of this book is to re-examine the epistemology of the exact sciences in the light of these developments. In order to do so, the various methods issued from Hussserl's original transcendental phenomenology will be used. These methods range from the analysis of intentionality to the conceptual and epistemological significance of scientific theory. For us today, epistemology means the revival of transcendental phenomenology in Husserl's sense.

The reasons for such a strategy are many. First the phenomenological method allows for a complete overview of the conditions of possibility of any scientific knowledge. Since science is ever more uprooted from its anchorage in the phenomenal lived experience of the world, the phenomenological method is indispensable in order to appreciate the entire range of its epistemological stakes. A reflection on the nature and scope of the Husserlian project itself is also involved in this strategy. From the *Logical Investigations* to the *Crisis*, Husserl's guiding thread has always been the problem raised by the decidedly formal character of modern science, especially the post-Galilean mathematization of nature. The emergence of the axiomatic method and the ever more symbolic dimension of physical theories have had a great influence over Husserl. In particular, they have determined the theoretical possibilities of phenomenology itself, which is therefore engaged in a continuous discussion with the sciences of its time. (Hilbert is a good example.)

Axiomatics, together with its various correlates (structure of theories of the universe, structuralist methodology, etc.), is seen very differently today. Does this invalidate Husserl's phenomenology? Is it still possible to redirect it in accordance with the new directions taken by the sciences themselves? What kind of tools does it provide contemporary science with? Could it not be the basis for the kind of epistemological renewal needed by the sciences today, beyond their technical accomplishments?

In order to answer these questions, it is also necessary to part ways with some recent interpretive moves in phenomenology, which unfortunately tend to disfigure it beyond recognition. The originality of the Husserlian heritage does not lie in the complete and dogmatic subjectivization of consciousness, with the dreadful implication of psychologism. Nor does it consist in the full naturalization of consciousness, which could be induced by the traditional use of the word "phenomenology" in the empirical sciences. Ultimately, phenomenology aims at providing a systematic articulation between the various modes of theoretical objectivity and the apprehension, followed by structuration, of the phenomenal world itself by intentional consciousness. Viewed from this perspective, phenomenology is always necessarily *transcendental phenomenology*.

The book is divided into three parts, each being preceded by a specific introduction. They reflect the guiding themes of phenomenological analysis, inasmuch as it is engaged in a dialogue with the sciences: spatial mereology, phenomenology of perception and intentionality, foundations and methods of the sciences, phenomenology of mathematics and its foundations.

PART I

SPATIALITY AND THE PHENOMENOLOGY OF PERCEPTION

FOREWORD

Luciano Boi

The first section of this book deals with the spatial properties of perception, the geometry of the body (its movements and the visual field) as well as with the formal relationship between the "whole" and its "parts." The essential idea underlying the different chapters may be summarized as follows: perception is first and foremost the perception of a phenomenal world that is endowed with a certain spatial (both geometrical and topological) organization. There is thus a close link between the geometric properties pertaining to the physical world and its objects and the constitution of perceptual structures. In fact, these geometric and topological properties, rather than being an accidental element – however important in itself – of the phenomenal world (as Husserl and the Gestalt theorists believed), play an essential role in the dynamic process of constitution of this world.

More specifically, the following has to be assumed: (i) from the outset, perception encloses the characters of space and time as fundamental features; (ii) the mechanisms inherent in perception cannot be understood independently of the properties and the objective physical laws that characterize phenomena, as such, in the natural world; (iii) there are spatial properties related to the notions of nearness, symmetry axis, orientation, group of movements, connectedness, which underlie the constitution of the perception of objects as "objective" phenomena.

Moreover, the objects and contents of perception could not assert themselves as autonomous units, as coherent global wholes, were it not for the intervention of these spatial properties; consequently, the latter appear necessary for the structuring of the phenomenal world. As a number of chapters show, these geometric features are in some sense the source of a great number of physical as well as psychic pregnancies which can invest a variety of objects localized in the surrounding space, and which can thus give rise to salient forms and a significant variety of related phenomena.

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However, such a process only seems possible if we admit a dynamic correlation between the physiological mechanisms inherent in perception and the geometric laws of transformations of rigid bodies in our three-dimensional Euclidean space.

In particular, the issues raised by the contributions of Part 1 lead to four groups of questions that we believe are fundamental to our understanding of spatial perception:

1. How is it possible to characterize the link between the geometrical organization of the neurophysiological structures responsible for perception and the spatial feature pertaining to the movements of our body in the physical space?
2. What kind of geometric models can be developed for the recognition of visual forms?
3. What is the relationship between perceived spatial forms and cognitive activity?
4. What kind of relationship exists between the "parts" and the "whole," or between "discreteness" and "continuity" with regard to the modalities and contents of perception?

These questions are addressed here for the first time in the promising form that they deserve. The authors agree that a satisfactory theory of spatial perception has to aim at explaining the connection between geometrical, physical and perceptual space. Husserl contributed to this explanation in a remarkable way, particularly by showing the type of proto-geometry that is involved in the structuring of phenomenal world or in the constitution of the "spatial things." In the present volume, it is suggested that the sensory space, which is the "natural environment" of our perception, constitutes a kind of "primitive" topological continuum which, in addition to being an essential datum of our intuition, presents a qualitative structure of primary importance. Husserl's analysis was limited by the fact that he could not seriously take into account the possibility of rendering mathematically intelligible the different phenomenal fields of perception. One of the main goals of the chapters in the first section of this book is precisely to fill this gap, by trying to develop a dynamic theory of perception interpreted in terms of interrelated sensory systems.

It is commonly believed that the senses of seeing, hearing, and touching are entirely separate "perceptual modules," each of which operating independently from the other in order to provide us with the relevant information about the external world. Recent studies, however, have revealed that our perceptual experience is in fact shaped by a multitude of complex interactions between sensory modalities. For example, a number of powerful multisensory illusions demonstrate that the senses are inextricably linked, and that our perception of visual, auditory, or tactile events can be either completed or dramatically altered when the information issued by the other senses is taken into account. When a sound is accompanied by a visual stimulus at another location, people tend to perceive this sound incorrectly at the same position as the visual stimulus – the ventriloquism effect. When two objects are lifted, different in visible size but equal in weight, the larger object is felt to be heavier – the size-weight illusion. When people see a life-sized rubber model of their own hand being touched at the same time as theirs, but hidden from view, they experience the touch on the rubber hand, and often report that the rubber hand feels as if it was their own.

Besides, there is more and more neuropsychological evidence indicating that action influences spatial perception. First, actions using a tool can modulate unilateral visual neglect and extinction, where patients are unaware of stimuli presented on one side of space. It has been showed, at least for some patients, that modulation comes about through a combination of visual and motor cueing of attention to the affected side. There is also evidence that action-relation between stimuli reduces visual extinction: there is less extinction when stimuli fall in the correct collocations for action, as compared to when they fall in the incorrect relations for action, or when stimuli are just associatively related. Finally, it can be demonstrated that action relations between stimuli can also influence the binding of objects to space, in the case of patients with certain syndroms (e.g., the Balint's syndrome). These neuropsychological data indicate that perception-action couplings can be crucial for our conscious representation of space.

From a more global perspective, we would like to stress some very interesting and promising issues closely related to Husserl's. They highlight the rich and meaningful phenomenological constitution, as well as the inner and dynamic geometric organization of objects, events and bodies, both in experienced space and in the living environment.

1. A first fundamental issue worth of mention concerns the investigation of global effects in visual occlusion. "Classical" occlusion examples, such as a square partly occluded by a rectangle, have given rise to so-called local and global accounts of *amodal completion* (in the terms of Kanizsa). Without denying the influence of local configurations, we are of the opinion that, in the long run, any theory of amodal completion should account for global properties. Recently two extensions of the stimulus domain have been proposed, which add weight to the necessity of global accounts. The first is the domain of so-called fuzzy regularities, i.e., regularities which are not based on metrical identities. It has been demonstrated that observers react to these fuzzy regularities and that they complete partly, occluded shapes accordingly. The second extension has to do with three-dimensional object-completion. Theories of object-representation that describe intrinsic regularities of objects appear to be most suitable to predict relative preferences in alternative object completions. Consequently, fuzzy object completions such as the completion of the back of a tree-trunk can be explained more satisfactorily by global constraints.

2. A second issue is related to the way in which visual object constancy across plane rotation and depth rotation can be achieved. Visual object constancy is the ability to recognize an object from its image despite variation in the image when the object is viewed from different angles. Research probes into the human visual system's ability to achieve object constancy across plane rotation and depth rotation. In some cases, the recognition of invariant features allows objects to be reorganized irrespective of the view depicted, particularly if small, distinctive sets of objects are presented repeatedly. By contrast, in most situations, recognition is sensitive to both the in-plane and in-depth view from which an object is depicted. This result suggests that multiple, view-specific, stored representations of familiar objects are accessed in everyday, entry-level visual recognition, or that transformations such as mental rotation or interpolation are used to transform between retinal images of objects and view-specific, stored representations.

3. A third issue regards the influence of spatial reference frames on imagined object and viewer rotations. It is an important fact for perception that the human visual system can represent an object's spatial structure with respect to multiple frames of reference. It can also use multiple reference frames to mentally transform such representations. Recent research has

shown that imagined object-rotations tend to be more difficult than imagined viewer rotations. How are we to understand this discrepancy in terms of the different reference frames associated with each imagined movement? An examination of many mental rotation situations reveals that the difficulties for an observer to predict an object's rotational outcome might stem from a general deficit with regard to imagining the cohesive rotation of the object's intrinsic frame. Such judgments are thus more reliant on supplementary information provided by other frames, such as the environmental frame. By contrast, motor imagery and other studies prove that imagined rotations of the viewer's relative frame are performed cohesively and are thus mostly immune to effects of other frames.

4. A fourth issue has to do with the problem of the visual representation of three-dimensional, rotating objects. Depth rotations can reveal new parts of objects, which results in poor recognition of "static" objects. Recent studies have suggested that multiple object views can be associated through temporal contiguity and similarity. Motion may also play an important role in object recognition since observers recognize novel views of objects rotating in the picture plane more readily than novel views of statically reoriented objects. The most interesting experiments presented in the literature investigated how different views of a depth-rotated object might be linked together even when these views do not share the same parts. The results suggest that depth rotated object views can be linked more readily with motion than with temporal sequence alone to yield priming of novel views of three-dimensional objects that fall in between "known" views. Motion can also enhance path specific view linkage when visible object parts differ across views. Such results suggest that object representations may depend on motion processes.

5. Lastly a fundamental issue is related to the problem of subjective contours. The phenomenon of perceptual closed contours cannot be psychophysically predicted by local rules of grouping. This indicates, e.g., that linkage of collinear segments is strongly affected by the global arrangement. In other words, equally aligned line segments are easily segregated from the background if they compose a circle, but they blend into the background when they are not closed. This robust "pop-out" effect requires that adjacent line segments ought to be quasi-collinear. For example, if the closed curve formed a half-moon, closure enhancement would disappear, although both a circle and a half-moon are topologically closed.

This implies that the closed curves cannot contain "kinks." In the case of contour detection, one can demonstrate that contour closure does have perceptual significance in binding spatially separate features: oriented segments group together to form a closed contour outside the range of local grouping constraints. Recent psychophysical studies showed that the detection of line continuity is supported by a well-defined spatial range of interconnection between neighbouring detectors, where interconnection is constrained along the major orientation axes of no-overlapping filters.

CHAPTER 1

HUSSERL AND THE PHENOMENOLOGY OF ATTENTION

Bruce Bégout

> "Millions of item of the outward order are present to my senses which never properly enter into my experience. Why? Because they have no *interest* for me. *My experience is what I agree to attend to*."
>
> W. James, *The Principles of Psychology* (1890)

According to Husserl, attention is one of the most difficult issues that a philosophy of consciousness has to understand and clarify. Although on the one hand its ordinary sense seems clear and relatively determined, on the other it presents unique problems and so continues to be discussed by philosophers. These difficulties originate in the various factors that intervene in the fact of being attentive, but also in a false understanding of attention. Attention is certainly, says Husserl, "one of the chief themes of the modern psychology,"[1] but nobody thinks to join it to intentionality, for the connection between attention and intentionality has been always thoroughly overlooked. According to him, a sensualistic understanding of consciousness blocks the way to a phenomenological account of attention, insofar as it considers attention just as the outcome of sense-data discrimination. During all his philosophical work, Husserl has attempted to explain this complexity of attention and to propose a new conception of it, namely a "phenomenology of attention."[2] We are, says Husserl in the first book of the *Ideas*, at the gates of "the radically first beginning of the theory of attention." So a systematic inquiry into the essence of attention

[1] *Ideas pertaining to a pure Phenomenology and to a phenomenological Philosophy*, Book I: *general Introduction to a pure Phenomenology* (*Ideas I*), translated by F. Kersten, Den Haag, Nijhoff, 1982, p. 226.

[2] *Logische Untersuchungen, Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, Halle, Niemeyer, 1913 (1901), Zweiten LU, §§ 18–23.

L. Boi et al. (eds.), Rediscovering Phenomenology, 13–32.

is "among the fundamental tasks of general phenomenology."[3] This investigation, which must be conducted within the limits of intentionality, has to deal with the general problem of how consciousness selects, by its own means, its objects from the background of the perceptual world. What are the principles of this elective discrimination of consciousness? Why do we focus our attention on this thing instead of another?

Since his *Philosophy of Arithmetics* (1891), Husserl has insisted on the fact that attention is always conjoined to the lower levels of consciousness, which have their own degree of *being-directed-towards*. Besides it is evident for him that attention is from the outset related to the general characteristic of consciousness: intentionality. Every sort of attention is "nothing else than a fundamental *species* of intentive modifications." Attention concerns at first sight the *consciousness of something* and cannot escape from it. But it will be false to believe that it is just a particular mode of intentionality as is perception or imagination. Actually attention forms a new feature of consciousness that one could distinguish sharply from the intentional functioning of the ego.

But even then, it is still hard to say what attention is exactly, because it stretches out as far as consciousness extends itself. To quote the *Logical Investigations*, attention, as far as it has no specific component, "embraces undoubtedly all the province of consciousness."[4] It has "an extension as wide as that of the concept of *consciousness of*."[5] It disappears in a way under the theoric investigation, because it is interwoven with every mental act. In an ordinary perception, whether it is external or internal, we only pay attention to the objective differences, but the act of attention in itself stays always invisible behind. To understand its nature and function, Husserl reckons that one must clearly mark out the legal borders of attention, especially with regard to affection and intention.

If phenomenology, as Husserl conceives it, claims a privilege to be a radical reform of the science of consciousness, it is evident then that attention must be clarified first, for it is the main medium to acquire a such science. The question of attention amounts in the end to the question

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[3] *Ideas I*, p. 225.

[4] *Logische Untersuchungen*, *Zweiten LU*, Halle, Niemeyer, 1913, p. 163.

[5] *Ibid.*

of the possibility of a true knowledge of what is consciousness and what is it doing.

1. Attention and Intentionality

According to the fifth *Logical Investigation*, there are three principal significations of consciousness. Consciousness means at first the internal perception of states, ideas, feelings, i.e., the ability to see what occurs in the mind; it means also the living set of those internal components of the mind, the unity and the stream of consciousness; and finally a specific operation that sets consciousness apart from the other natural things: intentionality. When Husserl speaks of attention as a function of consciousness, he refers essentially to the *third signification* of consciousness. Attention is therefore always considered as a "modification" of a mental intention. At first sight, attention and intention seem almost synonymous for him. They both mean to *be directed towards* an object. Besides they are specific acts of consciousness, which require a certain mental activity, so that the theory of intentionality includes, as it appears, the complete issue of attention. But if each attitude of attention implies an intentional act, the opposite is not necessarily true. Despite their apparent likeness, Husserl considers that attention and intention must both be clearly distinguished lest consciousness at large may be reduced to "attentionality."

So the main difficulty which Husserl is confronted by is to separate attention from intentionality. It is already conspicuous in the *Psychological Studies on Logics* (1894). In this text, Husserl tries for the first time to clarify what he calls "the subjective circumstances of seeing." Making the distinction between intuition and intention, he remarks on that occasion that attention belongs before anything else to the former. When I pay attention to something, an idea, a sound or a man, I am directed towards it as it is given in person. At this stage, intention refers only to a mental process of meaning which doesn't seize its object in person but, for this very reason, only aims at it. That is to say attention represents a particular orientation to the object, but surely not a genuine intention. Although attention is certainly a consciousness mode of being, yet it is not a simple intentional act. It is instead a "particular act-mode."[6] Even if Husserl

[6] *Ideas I*, p. 76.

changes his conception of intentionality at the turn of the twentieth century, as it can be seen in the *Logical Investigations*, he remains faithful to the distinction between attention and intention.

To pay attention is not to take position in the sense of an act. Attention is not intention.[7]

Despite their proximity as both mental attitudes, Husserl warns his reader against the confusion between attention and intention. In others words, attention has nothing to do with a *position-taking*, because it bestows no existential position upon its object as real or doubtful. It's not really an act in which a doxic position is taken. It is therefore ontologically neutral. One can't even say that attention is a particular case of intentionality.

But of course attention, as a presupposition, requires a such act of position-taking. That which with attention has to deal is already there, tacitly presupposed: intentive acts. Husserl notes in the first book of the *Ideas* that attention is a "fundamental *species* of intentional modification."[8] On its own, attention cannot produce the intentional relation to the object, but, in order to change this *intentive* attitude of the consciousness respecting the object, it has to be anchored to the ground of intentional acts. This is the reason why it always adds a conscious modification to the present intentions of the actual consciousness. In fact, attention superimposes itself upon intentions. It covers it up. As Husserl sees it, every new attitude of attention demands a previous intentional act with a previous doxic position-taking. It follows then that attention stresses or reduces the connection to the noematic correlate, but doesn't make it up. In fact, Husserl often gives two different meanings to intention. If first intention means *to be directed towards* an object, then attention is as well intention; but if secondly intention means only *an act in which a thetic or doxic position is taken*, then attention differs entirely from intention. Everything occurs as if attention was between a mere intention and a complete act of position-taking. Attention is embedded in a twofold intentional horizon.

[7] *Einleitung in die Logik und Erkenntnistheorie. Vorlesungen 1906/07* (*Introduction to Logics and Theory of Knowledge*), ed. by Ulrich Melle, *Husserliana XXIV*, Kluwer, Dordrecht, 1984, p. 250.
[8] *Ideas I*, p. 76.

For that reason, Husserl can venture to say that there could be attention without a doxic act, what he calls a "pure intention."[9]

But if attention is not a complete "act-character," as Husserl claims in the Lectures of 1906 (*Introduction to Logics and Theory of Knowledge*), what is it then? Since, after Husserl, an act means only a position-taking with a doxic attitude, attention can't be related to the genuine activity of consciousness. It pertains to the object but not as an intentional act does. This is therefore a conscious attitude which is to *direct oneself towards* something or to *adjust to* something. As "orientation by" an object, attention simply "modifies" the intentional act of consciousness.

By emphasizing the difference between modes of intentionality and modifications of consciousness, Husserl presents hereafter attention in the *Ideen* as a "mental regard." This "mental regard" means that attention is not really an intentional form of consciousness, but an attitude of consciousness that combines regularly with it. A "mental regard," which defines attention more than any other term, signifies for him that "this having the mind's eye on something which pertains to the essence of the act as act is not itself in turn an act on its own right."[10] In any intentional act "a mode of heedfulness dominates."[11] When I give heed to something, I am not necessarily attentive to the other things which are co-present with it, and yet they are intended as such. That means attention is surrounded by an intentional consciousness which is not previously *directed towards* something determined. An attentive object is not just an intented object to which the intentional consciousness is directed, but it is rather "an object seized upon, heeded."[12] . In addition, this particular mode of intending to something sprawls itself to every intentional modes of consciousness.

Given the different intentional modes (perception, imagination, memory, and mere meaning), attention could at every moment combine itself

[9] *Hua. XXIV*, p. 251.
[10] *Ideas I*, p. 76. It is evident that, if the whole consciousness is intentional, it is not in the same way attentional. Consequently there are many intentional acts that are not usually accompanied by an attentional advertence. Husserl's theory of attention consists essentially in putting this matter of fact in the foreground.
[11] *Ideas I*, p. 77.
[12] *Ideas I*, p. 76.

entirely with them. It then becomes evident that an intentive consciousness is not necessarily an attentive one. In a conversation with D. Cairns, Husserl states that "these modes of attention don't confine themselves to the perception, but take place in every Ego's activity."[13] There could be a "modification of attention" in a perceptual act, or in a meaning act. It may be remarked here that attention is not strictly related to mere perceptions. There could be also an attentional attitude in imagining, remembering or simply meaning. This is a characteristic feature of attention, to be associated with every intentional act in every intentional realm. Since attention is not a new intentional act, it can be joined to every perception, imagination, memory or meaning. It doesn't make up an act, but it can belong to every act. As Husserl points out in the first book of the *Ideas*, "I can let my attention wander away" from place to place," shift it *ad libitum* towards another object. To say it briefly, *to be directed towards* an object doesn't set up an new act, but it reveals itself as a possible characteristic of each intentive act.

Here Husserl has to account for the special quality of attention without overlooking its close relationship with the intentionality of consciousness. Is attention a new mode of intentional consciousness, or is it something completely different? Generally speaking, attention enters under the general heading: "changes of consciousness." *To pay regard to* implies, according to Husserl, a certain change in conscious attitude and attention means then this very result. Although they are merged into the intentional operations of mind, these "attentional changes" form "a quite universal structure of consciousness having its own peculiar dimension."[14] But attention can't be isolated from the intentional acts, it lives within them, without being them. In fact, attention "cuts across all other species of intentional events," so that there could not be attention qua attention without a previous activity of consciousness. To change consciousness somehow, attention must be before prior acts of consciousness. These attentional changes play a major role in cognitive consciousness without being separated phenomenologically from certain other phenomena. It is just when

[13] D. Cairns, *Conversations with Husserl and Fink*, Dordrecht, Kluwer Academics Publishers, 1976, p. 30.
[14] *Ideas I*, p. 222.

they are mixed with these others that "they are usually designated as modes of attention."[15]

But there are certainly different manners of *to be directed towards*. The word *attention* covers consequently several senses. First attention means *to be adjusted to* an object. The object represents the focus of its intention. In this case, it is brought to a complete presence; it is litteraly under the light of the mind, seized upon and singled out. The object of attention is what is given itself, picked out, so that attention is equal to the giving of something itself. But, around the object, in its surroundings, many other objects are apprehended also by consciousness. They are not just intended as empty representations, but are given to consciousness. However, as Husserl states it, they are not under the spotlight of "primary attention" (*Aufmerken*) which renders them fully present and lived, but they stand themselves only under the "secondary or incidentally attention" (*Bemerken*). That means that the surrounding objects are "just remarked," but not necessarily noticed. These objects are here related to consciousness as intentional correlates but not seized or singled out as relevant features of our actual perception. The field of consciousness embraces then two main places, the central place occupied by the primary attention and the peripheric place occupied by the secondary attention. An objectity of any kind could be present in two different manners: as noticed directly in primary attention or just remarked in secondary attention. These are for Husserl the two principal functions of attention. And "phenomenologically the consciousness of perception becomes other if we pay attention in a primary way to the perceived thing instead of remarking it in a secondary way."[16] For Husserl *becoming other* is a phenomenological fact that, in virtue of its ability to modify the intentional connections to the objects, alters every conscious component.

But sometimes Husserl wonders if there is not still a lower level of attention, that of the perceptual horizon in itself. Actually the secondary or incidental attention covers only the objects closely co-given with the primary object, viz. with the focus of our consciousness. But what happens to the other objects lying in the background and not directly linked to the

[15] *Ideas I*, p. 222.

[16] *Vorlesungen über Bedeutungslehre. Sommersemester 1908* (*Lessons on the Theory of Signification*), ed. by Ursula Panzer, *Hua. XXVI*, p. 20.

focus of the mind? Could there be a specific attention belonging to the halo of consciousness? For Husserl there are good grounds for supposing that attention doesn't confine itself to primary and secondary attention. To this inactual field of consciousness, namely the obscure background, "we are not yet directed to it with the mental regard, not even secondarily."[17] But it is still there and besides it could become seized in its main features, for instance in its infinite givenness. As Husserl himself points out, we must consider a "third level"[18] of attention. In other words, he leaves open the possibility of a marginal attention of the horizon itself, namely a tertiary attention. Inherent to the essence of the halo there is then this possibility to be noticed. Here Husserl discovers nothing else than the ternary structure of the field of consciousness that Gurwitsch will develop further in his works especially in his *The Field of Consciousness* and in his famous article "Phenomenology of Thematics and of the pure Ego: Studies of the relation between Gestalt Theory and Phenomenology (1929)."[19] But, unlike Gurwitsch who sees them as objective components of the field itself which would retrace the ways of mental regard,[20] Husserl considers these distinctions as belonging merely to the attitudes of consciousness. Hence attention remains for him an attitude of the Ego and it is independent as such of objective components of the field of consciousness.

Anyway, it is taken for granted that there are conscious differences, albeit often very smooth and gradual, in the way of being heed to something. I can always turn my attention towards the things not seized upon until then and convert them "in the mode of actional advertence." "A free turning of regard,"[21] Husserl claims, could always modify those unseen components of the background into attentive ones. But could the horizon be noticed

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[17] *Ideas I*, p. 72.
[18] *Hua. XXVI*, p. 19.
[19] "Phenomenology of Thematics and of the pure Ego: Studies of the relation between Gestalt Theory and Phenomenology (1929)"in *Studies in Phenomenology and Psychology*, Evanston, Northwestern University Press, 1966, pp. 214–215: "acts in which something is primarily noticed, acts of attention in the pregnant sense, (. . .), is consciousness whose objective correlate presents itself as theme."
[20] Gurwitsch, *Studies in Phenomenology and Psychology*, ibid., p. 223: "the possibility of thematic modifications is grounded in the essential situation that the theme has constituents and lies within the field."
[21] *Ideas I*, p. 71.

as horizon? In the *Lessons on the theory of signification* (1908), he is noting that "it remains yet an objective background where what is conscious in a primary and secondary manner is in a certain way extracted from."[22] To follow Husserl here, this background doesn't fail to be remarked as well; but it is remarked in a different manner, namely that of the background itself. As Husserl states it in the *Ideas*,

> It is likewise obviously true of all such mental processes that the actional one are surrounded by a halo of non-actional mental processes; the stream of mental processes can never consist of just actionalities.[23]

There are two manners to be attentive to this horizon of consciousness. One is by the horizontal attention as such, a sort of tertiary attention obscure and vague; but this mode is very difficult to single out because of its own nature to be not singled out. Another is by the "attentional changes," for when I give heed to the previous unseen components of the horizon, I seize upon them as they were just before as unseen and rightly out of the attentional cone of light. That is, I am at once aware that I was before entirely heedless of them or that they were barely noticed if not completely unnoticed though still appearing. These structural features of the field of consciousness determine in general the different ways of which attention could take place. That is, attention always slips into the ready-made bed of the structural field of consciousness that existed as such before it.

But one can ask if it is indeed attention that, by virtue of its faculty to select something already given, creates these structural differences in the field of attention or if it is rather these structures that underlie from the outset the way attention happens. Is attention the privileged factor in the perceptual discrimination or is it simply a subjective consequence of it? Is there, before attention as such takes place, a prior organisation of the field of consciousness that drives in a way the rays of the Ego? It is very difficult to answer now because the only theoretical tool that we have at hand is attention itself, so that a *circus vitiosus* could threaten our investigation. The only thing we can say for the moment is that to the

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[22] *Hua. XXVI*, p. 19.
[23] *Ideas I*, p. 72.

three modes of attention are certainly correlative to three places in the field of consciousness: the given, the co-given and the horizon of givenness.

In the *Ideen*, Husserl insists many times on the obvious fact that, although they do modify the conscious connection to the object, the "attentional changes" leave nevertheless the *noema* entirely unchanged in its internal composition. Consequently each "attentional mutation" changes solely the consciousness attitude towards the object, but not the object in itself.

> It is clear that throughout such alterations, the noematic composition of the mental process remains the same in so far as one can always say that the same objectivity is continuously characterized as being there in person, presenting itself in the same modes of appearance.[24]

Whereas attention brings constantly to light new aspects of the object, Husserl considers however that the noematic core on its side doesn't change. The attentional mutation affects directly the relation between noesis and noema, but not the internal components of the noema. It is then a more subjective than objective feature of consciousness since the noematic nucleus is still the same. The noema, as taken in its essence, is not altered by the *attentional mutations* not much more than an perceptual object is internally modified by the sunlight. The alterations affect only "the distribution of attention and its modes."[25] One can say that the noema, as it is taken by a lived apprehension, is not the always same, but the noema, as it is in itself, can't change. Therefore the Husserlian theory of attention always presupposes the invariability of the object and that attention in itself pertains essentially to a modification of consciousness but not of conscious objects. The noema must be surely changed, but not by attention. Only a real intentional act can modify the noema in its core. For its part, attention modifies nothing more than our conscious connexion to the object. This means that attention depends first of all on internal factors of consciousness.

Briefly, the attentional changes presuppose:

1. the presence of a noematic core
2. that they do not alter the correlative noematic production

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[24] *Ideas I*, p. 223.
[25] *Ideas I*, p. 223.

3. that they exhibit alterations of the whole mental process with respect to both its noetic and noematic sides

These are the structural factors of attention which depend on the general field of consciousness. For Husserl, correlative to structural features of the field of consciousness, there are different modes of *being attentive to*. The three modes of the consciousness of the field echo the three main spots in the field of consciousness (theme, thematic field and unthematic or marginal field). Attention can move from the core to the margins of the field and take place where it wants to be, but it can't really alter the object *qua* object. Although the attentional changes modify our apprehension of the object, they don't come under the reach of noematic correlates. They are specific features of consciousness *qua* consciousness. When I see the pencil before me, on my table, I don't pay attention to the table itself, its colour, its matter, not event to the room nearby, its form and its background lighting. In such a case, the object of attention overshadows usually its objective environment. Therefore this distinction between primary and secondary attentions doesn't square with the objective facts. It belongs only to the attentional mutations of the ego-ray. So one can say that the attentional change "consists merely of the fact that, in one of the compared cases, one moment of the object is favoured and, in another case, another."[26] These attentional alterations affect surely the noema but, adduces Husserl, "without touching the identical noematic core." Then the phenomenology of attention reaches here an important element of the intentional analysis: the noema is always divided between the noema as it is for me and as it is in itself, in its independent core.

Just in this way, remarks Husserl, the metaphor of light concerning attention and its modes is fully appropriate:

> Attention is usually compared to a spot light. The object of attention, in the specific sense, lies in the cone of more or less bright light; but it can also move into the penumbra and into the completely dark region.[27]

This metaphor which is, according to Husserl, far from relevant when it is a question of distinguishing phenomenologically all the modes of attention, is still designative insofar as it indicates alterations in what appears as what

[26] *Ideas I*, p. 223.
[27] *Ideas I*, p. 224.

appears. In other words, this metaphor makes us understand that attention does not alter at all what appears with respect to its own sense-composition, but just modifies "its mode of appearance"[28] from brightness to darkness.

In this light, the structures of the field of attention don't only concern the perception, but affect as well imagination, memory and pure meaning. It's worth here noting that every conscious object as a thing, a logic ideality or a person could be affected differently by attention.

2. Affection and Interest

But in his last works, since 1920, Husserl attempts to introduce new elements in his phenomenology of attention. Among them affection and interest play a significant role.

It is quite interesting to note that, in the *Ideen*, Husserl always understands and defines consciousness only as a pure activity of the ego. Acts of attention refer essentially to the actual performances of the *ego cogito*. Furthermore attention belongs to this activity, insofar as it could happen only by the means of the ego-acts. It deals with the different acts through which something is given as the target of the ego's intentionality. Each ray of attention, indicates Husserl, "presents itself as emanating from the pure Ego and terminating in that which is objective." Attention can't ever be detached from the Ego, so that it always remains related to an ego-ray, even when it occurs in its incidental manner. It is an "actional" (or actual) mode of consciousness.

> If an intentive mental process is actional, that is, effected in the manner of the cogito, then in that process the subject is directing himself to the intentional object.[29]

Each attentional attitude pertains to this activity of the aware and actual ego cogito. The ego has the freedom to direct itself to anything whatsoever. Attention is accordingly understood as a ray emanating from the ego's spontaneity, so that it is always said to be "actional." The actional ego lives

[28] *Ideas I*, p. 224.

[29] *Ideas I*, p. 75. Commenting on this text, Gurwitsch notes that "problems of attention concern the peculiar nature of acts through which something is experienced as theme," ibid., p. 215.

within its acts as free to do what it wants, as the pure subject of the acts. In this light, attention depends exclusively on the free ability of the ego.

> It is in their actionality modes that attentional formations have, in a pre-eminent manner, the characteristic of subjectiveness.[30]

This leads us to conclude that, conversely, all the passive processes of mind are inattentive and *vice versa*. Husserl confirms this point: "what goes on in the stream of mental process outside the ego-ray or the cogito is essentially characterized otherwise"; it lies outside the ego's actionality and yet it is relevant to him insofar as it represents "the field of potentiality for the ego's free acts."[31] Husserl then calls this non-actionality mode of consciousness: "inattention."

But one may ask if the secondary attention and even more the horizontal attention don't entail the presence of passive states of mind, of something undergone mentally, that does not amount to pure activity of the ego. On closer inspection, one may observe that the presence of the halo of consciousness implies a passive mode of being, given the fact that it is not present in the actional or actual ego-rays and that all the attentional modes of consciousness are said to have the actional form of the ego cogito. Husserl calls this peculiar presence of the co-objects before the ego converts himself to them: "affection." In *Erfahrung und Urteil* like in *Analysen zur passiven Synthesis*, Husserl clarifies the relations between actional attention which depends exclusively on the free ability of the pure Ego and affection which embodies a sort of passive and pre-reflexive attention. It is not our intention here to analyze these complex relations further but we can say however that the horizontal mode of consciousness is now interpreted by Husserl as a "passive synthesis," since the ego-activity concentrates itself only upon the primary and secondary attention. The structural distinction between *primary attention* which corresponds to the "seizing upon," *secondary attention* as "just noticing something" and *horizontal attention*, where we can already find the future specifications of consciousness made by Gurwitsch in *Field of Consciousness* (theme, thematic field, and marginal field) turns now into a modal difference between ego-activity and ego-passivity.

[30] *Ideas I*, p. 225.
[31] *Ideas I*, p. 225.

In a proper sense, attention is defined by the conscious advertence of the ego to the affecting object. To pay heed to something means to turn the mental regard towards what it is already given albeit unnoticed. On its side, affection pertains to all the sensitive stimuli that excite the Self, but it is still a mode of intentional consciousness, so that it refers itself to a certain *being directed towards something*. As Husserl said in an unpublished manuscript (M III 3 III 3 II, *Studien zur Struktur des Bewusstseins*), written before the *Ideas*, circa 1910:

> In passivity the Self is also part of it in the mode of affection and then we may distinguish the modes of objective consciousness after the kind of affection which is not still become attention. Affection is at once an egoical mode of consciousness' operations.

Every consciousness, goes on Husserl, lives only either as affection or as attention, *tertium non datur*. The field of affection itself is already organized after a steady order similar to that of the attentional consciousness. The distinction between passivity and activity, or affection and attention, depends exclusively on the degree of the ego's commitment.

It would be false however to consider these structures of the field of consciousness, even thought as genetic modes of consciousness as activity and passivity, as the sole factors of attention. In fact, according to Husserl, they constitute a necessary but not sufficient condition of it. When I pay attention to something, I am not only directed towards it in a primary or secondary way, with an active or passive attitude, but I am specially *interested in* it, busied with it. The object which I am directed to becomes then, with the supervening of many sorts of interests, the theme of my attentional rays of consciousness. For Husserl, contrary to Gurwitsch's statements,[32] the theme doesn't amount to the mere object as it is given and present before our mind. Because of its close relatedness to the doxic interests of the ego, it endorses above all a subjective commitment that the position-taking discovers. In a word, the thematic consciousness implies much more than *being turned to* something given.

For Husserl then, it seems that the structural factors which consist in primary, secondary, and background attention don't really account

[32] Cf. "Phenomenology of thematics and of the pure ego," in *Studies in Phenomenology and Psychology*, p. 183: "When we speak of the theme of an act of consciousness, we mean, accordingly, the object as it stands before our mind, as it is meant ad intended through the act in question."

for the theoric or affective reasons which henceforth drive me to focus on this particular thing and no other. They certainly draw the dotted lines that attention is going to follow, but they can't really explain either the reason of the attention or its goal, because they are not the main causes of it. Already before the *Ideen*, Husserl defines attention in terms of *being interested in*. In the *Lessons about Signification* (1908), Husserl is observing that attention can't reduce itself to remark something in a primary or secondary way. This claim allows us therefore to consider that attention is not just a matter of intentive *directionality* towards something, but entails also a specific attitude of mind. Husserl introduces then a new distinction between, on one side, the different ways to remark something that include primary, secondary and horizontal attention, and on the other side, attention conceived now as a selective interest, as the variable busyness with something. Therefore attention means actually two different things: to *be directed towards* something and to *be interested in* something. I could *be directed towards* a landscape without *being interested in* it or more exactly *being interested in* something else. This attentional interest implies a peculiar preference. Here we deal with a special meaning of attention. As notes Husserl: "There is something new here to come out: to pay attention in a particular sense."[33]

What's new that interest brings to light? Actually when a consciousness is *interested in* something, it makes, says Husserl, this something given to it as its "theme." This is consequently a "thematic intention," an intention that turns the intended object into a theme, into something posited at first by the doxic consciousness and then that remains for a while one and the same. When I pay attention to something given in this thematic sense, I am concerned with it. It's not enough, says Husserl, to *be directed to* something, to remark it as such in a primary way, this privileged thing

[33] *Hua. XXVI*, p. 21: "if the schoolmaster says "let be attentive to this object" he aims at a such kind of attention. He doesn't aim at the fact of *being directed to* in a primary way, in the sense of the primary remarking, but the fact of living-within-this-being-directed-to-the-things or better the fact of turning-them-into-something-thematic." For this reason, the fact that attention lends itself to both a structural and a thematic explanatory betrays its ambiguous nature.

must be also the "exclusive theme" of your concern.[34] For this very reason, something could attract my whole attention and become remarked, as a sudden noise, but nevertheless I am not occupied with it, my interest goes still to something else. When my attentive interest is absorbed in a thought, a memory or an image, I am resisting every other solicitation, however strong it is. This distinction between attention as remarking and attention as interest shows that, for Husserl, just remarking something is completely void of interest. But on the other hand thematic interest doesn't deny the structural factors of attention and its ternary structure, but uses them. So, by virtue of this connection, Husserl can adduce here that interest makes its theme as a primary remarked thing.

At this stage, Husserl argues that there are actually two major factors of attention: the structural ones that depend on the "autochthonous organisation of the field of consciousness,"[35] and the thematic ones that are solely a matter of interest,[36] namely a matter of a certain way of intending something. In every case, the factors combine themselves partly without contradicting each other. Interest can only pick out in the field of consciousness what is already singled out on its own. This distinction between remarking and interest made in 1908 and surprisingly forgotten in the *Ideen* is retaken at new cost in *Erfahrung und Urteil*. In this text, published in 1938 by his assistant L. Landgrebe, Husserl insists almost exclusively on these thematic factors of attention. Attention is defined nevertheless as a "doxic orientation." We must recall here that 20 years before Husserl was distinguishing attention and doxic orientation, specially in

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[34] A. Gurwitsch elucidates this point as follows: "calling the something with which we concern ourselves the theme of our busiedness, we accordingly designate by "cogito" those acts in which we actually busy ourselves with a theme and for "cogito" the expression "thematic consciousness," "consciousness of a theme," can be substituted, so that the terms "theme" and "thematic" always connote actual busiedness," in *Studies in Phenomenology and Psychology*, p. 177.

[35] A. Gurwitsch, *The Field of Consciousness*, Pittsburg, Duquesne Univ. Press, 1964, p. 31.

[36] As followers of Husserl, one can assert that Gurwitsch and Schütz have sought in these two directions: the structural components of the field of attention for the former and the essence of the relevant interest of consciousness for the latter. We refer here to the main book of Schütz on this problem, *Reflections on the problem of Relevance*, ed. by R. Zaner, New Haven-London, Yale University Press, 1970.

the lessons of 1906. Related now to interest, attention becomes anew doxic and thetic. It includes consequently a position-taking respecting the already remarked object. This is then an orientation guided by an interest, namely a "belief in action." This approach allows one to conclude that the structural factors of attention, those belonging to the "autochthonous organization of the field of consciousness," are not sufficient here to account for the emergence of the phenomena. A selective process underlain on interest must occur to explain the thematical presence of something.

But this privilege of interest in order to give an accurate definition of attention leads Husserl to question about the nature of interest. According to him, interest could be understood in two manners: (1) as *inter-esse*, i.e., as *being beside*. In that sense, interest means to *be directed towards* something and to look for an intuitive fulfilment within it. It amounts to a mental tendency to seize the object as given in itself (*selbstgegeben*), namely to a cognitive "curiosity"; (2) interest in a broad sense means "making the object a theme of our attention."[37] Here Husserl recalls that interest is not equal to an objective orientation, because my theme could be interrupted by a strong noise coming from the street and yet remain my only interest, so that when the noise fades away I will turn back towards my theme. For a while my theme has melted into the perceptual background but it is still there at hand, maintained in grasp. We must underline besides that, for Husserl, there are always many kinds of interest, not specifically pragmatic or merely linked to a practical action linked to vital needs. An attentional interest can belong to thought, affection or will. Despite appearances Husserl doesn't claim a pragmatic conception of attention. He just points out that attention is always related to a doxic attitude of the ego which he calls "interest." We must recognize here that attention for Husserl is both "structural" and "thematic," on one side connected to the specific organisation of the field of consciousness and on the other dependent upon the intervention of sundry mental motives as ideas, feelings of pleasure and pain, practical interests and so forth. To the question asked before whether attention is the cause or simply the effect of the organisation of

[37] *Erfahrung und Urteil, Untersuchungen zur Genealogie der Logik*, Hamburg, Claassen & Goverts, 1954, p. 92.

consciousness, we have now to answer: both. For there are two factors of attention, structural and thematic.

One could ask here however if the thematic factors of attention don't represent actually another way to understand the structural factors. In the case of perception disturbances, is there really a conflict between primary attention and interest, between *being directed to* and *being interested in*? But rather than distinguishing two factors of attention, mustn't we say that the perceptual field of consciousness is competing here with the intellectual field? In this way there wouldn't be in fact a difference between structures and interest, but just a mere superimposition of two fields of consciousness. In other words, when I am disturbed during my attention on a specific object by another object coming from the background, I am experiencing then the possible *competition* between two fields of attention. Unfortunately Husserl doesn't dig in this direction and stops his analysis at the mere distinction between *being directed towards* and *being interested in*. The main drawback of the Husserlian presentation of attention is therefore that it distinguishes two sorts of attention as if the living ego in its performing acts was experiencing the crossing from one to the other.

It is easy to show however in this case that there is already an interest in the background's attention, so that my attention could ever *divide itself* in two directions, namely in two different fields of consciousness (why not even two attentive objects in the same field of consciousness, for instance the perceptual?). When my activity of thinking is interrupted by a sudden noise, I shift my mental regard from a field of attention (thought) to another (external perception). But I cannot move from a field towards another by the core, i.e., the central theme, but only by the margins. The theme cannot be replaced immediately by another theme from another field of consciousness. For reaching the central interest of my mind, the next theme has to go first through the margins of the former. Consequently the four different attentional fields of consciousness communicate with each other only by the marginal zone of the field of attention. In the case of attentional disturbances, the sudden noise coming from the marginal zone tries to get the central place. This is not here a conflict between *being interested in* and *being directed towards*, because it's the same thing, but rather between two central themes of different fields of attention. When Gurwitsch draws our attention on the distinction between the "object" and the "topic" he wants to make us sensible to the conflict between two attentional fields.[38] When I speak a foreign language, I am attentive both to the perceptual letters and to the sense. The material word is the object of my perceptual attention and its meaning represents the topic of my intellectual attention; but if I were a poet it could be the contrary, and the meaning of a word could become just the object of my attention and its appearance its pure topic. In this case, there aren't two sorts of attention, but a superimposition between two fields of attention. Attention remains always the same, but in different fields of consciousness.

As we have seen before, attention could merge into an intentional act: perception, memory, imagination, and meaning. There is not on one side a structural field of attention and on the other many mental interests, but my interest could go through several fields of attention. That is to say that the structural factors and the thematic factors are *in fine* perhaps the same. I can live at the same time in different fields of consciousness, for instance in a perceptual and in a intellectual fields, but I am only interested in one. From a perceptual point of view, I am directed to the letters on my paper, to physical things. But my interest is turned exclusively towards the meaning of the words. Accordingly that attention, though it is related to the structural factors of the field of consciousness, depends first and foremost on the living interest of the mind. Notwithstanding its ability to single out something in the unnoticed perceptual background, attention doesn't create the organization of the field of consciousness, but it does use it in order to pick out what is relevant to it. Unlike Gurwitsch, it is difficult to admit that the relation between the given and its thematic field form the only determining factor of attention, for the structural factors of the field are unable to favour one thing rather than another.[39] Even if

[38] It is to be noticed that Gurwitsch takes attention into account above all in accordance with its possible alterations. That is to say, it is not attention as having something in view, as amounting to an attitude of position-taking, that interests him mostly but the ways attention modifies itself when the theme changes its own relation to the thematic and the unthematic fields of consciousness.

[39] Owing to his distrust pragmatical arguments, based on the supervening of memories, affective motives and so forth, Gurwitsch considers that attention is mainly dependent upon the internal relations between the theme and its thematic field, so that our *attentional attitude* follows essentially from the thematic organization of the field in which our theme is inserted. Cf. "We never deal with a theme *simpliciter*; instead, we confront

he overlooks the possibility of simultaneous fields of attention pertaining to different intentional realms, Husserl has nevertheless the inkling here that the organization of the field of consciousness is not equivalent to the selectivity of the mind.

In addition, it is important to note that attentional and thematic interest cannot contradict for him the structural factors and that it is then continuously bound to them; but, on their side, these essential factors cannot produce in turn a such thing as attention. The structural factors are just topical components of the consciousness that help to understand how perception is ordered, but that can't explain why it is this thing and no other that is aimed at by the attentional ray. On the contrary, the thematic factors form the dynamic components of attention. For Husserl there is absolutely no doubt that the fields of consciousness are already organized with regard to the ternary structure that Gurwitsch will study thoroughly, but it is much more evident for him that this previous structural order is not enough to trigger by its own means the specific attitude of consciousness that reveals itself in attention. To conclude, it seems that, in explanation of the origins of attention, Husserl bestows his favour mainly on the thematic factors of interest, for they are, in the last resort, the key-component of the mind's selectivity.

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a theme standing in a field. Our attitude is determined by the thematic field, and we deal with the theme as pertaining to this field," ibid., p. 203.

CHAPTER 2

PHÉNOMÉNOLOGIE ET MÉRÉOLOGIE DE LA PERCEPTION SPATIALE, DE HUSSERL AUX THÉORICIENS DE LA GESTALT

Luciano Boi

«Une seule perception peut (. . .) englober dans son unité une grande variété de modifications.»

E. Husserl

«Unsere Raumwarhnehmungen haben im allgemeinen den Charakter einheitlicher Verbände.»

W. Köhler

«I have the perception, I live in it. I do not perceive my perception, but rather its intentional object. The perception itself is no object but an act. However, it is a fact, and a fact of momentous import for the structure of consciousness, that I am able to become introspectively aware of my perception. So to speak, I then split myself into two parts and gaze with the eyes of mind upon my own perceiving activity. The perception itself changes, by this process of reflection, into the object of a new act, of an act of presentation or introspective perception. But then again I have this new inward perception – my life is immersed in it – that refers to the first perception as its intentional object.»

H. Weyl

Abstract. The aim of this chapter is to analyse some aspects of the relationship between geometrical properties of both physical surrounding space and neurophysiologic space, and the structures of perceptual systems. Our main hypothesis is that perception represents a fundamental way of knowledge of the phenomenal world supplied with certain spatial features and of the objects in it that are provided with variable (context-dependent) geometric relations. Furthermore, these geometric properties play an important role in the processes underlying the formation of "objects' perception." According to this perspective, the geometry of the phenomenal world shall be viewed not more as a static given concerning solely the pattern of physical stimuli, but rather as a very dynamic process directly involved in the constitution of that world. For example, receptors are *stimulated* whereas an organ is activated: the eye is part of a dual organ, one of a pair of eyes, and they are set in a head that can turn, attached to a body that can move from place

L. Boi et al. (eds.), Rediscovering Phenomenology, 33–66.

to place and this body actively interacts with some natural environment in which our actions take place. These make a hierarchy and constitute what Gibson has called a *perceptual system*. Further, we stress, as a good example of Wertheimer vision of a whole determining the behaviour of its constituent parts, the experimental verification of the fact that what a visual critical neuron responds to best depends more on the properties of the overall configuration in the visual field than on the parameters of the stimulus in its receptive field. More generally, we argue that consigning perceptual imperatives in the process, e.g., of patterns vision, solely to neural operations in the primary visual cortex cannot be the whole and satisfactory explanation. In particular, spatial and globally invariant rules (good continuity, inner connectivity, colinearity, closure, or other global or topological arrangements) between perceptual components rather than within physical or neural stimuli can showed to primarily act in perception.

Introduction

Le propos de ce travail est d'analyser quelques aspects du rapport entre géométrie et perception et, en particulier, du phénomène de la perception spatiale. On cherchera à développer l'hypothèse selon laquelle la perception est d'abord et avant tout un mode de connaissance du monde phénoménal pourvu d'une certaine organisation géométrique. C'est pourquoi l'éclaircissement du rôle que jouent des propriétés géométriques fondamentales du monde physique et des objets qui l' «habitent», peut aider à mieux comprendre comment les structures perceptives se constituent. Un premier point à souligner est que ces propriétés géométriques (métriques, topologiques, texturales, optiques, etc.) des «choses spatiales», au lieu d'être un élément somme toute secondaire par rapport à la constitution intentionnelle «objective» du monde phénoménal, comme encore Husserl et les théoriciens de la *Gestalt* en quelque sorte le croyaient, prennent una part fondamentale dans le processus de formation des «objets de perception».

Rappelons les principales positions théoriques relatives à cette question. Le physicien et psychologue Ernst Mach (l'un des fondateurs, avec Herbart, Weber et Fechner, de la psychophysique) soutint notamment l'idée que l'espace et le temps sont deux attributs primordiaux inhérents à la perception, mais il avait pensé une telle relation dans un cadre empiriste, où la sensation provoquée par des stimuli physiques élémentaires constituait le donné premier. Carl Stumpf et Edmund Husserl sont allés, à cet égard, beaucoup plus loin, et on peut d'ailleurs les considérer à juste titre comme les vrais fondateurs de la *Gestalttheorie*. Le premier a insisté sur le fait que les caractères propres à la perception ne peuvent pas être compris indépendamment des propriétés et des lois physiques objectives qui caractérisent les phénomènes comme tels dans le monde «naturel».[40] En ce sens, il s'est démarqué de la conception psychologisante des *actes intentionnels* de Franz Brentano. Le second, qui du premier a été disciple, s'est en particulier attaqué à montrer, notamment dans ses recherches fondamentales sur la «chose spatiale» et sur la constitution des mondes naturel et psychique,[41] que s'il est vrai que des groupes de relations géométriques sous-tendent la formation de la chose spatiale en tant que phénomène «objectif», celle-ci ne pourrait pas cependant s'affirmer comme unité autonome, comme totalité cohérente englobante sans l'intervention de la conscience intentionnelle ou noétique pour structurer le monde phénoménal pré-spatial.

Nous chercherons ici à montrer qu'il est possible de réconcilier ces deux perspectives. Il faut pour cela concevoir la géométrie du monde phénoménal, non pas comme un donné ou un élément statique ne concernant que la configuration des stimuli physiques ou la simple forme apparente des objets, mais bien plutôt comme un processus profondément dynamique directement impliqué dans toutes les phases de la constitution de ce même monde phénoménal.

1. Remarques sur quelques Modèles Géométriques de la Perception

On peut distinguer quatre niveaux d'investissement de la géométrie dans la perception.

(*i*) Le premier niveau a trait à des propriétés géométriques fondamentales qui caractérisent la structure et les mouvements des objets tridimensionnels dans notre espace ambiant, comme leurs symétries et

[40] Stumpf a approfondi cette question théorique dans ses recherches théoriques et expérimentales sur les sons musicaux, dont les résultats sont réunis dans l'ouvrage *Tonpsychologie*, 2 vol., Hirzel, Leipzig, 1883–1890.

[41] Respectivement, dans *Ding und Raum – Vorlesungen 1907*, et dans *Ideen II: Phänomenologische Untersuchungen zur Konstitution* (1e éd., 1952).

les lois selon lesquelles elles se composent pour former des groupes de transformations (de ces mêmes mouvements). Ce sont là des propriétés objectives et indépendantes du sujet, mais qui se trouvent à avoir été en quelque sorte «intériorisées» par les mécanismes de la perception au cours de l'évolution. De plus, elles peuvent être fonctionnellement modifiées et/ou étendues par des séries de mouvements de notre corps et de déplacements moteurs volontaires que l'on peut exécuter par rapport aux objets et à leurs positions dans l'espace.

(*ii*) Le deuxième niveau est relatif aux propriétés géométriques des surfaces qui servent d'interface entre le monde des objets physiques et les systèmes sensoriels du sujet percevant. Il s'agit en particulier des lois de l'optique géométrique en accord auxquelles les rayons lumineux se propagent dans l'espace (lois de la réflexion et de la réfraction, principe de Fermat, etc.), ainsi que des propriétés physiques de la lumière et plus précisément du champ électromagnétique. On a affaire ici aussi à de propriétés objectives, mais qui en même temps influent sur la façon dont on perçoit les objets du monde extérieur, tout particulièrement la formation des images visuelles.

(*iii*) Un autre niveau important inclut, en plus des propriétés géométriques, les caractéristiques physiologiques des systèmes perceptifs, et spécialement du système visuel. Il faut bien voir à ce propos que tout champ perceptif, et tout particulièrement le champ visuel, se constitue non seulement à partir de la nature physiologique de l'organe sensoriel concerné, mais encore et surtout d'après les propriétés géométriques (métriques et topologiques) propres au champ.

(*iv*) Il faut enfin souligner que la perception d'un objet extérieur quelconque, et plus généralement la reconnaissance de n'importe quelle forme dans l'espace ambiant, nécessite de la part de notre organisme la mise en place puis l'exécution d'une ou de plusieurs stratégies spatiales relatives, par exemple, à la locomotion et/ou à la tactilité. Or, la mise en place de telles stratégies comporte que l'on soit capable de reconnaître (en vue notamment de l'action et de la préhension) un certain nombre de propriétés géométriques fondamentales de notre espace ambiant.

On peut dès lors poser trois questions qui nous paraissent fondamentales: Quels sont les modèles spatiaux qui sous-tendent la reconnaissance des formes visuelles? Quel est le rapport entre formes perçues et cognition?

Quelle est la nature de la relation du «tout» aux «parties», ou entre le continu et le discret?

Pour tenter de répondre tant soit peu à ces questions, on va commencer par admettre l'existence de quelques modèles géométriques qui très probablement participent des mécanismes perceptifs impliqués dans la reconnaissance de la forme et des propriétés spatiale(s) des objets. De plus, on supposera que ces modèles sont pour ainsi dire des *invariants* de la perception. Une telle hypothèse nous paraît nécessaire pour expliquer le fait que, malgré la grande variété des circonstances physiques variables dans lesquelles ces objets peuvent se trouver, et des états subjectifs différents pouvant affecter notre perception, le système perceptif central est capable d'extraire les quelques caractères réguliers et constants des objets, et tout d'abord leurs *formes* qui sont des organisations dynamiquement stables. Or cela n'est possible que si l'on postule une corrélation entre les mécanismes inhérents à la perception et les lois géométriques de transformations des objets rigides dans E^3.

(i) Un premier modèle est celui des *cut-locus* élaboré par Harry Blum et généralisé ensuite par René Thom.[42] . Les auteurs proposent d'étudier les propriétés globales des formes pour lesquelles la configuration spatiale spécifique est particulièrement significative, notamment des propriétés topologiques comme l'ordre de connexité, la nature homologique ou le type de bords. Ce modèle suggère que la forme intrinsèque d'un objet, au lieu d'être définie analytiquement par un système de coordonnées cartésiennes, doit être reconstruite à partir de la nature qualitative de la courbe et de la surface formant le «squelette» de l'objet. En d'autres termes, les aspects les plus saillants de la forme d'un objet peuvent être obtenus une fois qu'on a caractérisé l'ensemble de ses singularités définies sur un espace approprié. D'une manière générale, ce modèle présuppose l'existence de mécanismes simples à la base du processus de reconnaissance des formes

[42] Voir: H. Blum, "Transformation for Extracting Descriptors of Shape," in *Models for the Perception of Speech and Visual Form*, édité par W. Wathen-Dunn, The M.I.T. Press, Cambr., Mass., 1967, pp. 362–380; R. Thom, "Perception et Préhension," in *Apologie du logos*, Hachette, Paris, 1990, pp. 162–182. Pour une présentation de la théorie mathématique des cut loci, cf. S. Kobayashi, «On Conjugate and Cut Loci», in *Global Diffential Geometry*, Studies in Mathematics, vol. 27, S. S. Chern (éd.), Math. Assoc. Amer., 1989, pp. 140–169.

qui permettent d'en explorer la géométrie sous-jacente et les fonctions visuelles.

(*ii*) Un deuxième modèle, qui occupe une place fondamentale dans plusieurs théories de la perception, fait intervenir le concept mathématique de *groupe de transformations*. L'idée en gros est que l'organisme humain pourrait avoir «intériorisé» certains principes qui gouvernent les transformations rigides des objets dans l'espace usuel. Il s'agit notamment d'un principe de moindre action, et d'un principe relatif au groupe de transformations dont sont susceptibles les corps rigides dans l'espace. Ces principes, qui pourraient s'être conservés invariants à travers l'évolution de notre espèce, peuvent être formulés abstraitement en les termes de la structure qu'ils permettent de conférer à la variété des positions possibles d'un objet dans notre espace tridimensionnel. Puisque la situation générale d'un objet dans l'espace euclidien présente trois degrés de liberté relativement à son mouvement, et trois autres degrés de liberté quant à son orientation, l'ensemble de ses positions possibles correspond alors à une variété à six dimensions. À chaque paire de positions arbitraires d'un objet dans l'espace, correspondent deux points distincts sur la variété, et les différentes structures que ces principes physiques et géométriques permettent de définir sur la variété, prescrivent des familles de chemins géodésiques, c'est-à-dire les déplacements «naturels» de l'objet d'une position à une autre. Ajoutons que l'observateur a tendance à se représenter le mouvement d'un objet étendu dans l'espace principalement autour des axes qui sont centrés en l'objet et qui sont déterminés par sa structure géométrique (système égocentrique), au lieu qu'autour des axes qui sont fixés par rapport à l'environnement (système héliocentrique). On peut ainsi affirmer que la reconnaissance des formes dépend principalement (mais pas seulement) de ce que les propriétés perceptives des objets sont conservées par le groupe d'isométries relativement, en ordre d'importance, au parallélisme, à l'orientation, à la symétrie principale et au référentiel spatial (ou repère mobile) privilégié.

(*iii*) Un troisième modèle fait intervenir le concept géométrique de *connexion*. En bref, il y a des bonnes raisons de penser que les stratégies adoptées par l'organisme humain pour la discrimination, la reconnaissance ainsi que pour la saisie des objets divers situés dans son espace environnant, pourraient obéir aux mêmes lois géométriques fondamentales (métriques et topologiques) qui caractérisent *la connexion sur une variété*. L'ensemble de ces structures définissent ce qu'on peut appeler *l'espace global de la perception* (voir plus loin pour des considérations plus précises à ce sujet).

2. Variétés Géométriques et Variétés Perceptives: de Riemann à Weyl

Originellement, le concept mathématique de *variété* recèle deux idées fondamentales: celles de «grandeurs connexes continues» et de «formes sérielles continues», introduites respectivement par Gauss et Herbart dans des contextes différents.[43] Ce qu'on voudrait montrer ici, est que les notions d' «objet connexe» et de «forme sérielle» jouent un rôle important dans la perception spatiale. Selon Riemann, la notion de continuité constitue la propriété fondamentale des «objets» mathématiques appartenant à la théorie des variétés différentiables. En particulier, il s'agit d'une propriété qui *a priori* détermine toutes les variations dont sont susceptibles les «objets» géométriques définis sur la variété. Mais on peut aussi parler de la continuité primitive d'un objet mathématique qui, ayant été éventuellement déplacée ou brisée à cause de l'apparition de certaines discontinuités (singularités), peut être en principe restaurée, notamment par recollement des morceaux qui composent l'espace (bien qu'il existe d'autres techniques de recollement et de recomposition de l'espace). Le premier cas comprend essentiellement les objets métriques (soumis à des relations plutôt rigides entre distances); le second, les objets topologiquement connexes (admettant des déformations plastiques).

Supposons donné le concept général de *grandeur* (mais rien n'empêche de prendre à la place une *quantité* physique ou une *qualité* sensible quelconques), alors, d'après Riemann, plusieurs modes de déterminations sont possibles, suivant qu'on peut ou non passer de l'une de ces déterminations à l'autre de façon continue, ou discrète. Les exemples de variétés discrètes sont très fréquents, aussi bien dans les mathématiques que dans la réalité. Il n'en est pas de même des variétés continues qui, tout en étant nombreuses en mathématiques, sont plutôt rares dans la nature. Riemann cite les exemples fournis par les lieux des objets sensibles (le champ de la perception dira-t-on) et les couleurs.

[43] Cf. L. Boi, *Le problème mathématique de l'espace* (Springer 1995), pp. 129–136.

Dans l'ouvrage *Espace-Temps-Matière* de 1918, H. Weyl reprend cette remarque de Riemann et montre que le concept de variété a une portée très grande dans le domaine de la nature. En effet, tout domaine d'objets, notamment en physique, peut être conçu comme une variété qui possède une certaine structure géométrique et dont les lois physiques sont déterminées par quelques paramètres fondamentaux définis dans l'espace de phases; ces paramètres doivent être pensés comme des fonctions qui varient de façon continue et différentiable sur la variété. Ainsi, les états uniformes d'un gaz parfait représentent une variété à deux dimensions d'après la pression et la température. Suivant le nombre de ces paramètres, on dira que la variété est à une, deux, trois ou plus de dimensions. Weyl va jusqu'à affirmer qu'on peut se figurer la plupart des domaines sensibles de la perception comme autant de variétés riemanniennes à deux ou trois dimensions. Il en est ainsi du système tonal, une variété déterminée essentiellement par l'intensité et le timbre (la hauteur du ton). Les qualités de couleurs comme telles (sans référence ici à la théorie des trois processus physiologiques: le rouge-vert-violet d'après Young-Helmholtz, et le noir-blanc, rouge-vert, bleu-jaune d'après Hering) forment aussi une variété déterminée par la qualité et l'intensité.

Sur ce point, Stumpf objectera à Weyl qu'il ne s'agit là nullement de variétés riemanniennes courbes et fermées.[44] Pour lui, les hauteurs du ton ne forment pas une *forme cyclique*, car on ne peut pas revenir au même ton simplement en élevant le nombre de vibrations ni on peut restaurer la même intensité du ton en augmentant la force d'oscillation. Les deux s'écoulent de façon linéaire dans l'infini, bien que, à cause de la constitution de notre organe sensoriel, elles aient le haut et le bas comme limites. Stumpf est pourtant forcé d'admettre que dans le disque des couleurs de Hering les qualités de couleurs reviennent au rouge après avoir passé par le jaune, le gris et le bleu, ce passage étant rendu possible par des nuances continûment graduées de certaines couleurs secondaires. Les tons aussi reviennent, en doublant le nombre d'oscillations, à un degré plus haut (dans l'échelle diatonique): l'*octave*. Cela étant, Stumpf conclut à une différence essentielle entre les attributs des impressions sensorielles et les dimensions d'ordre spatial: celles-ci sont homogènes et commensurables;

[44] Cf. C. Stumpf, op. cit.

ceux-là sont hétérogènes et incommensurables. Par conséquent, on ne peut pas définir d'une manière générale les domaines des perceptions sensibles en termes de variétés continues et courbes à plusieurs dimensions, en dehors des configurations spatiales particulières.

Husserl, ayant analysé à fond la même question dans *Chose et Espace*, parvient à des conclusions assez différentes. Il montre, entre autres, que le champ visuel est formé d'un système clos de mouvements oculaires (domaine fini) et d'un système ouvert de mouvements kinesthésiques (domaine infini). Le premier se comporte *grosso modo* comme un espace sphérique (riemannien) fini; le second, comme un espace euclidien (tridimensionnel) infini. Weyl montrera que les lois suivant lesquelles notre système visuel perçoit les objets extérieurs ressemblent plus précisément à celles qui caractérisent la géométrie projective bidimensionnelle.

3. L' Invariance et le Caractère Constant des Objets de Perception

On sait qu'un des problèmes les plus importants auquel toute théorie de la perception et en particulier de la perception visuelle se trouve confrontée, est celui du caractère constant des phénomènes perceptifs, autrement dit, de l'invariance de la position d'un objet dans la perception lors de mouvements volontaires des yeux ou de la tête, ou bien des deux simultanément. Or, plusieurs hypothèses théoriques et données expérimentales suggèrent que ces deux types de mouvements sont en fait géométriquement et fonctionnellement corrélés en accord avec un schème cognitif central. Phénoménologiquement cela peut être décrit comme suit: un sujet bouge le visage ou tourne la tête, l'image des objets dans le champ visuel proche se déplace sur notre rétine. Ce mouvement n'est pas perçu cependant comme un mouvement propre aux objets; ceux-ci sont perçus comme étant stationnaires. Une telle compensation du mouvement de l'image réalisée par nos systèmes visuel puis central est parfois à tel point involontaire qu'il n'est même pas enregistré consciemment comme un mouvement de la tête ou des yeux. Et il n'est pas enregistré parce que le changement de position qu'a subi l'image de l'objet sur la rétine, est traité au niveau des systèmes eux-mêmes. Certaines expériences que l'on peut faire sur notre propre œil sont à ce propos très instructives. Une première expérience montre que la constance de la position spatiale que l'on perçoit normalement lors d'un mouvement volontaire, résulte du fait que ce mouvement est «pris en charge» par l'activité intégrée de nos systèmes sensoriels.

Une autre expérience prouve que ce qui est traité par notre système visuel central n'est pas le mouvement volontaire actuel lui-même, mais la «commande» que reçoit le muscle chargé de l'effectuer. Supposons pour un instant que le muscle devant exécuter le mouvement ait été momentanément paralysé par l'injection d'une drogue. Dans ces conditions, où on a seulement l'intention de bouger l'œil mais non pas son mouvement, la «commande» transmise à l'œil d'un mouvement fait en sorte que l'image stationnaire d'un objet est incorrectement perçue comme un mouvement de l'objet. Ainsi, tout se passe comme si l'œil eût été bougé volontairement, et ceci parce que les systèmes visuels périphériques et central interviennent pour ajuster la position fixe de l'image de manière à compenser l'intention de mouvement. Cela prouve non seulement que certaines propriétés d'invariance que nous avons tendance à attribuer aux objets eux-mêmes sont en fait le résultat d'une activité compensatoire importante de nature proprioceptive, et que lors de ce processus la réponse de chaque système sensoriel se trouve *de facto* intégrée à l'action du système perceptif central, mais encore, que la perception est animée par un mouvement qui redéfinit et associe les paramètres de l'espace et du temps en fonction de ses diverses modalités et des différents contextes.

Outre les mouvements réflexes ou induits par certains mécanismes physiologiques, il faut considérer la classe très large des changements de position des objets qui sont compensés par un mouvement volontaire de notre corps. Ces mouvements peuvent être caractérisés par ce qu'ils permettent de ramener l'objet à sa position initiale à la suite d'un déplacement quelconque. De plus, ils permettent de répéter le même mouvement un nombre en principe infini de fois, de sorte que l'objet, ayant entre-temps subi plusieurs types de déplacements dans l'espace, nous apparaît néanmoins toujours comme invariant. (Naturellement, on fait ici abstraction de la possibilité que l'espace tout entier puisse aussi subir tel et tel déplacement, par exemple, changer d'orientation ou de symétries). Ces objets forment la catégorie des *corps solides*. De ce point de vue, notre corps avec ses divers organes sensoriels constitue un système de référence «idéal» auquel peuvent être rapportés tous les objets situés dans son espace avoisinant, de même que leurs positions relatives mutuelles, dans les limites bien sûr de quelques contraintes objectives et subjectives. Ils forment donc un groupe, le groupe des déplacements des objets dans l'espace de la perception. Ce groupe peut, à son tour, donner lieu à des sous-groupes qui peuvent être ou ne pas être isomorphes au groupe «principal». Remarquons que plusieurs grands physiciens et mathématiciens, notamment Mach, Poincaré et Weyl, ont cherché à expliquer la genèse de l'espace géométrique à partir de la notion physiologique de groupe de déplacements qui agit sur l'ensemble des impressions oculo-kinesthésiques. Cette idée conduit, en effet, à retrouver l'espace euclidien comme espace homogène du groupe des déplacements kinesthésiques.

Soulignons, dans ce même ordre d'idées, le rôle indispensable que joue l'ensemble des sensations musculaires dans la constitution de l'espace perceptif. En effet, elles fonctionnent comme un véritable organe des sens, le sixième sens a-t-on dit. Non seulement les sensations musculaires aident à l'action d'autres systèmes sensoriels, comme le toucher, la vision, et aussi les systèmes kinesthésique et vestibulaire, mais elles sont également la source de la sensibilité proprioceptive, qui joue un rôle important dans l'(auto)-représentation du corps.[45]

Husserl a bien décrit ces phénomènes relatifs à l'espace sensible et à la perception phénoménale, entre autres, dans *Recherches phénoménologiques pour la constitution* (1912-1928) et dans *Chose est Espace* (1907). Il a montré que toute une série de mouvements de l'image d'un objet s'effectuant à chaque fois dans des circonstances kinesthésiques précises ainsi que les phénomènes de compensation, d'illusion et de distorsion qui les accompagnent, contribuent à la structuration objective de ce qu'il appelle la «chose spatiale». Il est à noter également que le «corps spatial» se trouve ainsi déterminé par l'ensemble cohérent de tous ces mouvements effectivement réalisés et qu'à chacun d'eux correspond, en général, un acte perceptif de nature intentionnelle se déroulant dans le temps et dans l'espace. Ce sont principalement ces actes qui, se succédant d'une manière continue, «remplissent le corps spatial», et c'est grâce à eux que nous déterminons les qualités réelles d'une chose dans son devenir (intentionnellement) objectif.

[45] Voir à ce sujet l'intéressant ouvrage d'A. Berthoz, *Le sens du mouvement*, Odile Jacob, Paris, 1992.

Il est tout à fait remarquable que Husserl ait fait la distinction entre deux sortes de sensations qui ont des fonctions fort différentes et qui participent nécessairement entièrement à la constitution de la chose spatiale. Il conçoit le «corps propre» dans son double rôle de centre de toute orientation actuelle et virtuelle à partir duquel on a l'intuition de l'espace et du monde sensible, et de système de référence idéal (l'origine d'un repère mobile) auquel sont rapportés tous les déplacements possibles dans l'espace et par rapport auquel s'effectue la coordination cohérente de tous les mouvements actifs des parties de notre corps. Ces deux types de sensations sont cependant interdépendants et forment ce qu'on pourrait appeler, dans un langage moderne, un espace doué d'une *connexion*. Cette interprétation n'a été certes pas donnée, ni explicitement ni implicitement, par Husserl lui-même, mais il n'en demeure pas moins qu'elle s'accorde pour l'essentiel, comme nous le verrons, avec sa pensée sur le sujet.

Il y a en effet des sensations qui, "par les appréhensions qui leurs sont imparties, *constituent* dans des esquisses les traits correspondants de la *chose* comme telle. Il en est ainsi des couleurs données à la sensation avec leur déploiement sensible, dans l'appréhension desquelles apparaissent les colorations des corps avec l'étendue corporelle de ces colorations."[46] Il s'agit donc des impressions sensibles rattachées à différents moments du processus de perception. Il y a ensuite des sensations qui, tout en ne faisant pas l'objet de telles appréhensions, sont par ailleurs "*parties prenantes* . . . de toutes les appréhensions de ce type concernant d'autres sensations, dans la mesure où elles *motivent* celles-ci d'une certaine manière, en quoi elles font elles-mêmes l'objet d'une *appréhension d'un tout autre type* qui, ainsi, appartient *comme corrélât* à toute appréhension constituante." S'appliquant à des circonstances qui sont liées les unes aux autres, ainsi qu'à ce qui leurs est inhérent, ces «sensations *motivées*» (ou perceptions conscientes) obéissent à des schèmes tels que «si, alors » ou «parce que, donc». Ces «sensations *motivées*» se rapportent en particulier, dans l'aperception,[47] "à des systèmes de sensations kinesthésiques qui

[46] Cette citation et celles qui suivent proviennent de l'ouvrage *Chose et Espace (Leçons de 1907)*, chap. III, pp. 76–77 et sq.

[47] On voit que Husserl fait une distinction subtile entre «appréhension» (*Auffassung*) et «aperception» (*Apperzeption*): "L'appréhension . . . c'est le caractère d'acte qui anime

se déroulent librement dans leur ordonnance d'ensemble bien connue." Mais bien d'autres systèmes de sensations motivées entrelacées doivent à ce moment-là avoir lieu, et "c'est de cette manière que telles ou telles séries se déroulent dans la vision à partir du système ordonné des sensations des mouvements oculaires, des mouvements de la tête librement mue, etc. Tandis que cela se produit, se déroulent *dans l'ordre du motif* les «images» de la chose appréhendée par la perception au début de ce mouvement, ce qui inclut dans tous les cas les sensations visuelles qui en relèvent." Il faut ainsi insister avec Husserl sur le fait qu'"une *perception* est partout une *exécution unitaire* résultant par essence de la façon dont jouent ensemble *deux fonctions qui sont en relation de corrélation*. Il en ressort en même temps qu'à *toute perception* appartiennent *des fonctions de la spontanéité*. Les processus des sensations kinesthésiques sont ici des processus libres et cette liberté dans la conscience du déroulement est une pièce essentielle de la constitution de la spatialité."

Soulignons encore deux points. Deux moments fondamentaux contribuent au développement du processus de la perception. Ils sont à tel point liés qu'il est impossible de regarder l' «avant» et l' «après», la «sensation brute» et la «représentation mentale» comme des moments séparés ou superposés. Il s'agit d'*actes de perception* auxquels correspondent certains systèmes de sensations mais aussi des opérations de lissage puis d'interprétation de cette même information sensorielle. L'autre mérite de Husserl est d'avoir compris quelques caractères globaux de la perception.

(*a*) Le processus de la perception est un tout cohérent où chaque moment se trouve anticipé ou posé dans le cadre d'un système de sensations auquel correspond une série de remplissements objectivants de la chose spatiale.

(*b*) La perception forme une unité fonctionnelle qui se met en place à partir des interactions dynamiques diverses entre les observateurs et les propriétés phénoménologiques et spatiales que les objets inanimés et animés recouvrent dans leur environnement fixe et mobile.

(*c*) Bien que ses fonctions soient en partie spontanées, les actes de perception sont par essence de nature intentionnelle et ils impliquent une action consciente de nature intégrative de la part du sujet.

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pour ainsi dire la sensation et qui, selon son *essence*, fait en sorte que nous percevons tel ou tel *objet* . . . ."

4. La Structuration Intrinsèque du Monde Phénoménal Selon Husserl

Ce qui intéresse par dessus tout Husserl c'est d'analyser la structuration intrinsèque inhérente au monde phénoménal. Cette structuration ne peut pas être réduite à sa composante physique (matérielle), bien qu'elle en soit partie prenante, mais elle ne peut pas non plus être vue comme s'effectuant uniquement du côté de la pure subjectivité. Car c'est précisément cette organisation du monde phénoménal qui constitue le «milieu» à partir duquel se forment les structures signifiantes et les prégnances psychiques. Mais en même temps, cette organisation n'est autre chose que le monde spatial qui se constitue, selon diverses modalités temporelles, à travers le processus de la perception. On pourrait dire en définitive que la réalité phénoménale se situe à l'interface entre l'univers proprement physique et le monde psychique des représentations mentales et de la pensée conceptuelle. Le monde des phénomènes, qui est aussi celui des objets perçus et des formes apparentes, tout en étant tributaire des mondes physique et psychique, participe des deux en manifestant des structures signifiantes (ce sont les *remplissements* de Husserl) qui viennent s'ajouter aux modalités de l'étendue géométrique et physique et de la pensée.

Mais en dépit de la place importante que ce rapport entre constitution matérielle des corps et perception phénoménale occupe dans la conception de Husserl, il n'a pas pour autant l'idée que ce monde phénoménal puisse présenter une géométrie spécifique (voire des géométries spécifiques) qui lui confère une organisation et une signification propres. Il conçoit encore les deux réalités, celle des entités physiques et celle des phénomènes (c'est-à-dire le monde de l'être et le monde des apparences) comme séparés. Et il ne semble pas prendre en considération le fait que précisément ces apparences sont autant des modalités essentielles de l'être, ce par quoi en fait les substances reçoivent une forme. "Les déterminations géométriques, écrit-il, appartiennent à l'objet de la physique lui-même, le géométrique appartient à la nature en soi de la physique ; mais non pas les qualités sensibles qui relèvent entièrement de la sphère de la nature apparaissant." Husserl tire une conclusion encore plus radicale sur le plan philosophique lorsqu'il parle de l'opposition entre «concepts géométriques» et «concepts descriptifs»: les uns "sont des *concepts «idéaux»* ou exacts, ils expriment quelque chose qu'on ne peut pas «voir» et ont pour corrélât des essences qui ont le caractère *«d'idées» au sens kantien* du mot"; les autres sont "des *concepts morphologiques* portant sur des types vagues de formes qui seraient directement saisis en se fondant sur l'intuition sensible . . .." C'est dire, selon Husserl, que la géométrie n'a pas de fonction descriptive, et la morphologie n'est pas de nature géométrique. Mais c'est là une limite du projet phénoménologique, car, s'il est vrai que la géométrie n'est pas de nature descriptive, le monde des phénomènes et des formes peut, lui, posséder une structure géométrique à proprement parler, et c'est ce que nous tâcherons de montrer dans les pages qui suivent.

Les considérations qui précèdent doivent être cependant quelque peu nuancées. On peut, nous semble-t-il, distinguer deux phases fondamentales dans l'élaboration par Husserl du programme phénoménologique. Dans la première, qui coïncide avec les recherches aboutissant dans *Chose et Espace*, et dont on retrouve encore quelques développements dans les *Recherches phénoménologiques pour la constitution*, Husserl cherche à mettre en évidence les différentes structures géométriques qui sous-tendent la constitution phénoménale de la «chose spatiale». En ce sens, on peut dire que la signification des contenus de la perception est inséparable du type de géométrie que présente le champ phénoménal; celle-ci conditionne en quelque sorte les modes d'apparition de ceux-là. Dans les *Ideen* on assiste à un changement de perspective, et les contenus de perception sont conçus séparément des conditions objectives qui pourtant les accompagnent. Husserl oppose le statut sensible de l'apparaître au statut physique de l'objet perçu, et lorsque même il maintient un lien entre les deux, c'est pour revenir en quelque sorte à l'idée, remontant à Helmholtz, selon laquelle les perceptions ne seraient au fond que le *signe* de rapports réels existant entre les faits physiques, encore qu'il existe de profondes différences entre les deux conceptions.

Husserl écrit que "*le statut eidétique de la chose perçue*, par conséquent tout ce qui est là corporellement avec toutes ses qualités, et tout ce qui peut être perçu, est «*pure apparence*» et que la «*chose vraie*» *est celle que détermine la science physique*." "«*L'être vrai*» *aurait par conséquent des déterminations totalement différentes par principe de ce qui est donné dans la perception à titre de réalité corporelle*, celle-ci étant exclusivement donnée par des déterminations sensibles, au nombre desquelles appartiennent celles de l'espace sensible. *La chose proprement expérimentée fournit le pur «ceci», c'est-à-dire* *un* X *par lui-même vide*, *qui devient le porteur de déterminations mathématiques ainsi que de formules mathématiques correspondantes*; et cet X n'existe pas dans l'espace de la perception, mais dans un «*espace objectif*» dont le premier est simplement le «signe», à savoir *une multiplicité euclidienne à trois dimensions dont on ne peut avoir qu'une représentation purement symbolique*." On pourrait dire, en d'autres termes, que pour Husserl tout ce qui appartient à la sphère psychique proprement dite (actes mentaux, représentations cognitives, etc.) ne possède pas de caractère spatial, tandis que tout ce qui relève du monde phénoménal et qui est rempli par des qualités sensibles apparaissant par esquisses, n'est possible que comme spatial. Pour mieux illustrer une telle différence, il se sert de l'image suivante: la signification immanente de la qualité sensible (par exemple, de la couleur) est à l'unité identique (invariable) ce que le contenu empirique et physique de cette même qualité est à la variété continue de ses différents aspects sensibles changeants. Mais, en réalité, on peut penser cette «variété continue» dans les termes du concept mathématique exact de variété, douée d'une certaine structure géométrique et pouvant ainsi servir à rendre compte des propriétés phénoménologiques de divers champs sensibles. Pour Husserl, le concept de variété continue est moins un modèle mathématique, fondement d'une explication des phénomènes naturels, qu'un concept logique ou axiomatique régi par de pures relations formelles. Une variété (mathématique) abstraite est caractérisée par ce qu'un nombre fini de concepts et de propositions détermine totalement et sans équivoque l'ensemble de toutes les configurations possibles du domaine; cette détermination réalise le type de la nécessité purement analytique. Dans une variété définie de type mathématique on peut poser l'équivalence des deux concepts: «vrai» et «conséquence formelle des axiomes», de même celle des concepts: «faux» et «conséquence contraire formelle des axiomes». Par le concept de «variété» Husserl entend un domaine formel d'objets, comme corrélât objectif d'un système formel d'axiomes. Il n'admet pas qu'une variété, en tant que concept mathématique abstrait, puisse être le «support» d'un espace dans lequel se trouve réalisée une certaine géométrie phénoménale, ou même une morphologie.

Husserl s'est appuyé sur l'analogie entre la catégorie «qualité» couleur et le concept formel de «variété» à plusieurs reprises pour montrer que tout comme on ne passe pas de la variété à l'espace par de simples déterminations formelles mais uniquement à la variété euclidienne, de même, on ne passe pas de la qualité à la couleur ou à telle et telle autre couleur déterminée. Ici et là il faut que des «remplissements» d'une autre nature interviennent, au lieu d'opérations purement formelles. Autrement dit, ce qui transforme en espace la variété euclidienne est tout aussi peu un problème de principes formels que la question de la transformation de la qualité en couleur. En effet, comme Husserl le fait ici remarquer, et comme il le montrera amplement dans *Chose et Espace*, la figure spatiale est un moment dans l'intuition dans la mesure où elle détermine une constitution intrinsèque de l'objet phénoménal, aussi bien que sa couleur ou son poli. Mais on pourrait alors penser que "comme la catégorie «qualité» correspond à la couleur (c'est-à-dire le genre de constitution intrinsèque réservé de manière identique et indivise à l'objet tout entier, à toute partie de l'objet), de même la catégorie «variété» correspond-elle au «spatial», et cela dans la formation particulière que nous appelons variété homogène et, plus exactement, variété euclidienne tridimensionnelle." D'un certain point de vue, cette position peut sembler tout à fait justifiée du moment que l'on admet non seulement qu'aussi bien la notion d' «espace» que celle de «qualité sensible» sont d'un genre logique suprême, mais encore qu'elles sont des «substances premières» ontologiquement indivisibles. Mais on objectera à un tel point de vue ce qui suit:

(*a*) que c'est le réel qui est géométrique, et non pas la géométrie qui est réelle; c'est dire que même l'espace phénoménal peut présenter des propriétés géométriques intéressantes, comme il semble bien être le cas;

(*b*) que rien n'empêche de considérer que l'espace réel soit support de plusieurs «objets géométriques» abstraits définis sur des «variétés» douées de bonnes propriétés mathématiques, qui pourraient servir de modèles pour des espaces substrats dans lesquels on définit un ensemble de qualités sensibles et leurs transformations. Autrement dit, le concept de variété, bien qu'il soit le résultat d'une genèse idéale, est un principe d'intelligibilité apte à expliquer des phénomènes physiques fondamentaux ainsi que leurs modes de manifestation.

Une telle conclusion de Husserl qui conduit, comme on vient de le voir, à une conception dichotomique de l'*espace* et de la *variété*, est d'autant plus surprenante qu'ailleurs, et notamment dans *Chose et Espace*, il effectue constamment un déplacement du concept de variété hors de son contexte mathématique d'origine, pour l'appliquer au domaine phénoménal, celui de la constitution de la *res*. En tout cas, c'est grâce au concept de variété qu'il peut caractériser la structure du champ sensible phénoménal (spécialement du champ visuel), dans lequel se constitue par identification l'apparition unilatérale d'une «chose» et se définit pleinement la forme du «corps spatial». C'est-à-dire qu'il projette en quelque sorte les caractéristiques propres au concept de variété sur les propriétés générales des champs sensoriels (champ visuel, tactile et oculomoteur) qu'il considère comme autant de systèmes formels de référence corrélés les uns aux autres. Par ailleurs, il reste à savoir s'il y a indépendance de ces champs par rapport à la structure et aux lois d'organisation de l'espace objectif tridimensionnel. De même, au système fini et clos d'axiomes et de règles définissant le domaine d'objets d'une variété correspondrait le système unifié du champ, organisé et contrôlé en permanence par la légalité *a priori* que comporte l'unité de motivation kinesthésie/intention. Ainsi, les deux systèmes réalisent des *synthèses globales continues*.

Or il s'agit là d'un point nodal qu'il faut absolument saisir. En effet, supposons une série d'images (du champ visuel) i_p s'écoulant de façon continue dans le temps en coïncidence temporelle avec une série continue de mouvements (et sensations) kinesthésiques k_q. Cette continuité d'images (série de remplissements) est une variété linéaire, extraite d'une variété multidimensionnelle d'images possibles (d'une manière analogue, si l'on peut dire, on obtient l'espace euclidien abstrait R^3 à partir de l'espace euclidien de dimension n R^n, et d'ailleurs, toute surface ou variété abstraite se définit par un atlas de cartes et chaque carte est un ouvert de R^n), "laquelle, outre celle-ci, comporte encore d'autres variétés linéaires d'images en nombre infini, chacune étant, d'après son type déterminé, englobée dans le type d'ensemble déterminé de la variété d'ensemble. Celle-ci est d'une *puissance* pareille à celle de la variété continue des k possibles. Toute variété double d'images i et de kinesthéses k qui s'écoule actuellement est unifiée par l'unité de la continuité d'appréhension, laquelle unit fonctionnellement les (k, i) appartenant à chaque phase temporelle en une unité d'appréhension (en une apparition)." C'est bien ce genre de processus qui sous-tend la constitution de la chose spatiale. En effet, "la conscience d'unité qui se développe dans cette continuité d'apparitions, avec la continuité essentiellement afférente de la motivation-k, pose l'unité de la chose, constitue la chose. Il appartient au sens de cette unité que nous appelons chose, d'être unité dans une variété d'apparitions, dans une continuité d'apparitions d'un type idéel déterminé, infini."

En liaison avec ce qui précède, on peut affirmer que l'espace global de la perception se constitue à partir d'un certain nombre d' «êtres primitifs». Par exemple, la présence d'une métrique dans les espaces «sensori-moteurs» des animaux est une nécessité conceptuelle évidente. De manière plus générale, le problème est le suivant:comment expliquer la constitution de l'espace perceptif, c'est-à-dire d'un système qui assure la coordination des différents «espaces sensori-moteurs»? Si l'on écarte comme peu probable l'idée qui veut que l'espace sensible soit, pour notre espèce, entièrement constitué génétiquement à la naissance, il faudra admettre que plusieurs «espaces primitifs» jouent un rôle: espace postural de toutes les positions de l'organisme; espaces de l'organogenèse dont certains conduisent à l'édification d'organes métriquement très contrôlés, comme l'œil ou les muscles; espace neural dont les capacités psychiques pourraient résulter du type d'organisation positionnelle dans l'embryon au cours de l'ontogenèse; autres espaces physiologiques. Tous ces espaces ont naturellement une structure d'espace euclidien multidimensionnelle, beaucoup ont une structure métrique, d'autres présentent une structure d'espace fibré, voire stratifié. On se trouve donc en fait devant une opération de «synthèse» exigeant l'élimination d'un grand nombre de paramètres superflus: l'espace n'est pas «construit», comme une pièce articulée, par compositions des morceaux, mais bien plutôt comme un quotient par identification du produit d'un grand nombre d'espaces physiologiques, la plupart d'entre eux pourvus de toutes les structures désirables, parfois très complexes.

5. Constitution de l'espace Sensible et des Objets de Perception

Husserl a considéré une notion plus élargie de spatialité, dans les limites toutefois d'une position privilégiée qu'il ne cesse d'accorder à l'espace euclidien. C'est dans *Chose et Espace* (*Mémoires*), là où il traite de la constitution systématique de l'espace. D'abord, tout corps «baigne» dans l'espace. D'après Husserl, cela veut dire deux choses.

(*i*) En tant que schème sensible, le corps est un support spatial sur lequel se déploient des qualités sensibles. Ceci annonce un autre thème important, à savoir le rapport entre étendue spatiale et qualités sensibles: celles-ci se fusionnent nécessairement avec une certaine extension qui en constitue le substrat. Tout schème admet une ou plusieurs places variables, "de telle sorte qu'avec l'identité du schème un système clos de places (orientations) est idéalement possible."

(*ii*) Un corps est constitué en tant que schème sensible par le sens tactile et le sens visuel, et chaque sens réalise une liaison aperceptive des données sensibles avec des données kinesthésiques correspondantes.

Passons maintenant à la constitution de l'espace proprement dit. Elle comprend quelques niveaux fondamentaux, qui ont été bien décrits par Husserl.

À propos d'abord du rapport entre étendue spatiale et qualités sensibles, il importe de voir que la qualité participe d'une certaine façon à la modification de l'extension, où par cette notion on entend ici une forme localisée dans l'espace et susceptible de se déformer (par exemple, de se dilater ou se contracter) sous l'action de certaines forces (de prégnances physiques) au cours du temps. De fait, la qualité est affectée par des modifications de l'extension, encore que le genre de modifications qui lui est propre en soit (en partie) indépendante. Il s'ensuit que les deux contenus, de par leur nature, sont indissociables, qu'ils forment d'une manière ou d'une autre un contenu unique dont ils ne sont que des contenus partiels. C'est en ce sens que, par exemple, la forme et la texture de la surface de la toile et la couleur se trouvent être indissolublement associées; en effet, la couleur dynamise la forme et rend vivante la texture et, réciproquement, la forme et la texture différencient et multiplient les effets et les qualités esthétiques de la couleur, comme propriété de la matière et comme capacité de la sensibilité. Mais il ne s'agit pas là d'une simple question quantitative, en ce sens qu'à chaque fois qu'on agrandirait ou réduirait la «quantité » de l'extension, il en irait de même de la qualité. Il faut plutôt voir les choses qualitativement. De ce point de vue, il serait tout à fait inintelligible que la qualité diminue de la sorte et s'efface par simple diminution et disparition de la quantité sans se modifier alors à sa manière en tant que qualité. En d'autres termes, elles ne peuvent pas être des contenus autonomes, c'est-à-dire qu'elles ne peuvent ni être séparées dans l'objet de perception ni exister dans la représentation qu'on s'en fait indépendamment l'une de l'autre. Il faudra dès lors admettre que l'espace est aussi originaire et qu'on le perçoit aussi directement que la qualité. Autrement dit, un espace quelconque est immédiatement donné avec et dans la perception de la qualité. L'espace porte en soi la qualité sensible, dont elle émerge comme une propriété singulière saillante, comme un contenu concret de sa structure originaire dynamisée par l'action d'une ou plusieurs prégnances objectives et/ou subjectives, en tant que variation dans le champ de ses manifestations phénoménales possibles. Tant qu'il est question de l'espace phénoménal, sa propriété caractéristique consiste en ce que sa continuité correspond à la continuité des qualités, et que les diverses qualités (visuelles, sonores et tactiles), en elles-mêmes dépourvues de liaison, reçoivent par elle l'unité.

Supposons, avec Husserl, la coloration globale d'une surface composée d'éléments de couleur, de qualités qui se distribuent partiellement d'une région à une autre, et qui présentent également des sauts discontinus en des points isolés. Ces qualités sont alors ordonnées dans l'étendue, et s'y fondent en des colorations homogènes qui ont leur forme d'ordre unitaire. La coloration est la façon dont les régions de l'étendue sont colorées au sein de leur ordre, et dont ils se fondent en une unité grâce à cet ordre, exposent une unité de coloration différente. Mais la liaison ne réside pas dans les couleurs en elles-mêmes, bien plutôt dans le genre de relation spatiale unissant l'étendue à la couleur qui la recouvre. Pour bien saisir ce qui précède, il faut insister sur le fait de portée générale que la perception de l'espace confirme l'hypothèse selon laquelle le champ visuel n'est pas une étendue partout indifférente au caractère organisé en domaines du champ visuel lui-même, mais plutôt que le champ a tendance à se manifester de plus en plus comme homogène et uniformément unitaire. Une telle unité n'est d'ailleurs pas composée de points ni de pures relations formelles entre ces mêmes points. Il s'agit en fait de *formes saillantes*: nous voyons alors des domaines limités par des contours se détacher spontanément de façon vive et de plus en plus accentuée du reste du champ, bien qu'en même temps le champ tout entier continue de conserver l'aspect d'un «connexe» phénoménal.[48]

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[48] Sur cette question, cf. Thom, R., *Esquisse d'une Sémiophysique*, InterEditions, Paris, 1991, chap. 1.

Revenons maintenant à Husserl et à la manière dont il conçoit la formation de l'espace psychophysiologique de la perception.

(*i*) Premièrement, on a l'espace visuel (de premier niveau) et l'espace oculomoteur. Ici seul l'œil est mû. Cet espace est limité d'une manière fixe. Les corps aussitôt qu'ils y apparaissent, y disparaissent. Il y a dans cet espace un point-zéro privilégié, auquel est kinesthésiquement coordonnée la position fondamentale des yeux: la position frontale. En ce dernier se croisent les deux principaux axes du système de coordonnées oculomoteur: l'axe haut-bas et l'axe droite-gauche.

(*ii*) Ensuite, se constitue un espace bidimensionnel clos par le mouvement de rotation de la tête autour de son axe fondamental, avec par ailleurs le reste du corps immobile et fixe. L'espace visuel oculomoteur est une forme qui subsiste, mais qui reçoit, à chaque position de la tête, un nouvel index. Ce nouvel espace n'a pas de limites vers «la droite et la gauche», bien qu'il soit borné. Ce n'est pas ici un repère axiale qui fonctionne comme système fondamental de l'orientation, mais une ligne close de coordonnées de droite-gauche en abscisse, et une ligne close de haut-bas en ordonnée. À ce niveau déjà, nous n'avons plus affaire à un espace perceptif euclidien à proprement parler, mais plutôt à un espace bidimensionnel qui en est globalement différent, bien que localement équivalent: il s'agit précisément d'un espace en forme de cylindre. Comme Husserl le remarque fort justement, "vers le haut et vers le bas, le champ spatial serait délimité, et nous aurions une ligne-zéro, l'axe d'abscisse clos et deux lignes parallèles $y = +b, y = -b$. Pour $a = 0$ nous aurions un intervalle à deux sens opposés: de bas en haut et de haut en bas."

(*iii*) Considérons maintenant l'espace céphalomoteur tout entier, mais feignons que tout ce qui peut constituer la profondeur devienne imperceptible, c'est-à-dire sans mouvements kinesthésiques proprement dits (rotations dans différents sens), on aura alors un espace sphérique (fermé), en supposant que la mobilité de la tête se trouve idéalisée de manière appropriée.

(*iv*) Tous les corps considérés jusqu'ici sont des «êtres de surfaces» de nature bidimensionnelle et, dans le meilleur des cas, des «êtres sphériques». Il s'agit donc de la constitution d'un espace riemannien homogène à deux dimensions. Dans un tel espace, une troisième dimension (une profondeur), une corporéité tridimensionnelle n'aurait pas de sens. Mais celle-ci se constitue, selon Husserl, lorsque des groupes de mouvements, ou groupes de data kinesthésiques, sont coordonnés à des changements d'image d'espèce nouvelle. C'est ainsi que se formerait l'espace visuel comme espace euclidien, ou l'espace visuel complet.

Remarquons que, pour Husserl, l'espace géométrique au sens euclidien est un schème formel pour toute corporéité possible, et il n'y a en cela aucune différence entre le «corps propre» (c'est-à-dire notre corps) et les autres corps solides. Les différences entre ces types de corps, qui persistent encore dans l'espace visuel, ne disparaissent que lorsque le corps propre peut se mouvoir librement partout dans l'espace.

La constitution d'un espace kinesthésique, qui complète l'espace visuel proprement dit (le champ visuel sur la rétine est bidimensionnel), est l'une des conditions fondamentales (l'évaluation de la parallaxe en vision binoculaire en est une autre) qui permet de voir en trois dimensions. Des études récentes en psychophysique et en neurophysiologie ont confirmé le rôle essentiel que joue le système des mouvements kinesthésiques dans la perception, et non seulement visuelle.

6. Propriétés Mathématiques du Champ Visuel, Continuité de l'espace Intuitif et Géométrie des Qualités Sensibles (des Couleurs)

Par son examen des caractéristiques du champ visuel, Husserl a voulu montrer qu'il est une *variété* bidimensionnelle congruente avec elle-même, continue, simplement homogène, finie, et bien sûr bornée. Le champ visuel ne fait pas partie de l'espace objectif, pas plus que des points et des lignes dans le champ ne sont des points et des lignes dans l'espace objectif, ils n'entretiennent aucune espèce de rapports spatiaux avec des points et des lignes spatiaux (pas d'isomorphisme donc). Néanmoins, une certaine homologie doit se conserver entre les objets existant dans l'espace usuel et les images correspondantes qui se forment sur la rétine dans le champ visuel, modulo un certain type de transformations. Que le champ visuel soit une variété bidimensionnelle signifie que chaque partie du champ est circonscrit par des limites qui en dépendent, qui sont elles-mêmes à leur tour des variétés continues, donc à nouveau décomposables de telle sorte que les parties ont des frontières communes.[49] Mais les limites ne sont plus divisibles, elles sont de simples éléments de l'étendue, des «points». Les limites des zones du champs visuel sont des variétés de points continues, dont les parties sont délimitées par des points, c'est-à-dire que ce sont des lignes. La cohésion du champ ne se trouve ainsi nulle part rompue. Chaque partie du champ est décomposable, a un intérieur et un extérieur, et on peut passer par des chemins continus depuis l'intérieur de chaque domaine dans l'intérieur de chaque autre domaine. Autrement dit, ces parties seraient des *domaines connexes* de l'espace visuel global, et les chemins permettant de relier chaque point d'un de ces domaines à chaque point de n'importe quel autre domaine, seraient entre eux *isotopes*. Le dernier terme, qui n'est plus décomposable ni délimitable, est le point.

À ce propos, Husserl se demande "si la divisibilité *in infinitum* ne repose pas par principe sur l'essence du champ, ou bien si la décomposition de fait, qui conduit à des *minima visibilia*, donne sous cette forme des éléments essentiellement derniers, si par conséquent des points et des atomes visuels sont une seule et même chose."[50] Pour répondre à cette question, continue Husserl, il faudrait noter (dans les termes d'aujourd'hui) l'auto-similarité, l'homogénéité et l'isotropie du champ visuel, c'est-à-dire l'invariance d'échelle, la cohésion du local et du global, l'équivalence (du moins conforme) des déplacements et l'égalité symétrique des rotations. Il y aurait également un autre élément de réponse, dès lors que l'on considère que les «points» du champ visuel et les «*minima visibilia*» ne sont pas la même chose. Car ce que nous percevons à un instant donné sur notre rétine comme un point, n'est pas forcément un point *réel*, mais la limite au-delà de laquelle notre perception visuelle se révèle impuissante, autant dans le sens de l'infiniment petit, que dans celui de l'infiniment grand; elle doit être alors remplacée par la vision de l'esprit. Ainsi, même dans notre monde macroscopique ce que nous voyons à un moment donné comme un point, indépendamment du fait que ce soit ou non un point

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[49] Il s'agit ici, au fond, de la même méthode par laquelle on définit d'ordinaire le continu spatial. Elle énonce qu'une région d'espace, la surface qui limite ce sous-espace, une région de cette surface, et à nouveau la ligne qui limite cette région, sont des structures douées de la propriété que la totalité des points situés à l'intérieur se construit arithmétiquement comme un ensemble à trois dimensions de nombres réels.

[50] Husserl, E., *Ding und Raum*, op. cit., chap. 9, pp. 202–203 et sq.

qui est projeté sur notre rétine, pourrait changer complètement d'étendue et d'apparence si nous nous approchions de lui pour le regarder de plus près, ou si nous le focalisions par des moyens qui permettent d'en affiner la vue, etc. Ce fait est suggéré par les expériences quotidiennes les plus intuitives et il n'est donc pas nécessaire de nous y attarder.

Soulignons encore quelques points sur lesquels Husserl a insisté tout particulièrement.

(*i*) Un contenu du champ visuel peut changer de façon *quasi*-matérielle selon la qualité, ou le degré de clarté, ou bien encore la saturation, bien que ses rapports de grandeurs, d'étendue et de position puissent demeurer inchangés.

(*ii*) Mais il peut aussi s'altérer sous ses rapports métriques, on bien se modifier quantitativement et qualitativement à la fois. Donc le *quasi*-déplacement, la *quasi*-rotation sont possibles, et également la *quasi*-contraction ou expansion et la *quasi*-distorsion, et ainsi de suite. Les limites de l'objet peuvent ainsi devenir vagues quand l'acuité des discontinuités qualitatives (des singularités) s'atténue.

(*iii*) Il y a des changements qualitatifs significatifs des formes visuelles des objets qui se produisent lorsque ceux-ci se déplacent sur le bord du champ de vision. Ce qui prouve que ce qu'on appelle, en langage topologique, les «conditions au bord» jouent un rôle important dans la perception. La structure et la fonction des cellules rétiniennes (cônes et bâtonnets) ne sont pas en effet les mêmes à la périphérie de la rétine qu'en son centre. Et d'ailleurs, la formation des contours apparents des objets au niveau des divers systèmes visuels, constitue l'un des mécanismes fondamentaux de la perception humaine et animale. Rappelons en outre les changements qu'on désigne objectivement comme changements de l'accommodation, car c'est surtout grâce à ce dispositif physiologiquement inné que l'on peut avoir une perception à peu près correcte des images des objets.

Les changements ci-dessus ne doivent pas être interprétés dans un sens empirique, c'est-à-dire que dans ces changements qualitatifs, l'image qui se déplace vers le bord du champ visuel n'est pas un objet concret en mouvement. On voit par là que nous n'avons pas affaire à une simple représentation «objective», terme à terme et directe des objets du monde extérieur par notre système visuel. À ce niveau, il intervient un type de perception représentationnelle de nature déjà en partie symbolique. Les objets du monde extérieur ne sont plus perçus, à peu de choses près comme tels, mais conformément au type de structure qui caractérise les différents systèmes sensoriels périphériques et centraux ainsi qu'aux lois physiques et physiologiques qui les régissent.

Husserl analyse également ce qu'il appelle le terrain *solide* du spatial en quelque sorte concret. Il remarque que la «chose spatiale» ne cesse pas d'être identique à soi-même, malgré le flux de ses changements qualitatifs et quantitatifs. Mais pour cela, il faut qu'elle se conserve quant à sa propriété essentielle: *la continuité*. Toute continuité s'écoulant dans un flux temporel réel «implique» une identité,

> c'est-à-dire que l'on trouve la possibilité d'accomplir la continuité dans la conscience d'unité, donc de regarder et établir comme donnée, dans le flux des phases, l'identité d'un objet; ainsi que la possibilité d'unir dans une conscience synthétique d'identité des phases analytiquement extraites de l'unité, et de les élever à part jusqu'à la conscience évidente de l'identité de l' «objet» qui, dans ces phases, «s'expose». C'est une loi d'essence, valable pour tous les continua s'écoulant dans le flux temporel empirique. [. . .] Le son qui s'altère est une unité dans la multiplicité fluente et changeante. [. . .] Le son modifie par exemple sa hauteur ou son timbre, ou bien l'un reste inchangé tandis que l'autre s'altère. Évidemment les moments des phases individuelles prennent part à la substantialisation, ils reçoivent l'unité de la conscience d'unité que les lie: la hauteur du son est l'identique de tous les moments de hauteur qu'on trouve dans les phases du son.[51]

Cependant, comme Husserl le souligne, "si importante que soit la part de l'objectivation de chose que l'on présente avec l' «identité dans la continuité», ce n'est pas encore l'identité chosique, et il reste encore un chemin immense à faire pour avancer jusqu'à elle."

Il s'agit là d'un point important pour saisir la conception de Husserl. Il est question de la distinction entre continuité spatiale *phénoménologique*, de «premier degré», et continuité spatiale *mathématique*, idéalement posée et scientifiquement déterminée. Et également, de l'écart entre identité *chosique* et identité *objective* ou, en d'autres termes, entre connaissance *descriptive* et connaissance *scientifique*. Cet écart et l'impossibilité de confondre l'identité *objectale* avec l'identité *objective* tient à ce que, pour reprendre le même exemple de tout à l'heure, l'identité du son est identité dans le flux phénoménologique actuel, et ne porte pas au-delà. Mais il

[51] E. Husserl, *Chose et Espace*, pp. 313–314.

importe de voir, comme Husserl le souligne, que “la chose n’est pas seulement dans et avec le flux de ses changements actuels, mais aussi de ses changements possibles, et les possibles sont certes infinis, mais fixement circonscrits.”

7. Réversibilité et Irréversibilité de l’apparition des Objets dans l’Espace

On voudrait maintenant montrer que les intuitions de Husserl sur la constitution de la chose spatiale dans la perception et de l’espace phénoménal objectif sont d’une grande portée et qu’elles ont été à l’origine de plusieurs développements récents. Il a en effet anticipé quelques idées fondamentales que seulement aujourd’hui on commence à comprendre pleinement (notamment grâce aux travaux remarquables de l’école néo-gestaltiste de la perception), en plus d’avoir posé des problèmes mathématiques délicats relatifs à l’organisation des structures perceptives. L’un de ces problèmes concerne les positions possibles que les objets tridimensionnels peuvent occuper dans l’espace, ainsi que l’analyse de leurs structures intrinsèques. Ce problème peut en principe, comme nous allons le voir, s’énoncer sous une forme mathématique rigoureuse à l’aide du concept de groupe de transformations.

Husserl commence par mettre en évidence que si la série d’apparitions d’un corps spatial avec ses qualités sensibles dans une perception se déroule en son unité continue, car le premier ajout de changement, pour ainsi dire le différentiel de mouvement, détermine déjà la «direction» de l’écoulement, ce par quoi est donné un système d’actes psychophysiologiques intentionnels qui s’engagent et se remplissent continûment, on remarque alors qu'il est possible d’inverser toute direction de changement comme fondement essentiel du processus considéré. C’est-à-dire, en termes phénoménologiques, que l’objet qui s’expose comme donné dans la continuité de l’apparition reste le même dans l’inversion de l’ordre continu, à savoir lorsque le cours temporel est rempli de séries d’apparitions d’une façon exactement inverse. Pour mieux dire: à l’essence de cette apparition constituante appartient la possibilité idéale d’une apparition inversée, donnant le même objet, et cette dernière forme un élément essentiel de l’opération par laquelle se constitue la donnée pleine. En formules, cela peut se mettre sous la forme suivante: soit G le groupe de tous les mouvements rigides de l'objet θ_j, où j sont ses différentes positions, et soit $\sigma_{i=j}$ l'ensemble des rotations de θ, alors on a $G\sigma_i(\theta_j) \approx G_{.}^{-1}\sigma_i(\theta_j)$.

Husserl observe fort justement qu'une telle propriété ne s'applique pas à tous les objets de perception, et notamment aux objets qui se déroulent dans le temps, comme par exemple une mélodie (conçue comme une unité non statique, une suite de sons ordonnés perceptible comme forme), bien que ce type d'objets constituent, tout comme ceux d'ordre spatial, des séries continues ou des unités globales. Cette propriété de l'inversion ou de la réversibilité caractérise de façon spécifique les objets solides dans l'espace. D'un point de vue phénoménologique, on peut ainsi affirmer que, quel que soit le point de vue sous lequel nous voyons un objet solide ou, par ailleurs, quel que soit l'ordre dans lequel nous le faisons tourner devant l'œil, il ne cessera pas de nous apparaître toujours comme le même objet. Autrement dit, "il n'est ce qu'il est qu'en tant que l'identique dans l'unité systématique des changements d'esquisses (d'apparitions) ou possibilités d'esquisses de cet objet spatial." À ce propos, Husserl souligne que "toutes ces séries continues entretiennent les unes avec les autres une relation d'essence conforme à une loi, elles sont elles-mêmes reliées entre elles continûment, et ce n'est que dans l'unité englobante de ces séries que l'objet spatial accède intégralement, de façon effectivement «omnilatérale» à la perception."[52]

8. L'«Espace Quotient Global» de la Perception et les Développements Gestaltistes

L'idée de l'espace de la perception comme d'un espace quotient global produit de plusieurs sous-espaces physiologiques – qui n'est pas donné d'avance, mais résulte d'une genèse –, s'est révélée très féconde dans les recherches récentes sur les rapports entre espace et perception. Il y a d'abord un aspect philosophique qu'il faut dégager de ce concept avant d'en analyser les caractéristiques plus proprement psychophysiques. L'aspect philosophique est au cœur de la conception husserlienne de la perception spatiale. Celle-ci est conçue comme une «formation continue du sens», ce qui signifie que chaque modalité de la perception sensible (de la vue, du toucher, de l'ouïe, etc.) peut être considérée comme perception de la

[52] Ibid., p. 323.

même chose, comme *ostension* d'une figure spatiale et d'une couleur déterminées qui se recouvrent et se fusionnent. Dans leur déroulement, toutes ces modalités "fonctionnent de telle sorte qu'elles forment une synthèse de l'identification, ou, pour mieux dire, de l'*unification*, qui est tantôt une synthèse continue, tantôt une synthèse discrète. Cela ne se produit pas comme un mélange extérieur, mais, en tant qu'elles portent en elles-mêmes à chaque phase un *sens*, en tant qu'elles visent quelque chose, elles se nouent en un *enrichissement progressif du sens*."[53] Ce mouvement de formation continue du sens accompagne chaque nouvelle étape de la constitution de la chose spatiale, en tant qu'objet de perception. Nous remarquons ainsi que la diversité des ostensions partielles d'une seule et même chose forme un système et que celui-ci doit être mis en corrélation avec la diversité des processus kinesthésiques. Ceux-ci forment, à leur tour, un nouveau système qui comprend deux unités particulières distinctes lui appartenant: les kinesthèses internes (accompagnées de sensations musculaires) et les mouvements extérieurs réels du corps.

Dans la continuité de la perception un corps spatial se maintient identique lorsque, suite à certaines sensations kinesthésiques, on éprouve qu'elles sont accompagnées par des ostensions qui leur appartiennent, c'est-à-dire qu'il se montre de diverses façons tout en étant le *même* corps spatial. Mais cette appartenance des ostensions du corps spatial aux kinesthèses changeantes recèle un fait encore plus significatif, à savoir que ce processus cache un enchaînement intentionnel du type «si, alors»; par exemple: «si j'effectue tel et tel mouvement, suivant tel et tel chemin dans l'espace ambiant, je suis alors à peu près sûr d'atteindre tel et tel objet qu'y est situé à une certaine distance de mon corps et dans une position donnée par rapport à d'autres objets». Il s'agit donc moins d'une pure relation logique, que d'un acte sémiophysique effectué par notre conscience, spatialement et temporellement déterminée, en vue d'une action ou d'un but. Husserl souligne que "c'est dans un certain contexte systématique d'accompagnement que les ostensions doivent se dérouler; ainsi sont-elles pré-indiquées dans le déroulement de la perception comme signifiant la cohérence de celui-ci. Les kinesthèses actuelles prennent ainsi place dans le système des kinesthèses possibles, en corrélation avec lequel

[53] Cette citation et les autres qui suivent dans ce paragraphe sont extraites de l'ouvrage d'Husserl, *Chose et Espace*, chap. VIII, pp. 189–190 et sq.

se trouve le système des accompagnements possibles qui leur appartiennent de façon cohérente." C'est, en effet, à partir de cette chaîne de corrélations se déroulant dans un horizon actuel et dans un horizon possible que se constitue la chose spatiale identique. En d'autres termes: "dans toute perception de chose est impliqué un «horizon» entier de modes d'apparitions et de synthèses de validation non-actuelles et cependant cofonctionnantes." Ainsi, il n'y a pas vraiment de place pour le *singulier* dans la perception, la perception d'une chose ayant toujours lieu dans un *champ perceptif*. Husserl a souligné à plusieurs reprises qu'une perception, qui est bien autre chose qu'une ou plusieurs sensations juxtaposées, est toujours une exécution unitaire résultant essentiellement de la façon dont jouent ensemble des fonctions (et modalités) perceptives qui sont en relation de corrélation.

Un point de vue très répandu parmi les neurophysiologues et les psychologues a été d'admettre que la perception et l'action doivent être regardés comme des processus complètement distincts: la perception permet de traiter l'information au niveau de l'image rétinienne en vue d'obtenir une représentation symbolique du monde; l'action, elle, s'occupe essentiellement de donner les commandes aux muscles. Contrairement à ce point de vue, dans l'approche écologique, qui a repris et développé de manière originale plusieurs idées de la *gestalttheorie*, la perception et l'action sont des processus profondément liés. Les humains et les animaux ne perçoivent pas passivement le monde qui les entoure, mais ils se déplacent et agissent dans ce monde afin de sélectionner l'information nécessaire pour guider leurs mouvements. Il y a donc un échange cyclique continu entre l'organisme et le monde. De ce point de vue, le rôle de la perception consiste à fournir l'information nécessaire afin d'organiser l'action, laquelle implique à son tour qu'une compréhension de la perception exige de pouvoir comprendre les systèmes qui contrôlent l'action.

Récemment, des chercheurs comme Alain Berthoz, Jean-Pierre Roll et Christian Xerri, entre autres, ont repris l'idée esquissée par Husserl dans ses Leçons de 1907 *Ding und Raum* que «sans le concours des systèmes kinesthésiques, il n'y a pas de corps, et pas de chose», en soulignant que plusieurs faits neurobiologiques attestent chez l'homme du rôle fondateur de la sensibilité proprioceptive, à la fois pour l'intelligence du corps et pour la nécessaire coalescence des espaces corporel et extracorporel. Ils ont notamment avancé l'idée que les informations proprioceptives, nées de l'action même, participent à des fonctions mentales de niveau élevé, fonctions qui émargent clairement au répertoire des activités cognitives.

Une autre idée importante, défendue par ces mêmes chercheurs, et qui peut être en quelque sorte rattachée aux points de vues gestaltiste et écologique, est que le cortex pariétal contient des cartes somatosensorielles résultant de la projection topographique des surfaces sensibles corporelles, après relais des afférences périphériques (cutanées et proprioceptives) dans la moelle épinière et le thalamus. Chacune des cartes compose une figurine, image déformée du corps (homonculus) dont la configuration interne est somatotopique, et dont la surface et le degré de résolution dépendent de la richesse de l'innervation périphérique des territoires représentés, ainsi que de leur importance fonctionnelle (la main et la bouche sont exagérément représentés, et leur acuité sensorielle particulièrement fine).

Il est clair que cette conception husserlienne annonce des recherches récentes sur la perception et la cognition spatiales, dans la mesure où elles mettent surtout en évidence que toute modalité nouvelle qui intervient dans le processus de la perception exige un mouvement d'appréhension intentionnelle de l'objet (il s'agit là d'une véritable constitution de l'objet à la fois dans l'intuition et dans l'action), sans quoi d'ailleurs il ne pourrait pas y avoir émergence de nouvelles prégnances subjectives. De plus, ce mouvement intentionnel est enraciné dans une structure et une dynamique spatiales et temporelles dans lesquelles le sujet se trouve en quelque sorte «immergé », si bien que toute série de perceptions et d'appréhensions constituantes, et tout nouveau champ sémantique qui l'accompagne, ne vont pas sans l'apparition de nouvelles dimensions de spatialité et de temporalité. On peut dire alors que la géométrie propre au monde phénoménal est porteuse d'autres couches de sens, qui peuvent être tout à fait absentes dans la simple configuration des stimuli physiques élémentaires. Si l'on considère la perception comme un phénomène dynamique et global qui, en même temps que des mécanismes pour la discrimination des objets et de leurs formes, comporte différents types de mouvements, ainsi que les modalités de l'orientation, de la localisation, de l'identification, de la préhension, de la mémoire topographique et/ou topologique, etc., on est alors forcé d'admettre que toute représentation intelligible que l'on cherche à avoir des objets externes doit se former parallèlement à une «conquête» physique et sensible de l'espace et du temps.

Les recherches menées par les théoriciens de la *Gestalt* doivent être mises en relation avec les idées de Husserl, dont ils ont développé surtout leurs conséquences psychologiques et physiologiques expérimentales. Un point théorique capital est le fait d'avoir montré que les qualités perceptives des parties dépendent de la position, du rôle et de la fonction que chacune d'entre elles revêt dans le *tout*. Ce principe de *détermination relationnelle* a été et demeure fondamental dans les recherches actuelles sur la perception et la reconnaissance des formes. Il ne s'agit pas seulement du fait que le *tout* est plus que la somme des *parties*, mais de ce qu'il est tout à fait autre chose qu'une simple donnée additive statique. En d'autres termes, une forme globale n'est pas un plus qui s'ajouterait aux composants individuels. Les qualités géométriques, physiques et phénoménologiques de cette forme déterminent les propriétés et le comportement des composants, et le rôle qu'a chacun d'entre eux dépend essentiellement de ses relations au *tout*. C'est ce phénomène que l'on appelle la *loi de la prégnance*, indiquant qu'une forme globale émerge *ab novo*, à partir d'un agrégat d'éléments séparés et apparemment sans liens, comme un *tout* organisé et doué de structures qui font sens, car des connexions dynamiques et des processus de coopération leurs confèrent de nouvelles propriétés qui s'exprimeront de la façon la plus accomplie dans l'évolution morphologique et dans la manifestation d'autres qualités sensibles.

9. Quelques Remarques en Guise de Conclusion

Une question qui s'est posée depuis longtemps dans les diverses théories de la perception, notamment post-husserliennes et neo-gestaltistes, peut être formulée ainsi: de quelle façon un stimulus (par exemple, un input lumineux), qui peut être hautement variable et très dispersé dans l'espace ambiant, est-il projeté par l'organe et le système visuels sur des entités déterminées telles que les images ou les formes visuelles correspondant à l'objet réel ? En d'autres termes, qu'est-ce qui fait qu'un carré est reconnu perceptivement comme un carré, en dépit des changements de grandeur, de position, d'orientation et de luminosité qu'il pourrait avoir subi dans l'environnement physique?

À ce propos, il nous apparaît d'abord nécessaire de distinguer le processus qui consiste à coder l'information visuelle provenant de l'extérieur du processus qui permet de reconnaître les objets et leurs formes, en sachant que ces objets et ces formes peuvent présenter des caractéristiques qui demeurent inchangées. On sait, par exemple, que la clôture, la connexité, l'orientation, et d'autres propriétés spatiales de nature topologique globale, exercent de fortes contraintes sur la manière dont se réalisent les premiers processus conduisant à la perception des objets dans l'espace. Mais on ne dispose toujours pas d'une réponse satisfaisante à la question de savoir comment la reconnaissance des objets et de leurs formes s'opère-t-elle lors du passage d'une information locale à une connaissance de type global. Autrement dit, grâce a quels mécanismes telle et telle image qui se constitue dans notre système visuel en relation avec d'autres systèmes perceptifs correspond-t-elle à tel et tel objet existant dans le monde extérieur?

Il faut bien sûr reconnaître que les découvertes faites à la suite des travaux de Hubel et Wiesel au début des années 1960, ainsi que les nouvelles méthodes développées par la psychophysique, ont permis d'élucider un certain nombre de mécanismes fondamentaux impliqués dans la vision et dans d'autres activités perceptives chez les primates supérieurs et chez l'homme, comme le codage et le traitement des images, ou la détection et la reconstruction des contours apparents. Cependant, ce qui fait encore défaut dans ce genre d'approche de la perception, c'est la compréhension des processus intégrateurs qui permettent de former des images cohérentes et globales à partir de ces éléments individuels sur lesquels notre système visuel fait reposer le premier lissage et codage de l'information visuelle. Ajoutons que l'une des plus graves limites qui caractérise les approches dites computationnelles de la vision tient, nous semble-t-il, au fait de croire que l'information relative aux images visuelles puisse être obtenue uniquement grâce à l'application de quelques principes calculatoires de nature logarithmique incorporés (on ne sait pas trop comment!) à notre cerveau.

Or, le problème vient de ce qu'aucune de ces approches ne reconnaît explicitement la possibilité qu'il existe des propriétés organisationnelles spécifiques au processus de la perception, et de nature différente par rapport aux mécanismes des organes sensoriels comme tels. Il se trouve en effet que de telles propriétés aient pu être intégrées de façon corrélée par les différents systèmes perceptifs, puisqu'elles influencent le processus local de traitement de l'information visuelle, et d'autres types d'information sensorielle. Il ne s'agit pas uniquement des effets dits «contextuels» mis en évidence par les gestaltistes et néo-gestaltistes, pour lesquels on a pu montrer expérimentalement qu'ils jouent un rôle important dans la détection des objets, dans la restitution dun certain caractère invariant des formes perceptives, et dans d'autres processus importants qui interviennent dans la cognition spatiale. Mais également et surtout de propriétés plus générales, dont l'action a été confirmée par de nombreuses études expérimentales, qui ont notamment permis de mettre en évidence le rôle fondamental qu'ont certains mécanismes d'intégration multisensorielle et multimodale pour la perception des propriétés objectives de l'environnement et de l'espace tridimensionnel.[54]

Cela montre que le processus de la perception doit être pensé sous le mode d'un système dynamique complexe comportant une certaine évolution biologique et aussi psychologique (irréductible à la première), ainsi qu'une organisation phénoménale et des propriétés saillantes émergentes propres à la perception. Ces structures sont le résultat d'une interaction active entre les perceptions et les actions des sujets et les propriétés objectives de l'environnement. La construction perceptive de l'espace comporte une intégration spatio-temporelle et cognitive constante non seulement entre les divers systèmes sensoriels, mais également entre les structures neurophysiologiques, les propriétés de l'espace physique ambiant et les lois phénoménales sous-jacentes à la constitution des objets de perception.

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[54] Il existe une littérature riche et variée sur cette question. Nous pouvons renvoyer ici à: B. E. Stein et M. A. Meredith, *The Merging of the Senses* (MIT Press, 1993); M. Eimer, B. Fortser et J. Van Velzen, «Anterior and posterior attentional control systems use different reference frames: ERP evidence fron covert tactile-spatial orienting», *Psychophysiology*, 40 (2003), pp. 924–933 ; A. Pascual-Leone et F. Torres, «Plasticity of the sensori-motor cortex representation of the reading finger in braille readers», *Brain*, 116 (1993), pp. 39–52.

CHAPTER 3

ON THE RELATIONSHIP BETWEEN PARTS AND WHOLES IN HUSSERL'S PHENOMENOLOGY

Ettore Casari

Abstract. In his *Third Logical Investigation* Husserl worked out, from an "ontological" perspective, the "psychological" insights of his teachers Brentano and Stumpf about the distinction, in a whole, of *dependent* vs. *independent parts*. The resulting outline of a theory may be considered as one of the deepest contributes ever offered to the understanding of the structure of the objects.

In this chapter the attempt is done to account for the most relevant features of the original project as well as of its further developments in Husserl's thought.

After some introductory remarks, the first sections (1–8) present a sketch of a formal treatment, by means of essentially topological tools, of the theory of the dependency relations among individuals, paying due attention to the rich variety of such relations. The following sections (9–12) show possibilities and difficulties to be faced in extending this treatment to the dependency relations among species. The sections (13–14) are concerned with the important application of this theory to the theory of meaning presented by Husserl in his *Fourth Logical Investigation*. The final section (15) is dedicated to the rethinking of some parts of the theory which Husserl did in his later work and which may be found in *Experience and Judgement*.

Introduction

That the descriptive analysis of psychic contents apparently required a much finer notion of part than the one usually employed – an idea which occupied so much both his teachers, Brentano and Stumpf – does not seem to have originally struck Husserl's attention very much. In the *Philosophie der Arithmetik* there is no significant trace of this idea.[55] Later on, however, Husserl started to be interested in this theme, to which in the

[55] See, for instance, pp. 19–20, 72, 195 ff. of E. Husserl, *Philosophie der Arithmetik. Mit ergänzenden Texten (1890–1901)*, herausgegeben von Lothar Eley, Husserliana XII, Nijhoff, Den Haag 1970, referred to by [P] in the sequel of the article.

L. Boi et al. (eds.), Rediscovering Phenomenology, 67–102.

meantime also Twardowski had contributed,[56] and he very soon realized that this theme concerned a much wider field than the contents of representations[57]: in fact it belonged to the general theory of objects which he later called "formal ontology." No doubt, an important role in this shift of perspective was played by his mathematical education and in particular by his familiarity with the "topological" insights hidden in the "theory of functions" of his other teacher, Weierstraß.

Thus the first chapter of the *Third Logical Investigation: On the Theory of Wholes and Parts*, somehow recapitulates Husserl's journey from concrete psychological analyses and their treatment with the conceptual instruments elaborated by Stumpf to the establishment of the basic abstract notions which, deprived of all particular psychological connotations,[58] are set forth in the second chapter as the basis on which to construct that special branch of the general theory of objects to which he had assigned a very specific place in the *system of pure logic* envisaged in the final sections of the *Prolegomena*.

There Husserl had depicted the new science to be created as a two-sided, three-layered building, which, simplifying and using also later terminology, can be described as follows.

The two sides contain the "apophantic" and "ontological" aspects of logic, respectively, while the layers contain notions and problems of increasing complexity, founded on previously attained notions. In particular: the task of the apophantic first layer is to disclose the basic semantic categories and the structure of the world of meanings resulting from their compositions; the task of the ontological side is to disclose the determination of the basic formal categories such as object, state of affairs, unity,

[56] See K. Twardowski, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*, Wien 1894; repr. Philosophia Verlag, München 1982, referred to as [T]. On this also: J. Cavallin, *Content and Object: Husserl, Twardowski and Psychologism*, Kluwer, Dordrecht 1997.

[57] Referring to Twardowski's distinctions in his (unpublished) review (1895/6). He wrote «*In Wahrheit gehören die obigen Unterscheidungen in ein ganz anderes Gebiet, sie beziehen sich auf Notwendigkeitszusammenhänge zwischen Beschaffenheiten (also Begriffen), und nur insofern als auch Inhalte Beschaffenheiten haben sind damit Gesetze ausgesprochen, die auf Inhalte Anwendung finden*», Husserl, Edmund, *Aufsätze und Rezensionen (1890–1910)*, herausgegeben von B. Rang, Husserliana XXII, Nijhoff, Den Haag., pp. 349–56, referred to by [R].

[58] Only the term "*content* [*Inhalt*]" is preserved for what he later usually calls "*object* [*Gegenstand*]."

multiplicity, part, and so on. The themes to be treated on the second layer are the systems of laws founded on the categories just mentioned. Thus on the apophantic side the task is the working out of the theories of inference, whereas the ontological side concerns itself with the theories of sets, of ordinal and cardinal numbers, of whole-part relations, and so on. Finally, the tasks on the third layer are very close to what was later to be called "*methodology of deductive sciences*" on the apophantic side and to what was later to be called "*theory of models*" on the ontological side.

Incidentally, let us remark that this view of the building of logic, which closely corresponds to what in the Twenties and Thirties developed as the standard view, was, at the time when Husserl conceived it, something completely new, very far indeed from anything people such as Frege, Peano or Russell ever thought.

Thus, early in the first chapter, entitled precisely "*The difference between independent and dependent objects*" Husserl introduces, with respect to the descriptive analysis of representations, the basic dichotomy "*independent – dependent*" by saying that «independent contents are there where the elements of a representation complex (a content complex) *by their nature can be represented separately*; dependent contents, when this is not the case».[59] In the following sections, much work is carried out in order to clarify this idea, by eliminating possible misunderstandings, discussing and dispelling possible objections, and gradually shifting the issue from the psychological to the ontological level. This process yields fundamental insights on the role played in that dichotomy by the species and by both the formal and material laws which govern their relationships, thus identifying the basic elements on which that theory should be constructed. An account of these elements is attempted in second chapter "*Thoughts on a theory of the pure forms of wholes and parts.*"

Although Husserl did not systematically develop any further the theory he had envisaged at the time he wrote the *Logical Investigation*, and in spite of the fact that, as we will see later, he only sporadically returned to the

[59] «*Selbständige Inhalte sind da vorhanden, wo dei Elemente eines Vorstellungskomplexes [Inhaltskomplexes]* ihrer Nature nach vorgestellt werden können; *unselbständige Inhalte da, wo dies nicht der Fall ist*», p. 233 in E. Husserl, *Zur Lehre von den Ganzen und Teilen*, in *Logische Untersuchungen.* Bd. II, *Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, herausgegeben von Ursula Panzer, Husserliana XIX-1, Nijhoff, Den Haag 1984, pp. 227–300, referred to by [3].

subject, nevertheless what he has left us with his *Third Investigation* is by far the most important work ever done on the theory of wholes and parts. Accordingly we cannot but endorse Kit Fine's words: «Such is the range of the work that it is with a growing sense of excitement that one discovers the riches that lie beneath its rough and seemingly impenetrable exterior».[60]

1. A Tentative Formalization

Concluding his sketch of the theory, Husserl said «These ideas will and can count only as mere hints to a future treatment of the theory of wholes and parts. A real working out of the pure theory we here have in mind would have to define all concepts with mathematical exactness and to deduce all theorems by *argumenta in forma*, i.e., mathematically. . . . That this end can be achieved has been shown by the small beginnings of a purely formal treatment in this chapter. In any case, the progress from the vague to the mathematically exact conceptual formations and theories is here as everywhere the precondition for full insight into the *a priori* connections and an inescapable demand of science».[61]

In "On Husserl's Theory of wholes and parts"[62] I strived to make an initial contribution towards the carrying out of this "testamentary disposition"; in the next ten sections I will present, with some integrations, and many omissions, the results I believed I have reached in this direction.[63]

The construction of the formal theory envisaged and auspicated by Husserl requires two preliminary choices. The first one arises from the fact that there seem to be two main ways of expressing Husserl's suggestions in a formally satisfactory manner: one is to privilege, in the presentation of the theory, the "logical side" and so use "modal means"; the other is to

[60] K. Fine, *Part-whole*, p. 464, in B. Smith and D. Woodruff Smith (eds.), The Cambridge Companion to Husserl, Cambridge University Press, Cambridge 1995, pp. 463–485, referred to by [F].
[61] [3], p. 294.
[62] E. Casari, *On Husserl's Theory of Wholes and Parts*, «History and Philosophy of Logic» 21 (2000), pp. 1–43, referred to by [C].
[63] Refinements of the arguments as well as proofs of the asserted propositions may be found in [C].

privilege the "ontological side" and so use "topological tools." We chose the second possibility.[64]

The second preliminary choice concerns the basic level of the construction: objects or species? In Husserl's analyses, in fact, the arguments often proceed by mixing together notions from both realms; moreover, the propositions concerning the two realms are not as immediately interchangeable as Husserl sometimes seems to believe. Although there is little doubt that, from the philosophical point of view which motivated the whole investigation, the really important level was for Husserl the "specific" one, which contains the *a priori* laws which determine the essential interconnections among the objects; nevertheless in my view[65] the mathematically most practicable way is that one which begins by developing the "objectual theory." Accordingly, this is the way we will take. So, first of all, we must face the two main questions:

1. Which properties of the part-relation among objects are to be assumed?
2. How can we distinguish dependent and independent objects?

2. The Part-Relation Among Objects

As to the first question, there is no doubt that Husserl's part-relation was the traditional relation of "*proper part*," i.e., a relation both irreflexive and transitive. To simplify the arguments to follow, it is however more convenient to use his cognate "equal or proper part" relation, for which we will use the notation "$\leq$."[66] So we assume the usual axioms of reflexivity, transitivity and antisymmetry:

(A1) $x \leq x$ (A2) $(x \leq y \wedge y \leq z) \rightarrow x \leq z$

(A3) $(x \leq y \wedge y \leq x) \rightarrow x = y$.

[64] The same choice is made in [F]. For a treatment according to the second possibility see P. Simons, *The Formalization of Husserl's Theory of Parts and Wholes*, in B. Smith (ed.), *Parts and Moments: Studies in Logic and Formal Ontology*, Philosophia Verlag, München 1982, pp. 113–158. Also P. Simons, *Parts: A study in Ontology*, Clarendon Press, Oxford 1987; both referred to by [S].

[65] Kit Fine holds the same opinion. See [F].

[66] Henceforth, "part" used without specification will always mean "equal or proper part."

Furthermore Husserl considered the relation of "disjointedness" obtaining between two objects when they have no part in common and the triadic relation of "combination [*Verknüpfung*]," as he called it, obtaining between three objects the first two of which are disjoint and parts of the third.[67] From the use of this concept it is possible to conclude that Husserl accepted that, given two disjoint objects, there was not only a combination, but also a *least* combination of them, that is to say the object resulting by putting together the given objects and nothing more. In fact, it seems possible to argue further that Husserl accepted the possibility of putting together two objects, and them alone, even if they were not disjoint; in other words, he accepted the possibility of making of any two objects what we call their "join." Writing "$\amalg$" for this join-operation, we thus assume as a new axiom:

(A4) $x \amalg y \leq z \leftrightarrow (x \leq z \wedge y \leq z)$.

Husserl also used freely of the idea of an "object which is *the* common part of two given objects," i.e., he accepted that, given two objects, there always is what we call their "meet." Writing "Π" for this meet-operation, we thus assume as a new axiom:

(A5) $x \leq y \Pi z \leftrightarrow (x \leq y \wedge x \leq z)$.

The axioms (A1)–(A5) ensure us that the part-relation is in fact what is usually called a "lattice." What may be said, besides this, about the properties of Husserl's part-relation is a more delicate question. It is obvious that, in Husserl's perspective, there is no null-object; nevertheless, the assumption of a least element of the lattice, to be denoted by "0," enormously simplifies the formal manipulations. Accordingly we assume

(A6) $0 \leq x$.

In order to avoid misunderstandings, however, only elements different from 0 will be called "objects."[68] Note that disjointedness of x and y can now be presented as $x \Pi y = 0$.

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[67] [3] p. 230 «...wofern wir das allgemeine Teilungsverhältnis... disjunkter Teile innerhalb eines Ganzen, eine *Verknüpfung* nennen».
[68] In [C], following Husserl, they are called "contents."

On the contrary, the assumption of a "total object" is not only incompatible with Husserl's ideas, but also quite superfluous from a technical point of view. It follows immediately from this refusal that given any object, it is never possible assemble into a unique object all the objects disjointed from it; more generally, our part-relation is not, as we say, "complete," that is it is not true that it is always possible to assemble arbitrarily many objects obtaining each time a well-determined object.

Many of Husserl's arguments more or less implicitly use the fact that if x is part of $y \sqcup z$ but is disjoint from y, then it is part of z alone. This condition holds indeed in every "distributive" lattice, but, for our present aims, it will be enough to assume it directly; so

$$\text{(A7)} \qquad (x \leq y \sqcup z \wedge x \sqcap y = 0) \rightarrow x \leq z.$$

An important point is that, at the very beginning of the *Investigation*, Husserl stresses the necessity of distinguishing between the relation "*having or not having parts*"[69] and "*having or not having at least two disjoint parts*."[70] This makes it quite clear that his part-relation is weaker than the relation currently used in "classical mereology."[71] Indeed, it is obviously true that if something has at least two disjoint parts, it has at least one part; but in classical mereology also the converse holds, because there it is accepted that the removal of a part from an object yields another object and so if x is a part of y, then what remains of y, when x is removed from it, is also a part of y and obviously a part disjoint from x.

3. (Absolutely) Dependent And Independent Objects

As far as the second question is concerned, the basic idea is to identify "absolutely independent objects" with objects which are "closed" with respect to a suitable closure operation, and consequently to identify the total "need of integration" of a "dependent object" with what it needs in order to become closed, i.e., with the difference between its closure and itself. Now, quite generally, a *closure on a lattice* is an operation which associates to every element x an element x° in such a way that the following

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[69] [3] p. 229 «keine Teile habend».
[70] [3] p. 229 «worin nicht mindestens zwei disjunkte Teile zu unterscheiden sind».
[71] In our frame, where there is a 0, this means that $\leq$ is not the part-relation of a Boolean Algebra.

conditions are satisfied

$$\text{(C1)} \quad x \le x^\circ \quad \text{(C2)} \quad x \le y \to x^\circ \le y^\circ \quad \text{(C3)} \quad x^{\circ\circ} = x^\circ;$$

and an element *closed with respect to it* is an x such that $x^\circ = x$. The weakness of our part-relation mentioned above, given by the impossibility, in general, of removing a part from an object obtaining a "rest-object," prevents us, however, from obtaining the object's need of integration as the difference between its closure and itself. The way out of this impasse may be found in taking the "need of integration" as a primitive operation, to be called in short "defect" and denoted by "∂," and satisfying some very natural conditions which we will discuss shortly. It is clear that this operation (provided of course, that the conditions we impose on it, match the basic ideas concerning the questions at issue) gives us a simple way of distinguishing between (absolutely) independent and (absolutely) dependent objects. Indeed we may define:

(D1) An object x is (absolutely) independent iff $\partial(x) = 0$

(D2) An object x is (absolutely) dependent iff $\partial(x) \neq 0$ (i.e., iff $\partial(x)$ is an object).

As to the conditions: First of all, it is clear that $\partial(x)$ being just what x needs in order "to exist," it cannot have as its parts anything which x already has; i.e., x and $\partial(x)$ must be *disjoint*. Thus we assume the axiom

$$\text{(A8)} \quad x \sqcap \partial(x) = 0.$$

It is also clear that $\partial(x)$ being just what x needs in order "to exist," as soon as we join it with x, the resulting $x \amalg \partial(x)$ has no more needs, it is (absolutely) independent. Thus we assume

$$\text{(A9)} \quad \partial(x \amalg \partial(x)) = 0.$$

Finally, supposing that x is a part of y, it seems natural to assume that also the "smallest" independent object which contains x, i.e., $x \amalg \partial(x)$, is a part of the smallest independent object which contains y, i.e., $y \amalg \partial(y)$. Thus we assume

$$\text{(A10)} \quad x \le y \to x \amalg \partial(x) \le y \amalg \partial(y).$$

It is now easy to show that, if we define an operation° by

(D3) $\quad x^{\circ} = x \amalg \partial(x),$

then this operation satisfies the above conditions (C1)–(C3); that is to say, the operation thus defined is a closure and, as envisaged earlier, (absolutely) independent objects are just those objects which are closed with respect to it.

4. Dependency Relations Among Objects

Having answered the two preliminary questions, we move on to the outline of the theory of individual dependencies.

Now Husserl says, brought down to objects, that x *needs foundation* or *integration by a* y iff it cannot exist except in a more comprehensive unit which *combines* it with y.[72] So *x needs foundation* or *integration by a* y iff x is disjoint from y and every unit in which x exists must have y as its part. Since x° "exists," and is of course the "smallest" object in which x may exist, it follows that, in order to be something by means of which x needs foundation or integration, y must be a part of x°, disjointed from x, i.e., by (A7), it must be a part of $\partial(x)$. Writing in short "*Needs*" for "*needs foundation* or *integration by*" we thus have:

(D4) $\quad$ x *Needs* y iff y is a part of $\partial(x)$.

Of course, it may well happen that y alone can satisfy the needs of integration of x, i.e., that $y = \partial(x)$; in this case we say that x *Needs exclusively* y.[73] So

(D5) $\quad$ x *Needs exclusively* y iff $y = \partial(x)$.

[72] [3] p. 267 «Kann wesensgesetzlich ein α als solches nur existieren in einer umfassenden Einheit, die es mit einem μ verknüpft, so sagen wir, *es bedürfe ein* α *als solches der Fundierung durch ein* μ, oder auch, *es sei ein* α *als solches ergänzungsbedürftig durch ein* μ».

[73] [3] p. 267 «und zwar *ausschließlich* ... wenn die Ergänzungsbedürftigkeit von α_0 durch μ_0 allein gestillt wird».

As well as *Need* and *Exclusive need*, Husserl considers other, more general notions of dependence. The most important one is apparently that of *relative dependence of an x on a* y. Reduced to its "objectual form" Husserl's definition is as follows: a content x is relatively dependent with regard to a content y, respectively with regard to the entire collection of contents determined by y and all its parts, if x may subsist only in or combined with other contents from the entire collection of contents determined by y.[74]

It is easy to see that this definition defines in fact two notions: *dependence of an x relatively to* y and *dependence of an x relatively to the entire collection of contents determined by y and all its parts*.

The first notion, which we will call "*Strict dependence*," is clearly defined by: x *Strictly depends on* y iff x may subsist only in y. Thus in order to depend on y, x must be a proper part of y and, since x° "exists" in any case, y must also be a part of x°.

In other words:

(D6) x *Strictly depends on* y iff x is a proper part of y which is a part of x°.

The second notion, which we call simply "*Dependence*," is apparently defined by: x *depends on* y iff x may subsist only combined with other contents from the entire collection of contents determined by y and all its parts. Thus in order to depend on y, x may subsist only combined with some element of that collection, i.e., with some part z of y. But, as already seen, to be combined in a unit means to be simultaneously disjoint and part of the unit; therefore, since x° "exists," z must be a part of $\partial(x)$.

In other words:

(D7) x *Depends on* y iff x *Needs* some part of y.

It is well-known that Husserl, moving towards a formal development of his ideas, went so far as to formulate and prove six *Sätze*, all of which, with the exception of the first, have to do, remarkably enough, with individuals, not with species.

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[74] [3] p. 264 «Ein Inhalt α *ist relativ unselbständig zu einem Inhalt* β, bzw. zu dem durch β und alle seine Teile bestimmten Gesamtinbegriff von Inhalten, wenn . . . α nur in oder verknüpft mit anderen Inhalten aus dem durch β bestimmten Gesamtinbegriff von . . . Inhalt..en bestehen kann.»

Let us consider the first one.

> *Satz* 1. If an α as such requires foundation through [*bedarf Fundierung durch*] a μ, then every whole having an α but not a μ as a part requires a similar foundation.[75]

Brought down to the objectual level this *Satz* becomes:

> If x requires foundation through y, then every whole having x but not y as its part requires a similar foundation.

An extremely interesting fact about this theorem is that Husserl apparently did not use in it any of the four notions we already encountered, but a new one – one that is quite natural, as we will see.

In order to substantiate this claim, let us first of all show that if the relation involved is any of those already encountered, then the theorem admits rather simple counterexamples.

1. The notion used in the *Satz* cannot be *Need* as defined in (D4).

Take an x and a y such that y is a part of $\partial(x)$ and suppose that y has two disjoint parts u and w. Consider the object z obtained by combining x and u. Clearly: (i) x *Needs* y; (ii) x is a part of z; (iii) y is not a part of z. Now if the relation used in the *Satz* were *Need*, z should *Need* y. But this cannot be, because y has a part u which is also a part of z and therefore y cannot be a part of $\partial(x)$.

2. The notion used in the *Satz* cannot be *Exclusive need* as defined in (D5).

Take an x and a y such that $y = \partial(x)$ and suppose that y has two disjoint parts u and w. Consider the object z obtained by combining x and u. Clearly: (i) x *Needs exclusively* y; (ii) x is a part of z; (iii) y is not a part of z. Now if the relation used in the *Satz* were *Exclusive need*, z should need exclusively y, i.e., it should be $y = \partial(z)$. But this cannot be, because $\partial(z) = w$ which is only a proper part of y.

3. The notion used in the Satz cannot be *Dependence* as defined in (D7).

Take an x and a y such that y has a part z in common with $\partial(x)$ and another part u disjoint both from x and $\partial(x)$. Clearly: (i) x depends on y; (ii) x is a part of x°; (iii) y is not a part of x°. Now if the relation used

[75] [3] p. 268 «*Bedarf ein* α *als solches der Fundierung durch ein* μ, *so bedarf eben solcher Fundierung auch jedes Ganze, welches ein* α *aber nicht ein* μ *zum Teile hat.*»

in the *Satz* were *Dependence*, x° should depend on y. But this cannot be, because x° has no need at all.

4. Finally the notion used in the *Satz* cannot be *Strict dependence* as defined in (D6).

Take x such that $\partial(x)$ results from the combination of the two disjoint parts u and w. Let y be the combination of x and u and let z be the combination of x and w. Clearly: (i) x is a proper part of y and y is a part of $\partial(x)$, i.e., x *Strictly depends on* y; (ii) x is a part of z; (iii) y is not a part of z. Now if the relation used in the *Satz* were *Strict dependence*, z should strictly depend on y, i.e., in particular, z should be a proper part of y. But z is not even part of y.

Having ascertained that none of the notions encountered up to now satisfies *Satz* 1, the charity principle requires that we try to find, in the cluster of notions that gravitate around the idea of dependence, one for which *Satz* 1 holds. As hinted before, it turns out that such a notion may be obtained by strengthening *Dependence* stopping short however of both *Strict dependence* and *Need*.

In fact for there to be a reasonable relationship of foundation of an x on a y, x must, of course, *depend on* y, i.e., at least a part of its defect $\partial(x)$, must be a part of y. But a y satisfying this condition has in general many more parts than those which would allow x to "exist." Such are certainly those parts of y which do not occur in x°. So let us introduce this new relation , which we will call "*Foundation*," by stating:

(D8) x *is Founded on* y iff x *Depends on* y and y is a part of x°.

Note that both *Need* and *Strict dependence* are stronger notions than *Foundation*. Since *Need* requires that y be a part of $\partial(x)$, *Need* not only requires that y contain a part of $\partial(x)$ as well as no parts extraneous to x° (i.e., just that x be *Founded* on y), but also that y be disjoint from x, i.e., that it contain no parts which x already has. *Strict dependence*, on the other hand, adds to the requirement of *Foundation* just the opposite of what *Need* adds: i.e., it requires that y already contain x and so all the parts of x.[76]

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[76] Let us remark that this notion is in fact, after some rewording, the relation used, under the same name "*Foundation*," by Kit Fine in his analysis of Husserl's theory.

Let us now show that this new notion satisfies *Satz* 1. To this end, suppose x is both *Founded on* y and part of a z which does not have y as a part. By the *Foundation* hypothesis, ∂(x) and y have some part in common and y is a part of x°. On the other hand, as x is a part of z, so also x° is part of z° and therefore also y is a part of z°. But y must also have some part in common with ∂(z), otherwise it would be entirely a part of z, which is excluded by hypothesis. So y is a part of z° and, moreover, it has some part in common with ∂(z), i.e., by definition, z *is Founded on* y.

5. Fragments and Moments of a Whole

The next three *Sätze* concern a different kind of dependence relations, those which may occur between an x and a y *of which* x *is a part*. Among the parts of a whole, indeed, Husserl distinguishes *(relatively) independent parts* [*selbständige Teile*] (or *pieces* or *fragments* [*Stücke*] or *concrete parts* [*konkrete Teile*]) from *(relatively) dependent parts* [*unselbständige Teile*] (or *moments* [*Momente*] or *abstract parts* [*abstrakte Teile*]).

A *fragment of* y is a part of y which does not *Depend* on y; whereas *a moment of* y is a part of y which does *Depend* on y. It is important to realize that in these definitions it is neither assumed that y be, in itself, independent nor that a fragment of y be independent in itself; it just *does not Depend on* y. In other words: a fragment of y may well *Depend* on some other object, but in y there is no part which satisfies, partly or totally, its need of integration.

Satz 5 just makes explicit the previous remark.

> *Satz* 5. A relatively dependent object is also absolutely dependent, whereas a relatively independent object may be dependent, in the absolute sense.[77]

Satz 3 and what Husserl call "an analogous formulation of *Satz* 4," state, respectively, the transitivity of the relation *Being a fragment of* (which is not completely trivial) and the transitivity of the relation *Being a moment of*. *Satz* 4 is in fact a slightly more general proposition than *Satz* 4'.

[77] [3] p. 269 «*Ein relativ unselbständiger Gegenstand ist auch absolut unselbständig, dagegen kann ein relativ selbständiger Gegenstand in absolutem Sinne unselbständig sein*».

Satz 3. If y is an independent part of (and thus also independent relatively to) z, then every independent part x of y is also an independent part of z.[78]

Satz 4. If x is a dependent part of the whole y, then it is also a dependent part of every other whole of which y is a part.[79]

Satz 4'. If x is a dependent part of y and y a dependent part of z, then also x is a dependent part of z.[80]

Clearly if neither x nor y find any satisfaction of their needs within a z of which they are parts, they also find no satisfaction in each other. This is the content of

Satz 6. If x and y are independent parts of some whole z, then they are also independent relatively to one another.[81]

6. Reinforcements of Disjointedness

Much deeper insights into the structure of objects are gained by trying to answer the question: how are parts held together in a whole?

An important role in answering this question and, more generally, in the whole of Husserl's theory is played by some reinforcements of disjointedness. So we say that x is *external* to y when it is not only disjoint from y but also its closure is disjoint from y. Note that x is external to y iff it is not only disjoint from it but it also does not *Depend* on it. The relation of externality is, in general, not symmetric; if, however, both x is external to y and y is external to x, then we say that they are *separated*. So an x and a y are separated iff they are not only disjoint but also reciprocally independent. Finally we say that they are *strongly separated* when also they closures are disjoint. Clearly strongly separated objects are separated and thus both disjoint and reciprocally independent; but also their defects

[78] [3] p. 268 «*Ist G ein selbständiger Teil von (also relativ zu)* Γ*, so ist jeder selbständiger Teil g von G auch ein selbständiger Teil von* Γ».

[79] [3] p. 269 «*Ist* γ *ein unselbständiger Teil des Ganzen G, so ist es auch ein unselbständiger Teil jedes anderen Ganzen, von welchem G ein Teil ist*».

[80] [3] p. 269 «*Ist* α *ein unselbständiger Teil von* β*,* β *ein unselbständiger Teil von* γ*, so ist auch* α *ein unselbständiger Teil von* γ*.*»

[81] [3] p. 269 «*Sind und selbständige Teile irgendeines Ganzen, so sind sie auch relativ zueinander selbständig*».

are independent, because if there were some object $z \leq \partial(x)\Pi\partial(y)$, then such a z would also be a part both of x° and y°, against the hypothesis.[82]

In order to grasp the simple intuitive content of such notions let us consider the following three pairs of subsets of the closed interval [0,1]:

$$[x|0 \leq x \leq 1/2] \text{ and } [x|1/2 < x \leq 1];$$
$$[x|0 \leq x < 1/2] \text{ and } [x|1/2 < x \leq 1];$$
$$[x|0 \leq x < 1/4] \text{ and } [x|1/2 < x \leq 1].$$

Remembering that the singleton $\{1/2\}$ is what both $[x|0 \leq x < 1/2]$ and $[x1\,1/2 < x \leq 1]$ need in order to be "independent," i.e., to be closed in the usual topology of the real line, we see that, in the first pair, the first set is *external* to the second one, because it is closed and its closure (i.e., itself) is disjoint from the second; the second set, however, is *not external* to the first, because its closure (i.e., $[x|1/2 \leq x \leq 1]$) is not disjoint from $[x|0 \leq x \leq 1/2]$. So the sets are indeed disjoint (and even something more) but not separated. In the second pair, on the contrary, the first set is external to the second – its closure $[x|0 \leq x \leq 1/2]$ is disjoint from $[x|1/2 < x \leq 1]$ – and the second is external to the first – its closure $[x|1/2 \leq x \leq 1]$ is disjoint from $[x|0 \leq x < 1/2]$. So the sets of this pair are not only disjoint but also *separated*. Their closures (i.e., $[x|0 \leq x \leq 1/2]$ and $[x|1/2 \leq x \leq 1]$), however, are clearly not disjoint; in fact their defects, i.e., $\{1/2\}$, coincide; so, although separated, they are *not strongly separated*. *Strongly separated* are, on the contrary, the sets of the third pair, because this time also their closures $[x|0 \leq x \leq 1/4]$ and $[x|1/2 \leq x \leq 1]$ are disjoint.

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[82] It can even be shown that disjointedness of $\partial(x)$ and $\partial(y)$ is just what makes the difference between strong separatedness and separatedness. Suppose that x and y are separated and moreover $\partial(x)\Pi\partial(y) = 0$ but that their closures are not disjoint. Then there is an object z which is a common part of both $x\amalg\partial(x)$ and $y\amalg\partial(y)$. Such a z is disjoint from x; otherwise, there would be an object u which is a common part of both a z and x; but $u \leq x$ and $u \leq z \leq y\amalg\partial(y)$ imply $u \leq x\Pi(y\amalg\partial(y))$, against the hypothesis that y is external to x. From the disjointedness of z and x and $z \leq x\amalg\partial(x)$ it follows, by (A7), $z \leq \partial(x)$. Symmetrically we prove that $z \leq \partial(y)$. Therefore $\partial(x)$ and $\partial(y)$ are not disjoint, against the hypothesis.

Another basic notion of Husserl's theory is "fragmentation of a whole [*Zerstückung eines Ganzen*]," i.e., *presentation of an object as a combination of fragments*. Although in the *Third Investigation* there is no indication that fragmentations must consist of only a finite number of pieces,[83] the restriction to them, at this stage of our work, is harmless. Thus suppose that x has been fragmented into $x_1, \ldots, x_k$, that is to say: (1) all $x_1, \ldots, x_k$ are fragments of x; (2) each two of them are disjoint; (3) $x = x_1 \amalg \ldots \amalg x_k$. Now take two different x_i and x_j. By hypothesis they are disjoint, but let us suppose that x_i is not external to x_j, i.e., let us suppose that x_i *Depends* on x_j; but x_j is a part of x and so, by Satz 4, x_i *Depends* on x which contradicts the hypothesis that x_i be a fragment, i.e., an independent part of x. So all $x_1, \ldots, x_k$ are external to one another and so also separated from one another. On the other hand, suppose that we are given parts $x_1, \ldots, x_k$ of x such that each two of them are separated from one another and $x = x_1 \amalg \ldots \amalg x_k$, but at least one of them, say x_1, *Depends* on x, i.e., that there is an object z which is a common part of both $\partial(x_1)$ and x.

As $x = x_1 \amalg (x_2 \amalg \ldots \amalg x_k)$ and, by (A8), $\partial(x_1) \Pi x_1 = 0$, so, by (A7), z is a part of $x_2 \amalg \ldots \amalg x_k$; then among $x_2, \ldots, x_k$, there must be an x_i which is not disjoint from z, because otherwise, being clearly $x_2 \amalg \ldots \amalg x_k = (x_2 \amalg \ldots \amalg x_k) \amalg 0$, z would be, again by (A7), part of 0, against the hypothesis that it is an object. So let u be a common part of z and x_i; such a u is then a common part of both $\partial(x_1)$ and x_i, so x_1 *Depends* on x_i, against the hypothesis of their separatedness.

We have thus obtained the important equivalent reformulation of the notion of fragmentation:
A fragmentation of an object is a presentation of it as the join of a finite set of separated objects.

7. A Surprising Assertion

In the last section of the *Investigation*, under the title *Additional propositions about the fragmentation of wholes through the fragmentation of its moments*,[84] Husserl makes what he calls "a possibly not uninteresting remark" and which could be better called "a rather surprising remark." The argument,

[83] See however the two final notes of this paper.
[84] [3] p. 295 «§25. *Zusätze über die Zerstückung von Ganzen durch die Zerstückung ihrer Momente*».

which is very involved, has been analysed by Kit Fine and more at length in [C]. Omitting all discussion, we simply present our conclusions. Husserl's "remark" seems to contain two different assumptions: a *logical* one and a *phenomenological*, *factual* one.

The logical assumption is the "*additivity (at least) of strongly separated objects*": i.e., the assumption that (at least) when two objects are strongly separated, then the join of their closures coincides with the closure of their join, that is:

$$x^\circ \amalg y^\circ = (x \amalg y)^\circ \text{ [85]}$$

The factual one (the truly surprising part of the "remark") is that Husserl assumes that the relations of being separated and being strongly separated although logically distinct, coincide at the phenomenological level, i.e., that «*de facto*, we do not find in the field of pure intuition and evidence to which we have access any example» of separated but not strongly separated objects; moreover he suggests that «in that field this fact is connected with remarkable part-relations».[86] The extremely doubtful character of this idea has been pointed out by Kit Fine, who has given a rather convincing counterexample to it.[87]

8. Pregnant Wholes

Besides the quite general notion of a whole simply as the correlate of a proper part, i.e., as an object which may occupy the second position in the relation $<$, Husserl in the course of the paper singles out, as we have seen, another, more restricted notion of whole, that of (absolutely) independent object. In § 21 a third notion of whole, explicitly distinguished from the

[85] Note that the inclusion $x^\circ \amalg y^\circ \leq (x \amalg y)^\circ$ is always true; the assumption concerns $\geq$.

[86] [3] «*De facto* finden wir aber kein Beispiel in dem uns zugänglichen Gebiete reiner Anschauung und Evidenz, und damit hängen in eben diesem Gebiete merkwürdige Teilverhältnisse zusammen».

[87] [F] p. 477 «. . . suppose that I have mixed feelings towards someone. Then that complex of feelings may be taken to consist of a presentation x of the person and of two attitudinal moments, y_1 and y_2 - one of love, let us say, and the other of hate. Now y_1 and y_2 are reciprocally independent: for the closure of y_1, is a feeling of love towards the person and hence is disjoint from y_2 the attitude of hate; and similarly for y_2. But the closures of y_1 and of y_2 are not disjoint, since they have the presentation x in common». Note that "reciprocally independent" is the same as "separated."

previous ones by the qualification "pregnant [*der prägnante Begriff des Ganzen*]," is introduced by the definition: «by a *whole* we understand an aggregate of contents which are spanned *by a unitary foundation* and precisely without any resort to further contents. The contents of such an aggregate are called parts. When we speak of the *unity of foundation* we mean that *every content is connected, directly or indirectly, with every other through foundation*».[88]

The exegesis of this definition presents an easy and a difficult part. Clearly "without any resort to further contents" means that the whole is "self-sufficient," that is to say: *it is an (absolutely) independent object*. But what precisely should be understood by "every content is connected, directly or indirectly, with every other through foundation" is a rather difficult question. Immediately after the above definition, Husserl adds that a pregnant whole may present itself in two basic different forms: «It may so happen that all these contents are founded *on each other* immediately or mediately without external help; or even that, on the contrary, *all together* they found a new content, once again without external help».[89] Later on, he describes the first of these forms by saying that, in this case, the parts "*interpenetrate each other* [*durchdringen sich*]" and the second by saying that in this other case, the parts are "*external to each other* [*außer einander*]" but are held together by "*linking forms* [*verbindende Formen*]."

Simplifying things, the problem at issue seems to be: what makes a unitary object from a collection of parts? And the answer seems to be: there are only the two following basic ways, although of course intermediate ways, resulting from combinations of them, are possible.

Either the object is such that if we take away from it any of its parts then either it or some of its parts depends on what remains or on some of its parts or vice versa; in other words, *it is not possible to present the object as*

[88] [3] p. 282 «*Unter einem* Ganzen *verstehen wir einen Inbegriff von Inhalten, welche* durch einheitliche Fundierung, *und zwar ohne Sukkurs weiterer Inhalte umspannt werden. Die Inhalte eines solchen Inbegriffs nennen wir Teile. Die Rede von der* Einheitlichkeit der Fundierung *soll besagen, daß* jeder Inhalt mit jedem, sei es direkt oder indirekt, durch Fundierung zusammenhängt».

[89] [3] p. 282 «*Dies kann so statthaben, daß alle diese Inhalte ohne aüßeren Sukkurs unmittelbar oder mittelbar* ineinander *fundiert sind; oder auch so, daß umgekehrt* alle zusammen *einen neuen Inhalt, und zwar wieder ohne aüßeren Sukkurs fundieren*».

the join of two separated objects. Such a property is well-know in topology where it is called "connectedness"; so we may say that the first kind of wholes in the pregnant sense consists of *connected objects*.

Or the object results from a collection of separated, or even strongly separated objects tied together[90] by a "*unity moment* [*Einheitsmoment*]," which is defined as: «a content which is founded through a multiplicity of objects and precisely through all of them together and not merely through some of them».[91]

The difference between the two basic forms is stressed by saying: «according to our definition it is not necessary that to every whole[92] there correspond a form, in the sense of a particular unity moment linking all parts».[93]

Many other ideas concerning objects, are buried in this extraordinary *Third Investigation*; only some of them have attracted attention and only a few have been assessed formally in a way that is at least partly satisfying.[94]

It is, however, time to move on to the even more difficult problem of providing a not excessively improper account of Husserl's ideas about species and species-determined dependencies.

9. Species

In [C] we worked out a formal treatment of species and their dependence relations, which has some rather satisfying features although, as we will see later, it does not completely match Husserl's ideas. The leading idea is the following. Species are in general partitions of genera and the latter, in turn, partitions of higher genera. So if we can introduce among objects a basic partition, whose components could be regarded as *ultimate*

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[90] Let us remark that, in order to formally account for this, our notion of fragmentation should be slightly modified.

[91] [3] p. 287–288 «Einheitsmoment . . . *verstehen wir darunter* einen Inhalt, der durch eine Mehrheit von Inhalten fundiert *ist, und zwar* durch alle zusammen *und nicht bloß durch einzelne unter ihnen*».

[92] Of course, in the pregnant sense.

[93] [3] p. 283 « … *ist es gut, ausdrücklich darauf hinzuweisen, daß* nach Maßgabe unserer Definition nicht zu jedem Ganzen eine eigene Form im Sinne eines besonderen, alle Teile verbindenden *Einheitsmomentes* zu gehören braucht».

[94] For some suggestions, see both [F] and [C].

species, then the other genera can be introduced by successive unions. The basic partition, and the derived unions, should however correspond to the notion of species involved in the typical Husserlian locutions of the form: "x *as an* (object of the species) A *needs, depends, etc. ... .*" This seems to require that the equivalence relation which induces the basic partition should be endowed with sufficient compatibility properties with respect to the structure of the world of objects. Therefore we require the existence, among objects, of an equivalence relation (i.e., a reflexive, symmetric and transitive relation, to be called "isogeneity" and denoted by "$\sim$") which is compatible with both the defect and the closure operations, i.e., it satisfies both:

$$\text{(S1)} \quad x \sim z \rightarrow \partial(x) \sim \partial(z) \ \text{ and } \ \text{(S2)} \ x \sim z \rightarrow x^\circ \sim z^\circ,$$

and is "weakly compatible" with the relation $\leq$ in the sense that if x is a part of z then: (1) if x $\sim$ y, then there is a u which has y as its part and is isogeneous to z; (2) if z $\sim$ u , then there is a y which is a part of u and isogeneous to x; i.e., it satisfies both

$$\text{(S2)} \quad x \sim y \rightarrow (x \leq z \rightarrow \exists u(y \leq u \land z \sim u)) \text{ and}$$

$$\text{(S3)} \quad z \sim u \rightarrow (x \leq z \rightarrow \exists y(y \leq u \land x \sim y)).$$

For the sake of simplicity we agree to call "species" the equivalence classes determined by the relation of isogeneity, and endow the realm of such species with a *defect operation* and a *closure operation* as well as a *part relation* by assuming:

1. the *defect of* (species) A is the set, indicated with '$\partial(A)$', of all defects of elements of A; i.e., $\partial(A) := [\partial(x) | x \in A]$;
2. the *closure of* (species) A is the set, indicated with 'A°', of all closures of elements of A; i.e., $A^\circ := [x^\circ | x \in A]$;
3. (species) A *is a part of* (species) B, and we write '$A \subseteq B$' iff there are an $x \in A$ and a $y \in B$ such that $x \leq y$.

The most important fact is that, on the basis of the assumptions above, it can be shown that *such definitions really endow the realm of the species with a structure*, because if A is a species then also $\partial(A)$ and A° are species and if A and B are species then A $\subseteq$ B iff every object in A is a part of an object in B and, vice versa, every object in B has an object in A as its part.

In this way it becomes possible to try to "replay" on the specific level the dependence relations already known in the world of objects. So we can define, in particular, the specific "counterparts" of the three basic notion of *Need*, *Dependence* and *Foundation*, by assuming:

1. A *NEEDS B* iff B $\subseteq \partial(\mathrm{A})$
2. A *DEPENDS ON B* iff there is some C such that C $\subseteq \partial(\mathrm{A})$ and C $\subseteq$ B
3. A *IS FOUNDED ON B* iff A *DEPENDS ON B* and B $\subseteq$ A°.

As expected, it turns out that the *NEED* relation among species may be reformulated in terms of *Need* among their elements; i.e., that

A *NEEDS B* iff every element of A *Needs* some element of *B* iff there are an x $\in$ A and a y $\in$ B such that x *Needs* y.

The same happens with the *DEPENDENCE* relation; i.e., A *DEPENDS ON B* iff every element of A *depends on* some element of *B* iff there are an x $\in$ A and a y $\in$ B such that x *Depends on* y.

An important remark is that, contrary to what happens among objects, it may well be that a species *DEPENDS ON* itself. As shown in [C], there are models of the theory in which such "self-dependence" of species may be exemplified. As I remarked there, «the requirement for this possibility has been sometimes exemplified resorting to symmetric relational notions (the species of married persons is founded in itself because the integration a married person needs in order to be such, is just another married person). Although suggestive, such examples do not seem to be really significant from Husserl's point of view. He repeatedly stresses that the kind of dependence he is trying to describe is material, not formal, and the dependence induced by relational notions is for him a typically formal one».

With the *FOUNDATION* relation things are not so simple. It can indeed be shown that if every object in A is *Founded on* an object of B, then A is *FOUNDED ON* B; but, in general, not vice versa. This has to do with the fact that A may be *FOUNDED ON* itself (this is indeed always the case if only A *DEPENDS* on itself, because A $\subseteq$ A° is always true) and it is shown in [C], with the aid of a model, that there may be species which are *FOUNDED ON* themselves, but no content of which is *Founded on* another content of the same species.

10. Genera

As already remarked, *genera* are now defined as arbitrary non-empty unions of *species* (as defined above) and it is easy to show that, if we define the defect ∂ (G) and the closure G° of a genus G as the union of the set of the defects ∂(A), respectively of the closures A° of all species A included in G, then both ∂(G) and G° are genera.

Moreover, the basic dependence relations among species, can now be extended to genera G, F, . . . by assuming:

1. G *NEEDS* F iff every species included in G *NEEDS* some species included in F;
2. G *DEPENDS ON* F iff every species included in G *DEPENDS ON* some species included in F;
3. G *IS FOUNDED ON* F iff every species included in G *IS FOUNDED ON* some species included in F.

As desired it can then be shown that: (1) G *NEEDS* F iff every object contained in G *Needs* an object contained in G; (2) G *DEPENDS ON* F iff every object contained in G *Depends on* an object contained in G. As far as *FOUNDATION* is concerned, the analogue of the two preceding propositions can be proved only under the assumption that no species included in G *DEPENDS ON* itself; i.e., we have: (3) If no species included in G *DEPENDS ON* itself, then G *IS FOUNDED ON* F iff every object contained in G is *Founded on* some object contained in F.

An interesting consequence of this is that if in Husserl's *Satz* 1 we take "requires foundation through" to mean "*IS FOUNDED ON* and no species included in it *DEPENDS ON* itself," then we can prove it.

Another interesting consequence of the above definitions is that we are now able to account for such a basic Husserlian locution as "x *as a* G *depends on* y *as an* F," by identifying it with the following: "x is a G and y is an F and G *DEPENDS ON* F and x *Depends on* y" and sharply distinguishing it for instance from "x *which is a* G *depends on* y *as an* F" ("x is a G and y is an F and the species of x *DEPENDS ON* F and x *Depends on* y") or from "x *which is a* G *depends on* y *which is an* F" ("x is a G and y is an F and x *Depends on* y").

If we think of the theory sketched so far as a formal system written down in a suitable formal language, we may consider so-called "realizations" of this system, – i.e., assignments of meanings to the undefined symbols of the language in such a way that each formula becomes either true or false – and "models" of the system, i.e., those realizations which make all axioms true. If we distinguish, among the formulae of the language, the "purely generic" ones, i.e., those which do not contain variables for individuals, it becomes possible to somehow approximate Husserl's dichotomy between *formal* and *material a priori laws*. *Formal a priori laws* may be identified with those purely generic formulae which are true in every model of the theory and, for any given model μ, the μ-*material a priori laws* with those generic formulae which are true in the model μ.

11. A Strengthening of Isogeneity

The isogeneity relation discussed above has a certain weakness in that it does not really take into account the possible inner structure of objects. In particular, if an object x belongs to the same species as an object y which is, for instance, the defect of another object, there are reasons to think that also x is, on its part, the defect of some object, and precisely of an object which is of the same species as the object of which y is the defect; as defined above, however, the isogeneity relation does not allow us to make such a conclusion. We are thus induced to strengthen the defining conditions for isogeneity by means of the assumptions:

1. if x is isogeneous to an object which is the defect of a certain z then it is also the defect of an object which is isogeneous to z; i.e.,

   (S5) $\quad x \sim \partial(y) \rightarrow \exists z(x = \partial(z) \Lambda z \sim y);$

2. if x is isogeneous to an object which is the closure of a certain z then it is also the closure of an object which is isogeneous to z; i.e.,

   (S6) $\quad x \sim y^\circ \rightarrow \exists z(x = z^\circ \Lambda z \sim y);$

3. if x is isogeneous to an object which is the *sup* (the *inf*) of a certain y and a certain z then it is also the *sup* (the *inf*) of two objects which are isogeneous to y and z, respectively; i.e.,

   (S7) $\quad x \sim y \amalg z \rightarrow \exists uv(x = u \amalg v \ \Lambda \ u \sim y \Lambda v \sim z)$ and

   (S8) $\quad x \sim y \Pi z \rightarrow \exists uv(x = u \Pi v \Lambda u \sim y \Lambda v \sim z).$

This strengthening of isogeneity has important consequences on the theory of species and genera. So, for instance, that a species A *NEEDS* or *DEPENDS ON* the species B turns out now to be equivalent not only to the fact that every object in A *Needs* or *Depends on* some object in B, but also to the fact that for every object in B there is an object in A which *Needs* or, respectively *Depends on* it. In [C] two models are offered for the theory thus strengthened; a third model however shows that the strengthening is a proper one.

12. An Objection

Alessandro Torza, a former student of the Scuola Normale, made a serious objection to this theory being a faithful reconstruction of Husserl's ideas.[95] He started from the following text by Husserl: «The species tied together in the laws which delimit (just from the point of view of these laws) the spheres of accidental singularities, are often, but not always lowest specific differences. For example, if a law prescribes a connection between contents of the kind *colour* and contents of the kind *extension*, it does not prescribe to any definite colour a definite extension, or vice versa. The values of the lowest differences are here not in a functional relationship. The law mentions only lowest kinds (i.e., kinds which have the variety of the lowest specific differences immediately under them)».[96] Torza then argued that if the isogeneity classes are to be regarded as ultimate species, then we get the following paradox. Take a black dyed ball, of radius r; then the individual black of this ball is founded on the individual surface of the same ball and, moreover, black dye and surface of a ball of radius r are lowest species; so all black dye should be found on balls of radius r.

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[95] A. Torza, *A priori formale e materiale della cosa estesiologica. Un'analisi onto-fenomenologica*, Tesi di Laurea, Università degli Studi di Pisa, 2003, pp. 23–24, referred to by [L].

[96] [3] pp. 254–255 «*Die in diesen Gesetzen verknüpften Spezies, welche die Sphären der (vom Standpunkte eben dieser Gesetze) zufälligen Einzelheiten umgrenzen, sind mitunter, aber nicht immer, niederste spezifischen Differenzen. Schreibt beispielweise ein Gesetz Inhalten der Art* Farbe *Zusammenhang mit solchen der Art* Ausdehnung *vor, so schreibt er keiner bestimmten Farbe eine bestimmte Ausdehnung vor und ebenso auch nicht umgekehrt. Die Werte der niedersten Differenzen stehen hier also in keiner Funktionalbeziehung zueinander. Das Gesetz nennt nur niederste Arten (d. i. Arten, welche die Mannigfaltigkeit der letzten spezifischen Differenzen unmittelbar unter sich haben)*».

It seems to us that this objection does not undermine the isogeneity relation as defined above; what it certainly destroys is the idea that the residual classes modulo isogeneity are the ultimate species; they are possibly the "spheres of individual contents delimited by the laws" to use Husserl's words. Thus in order to obtain an exhaustive system of species, we must admit, besides systems of unions of such classes (what we called "genera"), also partitions of them. So far we have not found a completely satisfactory solution.

13. A Major Application: Independent and Dependent Meanings

The first, major and immediate application of the whole-parts theory is offered by Husserl in the *Fourth Logical Investigation: The Difference between Independent and Dependent Meanings and the Idea of a Pure Grammar*, where the basic insights gained in the *Third Investigation* are offered as the basis for the development of another well determined sector of the system of pure logic envisaged in the final sections of the last chapter of the *Prolegomena*. The application, which we will shortly discuss, concerns the first apophantic layer and, as we suggested earlier, is concerned with the determination of the semantic categories and the laws which govern their compositions; its aim is to result in a "*pure(logical) grammar*."

The starting point of Husserl's analysis is a seemingly irreducible opposition between two points of view he had come across in his reflections on meaning.

On the one side, there was Anton Marty's recent reprise and reformulation of the old idea, tracing back to the Stoic logical-grammatical tradition and current among Medieval logicians, that not all parts of a meaningful expression have a proper meaning, but that, quite generally, there are two kinds of expressions: *categorematic* expressions, which have an autonomous, proper meaning and *syncategorematic* expressions, which lack a proper meaning, but concur with other expressions (in particular with categorematic ones) to determine meanings.[97]

[97] Cf. Priscianus, *Institutionum grammaticarum libri XVIII*; Lib. II, 15 in *Grammatici Latini*, Vol. II Leipzig 1855–1859; repr. Hildesheim 1961, pp. 54, 5–8 «*Partes igitur orationis sunt secundum dialecticos duae, nomen et verbum, quia hae solae etiam per se*

On the other side, there was Bolzano's point of view which, reformulated in the semantic terminology, in fact implicitly refused this distinction, accepting that there were meanings corresponding not only to the traditional categorematic expressions, but also to the syncategorematic ones. So, for instance, Bolzano assumed that there were meanings corresponding to expressions like "not," "and," "or," "all," "some," and that the standard form of a proposition (in itself) was made out of three meanings: those corresponding to the *subject* and the *predicate*, as well as that corresponding to the *copula* (which, incidentally, he took to be "*has*" instead of "*is*"). As Husserl remarks, however, he had made these assumptions «without allowing himself to enter into deeper discussions».[98]

Convinced, with Marty, of the impossibility of denying the existence of a radical difference between the *expressions* classified as categorematic and those classified as syncategorematic, but not less convinced, with Bolzano, that, disregarding possible trivialities, *all* expressions have a meaning, Husserl saw precisely in his theory of *whole – parts relationships* the tool capable of reconciling these two seemingly incompatible views. Accordingly, all expressions have a meaning, but categorematic expressions have an independent meaning whereas syncategorematic expressions have a dependent one. Furthermore the difference which may be found among expressions is in fact nothing but a consequence of the much deeper lying difference among meanings.[99] Working towards a phenomenological characterization of the distinction, he specifies: «we will

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conjunctae plenam faciunt orationem, alias autem partes 'syncategoremata', hoc est consignificantia, appellabant. [Thus, according to the dialecticians, there are two parts of a sentence: name and verb; only these, indeed, even if put together alone, make up a full sentence; they called the other parts syncategoremata, that is, con-significant]».

[98] «*ohne sich übrigens auf nähere Erörterungen einzulassen*», p. 311 in E. Husserl, *Der Unterschied der selbständigen und unselbständigen Bedeutungen und die Idee der reinen Grammatik*, in *Logische Untersuchungen.* Bd. II, *Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, herausgegeben von Ursula Panzer, Husserliana XIX-1, Nijhoff, Den Haag 1984, pp. 301–351. Referred to by [4].

[99] [4] p. 314 «Wir müssen nicht bloss zwischen kategorematischen und synkategorematischen Ausdrücken, sondern auch zwischen kategorematischen und synkategorematischen Bedeutungen unterscheiden»; [4] p. 318 «der scheinbar so gleichgültigen Unterscheidung der Ausdrücke in kategorematische und synkategorematische eine fundamentale Scheidung im Gebiete der Bedeutungen entspricht»

call a meaning *independent* when it can constitute the *complete and entire meaning of a concrete act of meaning*, and *dependent* when this is not the case».[100]

Elaborating on this proposal, Husserl discusses also some finer questions concerning the Bolzanian triad: ⟨subjective representation – its stuff (objective representation or representation in itself) – object referred to⟩. This triad had entered the Brentanian milieu through Alois Höfler and Kasimir Twardowski as ⟨representation – its content – intended object⟩ and had shifted from the mental to the linguistic point of view as ⟨expression – meaning – intended object.[101] In particular, he discusses whether there is any correspondence between the structure of the meaning and the structure of the intended object. With respect to this question, he not only explicitly accepts the conclusions of Bolzano, who, in the *Wissenschaftslehre*[102] had maintained, against a widespread opinion, that there is absolutely no correspondence between the parts of a meaning and those of the intended object, but he goes further by stressing that not even such a basic property as that of being a dependent meaning is preserved when the meaning itself becomes an object, in particular, an object of itself. That is, quite analogously to the linguistic phenomenon the Scholastics called "*suppositio materialis*," when we say " 'and' is a dependent meaning" (i.e., "the meaning of the expression 'and' is a dependent meaning") the object intended by the meaning of the expression "and" is just the meaning of "and"; by this modification, the dependent meaning of the word "and" becomes the object and the meaning which refers to it as the subject of a proposition is an independent one. In fact – he maintains – «quite generally, *every dependent thing may be made*, and precisely also in a more direct way, the *object of*

[100] [4] pp. 320–321 «*Demgemäß werden wir eine Bedeutung* selbstständig *nennen*, wenn sie die volle und ganze Bedeutung eines konkreten Bedeutungsakte *ausmachen kann, und* unselbstständig, *wenn dies nicht der Fall ist*».

[101] Interestingly enough, neither Frege's triad, nor Frege's idea of the unsaturated nature of concepts are mentioned anywhere in the *Investigation*. Symmetrically no mention of Bolzano's triad or of Brentano's and Stumpf's ideas about dependencies is to be found in Frege.

[102] B. Bolzano, *Wissenschaftslehre*, §63. *Ob die Theile einer Vorstellung einerlei sind mit den Vorstellungen der Theile ihres Gegenstandes*. See also §64. *Ob die Theile einer Vorstellung einerlei sind mit den Vorstellungen von den Beschaffenheiten ihres Gegenstandes.*

an independent meaning: e.g., redness, figure, equality, magnitude, unity, be».[103]

Analysing the question about the possibility of understanding the syncategoremata when completely isolated from their context, Husserl reaches the conclusion that this is not possible, that a syncategorematic *meaning* as well as a syncategorematic *expression* (intended, of course, as a unity of sound and meaning), in order to be active in an epistemic situation always require the presence of a categorematic meaning or expression.[104]

14. Bedeutungskategorien

The really important and fundamental consequence of the introduction of the "dependency notions" into the realm of meanings is, of course, that – dependency relations being controlled, as seen above, by *a priori specific rules* – the whole realm becomes partitioned into *essential genera* [*wesentliche Gattungen*], i.e., "*semantic categories* [*Bedeutungskategorien*],"[105] some of which collect independent meanings and some of which collect dependent meanings, and where the laws ruling the dependency relations prescribe *a priori* the possible ways in which complex meanings may be obtained by simpler ones.[106] «Hence arises the great task, equally fundamental for logic and for grammar, of enucleating this *a priori* constitution which encompasses the realm of meanings and of investigating, within a "meaning morphology," the *a priori* system of formal

[103] [4] pp. 321–322 «So läßt sich überhaupt jedes Unselbständige, *und zwar auch in direkterer Weise*, zum Gegenstand einer selbständigen Bedeutung machen, *z. B.* Röte, Figur, Gleichheit, Größe, Einheit, Sein».

[104] [4] pp. 323 « . . . keine synkategorematische Bedeutung, nämlich kein Akt von unselbständiger Bedeutungsintention, in der Erkenntnisfunktion stehen kann, wenn nicht im Zusammenhang einer kategorematischen Bedeutung. *Und statt* Bedeutung *könnten wir natürlich auch sagen* Ausdruck, *normal verstanden als* Einheit *von Wortlaut und Bedeutung oder Sinn*».

[105] [4] pp. 326.

[106] [4] pp. 325 «*Wird der Unterschied der selbständigen und unselbständigen Bedeutungen auf den allgemeineren Unterschied der selbständigen und unselbständigen Gegenstände bezogen, so ist hierin eine der fundamentalsten Tatsachen des Bedeutungsgebietes eigentlich schon mit eingeschlossen, nämlich daß* die Bedeutungen unter apriorischen Gesetzen stehen, welche ihre Verknüpfung zu neuen Bedeutungen regeln».

structures, i.e., of those structures which leave open all material specificity of meanings».[107]

Among the "independent" categories, Husserl mentions, for instance, the categories of the meanings of "nominal expressions," of "adjectival expressions" and of "sentences." Husserl distinguishes two main kinds of dependent meanings (i.e., meanings which operate on meanings in order to generate – according to well determined *a priori* rules – other meanings): those which *compose* other meanings and those which *modify* them. Among the examples he makes of operations of the first kind there are the three meanings of the expression "and" which compose "nominal meanings," "adjectival meanings" and "sentential meanings," respectively, yielding – by means of specific laws – each time a meaning of the same category of the "arguments"; but Husserl also mentions the meaning, usually expressed in our languages by some juxtaposition, which composes an adjective and a nominal meaning to produce – by means of a specific law – a nominal meaning (e.g., *red house*) and so on. The most simple example he gives of a modifying operation he gives is that "analogue of the *suppositio materialis*" we mentioned above. Later on, he will also mention the basic forms of propositions, the categorical proposition and the whole taxonomy of its particular forms, the primitive types of complex propositions such as conjunctive, disjunctive, hypothetical, and the differences between universality and particularity on the one hand and singularity on the other, the syntaxes of plurality, negation, modalities, and so on.[108]

The system of meanings built according to the those specific rules is capable of sharply separating "proper meanings" from the mere assemblages of meanings which do not really constitute a "semantic unity" and which Husserl decides to designate – exclusively – as "*Unsinn*." After separation from the "senseless" assemblages of meanings, a further fundamental separation takes place among meanings. This happens when they are considered with respect to their "objectual validity." Thus on the one side, we have those meanings whose validity or invalidity is "analytic,"

[107] [4] p. 329 «Hieraus erwächst die große, für die Logik und Grammatik gleich fundamentale Aufgabe, diese das Reich der Bedeutungen umspannende apriorische Verfassung herauszustellen, das apriorische System der formalen, d. i. alle sachhaltige Besonderheit der Bedeutungen offenlassenden Strukturen in einer "*Formenlehre der Bedeutungen*" zu erforschen».

[108] [4] p. 347.

i.e., determined on *purely formal* grounds, and, on the other, those where this is not the case. An "analytically valid" meaning is, for instance, that expressed by "something which is round or not round"; an "analytically invalid" meaning – what Husserl calls a "*Widersinn*" – is "something which is both round and not round." The investigation of this kind of meanings and of the laws which govern their realm is precisely the subject of "pure logic," the science which, in the building we sketched at the beginning of the section, is located on the "apophantic second layer."[109] Among those meanings whose validity or invalidity is not formally, but *materially* determined, or, equivalently, is not analytic but "synthetic," a third fundamental distinction takes place. On the one side there are those meanings whose material "validity or invalidity" is determined *a priori*, and on the other side there are those meanings which are not so, i.e., those meanings whose material "validity or invalidity" is empirically determined. A synthetically "valid" *a priori* meaning is, for instance, that expressed by "a triangle having three sides"; a synthetically "invalid" *a priori* meaning is, e.g., "a round square." An example of an empirically "valid" meaning is that expressed by "the Mont Blanc," whereas the meaning expressed by "a gold mountain" is empirically "invalid." It is to be noted that the general properties of those theories in which synthetic *a priori* validity and invalidity find their proper place, constitute the major subject of that branch of logic which occupies the third level.

Husserl's firm belief, which pervades the whole *Investigation*, was that a meaning morphology such as the one outlined above, even though in itself a branch of pure logic, may also play a fundamental role in linguistic studies. Somehow realizing the old dream of a "*grammaire générale et raisonnée*" it appeared to him as an ideal framework [*ideales Gerüst*] which the different factual languages could fill in and coat with empirical matter, following a wide range of motivations (anthropological, social, historical, and so on).[110] Hence he proposed to call it "*pure logical grammar*" [*reinlogische Grammatik*].[111]

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[109] [4] p. 337 «In der reinen Logik der Bedeutungen, deren höheres Ziel in Gesetzen gegenständlicher Geltung der Bedeutungen liegt, soweit solche Geltung durch die reine Bedeutungsform bedingt ist, bildet die Lehre vom Wesensbau der Bedeutungen und den Gesetzen ihrer Formenbildungen das notwendige Fundament».

[110] [4] p. 348.

[111] In the first edition he called it simply "*reine Grammatik*"; the more restricted version is mainly due to Marty's not always fair critical remarks.

As is well-known, this idea had a sequel. Worked out, mainly for logical aims, by Stanisław Leśniewski and his pupil Kasimierz Ajdukiewicz (who had also followed from WS1912/3 to WS1913/4 Husserl's lectures in Göttingen), the categorial analysis of language presented in the latter's celebrated paper "Die syntaktische Konnexität,"[112] became, from the Fifties on, the standard reference for the development of "categorial grammar."

15. A Reconsideration and Some Refinements

The question of whole-part relations is taken up once more in what may be considered Husserl's last work: *Erfahrung und Urteil*. As is well-known, the problem at issue is here the phenomenological clarification of the origin of predicative judgements, of *apophanseis*. On the way to this goal, Husserl tries to find out the experiential basis of the fundamental insights about wholes and parts which he had worked out in the *Third Investigation*. Husserl takes as his starting point the analysis of the perceptual process and, from the formal-ontological point of view, the whole rethinking of the question turns out to be heavily conditioned by this choice. In fact, Husserl himself[113] explicitly opposes the "purely noematic" way of proceeding of the *Third Investigation* to the present way of proceeding from the "subjective side" and repeatedly stresses that the distinctions and insights acquired along the present way «refer firstly to simple . . . space-time objects of external perception; and cannot be immediately transferred by formalization to objectualities of a higher type, e.g., cultural objects, which are founded in them, although even in the latter objectualities relations such as that of whole and part or of properties and so on, must be recognizable, in a peculiar form, specific to them».[114]

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[112] K. Ajdukiewicz, Kasimierz, *Die syntaktische Konnexität*, in «Studia philosophica», 1(1936), pp. 1–27.

[113] E. Husserl, *Erfahrung und Urteil. Untersuchungen zur Genealogie der Logik*. Redigiert und herausgegeben von Ludwig Landgrebe, Claassen & Goverts, Hamburg 1948, p. 165, referred to by [E].

[114] [E] pp. 166–167 «*Es sei nochmals betont, daß sich alle diese Unterscheidungen . . . zunächst nur auf schlichte Substratgegenständlichkeiten, raum-dingliche Gegenstände äußerer Wahrnehmung beziehen; und nicht ohne weiteres durch eine Formalisierung auf die in ihnen fundierten Gegenständlichkeiten-höherer Art, z. B. Kulturobjekte, übertragen werden können; obzwar an diesen in einer spezifisch ihnen eigenen Art auch Verhältnisse wie die von Ganzem und Teil, Eigenschaftsbeziehungen usw. aufweisbar sein müssen*».

From the very complex phenomenological analyses carried out while pursuing the proposed goal, we shall now select and briefly summarize some points which seem to be particularly relevant to our present concern.

In his analysis of a single, unrestrained, perceptual process Husserl distinguishes three main steps. First of all there is the "simple catching" [*schlichte Erfassung*] of the object. The attention is captured by the object x and the perceptual interest [*Wahrnehmungsinteresse*] is oriented towards the object "as a whole."

Soon after, we pass to the second main step, the "proper explicative observation of the object" [*eigentliche, explizierende Betrachtung des Gegenstandes*] Here the perceptual interest, penetrating into the "inner horizon" of the object [*Explikation ist ein Hineingehen des Wahrnehmungsinteresse in den Innenhorizont des Gegenstandes*], gradually acquires a knowledge of the object as it is in itself, independently of its environment.

In the third step, finally, also the surrounding field enters into the cone of perceptual interest and the *external*, *relative determinations* of the object are detected. However, in general, not all objects that lie in this field are taken into consideration, but only a few of them.

Let us dwell a moment on the second, fundamental step. In it a basic distinction emerges: the perceptual interest on the one side thematizes the object as the *substratum* s (x) of the explication, while on the other side it gradually unfolds the object's *inner determinations* [*innere Bestimmungen*] $d_1(x)$, $d_2(x)$, . . ., that is to say, it acquires knowledge of s (x) as $d_1(x)$, as $d_2(x)$, and so on.

It is to this first distinction that we may apply the terminological pair ⟨*whole – parts*⟩ in its first, most general sense: the substratum is a whole the determinations of which are parts. Or, as Husserl says: «Every substratum for possible internal determination can be regarded as a *whole* which has *parts* into which it is explicated. . . . a whole is *every unitary object which allows partial catches* [*Partialerfassungen*], that is to say penetrating, explicating observation and . . . a part is every so resulting explicatum ».[115]

A most important fact which happens during the explicative process is that starting from the original substratum the determinations do not, in

[115] [E] p. 161.

general, become explicit straight away, but become, in their turn, substrata of new explications. So the explication of the (new) *object* $d_i(x)$ has the substratum s $(d_i(x))$ and unfolds it as $d_1(d_i(x))$, as $d_2(d_i(x))$, and so on. Now there are two possibilities. First: the perceptual interest leaves the original substratum s (x) and concentrates itself, almost exclusively, on the new one (in a bunch of flowers, the interest concentrates itself on a particularly red rose and "forgets" the bunch). In this case the explicative process of the object x is to be considered terminated. Second: the perceptual interest explicates the inner determinations of the new substratum, but preserves the original s (x) as the *main substratum* of its unfolding, so that the new explicative step in fact provides just a better comprehension of the object x (the rose and its particular colour are observed as a feature of the bunch). It is, we could say, a *second-level explication* of the object x. Clearly this process can be iterated and so a *chain* of unfolding steps is generated, all of which gradually enrich our knowledge of the object x. Of course, this chain-producing process may begin again, starting either from the main substratum s (x) or from some "secondary substratum," $s(d_k(\ldots(d_i(x))\ldots))$ in such a way that the explication of the original object x takes the form of a *tree*; in this case we speak of a "*ramified explication*" [*verzweigte Explikation*] of the object x.

From the previous arguments it seems to follow that the pair *substratum – determination* is in fact a completely relative one, every determination having the possibility of becoming the substratum of a proper explicative process. According to Husserl, however, this is not true. There are, he maintains – of course with reference to the field he is investigating, namely the perceptual experience of the real external world –, *absolute substrata* as well as *absolute determinations*; the substratification of a determination, indeed, does not completely obliterate its being originally a determination, and «so we come eventually and necessarily to substrata which did not originate from a substratification». Further analysing the issue, he concludes: «An absolute substratum is characterized by the fact that it may be simply, directly experienced, immediately caught and that its explication may immediately start»; in this sense, absolute substrata are «the singular objects of the external sensitive perception, i.e., *bodies*». Through a rather complicated and somewhat entangled discussion Husserl arrives also to the notion of *absolute determination*, as a determination which, even though it is, like any other determination substratifiable, is not an absolute substratum. The conclusion is «in this sense *absolute substrata* are *independent*, *absolute determinations dependent*».[116]

It is easy to see that it is precisely to these absolute substrata that it becomes possible to apply the second, more restricted sense of the terminological pair ⟨*whole – parts*⟩, the one in which 'whole' means *independent whole*.

But among absolute substrata in the sense specified above there are not only singular bodies, but also pluralities of bodies either arranged in a space-time configuration or even held together by some causal relationship (e.g., a machine). In the case of "plural bodies," the explication yields as determinations also bodies, possibly still plural bodies, but, of course, not only bodies. The bodies identified as determinations of a plural body are themselves absolute substrata in the sense above, but this does not change the original plural body's nature as an absolute substratum.

It is in the discussion of these plural objects that one can find the most interesting suggestions as to some refinements of the formal-ontological theory. In fact an independent plural object corresponds to what was in the *Third Investigation* the second possible form of a pregnant whole. There are however some difficulties. One of them depends upon the fact that the use of the terminological pair ⟨*mediate – immediate*⟩ is somewhat ambiguous; sometimes it is taken with its "formal-ontological" meaning, sometimes with its "descriptive-phenomenological" one; so, for instance, the *tie* [*Verbindung*] of a plural object x (what was called in the *Third Investigation* the *unity moment* [*Einheitsmoment*] of x) not being, clearly, a moment of the different fragments, but of the whole x, is, according to what is said on pp. 167–168, an *immediate moment* of x; on the other hand, that same tie, since it emerges in the explicative process of x only after the identification of its fragments, is said to be, on pp. 169–70, a *mediate moment*.

So let us just recall some of the other suggestions about the structure of a plural independent object which are to be found here. An *immediate fragment* of an independent object x is a fragment of x which is not a

[116] [E] p. 156 «In diesem Sinne sind *absolute Substrate selbständig, absolute Bestimmungen unselbständig*».

proper part of any (proper) fragment of x; a *mediate fragment* is a fragment which is not immediate. A finite plural independent object x having as immediate fragments the separated independent objects $x_1, \ldots, x_k$, is a combination of a moment, called "*tie*" or "*link*" u, with the object $x_1 \amalg \ldots \amalg x_k$, such that $\partial(u) = x_1 \amalg \ldots \amalg x_k$. To grasp the idea: take the two independent objects $[x|0 \leq x \leq 1]$ and $[x|2 \leq x \leq 3]$ and the dependent object $[x|1 < x < 2]$; then you can regard the independent object $[x|0 \leq x \leq 3]$ as composed from $[x|0 \leq x \leq 1]$ and $[x|2 \leq x \leq 3]$ tied together by $[x|1 < x < 2]$.[117] It is clear that, in general, for a given independent x, there are many z, with $\partial(z) \neq 0$ and such that $x = z \amalg \partial(z)$; every such z is say to be an *immediate moment* of x. An immediate moment of an independent finite plural object x which is not a tie of x is said to be an *immediate property* [*unmittelbare Eigenschaft*] of x. It is now possible to work out in detail these basic suggestions. There is however a critical point which cannot be overlooked: the assumption or not of additivity for (strongly) separated objects. If it is assumed and, for instance, $x_1, \ldots, x_k$ are (strongly) separated fragments of x, then $x_1 \amalg \ldots \amalg x_k$ is already independent and the (only) immediate fragment of x.

It is not completely clear whether or not the first form of the notion of a pregnant whole discussed in the *Third Investigation* – a pregnant whole as an independent connected object – finds application in this context. It could in fact be argued that such are those wholes which appear here under the notion of a singular body. In any case it is possibly worth noting that, already in the *Third Investigation* Husserl, though stressing that one should take care not to confuse non-finding with non-existing, inclined to accept the principle: «all wholes, with the sole exception of the fragmentable ones, fail to have linking unity forms».[118]

Finally let us emphasize that the major theoretical restriction which ensues from the previously mentioned choice of the field is that the inner part-whole relation of every object, and in particular of wholes in the pregnant sense, is a *finite tree*. «In real experience there is no infinite

[117] With a grain of salt, of course, because in the strict sense $\partial([x|1 < x < 2])$ is $\{1, 2\}$ and not $[x|0 \leq x \leq 1] \amalg [x|2 \leq x \leq 3]$.

[118] [3] p. 283 «*alle Ganze mit bloßer Ausnahme der zerstückbaren verbindender Einheitsformen ermangeln*»

subdivision and especially no experiential plurality which, as experience evolves (for instance in getting closer) decomposes itself infinitely in new pluralities».[119]

[119] [E] p. 154 «*In der wirklichen Erfahrung gibt es keine Teilung in infinitum und vor allem keine erfahrbare Mehrheit, die* in infinitum *in der fortgehenden Erfahrung (etwa in Näherkommen) sich in neue Mehrheiten auflöste*».

CHAPTER 4

SPACE AND MOVEMENT. ON HUSSERL'S GEOMETRY OF THE VISUAL FIELD

Giulio Giorello and Corrado Sinigaglia

Abstract. In 1916 Edmund Husserl devoted some pages to a *Systematic Analysis of the Constitution of Space* which can be considered as the most advanced stage of his work on the problem of space. The aim of our paper is to show, by means of close textual exegesis, how the phenomenological clarification of the origin of the representation of space can be achieved for Husserl only trough a "geometry of the sensuous fields", in particular of the visual field, and how such a "geometry" provides us with those "proto-idealizations" on which the phenomenological possibility of geometric idealization is based.

In 1916 Husserl devoted some pages to a systematic analysis of the constitution of space. Revised and corrected, they will be included by Edith Stein in her elaboration of the *Dingvorlesung* [Thing-Lectures].[120] This text, albeit short and incomplete, represents, in our opinion, the most advanced stage of Husserl's work on the problem of space bringing to completion a train of thought started with his lectures on geometry (in particular, on Riemannian geometry) of 1889/1890 and his projects (never accomplished) of a *Raumbuch*,[121] and developed in his 1907 lectures.

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[120] Cf. E. Husserl, *Ding und Raum. Vorlesungen 1907* (Husserliana XVI), hrsg. U. Claesges, Den Haag, M. Nijhoff, 1973, pp. 297–336, 377–379, 418–424. English translation by R. Rojcewicz, *Thing and Space. Lectures 1907* (Edmund Husserl Collected Works VII), Dordrecht, Kluwer, 1997, pp. 257–288. Henceforth, we cite first the page numbers in the German original, and then, in parentheses, the corresponding pages of the English translation.

[121] Cf. E. Husserl, *Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass (1886–1891)* (Husserliana XXI), hrsg. I. Strohmeyer, The Hague/Boston/Lancaster, M. Nijhoff, 1983, pp. 312–347, 402–406, 485–486. On this point see C. Sinigaglia, *La seduzione dello spazio. Filosofia e geometria nel primo Husserl,* Milano, Unicopli, 2000; Id., "La libera variazione delle forme. Husserl lettore di Riemann," in M. D'Agostino, G. Giorello, S. Veca (eds.), *Logica e politica. Per Marco Mondadori,* Fondazione Arnoldo e Alberto Mondadori, Milano, il Saggiatore, 2001, pp. 377–403.

L. Boi et al. (eds.), Rediscovering Phenomenology, 103–123.

In what follows, we will show – by means of close textual exegesis – how the clarification of "the deepest phenomenological sense" of the "problem of the '*origin of the representation of space*' "[122] can be achieved for Husserl only through a "geometry of the sensuous fields";[123] and how in such a "geometry" one can find those "proto-idealizations" on which the phenomenological possibility of geometric idealization is based.

I

Husserl's draft on the *Systematic Constitution of Space* rests on a double presupposition. On one hand, Husserl claims that our intuition of space is given by external perception or by external fantasy.[124] The clearest formulation of this claim is a passage of *Ideen* where he asserts that "we seize upon *the 'idea' of space* and the ideas included in it" in virtue of "the spatial aspect" of the thing: "it is capable [. . .] of infinitely multiple changes in form and, in the case where the shape and alterations in shape are retained as identical, of infinitely multiple alterations of place; it is 'moveable' in infinitum."[125]

On the other hand, Husserl never tires of reminding us that, although the phenomenology of space depends necessarily on the phenomenology of thinghood, it is only by means of a methodological abstraction that we can consider the thing as a mere *res extensa*, that is to say, that we can ignore the essential properties which characterize it as a *res materialis* – where by 'res materialis' Husserl means, as is well known, "a *substantial* unity [. . .], a unity of *causalities* and, with respect to possibility, of infinitely complex causalities."[126]

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[122] E. Husserl, *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie*. Erstes Buch: *Allegemeine Einführung in die reine Phänomenologie* (Husserliana III/1), hrsg. K. Schuhmann, Den Haag, M. Nijhoff, 1976, p. 351. English translation by F. Kersten, *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy*. First Book: *General Introduction to a Pure Phenomenology* (Edmund Husserl Collected Works II), Dordrecht, Kluwer, 1982, p. 362.

[123] E. Husserl, *Analysen zur passiven Synthesis* (Husserliana XI), hrsg. M. Fleischer, Den Haag, M. Nijhoff, 1966, p. 145.

[124] Cf. E. Husserl, *Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass (1886–1891)*, op. cit., pp. 275–276.

[125] E. Husserl, *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie*. Erstes Buch: *Allegemeine Einführung in die reine Phänomenologie*, op. cit., p. 348 (p. 359).

[126] ibid.

Admittedly, this is for Husserl a legitimate abstraction, for "the unity of a mere *res extensa* is conceivable without the unity for which the idea of the *res materialis* is a norm."[127] (Whereas the contrary does not apply). Yet, it is essential to make it explicit, if we are to lay hold of phenomena [*Erscheinungen*] "in which space is intuitively presented and is 'constituted' as the unity of appearances, of descriptive modes of presentation of something spatial."[128] To clarify the inner stratification of these phenomena, Husserl points out that "each thing-appearance [*Dingerscheinung*] necessarily includes in itself a stratum which we call the thing-schema [*Dingschema*]: it is the spatial shape merely filled with 'sensuous' qualities – without any determinateness of 'substantiality' and 'causality.' "[129]

With regard to this *Dingschema*, Husserl remarks in some notes dated 1910: "the thing occupies and fills a determinate sector of 'the' space. This sector of space is an 'inner' constitutive determination of the thing and has indeed a determinate structure (geometrical body; the best name for it is 'spatial schema') which, as geometrical, can be the same structure in different places (in different locations). This unity of structure (size included: the completely determinate geometrical body) and location is what we call the spatial schema."[130] Not only, however, does extension inherit to the essence of the thing, but it is *in virtue* of its extension that the thing fills the space *qua* unity of a "form in a location." This explains why for Husserl from a phenomenological point of view extension "is not a mere piece of space," and why he says that "it is not only that every alteration in magnitude, with a conservation of the same spatial form, implies an alteration of extension, and that the same is true for every alteration of the form with a conservation of the magnitude and for every deformation in whatever sense, but also every alteration of position is an alteration of the extension."[131]

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[127] ibid., p. 350 (p. 361).
[128] ibid., p. 351(p. 362).
[129] ibid., p. 350 (p. 361).
[130] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., pp. 341–342 (pp. 297–298).
[131] E. Husserl, *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie.* Zweites Buch: *Phänomenologische Untersuchungen zur Konstitution* (Husserliana IV), hrsg. M. Biemel, Den Haag, M. Nijhoff 1952, p. 30. English translation by R. Rojcewicz and A. Schuwer, *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy*. Second Book: *Studies in the Phenomenology of Constitution* (Edmund Husserl Collected Works II), Dordrecht, Kluwer, 1989, p. 32.

Yet, in addition to extension, the essence of the thing includes also what Husserl calls the "sensuous filling" [*sinnliche Fülle*] or the "sensuous matter" [*sinnliche Materie*], which, he claims, constitutes, along with the spatial schema, a further unity: the unity of the phantom of the thing [*Dingphantom*], or of the sensuous schema [*sinnliche Schema*].[132] By introducing this new stratum of the constitution of the spatial thing, with its attendant terminology, Husserl intends to account for the following aspect, namely that a form in any location is "qualified," i.e., it is "qualitatively filled." As he explains, "every quality of a thing 'fills the spatial body': the thing spreads itself out in the quality; in every one the thing fills its corporeality (its extension), and the same is true, at the same point in time, for all real qualities."[133]

In the beginning of *Systematic Constitution of Space*, Husserl alludes to this double modality of filling: "Each body, and, more precisely, each sensuous schema of a complete corporeality, is a spatial corporeality (a spatial form), 'over which' or 'in which' sensuous qualities are extended."[134] Furthermore, it is only for such a mutual implication of "spatial schema" and "sensuous matter," taken as "non-independent moments," of the unitary "sensuous schema," that it is possible to speak of space as "form," namely as a principle of order and individuation: "The sensuous quality can be given repeatedly in the unity of phenomenological – pre-empirical – extensions. But the *part of space* can be given only once. The ultimate specific difference of the sensuous quality can be duplicated; but the place, the extension that a quality fills, cannot be duplicated. The extension is *something never repeatable* and yet something *abstract*."[135]

The "place" is such insofar as it is filled by a quality which is, for this very reason, distinct from any other quality "completely equivalent to it in species." For "the place can never be overlayed at one and the same time

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[132] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., pp. 342–343 (pp. 298–299).
[133] E. Husserl, *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie.* Zweites Buch: *Phänomenologische Untersuchungen zur Konstitution* (Husserliana IV), op. cit., p. 30 (p. 33).
[134] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 297 (p. 257).
[135] E. Husserl, *Zur Phänomenologie des inneren Zeitbewusstsein (1893–1917)* (Husserliana X), hrsg. R. Boehm, Den Haag M. Nijhoff, 1973, pp. 250–251. English translation by J.B. Brough, *On the Phenomenology of the Consciousness of Internal Time (1893–1917)* (Edmund Husserl Collected Works IV), Dordrecht, Kluwer, 1991, p. 259.

by two (*visual* or *tactile*) qualities belonging to the same genus – whether by two *equivalent* or by two *different* qualities." That means that "if the species of the quality is determined and if the place is determined, then [...] the *concrete individual part* is determined. The *place* 'makes' the infima species of quality into an individual quality. The place is the *determination that determines individually*."[136]

This, however, is not the only possible spatial individuation. A more fundamental one is suggested by Husserl, according to which the "the determination that determines individually" transcends the absoluteness of the location, constituting the *spatial individual*: "The *spatial individual* is that which remains identical while the place varies [. . .]. If the spatial filling is preserved in its identity (in its species) and if it *moves* while what is specific in the filled spatial form (shape), then the spatial individual is the same. [. . .] The individual is that which is always determined as specifically the same and that which changes its absolute location. There is, of course, no qualitative individual; the possibility of the spatial individual depends on the peculiarities of the space and time."[137]

We must then distinguish between the extension (the spatial schema), which varies when its shape or location change, and the spatial individual, which preserves its shape even when its location changes. In this case too we shall talk of a "located shape," but now this term, "shape," comes to mean what is invariant with respect to "the manifold of its possible locations [*die Mannigfaltigkeit der möglichen Lagen*]."[138]

Before moving on to the constitution of the spatial individual, sometimes called also "*fixed* spatial thing [*starre Raumding*]" or "'geometric' body," [139] it is necessary to specify the types of qualitative filling of space. In the 1916 draft, Husserl makes a quick reference to it. He says: "A body is constituted as a sensuous schema by the sense of touch and the sense of sight, and every sense is a sense through an apperceptive conjunction of

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[136] ibid.

[137] ibid., p. 252 (p. 260).

[138] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 322 (p. 277). "Every location in a partial system and every location in a complete system [. . .] allows itself to be transformed into any location whatsoever, and ideally does so in a free movement" (Ibid., p. 325; pp. 279–280).

[139] E. Husserl, *Zur Phänomenologie des inneren Zeitbewusstsein (1893–1917)*, op. cit., p. 252, (p. 260).

the corresponding sense-data with kinaesthetic data."[140] Let us consider the first point Husserl makes here: both in the 1907 lectures and in the second volume of *Ideen*, he points out that, with reference to certain qualities (sound, taste, etc.), we can talk of "filling" only in an improper meaning or in a metaphorical one; he also draws a distinction between *materia prima* and *materia secunda*, i.e., qualities that intrinsically have an extension (like visual and tactile qualities) and qualities with regard to which spatial apprehension can be neutralized. While the former constitute the concretum of the thing, the latter are spatial only in an indirect way.

Thus, only visual and tactile qualities can truly be localized. In virtue of the individualizing determination of location, they can coexist in the same instant, even when they are identical. By contrast, "two tones, identical in species, can occur only at different times. In a given time there is only one tone with the same specific determination. Here there is *simultaneity* only of *what is different*, and even that is transformed into a unity, into a fusion."[141]

This implies that, in the case of tones, there isn't any space, or, in other words, the tonal field cannot be considered an authentic sensuous field. By sensuous field Husserl means "a continuous nexus," "a pre-empirical expanse," qualitatively "saturated," whose pieces are structured according to an order that makes it possible for them to be "presentational for some thing or other."[142] Insofar as it is the unity of possible locations, the sensuous field has the form of a system of positions ordered in a continuous way.[143]

It is for this reason that in the *Constitution of Space* draft Husserl restricts his analysis to "the primal fields, namely the field of the sense of sight and the field of touch." These fields are "without qualitative differentiations," i.e., they are "a continuum of equal qualities, distinct only in their position within the order of the field."[144] If the differentiation of a visual or tactile datum presupposes a qualitative discontinuity (think of the phenomenon

[140] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 298 (p. 257).
[141] E. Husserl, *Zur Phänomenologie des inneren Zeitbewusstsein (1893–1917)*, op. cit., p. 252 (pp. 260–261).
[142] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 83 (p. 68).
[143] Cf. E. Husserl, *Analysen zur passiven Syntesis*, op. cit., pp. 137,143.
[144] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 298 (p. 258).

of contrast, such as some red dots on a white background), such discontinuity does not depend merely on the homogeneity and heterogeneity of the contents, i.e., it is not explicable in terms of the passive syntheses of association. Neither can it be reduced to a simple gap between coexistent contents caused by ultimate differences within the same genus.

In line with *Logical Investigations*, [145] Husserl argues that "the places are in themselves distinct, but the qualities are distinct only in virtue of places. On the other hand, the places and complexes of places own their individual prominence to the quality, i.e., to their qualitative (specific) discontinuity."[146] It is thus clear that, if the system of locations is announced in the differentiation of discontinuous qualities, their discontinuity implies the order of extension, the unity of the sensuous field as "global system of locations." Without this implication, the possibility of movement (and rest), essential to Husserl's explanation of objective space, would be unintelligible.

Now, movement (and rest) show the crucial role played by the co-ordination between sensuous data and kinaesthetic data: this co-ordination determines "the orientation" by means of which the different sensuous adumbrations can be perceived as different appearances or aspects *of* the same object. "In the apperception of a body, this sensuous content is consciously co-ordinated to a kinaesthetic datum in such a way that if the kinaesthetic datum traverses (freely or unfreely), within its system, a 'line' [. . .] the adumbrational content of the sense-field traverses a certain appurtenant, characteristic modification."[147] Every movement of the eye

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[145] "Die Diskontinuität als solche bezieht sich auf die niedersten spezifischen Differenzen innerhalb einer und derselben nächst übergeordneten reinen Gattung; also z. B. auf Farbenqualitäten im Vergleich mit Farbenqualitäten. Wir definieren aber nicht etwa Diskontinuität als bloßen Abstand koexistenter Inhalte hinsichtlich solcher niederer Differenzen. Gleichzeitige Töne haben Abstand, aber es fehlt Diskontinuität im prägnanten Sinne. Diese bezieht sich auf die spezifisch differenzierenden Momente nur insofern, als sie über ein kontinuierlich variierendes Moment, nämlich das räumliche oder zeitliche, angrenzend ausgebreitet' sind." (E. Husserl, *Logische Untersuchungen.* Zweiter Band: *Untersuchungen zur Phänomenologie und Theorie der Erkenntnis. Text der 1. und 2. Auflage ergänzt durch Annotationen und Beiblätter aus dem Handexemplar,* Husserliana XIX/1, hersg. U. Panzer, The Hague, M. Nijhoff, 1984, p. 250.

[146] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 185 (pp. 155–156).

[147] ibid., p. 323 (p. 278).

that follows a line goes with a kinaesthetic datum in relation to which the line (or, more precisely, its visual schema) shows itself in a given orientation. Thus, it is the mutual coordination of sensuous contents and kinaesthetic sequences that offers the possibility to distinguish between movement and rest.[148]

The perceived object is regarded as "in objective rest [*objektive Ruhe*]," "if, along with kinaesthetic standstill, no changes in orientation take place." To stay with the case of the visual field, we can talk of rest when, the perceiving eye keeping still, the sensuous schema of the object will be oriented in the same location as the visual field. On the other hand, we can talk of objective rest even when, "with a free traversal of all the kinaesthetic series, ever the same appurtenant changes in orientation occur in cyclical nexuses."[149] Once again, if the eye, after moving in such a way as to let the image (schema) slip from the centre to the borders of the visual field, returns to the initial position and finds the image back to the centre, then the image is considered at rest and the sliding is attributed to the kinaesthesis.

Analogously, an objective movement occurs when "along with kinaesthetic sequences, no orientational changes arise" or when "the orientational changes do not elapse in parallel with the kinaesthetic changes, are not coordinated to them, and are not cyclically reproducible."[150] It is thus possible that the movement of the eye is not followed by any sliding of the image: the latter continues to be oriented in the same way as before. But it is also possible that the image, originally at the centre of the visual field, slips by itself to its borders to the point of reaching the peripheral zone in which the shape loses its edges.

Where the eye follows the gliding of the image, it can reorient the schema in order, e.g., to bring it back to the centre of the visual field. In such a case, visual contents and kinaesthetic data are coordinated, and

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[148] "We see that in fact the merely visual sequences are not sufficient for the apprehension and that they do not contain the means to bring rest and movement to an appearance where they would be discriminated. But that is to say that the constitution of objective location and of the objective spatiality is essentially mediated by the movement of the Body or, in phenomenological terms, by the kinaesthetic sensations, whether these be constant or changing kinaesthetic sequences." (Ibid., p. 176; p. 148).

[149] ibid., p. 327 (p. 281).

[150] ibid.

yet they do not elapse in parallel, since it is necessary to continuously correct the orientation of the image. It is because this correction belongs to the power of the kinaesthetic system of monocularity that the image can be constituted as "moveable," in the sense of the mere change of location. Could the eye not reset the original orientation of the image, and were the kinaesthetic system of monocularity not coordinated to the system of image changes, there would not be any awareness of movement.[151]

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[151] "The basic rule is: that which is constituted as movement must appear in such aspects that changes in the aspects can be offset by the appurtenant kinaesthetic motivations." (Ibid., p. 328; p. 281). Husserl's analysis have many points of contact with Poincaré's position in the article, "L'espace et la géométrie," first published 1894 in *Revue de Méthaphysique et de Morale* (3, pp. 631–646) and published later as chapter IV of *La science et l'hypothèse* (1902): "Whether an object changes its state or only its position, this is always translated for us in the same manner, *by a modification in an aggregate of impression*. How then have we been enabled to distinguish them? If there were only change of position, we could restore the primitive aggregate of impressions by making movements, which would confront us with the movable object in the same *relative* situation. We thus *correct* the modification, which was produced, and we re-establish the initial state by an inverse modification. If, e.g., it were a question of the sight, and if an object be displaced before our eyes, we can "follow it with the eye," and retain its image on the same point of the retina by appropriate movements of the eyeball. These movements we are conscious of because they are voluntary; and because they are accompanied by muscular sensations. But that does not mean that we represent them ourselves in geometrical space. So what characterizes the change of position, what distinguishes it from the change of state is that it can always be *corrected* by this means. It may therefore happen that we pass from the aggregate of impressions A to the aggregate B in two different ways. First, involuntarily and without experiencing muscular sensations – which happens when it is the object that is displaced; secondly, voluntarily, and with muscular sensations – which happens when the object is motionless, but when we displace ourselves in such a way that the object has relative motion with respect to us. If this be so, the translation of the aggregate A to the aggregate B is only a change of position. It follows that sight and touch could not have given us the idea of space without the help of the "muscular sense." Not only could this concept derive from a single sensation, or even from *a series of sensation*; but a *motionless* being could never acquire it, because, not being able to correct by his movements the effects of the change of position of external objects, he would have had no reason to distinguish them from changes of state." (J.-H. Poincaré, *Science and Hypothesis*. English translation, New York, Dover, 1952, p. 58). On the comparison between Husserl's and Poincaré's views on the structure of spatial representation we deal at greater length in a forthcoming paper.

II

The movement of the eye is obviously not the only possible kinaesthetic system: think, for instance, of the movement of the head, of the upper body, or think of walking. Each of these systems is a "system of power" [*System der Vermöglichkeit*],"[152] whose "basic directions of modification" are determined by the null-position of the system, which accordingly has the form of a "coordinate system of orientation."[153] Each defines a different space, to which corresponds a different geometry. It follows that, on the one hand, it will be necessary to emphasize the specific orientations of each system, on the other hand, it will be necessary to study the intersections to show how such multiplicity of spaces and geometries can coalesce into a total system, or, as Husserl writes, "into the unity of one constitution which makes possible the consciousness of something self-same."[154]

In the analysis of the different levels of the visual space, Husserl distinguishes the following: (i) the oculomotor system (delimited plane space); (ii) "the system of head movement around the basic axes" (cylindrical field of vision); (iii) "the complete cephalomotor system" (Riemannian space).

The oculomotor space is the space constituted by imagining that the eye alone is moveable and moving; it "has a null-point" to which the "basic position of the eyes (straight ahead) is "kinaesthetically co-ordinated."[155] The co-ordinate system has only two basic axes: the above-below axis, and the right-left axis. Husserl explains: "The oculomotor right-left is constituted by means of eye-movements from the null-position in the

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[152] Cf. U. Claesges, *Edmund Husserls Theorie der Raumkonstitution* (Phaenomenologica 19), Den Haag, M. Nijhoff, 1963, pp. 75–76.

[153] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 328 (p. 282).

[154] ibid., p. 324 (p. 279). In the Systematic draft Husserl focuses almost exclusively on the analysis of the visual field leaving aside the constitution of tactile space, which however, is extremely important as it shows the constituting function of Body [*Leib*] as an integrated system of partial kinaesthetic systems. Hence the problem brought up in the 1907 lectures: "what constitutes the identity of space, which is materialized once visually and at another time tactually and yet in this double materialization is the one identical space?" (Ibid., 156; p. 132). To the solution of this problem Husserl will work in the 20s, in several unpublished manuscripts. In connection with the above discussion, it is important to stress that Husserl locates in the possibility of a unitary geometry the *conditio sine qua non* of the constitution of "an identical spatial body."

[155] ibid., p. 309 (p. 266).

direction of the preferred side and in the reverse direction (L ← 0 → R)." Since in moving from L to 0 the "directional quality" is the same as in moving from 0 to R, whereas it is the opposite in moving from R to 0 and from 0 to L, here we have "a cyclical continuum of directional qualities." Further, since the same holds for every direction passing through 0, we have "a total continuum of directions which are exactly the same as the directional manifolds in a plane, specifically in a bounded one." Thus in the oculomotor system the kinaesthetic manifold is "a two-dimensional manifold, which can be 'generated' [*erzeugt*] only by traversing it, and it is a 'plane manifold.' "[156]

The oculomotor system is responsible for the constitution of the phenomena called *objective* movement and rest – but also of the constitution of the phenomenon called distance, "which remains the same in 'mere' movement within 'a body' or within two 'strictly conjoined' bodies."[157] Thus we can say that the constitution of the fixed spatial thing refers to this first level of visual space. And we can also say that here one can find the roots of what will make it possible for space to be both principle of individuation and condition of identification.

It must be noted that "identity" means here "invariant under *a* given group of transformations."[158] For it is not a coincidence that in some notes written to supplement the analysis of the oculomotor space, Husserl mentions explicitly the constitution of "Riemannian thing" [*Riemannsche Dinglichkeit*], meaning by this a limit-concept that finds its justification in the limit-case in which the geometrical body remains unchanged with respect to any movement, in the sense that "all the distances between its point and its straight expanses remain unchanged."[159]

Obviously, the Riemannian thing is not the thing in the objective space, nor is its "fixity" to be understood in the physical meaning of the term, nor can it be reduced – contrary to Helmholtz's opinion – to an empirical fact. The very same "objective" phenomenon of movement is,

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[156] ibid., p. 311 (p. 268).

[157] ibid., p. 331 (p. 284).

[158] Cf. L. Boi, *Questions Regarding Husserlian Geometry and Phenomenology. A Study of the Concept of Manifold and Spatial Perception*, Centre d'Analyse et de Mathématiques Sociales, Paris 2000, pp. 35–54.

[159] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., pp. 371–372 (p. 338).

with reference to its constitution in the oculomotor space, objective only within inverted commas. Indeed, it is a necessary methodological caution, since the phenomenological analysis of visual space takes place on a pre-objectual level. As a correlate of the oculomotor system, the visual space is a field of images that are open to different modifications concerning their intensity, their shape, etc. Nevertheless the limit-possibilities that Husserl links with kinaesthetic powers of the oculomotor system adumbrate those "proto-idealizations" without which geometrical idealization would not be conceivable. Not that the latter is reducible to the former; rather, the point is to emphasize the difference among them, so as to be able to acknowledge the various kinds of geometrization grounded in the original correlation between kinaesthesis and visual field.[160] For without such an understanding it would be extremely difficult, if not impossible at all, to make sense of how for Husserl experience can be *unitary* experience of a world: "Since the field, through its inner order, prescribes a fixed order to all the images, and since the posing of unity follows the continuity in the transformations of the individual images and of their reciprocal orientations, there arises the consciousness of a strictly ordered manifold of things and, ultimately, the consciousness of the world."[161]

As the second level of the constitution of space, Husserl refers to the visual field obtained by "the rotation of the head around its basic axis, while the remainder of the body is in its normal position, a position that is supposed to remain motionless and fixed."[162] It is immediately evident that now the visual space coordinated to the oculomotor system undergoes variations produced by the action of the other kinaesthetic systems. In this specific case, its overall form does not change; it acquires though "a new index with every position of the head." Since a new kinaesthetic system intervenes, with a new line of motivating data, oculomotor space becomes the "apparency [*Apparenz*]," i.e., "image" of a new space, and the same holds for the "oculomotor body." For instance, what appeared in motion

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[160] However, that the visual field is originally correlated to a kinaesthetic system does not mean – contrary to Ulrich Claesges' interpretation – that it is simply the correlate of a kinaesthetic system. (See U. Claesges, *Edmund Husserls Theorie der Raumkonstitution*, op. cit., pp. 72, 74).

[161] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 217 (p. 183).

[162] ibid., p. 309 (p. 266).

in oculomotor space can appear at rest in the "new" space: if, by turning the head, the oculomotor image is displaced but, through the reversal of this movement, it comes back to its previous orientation, then it will be regarded as an image of a stationary body, merely as a result of the new kinaesthetic system.

Now, exactly what kind of space is constituted by moving the head around its basic axis? And in what does the geometry of such a space differ from that of the oculomotor space? To begin with, it seems that if the latter is a "delimited space," the one determined by the rotation of the head has, by contrast, no limits either on the right or on the left. In fact, it is a "closed" space, for the basic system of orientation is not given by the intersection of axes, but it is given by "a closed line of coordinates in the right-left direction, as an abscissa, and an unclosed line in the direction above-below." More precisely, it is "a cylindrical field of vision," whose null-point is represented by "the kinaesthetic complex characterized by the normal position of the head and by the normal position of the eyes as they gaze straight ahead."[163] This means that from this basic position "a turning to the right [...] ideally could indeed continue so far that it would lead to the same oculomotor spatial and corporeal system as an ideally possible turning to the left would lead $(+a, -a)$." Yet, if the closed abscissa axis makes space unlimited in the left-right directions, thus allowing "an autonomous turning around $(0 + a + 0; 0 - a - 0)$," the unclosed ordinate axis delimits it "above and below," so as to get a null-line and two parallel lines $(y = +b, y = -b)$; or, in case the abscissa $= 0$ $(a = 0)$, "we would have a length with two opposite, possible ways of traversing it (from above to below and from below to above)."[164]

Eventually, the third level is represented by the "entire cephalomotor space" which is constituted by the "complete cephalomotor system." Given the appropriate idealization of the motility of the head, we are dealing here with a "spherically closed space," whose basic orientation system is composed of "two null-lines, which are closed 'circles', namely the closed right-left line (which was already constituted previously) and the closed

[163] ibid., p. 310 (p. 267).
[164] ibid., p. 332 (p. 285).

above-below line." Husserl explains further: "the one of their intersectional points is null, and has, as it were, a shadow, a counter-null."[165]

None of these widenings of the kinaesthetic system is sufficient, however, to reach the constitution of depth, for, as Husserl remarks, "all bodies have been, hitherto, 'surface beings', at best 'spherical beings.' " Even the total cephalomotor space would be but "a homogeneous Riemannian space of two dimensions," i.e., a two-dimensional spherical manifold.[166]

It must be kept in mind, at any rate, that anytime Husserl talks of levels of constitution he does not mean to establish a hierarchy of kinaesthetic systems, as though they could operate in isolation and the upper levels require that the lower ones have already constituted their own space. Or rather, he invites us to acknowledge that "the various systems of movement of our Body [*Leib*] can arise as partially substituting for one another, and so they do not have, individually, a different significance for the constitution of visual space." This is also the reason why Husserl asserts that we "cannot add up, without further ado, the number of the dimensions of the various systems of movement."[167]

Things do not change if we turn our eyes to the left or if we turn our head to the left keeping fixed our eyes: "Likewise, the up and down movement of the head and the up and down movement of the eyes can substitute for one another within certain limits. Accordingly, all the systems that constitute a closed visual space belong together, and the kinaesthetic system must correspond to the number of dimensions of this manifold and thus must be a two-dimensional cyclical manifold. Here belongs the visual sense-field, which is two-dimensional, with its 'centre.' "[168]

Appropriating for his own purposes the insights contained in Riemann's theory of manifolds, Husserl achieves two results: first, he gets rid of the thesis of those who, like Carl Stumpf, held that any visual content necessarily involved the third dimension;[169] second, he shows that the

[165] ibid., p. 310 (p. 266).
[166] ibid., p. 311 (p. 266).
[167] ibid., 336 (p. 288).
[168] ibid.
[169] Cf. C. Stumpf, *Über den psychologischen Ursprung der Raumvorstellung* (1873), Bonset, Amsterdam 1965, p. 182. While for Stumpf, if surface is immediately given in the visual representation, then it is also given depth, for Husserl such inference is unwarranted,

phenomenological constitution of three-dimensional space is not arrived at merely through the juxtaposition of the different fields and kinaesthetic systems, but requires the analysis of new kinds of movement. In dealing with the total cephalomotor field, Husserl asked us to "simulate that everything constitutive of depth is imperceptible," namely, that "no turnings, concealments, or rotations would occur."[170] Now, it is precisely these phenomena that the last pages of *Systematic Constitution of Space* mention, referring to the lectures of 1907 for its detailed investigation. He writes: "[a third dimension is] constituted when groups of movements, i.e., groups of kinaesthetic data, are coordinated to new sorts of changes in images. [. . .] There come into consideration: phenomena of concealment, perspectival expansion and contraction, and, in general, all sorts of perspectival changes in size and form, in which approaching and receding, as well as rotation in various directions, are constituted."[171]

III

Let us briefly examine the phenomenon of concealment that occurs where an image moves around in the visual field overlapping with another image, rendering it partially or completely invisible. According to Husserl, we have here "an objectivation which holds fast to the image after it is no longer seen."[172] Notice that if the image is concealed only partially, it continues to present the entire thing; but even if it were totally concealed, we could always reverse the movement of the concealing image to regain the entire thing. In line with this example Husserl describes the phenomenon of concealment as a constant process of "demolition and rebuilding" taking place according to "a system of modification which is strictly motivated by

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since it fails to realize the implications of Riemann's concept of manifold as an intrinsically defined space: "We don't see surfaces, but the visual field is a two-dimensional manifold. The mistake lies in the equivocal concept of surface: (1) surface = two-dimensional manifold; (2) surface = formation, and specifically a two-dimensional formation in the space" (stenographic remark on his copy of Stumpf's book).

[170] E. Husserl, *Ding und Raum. Vorlesungen 1907*, op. cit., p. 332 (p. 285).

[171] ibid., p. 336 (p. 288).

[172] ibid, p. 235 (p. 189).

the kinaesthetic circumstances."[173] To stay with the example, the concealing and the concealed images filled the same portion of the visual field and seem to break its fixed form – which defines co-existence only in terms of a difference of places – and thus prefigure the constitution of depth.

The latter becomes clearer as soon as we take into consideration the phenomenon of expansion and its correlate, contraction. Approaching and receding from an object, its image undergoes a modification with respect to its extension – it expands and contracts. This modification occurs according to a rule, the rule of "the preservation of the oculomotor figure as completely the same while its size is constantly altered."[174]

What we have described represents a new form of objectivation that is in accordance with *geometrical similarity*: "what is held fast [. . .] throughout all the expansions, and which is, as it were, intuited in them, is the thing. And the thing is posed at the same time as an identical being after it exits from the actual visibility of the currently actual oculomotor field, or in brief it is posed as identical in the extensional amplification or joining together of the oculomotor fields of images."[175] Here what is novel with respect to the phenomenon of concealment is not so much the nature of sensuous contents as the kind of modification they undergo. This is independent from the eyes' position; rather, it is connected with kinaesthetic alterations produced by the approaching and receding.

Now, although these alterations may appear to give "the full presentational material capable of presenting space," yet for Husserl such a presentation does not suffice to realize the three-dimensionality of the thing. The expansional modification has "the character of a kind of modification that goes on bilaterally ad infinitum"; in other words, "it has two and only two directions, which fuse as opposites into a linear manifold (an open, bilaterally infinite, and, as it were orthoid manifold)."[176]

In the case of approaching and receding we cannot speak of a loss of the presentational contents, as we could in the case of concealment; on the contrary, Husserl affirms that "it pertains to the essence of the modification

[173] ibid., p. 245 (p. 208). It is because of this specific motivation that Husserl can claim that "the appearance [. . .] requires that the visible refer to the invisible." (Ibid.).
[174] ibid., p. 230 (p. 195).
[175] ibid., p. 237 (p. 200).
[176] ibid., p. 253 (p. 215).

of mere receding that it always presents the Object from one and only one side and, furthermore, that if the stationary manifold had at its disposal only this mode of expansional modification there could be presented in it nothing like a 'closedness' of the form of a thing in a continuum of sides or in any more sides above and beyond the given ones. The very concept of side would then completely collapse."[177]

Thus, for the form to be closed, a further modification is necessary, namely rotation. For rotation constantly offers new presentational contents; indeed, the very phrase "the object is turning" for Husserl is tantamount to saying that "it constantly shows itself from new sides." From the phenomenological point of view, the essential trait of the visual object emerges here: the object is seen as having sides in which it presents itself, albeit incompletely. Only where a complete revolution is carried out, its sides "are joined to one another as continuous in the unity of sequential appearance," and they reveal "the closeness of the nexus of the sides and therefore make the complete corporeal surface appear as a closed one."[178]

Rotation involves a modification of partial images that is not reducible to their mere displacement, but represents instead a determinate kind of concealment – one in which the concealing and the concealed images intentionally belong to each other and refer to the same object.

The "un-concealment" does not necessarily require making the reverse kinaesthetic sequence: we can also go on along the kinaesthetic direction which led us to conceal a given part of the object, to the point of making it visible again. This allows Husserl to mark a difference also with respect to the phenomenon of receding: "it is a linear modification. The motivating circumstances vary infinitely in a linearly orthoid manner. Pure turning is a cyclical modification; the kinaesthetic circumstances vary cyclically, and in the system of pure modifications of turning they bring back the turning series of images."[179]

When we recede from an object, the same side appears to us. Yet that the object has other sides, or that it can make sense to speak of other sides of the object, depends only on the constitutive function of the possible

[177] ibid., p. 254 (p. 215).
[178] ibid., pp. 252–253 (p. 214).
[179] ibid., p. 249 (p. 212).

modifications relative to rotation. For it is only if the series of appearances of the object is cyclical, i.e., "if it cyclically transforms one side into another," it constitutes the closeness of sides, which is the closed corporeal surface": or better, as Husserl himself acknowledges, "a two-dimensional system of points."[180]

Rotating, all the object's points change "in a harmonious way" their orientation according to the rotation's direction. Thus, if a point in the visual field travels from left to right, "all the image-points may travel with it in a unitarily harmonious way."[181] Husserl takes this to mean that "they can be integrated into a determinate system of cyclical modifications, a system which constantly brings one image after another and, finally, brings back the original image itself. If we continue the turning in a constant direction, it then reverts back to itself. Thereby, however, the turning necessarily leads beyond the oculomotor field."[182]

With this we are on the threshold of the transformation of the two-dimensional oculomotor field into the three-dimensional field of space: it is not, yet fully accomplished. All the phases of rotation form a two-dimensional cyclical manifold, to which corresponds a two-dimensional cyclical kinaesthetic system. Since any rotation takes place according to a given direction, we can suppose that the objective point turns in accordance with the directional cluster of the oculomotor field. However, each of these directions contains further possibilities in case the phenomenon of rotation gests connected with that of expansion, that is, in case the two-dimensional cyclical manifold of turning and the one-dimensional linear manifold of receding coalesce functionally.

Due to its "functional" nature, the coalescence of these different kinaesthetic systems brings about different kinds of constitution that are bound to converge in the unitary constitution of the three-dimensional spatial object: "there are not, and cannot be, any other modifications, provided it is precisely a three-dimensional Objectivity that is to be constituted."[183]

To sum it up, Husserl's analysis of the various kinaesthetic systems and of their functional connections brings into light the first strata of the

[180] ibid., p. 250 (p. 212).
[181] ibid., p. 252 (pp. 213–214).
[182] ibid., p. 252 (p. 214).
[183] ibid., p. 255 (p. 216).

constitution of the visual object. It also shows the emerging of the third dimension, making it clear its necessary connection with the co-ordination between the one-dimensional linear manifold of sensuous data related to the expansion and contraction of the image, on one hand, and the cyclical two-dimensional manifold of sensuous data related to the rotation of the object around any of its axes, on the other. What characterizes the systems so far examined, is the fact that each of them has its *own* null-position, and, from this, its basic direction of modification: "every system has its extremes therein and thus also in the combinations. With each one, therefore, a horizon of the appurtenant level is constituted."[184]

But what if even the null-point and its relative co-ordinate system of orientation gets into motion? Imagine it is not just the eye or the head or the upper body that gets into motion, but it is the Body as a whole that moves or is moved. Up until now, this possibility had been neutralized for methodological reasons, since we were interested in the primary strata of the constitution of space. Indeed, there was no need to suppose that the Body [*Leib*] moves or is moved from *its* position in order to explain how "a closed system of possible aspects constitutes the complete apparency [*Vollapparenz*], i.e., the complete appearance of the surface of the body [*Körper*]."[185] Yet such a possibility must be taken into consideration if we are to sketch a plausible account of the constitution of objective space from a phenomenological point of view. It implies a new "fundamental distinction" within the kinaesthetic systems: that between "the system in virtue of which the closed horizon of objective space is constituted" and "the system in virtue of which this horizon is displaced and in its displacement becomes precisely the apparency of objective space."[186] In the passage from one to the other, the complete apparency of the spatial body in turn reveals itself as "one appearance [*Erscheinung*] in a manifold," and this manifold consists of "the universality of identical complete apparencies," but "with a new system of kinaesthetic motivations."[187]

To put it differently, whereas in connection with the kinaesthetic systems we have examined the complete apparency of the individual spatial body

[184] ibid., p. 328 (p. 282).
[185] ibid., p. 329 (p. 282).
[186] ibid.
[187] ibid.

presenting itself in the closed system of its aspects, now, thanks to the kinaesthetic system of walking, it is the objective space that presents itself in the identical complete apparencies of the body.

Thus, to this kinaesthetic system (walking) corresponds "the system of the ideal 'aspects' of the spatial horizon," a horizon which "is constituted as a unity."[188] This idealization depends on the fact that, unlike the other systems, the system of walking "does not lead on uniformly from a null-point to an extreme (possibly in several directions)."[189] To the visual space, as it is constituted in the kinaesthetic systems previously considered (oculomotor, cephalomotor, and entire cephalomotor), it pertains a null-point and a system of directions qualitatively different one from the other. A kinaesthetic sequence corresponds to each direction, and since this cannot go on ad infinitum along the same direction, here the visual field is finite.

By contrast, walking "is a periodic movement, with which, however, the delimited, remote image at the edge of the horizon changes continually and does so in such a way that the reversal of the periodic movement again restores that image. In such a periodic form, this new kinaesthetic modification continues on *in infinitum*, namely as *a b*, *a b*, *a b*, etc. Walking conditions a constant displacement of the horizon (= visual space) in 'homogeneous space', i.e., a degradation of visual space to mere 'appearance' of objective space."[190]

In the case where we limit ourselves to move our eyes, turn our head or bend our upper body, we reach an absolute limit of depth, with respect to which all positions get ordered. But as soon as we start walking, what beforehand appeared to us absolutely deep, now appears to us relatively deep. The quality "depth" acquires a merely relative meaning, and its modifications are kinaesthetically motivated with respect to the possible "here*s*" in which our Body is located. These "here*s*" have neither the oneness nor the univocal character that pertain to them when the body is not walking; accordingly, they come to possess a merely formal value: "In walking this form is maintained: always a near region and a remote one, in continuous mediation, and the null-region characterized by the greatest

[188] ibid.
[189] ibid.
[190] ibid., p. 329 (p. 283).

possible nearness and even by the limit-case. Since every remoteness is kinaesthetically transformable at will into a nearness, then every position in such an ordered 'world' can become the null-position, the null-position can become a form, and we would have identical objects and objective distances whose orientation, as mere form, is a matter of indifference." Indeed, Husserl is even more detailed: "[we] can walk toward every point of Riemannian sensuous space and [we] can walk back again: every point 'approaches' and, if it is not occupied by a body, becomes the null-point and 'disappears'. Ideally, each body [*Körper*] can coincide with the null-body," and our Body [*Leib*] "can coincide with each body. Accordingly, there is constituted an objective spatial point as well as a spatial region and an objective corporality."[191]

In the ideal coincidence of our Body with every other body the equivalence of places is announced, as well as their qualitative indifference on the basis of which they can all be the centre of the co-ordinate system of orientation. It is to be noted that even in this case we are dealing with a proto-idealization that finds its full accomplishment only in the geometric idealization. The latter confers on space the form of an ideal system of points within which the very notion of "place" loses its meaning. For this idealization to be possible, it is necessary that the visual space becomes the appearance of the objective space and that, accordingly, the null-region from which develops the opposition between a "near" and a "far" vanishes. Obviously, this opposition remains even when our Body is walking: but only as "form", as "continuous mediation" that puts back *ad infinitum* (ideally) the near region in which things are constituted in their spatiality, paving the way to the constitution of an open, infinite, homogeneous spatial world.

[191] ibid., p. 318 (p. 273).

CHAPTER 5

ON NATURALIZING FREE

Sonja Rinofner-Kreidl

Abstract. The paper starts with conceptual and metatheoretical issues referring to the current debate on naturalism. Here we ask, among others, whether modern science, due to its methodical autonomy, can be said to foster a naturalistic view although the former does not coincide with the latter. Following these introductory remarks which embed the topic of the paper within a wider horizon of philosophical dispute, we discuss two approaches which claim to naturalize free will, namely Gerhard Roth's neurobiological and Henrik Walter's neurophilosophical conception of free will. With regard to both theories it is argued that the naturalization rests on a fundamental re-interpretation of the philosophical issue of free will. In order to seize the basic intuition lying beneath this re-interpretation two theses are formulated (illusion-thesis (IT); hidden-agent-thesis (HAT)). Does Roth's and Walter's effort remove our philosophical uneasiness with regard to the issue of free will? Are the conceptual and methodical tools provided by their theories suited to meet the demands of an adequate conception of freedom? In contradistinction to naturalistic approaches we, finally, outline a phenomenological conception of bounded freedom focusing on the ideas of situational horizon, motivation, (inter)personality, demand and dependence.

Introduction

Disputes on the legitimacy of naturalism quickly get stuck. Both naturalists and anti-naturalists insist on their arguments without being able to convince their opponents. The same holds true for disputes on the existence of free will. Notwithstanding a host of subtle arguments weighing the pros and cons, no substantial progress in terms of a generally accepted solution has been achieved. In this situation it is advisable to pause and reflect upon the kind of problem we are faced with instead of going on to espouse solutions which turn out to be insufficient. What we need is not another approach to solve the problem within a well-known framework but a new way of framing it. This is what cognitive scientists claim to do, i.e., to reformulate the traditional problem of free will and, thereby, pave

L. Boi et al. (eds.), Rediscovering Phenomenology, 125–164.

the way for its ultimate solution. Anti-naturalists consider this claim to be vain. They point out that due to naturalistic reformulations the problem of free will has not been solved but, instead, tacitly dismissed. What, at first sight, presents itself as a new and successful approach to cope with the problem actually amounts to a petitio principii. Naturalists, however, raise the same objection to anti-naturalistically motivated defences of free will. So, the opponents mutually reproach each other with begging the question. How can we break this deadlock? Can we break it?

Trying to do so, I shall begin with shortly explaining what I take naturalism to be and why it cannot be refuted once and for all due to the inner dynamics of modern science although it ought to be refuted (I). Having sketched how to grapple with naturalism, I shall turn to a scientific project which has recently attracted much attention: naturalizing free will (II). Here, I shall concentrate on the underlying ideas of free will and some mistaken arguments used to corroborate a naturalistic view. These arguments, however, slip in due to a missing reflection on conceptual and methodical presuppositions of the theory at issue. Finally, I shall give an outline of a phenomenology of freedom (III) in order to supplement the foregoing critique with some constructive underpinning. At present, I cannot elaborate such an account which sets in by querying the scope and meaning of freedom (see Heidegger 1982, p. 290) and argues in favour of *bounded freedom.* I restrict myself to discuss some issues which are part of this project focusing on the ideas of situational horizon, motivation, (inter)personality, demand and dependence.

1. Naturalism

Any interesting form of naturalism excludes trivializations which result from equating "naturalism" with "empirical research," or "natural science." The philosophically most challenging and strongest version of naturalism is ontological naturalism claiming that there are natural (kinds of) entities which only natural science (or: a single natural science, e.g., physics) is authorized to discern.[192] These entities represent the entire scope of what may be called "reality." Ontological naturalism results from absolutizing a

[192] Weaker forms of naturalism are philosophically relevant, too. Since they are less vulnerable to objection than ontological naturalism it, pragmatically viewed, is even more important to show their deficiencies (see section II below).

particular perspective on reality and, correspondingly, a particular domain of objects. Naturalism is a philosophical thesis. Natural science does not automatically support naturalistic interpretations of its results. The peculiar character of naturalism can only be recognized by transcending the scientific view in favour of an encompassing conception of reality. Since tracing out such a conception is an essential task of philosophy, the explanations resulting therefrom may be called a *philosophically reflected idea of science*. The key to understand naturalism lies in the concept of science. Insofar as the methodologically biased modern conception of science lacks any attempt to limit itself, it has to be considered a *degenerated idea of science*.[193]

If we recognize a strong naturalizing impulse grounded in our modern scientific practice, we, thereby, do not refer to psychological matters. Talking about an "impulse" in this context rather indicates a particular idea of what it means to take a scientific attitude towards the world. This is a matter of the history of concepts as well as of the history of science – or, on a larger scale, of intellectual history – albeit there clearly are psychological habits that correspond to thorough-going changes of conceptual and institutional traditions. Conceptual and institutional transformations can only take place insofar as there are human persons acting in accordance with these traditions. It is thus not surprising to find the transformations in question manifesting themselves psychologically, too. However, focussing on psychological phenomena we could hardly hope to understand why from mid nineteenth century until today powerful naturalistic movements (e.g., biologism, psychologism) have cropped up again and again.

Notwithstanding the methodological "degeneration" of the idea of science we ought not to underrate the reflective power of modern science. Very often philosophical reflection turns out to be an internal factor. It is

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[193] See Rinofner-Kreidl, 2003c where I argue that science is *on principle* incapable of limiting itself, thereby presupposing a degenerated idea of science. However, according to its own nature, science has to "relate to that which is not accessible by its own methods and, therefore, at bottom, is hidden. Consequently, science is urged to dare into the circuit of the concealed which constantly surrounds it. The being-in-truth of science is precisely a being surrounded by concealment. Even this concealment of being is only a restricted concealment for every single science. Science is necessarily limited in such a way that it even does not have the concealment of limitation ('Grenzverborgenheit') which appears due to the real practice of science" (Heidegger, 1996, pp. 212–213). This and the following translations are mine (SR).

triggered by some unexpected turn within the normal course of scientific research. Considering modern science, we are faced with radically opposed tendencies. On the one hand, there is an emphatical abstention from metaphysical commitments and philosophical reflections in general. On the other hand, it is exactly the focussing on procedures and methods, strengthened by some pioneering discoveries (e.g., relativity theory), which presses for philosophical interpretations of natural science. Such interpretations usually occur when scientific research takes a self-referential turn. Whereas modern science excludes any philosophical questioning in favour of improving the methods for successfully predicting future states of affairs, it, nevertheless, reopens a direct path leading from methodology to philosophy. It is precisely the autonomy and purity of natural science (the latter resulting from its mathematical form of representation) that occasionally induces natural scientists to rediscover the need for a self-limiting reflection on science. (Just as the naturalization impulse mentioned above, this denaturalization impulse must not be considered in psychological terms. It concerns the underlying idea of science). The autonomy of science lies in a complete availability of those objects which are scientifically investigated. It results from the fact that a scientific procedure excludes everything which cannot be represented as a function of particular methods. In other words: The object appearing is a methodical construction. It "exists" only insofar as the methods in question prove to be applicable. *The autonomy of modern science*, in accordance with its idea and purpose, *manifests itself in the inseparability of scientific method and scientific object*. As soon as one becomes aware of this inseparability, it is near at hand to state the necessity of limiting the whole project of a scientific attitude towards the world[194] and, correspondingly, of denying that the scientifically treated objects could constitute the whole reality. *A self-limitation of science is necessary*

[194] This inseparability suggests that the modern scientific concept of nature has turned into a concept of our relation to nature. Thereto, we may argue that the philosophical impulses of science have been reawakened as soon as the implicitly realistic self-understanding of natural science broke down in course of the development of modern physics. "The scientific method of singling out, explaining and arranging becomes aware of the limits which are imposed on it due to the fact that the methodical grasp alters and reshapes its object, so that the method cannot be dissociated from the object. *Therefore, the natural scientific worldview ceases to be properly natural scientific*" (Heisenberg, 1955, p. 21).

because of its methodical autonomy. Insofar as this autonomy, pragmatically viewed, simultaneously impairs the attempt to self-limitation, i.e., encourages a self-forgetful ("degenerated") operation, naturalism is inherent to the development of natural science. Realizing this to be the case requires to transcend a naturalistic stance.[195] From this point of view we may argue that science necessarily leads to philosophy. Trying to answer the question "what does 'naturalism' mean?," therefore, requires to answer the question "what does 'philosophy' mean?."

2. Naturalizing the Human Mind: Is There Any Activity of Free Will to be Found in Our Brains?

Working on the issue of free will, cognitive scientists and brain physiologists seem to offer good evidence for supposing that there either is no free will at all, or, at least, that we can dispense with any strong assumption of free will without curtailing our theoretical and practical interests. So-called volitional experiments purport to demonstrate the narrow limits of human freedom.[196] Does this view bear closer examination? First of all, we have to find out what scientists are looking for when talking about "free will." What do they consider as the relevant philosophical notion of free will that should be given up in favour of a scientific approach? I restrict myself to sketching two neurophysiological theories of freedom which I take to be of paradigmatic meaning (G. Roth, H. Walter).[197] We shall start with some conceptual issues and go on by asking whether the theories in question show any inconsistencies. The critique I shall

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[195] In my view transcendental phenomenology is the proper philosophical attitude to lay bare the roots of naturalism. Here I cannot dwell on this. See Rinofner-Kreidl, 2003e, VIII–XIII (introduction).

[196] Of course, psychologists have been interested in volitional phenomena for a long period. After the decline of a behavioristic psychology the problem of volition has been rediscovered. In the meantime it had been banished due to its putative link with an obscure notion of mental privacy. See Heckhausen/Gollwitzer/Weinert, 1987. In the following I shall restrict myself to nonpsychological conceptions although my general line of reasoning with regard to the naturalism-debate could also be brought forward with a view to psychological theories.

[197] The following representation of these theories partly coincides with Rinofner-Kreidl, 2003a.

espouse may be summarized as follows. Scientific refutations of free will are based on considerable efforts to redefine the issue. These redefinitions imply strong presuppositions which are not (sufficiently) reflected upon. Traditional views are abandoned insofar as they exclude corroboration in terms of empirical evidences. The experiments which ought to show that the assumption of free will is untenable do not achieve what they are said to achieve. Scientifically (e.g., brainphysiologically) based rejections of free will are self-refuting.

Referring to traditional conceptions of free will cognitive scientists (Walter, 1999, pp. 31, 52, 65, 355) start from the assumption that there are three essential marks of free will:

(i) being able to decide or act *otherwise* than one actually does;
(ii) deciding or acting intelligibly (non-arbitrarily), i.e., deciding or acting on the basis of sound reasons;
(iii) functioning as an originator of one's own decisions or actions, i.e., causing them without thereby being caused by some natural event.

Cognitive scientists unanimously reject a libertarian conception of free will based on a strong interpretation of these features. A strong interpretation is said to advocate metaphysical dualism and to require indeterminism in order to do justice to the idea that we could decide or act otherwise *under the same circumstances*, thereby taking the active role of a prime mover ("Erstauslöser") of the decision or action at issue (Walter, 1999, pp. 13–14, 356). A neurophysiologist is expected to answer the question: How can one elaborate a weaker interpretation of (1)–(3) which can be tested and empirically corroborated within the framework of a theory of brain functions? Delving into this task we have to be aware of the fact that stipulating a particular definition of free will, largely anticipates the position we may take among defenders and critics of free will. Whether or not we are ready to confirm the existence of free will first of all depends on our concept of free will. Presenting a weak concept of freedom mainly leaves open two alternatives: either declaring oneself in favour of a compatibilistic view or defending free will on the basis of indeterminism. The theories we are going to discuss approve of a compatibilistic approach by introducing the idea of a *weak* or *conditioned freedom*. Discussing alternative concepts of free will indicates a still more fundamental level of the naturalization-debate. Finally, what is at stake in this debate are not conceptual conventions but certain kinds of experiences and phenomena. Do naturalists and anti-naturalists refer to the same phenomena when talking about free will? This obviously not being the case, we should reconsider our terminology in order to keep off false pretences of continuity. For this reason, cognitive scientists have rightly proposed the term "natural autonomy" instead of "free will" (Walter, 1996, p. 379 (fn 30); Walter, 1999, p. 354; Walter, 2000, p. 268; Roth, 2001, p. 449).

Gerhard Roth's Neurobiological Conception of Free Will

In order to illustrate the achievements of a neuroscientific approach with respect to the problem of free will Gerhard Roth follows the famous experiment of the neurophysiologist Benjamin Libet which, according to Roth, rocks the foundation of all our conventional intuitions concerning free will (Roth, 1998, p. 303; Libet, 1999, 2005). What processes emerge in our brain before and while we are intentionally effecting certain basic bodily movements? In answering this question Libet was interested in the so-called "readiness potential," i.e., a neuronal activity that occurs in special spheres of the brain whenever some simple kinds of motoric behaviour are performed, e.g., moving one's fingers or arms. The outcome of Libet's experiment is that there are brain activities localized in consciously inaccessible parts of our brain, i.e., the premotoric and motoric cortex (as part of the cerebral cortex) which, for a minimal period of time (a few hundred milliseconds), precede the actual movement. Roth, who is less cautious in interpreting this result than Libet himself (Libet, 2005, pp. 177–199), takes this to show that the real impulses of my behaviour and the real decision I am going to make lie in unconscious contents of memory and in those feelings which are linked up with them. Therefore, we should say that it is the brain and only the brain which decides to effect the movement at hand (Roth, 1998, p. 307). What we are aware of, i.e., our volitional experience, does not bring about the movement. It is nothing more than an accompanying feeling which occurs after the cortical processes have already set in. Our feeling that shortly before acting we have the volitional intention to perform the act in question is illusive. The same holds true for the assumption that it was this very volitional act which induced the ensuing action (Roth, 2001, p. 443). Free will does not play any identifiable role in directing our behaviour. What we usually call "free will" refers to high-level brain processes. A system operates autonomously if it is able to direct itself on the basis of self-evaluations and in accordance with its former experiences. If there were any free will activity, it would conflict with the activities of the limbic system which secure the survival of the organism (Roth, 2001, pp. 447–448). The distinction between reasons and causes is based on an illusion, too. It results from the fact that the causal genesis of our reasons is hidden to understanding. We believe to act in accordance with reasons because we do not know the real incentives of our actions.[198] Generalizing these results, we may introduce the following illusion-thesis (IT)[199]:

(IT) Brainphysiological investigations show that the features considered as essential for our (alleged) capability of free will (alternativity, intelligibility, origination) do not withstand examination. They are illusions whose emergence can be explained by referring to the structure and function of brain processes. The above-mentioned features merely reflect feelings of the person who is actually faced with a situation urging her to make a decision. From this subjective point of view it is impossible to remove the illusion of free will.

Objections. Roth does not analyse the concept of free will. Neurobiological investigations do not (contrary to Walter, 1999, p. 189) answer the question whether or not free will is real. They are occupied with finding out under what brainphysiological conditions we can osberve the emergence of the "accompanying" feeling to be wanting something (Roth/Schwegler, 1995, p. 76). The fixing of one's volitional intention (in Libet's experiment: of bending one's finger at a certain instant of time) falls beyond the experimental setting. The volitional intention is already settled when the experiment starts. It is settled by means of the instructions given by the conductor. The experiments in question are comprehensible and interpretable only on condition that we are acquainted with volitional

[198] When taking reasons to be "the conscious forms of experiencing brain processes" (Roth, 2004a, p. 82) Roth holds that intellectual entities, i.e., meaning contents, are real parts of my brain. They cannot be communicated to another person's brain. Every single brain has to generate them on its own and separately (Roth, 2006, p. 23). Consequently, Roth acknowledges the distinction between reasons and causes insofar as it can be interpreted in terms of brain processes. True reasons (causes) give rise to some kind of action, would-be reasons (by means of which an agent pretends to explain her behaviour) fail to do so (Roth, 2006, p. 24). In my view it does not make sense to maintain that the difference between truth and falsehood which is of prime interest with regard to *reasons* (as meaning entities) could be explained by referring to the predictable occurrence (truth) or non-occurrence (falsity) of certain brain activities.

[199] I give a detailed discussion of the difficulties inherent in this in Rinofner-Kreidl, 2003b.

experiences. In the following one inevitably has to rely on the introspection of the probationers. Introspection, of course, does not grasp brain activities but conscious contents which are expressed in a mentalistic idiom. On the contrary, using expressions like "decision" or "action" with regard to unconscious brain processes is highly problematic.[200]

Roth argues that having the disposal of free will would be detrimental to our survival: "an arbitrary decision of will is [. . .] not asked for" (Roth, 2001, p. 449). Why is it not asked for? Free will cannot be reduced to making one's choice between different modes of attaining *given purposes* which is a matter of rationality and not a matter of morality. (Notwithstanding the emphasis Roth at first lays on the distinction between freedom of will and freedom of action (Roth, 2001, p. 433), he keeps confounding these types of freedom (Roth, 2001, p. 453). Acknowledging free will means to acknowledge that the ends of our decisions and actions are subject to our choice. According to Roth there is no freedom involving choices about ends. "Consciousness and knowledge can only be transformed into action by means of the 'approving' of the limbic system" (Roth, 2001, p. 452) which is fixed to the end of survival. This holds true even if the organism is confronted with new situations and therefore depends on deliberation processes (Roth, 2001, p. 448).

If Roth is right then all reasoning is devoted to ensuring our biological survival. This being the case, we have to assume that whatever arguments we advance, whatever reasons we put forward in order to justify our decisions and actions are actually nothing else but functions of the purpose to survive independent of whether we come to know that or not, whether we are ready to accept it or not. If it is true that even our approving or disapproving of theories is subject to IT, then it is pointless to argue in favour of the existence or non-existence of free will. In this case, arguing, building theories and looking for truth is meaningless at all. It simply manifests an illusion. Whatever arguments we advance, we are necessitated to do

[200] Neurobiologists take their theories to be "fundamentally self-referential" because "we ourselves as brain states (ego, perception, consciousness, thinking) want to find out something about brain states (ego, perception, consciousness, thinking, planning one's actions and so on) by referring to brain states (perception, consciousness, thinking, planning actions which are required for scientific work). *Ultimately, I want to know how I myself have come about*" (Roth, 1998, p. 23).

so. If we change our mind and become naturalists or anti-naturalists in course of discussing with our opponents this will be a matter of necessitation (determination), too (Walter, 1999, p. 82; Roth, 2001, p. 454). All our reasonable activities are nothing but make-belief. In this way, IT which annulls the distinction between causes and reasons is self-refuting. Supposed that IT is true, we certainly should consider our present state of knowing that it is true, to be an illusion, too, since this knowledge equally has to be realized in brain processes. How to escape this situation? Roth tries to escape it by introducing ontological realism within his constructivistic approach. The brain which is said to gain knowledge about the construction of our *phenomenal world* is located in a transphenomenal world called "*reality*." What we normally consider to be the agent of our decisions and actions, namely the ego or person, is rather "a virtual agent acting in a world constructed by the brain which is given to us as our experienced world (*Erlebniswelt*)" (Roth, 2001, p. 452).

From what point of view is it possible to advance IT? Who is able to recognize the illusion in question? Defending IT obviously requires an additional hidden-agent-thesis (HAT):

(HAT) Contrary to our common view the person is not the last authority on what we think and do. It is the brain (or the limbic system) which is the true agent of volitions, decisions and actions.

Henrik Walter's Neurophilosophical Conception of Free Will

Neurophilosophy is a discipline whose task is to bridge between neurosciences and philosophy, between subjective experience, philosophical theorizing and empirical research (Walter, 1999, p. 161). On the one hand it works on philosophical problems resulting from neuroscientific research. On the other hand it uses its outcome for better understanding or solving philosophical problems. Walter endorses a "minimal neurophilosophy"–program which proceeds from the thesis that mental states are based on brain processes in terms of a supervenience relation and that it is reasonable to expect different types of mental states to be related to brain processes in different ways (Walter, 1999, pp. 162–172, 357). Although Walter declares his conception of free will to be grounded in a physical monism of substance, he takes the view that a minimal neurophilosophy is not irrevocably committed to a particular metaphysical theory (Walter, 1999, p. 190). He considers his approach to be naturalistic because it depends on empirical investigations, rejects metaphysical dualism and endorses externalism. The term "natural autonomy" refers to a special type of self-determination which is said to dispense with all supranatural forces and elements. "We have the disposal of natural autonomy if we are able to choose otherwise under very similar conditions (due to the chaotic nature of our brains), if this choice is intelligible (i.e., determined by the past, by ultra-rapid brain processes of adaptation and by our environment which is partly structured by language) and if it is authentic (in terms of identifying ourselves with it by means of iterative reflection and emotional adjustment)" (Walter, 1999, pp. 361–362).

The author's prime interest is not to refute a strong notion of free will but to find out how the components (i), (ii), and (iii) have to be interpreted in order to be compatible with our knowledge of the brain (see p.130). A strong idea of free will proves unnecessary even if its existence, strictly speaking, is not refuted (Walter, 1999, p. 103; Walter, 2000, p. 268). In the following, I shall concentrate on Walter's idea of how one should weaken condition (iii) which is closely connected to the issue whether a naturalistic conception of free will is able to ensure the ascription of moral responsibility. Walter's conception of free will is intended to present the view of a philosophy of nature (instead of a moral philosophy). However, he is interested in its ethical consequences (Walter, 1999, pp. 13, 25, 87, 328) and claims to answer the question what it means to act according to principles (Walter, 1996, p. 364). The author claims to re-evaluate the relation holding between freedom and responsibility in such a way that the ascription of responsibility (contrary to Roth's view) can be defended even on condition that the only tenable notion of free will is the notion of determined freedom (Walter, 1996, p. 377).

The hidden-agent-thesis: weakening the idea of origination. Walter agrees with Harry Frankfurt that free will is bound to our ability to comment on volitions we actually have (*first-order-volitions*). This involves reflexivity which is considered indispensable in order to maintain free will. According to this conception it is, on principle, possible to question our *second-order-volitions*, too. Consequently, there is a problem of infinite regress we have to cope with. The mere fact that we are able to reflect on our volitions does not ensure that we consider a decision we have reflected upon in any substantial sense to be our *own* decision

(Walter, 2000, p. 269). So, what allows for stopping reflexivity at a certain level? We have to stop it somewhere (apart from time-limits imposed by practical needs). Stopping our reflective activity amounts to committing ourselves to certain values or valuations. This commitment is necessary because all the reasons we may claim in favour of our decisions have to be evaluated (Walter, 2000, p. 280). Whereas Frankfurt proposes to stop reflexivity by means of decisively acknowledging some first-order-volition, Walter takes this to be a rationalistic and insufficient response to the problem. His conception of an emotionally based neuronal self is said to show how the reflexivity-regress can be stopped in a non-rationalistic and non-arbitrary way.

The neuronal basis of the self consists in a feeling of authenticity conveyed by the continuity of my bodily presence as well as by particular emotions closely connected with this presence. In this sense all our acts of thinking are embedded in our personal experiences and the past we have lived through (Walter, 2000, pp. 265–266). Decisions and actions will be authentic if we are able to identify ourselves with them. They become *my* decisions and actions if they harmonize with my emotionally based valuations. The latter are neurophysiologically realized in terms of emotional approvals resulting from past experiences of learning and, therefore, reflect my self as a historically grown person (Walter, 2000, pp. 280–281). "Decisions [..] do not become authentic by means of reflection or rational deliberation *alone*. What is needed in addition to that is an adjustment to and correspondence with one's own feelings" (Walter, 1999, p. 327). Emotions result in a suitable pre-selection and help to appraise the consequences of future events.[201] Furthermore, they are necessary for making rational cognitions effective and associating them with our personal state as well as the state of others (Walter, 2000, p. 277). The assumption that emotions play an important part in decision processes becomes plausible if we consider that real decisions take place under the condition of cognitive intransparency (the information at hand concerning the circumstances

[201] What does that mean with regard to our understanding of free will? We certainly can reasonably approve of choices supported by corresponding emotional tendencies. Being emotionally inclined to make a certain decision does not impede free choices as long as inclination is not interpreted in terms of determination. This whole matter requires a discussion of the issue *reason vs. cause*.

and consequences of our behaviour is incomplete) and temporal limitation (decisions have to be made under deadline pressure). According to Walter, especially the latter moment is often neglected when reflecting on the problem of free decisions (Walter, 1999, pp. 228, 346). "Decisions are not processes which take place in a particular moment of time. They *are temporally extended processes*" (Walter, 1999, p. 227). However, doing justice to the temporal aspect of decision-making does not require to consent to naturalization programs (see below, section III). On the contrary, taking into account temporality *in terms of distinguishing past, present and future, and thereby referring to someone's first-person-perspective* requires to transcend the subpersonal level of brain physiology and to talk about full blooded persons and their self-perception and self-reflection. It is unwarranted to hold that the temporal dimension of human decisions can be adequately articulated at the level of brain physiology as Walter claims. "Here the network of the brain needed somewhat longer to relax at a minimum-energy-level" (Walter, 1999, p. 347). This is certainly not a satisfying equivalent to statements like: "My decision had to mature." What we consider as "authentic" depends on the underlying concept of a self or a person. This is obvious if we, for instance, compare the neurophysiological concept of authenticity with the problem of authentic and inauthentic modes of Dasein in *Being and Time*. Taking notice of the prominent role of caring (*Sorge*) in Heidegger's analysis reminds us of an important aspect which is missing in the conception of a neuronal self, namely self-evaluation based on reflexivity as it is characteristic of self-conscious agents. Walter does not tell us how the "experiences" stored in particular brain areas and mechanisms are connected with the self-experience of conscious agents. He merely points out that he does not support the thesis that "the self is nothing else but a bodily represented personal continuity" and that he is ready to acknowledge a full-blooded human self ("angereichertes menschliches Selbst") allowing for rationally planning one's actions and suchlike (Walter, 2000, p. 281). Why can't we be satisfied with that? Can I myself as this individual mind-body-unity, this full-blooded human self invalidate an "authentic" decision which has been effected by means of brain processes? Does it really make sense and is it legitimate to introduce the term "authenticity" at a subpersonal level of discourse? Notwithstanding the fact that I am "embodied mind" (Walter, 1999, pp. 334) and that emotions and cognitions are "closely, if not inseparably, connected with one another" (Walter, 2000, p. 278), we have to realize that this entirely leaves open – or: systematically ignores – the problem of moral-practical self-determination which cannot be reduced to a problem of cognition. However, Walter maintains "that our true character mainly lies in our emotionally supported evaluations and that this is decisive with regard to our moral considerations. [..] This is due to the fact that, thanks to our emotions, we are reliably committed to certain modes of thinking and behaving" (Walter, 2000, p. 280). What we are confronted with here is a special variant of the HAT:

$(\text{HAT})_\text{i}$ Contrary to our common view, decisions and actions are not (primarily) grounded in higher-level mental activities (e.g., weighing alternatives, considering the consequences of possible modes of behaviour) that are constitutive of a *rational ego*. Instead, it is the *neuronal self* based on the unconscious emotional structure of our organism which functions as the true agent of our decisions and actions.

Objections. Even if we agree that intellectualistic approaches are unreasonable insofar as they ignore the real (practical) conditions under which decisions take place, we have to be careful not to lapse into the opposite extreme of a radical emotionalism. The latter is equally unsuited to grasp the phenomenon in question. Emotions are indispensable with respect to embody and practice our approval concerning the principles of deciding and acting. However, they cannot replace the latter with regard to their proper function.[202] Granted that moral behaviour without moral feelings is hardly imaginable (Walter, 1999, p. 349), this does not amount to considering moral feelings as the essential basis of moral behaviour.

Talking about "unconscious decisions" is problematic though we may concede that there are necessary neurophysiological conditions underlying decisions as well as hidden motives, preferences and tendencies which may be called "unconscious." We rightly hesitate to use the term "decision" with respect to such components that function as a "tacit background" for

[202] Occasionally, Walter seems to agree with this view: "Emotions do not replace rationality or reflection but they help to bring about a decision" (Walter, 1999, p. 350). Still he argues that "decision processes which are cut off from emotions result in bad decisions. It is only by means of the directing and stabilizing function of feelings, presentiments and valuations that we are obviously kept from being at the mercy of outer factors due to the great flexibility of our cognitive apparatus" (Walter, 1999, p. 335). For the concepts of emotion and cognition underlying this argumentation see Walter, 1999, pp. 329–330.

our explicit and conscious acts of assenting to some modes of behaviour which we are ready to identify as decisions. Making a decision includes being conscious of cutting off other possibilities of acting. Describing the "chaotic" nature of our brain by stating that even the slightest changes in circumstances will lead to deviating developments may be interesting in terms of necessary physiological conditions of higher-level-activities of the mind. It does not and cannot tell us what it *means* to make decisions (see Walter, 1999, pp. 226, 229).

Freedom and responsibility within a deterministic universe. According to Walter, his conception of authenticity helps us to understand why a personal attribution of decisions and actions plays an important role in moral behaviour (Walter, 1999, p. 316). "A person gives rise to her actions if she identifies herself with them even if she could not act otherwise. This I have called authenticity. The process of identification justifies to consider a person responsible for her deeds even if she could not have acted otherwise" (Walter, 1999, p. 326). Am I free not to identify myself with decisions or actions although they perfectly integrate in the history of my experiences? Following Walter's conception of determined freedom, we have to answer "no," since in this case we have to assume that the denial of identification (if not altogether arbitrary) is based on reasons which are not reducible to functions of past experiences. Referring to the above idea of authenticity and claiming that personal attribution is grounded in identification does not make sense if there is no real possibility to become inauthentic. Therefore, authenticity appears to be something we suffer from. A person inevitably becomes what she is determined to become according to the experiences she has accidentally gained in the past. There is no room for intervention and no possibility for self-creation in this mechanism of personal development. Our character depends on how our brain processes the information it gets due to its relatedness to a natural environment. The same holds true for our thinking. It depends on our environment (Walter, 1999, p. 288). "Within a deterministic universe no one, in an ultimate, absolute sense, can be blamed for what kind of person he is" (Walter, 1999, p. 353). This being so, how could we consider anyone to be responsible for what he does? And how could we pretend to show that persons are something more than "the place where external influences sum up?" (Walter, 1999, p. 18).

So in the end it turns out that a neurophilosophy of free will does not help us to escape from the extreme alternative of either acting as unmoved prime movers or being moved like puppets "whose thoughts and deliberations are of no effect with regard to what is going on with them" (Walter, 1999, p. 14). Walter does not offer any plausible naturalistic interpretation of phrases like "a person causes her actions." A person is built up in case that certain experiences take place. Authenticity is nothing to be achieved. It occurs as an outcome of a natural process. In the face of this, it is worth remembering Walter's announcement that he is going to transform "the phenomenon of origination ["Urheberschaft"] from a philosophically obscure thesis to an empirically explorable object" (Walter, 1999, p. 346). What has actually been achieved? Concepts like "self," "personal attribution," "origination" and others have been redefined at a subconscious and subpersonal level. In course of introducing these new concepts and discussing their neurophysiological application, the original problem – how to reconcile determinism and responsibility in a new way – has not been solved. It has been lost under way.

Walter's line of argumentation does not coincide with the usual compatibilistic strategy to fully detach the attribution of responsibility from the assumption of alternative possibilities (Walter, 1999, p. 235). However, in the end his considerations are thrown back to this strategy. Since the weak interpretation of alternativity which distinguishes Walter's approach from traditional compatibilism refers to an unconscious level, it cannot be effective with regard to mutual attributions of responsibility that (for lack of a brain-to-brain-communication) have to be described as conscious activities. So we are left with the old compatibilistic answer to the question what "responsibility" means within a deterministic universe. (Walter's weakening of (i) by means of investigating the chaotic structure of our brains does not annihilate the assumption of a macro-level-determinism). First, we are talking about responsibility in terms of attributions of responsibility. Secondly, responsibility is attributable if the agent to whom it is attributed is accessible to social measures expressing moral praise and blame (Walter, 1999, pp. 59). Now, if we ask *for what reason* we should attribute responsibility to anyone, the answer simply is: "Since this, in general, is the best strategy to make them [the others] behave morally" (Walter, 1999, p. 354). Our attribution is not grounded in an allegedly moral nature of the agents; it is not justified by referring to particular moral abilities. On the contrary, we attribute responsibility in order to induce others to behave morally. What it means to behave morally – or to commit oneself to certain convictions and values – remains unclear. Finally, this discussion of moral behaviour is burdened with the problem of self-refutation inherent in IT: "If we want that our fellows behave morally then we have to consider their neuronal 'construction' " (Walter, 1999, p. 352). From obvious reasons we should add the following: If we are interested in our fellows moral behaviour, this is due to our own neuronal construction. Shouldn't that give rise to moral optimism? Why shouldn't we try to manipulate our neuronal construction in order to make us behave morally (whatever that means)? However, we could not want to do that if our neuronal structure did not allow us to have this volitional intention. Of course, as conscious agents we remain notoriously unclear about our true "motives" to manipulate our neuronal structure. We, again and again, compile rational arguments in favour of supporting or ruling out such manipulations. So we end up with the ironical fact that our discussions about genetic manipulations turn out to be nothing but a "cunning of the limbic system" (Roth, 2001, p. 449) – if the neurophysiologist arguing in favour of his naturalization program is on the right track.

We may resume the above debate by setting forth some issues which a phenomenologically informed anti-naturalist considers essential, thereby transcending the theoretical framework of naturalistic conceptions.

1. A phenomenological-descriptive account points out the nature of different types of objects one may refer to by means of different modes of grasping and understanding. Correspondingly, different spheres of reality can be distinguished. A differentiated and sophisticated model of reality[203] is vital to any refutation of naturalism, e.g., by arguing (as we did above)

[203] In German one may distinguish "Wirklichkeit" and "Realität," the first referring to the sum total of all entities, moments and processes which are considered real *according to different modes of existence*, the second referring to the ontic equivalent of a particular (methodologically reflected) scientific mode of grasping things and processes. ("Wirklichkeit" may also include the idea of transcendence functioning as horizon within which different modes of existing can be discerned.) In the following I shall refer to "Wirklichkeit" and "Realität" by using the terms "reality" and "sub-reality." Ontologically viewed, emphasizing this difference is the gist of what an anti-naturalist has to say.

that basic bodily movements which can be arbitrarily controlled cannot be compared with those temporally extended complex actions and situations of (moral) deliberation we are driving at when discussing free will. Investigating how certain basic voluntary movements are neurophysiologically initiated (as Libet's experiment demonstrated), is not instructive with regard to higher-level actions that are grounded in higher-level volitional intentions.[204] It is unwarranted to presume that one could rightly judge upon the entire scope of reality from the point of view of a particular sub-reality. Following a phenomenologically sustained idea of different spheres of reality that are irreducible to one another, we cannot claim to explain the existence and behaviour of higher-order phenomena by referring to lower-order phenomena.[205]

2. Whereas the concept of a person touches upon an encompassing idea of reality (bringing into focus social and moral matters), there are several methodically induced subject-functions belonging to particular sub-realities. We refer to such ideas which can be considered theoretical fictions, if we, for instance, talk about the "homo oeconomicus" or the "average consumer" or the "transcendental subject." Accordingly, with regard to naturalistic theories of free will it is instructive to ask: who is acting? Or: is there anybody acting at all (given the methodical frame of volitional experiments)? The fact that there is no hint of a person in these experiments takes us back to (1). Decisions are complex phenomena embedded in complex situations. In order to allow for a scientific treatment, these phenomena are radically simplified and atomized. It is due to our self-experience as full-blooded persons that those truncated

[204] Thereto Roth holds that it is our character which determines our decisions (Roth, 2001, pp. 444, 456–457). Our character consolidates long before we realize to be able to behave voluntarily. In the following we shall take up this issue by proposing a non-deterministic account of a person.

[205] Two objections might be raised here. First, a naturalist will refute the above statement by arguing that it is question-begging. Therefore, we have to discuss whether phenomenological descriptions can corroborate the irreducibility-claim at issue. Secondly, referring to different domains of reality does not automatically rule out misguided ideas of complex entities that are constituted by several ontic layers. Particular sub-realitites are investigated by means of specific methodical approaches. Using these methods we run the risk of tacitly absolutizing the domains in question by disregarding their interconnection with other domains.

notions of the self or subject which are introduced by cognitive scientists and brain physiologists can be recognized as inadequate. Above we argued that the key to understand naturalism lies in the concept of science. Now we may add: the key to understand freedom of will lies in the concept of a person.

3. Rethinking how one should ask for free will unbiased by its scientific representation, we claim to think autonomously, i.e., practise our free will. This claim is to be acknowledged whether or not we succeed in theoretically demonstrating the existence of free will. Otherwise we could not explain what it means that our thinking is subject to the alternative true/false and for what purpose we advance arguments and look for evidences suited to corroborate our ideas. Scientists arguing in favour of the thesis that free will is illusive cannot explain their own theoretical activities, given that "theory" bears on truth, and that truth cannot be considered as necessarily resulting from antecedent natural occurrences. With respect to self-application and, especially, performative self-contradiction thinking presents itself as a kind of practice. Self-application is particularly under consideration as soon as we become aware of an essential epistemological asymmetry with regard to naturalistic and anti-naturalistic positions. Strong naturalism is by its very nature incapable of metatheoretically reflecting on the naturalism-dispute. In doing so naturalists are urged to abandon their point of view.

4. In order to lay bare the roots of the naturalism-debate, it is, in general, useful to take into account metatheoretical reflections. Here, we may argue that reproaching each other with circularity, petitio and self-application is effective only on condition that we have already mutually granted the ability to behave reasonably in terms of meaningful and logically correct thinking. Otherwise it would be pointless to demonstrate any argument or proof to be mistaken since our opponent could by no means recognize her error. What shall we make of this with regard to the issue of free will? (a) Every attempt to prove the non-existence of free will is, as indicated above, practically self-refuting. (b) Arguing that someone's reasoning falls prey to a petitio is reasonable only if the object of this reasoning is free from self-referring implications. This being the case, the unsoundness of reasoning itself would follow from the unsoundness of what it attempts to prove. Whenever the very possibility of reasoning is at stake (instead of particular instances of sound or unsound reasoning),

it is inadmissible to claim a petitio. (c) If we are right in advancing (a) and (b) then it is clear that freedom of will which is at the bottom of all our reasoning must not be considered as an arbitrary object of reasoning.

3. Outlines of a Phenomenology of Freedom

Given that we are successful in immanently criticizing naturalization projects, we nonetheless have to focus our attention on issues that open up a wider horizon. Amongst these are: From what epistemic motives do cognitive scientists fathom in human volitions? Which notion of freedom is involved in volitional experiments? What can be proved by means of these experiments? Are there any aspects of freedom which are inevitably ignored due to its scientific representation? Asking how one should ask for freedom, we are in a position to abandon the narrowly restricted approach of a brainphysiological investigation of free will. If we take seriously the above mentioned metatheoretical difficulties inherent in the naturalism-debate and if we consider naturalistic conceptions of free will to be misguided due to ignoring the real experience of freedom, then we should endorse a philosophical conception of freedom grounded in a phenomenology of freedom. Thereby, we advance the following thesis. The *real boundaries of freedom* do not lie in the field of scientifically explorable natural conditions of human action. We are faced with them whenever we realize that, within the sphere of meaningful action, there are things which are not at our disposal and cannot be placed at our disposal.

If we acknowledge the fundamental distinction between causally related entities and meaningful (intentional) experiences, we shall not be liable to narrow down the issue of free will to the issue of whether or not we are justified in assuming an unbroken causal chain of natural processes. (The argument in section I goes in the same direction. The problem of self-limitation is pressing whether or not the determinism/indeterminism-debate comes to a satisfying end). What is devoid of meaning in itself, can (and actually does) establish necessary conditions of the emergence of meaning entities. However, the former cannot determine the latter with regard to their intentional contents (Rinofner-Kreidl, 2004a, 2004b). To make one's decisions freely does neither require to be able to manipulate or eliminate the effects of those natural processes which represent the necessary conditions of our behaviour nor even to be conscious of these conditions while performing the action in question. Erroneously assuming this to be the case results in the *fallacy of latency* (Rinofner-Kreidl, 2003b, p. 37). Paying attention to the fact that causation and intentionality belong to different spheres, we realize that investigating the motives of an action has to be distinguished from investigating how these motives came into being (see Pfänder, 1900, p. 102). Equally, analysing the concept of free will has to be distinguished from analysing those physiological processes whose non-occurrence prevented volitional acts from being realized. I may (and usually will) be motivated to do something without having the slightest idea of how my motives have emerged. I am able to decide freely without knowing anything of the relevant brainphysiological processes. Contrary to the opinion of some brain physiologists who argue that natural processes, in a non-trivial manner, limit the exercise of free will, a phenomenologist holds that this view mistakes the idea of free choice and begs the question in favour of assuming the non-existence of free will.

How do fundamental unavailabilities manifest themselves? How do we encounter them? We encounter them, for instance, whenever we realize that the irrevocable course of time hinders us to change our past actions. Or whenever we realize the finitness of life and the vulnerability of human organism. Prima facie these conditions seem to be of a completely negative character. Nevertheless, they have a positive impact, too. Suppose our feelings, our thinking and behaviour were not subject to these conditions. In this case our actions, for instance the promises we make, could not have the meaning and value they actually have because we are aware of not being able to arbitrarily and carelessly renounce them. If we were not acquainted with unavailabilities of diverse kinds, we could not know what it means to decide on one's own ground either. An altogether unlimited freedom which could even annul natural laws is indistinguishable from arbitrariness. Supposed that our free will could annihilate natural laws, we had to face serious problems. In this case we could not claim to know anything about how to practically realize free decisions. At any given moment some other person's free will could interfere and interrupt the causal processes involved in the realization of our decisions. The ideal of an unlimited freedom, in general, amounts to the idea of negative freedom, i.e., the idea of not being subject to determination. On the other hand, if we are interested in elaborating an appropriate notion of positive freedom, we must take note of unavailabilities. We must conceive free will as something which can be achieved not in spite of unavailabilities but due to them. This is the basic idea of a theory of conditioned freedom. Experiencing the unavailable cannot be brought about at will. It happens to us like the culture, society, and family we are born into, physical and mental power and weakness, temper, states of mind, diseases, death, love, the persons we meet, the children we have. Limitations of this kind are of a completely different stamp than those restrictions we discover by means of methodically isolating objective variables in order to measure, for instance, the reaction speed involved in basic volitional experiences, i.e., the average temporal interval to perform a bodily movement one intends to perform. The difference in question can be explained by referring to the high or low complexity of the processes involved, the partaking or detached role of the subject getting to know the limitations in question and the presence or absence of a lifeworldly structured meaning horizon. Talking about "actions" in a full-blooded sense refers to the fact that an agent who is embedded in a specific situation is aware of a complex meaning structure which is given rise to or altered by the way she behaves.[206]

Lifeworldly embedded and interpreted experiences of freedom always include experiences of limitations. If we consider our decisions under real-life conditions, it appears that they are interwoven with a process of finding out what may fit into our character and what may be our peculiar way of meeting with others and handling situations. Real decisions do not occur in the shape of idealized theoretical models of decision-making. According to these models there are several (at least two) alternatives which can be completely enumerated and rated by referring to some nicely defined criteria which, in case of plurality, are hierarchically ordered. The agent is expected to reasonably settle on one of the alternatives in question, leaving the rest aside. What is wrong with models of this kind? Roughly speaking, we may say that they ignore the time element which is of prime importance both with regard to the complex state of affairs to decide on and ourselves

[206] For our present purpose we need not discuss the notion of action in detail which also comprises specific ways of abstaining from doing something. See Birnbacher, 1995.

as agents whose knowledge about the world and self-understanding gradually develops in course of time. Experienced temporality is part of any meaningful action. It does not coincide with objectively measured time which is the object of concern in Libet's experiment and similar investigations. The temporal order (milliseconds) involved in brainphysiological experiments is irrelevant to personal self-determination.[207] Idealized models of decision-making cut off the pre-given facticity of the world and of ourselves which is the sole place and medium of coming to decisions. These models retain a purely formal, narrowly conceived idea of decision, namely taking sides with a particular alternative from the point of view of ratiocination. Accordingly, a decision manifesting one's free will either cannot be considered motivated at all or the motivation in question is subject to an idealization, too, following the idea of rationally ordered preferences. The agent to whom these preferences are ascribed is a methodological construction, i.e., a theoretical fiction.

Real motivations can only be made intelligible by referring to individual biographies. The fact that we are always driven by particular motives is not only relevant with regard to assessing given alternatives. The motives we actually have also direct our attention to matters of fact which *could* function as alternatives. We are always biased towards certain alternatives. And we are biased in terms of considering to what degree they are attractive in a present situation. As soon as we grasp an alternative it already has its specific weight. Of course, we can expect its weight to alter. However, there is no neutral givenness of alternatives at any stage of the process in course of which our decision works out.[208] The time element enters into in another sense. It is a very common case that alternatives gradually become perspicuous while the overall situation of deciding and acting gradually takes shape. Moreover, considering real lives, real persons with

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[207] "Volition is level-headed determination. It is the ability to establish priorities in acting with regard to overall and long-term goals. It is stability of behaviour due to reason, not due to accidental drive, in accordance with a person's seriousness and faithfulness" (Kornhuber, 1987, p. 388).

[208] A fictitious neutrality of this kind which results from a complete lack of motivation marks the situation of *Buridan's ass* which is intended to represent the idea of *liberum arbitrium indifferentiae*. While discussing this issue may be of aporetic benefit, it certainly is misguided to define the notion of free will with a view to this situation.

real biographies, we may hesitate to assume there being any decisions made at particular moments. It seems to be more adequate to talk about decisions which are brought to maturity in course of time. The process of forming decisions cannot be separated from our passing through changing situations. This includes (implicit) definitions of the relevant situations from the point of view of that person I consider myself to be or like to become respectively.

In the end there may or may not occur what we usually call "decision," namely an act of volitional declaration in favour of one alternative preceded by a more or less extended period of deliberation. Whenever this explicitly choosing among alternatives, i.e., making *decisions (in a narrow sense)*, occurs, the person choosing will be immediately aware of it although she is not directed to her choice but straightforwardly behaves in accordance with the alternative she has chosen.[209] When the relevant process of making one's *decision (in a broader, temporally extended sense)* starts, it normally will not be clear whether or not we are going to run through any explicit reflection and whether we, afterwards, shall feel pressed to explicitly take our stand. Among other things, this depends on the peculiar atmosphere and character of the social relations which are involved in the decision at hand. Decisions (in the broader sense) do not necessarily become apparent in explicit reflections and declarations. They can also be made by tacitly going on as in the past or by tacitly and continually growing into some new form of life. On the other hand, it has to be noted that not every choice is accompanied by a feeling of freedom. This, for instance, is not

[209] See "He does not experience something happening which he can identify as the choice itself. A person does not encounter his choices; he makes his choices. The experience of choice is an experience of doing something; it is not an experience of undergoing anything. The connotation of passivity in the word 'experience' is misleading [...]. A person's own choosing is not given to himself; in this sense, choice is not a datum. Even if choosing is not a datum at the moment of choice, one is directly aware of it" (Boyle/Grisez/Tollefsen, 1976, p. 20). From a phenomenological point of view we may add the following. First, the connotation of passivity is owing to a positivistically biased notion of experience. Secondly, in analysing decisions in the broad sense as well as in the narrow sense (as the authors do) it is helpful to put more emphasis on two moments which are inseparably linked up with one another in every intentional experience: being (actively or passively) directed to the object intended ("experience"); being immediately aware of ("living through") this present intention.

the case if we choose between different means to realize an end which we do not question in this particular situation. I use the term "decision" in a more restrictive and emphatic sense (compared with "choice"). "Decision" is reserved to those cases where we have to choose between alternative ends and in doing so presuppose our will to be free. There are choices that do not manifest decisions (in this strict sense) whereas every decision implies choosing between alternatives. Only the latter may be called "free choice." Human agents can be considered responsible only for those actions which they perform voluntarily. Idealized models of decision-making are not appropriate to connect the ideas of free will and responsibility. Connecting these ideas requires to take the situational horizon of acting into account. If we refer to voluntary actions (in terms of free will as distinguished from free action) we may ask what it is that we are responsible for. What is the nature and scope of our responsibility? Responsibility cannot be restricted to present acts of choosing between alternatives. It is not only related to future events whose occurrence should be taken into consideration by the agent as possible consequences of her action. We are responsible for the things we do and effect, thereby affecting the lives of other persons, too. If our voluntary actions are motivated by past experiences which have been granted significance and have been integrated in our personality, then we should take our responsibility to imply a self-reflective turn. In this view it is the development of my character which I am responsible for.[210] Since it is essential for persons to live in social relations their responsibility cannot be restricted to the development of their own character.

In the above there seems to lurk a dilemma. On the one hand, we should consider ourselves as responsible for the development of our character. On the other hand, we are called upon to find out what our nature is and what kind of behaviour it requires. But how could we be responsible for what is implied in our nature? We are responsible for bringing to light what lies in our character in terms of dispositions to behave in this or that way.

[210] This is one of the problems a phenomenological ethics is faced with: how to profoundly and comprehensibly widen the scope of responsibility without thereby abandoning the idea of individual accountability. Sartre's idea of *mauvaise foi* is a (rather peculiar) attempt to meet this demand.

Insofar as it is up to every person to strengthen or weaken the propensities she has been endowed with by nature, we may say that freedom of will becomes real within an individually varying range of possibilities to shape oneself. It becomes real by approving or disapproving of dispositions. To what extent we succeed in governing given dispositions in the long run is of utmost importance with a view to the consistency or inconsistency of our behaviour. However, it is irrelevant with regard to the issue whether or not we are on principle able to decide freely. In order to resolve the dilemma mentioned above we, again, have to consider how temporality comes into play in our decisions and actions. Psychologically viewed, human persons are built up of diverse dispositions, motives and interests whose (partial) incompatibility is an outstanding matter of self-knowledge. From this point of view, what does it mean to make decisions, especially if these decisions are both of long-term effect and moral relevance? At least decisions of this kind which affect our life-plan may be described as follows: Coming to a decision implies to have the intention to strengthen particular aspects of one's character, i.e., to become the person I would like to be.[211] Approaching this ideal person can be said to be the tacit and indirect (self-referential) purpose of the decision and action at issue.[212] Referring to given dispositions and interests on the one hand and free will on the other hand, I am convinced to be able to realize the person who I would like to be. (This does not imply that I could become any person whatsoever or that I believed to be able to do so). However, with regard to motivation it is clear that decisions are made in a peculiar way because we are the persons we already are. Consequently, the above stated dilemma is due to the fact that every present decision includes the past as well as the future. Being able to decide freely requires to come to know one's

[211] Accordingly, vacillating between alternatives and hesitating how one should settle the issue is disquieting because it represents an uncertainty regarding one's *present* self.

[212] Taking this for granted we do not maintain that the morally good can (only) be intentionally realized, if we avoid confusing the intentional character of a present experience (implying a specific reference to an object which is not identical with the act itself) with intentional experiences directed at own (past) experiences. (Here, again, the object referred to does not coincide with the referring act.) Among phenomenologists, e.g., Max Scheler, Nicolai Hartmann, Hans Reiner, who tend to weaken the role and scope of volitional acts in ethics, the issue whether the morally good can be intentionally realized was intensively and controversially discussed.

own nature (past) and to creatively interfere with it (future). It requires both to acknowledge what is given and to bring to givenness what is not yet real.

Whether we consider someone responsible for what she wants and what she does is not only a question of voluntariness and mental health as usually taken for granted when one refers to the issue of accountability. Deciding and acting in a responsible manner is an ability we have to achieve and improve throughout our lifetime. It is something we have to learn not only in terms of rational implications (what consequences do we have to expect resulting from a decision or action of a certain type under normal conditions?) but also in terms of empathy, tact and knowledge of human nature. It is the latter which enable us to appraise what can be demanded from oneself and others *given that we are the persons we are*.[213] Deciding freely requires to become sensitive to the occasionally changing limits of freedom that also correspond to the peculiar demands of those situations we are faced with. People may be of sound mind and in this sense able to decide responsibly but, nevertheless, practically incapable to do so because of a lacking sensibility to human affairs as well as to the inner nature of themselves and their fellows. To be sure, arguing along these lines does not support the idea that freedom is tantamount to coming to know the rightness (and inevitability) of a deterministic view (see Pothast, 1978, pp. 413–484). Whereas this idea of freedom is a matter of metaphysics the above considerations are concerned with our experience of freedom that is given whatever arguments in favour of assuming or denying free will we may advance in the sphere of theory.

[213] The above reasoning does not amount to the thesis that freedom of will could be limited to the suitability of decisions to be harmoniously integrated within one's biography. This approach results in assuming a *gradually realized* free will depending on the extent to which I succeed in becoming a homogenous character as well as to the extent to which I succeed in gaining self-knowledge. See Bieri, 2001, p. 415 and passim. Contrary to this, my argument (as will be clear in the following) is *that gradually improving our handling of the ability to decide freely does not ensue a graduation of free will.* Here we face again the above stated dilemma. Demands can only be judged if we already know or feel what kind of person we are. On the other hand, it is owing to a lasting social, moral, sexual, and intellectual practice what kind of person we become in course of time.

Laying stress on the difference between accountability (*Zurechenbarkeit*) and demand (*Zumutbarkeit*) it is important to clarify the phenomenological concept of a situation in order not to call upon deterministic and naturalistic implications. "Situation" does not refer to a number of facts which are realized at the same time and place. In contradistinction to such objective occurrences, a situation, phenomenologically viewed, comprises phenomena that present themselves in light of some concrete interrelations between a self and her world. Situations do not emerge unless we take a first-person-perspective.[214] Consequently, it is misguided to say (see the formulation of IT in section II) that a particular situation urges me to make a decision. Rather, intending to make the decision in question and encountering the world in light of a particular situation go together. Insofar as I approach a decision, I am strongly inclined to interpret the present circumstances in a particular way. In this sense a situation neither occurs accidentally nor can it be considered as an "external factor" opposed to my "inner life" of expectations, feelings and volitions. I myself am part of the situation at issue. Equally, what may be described as a specific situation depends on the person who is actually living through it.

Here we have to note a shortcoming of naturalistic conceptions that cannot be reflected upon without transcending their theoretical framework. Due to cutting off the lifeworld horizon of volitional acts naturalistic conceptions tacitly presuppose an activistic idea of free will – even when trying to demonstrate that the activities in question cannot be demonstrated experimentally or do not have the impact one expects them to have. Contrary to this emphasis on efficiency (*Leistungskonzept der Willensfreiheit*), a phenomenological investigation formulates the problem within a wider

[214] In this context we should reconsider our usual understanding of "world" as referring to the whole of all things and processes (occurrences). Contrary to this, a phenomenological idea of the world points out that everything that may be given presents itself within a horizon which on its turn, does not appear in the same mode than the things and processes whose appearance it makes possible. Following the notion of horizon, there is an essential link between facticity and subjectivity which has to be kept in mind when phenomenologists debate on *givenness* and *reflexivity*. See Rinofner-Kreidl, 2003f. So far as things are concerned, it has to be emphasized that relating to human action neither things in themselves nor scientifically shaped objects are relevant but solely environmentally encountered things. See Scheler, 1954, pp. 158–159, 161 ("Milieudinge").

horizon of all those "active" or "passive" experiences that are formative with regard to our human life. In this sense a phenomenologist puts the emphasis on formative aspects when analysing freedom of will (*Gestaltungskonzept der Willensfreiheit*), thereby recognizing that the issue requires to go beyond the sphere of volitional phenomena. (Analysing volitional acts is an obvious starting point for discussing freedom although it does not automatically lead to or include an analysis of *free* will. See Mertens 1998). Freedom of will does not refer to decisions and actions which are performed exclusively by one's own authority. There is always a moment of dependence that has to be acknowledged. Deciding and acting freely depends on how the world (including ourselves) is actually structured, what it "allows" us to want. It depends on a pre-given latitude of subjective formation in concrete situations. Assuming that we succeed in deciding and acting freely within a certain latitude does not back up the idea that we are equally free to acknowledge or refute the latitude at hand.

"Latitude" refers to different kinds of variables. With regard to free choice, the most interesting materialization of latitude is the number und character of available alternatives. Since the person choosing is for the most part incapable of multiplying or altering the alternatives presently given to her, it seems to be near at hand to interpret this fact in terms of an essentially gradualized free will. There is a very common experience endorsing this understanding. The motivational bias and strength of our will obviously is strongly influenced by the fact that there would have been additional alternatives (as we *subsequently* notice) which we did not or could not realize while making our decision. This "blindness" or insensitivity to a wider scope of possible alternatives can be due to, for instance, some kind of unconscious reluctance (as we claim to recognize now). According to such a retrospect, it seems that we would have been more free in our past decision, if only we had been able to enlarge our view at that time. Or: we would have been more free, if we had been more successful in controlling the conditions our decision was subject to. In this way the gradual nature of free choice seems to result directly from the fact that real freedom is always conditioned freedom. Is this argument convincing? I do not think so. It is based on a confusion of choice and scope of choice – *Wahlsphäre* as Max Scheler calls it – and, albeit unintentionally, confirms a misguided idea of free will. An increasing number of alternatives does not correspondingly "increase" our freedom. We either choose according to our own will or we do not. *There is no graduation with respect to free will*. Graduation does not make sense here.[215] If we are inclined to hold the opposite view, this is owing to the fact that we equate free will with absolute autonomy, i.e., free choice which includes an unlimited disposal of the circumstances of choosing. (I do not dwell on the problem that in case of such a complete independency we could hardly have any idea what "decision" meant and why there should be any need for decisions at all. Following this line of reasoning we may wonder whether God's free choices are incomprehensible. Decisions seem to be an essentially human affair). If we compare our real decisions to the idea of an unrestricted independency, it looks plausible to consider the former as only gradual manifestations of free choice. However, as soon as we abandon this idea the plausibility of assuming graduations of free will equally vanishes. *Facticity* in terms of given situations and conditions of decision-making does not imply any graduation of free choice. Nevertheless, free choices are subject to conditions that are not within the power of the person choosing. It is in terms of these conditions that we should talk about "conditioned freedom."

One remarkable limitation of human freedom refers to the fact that we cannot consider ourselves free to renounce our free will. Doing so would require a decision that presupposes free will because otherwise we could not take it as a genuine decision. Free will that manifests itself as free choice cannot be annulled by means of free choice. We are condemned to be free,

[215] There is another misguided idea closely related to the issue discussed above, namely the idea that "acting from motives" would be tantamount to "being determined by one's motives to act in a certain way." It is this idea which normally lies beneath when psychologists or phyiologists refer to the so-called "battle of motives," thereby assuming a determined process of conflicting motives (e.g., Roth, 2006, p. 15). On the other hand, it has been argued that our feeling of freedom is only due to the fact that we do not know our unconscious motives. We consider our decisions free insofar as they are based on conscious motives, i.e., (!) rational deliberation; we consider them unfree insofar as they result from unconscious motives, whereas in fact all human behaviour is determined (Singer, 2004, pp. 51, 58–62). For a more differentiated approach see Stederoth's contribution in Köchy/Stederoth, 2006, pp. 219–236 and Keller, 1954, pp. 92–98 who presents a sustained phenomenological critique of the above views.

as Sartre said. (Of course, this is not equivalent to "we are determined to be free.") Our free choice is part of the facticity we encounter "in" the world.[216] From this point of view we may resume the issue of whether emphasizing the role of personal integration urges us to acknowledge a graduation of free will. Whatever choices I make, I seek to integrate them into my personal life. Notwithstanding this bias towards homogeneity (or: harmony), it has to be noted that freedom of will does not vary in accordance with a more or less incomplete personal integration. Of course, I cannot claim a choice to be *my* choice unless there is a sufficiently strong and recognizable connection with my character and biography. Otherwise I certainly feel alienated and will try to dissociate myself from the decision at issue. In this case I could, for instance, argue that, while making my decision, I had been extraordinarily ill-disposed, confused, or narcotized. Granting that these are relevant informations which should be noticed, we nonetheless have to stress that *the "mineness" of a choice is not to be confused with its freedom*. At every moment I can struggle to avoid alienation, self-deceit, and self-denial in the above sense. I cannot struggle to imbue my choices with freedom (whatever that meant). *I cannot decide to decide freely*.

Along these lines of argumentation a phenomenological conception of autonomy will have to include relational and contextual moments. If we

[216] Relating to this it has to be noted that Husserl's *paradox of subjectivity* indicates that transcendental-phenomenological theory is grounded in a peculiar kind of practice. See Rinofner-Kreidl, 2003e, pp. 125–205. From the present point of view this problem manifests itself in the paradoxical character of a decision to perform the epoché. Here "conditioned freedom" means that abandoning the natural attitude in favour of the phenomenological attitude is not exclusively a matter of individual arbitrariness. This change of attitude rather is made possible by our being-in-the-world. Otherwise it could not be considered as motivated. Motivations cannot be arbitrarily produced. "In *Ideen I* and *Erste Philosophie* 'attitude' is not meant to refer merely to something I *do*, it does not flow complete from my will; rather, it has to do with the world that I, as both consciousness and will, am 'in'. 'Attitude' is a description of my surrounding world from the standpoint of my 'being-in' this world; or, expressed in another way, 'attitude' is part of a description of the world from the perspective of the complex of relations operative in the situation where things appear to the subject, the one who is directed towards them, engaged with them in such a fashion that they constitute the 'surrounding world.' " (Dodd, 1998, p. 61)

rightly argue that the issue of free will leads us to investigate the fundamentally relational structure of self and world, we, moreover, should realize the *common ground of freedom and transcendentality*. Raising the issue of demand and latitude in the above sense also suggests that a phenomenology of freedom challenges the widely accepted idea that the so-called is/ought-problem, exclusively or primarily referring to different types of sentences, represents one of the most fundamental ethical problems. If we agree on the fact that any serious attempt to realize the morally good needs some basis in real life, we are directly led to recognize the frailty of human nature and, nonetheless, strive for improving our character. On the contrary, if the ideas of moral perfection we are ready to endorse could only be realized by ignoring our actual imperfect moral state, they could not be realized at all.[217] Considering free will to belong to a human way of living one's life does not mean to take volitional acts to be imbued with some mysterious absolute volitional power. Grasping free will in this way amounts to denying its possibility altogether. In contrast to this, a phenomenological approach picks up the problem as we really encounter it, namely as conditioned freedom (in the above sense). However, it is important to realize that endorsing the idea of conditioned freedom and recognizing that there might be nothing more to prove (in terms of an allegedly absolute autonomy) does not corroborate a brainphysiological or any other scientific approach to the issue of free will. According to the idea of conditioned freedom we have to explain freedom with a view to facticity. Doing this requires to make plausible that the term "facticity" does not refer to the totality of certain states of affairs. The notion of facticity does not match with a positivistic idea of the given. Brainphysiological investigations cannot help us to discover those unavailabilities that limit our freedom. Idealistic as well as naturalistic misconceptions of free will have to be rejected. Both approaches leap over our concrete, lifeworldly embedded experience of freedom. Following these approaches the latter should be considered essentially insufficient or illusive.

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[217] A phenomenological ethics based on the idea of (inter)personality will reverse the procedure. It will start with asking what idea of personal being we have to acknowledge in order to make sense of those moral actions we actually perform.

From ancient times wisdom has been understood in terms of knowing oneself and the world in such a way that allows to reconcile theoretical and practical needs. If we take a phenomenological view, wisdom and freedom become closely connected. Deciding against my inner nature, my character, my biography *as far as I have understood them up to now*, does not make sense according to the above considerations. Doing so does not in any interesting sense demonstrate the freedom of my will. On the other hand, considering the relevant real conditions of freedom does not point to and does not give reasons for passivity, fatalism, or moral indifference. Taking the above view rather suggests to decide what can be decided and confidently leave the undecidable to fate. This kind of composure must not be mistaken for a callous disengagement. It represents an essential trait of wisdom. We certainly should try to find out what can be subject of decisions in a concrete (moral) situation and what, by its own nature, eludes decision. This is an important aspect of learning what it means to act freely in theoretical as well as practical contexts. (In a broader sense both these contexts, as indicated above, are practical.) With regard to both traditional philosophical reflections and scientific views on the issue of free will it is of central concern whether they ignore the practical unity of freedom and wisdom, either by equating autonomy with a radical and unrestricted independency or by eliminating the real agent from methodical reasons.

Contrary to physical occurrences meaningful experiences, per definition, require the assumption of an agent (an ego) who is immediately aware of having the experiences at issue whenever they occur. With regard to meaningful experiences consciousness and self-consciousness are inseparably linked up with each other insofar as we take the latter in terms of immediate awareness. (There is a basic form of self-consciousness which does not involve a reflective turn. It solely consists in living through one's experiences. It, nevertheless, is presupposed in any higher-order, i.e., reflective self-consciousness). A similar situation occurs with regard to our experience of freedom which includes, in an irreflective or reflective mode, the acknowledgment of real conditions. If we are right in assuming a conditioned freedom that has to be discovered and improved by the agent herself, then we equally have to assume that every realization of free will, to some degree, includes being aware of individual boundaries of freedom. We simultaneously become acquainted with freedom and its limitation. We encounter situations of free choice, *and thereby* grasp the necessity to enter into commitments and to appreciate the unavailable. *There is no (real) freedom without limitation*.[218] Although these limitations shift in accordance with changing characters and situations, there is no evidence that, under certain circumstances, they could ever completely vanish. Supposing that they could, is owing to an idealization which demands to consider volitional acts in isolation from the character of the agent and the situational horizon of the action in question. Still, we have to keep in mind that whatever limitations of freedom we face in a particular case, they cannot be imposed from outside. They can only be known from within, i.e., from the point of view of the agent herself. There is a first-person-authority implied in the notion of free choice.

What does that mean? Doing justice to the peculiar demands of moral issues first of all requires to note that different moral facts may manifest themselves in the same phenomenal structures. What looks like weakness of will, laziness or cowardliness from a third-person-perspective can actually be, from the first-person-perspective, a deeply felt regret for one's individual limitations with regard to realizing the ability of free will. What presents itself as licentious hedonism can be the result of a constantly frustrated endeavour to bring to bear other motives of one's behaviour. What looks like resoluteness and courage from the outside, can actually be, from the first-person-perspective, an arbitrary whim as well as disrespect of or lacking sensitivity to some kind of unavailability. It is this experience of being thrown back to oneself which makes problems of moral life deeply challenging and intriguing.

There is nobody else whom we could charge with being responsible for our behaviour, both in terms of its bearings on the life of other persons and in terms of truly recognizing what kind of behaviour it is at all. With regard to these issues we cannot comfort our moral concerns by imagining what another person would do given that she took my place now. In moral

[218] On the other hand, the majority of our real experiences lacks any immediate feeling or reflective awareness of freedom. If we behave out of habit, as we very often do, there is no choice involved and, a fortiori, no free choice. Nevertheless, what at first sight looks like a habitual action *may* be grounded in free choice. This touches on the issue of how first-person-perspective and third-person-perspective are interrelated here (see below).

affairs we, in some serious sense, are on our own. At most, if we are lucky to live in intimate relations with others (lovers, friends, family members), two or more persons may share the responsibility for their moral development.[219] If limitations of freedom are subjective insofar as they do not exist unless they are recognized from a first-person-perspective, then this is of far-reaching consequences with a view to our common moral practice. The latter ultimately is, as every kind of practice, a social practice. It has to be interpreted in social terms. What could that mean given that we acknowledge the decisive role of our first-person-perspective? There is no objective measure and no generally binding authority enabling us to judge on the endeavour a single person undergoes in becoming aware of how to make her decisions. Whether I, in a particular case, have fallen behind my individual possibilities to realize freedom of will cannot be stated objectively and reliably by any scientific method or ratiocination. It is left to my "inner" moral monologue and the dialogue with my intimates. What cannot be traced in this way does not have any reality in our moral life.

Freedom of will *as we experience it* cannot be separated from our consciousness of deciding and acting freely (this being nothing more than a tautology).[220] Consequently, so far as our moral practice is concerned, it

[219] Since we do not have direct access to the moral endeavours of our fellows, *confidence* turns out to be of vital importance. If a phenomenology of freedom is bound to an interpersonalistic approach (which I take for granted), we even have to consider whether freedom of will can only be realized in interpersonal relations. (Freedom of action clearly is an originally social phenomenon albeit from other reasons. Here, it is due to the fact that any individual behaviour, regarding its consequences, interferes with the behaviour of other persons by restricting their scope of real possibilities to act.)

[220] We do not claim that going beyond our experience of freedom in favour of a metaphysics of free will is dispensable. Delving into our experience of freedom which is the starting point and whole project of a phenomenology of freedom, may throw light on how to ask for metaphysical freedom. Adequately describing our experience of freedom does not amount to and does not substitute for proving its existence. See e.g. Hartmann, 1962, pp. 712–725. On the other hand, we must not misinterpret the fact that such a proof is unavailable in terms of the thesis that assuming the existence of free will would be unwarranted. See Nida-Rümelin, 2005, pp.33–105, 154–171. It is certainly true that our experience of freedom is susceptible to deceptions (as brainphysiologists remind us of). However, it is quite another thing to argue that brainphysiology could tell us what it *really means* to have a feeling of freedom. The latter is assumed if one holds that

is of equal irrelevance trying to reason away or to prove freedom of will by proceeding from an objective point of view, i.e., from a scientifically shaped third-person-perspective.[221] Theoretically considered, it is impossible to consider one's freedom illusive if it turns out that every intellectual act, e.g., considering one's freedom illusive, presupposes this very freedom (see Nida-Rümelin, 2005, pp. 33–43).[222] This being true, it does not hit the mark to argue from an evolutionary point of view that being subject to illusions might produce positive effects in terms of improved abilities of adaptation (see Walter/Goschke, 2006, pp. 114–115). Furthermore, if we are right in maintaining that freedom belongs to the sphere of making sense of one's life, then any argumentation to the effect that necessary conditions of deciding and acting function as objective restrictions of free choice must be misguided. Referring to real limitations of freedom we have to concede that if a person is not sensitive to these limitations, e.g., does not feel obliged to keep the promises she has given, then she may feel widely unrestricted with regard to her decisions whatever she likes to decide. Two points should be noted here. In the first place, we cannot consider an arbitrary decision to manifest our own will except if we hold that it is our own will to have no (firm) will at all. Secondly, the supposedly most ambitious attempt to clarify the issue of free will, namely Kant's moral philosophy, establishes a principle which explains why it is impossible to fix our volitional intention in any other way than meeting the promises we have given *if we consider ourselves as rational beings*. What does a phenomenological approach aim at compared with this Kantian view? Phenomenologists are interested in finding out what we presuppose when arguing that we are able to act according to principles. They want to find out what it means to consider ourselves and others as persons who

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"certain feelings of freedom are meaningless (*sinn- und bedeutungslos*) because they occur on condition of manipulations as well as on condition of absent manipulations." (Grün, 2006, p. 57) The knotty point rather is, as argued above, what kind of manipulations or conditions we consider relevant with regard to the issue of free will.

[221] Supposing the distinction between causally related entities and meaningful experiences to be well-sustained, it is obvious that the fact that we are not (immediately) aware of any causes of our decisions is not relevant to the question whether we can decide freely. See above (fallacy of latency).

[222] For an elaboration of this issue relating to transcendental phenomenology see Rinofner-Kreidl, 2003d.

are endowed with specific rational and emotional abilities. Although Kant obviously expected his categorical imperative to improve our moral sensitivity, step by step, this sensitivity must have already been formed up to a certain degree in order to be able to apply the imperative as well as to feel obliged to it. Within his theoretical frame, Kant discusses these issues under the headings "power of judgment (*Urteilskraft*)" and "respect for the moral law (*Achtung für das Sittengesetz*)." However, if he describes the latter as the only rationally effected feeling, there is some explanation required of how a feeling could be grounded in reason.

It is an important aspect of any attempt to lay the foundations of moral theory to consider how reason can be implemented in our concrete lives. How should a phenomenologist cope with this problem? Reason can be implemented in our lives because it has always been "therein," prior to its reflective discovery. The same holds true for freedom. A phenomenological description of our lifeworld-practice should give evidence of both claims. In terms of presenting a detailed and adequate description of our *experience* of freedom, a phenomenologist will be critical with regard to traditional philosophical as well as scientific conceptions of free will, especially if these conceptions have rationalistic or naturalistic implications. Whereas brain physiologists may ask whether there is any weak notion of free will compatible with our knowledge of the brain, a phenomenologist is interested in finding out what idea of free will corresponds to our lifeworld-practice. Thereby she escapes begging the question by going beyond the usual claim to theoretically deny or defend free will. A phenomenologist does not intend to prove the existence of free will. Rather than that her claim is to make comprehensible what "free will" means by describing how freedom manifests itself in a person's reasoning and acting.

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PART II

PHENOMENOLOGY AND THE FOUNDATIONS OF NATURAL SCIENCES

FOREWORD

Pierre Kerszberg

In the last lines of the *Crisis of the European Sciences*, at the very point where the text of the original manuscript breaks off, Husserl rehearses the main points of his critique of modern natural science, and then leads his reader to ponder over an abysmal conclusion. Even though it is profoundly infatuated with the possibility of a progressive understanding of the real properties of the natural world, modern physics is dominated by the idea that "everything is decided in advance as pure mathematics and as nature itself." However hypothetical it may be, this idea is so powerful that it is supposed to precede the being of the world; it does so in such a way that it cannot influence this being, which is what it is (a being mathematical in itself), irrespective of the progress of knowledge. Yet from the standpoint of phenomenology, such an ontology of the world is nonsensical, since the world also contains spiritual beings for whom being, rather than being given in advance, unfolds itself in the time of a subject's life. The life of the soul has become the unsurpassable wall for modern natural science, since physics could not but devise an ontology of the soul analogous to physics. In this way, philosophy expects from a physicalistic conception of the world a response to its own perplexities concerning the nature of knowledge. But as a matter of principle, the nature of knowledge owes nothing to nature understood as a definite manifold of "being-in-advance." Therefore, if nature is thinkable in this way (as the progress of science demonstrates), the phenomenological critique of natural science indicates that what has been thus thought is an absurdity. A thinkable absurdity: this is the sense of nature bequeathed on us by the modern sciences. Are we to accept this diagnosis and think through its ultimate consequences, as Husserl seems to do in a celebrated manuscript dealing with the experiential fact that "the earth does not move," where he proposed a reversal of the Copernican reversal that would "not touch upon physics," or are we to re-think the exact sciences themselves in light of this critique?

L. Boi et al. (eds.), Rediscovering Phenomenology, 167–172.

It seems that quantum theory suggests a re-interpretation of the metaphysics underlying the whole of modern science. Thus, unlike most physicists since Galileo, Bohr argued that nature does not follow a mathematical scheme. Heisenberg claimed that we cannot disentangle ourselves from the mathematical garb of ideas with which we dress up the real; in this, quantum mechanics is profoundly different from classical physics, which is why we should be ready to abandon such concepts as those of velocity and momentum. By contrast, Bohr maintained the need for classical concepts and common language in quantum mechanics, for otherwise the theory could not be made intelligible. However, he refused to extend to quantum mechanics the interpretation of classical physics in terms of the Galilean metaphysical thesis, according to which the universe is a book written in mathematical language. Inasmuch as Bohr breaks with this metaphysics, and argues that the non-independence of atomic phenomena with regard to the agency of observation plays a constitutive role in the theory, his original interpretation of quantum mechanics echoes the phenomenological critique of the metaphysical foundations of modern science. Husserl pointed out that modern science conflated its own method with the true being of nature; the abstract construction of ideal entities was assumed to mirror reality as it is in itself. Obviously, this conflation brings out an extraordinary clarity in the ordering of natural phenomena, since the constructive procedures are visible in their final products. When applied to quantum mechanics, however, this alleged clarity turns into the well-known, outright obscurity that surrounds such concepts as “observer,” “trajectory,” or “measurement.” Bohr cannot be satisfied with the fashionable view of classical concepts being merely approximate. Consequently, one has to acknowledge the existence of separate regions of reality, i.e., essential differences between such things as galaxies or atoms. This tallies beautifully well with Husserl’s recommendation that the mode of givenness of objects of experience is not the same for all objects, but admits of differences which are themselves essential as regards the being of these objects.

Traditional epistemology is thus inefficient to account for the subtleties of quantum theory. If phenomenology is taken seriously as a plausible candidate to make sense of quantum theory, then some over-arching conclusions suggest themselves beyond this theory in particular. From the start, the motive of modern natural science was drawn from the rejection of the subjective realm, which was thought to be merely relative and unreliable for rational, universal statements. But in this way science undermines itself. The subjectivity of the working scientist is itself put out of play, so that the allegedly absolute rationality of the exact sciences is now on a par with the rationality of, say, the Egyptian pyramids.

One physicist who took phenomenology seriously as the only possible ground for natural science was Hermann Weyl. In the wake of relativity and quantum mechanics, Weyl was among the first to realize the new function assumed by epistemological critique, inasmuch it should from now on form the basis of an actual transformation in and of natural science. Weyl opposed the idea of perseverance, which is captured in the principle of inertia of classical physics, to the idea of adjustment of magnitudes to one another. Adjustment is required in order to remove the remaining arbitrariness in the values of such fundamental quantities as mass or charge; in fact, this arbitrariness is a residue of an inner vision of nature, a vision that ought to be eliminated from the realm of science. In order to implement this idea in a new physical theory, Weyl referred to phenomenology as the appropriate conceptual tool, which he viewed as allowing for an "open place" of meaning in the realm of being; i.e., phenomenology allowed for a much higher degree of permeability between the subjective and the objective components of experience, so that all that is supposed to be inner to nature is transferred to the inner life of the absolute ego. Thus, his attempt to unify the physics of gravitation and electromagnetism, which culminated in the late 1910s, gave rise to gauge invariance physics, in which the metrical essence of space is conceived purely infinitesimally, not through relations at a distance. This was supposed to provide a geometry of all possible spacetime manifolds, i.e., a "world geometry" for which the actual physical world is to be singled out as a special case. The hypothesis to be tested was whether this world geometry could be regarded as an acquisition of the transcendental self-constitution of the I. If this were to be the case, then the world geometry would be equivalent to the world horizon in the phenomenological sense.

To be sure, gauge invariance physics did not quite meet with the expected success. The ghost of arbitrariness quickly reappeared. Yet, irrespective of its merits or shortcomings as a tool for physical theory, the idea of adjustment raised explicitly the question as to what remains of the ego in science, i.e., the always individual and particular cognitive acts leading to the constitution of a transcendent realm of things. According to Weyl, the transcendent object can never be more than an intentional object in the sense of Husserl, and the capacities of the ego are eidetic structures of consciousness. Saving and re-asserting the infinitesimal credo demanded a grand metaphysical insight, according to which the infinitesimally small is reflected in the infinitely large. Weyl expressed the latter in his famous cosmological principle according to which all galaxies form a bundle of diverging world-lines from a common origin. This principle, which is still at the basis of all dynamic models of the global relativistic space-time, implies dependence of the state of the universe on its past history, whereas gauge invariance makes the metric "essentially one and absolutely determined."

Despite their irresolvable tensions, Weyl's contributions indicate an interesting way in which the relations of transcendental critique to natural science could be re-appraised. They suggest that the transcendental discourse is absorbed in physics, because the latter provides a mathematical construction of the non-representable process according to which reality becomes manifest. Where Husserl saw indirect mathematization of sense-qualities, i.e., transposition of intuitive experience in symbolic language, Weyl thought that the new mathematics is itself an actual condition of possibility in the transcendental sense. Just as the infinitesimally small does not quite fit in the infinitely large, the objective world is relative whereas immediate experience is subjective and absolute. The lesson to be drawn from the new conception of science suggested by Weyl is perhaps that the opposition of the subjective/absolute to the objective/relative cannot be absorbed in any possible science of the lifeworld, which Husserl conceived as subjective and relative.

Husserl's conception of epistemology is definitely at variance with the Fregean type of distinction between "foundation" and "genesis" of knowledge. Basing himself on an enlarged sense of historical explanation, he calls for the elimination of the dogma according to which epistemology and history ought to be severed from one another. That is why, in mathematics, axiomatic conventions and logico-formal proof are the final stages of a complex process of meaning constitution. The analysis of this process must be carried out in parallel with the transcendental constitution of the physical object in perception, so that ultimately the transcendence of the object (a "fuzzy" notion if taken in the usual sense) can be made intelligible. Thus, against Frege or Gödel, transcendence is not to be separated from transcendental constitution. Any realist epistemology of mathematics is bound to assert the transcendence of mathematical objects and ignore transcendental constitution. Mathematics belongs to the world in which we live, there is no such thing as a pre-existing set of "laws of thought." For example, the concept of infinity turns out to be the result of ancient, yet profound, non-arbitrary conceptual practices. In any event, both logicism and formalism are based on the absolute certainty of proof, but Gödel has already taught us that any formal theory in which proof is decidable is incomplete. Moreover, because in modern science the mathematical structure of the real world is thought to be constitutive of all physical knowledge, it is possible to assert within certain limits that the physical object "is" mathematical. Consequently, Husserl's remarks on the relative rationality of the exact sciences can be taken seriously, and pushed beyond what he envisaged for himself: the realist epistemology of mathematics blocks the way to a decent epistemology of physics.

Now, even Gödel, the mathematical Platonist, did not remain indifferent to the new foundations of knowledge laid out in transcendental phenomenology. When, on the basis of his own technical results, Gödel developed an epistemological reflection on the status of logic and mathematics, he re-discovered two of the main tenets of Husserl's logic. On the one hand, Gödel argues that mathematical objects exist independently of our cognitive acts, they form an objective reality of their own, yet they are given in accordance with a type of intuition which is different from sensible intuition. On the other hand, for mathematical intuition, the question of the objective existence of its objects mirrors the question of the objective existence of the external world. Both Husserl and Gödel can be said to agree on the structure of a formal edifice, namely, the double orientation of logic which divides itself into: (i) a theory of concepts, propositions, and proofs (grounded in a grammar); (ii) a theory of objects, which turns out to be the theory of sets (guided by a rationalist ideal, according to which any proposition can be demonstrated or refuted). Moreover, Gödel calls for phenomenological reflection inasmuch as it alone can fix the intuitive sense of mathematical objects. But Gödel's theorem of incompleteness implies a revision of Husserl's logic, inasmuch as this theorem rests on the non-paradoxical circularity of concepts.

Thus, whereas in natural science Husserl's original epistemology could suggest a reform of its underlying metaphysics, in mathematics and logic Gödel's reading of phenomenology could be the basis for a reform of Husserl's own logic.

CHAPTER 6

PERSEVERANCE AND ADJUSTMENT: ON WEYL'S PHENOMENOLOGICAL PHILOSOPHY OF NATURE

Pierre Kerszberg

Abstract. Basing himself on a phenomenological interpretation of the history of modern physical science, Weyl argued that in contemporary physics the idea of adjustment played the fundamental role which was assigned to perseverance (inertia) in classical physics. Adjustment between the basic quantities of the world would do away with arbitrariness, which was the last residue of an inner vision of nature. This substitution was only suggested by relativity and quantum mechanics. Its full implementation into gauge invariance physics required further conceptual developments. Weyl did provide such developments by following a suitably modified Husserlian perspective on the place of meaning in the realm of being. Even though Weyl's gauge invariance theory seemed to clash with his own metaphysical view of the universe at large, his phenomenological philosophy of nature proves that epistemological critique could form the basis of an actual transformation of physical theory.

1. Two Great Ideas

In the twentieth century the relations between physical theory and epistemological critique have changed radically. Consider physics in the seventeenth century: even if space were to be relative or relational in the sense of Leibniz, mechanics would still be Newtonian. Think of Mach's critique of absolute space in the nineteenth century: the redefinition of the inertial mass of a body in terms of the effects of all the other masses in the universe did not lead then to a new physics. It would seem that, in the wake of relativity and quantum mechanics, Weyl was among the first to realize the new function assumed by epistemological critique, inasmuch as it forms the basis of an actual transformation in and of natural science.

Weyl's epistemology is perhaps best captured in his own account of the need for the general theory of relativity, which differs markedly from Einstein's. Einstein had argued that Newtonian mechanics is incomplete inasmuch as it does not explain the local equivalence between

L. Boi et al. (eds.), Rediscovering Phenomenology, 173–194.

inertial and gravitational mass; moreover, the distinction between inertial and non-inertial reference frames (i.e., the restriction of coordinate transformations to Cartesian transformations) is justified by nothing empirical, since absolute space is not observable. As Einstein showed, arbitrary coordinate transformations bestow an "identity of essence" to the two fields. Weyl's justification is different: when one looks at the origin of inertial forces, one finds that Newton's theory is not incomplete, but incoherent. The Newtonian law of action and reaction already requires that if a physical entity (such as absolute space) exerts observable effects upon matter (inertial forces), then in turn it suffers such effects.[223] The only way in which this requirement of coherence can be met is by substituting the metrical field in Riemann's sense to absolute space. To be sure, the metrical field is no more observable than absolute space; but its fusion with gravitation assigns at least an observable *origin* to it, namely, the distribution of matter and energy in the universe.

The idea that a physical cause must itself be affected by that upon which it exerts its effects indicates the epistemological priority of a principle of universal reciprocity, which according to Weyl forms the basis for all future physical theory. Later in his career Weyl reflected on what he claimed to be the ultimate philosophical implications of science in the modern world.[224] In classical physics the fundamental idea concerning the natural world is *perseverance*, which is captured in the principle of inertia. Perseverance is the last residue in modern science of an inner vision of nature, since an inner tendency for bodies to persevere in their (original or acquired) state is ascribed to the whole of nature. Yet this inner vision is incomplete as far as the intelligibility of nature is concerned, since according to this view physical bodies could still have arbitrary mass or charge. Contemporary physics attempts to overcome this arbitrariness by introducing a new fundamental idea at the basis of our understanding of nature: this is the idea of *adjustment* of magnitudes to one another. How does adjustment overcome arbitrariness? From the standpoint of perseverance, as the transition from the first to the second Newtonian law of motion shows, perturbation

[223] H. Weyl, *Philosophy of Mathematics and Natural Science*, trans. O. Helmer (Princeton: Princeton University Press, 1949), p. 105. Hereafter PMNS.

[224] H. Weyl, "The Main Features of the Physical World; Morphe and Evolution," in PMNS, p. 288.

due to the contingent distribution of bodies must occur at any time and anywhere, and deviations from the initial values of the supposedly conserved quantities will certainly occur in the course of time; therefore, they depend on the past history of the universe, which exceeds the laws of nature alone. Not so in the case of adjustment, because the initial values are now independent of past history: as Weyl puts it, the definite value cannot be arbitrary because it "reasserts itself after any disturbances and any lapse of time as soon as the old conditions are restored." Thus the idea of adjustment goes hand in hand with a certain restriction on the conception of temporal development of all natural processes, since it allows for something like an eternal return of the same. Indeed Weyl was finally to cast doubt on the idea of evolution as the keystone of scientific knowledge: whether the laws of nature are those of classical mechanics, statistical thermodynamics, or quantum mechanics, "chance factors are never missing in a concrete development."[225] Unlike Einstein's theory, in which the actual behaviour of rigid rods and clocks still needs to be preserved in order for the ds^2 to remain the fundamental quantity, a systematic theory of the sort that Weyl envisioned for physical science would start with a metric ground form; then a primitive field quantity would be introduced, to which the particles adjust themselves.

The idea of adjustment reflects Weyl's attempt to unify the physics of gravitation and electromagnetism, which culminated in the late 1910s in his gauge invariance physics. Almost simultaneously (1923) he designed a grand cosmological principle of common origin for all the fundamental building blocks of the universe, according to which galaxies define a bundle of diverging world lines from a common origin that provides natural synchrony calibration for all events. At the level of global spacetime temporal evolution would thus be fixed in some way, whereas gauge invariance rules out past history as a determining factor. Historically speaking, Weyl's cosmological principle proved to be the appropriate conceptual basis for all relativistic models of the dynamic universe, whereas the gauge invariance theory remained problematical to the end. Rather than merely reflecting some peculiarities – if not glaring inconsistencies – of Weyl's own thought, the turbulent relations between the diverse principles of contemporary

[225] Ibid., p. 294.

natural science open up a vast field of renewed interrogation with respect to the role of epistemology in physics.

2. Phenomenology as the Key to Natural Philosophy

In a letter to Hermann Weyl, dated 10 April 1918, Husserl praised Weyl for having resurrected the need for a phenomenological consideration of the foundations of the exact sciences, by which he meant the clarification of its fundamental concepts which lead us back to the primal ground (*Urboden*) of logico-mathematical intuition. Later on, in a letter dated 9 April 1922, Husserl went on to praise Weyl's book *Raum, Zeit, Materie* for showing that Nature is to be understood in accordance with the demands of a priori grounds (that is, transcendental and constitutive), not in accordance with the positivistic principles. Obviously he was referring to Weyl's own interpretation of relativity which was supposed to go beyond Einstein's original theory of gravitation, which was itself based on Riemann's geometry: the measurement of lengths changes from point to point on the spacetime manifold, i.e., the metric is modified from point to point; the alleged identity of rigid rods was thought by Weyl to be the undesirable relic of the old way of thinking in terms of action at a distance. Weyl conceived a purely infinitesimal geometry whose basic relations are valid only in the infinitesimal neighbourhood of a point of a manifold. The mathematical construction yielded new variables that Weyl identified at first with the electromagnetic potential, only to discover later on a broader and deeper justification for gauge geometry, where the idea of gauge invariance was reinterpreted as pertaining to a factor of complex phase, not of scale.[226] These comments by Husserl raise a series of problems, both scientific and philosophical.

In the introduction to *Space, Time, Matter*, Weyl paid tribute to Husserl by arguing that the very advent of modern science was conditional on the rejection of all sensible and individual perception from the properties belonging to real things in themselves. The realization of the subjectivity

[226] See E. Scholz, "Hermann Weyl's Analysis of the 'Problem of Space' and the Origin of Gauge Structures," *Science in Context*, 17 (2004), pp. 165–197; T. A. Rykman, "Surplus Structure from the Standpoint of Transcendental Idealism: The World Geometries of Weyl and Eddington," *Perspectives on Science*, 11 (2003), pp. 76–106.

of sense qualities appeared together with the fundamental thesis of the mathematico-constructive method. In accordance with the allegedly universal scope of this method, the essence of the real was declared to be deprived of these qualities. Weyl addressed himself to the question of the ultimate validity of this claim, especially since in contemporary physics even space and time were dealt with as purely symbolic constructions. The question arises as to what really remains of the I in modern and contemporary science, of the always individual and particular cognitive acts leading to the constitution of an independent, transcendent realm of things. His answer is that all the determinations of the real world "are, and can only be given as, intentional objects of acts of consciousness."[227] Independent things *are* such objects. So, the first question that comes to mind is whether Weyl's own infinitesimal geometry is the expression of the transcendental ego thus understood.

It is noteworthy that in his later work, principally the *Crisis of European Sciences and Transcendental Phenomenology*, Husserl conceded to science a power of philosophical reflection which draws it back to its intentional foundation. Arguably the theory of relativity could be said to do just this, since it required that space and time be redefined in terms of the actual observational capacities carried out by a living subject. Yet these capacities could all too easily be confused with the well-known positivistic demands of verifiability. In Husserl's view, the danger involved in this confusion is best captured in the fact that the scientific way of thinking, by its very nature, falls short of the transcendental ideal because its retrospection (*Rückbesinnung*) stops too soon; only transcendental phenomenology could pursue this retrospection to the end. Is Weyl's amended relativity, then, a further step undertaken by science in the direction of transcendental constitution? If this were so, the relation of science to philosophy would be fundamentally changed. Indeed, irrespective of its own merits or shortcomings, Weyl's contribution to physics echoes a much broader perspective. As some prominent physicists of our own time have explained, the rise of relativity and quantum mechanics amounts to an intrusion of physics into the domain traditionally occupied by philosophy: "matter, space, and time are no longer dealt with as scientific concepts, but rather

[227] H. Weyl, *Space, Time, Matter*. Trans. H. Brose (New York: Dover, 1952), p. 4. Hereafter STM.

as gnoseological categories."[228] On this view, physics would become more and more transcendental since the conditions of manifestation of phenomena would be captured in physical theory itself. What is it that philosophy can do with this intrusion?

One suggestive interpretation goes like this.[229] The symbolic world construed by science does not force us to leave our own immediately intuitive world; rather, it reveals new dimensions in the process whereby the one world in which we live comes to manifest itself. Physics would then be, not a denial, but an amplification of perception because it would partake in the progress of perceptive intuition. Much like intuitive experience, physics seeks new data in the aperceptive horizon of experience, assuming that a part of this horizon is always to be discovered anew. Thus, unheard-of phenomena revealed by physical theory and experimentation would bring to manifestation those properties of the real world that were already present, yet not immediately apparent in the perceptual type of experience. The problem, however, is that this manifestation is quite peculiar since it is represented in purely symbolic terms. It is suggested that physics absorbs the transcendental discourse by providing a *mathematical construction of the non-representable process according to which reality becomes manifest*.

This seems to be a most plausible interpretation of the kind of strategy followed by Weyl. But it is a very problematical interpretation, because the *sense* of this construction is no more verifiable in intuition than the process itself. Phenomenologically any conceivable object, as object of a rational proposition, *is* an object only in virtue of the sense-bestowal of an intentional act of consciousness, in which consciousness in related to its object inasmuch as it is intuitable in some way. But how could the proposed mathematical construction provide this sense, since what is represented is precisely that which is supposed to make manifestation possible? Weyl's insight is precisely that this works in some way, since he argues that "there is no reason to see why the theoretical symbolic construction should come to a halt before the facts of life and of psyche."[230] On the

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[228] G. Cohen-Tannoudji and M. Spiro, *La matière-espace-temps* (Paris: Fayard, 1986), p. 356.
[229] See J. Ladrière, "Physical Reality. A Phenomenological Approach," *Dialectica*, 43 (1989), pp. 125–139.
[230] PMNS, p. 214.

other hand, according to Husserl himself, the exact sciences proceed by indirect mathematization, by which he understands the idea that "everything which manifests itself as real through the specific sense-qualities must have its mathematical index in events belonging to the sphere of shapes – which is, of course, already thought of as idealized."[231] If Weyl's insight were to be correct, then the new developments of physics should force upon us an interpretation of the mathematical index, not as some kind of transposition of intuitive experience in symbolic language, but as an actual condition of possibility in the transcendental sense. The problematical character of the above-mentioned interpretation needs to be examined inasmuch as it lies precisely at the core of the philosophy of nature that Weyl drew from his own renewed vision of natural science.

Let us first consider *space*. Weyl writes: "The problem of space is . . . a very instructive example of that question of phenomenology which seems to the author to be of greatest consequence, namely, how far the delimitation of the essentialities perceptible in consciousness expresses the structure peculiar to the realm of presented objects, and in how far mere convention participates in this delimitation."[232] Convention is a posit of the mind that looks habitual once it has been effectively put to use. Despite the strength of habitualities, convention is not given for once and for all. It is historically determined, but its historical development is not itself arbitrary. Weyl sets himself the task of understanding this development, from Euclid to Riemann and beyond, as an effort to force convention to evolve so that its participation in intuitive experience presents itself more fully to consciousness. As regards space, the task is to remove "the last remnant of geometry 'at a distance,' a remnant of its Euclidean past."[233] When mathematics is informed by the phenomenological concern for the essential structures of the given in experience, it teaches us how to do away, not so much with the I as an absolute self, as with these habitualities of the mathematical mind that *pass off* as self-evident. The absolute self is not like an all-powerful sovereign standing above the non-essential

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[231] E. Husserl, *The Crisis of European Sciences and Transcendental Phenomenology*, trans. D. Carr (Evanston, Ill.: Northwestern University Press, 1970), §9c, p. 37. Hereafter *Crisis*.

[232] STM, p. 148.

[233] Ibid., p. 102.

characteristics of experience, dictating how to overcome them; rather, it has a life of its own, which is deeply anchored in human nature. The way is thus cleared up to re-assess the actual role of the I in natural science.

In a late account of his philosophical motivations, Weyl spoke of intuition as mind's originary act, "limited in science to the *Aufweisbar*," i.e., that to which we can point in concrete, "but in fact extending far beyond these boundaries." One would expect that what lies beyond is precisely the life of the self that we are interested in, yet Weyl immediately went on to observe that "how far one should go in including here the *Wesenschau* of Husserl's phenomenology, I prefer to leave in the dark."[234] This darkness is not merely a confession of ignorance, but also a tribute paid to an authentic phenomenological way of thinking, since Weyl acknowledges that the contact between the world and consciousness occurs through mutual penetration between being and the *general form* of consciousness, which is not the mere stuff of external perception impinging on the senses. Obviously that which goes beyond intuition thus understood not only enlarges the field of immediate perception, but also conceals the transcendental sense of mathematical construction and physical experimentation. Therefore what Weyl finally decides to leave in the dark is the answer to the fundamental question as to what the mathematical construction of a non-representable process of manifestation could possibly mean, even if it were to exist. Yet this decision also preserves the enigma by making it philosophically fruitful, since at different points of his writings Weyl argues that the fundamental starting point of all inquiry about the world is the *tension* between subject and object. There can be no release of this tension in favour of the allegedly exclusive constitutive capacities of the absolute subject. Thus, at the bottom of both philosophical idealism and scientific construction, the separation between aprioristic and material features of the world can never be absolute: just as the mind does not and cannot exhaust the world, the world is not the residue left after the mind has been subtracted from it.[235]

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[234] H. Weyl, "The Unity of Knowledge" (1954), address delivered at the Bicentennial Conference of Columbia University, in *Gesammelte Abhandlungen*, vol. IV (Berlin: Springer, 1968), p. 629.
[235] PMNS, p. 135.

Weyl's epistemological program leads to a geometry of all possible manifold curvatures, and as such, a geometry of all possible space-time manifolds, a "world geometry" for which the actual world of physics is to be singled out as a special case. The question is thus whether the world geometry is an acquisition of the transcendental self-constitution of the I. Is this the physicist's account of Husserl's transcendental idealism, according to which the self-constitution of the I is tantamount to the constitution of the world? The question is all the more difficult that, from a technical point of view, what Weyl has actually done could still be interpreted as the mere characterization of another of family of geometries.[236]

3. The Double Life of the Ego in the Natural World

In Husserl's phenomenology, the experience of the natural world is distrusted in favour of the constitutive power of the transcendental ego. In accordance with the so-called "phenomenological reduction," our belief in the existence of a transcendent world (otherwise referred to as the "natural attitude") is put out of play, so that the self-reflection of the transcendental ego is preserved as the only thinking activity capable of relating to the world as its own *cogitatum*. As phenomenon, the world has not lost its real existence, but it has retained only its claim to be. To say phenomenologically that there is a world is to say that experience is not a chaotic mess, or that there is a general reliance on the possibility of confirming or disconfirming anything. Husserl brings the self to the threshold of the experience of the world, and the world itself to its minimal sense, when he argues that the unreality of the world does not proceed from its denial, but from the supposition that experience degenerates into chaos. Indeed, such chaos is a situation in which nothing can be either confirmed or disconfirmed. Once belief in the world's existence has been suspended, a real object is always present through multiple modes of givenness, each of which forms an adumbration of the object. The experience of adumbrations is such that, by virtue of their sense, the coming adumbration is prefigured by the adumbration in focus. Things are always situated in a field of possible appearances, i.e., in a determinate horizon.

[236] See E. Scheibe, "Hermann Weyl and the Nature of Spacetime," in *Between Rationalism and Empiricism*, ed. B. Falkenburg (Berlin: Springer, 2001), pp. 475–489.

The nexus of mutually implied horizons constitutes the world in the phenomenological sense.

Now, the concept of world from the standpoint of gauge invariance physics is also an attempt to bring it to its nascent sense. For Weyl metric is the essence of space, and congruence, which is the fundamental metrical concept, should not be assessed directly through relations at a distance, as in Riemann's geometry. Rather, congruence must be conceived purely infinitesimally, something which is possible only if the units of magnitude are allowed to vary independently (and yet continuously) from point to point. World geometry is one in which the Riemannian line element is defined only up to an arbitrary positive function of position. The physical world is to be singled out within the world geometry through the choice of a gauge invariant action function. One could say that this choice expresses the fundamental possibility of confirmation or disconfirmation, which prevents the experience of the world from falling into chaos. From this viewpoint, world geometry is tantamount to world horizon.

Technically, the problem with Weyl's theory was that there are many functions available to express the physical world as a special case of world geometry. Ultimately the choice of the appropriate function appeared to be arbitrary. This did not prevent him from defending to the end the epistemological superiority of his own principle of relativity of magnitude. This is precisely where his debt to phenomenology came into relief.

There is a fundamental difference between Husserl and Weyl, since Weyl interprets Husserl's adumbrations as progression from level to level ending with the objective world represented by symbols; this progression does not remain confined to the world as phenomenon. In other words, Weyl seems to have interpreted the perceptual process by accumulation of adumbrations as a feature of the natural attitude itself, not as a consequence of its suspension. On this basis, he goes on to argue that scientific explanation properly speaking *reverses the order of the natural attitude*, as the symbolic world is now regarded as a realm by itself for which the relation with the corresponding data of consciousness must be described.[237] Obviously, reversing the order of the natural attitude is not typical of scientific explanation according to Husserl, for whom such explanation takes place within the natural attitude. This difference testifies

[237] PMNS, p. 113.

to Weyl's relative dissatisfaction with the Husserlian concept of world. He does not wish to leave the question of the reality of the world in phenomenological brackets, and describe the concordant nexus of implied horizons in terms of laws of the world as a mere phenomenon, until a reason is provided for its lawful *mathematical* harmony.[238]

Nevertheless, even if the latter harmony sends us back to the ultimate transcendence (God) which Husserl himself wished to put in brackets as well, phenomenology is still according to Weyl the only philosophical way of thinking capable of acknowledging that the mastery of the natural world by the mind is limited by the fact that there is an "open place"[239] of Meaning in the realm of Being. The independence of the ego with respect to nature, which was denied by science as it developed since Galileo, is that which needs to be retrieved. But, at least to some degree, the dependence of the ego with respect to the exact natural laws cannot be denied either. Hence, "the real riddle" which, according to Weyl, "lies in the double position of the ego: it is not merely an existing individual which carries out real psychic acts but also 'vision,' a self-penetrating light (sense-giving consciousness, knowledge, image, or however you may call it)."[240] In other words, whereas real psychic acts are those that are not exempt from the natural laws, the intentional or sense-giving acts are those that free themselves from their overpowering rule. This duality reflects the dual status of the ego in Husserl's phenomenology, which for Husserl holds between natural and transcendental, not eidetic, consciousness. Indeed Husserl explains that "as an ego in the natural attitude, I am likewise and at all times a transcendental ego, but [...] I know about this only by executing phenomenological reduction."[241] Be this as it may, Weyl's phenomenological insights proceed as follows.

Because of the open place occupied by Meaning in the realm of Being, the double position of the ego exemplifies an antinomy between causality and freedom.[242] At first, it looks as if the freedom constitutive

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[238] Ibid., p. 125.
[239] Ibid., p. 209.
[240] Ibid., p. 215.
[241] E. Husserl, *Cartesian Meditations*, trans. D. Cairns (The Hague: M. Nijhoff, 1960), p. 37.
[242] PMNS, pp. 210–211.

of intentional acts does not contradict the physical world, since this world too allows for freedom in the space left open because of the incomplete determination of the world by a given physical state. Is this open place in the sense of physics that of freedom *from* natural causality as well? In Newtonian physics, as interpreted according to Laplacean determinism, there should be exact mathematical laws according to which the state of the world at a given moment (a three-dimensional cross-section $t = cst$) determines the entire course of past and future events. Now, if the future can be calculated by an individual located at a world point O of this cross-section, then the future is like the past, since what is open to perception and accessible to direct knowledge from O is only the past.[243] Thus, the freedom of the observer at O, which enables him to calculate the future, defeats itself by virtue of being exercised in order to calculate this future. This expresses the well-known fact that the classical laws of physics are time-reversible, as if the sheer passage of time could change nothing to the course of nature. Weyl goes on to argue that this antinomy has been overcome by relativity theory, in which no present Now extends everywhere. The backward light-cone does not uniquely determine the sum total of events since it defines a well-ordered sequence of events only within the section limited by the absolute spatio-temporal elsewhere. At any given time of an observer's world-line, the passive past, and the active future are completely disjointed from one another by virtue of the velocity of light being an upper limit for the propagation of all signals. What separates the known from the unknown is the light track itself, and since a spatio-temporal interval happens to vanish along such a track, the repetition of the same is confined to light only. This resolves the classical antinomy by eschewing the reduction of freedom to a factual matter: the *partial* determination of any knowable present is now constitutive of this present by virtue of the laws of nature themselves. According to pre-relativistic physics, the propagation of effects is instantaneous, so that the distinction between cause and effect is absorbed in the functional or symmetrical character of the laws of nature. In relativity physics, things are different: the effect really occurs later than the cause, because the velocity of light is the upper limit for the propagation of all signals.

[243] Ibid., p. 194.

Retrospectively the Laplacean antinomy turns out to be not a real antinomy at all. This can be understood in the sense that in Newtonian gravitational theory, claims about *all* future and past times are simply refused: the unique solution compatible with Laplacean determinism may fail to exist after some time because particles may have accelerated to such a degree that they escape to spatial infinity. This can happen because the theory does not incorporate any upper bound for the velocity separating inertial frames.[244] Thus Weyl is certainly justified in claiming that in any cross-section of the Laplacean world, there exists what he calls an "open place." This development in physical theory teaches us that the independence of sense-giving conscious acts is supported by the laws of nature. Weyl goes one step further as he explains that, at least in one instance, the laws of nature may even account for this sense-givingness.

The rights of universal causality are re-established in the theory of relativity, because both world and mind are affected by the limitation of an open place. Is this limitation common to both? Are we sure that we are talking about the same place? Weyl is rather optimistic, as he interprets the theory of relativity as having overcome all antinomy. Indeed he argues that subjectivity and objectivity are at one in the theory: "Subjectively that part of the light-cone which opens toward the future plays an entirely different role from that which is directed toward the past. We travel along the world line of our body with 'screened-off' consciousness." This leads him to say that the theory of relativity is the first satisfying response to the "demand that that basic fact of consciousness, the one-way direction of the flow of time, have a physical foundation." In this case, one significant feature of our sense-giving consciousness is explained away as an aspect of the physical world. Could it be that Weyl's epistemological plan is meant to compel all other such features to undergo a similar treatment, so that ultimately a complete picture of the world would involve a complete interchange between the subjective and the objective? There are indications in Weyl's writings that abating the tension between subject and object ought to be accomplished by making the balance tilt toward the a posteriori components of experience. In fact this is the lesson to be drawn from the theory of relativity, since one fundamental consequence

[244] See J. Earman, *A Primer on Determinism* (Dordrecht: Reidel, 1986).

from the Riemann-Einstein view of the world is that the strict separation between a priori and a posteriori features is the gain over transcendental idealism, inasmuch as the distinction is now in a certain sense objective: it does not refer "like Kant [. . .] to their cognitive source."[245]

The elimination of antinomy is understandable in light of Weyl's profound belief in the unity of nature, which is mirrored in his view about the unity of scientific method and his belief in the principle of universal reciprocity. Yet the riddle of the double position of the ego still lurks in, since presumably only the real psychic acts of consciousness can be thought of as objective. What about eidetic consciousness? The persisting embarrassment caused by this double position is reflected in the way unity is connected with *totality*.

Fundamentally, infinitesimal geometry is justified on the basis that all laws of nature are differential laws, i.e., the connections that they establish in nature prevail "only among what is infinitely closely adjacent."[246] In this way, the contingent necessity of the laws of nature is associated with the inner necessity of the things themselves, since from one point to the next they cannot be other than they are. Any purposiveness over and above the infinitesimal nexus is thus discarded a priori as an explanatory factor of natural phenomena. But in order to save the very idea of one nature in which somehow "all is woven" beyond the immediately given, Weyl is then eager to capitalize on the notion of the world as one whole, which seems to work as a substitute for the notion of purposiveness, especially since the sense of totality is derived from phenomena pertaining to both to life and psyche. Thus he appeals to psychoanalysis in support of the view "that in nature all is woven into one whole."[247] That all the forces of the inanimate as well animate nature are connected cannot be proven, given the state of natural science today, but he hopes that it is only a question of time until it will be so. Therefore, it would seem that for him the proper way of saving and re-asserting the infinitesimal credo is to rely on some grand metaphysical insight, according to which the infinitesimally small is reflected in the infinitely large and vice versa. In a series of lectures in 1931, Weyl argued with no small measure of self-confidence that it is only

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[245] PMNS, p. 134.
[246] Ibid., p. 213.
[247] Ibid., p. 214.

now that we perceive the true perfection of the universe as it springs from the relation of its parts to the whole.[248]

The full realization of the principle of universal reciprocity would imply that the cosmic order is not to be explained in terms of perturbation to an otherwise given configuration (a pre-existing, persisting metrical field). It is the cosmic order itself that needs to be accounted for, not as empirically given, but as given in the phenomenological sense, i.e., open to some eidetic insight thanks to which it is fixed absolutely in accordance with its own essence. The epistemological lesson of general relativity was that the spacetime metric is contingent upon the distribution of matter. Therefore, Weyl concluded, the only thing left to be understood is what he called the Pythagorean *nature of the metric*. Indeed, prior to general relativity, the metric itself was assumed to be independent of the distribution of matter in space; now, if the nature or eidetic structure of the metric is still Pythagorean, then it must be assumed to be "essentially one and absolutely determined," thus providing a universal standard of adjustment. This capacity is due to the fact that it "does not participate in the irradicable vagueness of that which occupies a variable place on a continuous scale."[249] It is not in the metric, but in the nature of the metric that the a priori essence of space-time structure manifests itself. The new type of axiomatics required by this shift was provided by Weyl's own gauge invariance physics. On the other hand, the relative orientation of the metric at each point is given entirely a posteriori: it is accidental, ever-changing, ascertainable only by means of empirical intuition. In the final analysis, if Weyl's epistemological re-assessment of general relativity was supposed to lead to a new physical theory, in fact it only displaced the "great divide" between metric and matter to a new great divide between the a priori nature of spacetime and the a posteriori relative orientation of the metric field at various points. When confronted to the actual developments of physical theory, the epistemological assertion concerning the permeability between the subjective and the objective, the a priori and the a posteriori, boils down essentially to the fact that the line of separation between them is not absolutely fixed. Obviously the a posteriori component plays

[248] H. Weyl, *The Open World* (New Haven: Yale University Press, 1932), p. 6.
[249] PMNS, p. 134.

a much stronger role than Weyl would have liked to admit at the time of the inception of the gauge invariance principle.

4. Time and the Cosmos

This does not mean that the power of pure, eidetic intuition over the natural world gives out completely. In order to be carried out to its ultimate implications, the foregoing conclusion had to be extended from space to time. Weyl thought that the intelligibility of the cosmic order itself could be arrived at by using the concept of the history of the universe as the universal support for the mutual generation and adjustment of metrical field and matter-energy. But this was not an easy thing to do. Up till the fourth edition of *Raum, Zeit, Materie*, he held the view that "in so far as the state of the guiding field [*Führungsfeld*, or affine connection] does not persist, and the present one has emerged from the past ones under the influence of the masses existing in the world, namely, the fixed stars, the phenomena [such as centrifugal forces] are partly an effect of the fixed stars, relative to which the rotation takes place."[250] In a footnote, Weyl added this qualification: "We say 'partly' because the distribution of matter in the world does not define the 'guiding field' uniquely, for both are *at one moment* independent of one another and accidental." From the standpoint of the physics of fields, all we know is that when an initial state is given, all other states (past and future) necessarily arise from them. Now, if the initial state were to be that of nature itself, then the principle of universal reciprocity would be grounded beyond the mere epistemological demand for unity; nothing that exists would fall outside the origin, so that the possibility of universal adjustment would be actualized. The need arises to re-think the concept of time in accordance with this cosmogonic requirement.

The original breakthrough in our understanding of the concept of time was accomplished by the special theory of relativity. Einstein had explained that in this theory "becoming" in the three-dimensional space has been transformed into "being" in the four-dimensional world. Reflecting on the notion of becoming, Weyl says that "the objective world simply *is*, it

[250] STM, p. 221.

does not *happen*."[251] Becoming is not a deceptive appearance, however, as the theory suggests a re-consideration of the movement of consciousness toward the future. The world appears to become as it becomes manifest to our "blindfolded consciousness" (*abgeblendete Bewusstsein*), which creeps along its world-line of its own body into the area of the universe called "future." The theory allows for the fact that we meet the pre-existing future events on our way to the future; even if the future is completed, our way to the future is still going on and it is precisely this movement which gives the world its actual appearance. The amalgamation of space and time in the pseudo-Euclidean four-dimensional continuum exhibits the original relation of consciousness to the world. What is gained in the emphasis on the operation of consciousness is acquired at the expense of the formal aspect of the theory, which cannot take history as its theme.

Where does consciousness go from there, once its original relation to the world has been defined? Consider the connection between intuition and symbol with respect to space. The intuition of space does not conflict with its symbolic representation in the non-Euclidean terms of general relativity as long as the intuition is confined to infinitesimally small neighbourhoods at a point O. But then the connection "becomes increasingly vague the further one departs from O. This is analogous to a tangential plane (intuitive space) touching a point O of a curved surface (physical space)."[252] Should we not expect something similar with regard to time, when consciousness breaks in the four-dimensional continuum and meets it as history? Would not the history carried out by time-consciousness fade away as the world is conceived symbolically as a global spacetime model? This is the point at which Weyl takes an entirely different route, as he is the pioneer of a famous cosmological principle at the basis of the theory of comoving coordinates needed in all homogeneous and isotropic expanding spacetimes. According to this principle, at any point of spacetime, the galaxies form a bundle of diverging worldlines such that they are stationary in space that is perpendicular to the worldlines. In this space the galaxies have a common time (cosmic time) so that they display a common history. We have to ask whether the cosmic time function in relativistic cosmology

[251] PMNS, p. 116. Cf. also STM, p. 217.
[252] H. Weyl, "Erkenntnis und Besinnung" (1954), in *Gesammelte Abhandlungen*, op. cit., p. 632.

has anything to do with primordial time-consciousness. Is this function the full projection of consciousness in the world as a whole, or is it still merely a convenient tool for calculation?

The comoving coordinates are the technical expression for the a priori foundation of time as separate from space at the global level of spacetime. Even though this a priori foundation may not appear in the technical expression, Weyl's cosmological principle points to the need for decision of philosophical order at the basis of all natural science; the nature of this decision is masked by technical expertise. Cosmologists generally think of global spacetime as a solution yet to be found to the equations of general relativity, assuming that the option of cosmic time reflects certain factual evidence about the observed universe. But Weyl's principle aims at embedding cosmicality in the equations themselves, or even more deeply in the principles of natural science as such. Thinking about the universe does not merely amount to seeking the appropriate solution to the equations of general relativity; this search is meaningful only inasmuch as the nature of the metric itself is thought out conceptually in terms of the eidetic capacities of consciousness.

Weyl's decision to allow for time-consciousness to be fully projected in the world breaks through the straitjacket of mathematical formalism. At the origin, it is impossible to talk of perturbation of an otherwise independent field. But the actual appearances result from the universe being inhabited, as it were, by the "spirit of unrest" (*Geist der Unruh*)[253] which is matter. This restlessness appeared and went on growing as the universe expanded. Taken by itself Weyl's cosmological principle does not specify whether the origin is at infinity in the past or whether it defines a point in the finite past. As a mirror of the mystery of the origin, it compensates for the mathematization of nature, which provides the scientist with methods that debar him from questioning the greater depths of the origin of things. As a matter of fact, "all beginnings are obscure,"[254] yet they determine a genuine way of *comprehending* that precedes and constrains all processes of formalization that take place in natural science. Conversely, once effective, comprehension implies a retrospective look into the beginning of the world itself.

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[253] H. Weyl, *Raum, Zeit, Materie*, 5. Aufl. (Berlin: Springer, 1923), p. 297.
[254] STM, p. 10.

Already in the introduction to *Raum, Zeit, Materie*, Weyl was in a position to dramatically reassess the views on the relations of consciousness to transcendent reality. The bridge between the two is not merely perception. More fundamental than perception is what he calls the pair, action and passion, what there is in us as experience of strife and resistance, which echoes in the ego the epistemological assertion about universal reciprocity and gives it metaphysical garb. First and foremost, it is this form of causality, more primitive than perception, which is responsible for the indissoluble coupling of space (as form of material reality) with time (as form of consciousness). Furthermore, in the action of action and passion, "I become a single individual with a psychical reality attached to a body which has its place in space among the material things of the external world."[255] At the same time that communication between individuals arises, the ego "becomes a piece of reality." The pair action/passion is thus more fundamental than perception, because it allows for the real acts of the psyche to be redirected to the transcendental self from which they emerge.

5. Conclusion: Doing away with Arbitrariness

From the outset, Weyl's proposal to substitute the idea of adjustment for the idea of perseverance, as the fundamental insight at the basis of all laws of nature, was motivated by the desire to take away the last residue of arbitrariness in the classical picture of the world. In this picture, arbitrariness reflected the impossibility to satisfy the demands of an inner, yet inaccessible, vision of nature. Weyl's gauge invariance physics followed from the epistemological program to reconcile our intelligibility of nature with a thorough outer vision, so that the absolute I takes over the remaining demands of an inner vision, which exceed the scope of natural science. Does this imply that the subjective component of experience is completely divorced from the objective description of the world? This is not so simple, as Husserl's own later concept of world illustrates.

Enlarging the concept of world beyond the core of its primitive perceptions, Husserl finally developed the lifeworld, which is the world in

[255] Ibid., p. 6.

which things appear in their full experiential qualities, values, and uses. Despite its full integrity, the lifeworld is surpassed from within when the hypothesis of objective truth is set up as a goal to be reached by scientific methods.[256] That is why the lifeworld is characterized by a double view: the world that stands over against the quantified world of modern science, and the world from which the world projected in the natural sciences arises. In the first sense the lifeworld is subjective, in the second sense it is relative. But how could the lifeworld as relative present itself as the ground out of which science grows? Husserl argues that all that exists relatively originates from an invariant structure, which is not itself relative. This structure (spatiotemporality, causality, etc.) is in fact *the same* as that of the objective sciences, except that it is not concerned with theoretical idealizations. Thus the world horizon in the lifeworld "exists with such uniqueness that the plural makes no sense when applied to it,"[257] in contradistinction to the space of geometry which is always defined as a manifold. Yet the impact of the scientific ideal is so powerful that the objective a priori "is always immediately substituted" for the universal a priori of the lifeworld.

Weyl's own view about the lifeworld is one in which the substitution has been effected "naturally" as it were: "in the transition from consciousness to reality the ego, the thou and the world rise into existence indissolubly connected and, as it were, at one stroke."[258] Because there is no way to distinguish them, the world of the natural attitude enlarged to the lifeworld is none other than the objective world of the exact sciences themselves.[259] Hence, instead of the qualitative distinction advocated by Husserl between the a priori of the lifeworld and the a priori of the objective world, a certain *parallel* prevails between the subjective and the objective. Every change in immediate experience finds its counterpart in the objective world, and vice versa. Therefore, if there is any difference yet to be marked out, it will become perceptible when the idea of invariance is applied to both. For example, if we ask what remains of the ego in the invariant construction of the angle under which two stars will appear to a given observer O, the

[256] *Crisis*, §36, p. 139.
[257] Ibid., §37, p. 143.
[258] *The Open World*, op. cit., pp. 26–27.
[259] PMNS, p. 116.

answer is "the visual shape of the constellation" and the fact that "I myself am the point eye at O."[260] Invariance in the subjective sense means the absoluteness of what is given thus and not otherwise to a single subject. Invariance in the objective sense means the mapping out of what is so given in terms of definite symbols accessible to any collection of observers; that is, the objective world is relative to an arbitrary system of coordinates, thanks to which the observer of this world has become an abstract, disembodied entity. Even intersubjective communication is not enough to secure the transition from monadology to the objective world. The intersubjective observer is nowhere, and does not exist.

This account of the relation of the lifeworld to the objective world of science in terms of the subjective/absolute opposed to the objective/relative differs from Husserl's, where the subjective/relative is to be distinguished from the objective/absolute. The lesson to be drawn from the conception of science outlined by Weyl is that the opposition of the subjective/absolute to the objective/relative cannot be absorbed in any possible science of the lifeworld. With the help of this reversal, we might try to answer our preliminary question about the relation between intuition and symbolic construction, which was suggested by the physicist's account of the transcendental experience of the world. Inasmuch as the process thanks to which reality becomes manifest is representable, it bumps into irreducibly individual facts of subjective life, and therefore it does not lend itself to mathematical construction. Inasmuch as this process is not representable, mathematical construction applies to it, but then reality becomes manifest as symbols.

On the one hand, then, the world of immediate experience is both subjective and absolute; phenomenologically speaking, this refers to its irreducibility. On the other hand, the natural world is relative to certain abstract means whereby the observer loses its subjective singularity and things gain their definiteness; the objective world is relative to those means, and this expresses its reducibility. The latter point is the main acquisition of the theory of general relativity, which has taught us how and why the arbitrary system of coordinates is to be preferred to any privileged such system, where the unknowability of the inner perseverance of things

[260] Ibid., p. 115.

in space, time, causality, etc., was still taken in as an indication of our belonging to the world. This arbitrariness in the new worldview is the price to pay for one ultimate impossibility of comprehension: namely, the freedom to think myself in my capacity to see the world, and then interrogate it.

CHAPTER 7

MATHEMATICAL CONCEPTS AND PHYSICAL OBJECTS

Giuseppe Longo

Abstract. The notions of "construction principles" is proposed as a complementary notion w.r. to the familiar "proof principles" of Proof Theory. The aim is to develop a parallel analysis of these principles in Mathematics and Physics : common construction principles, in spite of different proof principles, justify the effectiveness of Mathematics in Physics. The very "objects" of these disciplines are grounded on commun genealogies of concepts : there is no trascendence of concepts nor of objects without their contingent and shared constitution. A comparative analysis of Husserl's and Gödel's philosophy is hinted, with many references to H. Weyl's reflections on Mathematics and Physics.

Introduction (with F. Bailly)

With this text, we will first of all discuss a distinction, internal to mathematics, between "construction principles" and "proof principles" (see [Longo, 1999; 2002]). In short, it will be a question of grasping the difference between the construction of mathematical concepts and structures and the role of proof, more or less formalised. The objective is also to analyse the methods of physics from a similar viewpoint and, from the analogies and differences that we shall bring to attention, to establish a parallel between the foundations of mathematics and the foundations of physics. The paper is introduced by a joint reflection with a physicist, F. Bailly, coauthor of the complete French version of this work, originally a dialogue in two parts ([Bailly, Longo, 2006]).

When proposing a mathematical structure, e.g., the integers or the real numbers, the Cartesian space or . . . a Hilbert space, we use a plurality of concepts often stemming from different conceptual experiences: the construction of the integers evokes the generalised successor operation, but at the same time we make sure they are "well-ordered," in space or time, to obtain this well-ordered "line of integer numbers" which we easily "see," within a mental space. And we construct the rationals, as ratios of integers modulo ratio equivalence, and then the real numbers,

L. Boi et al. (eds.), Rediscovering Phenomenology, 195–227.

as convergent sequences (modulo equiconvergence), for example. The mathematician "sees" this Cantor–Dedekind-styled construction of the continuum, a remarkable mathematical reconstruction of the phenomenal continuum. It is nevertheless not unique: different continuums may be more effective for certain applications, albeit that their structures are locally and globally very different, non-isomorphic to this very familiar standard continuum (see [Bell, 1998]). And this construction is so important that the "objectivity" of real numbers is "all there," it depends solely upon the well order of integers. One could say as much about the most important set theoretic constructions, the culmulative hierarchies of sets, the sets constructed from the empty set (a key concept in mathematics) by the iterated exponent operations, and so on. . .. These conceptual constructions therefore obey well-explicitated "principles" (of construction, as a matter of fact): successor, ordering in space (well order of integers, iteration, limits…).

But how may one grasp the "properties" of these mathematical structures? How may one "prove them?" The great hypothesis of logicism (Frege) as well as of formalism (Hilbert's program) has been that the logico-formal proof principles could have completely described the properties of the most important mathematical structures. Induction, particularly, as a logical principle (Frege) or as a potentially mecanisable formal rule (Hilbert), should have permitted to demonstrate all the properties of integers (for Frege, the logic of induction coincided, simply, with the structure of the integers – it should have been "categorical," in modern terms). Now it happens that logico-formal deduction is not even "complete" (let's put aside Frege's implicit hypothesis of categoricity); particularly, many of the integers' "concrete" properties elude it. We will evoke the "concrete" results of incompleteness from the last decades: the existence of quite interesting properties, demonstrably realized by the numerical structures, and wich formal proof is unable to grasp. But that also concerns the fundamental properties of sets, the continuum hypotyhesis, and of the axiom of choice, e.g., demonstrably true within the framework of certain constructions Gödel, in 1938, or demonstrably false, as shown by Cohen, in 1964, so unattainable by the sole means of formal axiomatics and deductions.

To summarise this, the distinction between "construction principles" and "proof principles" shows that theorems of incompleteness prohibit the reduction (theoretical and epistemic) of the formers to the latters (or also of semantics – proliferating and generative – to strictly formalising syntax).

Can we find, this time, and in what concerns the foundations of physics, some relevance to such a distinction? In what would it consist and would it play an epistemologically similar role? Indeed, if the contents and the methods of these two disciplines are eminently different, the fact that mathematics plays a constitutive role for physics should nevertheless allow to establish some conceptual and epistemological correspondences regarding their respective foundations. This is the question we shall attempt to examine here. To do so, we will try to describe a same level of "construction principles" for mathematics and physics, that of mathematical structures. This level is common to both disciplines, because the mathematical structuration of the real world is a constitutive element of all modern physical knowledge (in short, but we will return to this, the constitution of the "physical object" *is* mathematical).

However, the difference becomes very clear at the level of the *proof* principles. The latter are of a logico-formal nature in mathematics, whereas in physics they refer to observation or to experience; shortly, to measurement. This separation is of an epistemic nature and refers, from a historical viewpoint, to the role of logicism (and of formalism) in mathematics and of positivism in physics. We will therefore base ourselves upon the following table:

Discipline	**Matematica**	**Fisica**
Construction principles	Mathematical structures and their relations	
Epistemic reductions/separations	Logicism/Formalism	Positivism/Empiricism
Proof principles	Logic/formal languages	Empirical evidence/experience

1. Mathematics Discipline, Physical Aspect

Construction principles level Mathematical structures and their relationships Epistemic Reduction Logicism/Formalism Positivism/Empiricism. Proof principles level Formal/Logical Languages Experience/observation.

Let's comment this schema with more detail. The top level corresponds to the construction principles, which have their effectiveness and their translation in the elaboration of mathematical structures as well as in the various relationships they maintain (that these structures be relative to mathematics as such or to the mathematical models which retranscribe, organise, and give rise to physical principles – and by that, partly at least, the phenomena that these principles "legalise" by provoking and often guiding experiments and observation). This community of level between the two disciplines, in what concerns construction, does not only come from the constitutive character of mathematics for physics, which we just evoked and which would almost suffice to justify it, but it also allows to understand the intensity of the theoretical exchanges (and not only the instrumental ones) between these disciplines. Either physics obtains elements of generalisation, modelisation, and generativity from mathematical structures and their relationships, or else physics' own developments suggest and propose to mathematics the construction of novel concepts . . . of which physics already make use, without waiting that they be rigorously founded. Historical examples abound: be it the case of leibnizian infinitesimals which appeared to be so paradoxical at the moment they were introduced – and for a long while after that – and which were never theoretically validated elsewise than by non-standard analysis, be it Dirac's "function" which was rigorously dealt with only in the theory of distributions, be it the case of Feynmann's path integrals – which have not yet found a sufficiently general rigorous mathematical treatment, while revealing themselves to be completely operable – or be it the birth of non commutative geometry inspired by the properties of quantum physics.

The bottom level, corresponding to that of the proof principles, divides itself into two distinct parts according to whether it concerns mathematics or physics (in that their referents are obviously different). For mathematics, what works as such are the corresponding syntaxes and logico-formal languages which, since Frege, Russell, Hilbert, have been presented as the foundations of mathematics. In fact, the logicism and formalism which have thus developed themselves at the expense of any other approach never stopped to identify the construction principles level with the proof principles level by reducing the first to the second. The incompleteness theorems having shown that this program could not be fulfilled for reasons internal to formalisms, the paradoxical effect was to completely disjoin one level from the other in the foundations of mathematics, by leading syntax to oppose semantics or by refusing to satisfy oneself with proofs not totally formalised (in the sense of this formalism) as can exist in geometry for example. In fact, it appears, conversely, that, as all of the practice of mathematics demonstrates, it is the conceptual coupling and circulation between these two levels that make this articulation between rigour and the innovative imagination which characterises the conceptual generativity of mathematics and the stability of its invariants.

Now a similar table can be drawn for physics, where the emergence of invariants (and symmetries) also constitutes a methodological turning point, as well as the constitution of objects and of concepts (see, e.g., [Bailly, 2002], [Bailly, Longo, 2006]). But this time, at the level of the proof principles, we no longer find a formal language, but the empiricism of phenomena: experiences, observations, even simulations, validate the theoretical predictions of mathematical models and prove their relevance. As constructed as they may have been by anterior theories and interpretations, it is the physical facts which constitute the referents and the instruments of proof. And there again, a particular philosophical option, related to the stage of development of the discipline and to the requirement of rigour in relation to physical factuality, has played, for the latter, a similar role to that of logicism and especially to that of formalism for mathematics. It consists in the positivism and the radical empiricism which, believing to be able to limit themselves only to facts, attempted to reduce the level of construction, characterised, namely, by interpretative debates, to that of proof, identified to pure empiricity. The developments of contemporary physics, that of quantum physics particularly, of course, but also that of the theory of dynamical systems, have shown that this position was no longer tenable and that the same paradoxical effect has lead, doubtlessly by reaction, to the epistemological disjunction between the levels of construction and of proof (a transposed trace is its opposition between "realists" and "nominalist" in the epistemology of physics). While, there again, all the practice of physicists shows that it is in the coupling and the circulation between these levels that lies the fecundity of the discipline. And, since for us the analysis of the genesis of concepts is part of foundational analysis, it is this productivity itself that feeds off interactions and which takes root within cognitive processes, which must be analysed.

It is thus in this sense, summarised by the above schema, despite their very different contents and practices, that the foundations of mathematics and the foundations of physics can be considered as presenting some common structural traits. That is, this distinction between two conceptual instances are qualifiable in both cases as construction principles and as proof principles, and the necessity of their coupling – against their disjunction or conversely, their confusion – is important to also be able to account for the effective practice of researchers in each of these disciplines.

Moreover that they share the same level of mathematical structures characterising the dynamics of construction principles and feeding off the development of each of them.

If we now briefly address the case of this other discipline of natural sciences which is biology, it appears, in what concerns the structure of its own foundations, to distinguish itself from this schema, though we may consider that it shares with physics the same level of proof principles, i.e., the constraint of reference to the empiricity of observation and of experience. However, we are lead, at the level of this proof principle, to operate a crucial distinction between what is a matter of *in vivo* (biological as such in that it is integrated and regulated by biological functions), and what is a matter of *in vitro* (and which practically confounds itself with the physico-chemical). But what manifestly changes the most depends, it seems, on two essential factors. On one hand, the level of what we may call (conceptual) "construction principles" in biology still does not seem well characterised and stabilised (despite models of evolution, autonomy, or autopoiesis). On the other hand, it seems that another conceptual level adds itself, one specific to the epistemology of the living, and to which is confronted any reflection in biology and which we may qualify, to use Monod's terminology, as the level of the teleonomic principle. This principle in some way makes the understanding of the living depend not only upon that of its past and current relationships to its relevant environment, but also upon that of the anticipations relative to the future of what this environment will become under the effect of its own activity of living (an aspect of this third factor of temporality, shown in [Bailly, Longo, 2003]). And this temporality lays itself beside the usual physical temporality which regulates the physico-chemical action-reaction relation and the biological temporality specific to the organism which manifests itself as the existence and the activity of "biological clocks" which time its functions (also see [Bailly, Longo, 2003]). This conceptual situation then leads to consider, for biology, the characterisation of an extra, specific concept, in interaction with the first two, which one of the authors called "contingent finality"; meaning by that the regulations induced by the implications of these anticipations, and which themselves open the way to the accounting for "significations" (see [Longo, 2003]).

This paper contains no more remarks on systems of life: some work in this direction may be found in [Bailly, Longo, 2003; 2006]. A synthetic view is in [Bailly, Longo, 2006].

2. Genealogies of Concepts

Let's more closely tackle now the idea of a parallel between the constitution of mathematical concepts and of physical objects. We will only be able to respond partially to this inquiry and shall rather reflect upon the meaning of the relativising constructions specific to mathematics and to physics, within an explicative and foundational framework inspired by the arisen questions. But the project is wider, because it is a question of grounding the two "constitutive histories" within our worldly living being, to grasp this biological and historical "cognitive subject," which we share and which guarantees us the objectivity of our forms of knowledge. It is not a question of unifying by force the epistemologies of differing disciplines, but to make them "exchange between themselves," to reveal the reciprocal dependencies, the several common roots. The analysis we propose here will thus base itself upon the following principles:

- The problem of the foundations of mathematics is (also) an epistemological problem.
- Any epistemology (of mathematics) must refer to a conceptual genesis, as a "process of construction of knowledge."
- The epistemology of mathematics is an integral part of the epistemology of the sciences (the exact sciences, at least).
- A constitutive element of our scientific knowledge is the relationship, established in the different sciences, to space and to time.

In short, a sensible epistemology of mathematics must try to explicitate a "philosophy of nature," term which is dear to the great minds of the nineteenth century. As it is, mathematics are one of the pillars of our forms of knowledge, they help to constitute the objects and the objectivity as such of knowledge (exact knowledge), because they are the locus where "thought stabilises itself"; by this device, their foundation "blends" itself to other forms knowledge and to their foundations. Moreover, the conceptual stability of mathematics, their relative simplicity (they can be profound all the while basing themselves upon stable and elementary, sometimes quite simple, principles) can provide the connection which we are looking for with the elementary cognitive processes, those which reflect some of the world's regularities in our active presence within that same world, as living beings (and living in intersubjectivity and in history). For these same reasons, the theories of knowledge, from Plato to Descartes, to Kant, Husserl, or Wittgenstein, have all addressed the question of the foundations of mathematics, this "purified knowledge," both mysterious and simple, where notions of "truth" and of "proof" (reasoning) are posed with extreme clarity. The problem of the cognitive foundations of mathematics must therefore be analysed as an essential component of the analysis of human cognition. Within that framework, we will attempt to analyse in what sense "foundations" and "genesis" (cognitive and historic) are strictly related. The very notion of "cognitive foundations" explicitly juxtaposes foundations and genesis (see [Longo, 2005] for more in this direction).

In this study, the notions of time and space which we use do not refer to "natural entities," but rather to the play between sensible experience and conceptual frameworks which allow the natural sciences to manifest themselves. That was in fact the inquiry of the great geometers (Riemann, Helmholtz, Poincaré, Enriques, Weyl. . .) who tried to pose the problem of the foundations of mathematics within the framework of a philosophy of nature. But the analysis which came to dominate afterwards stemmed from a very clear division between logical (or formal) foundations and epistemological problems, particularly under the form of this relationship to time and space which ground mathematics in this world.

Frege explicitly denounces the "delirious" situation in which the problem of space finds itself, because of the emergence of non-Euclidean geometries ([Frege, 1884]), and proposes a "royal way out," by laying the bases of a new discipline, mathematical logic. Mathematics themselves are the development of "absolute laws of thought," logical rules outside of this world and independent of any cognitive subject. For that, Frege introduces a very clear distinction between "foundations" and "genesis," he breaks any epistemological ambition, all the while attacking "psychologism" (as of Herbart/Riemann) and "empiricism" (as of Stuart-Mill). The former try to understand which "hypotheses" (which "a priori") allow to make physical space (and time) intelligible to the knowing subject, while the latter relates mathematics to a theory, alas too naïve, of perception. Faced with all these first attempts at a "cognitive analysis" of mathematics, Frege proposes a philosophy centred upon a very inflexible dogma, the logicist dogma, according to which mathematics have no psychologico-historical or empirical genesis. They are, according to him, a constituted knowledge, concepts without conceptors. This philosophy, this dogma, is at the origin of the fundamental split, which will accompany all of the twentieth century, between foundational analysis and epistemological problems, between mathematics and this very world they organise and make intelligible.

Moreover, for Frege, geometry itself, as given by numerical ratios ([Frege, 1884]), bases itself on arithmetics; and the latter is but the expression of logical laws, because the concept of the number is a logical concept and induction, a key rule of arithmetics, is a logical rule. Finally, the continuum, this difficult stake of phenomenal time and space, is also very well mathematised, in Cantor–Dedekind style, from arithmetics.

So there are the problems of time and space and of their mathematisation, neglected to the benefit of their indirect foundation, via arithmetics, upon logic; pure concepts, with no relationship whatsoever to sensible experience nor to physical construction. Conversely, this relationship was at the centre of the inquiry of the inventors of non-Euclidean geometries: Gauss, Lobatchevsky, or Riemann did not play the logical negation of Euclid's Vth axiom and of its formal developments, but they proposed a "new physics," a different organisation of the world (see [Riemann, 1854], [Lobachevskij, 1856]). It also happens that the numerical relationships may possibly "found" Euclidean geometry, but surely not other geometries, because Euclidean geometry is the only one which "preserves" these relationships (it is the only one whose group of transformations – of automorphisms – which defines it, contains the homotheties).

Now it is doubtless that mathematics have a logical as well as a formal foundation (a distinction will need to be made here), but they are in fact a "three-dimensional" construction. They constitute themselves within the interactions of the logical and totally essential "if . . . then" (first dimension), of perfectly formal, even mechanic calculus (second dimension), but also in a third conceptual dimension, these constructions of (and in) time and space, which mingle it, even more so than the two others, to the different forms of knowledge. And the epistemological problem then poses itself as an analysis of the constitution of the invariants of language and of proof, these invariants which we call "logic" and "formal systems," as well as the invariants of time, and of space, upon which we construct our geometries, these "human constructs . . . in our spaces of humanity" as Husserl says in the "Origin of Geometry" (see below). The problem is thus posed from the analysis of this very peculiar form of knowledge which is mathematics, from their cognitive roots, be they pre-human, to their communicable display, with its thousands of mediating levels.

Axiomatic conventions and logico-formal proof are actually but the ultimate results of a constitution of meaning, common notations of concepts rooted in "our living practices," to put it as Wittgenstein would do, in our "acts of experience" (Weyl): logico-formal analysis is a necessary accompaniment to this latter part of the epistemological process, the analysis of proof, of certain proofs, but it is insufficient (it is essentially "incomplete," some theorems tell us). The foundational analyses of mathematics must thus be extended from the study of deduction and of axiomatics to that of the constitution of concepts and of structures; impossible without a parallel analysis of the constitution of the physical object and of perception.

3. Concerning the "Transcendent" in Physics and in Mathematics

There is no doubt that there exists a reality beyond ourselves, which enters into "friction" with our actions upon it and which, moreover, "canalises" them. Husserl uses a word from the idealist tradition to designate this reality: he recourses to the notion of transcendence. In a very common interpretation of this word, and quite independently from Husserl, the following deduction is usually made, first in physics, then in mathematics: the "properties" of the world (physical, numerical, mathematical. . .) are transcendent, and moreover, are not all known. They are therefore "already there," they pre-exist. The objects of the world around us have well established properties that are quite stable and invariable in connection to our senses: I look at this pencil, I touch it, even its odour confirms its "objectivity," independently of the specific sense I use to explore it . . ., it is thus already there, it pre-exists my explorations, with all its properties. In a completely analogous manner, the properties of numbers, of mathematical structures do not depend on notation (for numbers: decimal, binary . . .) nor on other details of representation, of the mathematician subject exploring them . . . therefore they pre-exist.

Now it is the word "property" – in physics, in mathematics – that must first be agreed upon: a property is "talked about," it is first of all an expression in these languages through which we try to speak of the world, to organise it and to give it meaning, a meaning shared with others. But the world canalyses our efforts to obtain knowledge and displays some "resistance" (causes friction) to our propositions to organise it. "Properties," as we render them through intersubjectivity by words, are not in themselves isomorphic to "absolute facts" that are "already there," possibly well established or that would manifest themselves under well established forms of linguistic structures; by our active gaze, in our exchange with others, we propose a structure with hints of a reality which is there, as unorganized frictional matter. Thus, through language, pictures, gesture, we unify certain phenomena, we draw contours upon a phenomenal veil which is an interface between the world and us. The transcendent is a constituted, it is the result of a constitutive activity, of a process which precedes the individual or that the individual performs mostly with others. This process is best synthesised as the result of a *transcendental* (and not transcendent) activity, and such is the lesson we draw from Husserl.

It is no coincidence if the many "examples of objects" proposed by ontologising philosophies, in mathematics, in physics, refer to "medium size manufactured objects," all the while attempting to escape the problem of cognitive relativism. These thinkers of ontology, of essences, rarely refer to the "objects" of quantum physics, e.g., in order to propose an ontology that is much more difficult to take on, of the electron, of the photon . . . But even these medium size manufactured objects, of an apparently such simple ontology, if it is true that they are really there, are just as much constituted as the concept they are associated to. The pencil is constructed, in history, at the same time as the "concept" of the pencil. Both are related to drawing, to writing, as human activities. They are pre-existent, the object and the concept, for the individual subject, they are not so for humanity, in its history. There was no pencil, nor table, nor a pot such as the one laid on Kurt Gödel's table, before the beginning of our human acting and thinking. On the other hand, there was surely already a physical "reality" (for Galileo, less so for Tales), but its organisation and its interpretation as photon, electron . . . solid, stable, in fact mathematical, was not yet there, nor was it's organisation into pots, pencils, and tables before the blossoming of our humanity. And this approach, we think, does not face the dangers of relativism, because the objectivity of the constructed, of the concept, of the object, lies in the constitutive process which is itself objective.

Cassirer, quoted by Parrini in a work whose goal is to overcome the fracture between absolutism and relativism, partially addresses this theme [Parrini, 1995]: "if we determine the object not as absolute substance beyond all knowledge, but as object which takes form within the progression of knowledge itself," then, "this object, from the viewpoint of the psychological individual, can be said to be transcendent," despite that "from the viewpoint of logic and of its supreme principles," it must "be considered as immanent." Ideality, the concept as "conceived," "a cut-out" ("decoupage") performed upon the world in order to give it contours, to structure it, will thus detach itself from subjective representation, despite that it may have its origins within the community of subjects, in what they share: similar bodies and brains from the start, in the same world, and all that which they build in common, in their common history. It is thus not a question of writing a history of individuals, but to trace back the origin of an idea; no historising relativism, but a reference to history as an explicitation of our "being together in the world," locus of the active constitution of all our forms of knowledge.

In the case of the objects of physics, of microphysics in particular, this activity of the construction of objects by "conceptual carving" is rather clear: electrons, muons, fermions, quantum field . . . are not already there. They are concepts that are proposed in order to unify, to organise, to understand the signals the world sends us. These signals are not arbitrary and they are also the result of an active exploration. In order to obtain them it was necessary to develop rather complex measurement instruments, which are themselves the result of a theory. All the instruments for physical measurement, and more so those of microphysics, are constructed after an enormous theoretical commitment: I want to measure this but not that, by using these materials but not other ones, I "look" here and not there. The "facts" which result from this, as Goodman would say, are thus "small-scale theories" themselves.

Let's consider, e.g., the wave-particle duality in quantum physics. The photon, the electron, present themselves as "waves" or "particles" depending upon the "experimental context": specific instruments are put into place, in fact the experiment is prepared from the viewpoint of a certain theory. . .. The object that will result from this will depend as much upon the theoretico-experimental framework as it will upon friction – "the canalisation of thought" that nature imposes upon and through these tools. A certain viewpoint will show us the particle, another will show us the wave. More precisely, we will obtain macroscopical properties on a screen, on any detecting device, and by a process just as important, we will interpret them as "symptoms" of the "existence" of a particle or of a wave. There is no duality as such for the physical object, but a context of reconstruction of the world where we are as present as the object under observation.

Properties, then, are the "explicitated" result of an organising of clues, of a group of facts, which are themselves "little theories." But reality is there, doubtlessly, because it canalises our efforts to obtain knowledge in non-arbitrary directions, it causes friction, by opposing itself to our theoretical propositions, great and small, these "properties" spoken of in our languages. The transcendency of these properties, as if they were already constituted, as "ontologies," is a "flatus vocis" and surely cannot be based on Husserl's views, because it is the constitutive process of the transcendental which is at the centre of his philosophy. It is our task, when referring to different forms of scientific knowledge, to enrich and to specify this so very fuzzy word, the notion of "property" for the physical world, as well as that of mathematical property.

Transcendence vs. Transcendental Constitution: Gödel vs. Husserl

So let's move on to mathematics. In this discussion we refer to one of the most interesting among thinkers having an "ontologising" tendency (and one of the greatest mathematicians of the twentieth century), K. Gödel. Actually, Gödel also proposes a strict parallel between physical objects and mathematical concepts, although from a perspective different from ours (similarity of "ontologies" or of "independent existence"): "It seems to me that the assumption of [mathematical] objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence" [Gödel, 1944] . . . "the properties of these concepts are something quite as objective and independent of our choice as physical properties of matter . . . since we can create [them] as little as the constituent properties of matter" [Gödel, 1947]. So, physical bodies, and constituent properties of matter, as well as mathematical concepts are all preconstituted entities, possibly the ultimate building blocks, independent of or transcending the cognitive subject (not "created"). Again, even the word property, as referring to outside objective states of affairs, is used in a naïve, ordinary way, even for constituents elements, it seems, whose analysis belongs to the entangled constructions of microphysics, where the constitutive polarity "subject/object" is at the core of the modern perspective in Quantum Mechanics (indeed, since the '30s).

In his masterpiece about the foundations of mathematics, "The Origin of Geometry," Husserl frequently emphasises the role of the transcendental constitution of mathematical "objects." The epistemological problem they pose is, for him, a "problem of genesis," a "historical problem" (see the footnote above). Geometry, as an attempt (and mankind makes many) to make space intelligible is the result of an activity by "our communicating community"; it is "the constituted," the result of a non-arbitrary process, which grounds our constitutive hypotheses within certain regularities of the world, regularities, "donations" which impose themselves upon us; these regularities are themselves "already there" (the connectivity of space, isotropy, symmetries – inspiring ourselves by Riemann and Weyl). But it is us who choose to see them.

I have a Jupiterian friend who has five legs, three eyes and a half and no, absolutely no symmetry to his body. He sees not or does not give any importance to the symmetries of light reflected by a surface, or to crystals, e.g., these symmetries which are before our eyes, before his eyes; and his mathematical structures are not imbued with symmetries like ours (from Greek geometry to the dualities and adjunctions so well described in the Theory of Categories). They are rather constructed around "zurabs," an essential regularity from his perspective, but which we do not see or which we neglect. It goes likewise for colours; he sees a bandwidth beyond violet, where one can find, as a matter of fact, splendid colours. He therefore cannot appreciate this marvellous human construction, rich in history, that we call "painting": Titian's colours are invisible for him. Just like we do not see his masterpieces, of such beautiful ultraviolet colours.

The two constructions are not arbitrary, light waves (or the reality we categorise as such) "are there," just as are the symmetries of crystals or of light bounces, but our active presence interacts with these elements of reality in order to choose, emphasise, correlate some of them, but not others, to gives names, not arbitrary names because they are rich in history and in meaning, to certain colour bandwidths and not to others. Moreover, our action interpolates the missing elements, proposes links by analogy, analogies derived from other experiences; it integrates a variety of acts of experience in order to create a new structure, an inexisting network between "the things" of the world. To figure out, among the regularities of the world and among the foundational acts of any form of knowledge, which ones are at the origin of mathematics is one of the tasks of the analysis of the cognitive foundations of mathematics. Husserlian phenomenal analysis may be one tool, if we do not limit ourselves to a fuzzy notion of "transcendence," but if we recover the richness of "transcendental constitution." Unfortunately, most anti-formalist mathematicians, and even the greatest of Mathematical Logic, such as Frege and Gödel, insist upon the "transcendent" ("the properties and the objects of mathematics pre-exist, just as do the properties and the objects of physics"). In fact, Gödel, while knowing Husserl, does not refer to the "genesis," to the "history" (in the sense of [Husserl, 1933]) of this constitution at the centre of our conceptual constructions. He thus remains, in mathematics and in physics, at a stage of a realism, which neither specifies the notion of property nor that of object: it is only the objects and the properties derived "from sensations"; properties of a physics of "medium size objects" (this table, a pencil . . .), physics which no longer exist, decades after work and debate in relativity, in physics of critical systems and in quantum physics. The failure of this "realist" epistemology of mathematics is parallel to the absence of an epistemology of physics.

It should be clear though that we have been mainly discussing of Gödel's "realist" position, not only as a tribute to the mathematician (of whom the work on types, in 1958, as well as that on recursion and incompleteness, in 1931, made its mark on twentieth century mathematical logic, as well as on the work of this author), but also because his philosophy is by far the most profound among philosophies of mathematical "realism/Platonism." Alain Badiou, [Badiou, 1990] emphasises the richness of this Platonism, alone, in mathematics, resembling that of . . . Plato: *thought envelopes* the object, while the idea *is* "already there," but as the name of that which is thought and which would remain unthinkable if not activated within thought. . .. Moreover, for Gödel, as we reminded, "the objective existence of the objects of mathematical intuition . . . is an exact replica of the question of the objective existence of the outside world" [Gödel, 1947]. This approach, all the while bringing the question of a mathematical ontology closer to that of an ontology of physics, is far more promising than the realism common in mathematics, a funny mix of vulgar empiricism and of idealism, with the worst shortcomings of each of these two philosophies. However, the difference, relative to the approach sketched here, is given by the understanding of the object as constituted; it is not the existence of physical objects or of mathematical concepts that is at stake, but their constitution, as *their objectivity is entirely in their constitutive path*. It is thus necessary to take Gödel's philosophy, for what it puts into mathematical and physical relation, and to turn its head over heels, to bring it back to earth: one must not start "from above," from objects, as being already constituted (existing), but from the constitutive process of these objects and concepts. This requires a non-naive analysis of the object and of physical objectivity, as well as a non-passive theory of perception.

Conceptual Constructions: History vs. Games

To summarise, the objects of mathematics are "outside of ourselves" (transcendent) only as much as they belong to a "constituted" which precedes our subject: they are a co-constituted, at the same time as the very intelligibility of the world, by our "living and communicating community." They are not arbitrary because they are rooted in the regularities of reality, to which are confronted our living beings in the world. They are (relative) invariants, first, of time and space, that we then develop by constructing a whole universe derived from conceptual structures, with the most stable tools of our understanding, these invariants of language and of intersubjectivity that we call "logic" and "formalisms": these as well are *the result of a praxis*, the practice of human reasoning, beginning with the Greek Agora, in human interaction. In this sense of a previous phylogenetic and historic constitution of their construction principles, and not any another, the objects of mathematics may have properties of which "we do not know," as not yet engendered properties within a more or less precisely given conceptual universe. Take the integers, for example. Once presented, by 0 and the successor operation, as the mental construct of an infinite sequence, discrete and well ordered (you can picture it, aligned from left to right in a mental space, right?), we can surely give ourselves a language (that of Peano–Dedekind, for example) and enounce an infinity of properties for the elements of this sequence which "we do not know." We will then need to exercise some "friction" between these properties, in that language, and the given construction; and to verify by the most varied methods or tools (arithmetic induction, but also complex variable functions, for example) if they are "realized" upon this well ordered, infinite structure. In other words, we need to "compare" construction principles and proof principles. It is thus like this that we may understand the essential incompleteness of the formal theory of numbers, see [Longo, 2002; 2005]. It should then be clear that this absolutely does not imply that this infinite sequence "pre-exists" as a conceptorless concept: if five stones were surely already there, at the foot of this mountain, one billion years ago, what was not there was the concept of the number of 5, something completely different, nor were the infinitary properties of that number, ordered within the infinite sequence with the others, as for example the solvability of fifth degree equations or the results of many other linguistic/algebraic constructions we know how to make; constructions that are far from being arbitrary, because rooted in a creative mix of significant conceptual methods (logico-formal, results of spatial invariants, regularities, etc.).

Also consider a variant of chess I am inventing right now: a 100×100 square, with 400 pieces that have quite varied but not arbitrary, very symmetrical finite movements, simulations of natural movements, say . . . I then scatter the pieces randomly; what must be demonstrated is that the configuration thus obtained is compatible with (attainable by) the given rules. Can we say that configuration (a property of the game) was already there, a billion years ago? What is the meaning of that sentence? Worse, I propose a game with an infinity of squares, ordered with great originality in the three dimensions, but effective (spirals, fractals. . .), I call them "spiralu numbers" or "zamburus," and give you infinitary relationships upon these conceptual objects (I describe, using words, infinite subsets, relationships upon this structure or I scatter the pieces randomly). What sense does it make to say that these properties/relationships were already there? That the compatibility of the distributions of the pieces thus obtained were already decided or were valid since ever? Surely, proof will be necessary in order to "verify" it (I prefer: to check if these distributions are "realized" upon the structure, that is to establish friction, by means of proof, between given properties in the language or the geometry of the squares and the game's construction principles). But as long as the infinitary structure, my construction, built in history, a non-arbitrary extension of a practice of squares and of order, is not posed with the rigour of its constructions principles, as the locus where to realize, by the friction of proof, this other construction given in the language of the properties to verify, what sense does it make to say that the conceptual structure and the properties of its infinite subsets "pre-existed"? Conversely to the games which I just proposed, which are my own individual construction, the grounding in the world, within a very ancient intersubjectivity, of the concept of the number, of zero, of the successor, of the infinite well order . . . gives them a "transcendent" status with respect to my individual existence. Yet, this must not lead to forget that also these mathematical "objects" are concepts, the results of a very structured, phylogenetic, and historical conceptual construction, determined by its constitutive hypotheses; they are not a "pre-existing ontology," they do not transcend our human, actually animal existence (as counting is a pre-human activity). The fact that we ignore the totality (?) of their "properties" (careful with this word) in no way demonstrates this ontology we so easily confer them: we ignore them, just as we ignore the totality of the scatterings of our whimsical chess games on the infinite chessboard above. There is no transcendence in mathematics, or, rather, there is no transcendence which is not the result of non arbitrary constitutive processes (e.g., the construction of algebrico-formal enunciations or of the well-ordering of integers), constructions needing to be compared (relatively realized) with one another, by means of this "friction" between and upon conceptual structures, which is called mathematical proof. More specifically, between principles of proof (that we give ourselves, by non-arbitrary choices) and construction principles (that participate in our own cognitive determination, in the relationship to the world).

Continue, for example, and start with the construction of the integers to pass on to the rationals, as ratios of integers, modulo an equivalence of ratios; then consider the convergent sequences (of Cauchy) of these new numbers, modulo equiconvergence. There are the real numbers, constituted using a mathematical method which reconstructs and links together, in its own way, different histories, by distilling the key concepts. The real numbers do not exist, in any sense of a plausible ontology, but their constitution is as objective as are many other conceptual organisation of the world which render it intelligible to us. And they propose us a very efficient conceptual structure for the phenomenal continuum of time and space.

4. Laws, Structures, and Foundations

In his inquiry, F. Bailly, from the perspective of Physics, poses other important questions, among which I now retain those concerning the terms of "structure" and of "foundation" (see [Bailly, Longo, 2006]). What I deny is that one can identify the notion of mathematical structure with its axiomatic presentation and, then, that the analysis of proof, within these axiomatic frameworks, can be a sufficient foundational analysis. To discuss this last point, we will also speak of "Laws."

Physicists sometimes confuse "formalism" with "mathematisation"; it is customary of their language. The mathematical structuration of the world, of a physical experiment, that they propose is often called "formalisation." That is quite understandable, because in what concerns the "very concrete" about which they are thinking (physical "reality"), the mathematical structure is surely abstract and symbolic. But with a bit of experience with the debate about the foundations of mathematics, where these terms are employed with rigour (and philosophical relentlessness, I would say), one understands that rigorous, abstract and symbolic does not mean formal (see [Longo, 2005] for a more detailed analysis of "rigorous," "abstract," and "symbolic" as differing from "formal"). In fact, a formal system must work without reference to meaning; it is constructed and manipulated thanks only to mechanical rules. These rules are also and surely used during a physico-mathematical calculus, but the formula about which the physicist thinks has nothing to do with that of logical "formalism": the formula is significative from the onset, because the physicist constructed it with permanent reference to its meaning, to his or her physical experience, he or she inserts it into a mathematical context rich with explicative connections. The physicist proposes mathematical structures to make his or her experience intelligible, the physicist does not invent a set of formal rules disconnected from the world, as would do the formalist, whose foundational analysis lies only in consistency. He or she thus proposes mathematical structures, and not formal systems. Between the two there are at least the great theorems of incompleteness, which separate structural construction principles from formal deductions.

Let's try to exemplify this distinction within mathematics themselves. Consider, as "construction principles," translations and rotations of figures constructed by rule and compass; if one fixes the unit of length, one will easily construct a segment of square root of 2 length . And there, a very first challenge for mathematical understanding: the theory of linear equations with integral coefficients, and with its formal rules of calculus, is demonstrably incomplete with regards to this construction (the segment is not a ratio between integers). With the same principles of construction, including the absence of gaps and jumps within the Euclidean continuum, construct the limit of the poligons inscribed in and circumscribed around a circle. It will then be the formal theory of rational coefficient algebraic equations, which is incomplete with regards to this construction of π. If we move on to the twentieth century, it is demonstrated that the formal theory of numbers, with its proof principles, is incomplete with regards to the well order of integers as construction principle. By analogy to the role of symmetries in physics, one could say in that regard that Hilbert's conjecture of the completeness of formal Artihmetics was a mirror-symmetry hypothesis between formal language and ontologising semantics (the first accurately reflects the second). Gödel's theorem of incompleteness breaks this alleged symmetry and initiates modern logic. In more constructive and recent terms, the breaking of the symmetry between proof principles and construction principles, of an essentially geometric nature, leads us to understand the insufficiency of a sole logico-formal language as the foundation of mathematics and brings back to the centre of our forms of knowledge a constitutive mathematics of time and space, thanks to its construction principles. There is concrete incompleteness, a modern version of gödelian incompleteness, a discrepancy or breaking in provable symmetry between construction principles and proof principles (see [Longo, 2002] for a detailed analysis of proofs of some recent theorems of incompleteness).

Mathematical structures are, in fact, the result of a reconstruction which organises reality, all the while stemming from concepts, such as the pre-mathematical concept of the infinite (the theological concept, for example), or, even, from pre-conceptual practices (the invariants of memory, the experience of order, of comparison, the structurations of the visual and perceptual in general Gestalts. . ., see [Longo, 2005]) which lead to a structuration, explicited in language, of these (pre)concepts and of their relationships: the well order of the integers, the Cantorian infinite, the continuum of the real numbers, . . . the notion of Riemanian manifold. The concept of infinity gets involved, because it is the result of a profound and ancient conceptual practice, as solid as many other mathematical constructions; these practices are not arbitrary and each may be understood and justified by the process of the construction of scientific objectivity to which it is related.

After the construction of these abstract structures, that are symbolic yet rich in meaning, because they refer to the underlying practical and conceptual acts of experience, we may continue and establish axiomatic frameworks that attempt to grasp at a formal level, whose manipulation may disregard meaning. This process is important, because it adds a possible level of generality and especially highlights certain "proof principles" which enable us to work, upon these structures, by using purely logico-formal deductions, within well specified languages. But these principles are essentially incomplete, that is what the great results of incompleteness of the last 70 years, in particular the recent "concrete" ones, tell us (see [Longo, 2002]). Moreover, as we said in the introduction, the analysis of proof, particularly if this analysis is only formal, is but the last part of an epistemology of mathematics: it is also necessary to account for the constitution of the concepts and of the structures which are "manipulated" during these proofs. But there is more to this usual and fallacious identification of "axioms" with "structures," of "foundations" with "logico-formal rules." In order to understand this, let's return to physics. Husserl, in an extraordinary epistolary exchange with Weyl (see [Tonietti, 1988]), grasps a central point of relativistic physics, highlighted, particularly, by the mathematical work of Weyl (but also by the reflections of Becker, a philosopher of physics and student of Husserl, see [Mancosu, Ryckman, 2002]). The passing from classical physics to the new relativistic framework first bases itself upon the following change in perspective: we go from *causal lawfulness* to the structural organisation of time and space (*structural lawfulness*), nay, from causal lawfulness to intelligibility as "normativity" by mathematical (geometric) structures. In fact, Riemann is at the base of this revolutionary transformation (all the while developing the ideas of Gauss). In his habilitation memoir, [Riemann, 1854], a pillar of modern mathematics and of their applications to physics, he aims to unify the different physical fields (gravitation and electromagnetism) through the geometrical structure of space. He throws out the hypothesis that the local structure of space (its metric, its curvature) may be "linked to the cohesive forces between bodies." "Divination" Weyl will call it in 1921, for it is effectively the viewpoint peculiar to this geometrisation of physics which at least begins with Riemann, finds its physical meaning with Einstein, and with Weyl, its modern mathematical analysis.

It thus seems to me that the attempt to mathematise the foundational analysis of mathematics by only referring to the "laws of thought" is comparable to a reconstruction of the unique, absolute classical universe in physics, with its Newtonian laws. It is not a priori laws that regulate mathematics, but they do constitute themselves as structures, conceptual plays, that are not arbitrary. The "cohesive forces," in mathematics, would correspond to an "interactive dynamic of meaning," a structuration of concepts and of deduction itself.

In category theory, e.g., we propose a new conceptual structure, by novel objects (invariants) and morphisms (transformations); we link it to other structures by using functors, that we analyse in terms of transformations ("natural," their technical name), all the while following/reconstructing the open dynamic of mathematics, of which the unity manifests itself through these reciprocal translations of theories (interpretation functors). And the relative (functorial) interpretations relate the ongoing conceptual constructions (categories): unity is an ongoing conquest and not given by a pre-existing set-theoretic background universe. Moreover, certain of these categories have strong properties of closure, a bit like rational numbers that are closed for multiplication and division, as real numbers are for particular limits . . . One of the logically interesting properties, among many others, is "small completeness," i.e., the closure with regards to products which interpret the universal quantification, among which, in particular, second-order quantification (quantification upon collection of collections). Through this device, some categories confer mathematical meaning to the challenges of impredicativity [Asperti, Longo, 1991], the great bogeyman of "stratified" worldviews and of logic (formal certitudes constructed upon elementary and simple building blocks, one level independent of the other. . .). The world however seems to build itself upon essential circularities, from the merest dynamical system (three bodies interacting in a gravitational field) or the local/global interaction (non-locality) in quantum physics, up to the "impredicative" unity of any living organism, of which the parts have no meaning and are out of place outside of the organism as a whole (see [Longo, 2000], [Bailly, Longo, 2003]). Maybe the emergence of that which is new, in physics, in biology, only takes place under the presence of strong circularities, sorts of internal interactions with complex systems.

Mathematics are thus not a logico-formal deduction, nicely stratified from these axioms of set theory that are as absolute as Newton's universe, but are structurations of the world, abstract and symbolic, doubtless, yet not formal, because significant; their meaning is constructed in a permanent resonance to the very world they help us understand. They then propose collections of "objects" as conceptual invariants, of which the important thing is the individuation of the transformations which preserve them, exactly like (iso-)morphisms and functors preserve categorical structures (properties of objects of a category).

There are no absolutes given by logical rules, beyond the world and the cognitive subject, by definite rules (but then why not those of scholastics or of Euclid's key rule: "a part always has less elements than does the whole," which is false in the case of our infinite sets?), but there is a dynamic of structures (of categories), emergent from a mathematical practice, then linked by those interpretation functors which unify them, which explain the ones by the others, which confer them meaning within a "reflexive equilibrium" of theories (and of categories, particularly those which correspond to deductive systems [Lambek, Scott, 1986], [Asperti, Longo, 1991]).

Surely there is a temporality in the construction of the meaning we confer to the world through mathematics; and it is a "rich" temporality, because it is not that of sequential deduction, of Turing Machines: it is closer to the evolution of space distributed dynamical type systems (see [Bailly, Longo, 2004]). We must let go of this myth of pre-existing "laws of thought" and immerse mathematics into the world while appreciating its constitutive dynamics of which the analysis is an integral part of the foundational project. The laws or "rules" of mathematical deduction, which are surely at the centre of proof, are themselves also the constituted of a praxis, of language, as invariants of the reasoning and of the practice of proof itself.

The foundation, so, as the constitutive process of a piece of knowledge, constructed responsively to the world, the physical world and that of our sensations. But . . . where does this process begin? It is surely not a case of reascending "to the mere stuff of perception, as many positivists assert," since physical objects are "intentional objects of acts of consciousness" [Weyl, 1918b]. There is a very Husserlian remark, a constitution of objects which we have called a conceptual "découpage" (cutting-off). And this découpage is performed (and produced) by the mathematical concept, conscious mental act towards the world. Then, reasoning, sometimes rooted in a whole different practice, in the language of social interaction, that of the rules of logical coherence or of the aesthetic of symmetries, e.g., generates new mathematical concepts, which may themselves, but not necessarily, propose new physical objects (positrons, e.g., derived from electrons by a pure symmetry between equations in microphysics).

The autonomy of mathematics, thanks to the generativity of reasoning, even of the formal type (calculus for example), is indubitable, there lies their predictive force in physics. The integration of these different conceptual dimensions, of these different praxis (geometrical structuration of the world, logical and formal deduction, even far removed from any physical meaning), also confers mathematics their explicative and normative character with regards to reality: one goes from a physical invariant, from space, let's say, to another by purely logico/formal means (an algebraic transformation applied to this invariant and which preserves it, a symmetry. . .) and a new physical object is thus proposed. The physical proof will be a new experience to invent, with instruments to be invented.

Obviously, in this grounding of our sciences into the world, perception also plays an essential role, but we must then develop a solid theory of perception, rooted in a cognitive science that allows to go far beyond the positivist's "passive perception," of which Weyl speaks about. We shall return to this point.

The approach we propose, of course, causes the loss of the absolute certitude of logico-formal, decidable proof. But we know, since Gödel, that any formal theory, be it slightly ambitious and of which the notion of proof is decidable, is essentially incomplete. So logicism's and formalism's "unshakeable certainties" (the absolute certification of proof) are lost . . . since a long time. There remains the risk of the construction of scientific objectivity, thoroughly human, even in mathematics, the adventure of thought which constitutes its own structures of the intelligibility of the world, by the interaction with the former and with the thoughts of others. The risk, for example, of acknowledging the foundational role of the well ordering of integers, by a geometric judgement constituted in history, action, language, and intersubjectivity, in order to certify the coherence of Artithmetics [Longo, 2002; 2005].

5. Subject and Objectivity

In various works, Weyl develops a very interesting philosophical analysis concerning the passage in physics from the subjective to the objective, on the basis of references to his own mathematical works in relativity theory. This analysis is emphasised by [Mancosu, Ryckman, 2002], who refer mostly to [Weyl, 1918b; 1927]. The importance of Weyl's remarks obviously extends way beyond the philosophical stakes in physics and in mathematics, because it touches upon a central aspect of any philosophy of knowledge, the tension between the "cult of the absolute" and "relativism." Husserl seeks to move beyond this split in all of his work and in his reading of the history of philosophy (see, e.g., [Husserl, 1956]). Twentieth Century physics can provide tools for contributing to that debate, and those are Weyl's motivations.

For Weyl, immediate experience is "subjective and absolute," or, better, it claims to be absolute; the objective world, conversely, that the natural sciences "crystallise out of our practical lives . . . this objective world is necessary relative." So, it is the immediate subjective experience which proposes absolutes, while the scientific effort towards objectivity is relativising, because "it is only presentable in a determined manner (through numbers or other symbols) after a coordinate system is arbitrarily introduced in the world. This oppositional pair: subjective-absolute and objective-relative seems to me to contain one of the most fundamental epistemological insights that can be extracted form natural sciences."

Following his works in relativity, Weyl thus gives a central role to reference frames. The subject lays, chooses, a reference frame and in this manner organises time and space. That choice is the very first step performed by the knowing subject. But the operation of measurement, by means of its own definition, also implies the subject: any physical size is relative to (and set by) a "cognising ego." The passage to objectivity is given, in quantum physics, by the analysis of "gauge invariants," e.g., one of Weyl's great mathematical contributions to this field: they are given as invariants in relation to the passing from one detection and measurement system to another. More generally, the passage from subjectivity to scientific objectivity implies the explicit and explicitated choice of a reference frame, including for mathematical measurements and invariants.

Weyl thus emphasises, in Husserlian fashion, that any object in the physical world is the result of an intentional act, of the awareness "of a pure, sense giving ego." For both thinkers, it is a matter of the Cartesian "Ego," to which Husserl so often returns to, which "is, since it thinks"; and it is, because, as a consciousness, it has "objects of consciousness" (consciousness is "intentional," it has an "aim"). It is the subject, this conscious

Cartesian "Ego," that chooses the reference frame and who, afterwards, is set aside. It poses the origin, the 0, and the measurement, and it mathematically structures time and space (as Cantor–Dedekind continuum, e.g., or as a Riemannian manifold with its curvature tensors); by that act (the construction of a space as mathematical manifold), it poses a framework of objectivity, independently of the subject, objectivity nevertheless consciously relativised to that choice. Because the choice of "viewpoint," of the frame, is relativising and breaks the absolute characteristic of the subject before the passage to scientific objectivity; this passing of subjectivity, which claims to be absolute, to relativising objectivity, is the meaning of the scientific approach central to relativity. Just as it is very well put in [Mancosu, Ryckman, 2002], "The significance of [Weyl's] 'problem of relativity' is that objectivity in physics, i.e., the purely symbolic world of the tensor field of relativistic physics, is constituted or constructed via subjectivity, neither postulated nor inferred as mind-independent or transcendent to consciousness." But this symbolic world of mathematics is in turn itself the result of an interaction of the knowing subject(s), within intersubjectivity, with the regularities of the world, these regularities which we see and which are the object of intentional acts, of a view directed with fullness and willing," as Husserl and Weyl say.

The subject is thus at the origin of scientific knowledge, and it is with the subject that any mathematical construction begins. However, it will be necessary to push the analysis of the subject's role further: today we can pose the problem of objectivity at the very centre of the knowing subject, because this subject is not the psychological subject, which is also disputed by the seekers of the absolute, of transcendental truths, of configurations or properties which are already there, true prior to any construction/specification, even in my infinite chessboard or in the sequence of integers. In fact, it is a question of the "cognitive subject," of this «Ego» that we share as living, biological creatures, living in a common history that is co-constituted with the world, at the same time as its activity in the world. There is the next issue we will have to deal with, in the dialog with cognitive sciences, basing ourselves on non-naive (and non passive) theories of perception, on theories of the objective co-constitution of the subject. The scientific analysis of the subject must, by these means, underline what is common to subjective, psychological variability: more than a simple "intersection of subjectivities" it is a question of grasping in that way what lies behind individual variabilities, what directs them and allow them to communicate and to understand/construct the world together.

Foundational analysis, in mathematics and in physics, must therefore propose a scientific analysis of the cognitive subject and, then, highlight the objectivity of the construction of knowledge within its referenctial systems or reference frames.

In what concerns the foundations of mathematics, a process analogous to this "choice of reference frame" is well explicitated, in Category Theory, by choosing the right "topos" (as referential category for a logic or with an "internal logic" [Johnstone, 1977]), to relate, through interpretation functors, other categorical constructions, in a dynamic of these structures by which we give mathematical meaning to the world (algebraic, geometrical, manifolds' categories. . .). This has nothing to do, as we have already emphasised, with the absoluteness of the axioms of set theory, a Newtonian universe that has dominated mathematical logic and that has contributed for a century to the separation of mathematical foundations from epistemology and from the philosophy of natural sciences. That was a matter, indeed, of an absolute, that of sets, intuition of which is compared, by the "realists" of mathematical philosophy, to the perception of physical objects (quite naively described in its passivity), sets and objects also being transcendent, with their properties all "pre-existing, since unknown" [Gödel, supplement to 1947]. A typical example of that which Husserl, de Ideen, and Weyl (taken up by Becker, see [Mancosu, Ryckman, 2002]) call the "dogmatism" of those who speak of absolute reality, infinite list of already constituted properties, constituted before any pre-conscious and conscious access, before the shared practices in our communicating community.

6. From Intuitionism to a Renewed Constructivism

Quite fortunately, within the same mathematical logic, we begin to hear different voices: "Realism: no doubt that there is reality, whatever this means. But realism is more than the recognition of reality, it is a simple-minded explanation of the world, seen as made out of solid bricks. Realists believe in determinism, absoluteness of time, refuse quantum mechanics: a realist cannot imagine 'the secret darkness of milk.' In logic, realists think that syntax refers to some pre-existing semantics. Indeed, there is only one thing which definitely cannot be real: reality itself" [Girard, 2001]. The influence of Brouwer, leader of intuitionism, and of Kreisel, as well as the mathematical experience with intuitionnist systems, is surely present in the mathematical work and in the rare philosophical reflections of Girard, but without the slip, characteristic of Brouwer, into a senseless solipsism, nor with the a priori limitations of our proof tools. Moreover, time and space are included in Girard's proof analysis: the connectivity, the symmetries of proof as network, time as irreversible change in polarity in [Girard, 1987; 2001], have nothing to do with "time as secreted by clocks" (his expression), the time of sequential proof, of Turing machines, which is beyond the world (see [Longo, 2002b], [Bailly, Longo, 2004]).

Brouwer's intuitionism, among the different trends in the philosophy of mathematics (formalist, platonician realist, intuitionnist), is possibly the only foundational analysis that has attempted to propose an epistemology of mathematics (and a role for the knowing subject). The discreet sequence of numbers, as trace of the passing of time in memory ([Brouwer, 1948], see also [Longo, 2003]), is posed as constitutive element of mathematics. It is exactly this vision of mathematics as conceptual construction that has made Weyl appreciate Brouwer's approach for a long time. In fact, the analyses of the mathematical continuum for Brouwer and Weyl (as well as for Husserl, see [Weyl, 1918], [Tonietti, 1988], [Longo, 1989 and 1999]) are quite similar in many respects. However, Weyl had to distance himself from Brouwer, during the 20s, when he realised that the latter excessively limits the tools of proof in mathematics and does not know how to go beyond the "psychological subject," to the point of renouncing the constitutive role of language and of intersubjectivity and to propose a "languageless mathematics" (a central theme of Brouwer's solipsism, see [Brouwer, 1948], [van Dalen, 1991]).

Conversely, and as we have tried to see, the relativity problem for Weyl, as passage from "causal lawfulness" to "structural lawfulness" in physics, as well as play between subjectivity-absolute and objectivity-relative, is at the centre of an approach that poses the problem of knowledge in its unity, particularly as relationship between physical objectivity and the mathematical structures that make time and space intelligible, thanks, among other things, to language. All the while following Weyl, we have made a first step towards an extension of foundational analysis in mathematics by a cognitive analysis of what should precede purely logical analyses: only the last segment is without doubt constituted by the logico-formal analysis of proof. But upstream there remains the problem of the constitution of structures and of concepts, a problem which is strictly related to the structuration of the physical world and to its objectivity. The project of a cognitive analysis of the foundations of mathematics thus requires an explicitation of the cognitive subject. As a living brain/body unit, dwelling in intersubjectivity and in history, this subject outlines the objects and the structures, the spaces and the concepts common to mathematics and to physics, on the phenomenal veil. In short, parallel constitutive history, in physics, begins with perception as action: we construct an object by an active viewing, by the presence of all of our body and of our brain, as integrator of the plurality of sensations (Merleau-Ponty's "vision as palpation by sight," perception as the result of a comparison between sensorial input and a hypothesis performed by the brain, [Berthoz, 1997]). In fact, any invariant is an invariant in relation to one or more transformations, so in relation to action. And we isolate, we "single out," invariants from the praxis that language, the exchange with others, forces us to transform into concepts, independently, as communicables, from the constitutive subject, from invariants constituted with others, with those who differ from us but who share the same world with us, and the same type of body. From the act of counting, the appreciation of the dimensionless trajectory – dimensionless since it is a pure direction – we arrive at the mathematical concepts of number, of unidimensional line and, then, of point, [Longo, 2005]. Invariants quite analogous to the physical concepts of energy, force, gravitation, electron . . .. The latter are the result of a similar process, they are conceptual invariants which result from a very rich and "objective" praxis, that of physics, inconceivable without a close interaction with mathematics. They organise the cues that we select through perception and through action upon the world, through our measurement instruments; the geometrical structuration of those invariants is the key organising tool, because it explicitates in time and space our action and our comprehension.

Individual and collective memory is an essential component to this process constitutive of the conceptual invariants (spatial, logical, temporal. . .). The capacity to forget in particular, which is central to human (and animal) memory, helps us erase the "useless" details; useless with regards to intentionality, to a conscious or unconscious aim. The capacity to forget thus contributes in that way to the constitution of that which is stable, of that which matters to our goals, which we share: in short, to the determination of these invariant structures and concepts, which are invariant because filtered of all which may be outside our intentional acts of knowledge, [Longo, 1999]. Their intercultural universality is the result of a shared or "sharable" praxis, in the sense that these invariants, these concepts, may very well be proposed in one specific culture (think about Greek geometry or Arabic algebra), but their rooting in fundamental human cognitive processes (our relationship to measurement and to the space of the senses, basic counting, and ordering. . .) make them accessible to other cultures. This widening of a historic basis of usage is not neutral, it may require the blotting out of other experiences specific to the culture which assimilates them, but confers them this universality that accompanies and which results from the maximal stability and conceptual invariance specific to mathematics. But this universal is posed with relation to human experience and does not mean absolute; it is itself a cultural invariant, between cultures that take shape through interaction. Because universality is the result of these communicating communities and because historical demise is also a factor; oblivion or expulsion from mathematics of magical numbers, of zombalo structures . . . of that which does not have the generality of method and results we call, a posteriori, mathematical. As for the mathematical organisation of space, both physical and sensible, it begins very early, probably as soon as space is described by gesticulation and words, or with the spatial perspective and width of the pictorial images of Lascaux, 20,000 years ago, or from the onstart of the play of Euclid's rigid bodies which structures geometrical space. Euclid's axiomatics indeed summarise the minimal actions, indispensable to geometry, with their rule and compass, their construction and measurement instruments: "trace a straight line from one point to another," "extend a finite line to a continuous line," "construct a circle from a point and a distance" . . . (note that all these constructions are based and/or preserve symmetries). His first theorem is the "vision of a construction" (in Greek, theorem means "sight," it is like a "theatre"): he instructs how to "construct an equilateral triangle from a segment," by symmetric tracing with a compass.

This history leads to Weyl's symmetries, regularities of the world which impose themselves (donations that, in this sense, pre-exist or that reality imposes us), but that we see or decide to see. We then transform them into concepts and choose to pose them as organising criteria of reality, even in microphysics, far removed from sensorial space. But now we turn to [Bailly, Longo, 2006] for more work in this direction.

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CHAPTER 8

UNDERSTANDING QUANTUM MECHANICS WITH BOHR AND HUSSERL

François Lurçat

There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.

[Niels Bohr[261]]

Abstract. As was shown by Husserl in *The Crisis of European Sciences*, modern physics was created by Galileo on a wrong basis. Namely, the following metaphysical axiom: the universe is a book written in mathematical characters. During three centuries or so, this mistake had apparently no serious consequences. It prevented, however, a real understanding of the nature of physics. A misunderstanding made even more important and more visible by the advent of quantum mechanics, which is often considered to be nonlocal or even unintelligible.

Two convergent lines of thought might help us to overcome this difficulty.

On the one hand, Bohr has worked out a coherent interpretation of quantum mechanics. He insisted that the concepts of classical physics have a limited validity, but are and will always be necessary to describe experimental results. This makes it impossible to maintain the dogma that the world is mathematical.

On the other hand, Husserl developed a new conception of physics, radically different from the ideas generally prevailing among physicists. For him, when we believe that the world is mathematical in itself, we take for the true being what is actually a method. In fact the physicist does not deal with a "true" physical thing, of which the experimental facts (the click of a counter, or a trace in a chamber) would be unessential appearances. He deals with what is actually perceived, i.e., the click or the trace.

Thus, the main obstacle to a real understanding of quantum mechanics is the persistence of Galileo's metaphysical axiom, to which - although it has been refuted long ago by the development of atomic physics - many physicists and mathematicians are passionately attached.

[261] Declarations of Bohr reported by A. Petersen, *The philosophy of Niels Bohr*, Bulletin of the Atomic Scientists, Vol. 19, pp. 8–14 (1963). See also M. Jammer, *The Philosophy of Quantum Mechanics*, New York, John Wiley & Sons, 1974, p. 204.

L. Boi et al. (eds.), Rediscovering Phenomenology, 229–258.

Quantum mechanics brilliantly succeeds as a mathematical formalism: the numbers it provides are always successfully compared with experimental results. But it is often said to fail as an explanatory theory allowing us to understand the laws of atomic processes.[262] Richard Feynman (1918–1988), author of essential contributions to both the theory and its applications, once declared: "I think I can safely say that nobody understands quantum mechanics."[263] According to Roger Penrose, it "makes absolutely no sense."[264] And René Thom described it as "the intellectual scandal of the century."[265]

There exists, however, an interpretation of quantum mechanics that makes it understandable. It was worked out by Niels Bohr, author of the quantum theory of atoms, in the course of more than thirty years of researches and discussions. During the 1930s and 1940s, there was a sort of loose consensus in its favour. But most physicists did not even try to follow the line of Bohr's subtle and deep arguments. Einstein's seminal discoveries were far behind, and he was presently busy with topics outside the mainstream. Bohr, on the contrary, was making essential contributions to nuclear physics. So he was necessarily right about everything, Einstein was a troublemaker, and the debate between both physicists was not really relevant. In the last decades, however, the situation has changed. Some theoreticians have proposed that we replace the "thought experiments" discussed by Einstein and Bohr with feasible ones. Thanks to advances made in measurement techniques and to the ingenuity of many experimentalists, crucial experiments have been done, and their results have supported Bohr's conceptions.[266] This might perhaps have made these conceptions

[262] I use the convenient term "atomic" to refer to such physical entities as atoms, molecules, nuclei, and particles.

[263] R.P. Feynman, *The Character of Physical Law*, Cambridge, Mass., MIT Press, 1965, p. 129.

[264] R. Penrose, in: R. Penrose, C.J. Isham (eds.), *Quantum Concepts in Space and Time*, Oxford, Clarendon Press, 1986, p. 139.

[265] R. Thom, *Prédire n'est pas Expliquer*, Paris, Flammarion, 1993, p. 86.

[266] Conclusive experiments about the violation of Bell's inequalities have been made by Alain Aspect and his collaborators in Orsay: A. Aspect, P. Grangier, G. Roger, *Physical Review Letters*, Vol. 47, p. 460 (1981); Vol. 49, p. 1804 (1982). More recently, I have been interested in the experiments of Nicolas Gisin and his group in Geneva: W. Tittel, J. Brendel, H. Zbinden, N. Gisin, *Physical Review Letters,* Vol. 81, p. 3563 (1998); A. Stefanov, H. Zbinder, N. Gisin, A. Suarez, *Physical Review Letters*, Vol. 88, p. 120404 (2002). Additionally, I have considered those of Anton Zeilinger and his group in Vienna:

more acceptable; it turned out, however, that they became less and less accepted. The whiff of heresy no longer lingers about Einstein's thought, but now it surrounds Bohr's. When physicists, philosophers, and historians of science write about such questions, as a rule they refer only vaguely to the Danish physicist's ideas without ever going explicitly and clearly to his texts.[267]

I would like to present here an approach that in my opinion should help to explain why Bohr's essential contribution has been rejected for its strangeness even after its successful confrontation with experiment. I rely on Husserl's definition of phenomenology in his article in the *Encyclopaedia Britannica* (1929). Phenomenology, he writes, "has established (1) an a priori psychological discipline, able to provide the only secure basis on which a strong empirical psychology can be built, and (2) a universal philosophy, which can supply an organum for the methodical revision of all the sciences."[268] My assumption is as follows: as far as physics is concerned, the kind of methodical revision Husserl had in mind was never undertaken; it is, however, needed even more urgently now that quantum physics has come to light, in fact, at the same time as phenomenology. The lack of revision, which has permitted the persistence of age-old confusions, can help to explain the paradoxical rejection of the Bohrian approach and the ensuing failure to understand quantum mechanics.

Of course one should not forget that there are many publications devoted to the interpretation of quantum mechanics, some of which are quite valuable. My aim here is not to discuss what has been done recently in this field of research, but to go back to the origins in order to understand why, despite so many efforts, confusion remains about several important aspects of the problem, for instance, about such notions

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G. Weihs, T. Gennevein, C. Simon, H. Weinfurter, A. Zeilinger, *Physical Review Letters*, Vol. 81, p. 5039 (1998).

[267] Such is the case, for instance, of the philosopher David Z. Albert's book *Quantum Mechanics and Experience*, Cambridge, Mass. and London, Harvard University Press, 1992; of the historian of science Mara Beller's book *Quantum Dialogue*, Chicago and London, The University of Chicago Press, 1999; of the physicist Frank Laloë's article "Do We Really Understand Quantum Mechanics? Strange Correlations, Paradoxes, and Theorems," *American Journal of Physics*, Vol. 69, pp. 655–701 (2001); and of the physicist Bernard d'Espagnat's book *Traité de Physique et de Philosophie*, Paris, Fayard, 2002.

[268] E. Husserl, article *Phenomenology*, *Encyclopaedia Britannica*, Vol. 17 (1947) (first publication: 1929). See also E. Husserl, *Collected Works*, Vol. 8, Kluwer Academic Publishers, Dordrecht/Boston/London, 1999.

as "observer," "Copenhagen interpretation." In my opinion, as long as Bohr is understood to be an incomprehensible author, the beginnings of quantum mechanics will continue to be shrouded in mist, and the same will probably be true for quantum mechanics as a whole.

Bohr's thought is generally considered difficult to grasp, a difficulty commonly ascribed to what he himself called an "inefficiency of expression."[269] One may wonder, however, whether the clarity of classical physics, implicitly contrasted with Bohrian obscurity, is not itself at least partly based on an illusion. Because we are familiar with mathematically formalized space and time, we assume a degree of clarity about it. This familiarity has a double origin. It comes from our technical environment, which embodies in some way Euclidean space and Newtonian time (or, when electromagnetic signals are exchanged, Einsteinian space-time). And it follows from a feature analyzed by Husserl: the substitution of the mathematically substructured world of idealities for the real world.[270] The lack of clarity commonly found in Bohr, and the inefficiency of expression for which he blames himself, are actually effects of thought exploring virgin territories. They can be understood as traces of Bohr's effort to free himself from old mental habits. But the feeling of obscurity experienced by Bohr's readers is also due to the result of that effort: a reasoning free from several – most often implicit – premises of common thought.

"We take," says Husserl, "for *true being* what is actually a *method*."[271] By retaining only those properties of things that can be geometrized, Galileo laid the foundations of modern physics, but at the same time he gave

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[269] Commenting on his answer to Einstein, Podolsky, and Rosen, Bohr writes: "Rereading these passages, I am deeply aware of the inefficiency of expression which must have made it very difficult to appreciate the trend of the argumentation." N. Bohr, "Discussions with Einstein on Epistemological Problems in Atomic Physics," in: P.A. Schilpp (ed.), *Albert Einstein, Philosopher-Scientist*, New York, Tudor Publishing Company, 1949. This text is reprinted in: N. Bohr, *Atomic Physics and Human Knowledge*, New York, John Wiley & Sons, 1958. Also in: J. Kalckar (ed.), *Niels Bohr Collected Works*, Vol. 7, Amsterdam, Elsevier, 1996, p. 234. (In the following the *Collected Works* is abbreviated as BCW.)

[270] E. Husserl, *Die Krisis der Europäischen Wissenschaften und die Transzendentale Phänomenologie*, §9h. Translation by David Carr: *The Crisis of European Sciences and Transcendental Phenomenology,* Evanston, Northwestern University Press, 1970.

[271] E. Husserl, *The Crisis* §9h. Translation quoted, p. 51. All the analysis in the following section is a simplified summary of this paragraph of the *Crisis*.

credibility to the prejudice that declared those properties to be the only real ones: nature is mathematical, and whatever cannot be mathematized is relegated to the swamp of preconceptions and subjective impressions.

What made classical physics understandable was the general acceptance of its metaphysical foundations, laid by Galileo and Descartes.[272] During the twentieth century, the development of quantum physics challenged those foundations – a major event in the history of scientific and philosophical thought. But the prevailing positivist view of sciences did not allow that event to be understood or even noticed. According to positivism, there are no metaphysical foundations of sciences; there is a scientific method, the only one that allows us to reach correct conclusions. One can thus understand the negative reception of the Bohrian conception of quantum mechanics. Bohr's thought has indeed challenged generally accepted ideas, but the status of those ideas is misunderstood. Their metaphysical character is not recognized: they are mistaken for basic and necessary methodological principles.

1. Classical Physics

Why is quantum theory reputed to contain paradoxes? Because the Galilean and Cartesian substitution of mathematical abstractions for the real world has been almost unanimously and uncritically accepted by scientists and philosophers – except, of course, phenomenologists. The strangeness of quantum physics might even follow from affinities with phenomenology that remain implicit. We must therefore begin this study by taking a brief look at the Husserlian critique of the metaphysical foundations of classical physics. According to Husserl,[273] Galileo's essential discovery is that of "nature, which is in itself mathematical." Summarizing the Galilean view, he writes again: "Nature is, in its 'true being-in-itself,' mathematical." Hence follows, for Galileo, the "law of exact lawfulness": every occurrence in nature must come under exact laws. Here we apparently have the basic principles of physical theory, used daily by scientists.

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[272] See E.A. Burtt, *The Metaphysical Foundations of Modern Physical Science* (1924; revised edition, 1932), Doubleday Anchor Books, Doubleday & Company, Garden City, N.Y., 1954.
[273] E. Husserl, *The Crisis*, loc. cit.

Yet the mathematical nature in question is a "methodical idea," so when we accept Galileo's discoveries as "straightforward truth," we repeat his "naïveté," a naïveté never overcome by his successors. Galileo is a genius: his idea of mathematical nature "blazes the trail for the infinite number of physical discoveries and discoverers." But he is "at once a discovering and a concealing genius." With the Galilean mathematization begins "the surreptitious substitution of an idealized nature for prescientifically intuited nature." In the world where we live, "we find nothing of geometrical idealities, no geometrical space or mathematical time with all their shapes." This is an important remark, Husserl observes, even though it is so trivial. "Yet this triviality has been buried precisely by exact science, indeed since the days of ancient geometry, through that substitution of a methodically idealized achievement for what is given immediately as actuality."

Physics, then, appears as "a particular technique, the geometrical and Galilean technique which is called physics." "In geometrical and natural-scientific mathematization, in the open infinity of possible experiences, we measure the life-world – the world constantly given to us as actual in our concrete world-life – for a well-fitting *garb of ideas*, that of the so-called objectively scientific truths." This garb of ideas allows us to make predictions relevant to our practical life, and this kind of prediction infinitely surpasses the accomplishment of everyday prediction. But, on the other hand, it dresses up the life-world as "objectively actual and true" nature. It is because of this substitution that the actual meaning of the method was never understood.

Since the days of Galileo, the substitution of a mathematical schematization of nature for the real world has obscured the nature of classical physics; this continuing confusion now hinders us from understanding quantum mechanics. Believing that the trouble started with the advent of quantum physics would be an illusion. Both the obscurity of quantum physics and the clarity of classical physics are grounded in the same initial mistaking of mathematized nature for the world we live in.

For three quarters of a century, the persistent Galilean substitution has been an insuperable obstacle in the way of the acceptance of Bohr's conceptions, which provide a rational and understandable frame for quantum mechanics. The fundamental criticism of substitution by Husserl has not been taken into consideration, and we still pay the price for this neglect or refusal.

2. Bohr's Dissertation (1911) and Bohr's Atom (1913)

Let us now set some milestones on the way followed by Bohr as he discovered and developed his interpretation of quantum mechanics. This will require placing his work in a historical context, in order to regain with their original strength arguments now covered with various layers of sedimentation.

In 1911, the young physicist defended his dissertation on the electron theory of metals. Its theoretical frame was the statistical mechanics of a gas of electrons. The main result was that magnetic properties of metals cannot be explained in this frame. As Rosenfeld commented in his biographical sketch, "the very rigour of his analysis gave him, at this early stage, the firm conviction of the necessity of a radical departure from classical electrodynamics for the description of atomic phenomena."[274] In 1921, the primary finding of the dissertation was rediscovered by a Dutch physicist, J.H. van Leeuwen; it is currently known as the "Bohr-van Leeuwen theorem."

Two years later, Bohr published his trilogy, *On the Constitution of Atoms and Molecules*. His starting point was Rutherford's hypothesis: an atom is composed of a central nucleus and peripheral electrons. In the first article, devoted to the hydrogen atom, he assumed that the single electron of this atom can follow only a discrete set of trajectories, characterized by a "quantum condition" involving Planck's constant. This means that when the electron is on one of those allowed orbits, it is in a "stationary state" and does not emit electromagnetic radiation. Each of the allowed orbits is characterized by its energy. Radiation takes place only in the transition from one orbit to a second one of smaller energy; the difference between the energies of both orbits is taken away by the quantum of radiation emitted. Such a process is called a transition. From these simple assumptions Bohr deduced the empirical Balmer formula, which describes the main features of the hydrogen spectrum. But, however simple they are, they represent a radical innovation with respect to classical mechanics and electromagnetism.

In classical mechanics, there is a possible trajectory for any initial condition (position and momentum): this precludes any quantization of

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[274] L. Rosenfeld, *Niels Bohr, Biographical Sketch*, in: J. Rud Nielsen (ed.), *Niels Bohr Collected Works*, Vol. 1, Amsterdam, North-Holland Physics Publishing, 1972, p. XIX.

trajectories or of energies. Bohr understood very early that this freedom of classical trajectories was incompatible with the stability of matter. As he later explained it to Heisenberg:

> What I mean by stability is the fact that always the same substances are found, with the same properties; always the same crystals are formed, the same chemical compounds are created, etc. This must mean that, after many modifications due to external influences, an iron atom becomes again an iron atom, with exactly the same properties as it had previously. This cannot be understood according to classical mechanics, above all if one admits that it is like a planetary system. Hence there exists in nature a tendency to produce definite forms – here I use the word "forms" in its most general sense – and to make these definite forms reappear again and again, even when they have been perturbed and destroyed.. . . This looks incomprehensible if one admits the basic principle of Newtonian physics, namely the strict causal determinism of phenomena; in other terms, if the present state of a system must always be determined uniquely by the state that comes immediately before it, and only by that one. This contradiction worried me very early.[275]

According to the trilogy, when a transition takes place the frequency of the radiation emitted depends on the energy of the initial state (before the emission of radiation) and on the energy of the final state (after the emission). If the transition is a process continuously unfolding in time, as are those described by classical physics, how can we understand the idea that the intermediate stage (the emission of radiation) is partly determined by the final stage?[276] To this, Bohr simply answered that while the stationary states of an atom follow the laws of usual mechanics, these laws do not hold for the transition from one state to another.[277] Later he was to develop this idea, specifying that a description of atomic processes in space and time is not always possible. A transition is not a process of which the physicist can write a history, but rather a "quantum jump." Here we have a radical break with the principles of classical physics, which declared a description in space and time to be a universally valid requirement.

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[275] From W. Heisenberg, *Der Teil und das Ganze*, Munich, R. Piper & Co Verlag, 1969, chapter 3. English translation: *Physics and Beyond*, New York, Harper and Row Publ., 1971. I translate from the French translation by Paul Kessler, *La Partie et le Tout*, Paris Albin Michel, 1972.

[276] The remark about the frequency determined not only by the initial state of the atom, but also by its final state can be found in Bohr's Nobel Conference (1922): *The Structure of the Atom*, in BCW, Vol. 4, 1977, pp. 467–482.

[277] N. Bohr, *On the Constitution of Atoms and Molecules*, Part I, §1; in BCW, Vol. 2, 1981, p. 167.

In the relation of physics to experiment we can see another aspect of the break. Newton lays down his axioms of mechanics and draws consequences from them about planetary motions, tides, etc. The refined calculations performed by his successors are almost always borne out by astronomical observations: the only exception, a detail in the motion of Mercury, was explained by Einstein when he laid down new axioms in his theory of general relativity. Similarly, Maxwell's equations prevailed until Bohr called into question their universal validity. This represents a new kind of challenge. The aim is no longer to replace one theory with a new one that better meets the Galilean ideal of identity with the physical universe. Rather, Bohr's atom inaugurated a new conception of the relations between theory and experiment. While classical mechanics and electromagnetism remain essential to describe the stationary states of the atom, some of their implications are rejected: stationary states are quantized and radiationless, despite classical impossibilities. Nature can no longer be identified with the theories of Newton (or Einstein) and Maxwell. They are not wrong, but they have only limited validity.

3. A Digression on Semiclassical Theories

As we shall see further, Bohr's atom – "the old quantum theory" as it is now called – is currently considered to have been a provisional theory, superseded by quantum mechanics. This is only a partial truth, however. Bohr's atom still survives under the rubric of semiclassical theories, an active field of research nowadays.[278] In atomic, molecular, and nuclear physics, for instance, it happens in many cases that the phenomena of interest involve a large number of quantum states: one is thus near the limit where the correspondence principle becomes relevant, as we shall soon see. On the other hand, as Miller puts it, semiclassical theories play an interpretative role: they provide spatiotemporal descriptions, of approximate validity, which nevertheless give tools for understanding that are more efficient than what "exact" quantum mechanical calculations can offer. In the case of the formaldehyde molecule, for instance, or of exchange

[278] W.H. Miller, "Semiclassical Methods in Chemical Physics," *Science*, Vol. 233, pp. 171–177 (1986). T. Uzer, D. Farrelly, J.A. Milligan, P.E. Raines, J.P. Skelton, "Celestial Mechanics on a Microscopic Scale," *Science*, Vol. 253, pp. 42–48 (1991).

collisions between a hydrogen atom and a dihydrogen molecule, we are told by specialists that "rigorous" quantum-mechanical calculations are untractable, while semiclassical description permits both an understanding of the phenomena and acceptable numerical results. We must consider semiclassical theory, then, as the truly relevant theory for such objects or phenomena.

Hence the idea of limited validity has not become obsolete with the advent of quantum mechanics. The epistemological lesson of this idea is as relevant as ever: our concepts are not written in the book of the universe, we ourselves devise them in an effort to understand the laws of natural and, more generally, physical phenomena.

4. Wave-particle Duality and its Consequences

There is no question here of summarizing the main stages of the development of quantum theory, or even of Bohr's contributions to it.[279] I would only like to show, relying on Bohrian texts, how phenomenological thought can give a meaning to quantum physics. In order to do that, some historical data will have to be recalled; let us begin with the debate about the quanta of radiation.

A photocell is now a familiar object (found, for instance, in every elevator). One century ago, the photoelectric effect was something new, unexplained by classical electromagnetism. In 1905, Einstein published an article: "On a Heuristic Point of View about the Creation and Conversion of Light."[280] He showed that the thermodynamic properties of thermal radiation could be described by comparing the radiation to a gas of light quanta (later called "photons"). Applying this assumption to the photoelectric effect, he obtained a simple relation (the "Einstein equation") between the frequency of incident light and the energy of the electrons freed from the metal. Einstein's "heuristic point of view" so obviously contradicted some known properties of electromagnetic radiation (such as interferences and diffraction) that it was received with scepticism. In

[279] The interested reader may refer to: M. Jammer, *The Conceptual Development of Quantum Mechanics*, New York, Mc Graw Hill, 1966; B.L. van der Waerden, *Sources of Quantum Mechanics*, Amsterdam, North-Holland, 1967; or to A. Pais, *Niels Bohr's Times, in Physics, Philosophy, and Polity*, Oxford, Clarendon Press, 1991.

[280] A translation of this article is given in: D. ter Haar, *The Old Quantum Theory*, Oxford, Pergamon Press, 1967.

1916, however, the careful experiments of Millikan confirmed the Einstein equation. Another experiment went further in the same direction. When a beam of X-rays falls on a sample of matter, a fraction of the beam is deflected (what is called "scattering") and its wavelength increases. This phenomenon cannot be explained by classical electromagnetic theory. In 1923, however, Arthur Holly Compton's experiments showed that it could be understood by making use of the hypothesis of light quanta, if they are endowed not only with energy but also with momentum. The scattering of X-rays (now called the Compton effect) is described as a collision between an X-quantum and an electron. From this point on, light quanta seemed to possess all the attributes of particles. The Compton assumption implies a definite relation, well confirmed by experiment, between the increase in wavelength and the angle between scattered and incident radiation.

Thus electromagnetic theory found itself in a strange situation: from interference and diffraction experiments followed the inescapable conclusion that radiation has a wave nature, while other experiments, such as those regarding the photoelectric and Compton effects, convincingly proved its corpuscular nature. While this mysterious duality remained unexplained, it was generally recognized as a fact. There were still opponents to the light quanta, however, and Bohr was one of them. His point of view was well expressed at a conference given in 1922:

> In spite of its heuristic value, however, the hypothesis of light-quanta, which is quite irreconciliable with so-called interference phenomena, is not able to throw light on the nature of radiation. I need only recall that these interference phenomena constitute our only means of investigating the properties of radiation and therefore of assigning any closer meaning to the frequency which in Einstein's theory fixes the magnitude of the light-quantum.[281]

What did Bohr have against the hypothesis of light quanta? The relation that gives the energy of a quantum (equal to the frequency multiplied by the Planck constant), whatever its experimental success or "heuristic value" may be, is meaningless. Indeed, the frequency can only be measured using interference phenomena that are incompatible with the hypothesis of light quanta.

But why should one look for the meaning of a physical concept? For classical concepts, such a problem did not arise: the concepts of classical physics were supposed to be inherent to the physical objects that they

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[281] N. Bohr, *The Structure of the Atom*, above quoted conference; see p. 470.

described; nature simply *was* mathematical. Classical physicists might have repeated Galileo's famous assertion that the book of universe is written in geometrical characters. As far as I know, the question about the meaning of a physical concept was first explicitly asked in Einstein's seminal paper about relativity theory.[282] He explained there that the usual description of the motion of a material point (based on coordinates as a function of time) "has no physical meaning unless we are quite clear as to what we understand by 'time.'" He then analyzed the experimental procedure allowing us to establish the simultaneity of two events located at two different places; this analysis led him to relativistic kinematics. The important point is that the meaning of simultaneity is disclosed by the experimental procedure that establishes it. The reasoning that goes from experiment to meaning is radically different from that of classical physics, which goes from a priori mathematical principles to the interpretation of experiments. It deviates from the metaphysical foundations of classical physics and gets closer to phenomenology, according to which one should abandon dogmatic certitudes and pay attention to modes of givenness.

When he looks for the meaning of frequency in the experimental method for its measurement, Bohr follows in Einstein's footsteps; but while Einstein came back to the Galilean identification of the universe with geometry, Bohr did not stop moving forward towards phenomenological thought.

5. Quantum Mechanics

One of the main tools used by Bohr to work out his 1913 theory of atoms was found in the relations between quantum and classical theories. The stationary states of the hydrogen atom can be numbered according to increasing energies. The number of a state is called its quantum number. Bohr noticed that the numerical results obtained by application of the quantum laws approach the classical results when the quantum number becomes very large. This "correspondence principle" was well

[282] A. Einstein, *Zur Elektrodynamik bewegter Körper, Annalen der Physik*, 4th series, Vol. XVII, pp. 891–921 (1905). A translation of this article is given in: H. A. Lorentz, A. Einstein, H. Minkowski, H. Weyl, *The Principle of Relativity*, New York, Dover Publications, 1952.

borne out afterwards. Between 1919 and 1925, Bohr and his followers worked out the consequences of the correspondence principle. It turned out that, in the simplest cases, important results could be obtained, especially about the interactions between atoms and electromagnetic radiation; but the "systematic guessing guided by the correspondence principle"[283] failed as soon as one dealt with any systems except the simplest ones. For instance the hydrogen atom, with its single electron, lends itself well to these methods, but for the helium atom, which has two electrons, they no longer work. Physicists needed to depart further from classical physics.

The decisive step was taken in 1925 by Heisenberg. In his article "On the Quantum-Theoretical Re-interpretation of Kinematic and Mechanical Relations,"[284] he showed how, by transforming the differential equations of classical mechanics into difference equations, one could get a formulation of the expected theory – "quantum mechanics," as it was called by Max Born. While in classical mechanics the physical quantities are represented by numbers, in the new theory they are represented by matrices. Very soon this "matrix mechanics" became a powerful tool, able to deal successfully with problems that had defied the physics of correspondence principle. But a theory that prescribes representing such familiar quantities as the positions and momenta of the electrons by mathematical objects other than numbers is not easy to understand.

Before dealing with this difficulty, some facts should be recalled. With Louis de Broglie's thesis (1924), the wave-particle duality had been extended to electrons. Experiments about the diffraction of electrons by a crystal soon confirmed that electrons could indeed behave as waves. In 1926, Erwin Schrödinger raised this physical idea to the rank of a coherent theory: wave mechanics. Starting with very different physical conceptions he found again the results of matrix mechanics, especially concerning the stationary states of the hydrogen atom. He hoped that wave mechanics would lead to a theory closer to classical physics. These hopes were to be disappointed by the ensuing developments: the Schrödinger wave is not a classical wave, as shown by the essential fact that the function that

[283] This phrase belongs to van der Waerden, see reference 279.

[284] W. Heisenberg, "Über Quantentheoretische Umdeutung Kinematischer und Mechanischer Beziehungen," *Zeitschrift für Physik,* Vol. 33, pp. 879–893 (1925). A translation of this article is given in van der Waerden, reference 279.

describes it is not real-valued but complex-valued. As Max Born showed, the squared modulus of this function can be interpreted as a "probability of presence."

6. Debates between Bohr and Heisenberg

Bohr at once recognized quantum mechanics as a father recognizes his child. "The whole apparatus of the quantum mechanics," he stated in 1925, "can be regarded as a precise formulation of the tendencies embodied in the correspondence principle."[285] Among all the debates of that time, the most relevant for us here is the one between Bohr and Heisenberg. While Heisenberg emphasized mathematical formalism, Bohr wanted to understand physical concepts and to explain them in common language. According to him, whatever the merits of quantum mechanics, the problem of wave-particle duality was not yet solved.

Heisenberg's discovery of the indeterminacy relation took place after long months of intense discussions with Bohr. In his article,[286] he first defined the notion of understanding: "We believe we understand intuitively a physical theory when we can imagine qualitatively its experimental consequences, and when at the same time we have recognized that the application of the theory never implies internal contradictions." From the mathematical formalism of quantum mechanics, he then deduced the indeterminacy relations: that the product of the imprecisions with which the position of an electron and its momentum are determined is of the order of the Planck constant (It will appear later that it is in fact a matter of inequality: the product is greater than or equal to the constant). He also gave a physical interpretation of the indeterminacy relation, in the case of the state of lowest energy of the hydrogen atom. To determine the position of the electron, one must use a microscope illuminated with a light of wavelength shorter than the size of the electronic orbit (a notion that has no precise meaning in quantum mechanics, but retains some sense as an order of magnitude). Therefore we shall have to use gamma rays. The

[285] N. Bohr, "Atomic Theory and Mechanics," Supplement to *Nature*, December 5, 1925, pp. 845–852. In BCW, Vol. 5, pp. 273–280.

[286] W. Heisenberg, "Über des Anschaulichen Inhalt der Quantentheoretischen Kinematik und Mechanik," *Zeitschrift für Physik*, Vol. 43, pp. 172–198 (1927). The original text is given in BCW, Vol. 6. There is a translation in: J.A. Wheeler, W.H. Zurek, *Quantum Theory and Measurement*, Princeton University Press, 1983.

gamma photon changes the position of the electron (Compton effect), from which comes an imprecision in the determination of the position; the product of the imprecisions about the position and the momentum satisfies the Heisenberg relation.

As will be seen, Bohr criticized this demonstration; but let us first have a look at both protagonists' conceptions of physical knowledge. They clashed during heated discussions during the winter 1926–1927. Unfortunately, "hardly any trace has survived of their conversations in the documents of that time."[287] There is, however, in an interview of Heisenberg in 1963, an analysis that agrees with what is known from other sources:

> The main point was that Bohr wanted to take this dualism between waves and corpuscles as the central point of the problem, and to say: "That is the center of the whole story, and we have to start from that side of the story in order to understand it." I, in some way, would say, "Well, we have a consistent mathematical scheme and this consistent mathematical scheme tells us everything that can be observed. Nothing is in nature that cannot be described by this mathematical scheme." It was a different way of looking at the problem because Bohr would not like to say that nature imitates a mathematical scheme, that nature does only things which fit into a mathematical scheme. While I would say, "Well, waves and corpuscles are, certainly, a way in which we talk and we do come to these concepts from classical physics. Classical physics has taught us to talk about particles and waves, but since classical physics is not true there, why should we stick so much to these concepts? Why should we not simply say that we cannot use these concepts with a very high precision, therefore the uncertainty relations, and therefore we have to abandon these concepts to a certain extent. When we get beyond this range of the classical theory, we must realize that our words don't fit. They don't really get a hold in the physical reality and therefore a new mathematical scheme is just as good as anything because the new mathematical scheme then tells what may be there and what may not be there. Nature just in some way follows the scheme."[288]

After quoting this interview in his book about Bohr, Abraham Pais adds: "Having talked countless hours with Bohr on complementarity, I could imagine that to Heisenberg's 'our words don't fit' he would have replied: 'Our words *have* to fit, we have nothing else.' "

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[287] I have used the book by Jagdish Mehra and Helmut Rechenberg *The Historical Development of Quantum Theory*, Vol. 1–6, New York, Springer-Verlag, 1982–2001. See Vol. 6, "*The Completion of Quantum Mechanics, 1926–1941*," chapter 2. The quotation is from p. 151.

[288] Interview of Heisenberg quoted by A. Pais, *Niels Bohr's Times*... quoted above, pp. 309–310.

The difference between both conceptions most clearly appears here. Heisenberg follows the Galilean tradition when he imagines a nature strictly obedient to a mathematical scheme. That can be seen also from his article, when he explains finally:

> The proposition that, for instance, the x-component of the velocity is "in reality" not a number but the diagonal term of a matrix, is perhaps no more abstract and no more unvisualizable than the statement that the electric field strengths are "in reality" the time part of an antisymmetric tensor of the spacetime world. The phrase "in reality" here is as much and as little justified as it is in any mathematical description of natural processes. As soon as one accepts that all quantum-theoretical quantities are "in reality" matrices, the quantitative laws follow without difficulty.[289]

The ideas of abandoning the classical concepts and giving up common language logically follow from Heisenberg's essential statement that "nature follows a mathematical scheme," which repeats the Galilean thesis that the universe is written in mathematical language. As will be seen, Bohr maintained and developed the idea that classical concepts and common language remain necessary.

Thus the current idea of a "Copenhagen interpretation" of quantum mechanics, supposedly worked out by Bohr and Heisenberg, should be clarified and even rectified. The discussions between Heisenberg and Bohr were very fruitful for both of them, and they gave rise to two different interpretations of quantum mechanics.[290] The Heisenberg interpretation continues the Pythagorean and Galilean traditions; it has been adopted by many theoretical physicists. Bohr's interpretation breaks with these traditions; it has deep similarities to the Husserlian critique of the metaphysical foundations of classical physics.

7. Complementarity

The close personal collaboration between Bohr and Heisenberg came to an end in the summer of 1927, when Heisenberg left Copenhagen to take up a professorship in Leipzig. In September 1927, an "International Congress

[289] W. Heisenberg, "Über den Anschaulischen Inhalt," article quoted above, end of §4; translation quoted, p. 82.

[290] See the books, already quoted, of J. Mehra and H.Rechenberg, and of A. Pais, and above all the introduction and comments of J. Kalckar to Vol. 6 of BCW.

of Physicists" took place in Como, on the centenary of Alessandro Volta's death. Bohr gave an address there: "The Quantum Postulate and the Recent Development of Atomic Theory."[291] He first characterized the situation of quantum theory with respect to classical physics: on the one hand it entails "a fundamental limitation in the classical physical ideas, when applied to atomic phenomena"; but on the other hand, "our interpretation of the experimental material rests extensively upon the classical concepts."

Here a brief comment is in order: we are thus, from the very beginning, at variance with the metaphysical foundations of classical physics. Concepts whose validity is subject to a fundamental limitation cannot be found in nature as one finds a character on a page; rather, the fact that they play an essential role suggests that they are built by physicists to allow an understanding of physical processes. Their privileged role stems, inseparably, both from the features of human knowledge and the nature of physical phenomena.

Bohr then expressed the *quantum postulate*: "to any atomic process [it] attributes an essential discontinuity or rather individuality, completely foreign to classical theories and symbolized by Planck's quantum of action" (by "individuality" Bohr means indivisibility).[292] Indivisibility is indeed completely foreign to the metaphysical foundations of classical physics; in Western science the division of processes or of objects into as many parts as may be necessary is generally considered as a self-evident methodological principle.[293]

Bohr then explains:

> The postulate implies a renunciation as regards the causal space-time co-ordination of atomic processes. Indeed, our usual description of physical phenomena is based entirely

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[291] N. Bohr, "The Quantum Postulate and the Recent Development of Atomic Theory," *Nature*, Vol. 121, pp. 580–590 (1928). Reprinted in: *The Philosophical Writings of Niels Bohr*, Vol. 1, *Atomic Theory and the Description of Nature*, Ox Bow Press, Woodbridge, Connecticut, 1987. Also in: J.A. Wheeler, W.H. Zurek, *Quantum Theory and Measurement*, book quoted above; and Vol. 6 of BCW.

[292] "The inability of the classical frame of concepts to comprise the peculiar feature of indivisibility, or 'individuality,' characterizing the elementary processes." N. Bohr, *Discussions with Einstein*. The passage quoted is at page 34 of: N. Bohr, *Atomic Physics and Human Knowledge*. op. cit.

[293] Descartes, for instance, states this principle explicitly in the *Regulae ad Directionem Ingenii* (Rule 13), and in the *Discours de la Méthode*, Second part.

on the idea that the phenomena concerned may be observed without disturbing them appreciably.... Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation.

As to the indeterminacy relations, Bohr did not deduce them from the interaction with the measuring device, but from classical considerations. In the theory of optical instruments, indeed, there are well-known relations between the duration of a wave train and the width of its frequency spectrum, and similarly between the spatial extension of the train and the indeterminacy of its wave number. Combining these results with the Planck relation between frequency and photon energy, and the de Broglie relation between wavelength and photon momentum, one gets the Heisenberg relations.

This reasoning is essentially different from Heisenberg's. According to Bohr, the indeterminacy relations should not be explained by the perturbing action of the photon used for observation, but by the mutual limitation of the possibilities of *definition* of the conjugated physical quantities.[294] This point lies at the centre of the misunderstandings so often met with, especially in pedagogical or popular accounts of the Heisenberg relations. The central idea is the limited validity of classical concepts, from which follows the limitation of the possibilities of definition. The approximate validity of the concepts of coordinates and momentum components allows us to make picturesque representations of atomic processes. But the very fact that their validity is only approximate requires us to be careful when we ask such questions as "What is the value of the coordinate?" or "What is the value of the momentum component?" Asking such questions carelessly means going no further than Galileo's mathematical nature, with material points having definite values of their coordinates and momenta. Heisenberg's proposal takes up again the idea of mathematical nature, simply replacing the old mathematical concepts with new ones. Bohr's

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[294] "Indeed, a discontinuous change of energy and momentum during observation could not prevent us from ascribing accurate values to the space-time coordinates, as well as to the momentum-energy components before and after the process. The reciprocal uncertainty which always affects the values of these quantities is (...) essentially an outcome of the limited accuracy with which changes in energy and momentum can be defined, when the wave-fields used for the determination of the space-time coordinates of the particle are sufficiently small." (N. Bohr, "The Quantum Postulate," §3).

proposal is more fundamental: the meaning of any question has to be clarified by defining the experimental device that allows us to ask it concretely. Such is the meaning of an idea that, from the point of view of the classical tradition, looks strange and even incomprehensible – the reciprocal non-autonomy of atomic processes and experimental devices.

8. The Debate with Einstein, Podolsky, and Rosen ("EPR")

In the years following the Como congress, a discussion began between Bohr and Einstein. The account of it by Bohr is the most illuminating of his texts.[295] I will retain here only some passages of particular relevance for a study of Bohr's way towards phenomenology. In 1935, Einstein, who had become a refugee in the United States, published an article with Boris Podolsky and Nathan Rosen entitled "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" (commonly known as EPR). Bohr's answer was published some months later under the same title.[296]

The first sentence in the abstract of EPR's article at once characterized the philosophical conception of the authors: "In a complete theory there is an element corresponding to each element of reality." This shows how opposite are the premises on both sides: as we have just seen, Bohr's quantum postulate states precisely that atomic processes are indivisible.

Einstein and his collaborators considered the case of two particles A and B that have interacted in the past, between which exists such a correlation that the measurement of a quantity (coordinate or momentum) belonging to A immediately gives the value of this quantity for B (That such correlations exist is a consequence of quantum mechanics). EPR states: "If, without in any way disturbing a system, we can predict with certainty . . . the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

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[295] N. Bohr, *Discussions with Einstein*. I have analyzed other aspects of the Bohr-Einstein debate in my book *Niels Bohr et la Physique Quantique*, coll. "Points Sciences," Paris, Editions du Seuil, 2001.

[296] A. Einstein, B. Podolsky, N. Rosen, *Physical Review*, Vol. 47, pp. 777–780 (1935). N. Bohr, *Physical Review*, Vol. 48, pp. 696–702 (1935). Both texts are reprinted in: J.A. Wheeler, W.H. Zurek, *Quantum Theory and Measurement*, quoted above. Also in BCW, Vol. 7.

Now, EPR continues, we can choose to measure of A either its coordinate or its momentum. Because of the correlation between A and B, these measurements will give the value of the coordinate or of the momentum of B. As we did not touch B, we must conclude that B has indeed determined values of the coordinate and the momentum. Since according to quantum mechanics this cannot be, it follows that the quantum-mechanical description is incomplete.

In his answer, Bohr first outlined the general frame of the debate. The apparent contradiction raised by EPR, he says, "in fact discloses only an essential inadequacy of the customary viewpoint of natural philosophy for a rational account of physical phenomena of the type with which we are concerned in quantum mechanics. Indeed the *finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails . . . the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality."

Bohr then shows that the correlations, presented by EPR in the abstract form of a mathematical expression for the wave function of the two particles, can be realized by sending the particles across suitably arranged diaphragms. He comes at last to the core of the argumentation. The criterion of physical reality proposed by EPR, he says, "contains an ambiguity as regards the meaning of the expression 'without in any way disturbing a system.' " Of course there is no mechanical disturbance of the system B. But there is "*an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system*" (Italics are Bohr's). And he concludes: "Since these conditions constitute an inherent element of the description of any phenomenon to which the term 'physical reality' can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete." He then points out that the experimental procedures permitting the definition of complementary quantities (such as coordinate and momentum) are mutually exclusive; this "provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science." The final conclusion then follows: "It is just this entirely new situation as regards the description of physical phenomena, that the notion of *complementarity* aims at characterizing."

We have here a further elaboration of ideas already presented at the Como conference. But the critical passage of Bohr's answer, quoted above, enjoys a special reputation because John Bell had explained why he did not understand it[297] (One should not forget how fruitful Bell's lack of understanding of Bohr's conceptions was: it gave rise to Bell's fundamental discovery, perhaps the most important one in theoretical physics in the second half of the twentieth century). It may be useful, therefore, to comment briefly on the italicized passage. In the first sentence, by "the very conditions that define the possible types of predictions regarding the future behaviour of the system" Bohr means the whole experimental arrangement. The sentence then means: choosing to measure either the coordinate or the momentum of A has practical implications, because, in accordance with this choice, we must modify the experimental device. The second sentence then could be stated: in the quantum domain, a relevant description of a real phenomenon is one that includes the whole experimental arrangement.

The core of the contradiction between Bohr's conception and what he calls "the customary viewpoint of natural philosophy" may appear even better in less formal accounts. The physicist Robert H. Romer tells us how, as a graduate student, he had the opportunity to spend an hour with Einstein.[298] "Do you really believe," Einstein asked him, "that if you would measure the z component of the spin of an atom here, that might instantaneously have an effect on the spin of another atom, perhaps miles away from here?" The question here is about the correlation between two atomic objects, already considered in EPR's article, but in a different form, one nearer to practically realizable experiments. Probably inspired by David Bohm's treatise on quantum mechanics,[299] what Einstein had in view was not the coordinates and momenta of two particles, but the spins of two atoms. To be even nearer to experiments that have been really carried through, we shall now speak about the correlated polarizations of

[297] J. Bell, "Bertlmann's Socks and the Nature of Reality," *Journal de Physique*, Vol. 42, Coll. C2, suppl. no. 3, pp. 41–61 (1981). Reprinted in: J. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, 1987.

[298] R.H. Romer, Editorial: *John S. Bell (1928–1990),* "The Man Who Proved Einstein was Right," *American Journal of Physics*, Vol. 59, p. 299 (1991).

[299] D. Bohm, *Quantum Theory*, Englewood Cliffs, N.J., Prentice Hall, 1951.

two photons, emitted by a calcium atom. The result of the measurement of polarization of photon A is random, and the same is true for photon B. But as the two photons have been emitted by the same atom, when we know the result of measurement A we can predict with certainty the result of measurement B, and conversely. What Einstein refers to, then, is the following paradoxical question: if the two photons are far apart, how can the result of one of the measurements have an influence on that of the other one?

The correlation of both experimental results has been well proved now; for instance, in Gisin's experiments both measurement devices are more than six miles apart.[300] Einstein had always considered such correlations (which during his lifetime were never experimentally proved) as unacceptable; this opinion justified his opposition to quantum mechanics. In 1947, speaking to his friend Max Born about the statistical interpretation of quantum mechanics (and hence about quantum mechanics itself), he wrote: "I cannot, consequently, believe it really, for the theory is incompatible with the principle according to which physics must represent a space and time reality, without spooky action at a distance."[301]

Let Bohr and Husserl answer Einstein in an imaginary dialogue:

"I had," says Bohr, "answered in advance in general terms in my Como report. What the quantum postulate says is precisely that quantum phenomena have a character of indivisibility completely foreign to classical physics. Of course one may be surprised at a feature so remote from our every day experience, but there is nothing unacceptable here."

Husserl adds for his part: "When you speak about atoms (or photons), you have in view small objects belonging to 'nature, which is in itself mathematical.' You do not take into account the way by which these supposed objects come to our knowledge. It seems that now, experiment is for you no longer anything else but a way of checking the theories. If the raw experimental facts (the click of a counter, or a trace in a Wilson chamber) were mere signs for elements of the mathematical nature, one could disregard the signs and care only about the signified. But as I have explained long ago, it is misleading to think that in physics we are dealing with 'true' physical things, while the appearances are only signs of these true things. As I said, 'the *perceived physical* thing itself is always

[300] See the articles of the Gisin group quoted above, reference 266.

[301] A. Einstein, letter to Max Born, March 3 1947, in: M. Born (ed.), *The Born-Einstein Letters*, London, Macmillan, 1971. French translation by Pierre Leccia: Albert Einstein, Max Born, Hedwig Born, *Correspondance 1916–1955, Commentée par Max Born*, Paris, Editions du Seuil, 1972.

and necessarily *precisely the thing which the physicist explores and scientifically determines following the method of physics*.'[302] Now there is nothing in the experimental results that would allow us to state that the part of the experimental arrangement located in A, for instance, deals with a definite partial object, named the photon A; we can only say that the complete experimental arrangement deals with the radiation (two photons per atom) emitted by calcium atoms. We must abide by what is or can be actually perceived."

"This is," Bohr adds finally, "precisely what you did at the time of your seminal article about special relativity, when you discarded Newton's absolute space (at that time an essential part of the supposed mathematical nature) because it has no counterpart in the experimental methods of measuring distances and durations."

To follow Bohr's and Husserl's wise advice, we should mention measurements carried out on the "biphotons" emitted by calcium atoms. The correlation between the results recorded by both parts of the experimental arrangement (photomultipliers and polarizers on both sides) shows that as far as the polarizations are concerned, the biphoton is not separable into two photons. This result is surprising, but it is the case. One of the great merits of EPR's article is that it raised the problem of distant correlations, a striking proof of the originality of quantum phenomena.

9. Bohr's Report about His Discussions with Einstein: Complementarity

This report was published in 1949, in a volume of the collection "The Library of Living Philosophers" devoted to Einstein.[303] Going back over the successive stages of the debate, Bohr improved or corrected several of his formulations, nearing a phenomenological approach. First he reviewed the role of classical concepts, which he considered essential for the description of experiments and their results to be communicable to other people. Bohr then critically examined the interpretation of the indeterminacy relation. "A sentence like 'we cannot know both the momentum and the

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[302] E. Husserl, *Ideen zu Einer Reinen Phänomenologie und Phänomenologischen Philosophie. I. Buch: Allgemeine Einführung in die Reine Phänomenologie* (1913), p. 99. Translation by F. Kersten: *Ideas pertaining to a Pure Phenomenology and to a Phenomenological Philosophy, First Book: General Introduction to a Pure Phenomenology*. Edmund Husserl, *Collected Works*, Vol. 2, The Hague, Martinus Nijhoff Publishers, 1982.

[303] N. Bohr, *Discussions with Einstein*, see note 269.

position of an atomic object,' " he explained:

> raises at once questions as to the physical reality of two such attributes of the object, which can be answered only by referring to the conditions for the unambiguous use of space-time concepts, on the one hand, and dynamical conservation laws, on the other hand. While the combination of these concepts into a single picture of a causal chain of events is the essence of classical mechanics, room for regularities beyond the grasp of such a description is just afforded by the circumstance that the study of complementary phenomena demands mutually exclusive experimental arrangements.

Let us comment on this passage. Energy and momentum, subjected to conservation laws, allow a causal description of atomic processes. In the Compton effect, for instance, the collision between a photon and an electron – which in the initial state are supposed to have definite values of energy and momentum – is the cause of a final state in which both particles take different directions ("scattering"). This description allows us to discover a relation, well confirmed by experiment, between the deflection of the X-radiation and the increase of its wavelength. But on the other hand a space-time description of the X-beam is necessary for the definition and measurement of its wavelength; such a description corresponds to experimental situations where diffraction phenomena, described in terms of waves, can take place. Bohr's comment, then, is that these two descriptions require mutually exclusive experimental arrangements. This allows a rigorous critique of the quoted sentence ("We cannot know both"), which implicitly assumes that the particles really have at any time, as in classical mechanics, determined values of coordinates and momenta. According to the Galilean conception of nature as in itself mathematical, one may speak about "the position (or the momentum) of the particle," without wondering whether these words have a precise meaning. Bohr requires us to become aware that a particle has such an attribute (coordinate or momentum) only as a part of a relevant experimental arrangement. As the arrangements relative to coordinate and momentum are mutually exclusive, the criticized sentence has no rigorous meaning.

This example shows how complementarity requires us to get rid of the "nature mathematical in itself" and relate any statement about atomic objects to the experimental situation in which it can be tested.

10. Bohr's Report about His Discussions with Einstein: The Observer and Objectivity

The report then described Einstein's objections to some quantum statements. In his answers, Bohr always implemented the above stated complementarity principle. He also dealt with the random aspect of atomic processes (This is an aspect which can be easily observed if one uses a radioactive source of weak intensity and a counter: the only reasonable description of the time sequence of the clicks of the counter is as a random sequence). According to Dirac, individual effects of that kind correspond to a choice on the part of nature. According to Heisenberg, on the other hand, in such cases "we have to do with a choice on the part of the 'observer' constructing the measuring instruments and reading their recording." As he quickly refuted both points of view, Bohr put forward an interesting argument about Heisenberg: "It is certainly not possible for the observer to influence the events which may appear under the conditions he has arranged."

This sentence briefly settles both sides of the question of the "observer." *The observer does not create or influence the phenomenon, but he creates the conditions of the phenomenon.* On the one hand, the laws of atomic processes are objective, independent of our desires and of fashions (at least in their essential content); but on the other hand, the properties of individual atomic objects can only appear when we, physicists, prepare an experimental arrangement and record the results. (Or else, when we prepare the arrangement for the automatic recording of the results). This statement is not a dispensable comment: it is inscribed in the very mathematical formalism of quantum mechanics. The basic element of the formalism is indeed an amplitude, made of two abstract vectors; one of them represents the preparation of an experiment, while the second one represents one of its possible results.[304] Without physicists, there is neither preparation nor results. Of course the experimental results are objective: if the experiments are correctly and honestly carried out, their results will be (approximately) the same in different laboratories and different countries. One can hardly

[304] This is explained in textbooks, for instance by Feynman: R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman Lectures on Physics*, Vol. 3, Reading, Mass., Addison-Wesley, 1965.

say, however, that they are independent of human beings, because without human beings there would be no results.

In that respect, atoms differ from physical objects of human or astronomical dimensions. One might try to summarize this essential difference by saying that *atoms are invisible.* This formulation, however, is too cursory. Microbes and remote galaxies seem invisible as well, but they are able to be seen. To see them, one magnifies their image with a microscope or a telescope; as Galileo replied to those who questioned the objectivity of the images given by his telescope, perhaps lynxes, with their sight better than ours, can see the satellites of Jupiter he had discovered. But atomic objects make their entrance in the sensible world of appearances thanks to a much more complex and roundabout process: in a counter or in a Wilson-type chamber, they come into contact with a macroscopic system in an unstable state, in which they trigger an avalanche-type process. For the believer of "nature mathematical in itself," such a difference does not deserve any attention: atoms, like stones and stars, are "in space." But it is an essential difference from the point of view of common sense, as well as for the phenomenologist, whose field of study can be defined as "the method of the analysis of essences within the sphere of immediate evidence."[305] Furthermore, the phenomenologist has something to add here, because for him the essential difference between the modes of givenness of atoms on the one hand, and of stones and stars on the other hand, points out that they belong to different regions of reality.[306]

The misunderstandings between Bohr and most physicists stem from his claim of an essential difference between quantum phenomena and the world of classical physics. The drama of physics, and more particularly of quantum physics, is that it never admitted the notion of regions of reality or, equivalently, the notion of essential differences. When Galileo

[305] E. Husserl, *Die Idee der Phänomenologie* (1907), p. 14. Translation by Lee Hardy: *The Idea of Phenomenology*, Kluwer Academic Publishers, Dordrecht/ Boston/ London, p. 70.

[306] E. Husserl, *Ideen zu Einer Reinen Phänomenologie und Phänomenologischen Philosophie. Drittes Buch: Die Phänomenologie und die Fundamente der Wissenschaften*, ed. by Marly Biemel, *Husserliana* 5, The Hague, Martinus Nijhoff, 1971. Translation by Ted E. Klein and William E. Pohl: *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy, Third Book: Phenomenology and the Foundations of the Sciences*. Edmund Husserl, *Collected Works*, Vol. 1, The Hague, Martinus Nijhoff Publishers, 1980.

annexed sensible appearances to the world of mathematical concepts, he put an obstacle in the way of understanding what physics really is, an obstacle still not overcome.

In his study "Einstein and the Quantum Theory," Abraham Pais recalls his conversations with Einstein.[307] There we find one of the most striking examples of a refusal of essential differences. I quote Pais:

> We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it.

There is no question that the moon existed before human beings were there to look at it; all the same, quantum mechanics describes phenomena as prepared and observed by human beings. Abiding by "nature mathematical in itself" leads to insuperable problems. It would probably be wiser to take into account the notion of regions of reality, admitting that the moon, on the one hand, and atomic objects, on the other hand, which are given to us in such radically different ways, belong to different regions.

This being said, one should never forget that without Einstein and some of his followers (David Bohm, John Bell), who resolutely and perseveringly opposed Bohr's views, we would have perhaps gotten no further than the Bohrian vulgate of the 1930s and 1940s, according to which there was essentially no problem. Today mimetism and unanimity are harmful to science.

11. Bohr's Report about his Discussions with Einstein: The Phenomenon

Finally the report recalled a review of some terminological questions, made by Bohr in his contribution to the conference, "New Theories in Physics" (Warsaw, 1938):

> In this connection I warned especially against phrases, often found in the physical literature, such as "disturbing of phenomena by observation" or "creating physical attributes to atomic objects by measurements." Such phrases, which may serve to remind one of the apparent paradoxes in quantum theory, are at the same time apt to cause confusion, since

[307] A. Pais, *Reviews of Modern Physics*, Vol. 51, p. 863 (1979). The quoted passage is at page 907.

words like "phenomena" and "observations," just as "attributes" and "measurements," are used in a way hardly compatible with common language and practical definition.

Both phrases are in fact quotations from Heisenberg. We have already discussed the first one, and the second one can be found in his article on the uncertainty relations: "The 'trajectory' first comes into being by the fact that we observe it" ("*Die 'Bahn' entsteht erst dadurch, dass wir sie beobachten*").[308] Bohr then explained how one should understand the words "phenomenon" and "observation":

As a more appropriate way of expression I advocated the application of the word *phenomenon* exclusively to refer to the observations obtained under specified circumstances, including an account of the whole experimental arrangement. In such terminology, the observational problem is free of any special intricacy since, in actual experiments, all observations are expressed by unambiguous statements referring, for instance, to the registration of the point at which an electron arrives at a photographic plate. Moreover, speaking in such a way is just suited to emphasize that the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinate or statistical character, pertaining to individual phenomena appearing under conditions defined by classical physical concepts.

Here, Bohr defines his rules: break with "nature mathematical in itself" and use "practical definitions," dealing with experiments and their results. Describe them in the language of classical physics, appropriate to the visible world. Keep to the definition of the phenomenon: it occurs in conditions determined by the physicist; it consists of macroscopic events; it is within the reach of senses, hence it can be described in "common language."

Finally, in the use of the notion of attribute, Bohr sees the danger of forgetting that position, momentum, and other quantities relating to an atomic object can only be defined in terms of a method of measurement, which implicitly supposes an indissoluble bond between object and measuring apparatus. Of course, it is not always easy to follow these rules, for an age-old tradition constantly incites us to ask: Where is the atom? Which path did the photon follow? But we know that we should answer these questions with practical definitions: if you want to know which path the photon has followed, build and use an apparatus designed to give an answer.

[308] W. Heisenberg, article quoted above (note 26), §3, page 185 of the original text.

12. Conclusion

In his Vienna lecture of 1935, Husserl summarized the situation of modern physical sciences:

> Mathematical natural science is a wonderful technique for making inductions with an efficiency, a degree of probability, a precision, and a computability that were simply unimaginable in earlier times. As an accomplishment it is a triumph of the human spirit. As for the rationality of its methods and theories, however, it is a thoroughly relative one. It even presupposes a fundamental approach that is itself totally lacking in rationality. Since the intuitively given surrounding world, this merely subjective realm, is forgotten in scientific investigation, the working subject is himself forgotten; the scientist does not become a subject of investigation (Accordingly, from this standpoint, the rationality of the exact sciences is of a piece with the rationality of the Egyptian pyramids).[309]

In classical physics, the oversight of the subject manifests itself in the form of belief in mathematical nature. Galileo began with the sensible world of appearances; he then amputated from it of most of its qualities, and, finally, denied the reality of the suppressed qualities (There is no essential difference between the Galilean illusion and the present-day naturalistic illusion, according to which the essence of subjectivity lies in the electrochemical processes that take place in the brain).[310]

With quantum physics, the negation of the subject began to exert its corrosive action within the physics itself. The classical physicist did not understand the nature of his science; the quantum physicist does not understand his very science, and, as we have seen, he is in many cases aware of this lack of understanding. Locked up in the Galilean prison, he does not see the key proposed by phenomenology, a key that Bohr, to a certain extent, rediscovered by himself.

For a long time, atoms could be reached only by speculation, first philosophical, then scientific. It became possible to perform experiments

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[309] E. Husserl, *Die Krisis des Europäischen Menschentums und die Philosophie*, published as an Appendix of *Die Krisis der Europäischen Wissenschaften.* Translation by David Carr in *The Crisis. Philosophy and the Crisis of European Humanity*, a lecture presented before the Vienna Cultural Society on May 7 and 10 1935.

[310] This prejudice had been already aptly criticized by Erwin Straus, *Vom Sinn der Sinne, Ein Beitrag zur Grundlegung der Psychologie*, 1st edition, 1935; 2nd edition, Berlin, Heidelberg, New York, Tokyo, Springer-Verlag, 1956.

on atoms only when scientific instruments designed to comply with theoretical notions about their nature could be made. To make counters and chambers of various types, one must have the electromagnetism of Maxwell–Faraday, notions about electrical discharge in gases or about thermodynamics, and basic notions of atomic theory. Atoms, molecules, nuclei, particles, which are not parts of the sensible world of appearances, can have perceptible effects and become accessible to experiment, but only in a society with a high degree of scientific and technical development. They are given to us in quite a different way than are stones, trees, or stars.

But the essential importance of this difference can be seen only by getting out of the metaphysical matrix inside which the growth of classical physics took place. As long as physics is understood as dealing with "nature mathematical by itself," perceptibility, or lack thereof, will continue to be reputed unessential. In the prevailing way of understanding quantum phenomena, quantum mechanics is a universal theory, while classical physics is merely an approximation valid for processes involving actions large with respect to the Planck constant. The subjective side of the difference thus falls out of sight, and both the way to Bohrian conceptions and the way to phenomenology are barred.

Quantum mechanics is really, as René Thom once put it, an "intellectual scandal": a theory successfully used in many different fields of science and technology, which most people accept without understanding it. Husserlian phenomenology proposes a frame in which this paradoxical lack of understanding might be understood, thereby opening a way towards recovery. It invites us to enlarge our horizons, especially by going back over the origins of physics (of modern physics, but not only of it). It suggests that our irrational attitude towards quantum mechanics is part of a wider historical fact: the global lack of understanding of the nature of science, which became an acute problem with the advent of modern science. In the present situation of the world, this problem has become particularly urgent.[311,312]

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[311] Such is the problem that I study in my book *De la Science à l'ignorance*, Paris, Editions du Rocher, 2003.

[312] I would like to thank Pamela Kraus for her kind linguistic help.

PART III

PHENOMENOLOGY, LOGIC, AND MATHEMATICS

FOREWORD

Frédéric Patras

«All roads lead to Hilbert».[313] The formula applies perfectly to Husserl, whose dependence upon the Hilbertian theory of science, although not always fully understood, is now well established. Too many superficial analyses still tend to consider phenomenology as a mere "philosophy of intuition" leading to a theory of knowledge radically different and divergent from the formalist approach. However, things are much more complex. A good deal of the historical evolution of Husserl's thought was motivated by the progress in his understanding of the power and limitations of axiomatics and formalism. Much of the future of phenomenology, and especially its ability to play a key role in the contemporary, post-structuralist, philosophy of mathematics, also depends on our understanding of how intentional and formal moments of knowledge fit together in order to make science possible.

However, even if we take into account Husserl's debt to Hilbert, and Husserl's reflections on syntactic completion and the categoricity of mathematical theories, it is still difficult to assess the extent of the agreement (or disagreement) of phenomenology with a formal conception of mathematics (or science in general).

It should also be pointed out that, throughout the twentieth century, the conceptual and methodological difficulties that Husserl faced in the 1920s have never received a satisfactory answer. The impact of Gödel's theorems has certainly been devastating: not only did it lead to giving up Hilbert's program that aimed at ascertaining the truth of mathematics on the basis of purely logical and finite arguments, but it also created a long-lasting suspicion inside the mathematical community with regard to any kind of logical or philosophical attempt to ground mathematics. The texts in the present chapter take up these problems, according to various

[313] See the article by J. Dodd, to whom we owe the expression.

L. Boi et al. (eds.), Rediscovering Phenomenology, 261–265.

modalities and kinds of strategy. They agree, however, inasmuch as they grant phenomenology a central role in the current epistemological debates.

It is best to recall that, about twenty years ago, mathematical thought had to refrain from relying on mathematical structuralism, which had been its epistemological foundation since World War II. Structuralism was a hybrid, yet powerful conception of mathematics, which received the legacy of Frege, Russell, the early Hilbert, and the "positive" spirit that had supported the foundation of the Vienna Circle. Systematically avoiding to deal with foundational or philosophical questions, its central idea was to retain from the debates of the beginnings of the twentieth century only that which could be useful for the efficiency of mathematics: the axiomatical method of *Hilbert's Grundlagen der Geometry*, Emmy Noether and Bartel van der Waerden's *Moderne Algebra*, set theory within the Zermelo-Fraenkel prescriptions, were the models for a method that rejected, as irrelevant to the core of mathematical knowledge, logical paradoxes, the limitations suggested by Brouwer's intuitionism and, more generally anything that could potentially hinder the progress of "real" mathematics.[314]

Under such circumstances, it is hardly surprising that phenomenology received a very limited echo inside the scientific community. The crisis diagnosed by Husserl in the 1930s didn't make any sense to the mathematicians and physicists of the 1950s who had survived the war. These scientists contributed to the remarkable subsequent expansion of science and technology. They had a renewed faith in their task, and for them "crisis" now meant atomic power and the military use of science. Mathematical structuralism even claimed to look beyond its own scientific sources, and became influential in many fields of the humanities.

But the situation has evolved dramatically in the past twenty years, so much so that a renewal of the phenomenological theory of science is now necessary. Among the changes that have occurred recently, the technical ones are not the less relevant to that purpose. To be sure, a number of Husserl's analyses turn out to be obsolete, yet they also provide new tools to understand the origins of mathematical knowledge. In the final analysis,

[314] For a systematic study of the mathematical thought in the twentieth century, see F. Patras, *La Pensée mathématique contemporaine* (Paris: P.U.F., 2001).

the value and meaning of phenomenology are strengthened. Phenomenology has never been quite at ease with the axiomatic method; in fact, it is suited more to the new concepts and methods of algebra, topology, geometry, even the post-Gödelian logic in which the chief theoretical interest has moved from formal moments of knowledge to the dynamics of constitutive processes.

The first step of this renewal lies in the notion of intuition. In the scientific community, there is certainly a wrong perception of the phenomenological theory, since it systematically underestimates its transcendental features – a bias that is most striking when it comes to intuition. Some technical reasons can be alleged for this: penetrating the transcendental theory of knowledge requires a philosophical background and a careful study of phenomenology hardly compatible with the practice of science. Though seemingly trivial, this neglect should not be underestimated in the case of the reception of Husserl's works, especially when compared to other contemporary theories of knowledge. The *Origin of Geometry*, which has been very influential in the scientific community, has probably contributed to this bias by emphasizing non-transcendental moments of the phenomenological quest for the foundations of mathematical knowledge.

The first chapter re-examines the function and nature of intuition in the works of Husserl. As soon as Hilbert's thought is considered in its whole complexity, the opposition between formalism and intuitionism cannot be reduced to the idea that formalism (as it appears in Hilbert himself) necessarily leads to bracketing intuition. Two conceptions of intuition, its power and its extension, have to be considered inasmuch as they pertain ultimately to two different conceptions of mathematics. Husserl's contributions to the discussion, examined in "*Husserl between formalism and intuitionism*," are numerous. It is worth quoting, among several examples, the crucial distinction between two conceptions of logic that underlie the various assessments of the role of intuition. Modern logic is not only apophantic, in accordance with tradition, but it is also (as was emphasized by Hilbert) a formal ontology, in accordance with the axiomatic method. This ambivalence disturbs the classical schemes and conceptual divisions: hence a necessary re-evaluation of the function of intuition in the cognitive processes.

Gödel's results are another source of uncertainty as far as the Husserlian theory of science is concerned. Whereas Gödel claimed to have been inspired by Husserl, his logical results could cast discredit on Husserl's logical analyses, the content of which is *de facto* limited to a pre-Gödelian understanding of mathematical logic. The chapter "*The two-sidedness and the rationalistic ideal of formal logic: Husserl and Gödel*" shows how the study of Gödel, particularly his unpublished documents, hints at a possible revival of the Husserlian approach to mathematical logic. Gödel shares with Husserl a certain optimistic and rationalistic conception of knowledge. Only a superficial understanding of the incompleteness theorems would undermine the possibility of its realization. On the contrary, they lead to taking into consideration the phenomenological method as a sound way to ground the strategical choices, especially the choice of axioms.

The other two chapters reconsider Husserl's works from the point of view provided by more recent scientific advances: they put the emphasis on the theory of categories. German mathematics in the 1920s and 1930s – the mathematics Husserl was in contact with, either directly or through a scientific *Zeitgeist* – was concerned with the raise of the structural method in algebra – the same method that would ultimately give rise to mathematical structuralism.

"*Mettre les structures en mouvement: la phénoménologie et la dynamique de l'intuition conceptuelle*" shows that Husserl's thought, despite its permanent tension between forms and intuitions, is intrinsically structural. However, the structural way of thinking rigidifies the processes of knowledge, whereas phenomenology, by its very nature, aims at understanding the movement of thinking inside a given phenomenal or eidetic horizon. The modern theory of categories, which substitutes the notion of category (a field of objects related to one another by transformations) for the notion of structure (a given set of axioms for a theory) makes evident the dynamics inherent to the mathematical progress, and therefore it seems to provide phenomenology with new tools and methods that were not available in the 1930s.

"*Pourquoi les nombres sont-ils naturels?*" follows the same general theoretical ideas, and chooses to emphasize the phenomenological potentialities of the categorical way of thinking by focussing on the internal eidetic structure of the theory of cardinal numbers. The phenomenological approach is confronted to recent writings, representative of the current implicit epistemology of many working mathematicians. It appears that, if mathematical objects do have a structure, which is one of the main lessons of the contemporary axiomatic method, so do the noetic moments of mathematical knowledge. Though not quite in the original sense of Husserl, "transcendental logic" is obviously the name that has to be given to the mathematical and philosophical theory of these noetic structures. The remaining task, however, is to give a concrete content to this theory. Some categorical notions (adjunction, natural number objects, universal problems) point to the first materials of this forthcoming theory.

CHAPTER 9

HUSSERL BETWEEN FORMALISM AND INTUITIONISM[315]

James Dodd

The subject for which I am asking your attention deals with the foundations of mathematics. To understand the development of the opposing theories existing in this field one must first gain a clear understanding of the concept "science"; for it is as a part of science that mathematics originally took its place in human thought.

L. E. J. Brouwer[316]

Abstract. Where does Husserl stand in the debate between Hilbert and Brouwer on the foundations of mathematics? This paper argues that, despite the fact of his sympathies with Brouwer's intuitionism, Husserl's mature conception of logical objectivity can allow us to appreciate the power of logical analysis as an instrument for mathematics, and by extension Hilbert's finitist program.

Introduction

How can we situate Husserl in the debate on the foundations of mathematics in the 1920s between Hilbert and Brouwer? A look at *Formale und transzendentale Logik* (1929) shows that this question is rather more difficult than it might at first seem. In one sense, all roads lead to Hilbert: it is clear that there is more than a little in common between Hilbert's formal axiomatics and Husserl's theory of manifolds (*Mannigfaltigkeitslehre*),

[315] Research for this paper was conducted at the Husserl Archives at the University of Cologne, and was supported by a generous research fellowship from the Alexander von Humboldt Foundation. I would also like to extend my thanks to Prof. Rudolf Bernet and to the staff of the Husserl Archives in Leuven, who assisted me in checking passages quoted from Husserl's unpublished manuscripts.

[316] Brouwer, L.E.J. "Intuitionism and Formalism," in: D. Jacquette, ed. *Philosophy of Mathematics* (Malden: Blackwell, 2002), p. 269.

L. Boi et al. (eds.), Rediscovering Phenomenology, 267–307.

right down to the notion of completeness, something that Husserl himself stresses.[317] And there is no hint of that disdain for the abuse of logical principles or non-constructive methods in mathematical proof and argumentation. At most one can find an echo of Brouwer's polemical accusation, invited one must say by Hilbert himself, that finitism amounts to a mere empty game with symbols devoid of all sense, and which in the end has no genuine connection to mathematical objectivity at all.[318] But that is mere posturing; a closer look at Hilbert shows a deep commitment to mathematical objectivity: the point of finitism was not to reduce the "thought-content" of mathematics to an empty game, but to navigate infinitistic mathematics with a finitistically secured "technique of our thinking."[319] Stephen Simpson, in a symposium on Hilbert's program a few years ago, put it succinctly:

> A balanced reading shows that Hilbert's overall intention was not to *divest* infinitistic formulas of meaning, but rather to *invest* them with meaning by reference to finitistic mathematics, the meaning of which was unproblematic. [...] The whole drama had the bad effect of lending undeserved respectability to empty formalism. We are still paying the price of Hilbert's rhetorical flourish.[320]

Yet there remains the central role of *intuition* in Husserl's writings on logic and mathematics. This alone does not slide towards Brouwer, for even in Hilbert's finitism, intuition plays a key role, as for example at the beginning of his 1927 "The Foundations of Mathematics." Here Hilbert, after citing the paradoxes generated by the logicist program, emphasizes that all scientific thinking requires the immediate intuition of elements that can be surveyed in their parts, recognized when they occur, follow one

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[317] See §31 in Husserl, E. *Formale und transzendentale Logik*. P. Janssen, ed. Husserliana XVII (Hague: Nijhoff, 1974) [herafter Hua XVII]; translation: *Formal and Transcendental Logic*. D. Cairns, trans. (1969) [hereafter FTL]. Cf. Majer, U. "Husserl and Hilbert on Completeness," in: *Synthese* 110.9 (1997): 37–56; Hill, C.O. "Husserl and Hilbert on Completeness," in: *Husserl or Frege? Meaning, Objectivity and Mathematics.* C.O. Hill and G.E. Rosado, eds. (Chicago: Open Court, 2000): 179–198. [Note: I will be adopting the convention of citing Husserl's works by Husserliana volume number, adding references to English translations when available.]

[318] FTL/Hua XVII §§33–34.

[319] Hilbert, D. "The Foundations of Mathematics," in: J. van Heijenoort, ed. *From Frege to Gödel* (Cambridge: Harvard, 1967), p. 475.

[320] Simpson, S. "Partial Realizations of Hilbert's Program," in: *The Journal of Symbolic Logic* 53:2 (1988), p. 351.

another, or are given as a group – it is just that, in the case of mathematics, such "intuited elements" can be limited to written *signs*. Intuition is also presented here as a kind of restriction, though in a sense different from Brouwer's: whatever the mind may think, whatever it may "reason," such thinking is ultimately dependent upon its ability to intuit elements ordered in such and such a way. Thus the meaning of infinitistic formulas is dependent on limitations of ordering inherent to an intuited, finite multiplicity of elements (signs), each of which is intuited in turn. There is no room in finitism for the mere suggestiveness of an expression: its sense must be unequivocal, which (in part) means, so to speak, *rigidly intuited*.

On a superficial level, this means that the difference between intuitionism and formalism is not so much between the argument that intuition represents a necessary limitation to mathematical thinking, and the argument that it does not, but between two different ways to understand the role of intuition as a condition of mathematical thinking. And the difference is more or less clear: in Brouwer's writings the limits of intuitivity are not identified with the intuitions of signs. Rather, thinking itself, prior to language, is *as such* intuition; for Brouwer, it is even originally *mathematical* intuition. The limitation to intuition in Brouwer is meant to keep the mathematician true to the original sources of the mind, or what makes it possible as mind, and defend it from what he took to be the degrading influence of language and communication.[321] Yet the disagreement with Hilbert does not really come down to the role of language, for the arguments over language (or symbolic manipulation) could be said to come down to a disagreement about the nature of intuition.

A deeper question thus suggests itself. What does intuition accomplish, that makes "mathematical thinking" possible? This question makes Husserl a potentially interesting contributor to the debate between formalism and intuitionism. This connection was already explored to some extent in Husserl's lifetime, most notably in Oskar Becker's *Mathematische Existenz*, which drew from Husserl's VI. Logische Untersuchung.[322] In general the tendency has been, for obvious reasons, to turn to Husserl's early writings when considering phenomenology in light of debates in logic

[321] See in particular Brouwer, "Mathematics, Science, and Language," in: P. Mancosu, ed. *From Brouwer to Hilbert* (1998): 45–53, especially pp. 50–51.
[322] Becker, O. *Mathematische Existenz. Jahrbuch für Philosophie und phänomenologische Forschung* 8: 439–809.

and mathematics. In this essay, however, the focus will be on Husserl's later writings, in particular *Formale und transzendentale Logik* and *Erfahrung und Urteil,* as well as some unpublished manuscripts from the same period. The reason is not because one can find a radically different conception of logical and mathematical thinking in the later Husserl when compared to the earlier; it is just that the phenomenological analyses are in many respects more nuanced and rich. And it is in the mature works that one can find the essentials for a significant contribution to the question about the role of intuition in logical and mathematical thinking.

These essentials can be summed up in the following theses, which will serve as an outline in what follows:

1. *Formal analytics is objectively oriented.* More, it is ultimately oriented to real objects that belong to the "natural" world. This is true even in its modern, mathematical form, a claim supported by theses (2)–(6);
2. *The fundamental structure of objective orientation is established in originary perceptual experience*;
3. *What enriches the sense embodied in perceptual experience is not limited to what has its origin in perceptual experience*;
4. *The intuited, perceptual sense of a given can itself be made into an object within perceptual experience*;
5. *The "object" of the intuited, perceptual sense of a given can be transformed into an intuited, but not originally perceived, "higher order" objectivity.* Such higher order objectivities can in turn enrich lived experiences in general (even originary perceptual experiences), in accordance with thesis (3) above;
6. *The formation and elaboration of formal logic is originally and ultimately, if not proximately, motivated by an interest in evidence (truth, confirmation) with respect to things ("objects") in the natural world.*

The procedure of this paper will be to outline the phenomenological-genetic analyses behind each of these theses, (§§1–6) then conclude with a suggestion with respect to how they may be employed to situate Husserl within the debate between intuitionism and formalism.

1. Formal Analytics is Objectively Oriented

Nevertheless, the objective orientation of formal analytics is fraught with ambiguity, which is Husserl's point of departure in *Formale und* *transzendentale Logik* (hereafter FTL). The ambiguity is twofold: (1) it is ambiguous with respect to the *identity of its object*. Traditionally, logic is the study of those structures that can be found in the products of theoretical activity; the study of *activity qua activity* is usually neglected, though not wholly absent.[323] It is not even absent where the focus is exclusively on the objective structure of expressed assertions (*apophantics*), in that even here judgments are understood to be "insights," thus involving a necessary subjective dimension.[324] The result is that an entire subjective problematic remains latent in the theme of validity.

The second set of ambiguities (2) has to do with the *formal character* of logical assertions. Husserl's preparatory considerations in FTL lead to a characterization of logic as the study of the forms under which an object can be spoken about meaningfully. But how are these forms made available to us? Presumably by way of a reflection in which we make assertions about the "formal sense" of judgments, independent of specific content. But what kind of sense is a "formal" sense? In the Aristotelian tradition, apophantics formulates concepts for and descriptions of judgment forms taken as syntactical structures; it does not, Husserl points out, form a formal concept of "what" it is that is being expressed in these structures, because it quite naturally presupposes that the only relevant sense of "what" can be taken to have already been decided on the level of a content-filled act of judgment. Formal analysis thus terminates in the expression itself as its theme; the apparently "algebraic" use of letters such as "A" and "B" in Aristotle's logical works *does not signify a formal conception of an object*, but instead acts merely as a kind of empty reference to a concrete (non-formal) objectivity, or the only *operative* sense of objectivity as such.[325]

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[323] Husserl cites the Stoic doctrine of λέκτον at FTL 82; Hua XVII 86:39–87:6. Cf. Sextus Empiricus, Adversus Mathematicos VIII 11, 38. Also Bochenski, J. M. *Formale Logik* (Freiburg: Alber, 1956), pp. 126–127. What is of interest here is the subjective focus as well as the denial that this subjectivity takes a psychic-corporeal form: "λέκτον" means: what is meant (σημαίνομενον), or a product of a logical act of presentation (φάντασιαν λόγικην), but which is non-corporeal, unlike the sensible formation "in which" the meant is meant (σεμαίνον) and the thing itself (τυγχάνον). The "object" of Stoic logic, therefore, even if it is the product of thinking, is not thereby something subjective as opposed to something objective, but an *ideality* opposed to both.

[324] FTL 42; Hua XVII 46:3–25.

[325] FTL/ Hua XVII §26a.

Husserl remarks that a purely formal conception or sense of the object was not developed in the history of apophantic logic, but in the history of *pure mathematics*. It is with the systematic re-introduction by Vieta of a line of thinking found in an ancient Greek treatise on methods of calculation by Diophantus that we have the first true example of a conception of an object that can be called properly *formal*.[326] "Object" here is no longer understood as a specific, determinate something to which we can refer arbitrarily, but rather as a "something" which itself has the sense of arbitrariness, thanks to which it is uniquely accessible as a "general something" (or, in mathematics, a "general magnitude") for the method or procedure of investigation. Here, unlike in Aristotle, the usage of an algebraic symbol to mean "anything whatever" actually designates "something" as the basis of various purely formal operations, without any implicit reference to non-formal "objects": its "sense" is thus uniquely expressed in symbols. This conception of object in the algebra of Vieta opens the way for understanding the "theory of arithmetic" as a pure construction of an explicit concept of a "something in general" (*Etwas überhaupt*). Husserl reads this as the first emergence of the notion of what he calls a "pure formal ontology" that finds its mature expression in the Leibnizian notion of a *mathesis universalis*, or the idea that apophantic forms are comparable to mathematical structures, in that we can perform "operations" with the former using calculative-type procedures with symbols.[327] Interpreted as a formal ontology, the laws of *mathesis universalis* would hold for all structures of meaning, as well as the mathematical manifolds, including the metamathematical manifolds of pure formal axiomatics.[328]

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[326] See Klein, J. *Greek Mathematical Thought and the Origin of Algebra.* E. Brann, trans. (Cambridge: MIT, 1968), pp. 150–185. Cf. FTL/Hua XVII §26a.
[327] Cf. Hua XXIV 79–95. On Leibniz's *mathesis universalis* ("universal" because it covers both quantitative and qualitative domains) see Bochenski, *Formale Logik* (New York: Barnes and Noble, 1965), pp. 320–323. The practical aspect of the "algebratisation" of reasoning is emphasized in Boole's *The Mathematical Analysis of Logic* (1847); also cf. Husserl's early review of Schröder's *Algebra der Logik* (1891–1895), where he makes the same point (Hua XXII 22–23). For a more detailed discussion of the history of the emergence of formal-mathematical logic, see Meschkowski, H. *Problemgeschichte der Mathematik* III (Mannheim: Bibliographisches Institut, 1986), pp. 187–268.
[328] For a concise history of the period after Husserl's *Formale und transcendentale Logik* (1928–1929) see again Meschkowski, *Problemgeschichte der Mathematik*, pp. 196–296.

The course of this historical development of the idea of a *mathesis universalis*, Husserl argues, led to a fundamental ambiguity: if the pure structural regularity of judgment forms is ultimately expressed not in purely *apophantic* concepts (such as assertion, or negation taken in an intuitive vs. symbolic-operative sense), but *ontological* concepts (the "something in general" and its derivations), then is it the case that the distinction between a *formal theory of judgments* and a *formal theory of objects* is no longer meaningful? The question is not one of the technical feasibility of collapsing apophantics into formal ontology. The question has to do with the conceptual means with which we are to articulate the rational essence of both disciplines respectively, and thus understand the significance of the reduction.[329] What would we lose, if we treated all judgment forms as derivation-constructs of the "something in general?" Structurally, nothing at all. All apophantic concepts, such as "assertion," "negation," and "conjunction," are *categorial forms* – structures that are the result of *syntactical activity*. But this is equally true of formal ontological concepts, i.e. the conceptions of the "something in general" and its derivations – they are all syntactical complexes that appear, or arise, *only in judgments*, in Husserl's sense of "judgment."[330]

More, that formal apophantics can be expressed (or constructed) using pure ontological concepts does not amount to the claim that these formations occur in the natural world. Any decision about relations with determinate objects in the world is precisely what the formal concept of the object evades. Instead what we have is the idea of an ontology

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For a more detailed consideration of Husserl's logic in light of this history, see in particular Lohmar, D. *Phänomenologie der Mathematik* (Dordrecht: Kluwer, 1989); as well as the classic discussions in: Bachelard, S. *La Logique de Husserl* (Paris: PUF, 1957), pp. 101f; and Cavaillès, J. *Sur la logique et la théorie de la science* (Paris: PUF, 1960).

[329] Cf. FTL/Hua XVII §25.

[330] (a) Judgments are syntactical structures that have their origin in cognitive activity: "Judgments are there for us originally in judicative activities. Every work of cognition [*alle Erkenntnisarbeit*] is a multiple and unitary psychic activity in which cognitional formations [*Erkenntnisgebilde*] originate." FTL 81; Hua XVII 85. This is the origin of the difficulty in fixing the ideality of apophantic formations (*Gebilde*) – they are too close to subjectivity. (b) Husserl argues in FTL/Hua XVII §39 for a wider conception of judging (not to be confused with the "broadest concept of judgment" in §21) that includes non-predicative acts, such as counting, collating, etc. Cf. Hua XII 84–89.

of thoughts, or a kind of "geometry" of thought.[331] The already weak connection of apophantics to any "objectivity" outside of the expression itself is replaced by what seems to be a hermetically sealed field of study that has no need to relate itself to the "world," even that of an extra-mental mathematical reality.

All of this, Husserl argues, requires a deeper reflection, in order to rediscover the sense of the difference between logic as apophantics and logic as formal ontology, even as the structural equivalence is rigorously maintained.[332] What is *structurally* equivalent is not always fully equivalent in *sense*. And in drawing this distinction, as has already been suggested, a decisive role will be played by the concept of *intuition*.

2. The Fundamental Structure of Objective Orientation is Established in Originary Perceptual Experience

Let us take a closer look at the analysis of perception in *Erfahrung und Urteil* (hereafter EU), which serves as an important supplement to FTL. Here,

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[331] (a) This, argues Husserl in FTL/Hua XVII, was Aristotle's principal shortcoming: "In the first place, Aristotle's establishment of analytics as apophantics, as a logic of the predicative statement and, correlatively, of the predicative judgment, proved itself a hindrance. However necessary that was as a beginning, it involved a deeply rooted difficulty […] of abstracting thematically from the judging activity and, while remaining consistent in so doing, regarding the judgment-sphere theoretically as a specific Objective field of apriori ideality, just as the geometer regards the sphere of pure geometrical shapes and the arithmetician regards the sphere of numbers." FTL 81; Hua XVII 85. The *kind of objectivity* proper to judgments is obscured in traditional logic due to a prejudice with respect to the *kind of subjectivity* that is "obviously" active in forming judgments. Only once the former has been developed are we then in a position to re-think the latter: a radicalization of the objective precedes a radical inquiry into the subjective. (b) On the difference between formal and material ontologies, see Ideas I/Hua III,1 §§9–10.

[332] FTL 111; Hua XVII 116:13–19: "We shall now attempt to clarify these two focusings [*doppelte Einstellung*] and to justify originaliter the consequent distinction between apophantic logic (in the broadest sense) and formal ontology – a distinction, however, that is at the same time an equivalence, since it will remain true that the two disciplines, even down to the last detail, stand in perfect correlation throughout and, *for that reason*, must be held to be a single science." For a clear and precise commentary on §§37–54, which will from this point on be our focus, see Lohmar, *Edmund Husserls >Formale und transzendentale Logik<* (Darmstadt: Wissenschaftliche Buchgesellschaft, 2000), pp. 88–112.

as elsewhere, the theme of intuitivity (*Anschaulichkeit*) is coupled with that of givenness (*Gegebenheit*). Givenness in Husserl has two basic senses: (1) *self-givenness* (*Selbstgegebenheit*), the givenness of evidence or clarity; and (2) *original givenness* (*originäre Gegebenheit*), a wider conception that does not necessarily imply clarity. The insight is that there is something about perception that is more basic than clarity, and which for Husserl gives perception the unique status of being "original consciousness" (*Original-bewusstsein*).[333] The two are intimately connected: self-givenness emerges out of originary givenness – not as a separate movement, but as a formation *within* originary perception. In Leibnizean language, the elaboration of the given does not take place outside of its intuitive presence[334]; likewise for Husserl, there is a progressive emergence of clarity accomplished within perception, making it an example of "thinking in the widest sense."[335] Furthermore, in EU all of the characteristics of thinking necessary for the *development* of sense – clarification, discursivity, and determination – belong to the intuitivity of originary perception, when considered from the perspective of its genesis.

This distinction between "originary" and "self" givenness is refined by Husserl in EU into a double sense of intuition: (1) On the one hand, the given is grasped (*erfasst*) as a "whole," a fundamental moment of non-discursivity in what is otherwise a movement of phases and differentiating moments,[336] while (2) on the other hand, grasping intuition does not

[333] Cf. Hua III,1 11, 81:3 (the expression here is "Urerfahrung"); Hua XI 4:16–17.

[334] Yet for Husserl, the non-intuitive background of ego-life can be "active," thus "articulate" of givenness as well, at least in a limited way. Cf. MS. D14, p. 49a: "Anschaulichwerden ist nicht Aktivierung, aktiv wird auch das Unanschauliche. Es kann in Unanschaulichkeit bald aktiv, bald inaktiv sein, es kann im 'Hintergrunde' sein, für mich gewohnt, also als dasselbe mich affizierend, ich kann in Beziehung darauf aktiv werden – mich darauf richten."

[335] FTL/Hua XVII §3.

[336] Husserl, *Erfahrung und Urteil* (1939), L. Landgrebe ed. (Hamburg: Meiner, 1999) [EU]; *Experience and Judgment*, J. Churchill and Karl Ameriks, trans. (Evanston: Northwestern, 1973) [EJ], §22. The first level (*Stufe*) of "contemplative perception" (*betrachtende Wahrnehmung*) is "the contemplative intuition [betrachtende Anschauung] which *precedes* all explication, the intuition which is directed toward the object 'taken as a whole." EJ 104; EU 114:9–14. Cf. MS. D5, p. 18a: "Die erste Einheit, die sich konstituiert, ist die kontinuierlich bewusste, vorgegebene, in sich teilungslose, noch keine

simply "continue on" as the unity of a static consideration (*Betrachten*): originary intuitivity lends itself, *necessarily*, to the inner development of sub-themes that nevertheless belong to the same unity of intuition. The whole is not left behind, but "continues on" in an "inner horizon." The thetic character of an *unfolding* interest in a given is thus, in its essence, *poly*-thetic.[337]

Originary givenness is already intentional fulfilment; but this fullness achieves complexity only thanks to its development in the polythetic interest of an I. This process is what Husserl calls *explication*: "The process of explication in its originality is that in which an object given at first hand is brought to explicit intuition."[338] Yet the intuition becomes more explicit only when the I *pushes aside* the "whole" and considers a particular feature or aspect, progressing along the lines of what Husserl calls a double formation of sense (*zweifache Sinnbildung*), a basic structure of the genesis of self-givenness: on the one hand there is that from which the I turns away, in order to approach it – the *substrate*; on the other hand, that towards which the I turns, but only in order to fix that which has been left behind – the *determination* (*Bestimmung*).[339]

Formed within the originality of perception, "determinateness" occurs thanks to the coincidence of moments within the intuitive fullness of original consciousness; and it occurs only to the extent to which its prominence also converges (*deckt*) with an initial theme – thus *carrying it along or forward*. The moment of determination is an *extension* of intuition that fills a place where, so to speak, the intuitivity of the perception is able to be taken. Likewise, "substrate" here means something *carried over* into its own determination within a unity of convergence (*Deckungseinheit*); on the other hand, it is also being "pushed back," and in this manner kept in

Stücke, noch keine Eigenschaft in sich abgeteilt und noch nicht durch Teilungen hindurch sich als Identität bewusst gebende Einheit; auch sie ist schon intentionale Einheit, aber die Bewusstseinsintention ist noch ungeschieden, völlig ohne 'analytische Synthesis,' der intentionale Gegenstand ist noch Einheit eines völlig unexplizierten Flusses; und sie ist noch Einheit vor aller 'Aufmerksamkeit.' "

337 EJ 112; EU 124:17–23. Cf. Hua XXXI 18:28–34.

338 EJ 114; EU 127:10–12.

339 EJ 114; EU 126:22–127:9. Cf. Hua XXXI 20:21–36.

tension with its "own" determination.[340] In other words, what Husserl is describing is a special form of temporal-ecstatic *association*: substrate and determination are bound within one consciousness; not identified, but kept within a unified tension with one another.[341]

The temporality (*Zeitlichkeit*) of this association is a kind of *memory*, what Husserl calls a "retaining-in-grasp" (*Im-Griff-Behalten*). Retained in grasp, the substrate is something "grasped" *qua* particular formation of the past; it bears on the present determination only to the extent to which a past in general can bear on something present. Likewise, determination is what it is in perception because its active apprehension is oriented *towards* the past, towards the substratum with which it is "associated."[342] This temporal-synthetic relationship of elements represents a basic condition for a *progressive* enrichment of sense within the intuitivity of an unfolding

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[340] EJ/EU §24b. Cf. Hua XXXI §6. (a) Explication is thus an originary, evident identification of the "being" of the object, where these identifications themselves can be explicated in turn: "[…] sein Sein 'bestätigt' im Rotsein, im Raumsein, in Verschiedenes-Sein und eben doch derselbe Gegenstand sein. Und jedes solche sein Sein Explizierende ist selbst ein Identisches und als das selbst wieder evtl. sich auslegend in seine Explikate." MS. D1, 5b. (b) Thus the substrate "is" only through its accidents: "[…] accidentiell heißt nicht zufällig, sofern jede Substanz notwendig diese Akzidenzien hat, und nur ist, was sie ist, als sie habende, und die bestimmte Substanz als diese nur ist, was sie ist, in diesen Akzidenzien. Substanz ist hier immer verstanden als die phänomenale Einheit selbst, das *tauton*, gegenüber dem thateron in dem jenes sich begründet und gibt." MS. D8, p. 39b.

[341] EJ 116; EU 129:29–130:6. There is striking resemblance to Brouwer's conception of mathematical intuition, e.g., "[Intuitionism] has recovered by abandoning Kant's apriority of space but adhering more resolutely to the apriority of time. This neo-intuitionism considers the falling apart of moments of life into qualitatively different parts, to be reunited only while remaining separated by time as the fundamental phenomenon of the human intellect, passing by abstracting from its emotional content into the fundamental phenomenon of mathematical thinking, the intuition of the base two-oneness." Brouwer, "Intuitionism and Formalism," in: *Philosophy of Mathematics*, p. 271.

[342] There is an important difference with respect to retaining-in-grasp on the level of explication when compared to the level of pure affectivity, which corresponds to the distinction between the temporality belonging to the *content* of the act and the temporality belonging to the *act itself*. The latter is the subject of Husserl's writings on "inner time consciousness" (Hua X, XXXIII), which is presupposed by the analyses in EU, and which provides an analysis of the temporal dynamic thanks to which consciousness is present to itself as a past.

perception, or the constitution of the *sense in which things are given* (their *Sinngehalt*). That is, temporalization provides the structure of an internal plurality of content with respect to a given "identical" object:

Exactly as in continuous simple apprehension, therefore, there is *at each stage of explication a retaining-in-grasp of the substrate*. But here the retaining-in-grasp is totally different from that which is likewise under consideration in simple apprehension. That is, the apprehension of the object which is included in the constant retaining-in-grasp of the substrate takes upon itself, step by step, all of the particularities which have been thrown into relief: the having-in-grasp of the object being explicated is not a having-in-grasp which is unchanged with regard to content, i.e., a still-having-in-grasp of the *same*, "such as" it was for consciousness before this stage; on the contrary, thanks to constantly new partial coincidences, it is an always different having-in-grasp.[343]

Temporalization provides for an inner plurality of content by setting into place a "past," not as a store of dead memories, but as a living past in which the present is itself continuously embodied; as such, the past (in this particular sense) becomes the bearer of sense (*Sinn*), insofar as it is constituted as the space of this embodiment:

In every step, what is gotten hold of as singular is incorporated [*einverleibt*] by the coincidence into the sense content of the substrate. The individual graspings are transformed, not into merely retentive individual graspings such as occur when something is still retained in simple contemplation or when one passes on to a new object, but into *modifications of a total grasp*, in other words, into enrichments of its content.[344]

For Husserl, intuitivity, in its most original form, that of perception, is constituted in such a way that not only does it bear "sense," but also contains the genetic structures necessary for its progressive development and enrichment. Yet this presence of meaning in perception is passive: however open to enrichment a perception may be, all of these moments of sense are present only as passive sedimentations within perceptual intuition. The I does not make the sense or meaning present in perceptual intuitivity its theme; thus originary perception is a kind of "thinking," but only because it is an activity that can become passively *laden with sense*; it is not yet a *making sense out of things*. Still, we have a fundamental structure of objective orientation, even active orientation, already in place: *all objective orientation puts into play a development, or progressive enrichment of sense*.

[343] EJ 118; EU 132:12–25.
[344] EJ 118; EU 132:25–33.

3. What Enriches the Sense Embodied in Perceptual Experience is not Limited to What has its Origin in Perceptual Experience

Basic to Husserl's analyses in EU is the argument that the unity of meaning in originary perception has both an *objective* and a *subjective* side. (1) The objective side is that of an *objective time*, or the time that belongs to objects insofar as they are objects. But time in all its forms is the condition of the being-given-"together" of a multiplicity; more, "objective time" is a structure of givenness: to be "given" is to be given time, or *individuated*. Thus in their individuation, or their "time," things are given "together," and this being-together structures the movement of their sense-determination.

All perceptual sense-content for Husserl bears the stamp of individuation, and can be studied from the perspective of individuation as a temporal process. This implies that at each and every step of its sense determination the given is "bound" or "coordinated" with the co-given, independently of any combining activities of the I:

> With these comments, it has become evident that a *plurality*, a mere coexistence of pregiven individual objects, is a *unity of connection*: not a categorial unity produced in a creative spontaneity, but a unity of the same sort as that of a particular individual. Certainly, it is not itself an individual, but it has the basic phenomenological property of all simply given objectivities: namely, that it must be given originally and as a sensuous unity and that, for it, all active apprehension requires a unitary pregivenness of sensuousness. […] Accordingly, the temporal form is not only a form of individuals, insofar as these are enduring individuals [*daurende Individuen*], but it also has, further, the function of uniting individuals in a unity of connection.[345]

This original passive multiplicity is an *original objectivity*. The "objective time" of individuation positions the given with respect to its coordination with others in an originary, but non-thematic "world-time."[346] In EU, the genetic analysis of the emergence of a world-time in originary consciousness hinges on a twofold interpretation of *impression*: on the one hand, (a) all egological orientation to the given is in the mode of an impressional consciousness; on the other hand, (b) the intuitive presence of the

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[345] EJ 158; EU 182:15–26, 182:28–32.

[346] Cf. MS C17 III, p. 51b: "Die 'objektive' Zeit ist selbst starre, verharrende Zeit gegenüber den zeitigenden Modis – aktuelle Gegenwart, aktuelle Vergangenheit und Zukunft."

given *belongs to objectivity* – impressional "position" in consciousness is not merely a mode of *consciousness*, but a position in the *world*.

Take, e.g., a memory. As an instance of (b), the remembered has a position in the world, one that represents a kind of break with the position in world-time that belongs to the field of what is perceived "now." The remembered no longer belongs to this nexus; it lies outside of "this" time, "this" collection of perceptions. The remembered table that used to be in this corner cannot appear in the same perceptual unity side by side with the table at which I am now working; I cannot "see" them both within the same perceptual field. The intuitive individuation of the table that I remember is in a different *place* than the one where I sit, there is an irremovable objective *distance* between the two *in time*.

Yet there is also a unity. Thanks to (a), the impressional modality of all consciousness, the I is "affected" by both positions "at the same time." Both the remembered and the perceived are given different positions within "one" time, but this objective order nevertheless unfolds within the impressionality of consciousness that is itself not located "in" objective time. The givenness of originary impression, of the originary consciousness of perception, thus *cuts across both phenomenological and objective time*.[347]

This leads Husserl to an appropriation of the Kantian thesis that time is a form of sensibility: intuitively given in the givenness of a given is a time that, in turn, fits a priori into the one given time:

> Prior to all questions about objective reality – prior to the question concerning what gives priority to certain "appearances," to intentional objects which are self-giving in intuitive experiences, by reason of which we bestow on them the predicate "true" or "real object" – is the fact of the essential characteristic of all "appearances," of the true as well as those shown to be null, namely, that they are *time-giving*, and this in such a way that all given times become part of *one time*. Thus, all perceived, all perceptible individuals have the common form of time. [...] But, from the first, "form" designates here the character which necessarily precedes all others in the possibility of an intuitive unity.[348]

[347] EJ 164; EU 190:21–32.
[348] EJ 164–165; EU 191:6–17, 191:20–22.

The result is that the context of intuition – the context into which what is intuited is placed, in accordance with the "form" of sensibility – necessarily transcends individual perception. For the full context of objective time, thus the full field of impressionality, can never be simply "seen" in a given perception, or even in the sum total of perceptions that I myself live through individually.

Let us now turn to (2), the "subjective" side of the unity of intuition. In EU, Husserl pursues a comparison of perception and imagination, or the contrast between a position within objective time with a position that lies outside of the context of objective time altogether. When we imagine something, there is again a "break" with perception, as was the case with memory; breaking with the actuality of the moment, memories, and imaginations are what Husserl calls non-genuine givens or presentations (*Vergegenwärtigungen*). Yet this is not a break with intuitivity. Imagination, like memory, has its own intuitive fullness – the imagined has its own particular features, characteristics, relations to its surroundings, though they are all "imagined." The imagined also has its own time, or duration in consciousness; the imagined is an appearance in the duration of its becoming. Imagination even has its own context, including possible lines of development. Developing the course of an imagination, possibilities become more and more fixed, the appearing of imagined things takes certain directions of which I am more or less in control. In short, imagination has all of the marks of a world, though it is an imaginary world in which imaginary events occur to imaginary people and so on.

Nevertheless, even if the imagined has its "time," it is a "time" that is not "in time"; it is a time, so to speak, which does not "give" itself as a position in the world.[349] Or rather, the given in imagination has a positionality, but it is position without a position; at the most it is a quasi-position, and thus has its "place" only in what Husserl calls a *quasi-time*:

But one thing which distinguishes actually existing objects is necessarily lacking in the mere fiction: *absolute temporal position*, "actual" time, as absolute, rigorous uniqueness of the individual content given in temporal form [die absolute Zeitlage, die 'wirkliche' Zeit, als absolute, ernstliche Einmaligkeit des in Zeitgestalt gegebenen individuellen Inhaltes]. To put it more plainly: time is certainly represented in imagination, and even represented

[349] EJ 168; EU 196:28–29: "It [the imagined] is a temporal object, it has its time. And yet it is not in time."

intuitively, but it is *a time without actual, strict localization of position* – it is, precisely, a *quasi-time*.[350]

Thus in imagination we have a unity of intuition (quasi-individuation, quasi-world) that not only passes beyond perception, but beyond world-positionality as well. And it is because of the peculiar contextless character of an imagined context, that the question of whether or not there can be a unity of intuition between perception and imagination will lead Husserl to the *widest conception of the unity of intuition as such*.[351] Or, to ask the question in another way: *is there another form of constituted time*, that is, *does time appear as a context in another form other than objective time*? If so, then this "context," intuitive as well, will encompass the broadest sense of what can be present in intuition, thus the widest scope of potential sources of sense-enrichment.

Husserl's answer to this question takes place in two parts. The first (a) is the identification of the basis of the unity of intuition (in the broadest sense) not in objective-phenomenal time, but in *subjective-phenomenal time*, or the time that belongs to the appearance of subjectivity itself:

All the lived experiences of an ego have their temporal unity; they are constituted in the absolute flow of internal time-consciousness and in it have their absolute position and uniqueness, their unique appearance in an absolute now, after which they retentionally fade away and sink back into the past. Naturally, *this time of the lived experiences* [*Zeit der Erlebnisse*] *is not the time of the intentional objectivities in the lived experiences* [*Zeit der in den Erlebnissen intentionalen Gegenständlichkeit*].[352]

But that is only the first step. And looking at Husserl's analyses of inner time consciousness, we can see why: the most original temporalization of subjectivity is not in and of itself a coordination or even, to push the point, a synthesis in an "objective" sense, where the end result could stand on its own as an ordering of the given.[353] This assessment lies behind Husserl's claim in EU that the temporal unity of the lived experiences of an I (their

[350] EJ 169; EU 197:13–21.
[351] EJ/EU §41. Cf. Depraz, N. "Imagination and Passivity," in: *Alterity and Facticity. New perspectives on Husserl*. ed. N. Depraz and D. Zahavi (Dordrecht: Kluwer, 1998).
[352] EJ 175; EU 205:11–19.
[353] This contrast is more nuanced in Husserl than we are making it here. (a) The positioning of the duration of the given in inner-time does not bind it to other positions within the same; but then again not even objective time does that: "Die Dauer verbindet

"inner time") is not in itself sufficient to provide for a unity of intuition – for it does not in and of itself represent a context of intuition.[354] But it does serve as a *basis*, insofar as time-consciousness is the basis for *associative syntheses*. This is (b), the second, decisive step: *association* as a passive synthesis within the horizon of internal time-consciousness, as a non-coordinated, *subjective* flow fixes the widest concept of the unity of intuition:

> *A unity of intuition*, a unified assemblage of objects of intuition (it being of small importance whether perceived or presentified), means, therefore (since we are in the sphere of individual or quasi-individual objects), *a unity of time in which these objects are intuitively together*. [...] This unity of intuition, originally established by association, is such, therefore, that it is possible, not only between perceptions and memories of the same ego, but also between positional and imaginary intuitions. With this we have attained the *broadest concept of the unity of intuition* [...].[355]

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nicht, wie überhaupt nicht die Zeit." MS. D5, p. 19b. (b) Insofar as this positioning of the given in time does not yield any "real" properties, this leads Husserl to a recognition of the prominence of *predication* in the constitution (or "appearance") of the real: "Jedenfalls birgt der Begriff des Realen eine gewisse Struktur in sich und weist uns hin auf eine Bevorzugung der Prädikationsrichtung in Hinsicht auf reale Prädikate als solche, welche den individuellen Gegenstand eigentlich 'ausmachen,' konstituieren, während die Aussagen über Zeitliches und besonders Zeitlage nicht Aussagen sind über das, was er ist, sondern über das Wie seiner Erstreckung in der Zeit und Lage in der Zeit, die die identisch eine und doch nicht verbindende Zeit ist." MS. D5, p. 20b.

[354] EJ 176; EU 206:22–207:2.

[355] EJ 181–182; EU 213:24–214:5. (a) Association is the unity of a subjective life, not just of subjective time – or better, life is the form subjective time takes in order to be a unity. And this unity of life not only includes the sphere of the originarily given, but of the given as sedimented as well. Husserl: "Die ganze Einheit des Lebens ist Einheit aus universaler Verschmelzung, also aus Assoziation. Aber dann ist eben ein besonderer Modus der Assoziation [...], dass sie weckende und besondere zeitliche Konfigurationen aus der Sedimentierung einheitlich weckende Assoziation werden kann." MS. D14, p. 12. (b) Unlike a "real" unity, however, this life-unity is "loose": "Dann hätten wir für die Mehrheit dieser Regionen eine losere Einheit und eine einzige Zeitlichkeit, an der dann alle Onta aller Regionen zusammen Anteil haben – eine Zeit als Form der universalen Koexistenz, die, wenn auch lose, durch Assoziation gestiftete Verbindung ist, hergestellt dadurch, daß alle Sonderzeiten dadurch assoziativ einig sind, daß sie noetisch eine einzige noetische Zeit, eben die noetische Form des urströmenden Lebens, konstituieren." MS. C15, pp. 3a/b.

Association constitutes a context; yet the intuitive unity that results retains the contextlessness of the non-worldly, comparable to the imagined. Thus when Husserl argues that there is a special type of constituted time in which an imagination can enter into an intuitive unity with a perception, whereby an element of the one is "associated" with an element of the other, this does not mean that what is imaginatively intuited now belongs to the context of the world. Rather, the "broadest concept of the unity of intuition" is that of a unity in which any given element or content "present" in original time, anything that has been "lived through" in the original impressional syntheses of time-consciousness, can be "given" in a non-objective, associative context.

The consequences of this move are important and diverse. For one, intuition is extended in such a manner that intuitivity or evidence can take a form which, unlike the cases of memory and perception, does not represent a break with the past.[356] Likewise, intuitive evidence in general does not need to break with horizonal consciousness in order to be something "established." There is no *absolute* separation within intuition between what is actual and merely potential, given and non-given. Intuition is not limited to the confines of originary givenness, nor even to an association in which one of the termini must be "originarily" given. The unity of intuition in the broadest sense is insensitive to all of these contexts, for it is founded not on the basis of the original givenness of content, but on the *subjectivity in which content is given*. And it is *generative* – i.e., its unity takes the specific form of the associative synthesis of a plurality of "contexts" within the stream of lived experiences.[357]

Nevertheless, perception retains for Husserl its pre-eminence as the standard for both intuition and evidence, the paradigm of the intuitively given; but at the same time intuition and evidence are extended to include those objectivities and unities of givenness which can never be "seen" or "given" in the form of a perception, though they can still be "intuited" in

[356] EJ 178–179; EU 209:25–29.

[357] EJ/EU §43c. Cf. MS. D5, p. 3b: "Doch genauer gesprochen: wir betrachten Deckungen, die sozusagen unempfindlich sind gegen den Unterschied von Erfahrung und Quasierfahrung, die also bestehen bleiben, wenn wir Erfahrungen in Quasierfahrungen verwandelt denken (was apriori immer möglich ist) bei den Gliedern der verbundenen Akte."

the broad sense defined. The "extension" of intuition thus passes beyond its own original, and in that sense "genuine" sense in perception; in association intuition is, so to speak, over-extended, so that it lies on the other side of itself. It is on this "other side" of perception that an activity which makes sense or meaning itself into an intuited object has its origin. But before we move to that thesis, we need to take a look at the rather surprising argument in EU that the thematization of meaning occurs, to some extent, *within perception itself*, thereby providing the foundation for the possibility of a thinking that operates outside of the confines of natural experience.

4. The Intuited, Perceptual Sense of a Given Can Itself be Made into an "Object" or "Theme" Within Perceptual Experience

The purpose of §3 was to prepare for the argument that judgment can be understood as having its *own unique intuitive content*, even if judging is not thereby identified as a mode of perceiving. The argument will be that there are passive syntheses of intuitive givenness even in the most highly developed activities of the I beyond originary perception.[358] Nevertheless, the original emergence of a content-rich judging activity takes place for Husserl within perception, and based upon its own resources.

The very origin of judgment for Husserl lies in a shift within the interested directedness of the perceiving I. The shift is from the interest that the I takes in *objects* to an interest in *knowledge of objects*. "Knowledge" is meant here in the most primitive of forms: the constitution of sense in explication described above, in which an objective sense of the object is established in a passive manner. Recall that for Husserl even on this level sense-enrichment has a permanence thanks to an inner tradition, thus a meaningfulness that is not left behind but continues to determine the horizon of expectation. Nevertheless, it cannot be said to be something

[358] Based on this argument, one could propose an alternative kind of "intuitionism" in mathematics to that of Parsons, where there would be more than an analogy between the cognitive relation of perceiving a world and the cognitive relation of thinking about mathematical entities. Cf. Parsons, C. "Mathematical Intuition," in: *Proceedings of the Aristotelian Society* 80 (1980).

that the I has as a *theme*; the "tradition" belongs more to the experiences of the I than to the I itself. To be sure, the I "has" a world of sense, insofar as it is the I of lived experiences; but it does not yet have a sense of the world that it can properly call its own. In other words, the interest of the I has not yet turned to *cognition* as a new formation of what it already has.[359]

The shift is towards an interest in the acquisition of a sense of the world in the form of a *confirmation*: "Confirmation of what exists [*Feststellung des Seienden*], how and what it is, is the sense of all cognitive activity."[360] Instead of being simply the pole of an attentiveness thanks to which the object of lived experience takes on sense, the I is now directed towards the possibility of an experience in which this objective sense itself becomes a theme. Instead of the interest remaining within the receptive explication of givenness, the direction of interest now turns towards the established content of receptivity, *as something established*. Yet this shift in interest is not a shift away from the original theme of perception; it is only that what is perceived is now being "aimed at" in a way that will "fix" its sense "once and for all." The I does not turn away from the object to its sense or meaning; it just casts, as it were, a broader arc around what can be considered as relevant, as "interesting" about the given: it is not simply the object in its determinations, but *that* the object is what it is, *that* in its givenness the given shows itself as "such and such" – and *that* this is something that the I "knows."

In other words, in this shift of interest the I *actively grasps the self of the thing*, the result of which is a more explicit "having" of the thing in its selfhood (*Selbsthabe*). Perception itself is a kind of self-having, but there self-having cleaves closely to the originary emergence of the self in consciousness. This "self" of the thing becomes more evident in the wake of an interest not in its original emergence, but in the fixing of its content in an explicit, creative act (*Erzeugung*) on the part of the I. Yet Husserl emphasizes the creative activity of the I while at the same time defending the notion that this creativity never leaves behind a relation to things, that it is even this very relation that is being generated.[361]

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[359] EJ/EU §47. Cf. Hua XXXI §4.

[360] EJ 197; EU 231:19–20.

[361] EJ 200; EU 235:20–30: "All cognitive activity [*erkennende Tätigkeit*] is ultimately referred to the substrates of the judgment [*bezogen auf Urteilssubstrate*] […]. The goal

There are two distinctions that should be kept in mind here. (1) The interest or occupation with things, in whatever form, is not the same as the occupation with the experience itself, that is, with the lived experience in which these things are given.[362] This leads to (2), the distinction between the relation to the object and the activity in which this relation is "produced" (*hergestellt*).[363] Unlike in the case of receptivity, where the formation of the relation to the given is nothing produced, in cognition all such formation is grounded in creative activity. What this distinction implies is that the question of the origin of judgment is not identical with the question of the origin of the act of judging; if we presuppose the act of judging, we have not, from a genetic point of view, presupposed the appearance of a judgment.

Take, e.g., the perception of a copy of Euclid's *Elements* lying on a table. As a theme, the given is given as a unity of substrate and determination established prior to the activity of judging. What takes place in the creative act of judging is not sense per se, but a particular manner in which sense is *formed*. This is a continuation, but not a simple extension; it involves a re-formation, or repetition of what is already in place. It is the repetition of the manner in which the substrate is "given as…," or present, as Husserl puts it, "in" its determinations. But in judging, this "given as…" *itself* becomes an explicit theme – and in this way the I, never losing sight of the object, creatively intensifies the richness of its focus. The object, no longer simply a passive retentional presence of something determined in its determinations, now becomes the theme of an activity that takes interest in its very "being-in" its determinations as their *subject*. The passive temporal determination of the substratum is replaced by the sense formation of a subject of determination: the I now takes up an interest in the thing as a *subject of possible predication*. What can be predicated? In its most basic form, within perceptual experience itself, whatever it was that had been passively synthesized as the determination of the self within the "Sich-bestimmen als…" of the substrate. *The judging*

of this activity is not the *production of objects* but a *production of the knowledge of a self-given object*, therefore the possession of this object in itself as that which is permanently identifiable anew [*also seiner Selbsthabe als eines dauernd wieder Identifizierbaren*]."

362 FTL 111; Hua XVII 116:30–35.

363 FTL 111–112; Hua XVII 116:36–117:14.

I repeats this passive self-becoming of the theme in its own activity of predicating. Thus: the I makes the determination of the thing its theme by *actively performing* what had been *passively accomplished* on the level of receptivity.[364]

It should be emphasized that, in original judging activity, the I is not turned towards the *judgment* but remains directed towards the *object*.[365] The object is that which is being judged in judging, and the givenness of which takes on the form of a judgment. The difference with receptivity lies solely in a "creative" modification of form – the passive synthesis of sense has been made into an active synthesis of sense, the implicit has been made explicit. *Thus what takes shape, what is originally "formed" in judging, is synthesis as such.* A judgment is a formed synthesis shaped by the act of judging within the unity of perceptual experience.

Characteristically, Husserl describes the formation of synthesis in judgment as *double*, reflecting the original passive synthesis in perception. In the latter, the substrate took shape as a formation of the given coordinated with the current moment of givenness in a special form of retentional grasp that Husserl called "Im-Griff-behalten"; vice versa, the current moment takes shape as a determination in a passive convergence (*Deckung*) with the substrate being "held-in-grasp." Since in judging this convergence is no longer passive, but active, it is no longer a convergence at all, but instead an *active attribution* of a determination "to" the substrate. And insofar as the relation can be described in terms of the place-forms of the relata within the relation, then the termini in the case of judgments are formed in such a way that the substrate becomes the *subject that bears attributes*, and the determination becomes the attribute ascribed to the newly formed subject – in short, the one takes on the form of a subject-theme, the other the form of a attribution-theme.[366]

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[364] EJ 208; EU 245:15–23: "[…] in order for the substrate of the explication to become a subject and for the explicates to become predicates, it is necessary that the regard turn back to the unity which is passively preconstituted within the receptive activity of the process of explication and is in a sense concealed. *Being turned toward this unity in order to apprehend it* implies *repeating the process in a changed attitude*, making an active synthesis form a passive one."

[365] FTL 112; XVII 117:18–23.

[366] EJ/EU §50b.

Neither of these place-forms, Husserl argues, are forms of *synthesis*, though they do have a law-like relationship to forms of synthesis or combination (syntaxes): for example, only that which has the form of attributability, or substantivity, can in turn take on the role of a subject in a judgment; vice versa, only that which has the form of transferability, or adjectivity, can take the role of a predicate.[367] This is the distinction between what Husserl calls "core forms" (*Kernformen*) and the function-forms within syntactically synthesized wholes (judgments) as such. One should be clear, however, about just what it is that is being formed, thus what develops in judging: it is *neither* the thing, *nor* the act of judging. It is the *sense* of the thing, its mode of givenness. The argument is not that things are being "given" sense, that only in judging do things have meaning; on the contrary, for Husserl meaning is more primitive than judgment. But sense *is* here being given a structure that it did not have before; it is being placed within an apparatus, called the "judgment," in which it enjoys a visibility, and with that a fixity, that it did not have in perception.

What does primitive judgment accomplish? The answer is twofold: (1) insofar as judgment actively repeats a passive synthesis, it instills an internal differentiation into the being perceived of the perceived – a *diairesis*, to evoke the traditional Aristotelian definition of judgment. Creative activity breaks an original unity, precisely in order to re-accomplish it in activity.[368] The result is that determination is no longer the passive moment of perceptual self-givenness, but the active formulation of the

[367] Also cf. Beilage I to FTL/Hua XVII.

[368] (a) EJ 209; EU 246:26–36: "But then there is something new; namely, the ego in its interest turns back to *S* and, for example, first taking *p* particularly in grasp again and directing a new ray of attention [*Blickstrahl*] toward it, becomes aware of the enrichment of sense and is saturated with it, while it again reproduces it by an original activity in a new passage to *p*; and thus for each of the determinations. *Determination always has two members*. Thus is described the process of predication which tradition always already had in view under the terms 'synthesis' and 'diaeresis' without actually being able to come to grips with it." (b) It is instructive here to recall the split between Husserl and Brentano on the question of judgment: Brentano rejects the synthesis/diaeresis definition, while Husserl wants to re-incorporate it back into an essentially Brentanian argument about the presence of the judged. Cf. on this point Benoist, J. *Phénoménologie, sémantique, ontologie* (Paris, PUF, 1997), pp. 84–97.

given in the new givenness of the judged as judged. We arrive at the same goal, but by a very different type of route. (2) This new givenness takes on the character of something that itself has been determined through the activity of judging. What is "given" becomes closely identified with what is understood, actively *grasped*, and not simply "received." For Husserl, this accomplishment, established within perceptual experience, represents a proto-knowledge that functions as a foundational structure upon which the edifice of all knowledge ultimately rests.

5. The "Object" of Intuited, Perceptual Givenness can be Transformed into an Intuited, but not Originally Perceived, "Higher Order" Objectivity, Which in Turn Enriches Lived Experience

A judgment is something independent; as such, it can in turn serve as the basis for further judgments.[369] However, as noted above, in the judging itself the judgment is not the theme, only accomplished in the course of the activity of judging. The "judgment," as Husserl puts it, is "pre-constituted." To make judgment a theme requires a new, special mode of regard, one that necessarily posits the judgment qua object over and against the original theme to which it owes its passive pre-constitution. Nevertheless, even as posited, the state of affairs (*Sachverhalt*) belongs *originally* to an activity directed *at objects*, and not to the activity of positing states of affairs:

> Whereas originally S was the substrate-object and was determined as having the property p, now the substrate-object is "this, that S is p": the affair complex [Sachverhalt] which was indeed constituted before but was not then the object-about-which [Gegenstand-worüber]. The same operation, when exercised on p, changes it into the nominalized P, the P that has become the judgment-substrate (the red, the quickness, or whatever it is). Accordingly these changes of form (syntactical changes), which are effected in the judging, do not in any respect alter the fact that we are directed to *something objective* [*gegenständlich* gerichtet]. The new syntactical forms themselves make their appearance in the judgment as forms shaping that which is objective; though we, as directed to this

[369] EJ 214; EU 253:11–16: "In any case, every judgment has a closing [*thematischen Abschluß*] in itself; it is in itself something thematically independent. And yet it is a member of an open and, according to ideal possibility, constantly widening thematic complex, which therefore it not closed." Cf. Hua XXXI 54:29–37.

itself, do not take them to be an intrinsic part of it. For example, we say: The same predicatively formed affair-complex is meant judgingly – only in different forms – as "S is p" and as "this, that S is p,…"; the same property is meant, now as the predicate "red" and again as the subject "this red."[370]

The development of this objectivity thus builds on itself, on its own formations; each step does not in and of itself really constitute a new theme, even if it does represent a new "objectivity" – each is a step within an progressive givenness that always remains the givenness of that theme which now "stands in" the judgment. This identity of a theme through the constitution of various objectivities leads Husserl to the distinction between the state of affairs (*Sachverhalt*) itself and the *proposition* (*Urteilssatz*): when one "thinks" of the same state of affairs in different ways, the *same objectivity* (of the given in the form of a state of affairs) is in play under the guise of different objective formations, which do not collapse into the "originary" objectivity of the given. We have, within the unity of judgment, the constitution of a *double objectivity*:[371] (a) the multileveled formation of sense (*Bedeutungsgehalt*), the complex deposit or sediment (*Niederschläge*) of what has been made objective ("judged") in previous phases of the experience; (b) a *total* propositional formation with respect to a *single unified theme*, the state of affairs, however complex the sedimented content of (a). However multifaceted the thinking of the states of affairs may be, it remains "the same," only "meant" in different ways.[372]

This *ideality* of the state of affairs vis-à-vis a manifold of sense is an achievement of the interest in the fixity of sense; the originary formation of the pre-constituted state of affairs is an *idealization* intrinsic to the becoming of judgment as such, part of its fabric.[373] And it is this already accomplished ideality that is *explicitly made a theme* when the

[370] FTL 112–113; Hua XVII 118:3–18.
[371] Cf. EJ/EU §60; Hua XXXI §14.
[372] FTL 113; Hua XVII 118:19–22, 118:25–30.
[373] The fabric of judgment is of course temporality. Cf. Hua XXXI 30:14–20: "Wie das Urteilen ein Werdensprozess ist, so ist das ursprünglich sich vorkonstituierende Objektive, das wir Urteil nennen, eine Werdenseinheit, das Werden ist ein Geschaffenwerden vom Subjekt her, natürlich aus Materialien der Passivität. Das ursprüngliche Selbstsein des Urteils, das der Konstitution, ist ein Sein im Modus des Geschaffenseins, also ein Sein in der Form der Zeitlichkeit."

state of affairs is "declined" (*entnimmt*) from the *given* judgment.[374] This ideal objectivity of the state of affairs is not isolated, but intrinsically related to its sense, which is in turn not an ideal *identity*, but an ideal *variability* that is expressed in the (re)formulations of judging. There is a twofold development within these variations: (1) the ideal identity of the state of affairs, as an objectivity (*Gegenständlichkeit*); and (2) the manifold of sense, or so to speak the manifold objectifications possible within the objectivity of the given. The state of affairs remains indifferent to the manifold of sense insofar as it is an ideal identity; but, on the other hand, it is also enriched, insofar as each formation of sense remains part of the state of affairs as the particular ways in which it has been "thought." This latter relation is the relation between a given and the sedimentations (*Niederschläge*) of sense which make up the full range of its givenness.[375]

In the case of a state of affairs, however, we do not have an object in the sense of an individuum. A state of affairs is a unity of objectification, it is not something "originarily given." But it does enrich the givenness of a given, insofar as the state of affairs represents a higher mode in which the given is explicated; more, it is an explication which is formed in such a way that it can in turn become explicated. The state of affairs is, in a sense, *an intuition which is in turn intuited*. To "know" or intuit a state of affairs, "that the book is blue," is to "see" an articulation of relations that were pre-constituted in perception – i.e., in receptivity.

Let us take a closer look at this process. Describing the givenness of the given, we follow the manner in which a determination enriches a given substrate (the blue book); we can also attend to the manner in which a given determined substrate is given in relations to other objects (the book on the table). These "relations" between book-blue and book-table, however, are present in receptivity only insofar as they belong to the passive situatedness of the elements; they belong, so to speak, to the *given situation of the matter at hand* (*Sachlage*).[376] This situatedness is enriched when articulated in thinking, which actively traces out the manner in which the situation (*Lage*) passively "situates" the matter at hand (*Sache*). This

[374] Cf. EJ/EU §58; Hua XXXI §7.
[375] EJ 243; EU 290:32–291:1. Cf. Hua XXXI 64:3–9.
[376] EJ/EU §59.

active relating (*verhalten*) of *things related one to the another* (*Sachverhalt*) is a sense enrichment (*Sinnzuwachs*) of *the given situation of the matter at hand* (*Sachlage*). But at the same time the tension remains: the articulation of a situation never truly belongs to the situation itself; it is "originary" only from the perspective of activity; in this sense the articulation is an objectivity that is never an "object" or a "given" in the world, but only in reason. States of affairs are, therefore, what Husserl calls *objects of the understanding* (*Verstandesgegenständlichkeiten*).[377]

Thus the "judgment" (*Geurteiltes*) formed in the judging of a given situation of the matter at hand is *both* a sense enrichment of the given *and* an object – it is an object that objectifies, or an "objectivity." However, the full scope of the sense enrichment achieved by judgment is not always objectified in the state of affairs. In fact, only in the most basic judgments of perception can we say that the judgment judged, or the proposition (*Urteilssatz*), and the state of affairs (*Sachverhalt*) are indistinguishable.[378] When thinking develops, when the judgment judged becomes part of an unfolding of different modes or ways of thinking the given, then these different ways in which the judged is "meant" is just as much an enrichment of the horizon of objectivity as it is the objectification of the state of affairs. The unfolding understanding, building on its own objectifications, introduces a dimension of givenness in judgments that is not limited to the unity of a given state of affairs: the "full judgment" involves more than the specific state of affairs that it articulates.

Nevertheless, the core sense is fixed, and formed categorially: e.g. "property" as the result of categorial determination, "collection" as the result of collecting (*Kolligieren*).[379] But syntactical activities can also function as structures into which already formed categorial objectivities such as "property" or "collection" can be "plugged in," so to speak. *The same active judging* can both form a *new* syntactical structure *and at the same time* "take up" already constituted categorial formations and establish them within higher level categorial forms. Thus the "things" that stand in relation to one another in a new syntactical structure can be categorial things; and these "things" retain their ideal objectivity, because, as Husserl argues, all

[377] EJ 238–239; EU 284:18–285:6.
[378] EJ 244; EU 291:8–16. Cf. Hua XXXI 64:27–33.
[379] Cf. FTL/Hua XVII §42c.

judgments are "complete," "once and for all," another thesis that Husserl shares with Brouwer. The "property" remains the same, whether it functions now as subject, now as object, now as state of affairs, now as state of affairs in the function of a premise, etc.[380]

As sense enrichments, states of affairs belong to the objectivity towards which the one who judges is "directed," not simply to see, but to invest the horizon of the thing with sense; thus to lend it a visibility that it did not possess:

> Judging and again judging, the judger acquires for this objectivity the multiple "How it is," individually or universally; he acquires predicatively formed affair-complexes pertaining to it, in which it stands thus and so, and the like – ever new categorial objectivities into which the substrate-objectivity enters in consequence of his judgments, and which themselves become *relatively thematic* and thus in turn undergo determination while at the same time, through them, the first substrate-objectivity, as the *ultimately thematic* one, becomes determined.[381]

The thing finds its conceptualization in the ongoing march of specific thematizations which, step by step and level by level, fix the thing in its "what in the end makes it up," or its *concept*:

> All the various particular formations that he actively acquires in doing so have categorial coherence by virtue of the identity of the substrate-objectivity (itself constituted in a judging identification); and they progressively constitute for the substrate the *determining concept* accruing to it precisely from all these judicative performances – the current "How it is, all told," a concept always in progress, always being further fashioned, and also refashioned.[382]

This is what Husserl calls the constitution of "logical sense."[383] The givenness of the given with respect to its logical sense is the givenness of an *object of knowledge* (*Erkenntnisgegenstand*). This is different from a "logical" determination, or the initial formation of the substrate in the copulative judgment; it is the formation (*Gebilde*) of a sense for something from the perspective of what can be judged about it – what, in short, becomes "true" of the given in the wake of an experience of knowing. This point

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[380] FTL 114–115; Hua XVII 120:1–10.
[381] FTL 115; Hua XVII 120:29–38.
[382] FTL 116; Hua XVII 121:9–17.
[383] Cf. Hua XXXI §13.

is important: the being-true of the given does not occur within judgment, but within an experience oriented by judgments, an experience in which judging builds on judgments that have been "arrived at" by previous activities of judging.

This also means that categorial formations are *transcendent*, in the sense of *valid*:

What the judger has fashioned now in his active thinking, he possesses from now on as an abiding mental acquisition [*bleibenden geistigen Erwerb*]: The active acceptance changes for him into habitual acceptance [*die aktuelle Geltung verwandelt sich ihm in eine habituelle*]. That which has once been given acceptance in an actually constituting activity can emerge again in memory; and it emerges not only as something that was once accepted but as something still accepted [*als noch Geltendes*].[384]

Thus the "interest" in the world develops not as a continuously extending articulation that remains unambiguously "active" in each of its phases. This interested directedness, building on itself, also transcends itself, then turns around to encounter its accomplishments as intuitivities of a particular type. Interest, and the context of what is of relevant importance or weight within an experience, thus rests not only on what the I has found to be valid with respect to such and such a matter at hand, but also on the continued, passive presence of relevant judgments as motivating elements in the movement of interest.[385] This passive, pre-given presence is

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[384] FTL 117; Hua XVII 122:29–35.

[385] (a) "They [interests] all depend on *habitual and reawakable acceptance*, as a continuing acceptance throughout any number of reawakenings [*habituellen und wiederzuweckenden Geltung als Fortgeltung durch alle beliebigen Wiedererweckungen hindurch*]; what exists is what exists 'henceforth' for the judger – as long as he does not give up his 'conviction' and does not cancel the acceptance, which is at the same time a *continuing acceptance* [*die Geltung, die zugleich Fortgeltung ist*]." FTL 118; Hua XVII 123:6–11. (b) Yet this validity is not "in place" as a continuing experience of truth; it is more that such an experience is "repeated," or "found again" as still being in force. Which raises questions: "Was sagt das – eine reproduzierte Geltung hat *noch Geltung*?" MS. C3 III, p. 36a). The key is to understand that this "continuation" is neither a single continuous experience nor a continuum of reproduced experiences, but a lasting functionality of the act throughout its temporal modifications as now, past, repeated, etc. within the living present: "Aber verständlich ist schon soviel, dass die jeweilige Aktgeltung in der strömenden Gegenwart ein Verharren hat, oder dass der Aktus sich zwar wandelt, aber in diesem Wandel doch ein Verharrendes hat als Aktus, als 'Funktion' des Ich." MS. C 3 III, p. 36b.

genetically "under" concepts, but also "in" the concepts themselves, for it is the pre-givenness to which concepts owe the transcendence necessary for their development.[386] Pre-givenness (*Vorgegebenheit*), a central theme in the *Crisis*, is not a state of being before active articulation; its actuality lies "in" its articulation, insofar as it is only "in" the actively articulated or conceptual that the pre-given has its genetic locus.

Because the given first has objective being only in actively thinking experience, we can already point to an Husserlian account of the *formal character* of logic. Logic is the study of categorial formations; and these formations are, as we have seen, not formations of *thinking* but rather the formations of the *conceptual articulations* of objects in experience.[387] Logic is formal in that its focus is purely the syntactical structures of this articulation, the combinations that generate the objects that are found only in the sphere of thinking, not posited in the world.[388] However, at least initially, logic need not have a formal-ontological calling; it can remain ontically directed to categoriality without explicitly developing a concept of the formal object "something in general" cited above. Nevertheless, such a science can serve as a first step towards the insight that judgment-systems are possible which, in their completeness, exhaust the possibilities of conceptuality as such – and in this sense formal considerations can lead from ontic descriptions to ontological claims, so long as the subjective dimension of logic is left untouched.[389]

Here the question arises: what motivates the move towards ontology, if formal-logical considerations can remain within a "naïve" thematization of structure? What is the source of a need for a formal conception of an object? The key for Husserl is the question of *truth*.

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[386] FTL 118; Hua XVII 123:24–29: "Nature as a judgment-formation [*Urteilsgestalt*] – in particular, as a natural-scientific cognition-formation [*Erkenntnisgestalt*] – will of course *have under it* Nature as an experience-formation [*Erfahrungsgestalt*], a unity pertaining to actual and possible experience, one's own and the pooled experience of a community that includes others: But *the under-it is at the same time an in-it* [*das Unter-sich ist zugleich ein In-sich*]."

[387] FTL 124; Hua XVII 124:22–31.

[388] FTL 119; Hua XVII 124:31–35.

[389] FTL 120; Hua XVII 125:16–25.

6. The Formation and Elaboration of Formal Logic is Originally and Ultimately, if not Proximately, Motivated by an Interest in Evidence (Truth, Confirmation) with Respect to Things ("Objects") in the Natural World

That logic is originally directed to objects seems to be, from Husserl's perspective, obvious:[390] "For judging is always believing something, having something 'before one' as existent (*etwas als seiend 'vor sich' haben*), whether one has it there intuitively or non-intuitively."[391] Yet judgments are constituted in the wake of an interest not in *given being*, but in being *secured* in its sense, or more precisely: in its *evidence*. Such an interest takes as its guide the intuitive givenness of categorial formations:

There arise occasionally, even in everyday judging, interests in cognizing in the pre-eminent sense: interests in assurative "*verification*," needs to convince oneself "*by the affairs themselves*" of "how they *actually* are." The categorial formations which previously were simply existing objectivities for the judger, and simply the same ones while the process of identification continues, must be verified by going over to the evidence, the "categorial intuition," in which they would be given originaliter as they "themselves," verified, cognized as *truly and actually existing*.[392]

Verification is in its essence an attunement – though not yet an explicit turning-towards – to the becoming of a judgment, to its fundamental relation to its own possibility within originary givenness as something that can be made, formed, arrived at, experienced. Like judgment itself, confirmation is retrospective; but at the same time it is also forward-directed: to confirm something is to establish that a judgment about the given is one that can be made "again," but now in an intuition that belongs not only to the horizon of *repeatable*, thus *available* knowledge, but to knowledge in a position to articulate a claim to "truth."

As a direct result, an important differentiation within the becoming of judgment first comes to the fore: the difference between *what is meant*, and *what is meant "as meant"*:

Thus a distinction arises occasionally, even for the judger, between the supposed objectivities *as supposed* – purely as the ones that have become posited in such and such a categorial

[390] FTL 121; Hua XVII 126:8–12.
[391] FTL 121; Hua XVII 126:33–35.
[392] FTL 122; Hua XVII 127:16–26.

form in his actions of judgment, purely as what, throughout the synthetic course of his positings, is *positum qua positum* – and the corresponding "*true*" or "*actual*" objectivities, that is: the categorial formations accruing in the distinctive phenomenological form of insights, in the *judging that "gives them-themselves,"* step by step, formation by formation [*kategoriale Gebilde, die in der ausgezeichneten phänomenologischen Gestalt von Einsichten in den Schritt für Schritt, Gebilde für Gebilde 'selbstgebenden' Urteilen erwachsen*].[393]

As a result, the I steps outside of a naïve acceptance of the course of a belief. The difference between the meant and the meant *as meant* arises because of a distance established in order to bring into relief the objective orientation of believing itself. This is nothing more, or less, than the distance of a question: "Is it really so?" "Is it the case that…?" Such questions ask whether *what is meant* in a belief *genuinely articulates* what is given. When the I turns to the given in the wake of such a question, it does not simply fulfil an intention; the I now seeks insight into the success of its previous judgment in articulating the self-givenness of the given. In doing so, it keeps active a distinction between *what had been meant* with *what is now being seen*. This newly structured judgment is fulfiled (or not) in a special sense: fulfilment now means the confirmation of something that *had been meant in a belief*, which had then been actively prepared by the I that forms it into the objective unity "the meant as meant," and *then* brought to bear on the present given, yielding insight. Before, fulfilment had simply been the correlate of *the act of intending*, one that need not have had as an explicit theme any particular objectified contents from its own past.[394]

Such activity, guided by a developed interest in adequation, is for Husserl definitive of the scientific attitude.[395] It is the interest of the scientist to bring to bear only those judgments that are fulfiled solely by an evidence the development of which has been *followed with insight*. Yet this interest alone does not necessarily imply the development of logic as an independent science. Sensitivity to this difference is instead pre-constituted within an attitude primarily directed at the matter in the self-givenness of its evidence. "The intention aimed at cognition passes clear through them [the provisionary judgments], as supposed, and aims precisely at the affairs themselves, at the givenness of them themselves or

[393] FTL 122; Hua XVII 127:26–35.
[394] FTL 123; Hua XVII 128:4–9.
[395] Cf. FTL 124; Hua XVII 129:14–24; also 124fn/129fn.

at the evidence of them [*eben zu den Sachen selbst, zu ihrer Selbstgegebenheit oder Evidenz*]."[396] It is only that now the interest in the self of the thing is focused in such a way that the "meant as meant" is passively pre-constituted in every judging activity guided by the interest in "genuine" knowledge.

The orientation towards confirmation and adequation shapes judgment from within: the structure of judgment now includes an identifying synthesis between the meant as meant and what is meant in evidence; the result is the characteristic "back and forth" movement of scientific thinking:

> His judgments must be verified by genuine, by maximally perfect, evidence; and only as so verified shall they be admitted among the results of science as theory. This brings about a *peculiar judging procedure on the scientist's part*, a *zigzag* judging, so to speak: first making straight for the givenness of something itself [*auf Selbstgebung lossteurendes*], but then going back critically to the provisional results already obtained – whereupon his criticism must also be subjected to criticism, and for like reasons.[397]

Seen from the perspective of the genesis of the particular form that judgments take within it, science is characterized by an alternation between the directedness to objective reality as such (*Gegenständlichkeit schlechthin/abgezielte Wirklichkeit*) and the objectivity meant, or the judgment (*vermeinte Gegenständlichkeit als solche/Urteil*). In other words, science for Husserl is a movement between belief (*doxa*) and true knowledge, where the former is never left behind, but remains in tension with the latter, even if it is merely the tension of being a pole within an identifying synthesis.[398] And it is doxa, understood as what is believed in the believing, or what is asserted in the asserting, that becomes the traditional object of logic as apophantics.[399]

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[396] FTL 125; Hua XVII 130:6–8.

[397] FTL 125; Hua XVII 130:16–24.

[398] FTL 126; Hua XVII 131:16–21: "With this *supposed as such* [*Vermeinten als solchem*], the mere correlate of the 'supposing' or 'opining' (often spoken of as the opinion, δόξα), we have now laid hold of *what is called the judgment (apophansis) in traditional logic* and is the theme of apophantic logic." And in a footnote: "It is the noema of the judging."

[399] (a) Logic is oriented to science, and the goal of science is to determine its object-region. "Consequently the *predicative judgment* (the apophansis as a *self-contained unity of determination*) is always given pre-eminence." FTL 126; Hua XVII 131:28–30. (b) Furthermore, all categorialia occur within determining judgments, thus are constituted within the unity of interest. FTL 126; Hua XVII 132:3–6.

The tension between belief and justified belief amounts to a break with the simple naïve interest in knowing things as they are; for to simply judge is not yet to make the truth of judgments into an object of study.[400] Science, like knowing in general, has as its goal knowledge of things, not the isolation or justification of its own standard of critique.[401] The latter is the goal of a "theory of science" or *Wissenschaftslehre,* which for Husserl represents the genuine form of logic, in that it expresses the original motivation that gives rise to science as such: that critical interest in the truthfulness of scientific thinking.

The purpose of Husserl's genealogy of logic in EU and FTL is to rediscover, and even re-conceive, this interest in truth at the origin of the formal sciences and, by extension, of all science as such. His goal is nothing less than to repeat the Platonic moment cited at the beginning of the Introduction to FTL: the discovery and formulation of the very idea of science, thereby re-discovering its essence in a genuine, critical fashion that replaces the naïve horizon of scientific pursuit with a critically self-conscious grasp of the meaning of science as such.

7. Husserl Between Formalism and Intuitionism

It may appear that in following Husserl's analyses in EU and FTL we have gone somewhat far afield from the debate on the foundations of mathematics. But that is not really the case. The point I want to make is that Husserl's philosophy has the resources to articulate what is ultimately at stake in the debate between Hilbert and Brouwer: the very idea of science itself, and more: an understanding of *how* science is compelling (or not), what gives its theories and proofs their value. In this sense, Husserl's contribution involves that aspect of the debate that remains relevant to this day, even after Hilbert's program has been discredited by the Gödel theorems[402] and Brouwer's revolution run aground on the insuperable technical difficulties

[400] FTL 129; Hua XVII 134:16–24.
[401] FTL 129; Hua XVII 134:24–35.
[402] Nevertheless, it appears that a partial redution of infinistic mathematics is in fact possible. See Simpson, "Partial Realizations of Hilbert's Program," §3, where he discusses the recent work of W. Sieg and H. Friedman.

in re-establishing classical analysis on exclusively intuitionist principles.[403] For the debate between Hilbert and Brouwer was really about the nature of scientific thinking, and not simply over whether analysis could do without the principle of excluded middle.

Husserl's reflections on logic do not have as their focus the consequences that the exclusion or inclusion of specific formal-logical principles or methods would have on actual mathematical work. But they do, in a very sophisticated manner, develop the question of the sense in which logical reasoning, or reasoning guided by logical constructions, has an evidential character. We saw above that in Husserl's account of the foundations of logic, evidence lies in intuition, but not where intuition would be an external touchstone or source that would lend linguistic formulations an added character of evidence. The role of intuition is visible only in a phenomenological reflection that shows it providing the medium for those movements of passivity and activity that give rise to formations of sense in accordance with complex, multiple formations of interests. Intuitivity belongs to articulation itself, on all levels, including all levels of formality.

Formal-logical sciences are higher order versions of the sense-structures and, more importantly, interest structures basic to all science. More, this logical interest originally engages thinking *on the level of meaning*, and for this reason Husserl insists that pure formal analytics ultimately be understood, whatever its ontological tendencies, as the closed, systematic theory of senses (*Sinne*).[404] In FTL Husserl presents a notion of the *mathesis* of pure sense as the study of the particular manifold-forms of scientific theory, e.g., the formal structure "Euclidean manifold" as the formalized system-form of Euclidean geometry. From a structural point of view, other individual systems of judgments can have certain general features in common with this Euclidean manifold-form; thus we can generalize

[403] Cf. Weyl, H. *Philosophy of Mathematics and Natural Science* (Princeton: Princeton University Press, 1949), p. 54: "Mathematics with Brouwer gains its highest intuitive clarity [...] It cannot be denied, however, that in advancing to higher and more general theories the inapplicability of the simple laws of classical logic eventually results in an almost unbearable awkwardness. And the mathematician watches with pain the larger part of his towering edifice which he believed to be built of concrete blocks dissolve into mist before his eyes."

[404] FTL 137; Hua XVII 142:12–17.

the Euclidean manifold as the form of a theory that can be modelled in multiple "Euclidean-type manifolds."[405] The articulation of such models takes place solely within the region of sense: *mathesis universalis* is thus interpreted as the analytics of possible categorial sense-forms.[406]

Yet the original emergence of sense as a theme, as we saw, has its origin in an interest in the truth (verification) of judgments; yet the objective status of *mathesis universalis* as a thematic field includes the possibility, even the necessity, of its development independently from all explicit questions of "truth" – the heart of its traditional ambiguity.[407] And in fact Husserl emphasizes that once the subject matter of pure *mathesis* is in place, there is no *technical* reason to ask whether individual multiplicities constructed under the guiding idea of a theory form "exist," whether they are "true" or not – the formation of their objectivity does not reflect any claims about the world, but is to a great extent founded in their being fashioned as a "world" or region in themselves. The world of formal axiomatics need not be taken to deal with "real" possibilities in order to articulate what is given within its sphere of concerns; concepts here can be formulated such "that their extension does not at all involve the assumption of such possibilities."[408] For *once* this level has been reached, *once* these possibilities are open, the mathematician can remain indifferent to the concrete – even if mathematics, as Brouwer puts it, has the place it has in human thought because of its emergence in an activity very much concerned with the concrete.[409]

It is worth stressing that this indifference is not only a possibility. It is a decisive, and from Husserl's perspective wholly legitimate step in the development of logic. Nevertheless, however important it may be, Husserl argues, it is a development that is only meaningful from within the

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[405] FTL 138; Hua XVII 143:16–24.

[406] FTL 138; Hua XVII 143:24–30.

[407] FTL 137–138; Hua XVII 143:2–6.

[408] FTL 139; Hua XVII 144:8–9.

[409] FTL 138; Hua XVII 143:37–144:5: "Equivalently, the *mathematician as such* need not be at all concerned with the fact that there actually are multiplicities in concrete 'actuality' ['*Wirklichkeit*'] […]; nor indeed need he be at all concerned with the fact that there *can* be something of the sort, that something of the sort with some material content [*Sachgehalt*] or other is thinkable."

philosophical perspective of the question of the essence of "true thinking," or what makes knowledge "genuine." And this requires a re-discovery of the structures of *mathesis universalis* as pure *meanings* that are only "pure objects" in an equivocal sense. Husserl understands this re-discovery of logic as a science of meaning not as a step beyond formal mathematical logic into something else, but as an extension of formal logic itself, in accordance with its proper, if obscured origin and motivation. For Husserl, logic *is* theory of science (*Wissenschaftslehre*), and if it remains within pure analytics, even pure theory of meaning indifferent to the problem of truth and evidence (say an analytics of expressions), it will remain an *incomplete* theory.[410] Or, to recall themes from EU and FTL, the potential sense-enrichment represented by the development of formal axiomatics will remain in a latent state.

It is just such an extension of logic into a philosophical articulation of the meaning of science that Husserl in FTL projects as the task of transcendental phenomenology. Again, the key is the theme of meaning, prepared by the analyses of intuition. Take again the example of the axiomatic formalization of Euclidean geometry.[411] As already noted, the axiomatic schemes of possible deductive sciences can be pursued along the lines of a theory of possible mathematical manifolds. To be sure, a mathematical manifold is not the same as a unity of sense, nor is a theory. Both manifold and theory can, however, be *reduced* to a pure formation of sense, the idea being that formalized theory forms provide the guiding clue for the reduction of the unity of science to the unity of meaning, and to the subjective interests and intuitivity that belongs to the life in which this meaning is constituted.

This reduction of formal structure to sense can be taken in two ways: (1) as a reduction of the *theory-form* of the science, yielding pure systematic formations of sense ("judgments"). The latter no longer constitute a general theory form of which something like a concrete science could be a "model," since we have moved from a generalized formal structure to the manner in which the thinking that constitutes such structures unfolds as an eidetic structure of pure consciousness.[412] Next (2), the reduction of

[410] FTL 140–141; Hua XVII 146:8–27.
[411] Cf. Weyl, *Philosophy of Mathematics and Natural Science*, pp. 25–28.
[412] FTL 141–142; Hua XVII 147:8–20.

formal structure to sense can be carried out on the level of the manifold of *objects* that fall under a given theory, taken as a formal unity – i.e., the multiple derivations of the "something in general." There is a correlation between theory and object, at all levels; thus the reduction of theory form to structures of sense implies the possibility of a reduction of its object to sense, though in a manner in which the correlation of object and theory remains in a hyper intensified form: "objects" are now "substrate senses, that are adapted to function harmoniously in a judgment-system as substrates of predications."[413] *And they are only this*: for they are reduced to what is meant "as such" in a theory when the theory itself is reduced to the unity of meaning.[414] This in effect allows Husserl to describe the complex relationship of the idea of *method* and the idea of the "something in general."

As a countermovement to formal ontology, this double reduction amounts to the reduction of formal logic to apophantics. The mathematization of logic had led reflection in a formal-ontological direction; the reduction of a fully developed formal logic returns it to apophantics, but now with a far more nuanced, and critical character. After the reduction, formal logic is no longer "ontological," but it is still a logic of being, though now of the being of the region of "meant objectivity" (*vermeinte Gegenständlichkeit*), and not of the "something in general." Thus apophantics, qua apophantics, reveals itself to be a sub-discipline within a *material ontology of consciousness, or of the being of lived experience as such*.

Yet this "being of lived experience," as we have seen in the case of thinking, is *intentional*, or object-directed; it is a being which is not closed, but has a structure in which a being not its own becomes apparent. If we are to talk meaningfully about the "being" of intentionality, then this cannot be understood as a simple description of a given ontic region, for the directedness of this particular kind of being to the given is not simply an external relation between two spheres of being: the being of consciousness, and the being of the world. Lived experience is more than being; it "is," but in such a way that the other, the given, comes to its own being through its being-given in consciousness. "Givenness" is thus

[413] FTL 142; Hua XVII 148:5–7.
[414] FTL 142; Hua XVII 148:7–9.

something which, as formations of sense, makes its "appearance" in the order of objectivity or being, but it cannot ultimately be reduced to it – for it itself is an origin, which means a beginning, or an emergence of something else. Thus however "closed" formal analytics may appear to be, as an event within the life of consciousness its ultimate significance is determined by its role within an experience of things – there is always a material side to the most formal of formalisms.

This is the insight that, after the double reduction enables Husserl to determine *mathesis universalis* as the science of apophantic sense,[415] guides the return, and further development, of logical interest in the broader sense of an interest in possible truth. On one level, an immediate benefit of pure analytics conceived in this manner is evident: there is now a clearly defined functional role for *mathesis universalis* within the articulation of the idea of science, in accordance with which formal analytics can be meaningfully extended to a logic of possible truth (*Wahrheitslehre*) in Husserl's sense, all the while retaining its formal character, at least insofar as "objects" or "concepts" (*Kerne*) of formal propositions remain indeterminate and are thought of only as "possible objectivities."[416]

And when we again turn to the question whether there is anything lost when we reduce logic as apophantics to logic as formal ontology, our new understanding of pure analytics as a *mathesis* of sense shows us that it *cannot* be a formal ontology, because it is not an ontology of a something *in general*, but of a *particular region* of objects, namely pure senses (*Vermeintheiten*).[417] Possibility of sense, above all of evident, verified sense (the evidence of clarity, or the "evidence" of non-contradiction), is not the same as the possibility of a particular determination of the something in general; the latter remains within the scope of the "purity" of sense taken as such. The full scope of the problem of the objectivity of

[415] FTL 143; Hua XVII 148:31–35.

[416] FTL 143; Hua XVII 148:36–149:16.

[417] FTL 144; Hua XVII 150:1–7: "Consequently we must say: *the aforesaid pure mathematics of non-contradiction*, in its detachment from logic as theory of science, *does not deserve to be called a formal ontology*. It [pure mathesis of non-contradiction] is an ontology of pure judgments *as senses* and, more particularly, an ontology of the *forms* belonging to non-contradictory – and, in that sense, possible – senses: possible in distinct evidence [*Evidenz der Deutlichkeit*]."

objects, the real game of science, plays itself out beyond the ontological boundaries of pure sense:

> To any possible object there corresponds its object-sense. Every sense-form pertaining to possible objects naturally occurs among the possible sense-forms dealt with by "extra-logical" mathematics. But the just-mentioned possibility of a sense-form ⟨as distinctly non-contradictory⟩ does not contain, in and of itself, the least possibility of objects with a sense corresponding to it; and indeed even this "corresponding" itself takes us beyond the sphere of pure senses.[418]

Thus genuine logic must *go beyond the pure sphere of sense*, precisely in order to pose in a genuine fashion the question of truth. If so, then the pure *mathesis* of judgments does indeed have ontological *significance*, insofar as it plays a key role in orienting a genuine posing of the question of truth; but it is not yet without further ado an ontology of being in its truth ("genuine being"). Even the thematization of *true judgments* is not yet at the level of ontology, as long as the focus remains *judgments alone*,[419] and fails to ask in a more radical manner the question of the possibility of the truth that becomes visible in judgments.

The broader, "genuine" logic of truth in this way reflects science itself: just as in science there is the "back and forth" (*Zickzack*) between judgments and objectivities, so too here: logic as *Wissenschaftslehre* has its own "back and forth" between apophantics and ontology, where the former plays a function in the development of the latter from the perspective of an interest in possible true being. The ultimate purpose of logic is the thematization not merely of the forms of true judgments, but of true objects, or objects of truth – categorially formed objectivities: "*Categorially formed objectivity* is *not an apophantical concept*; *rather* it is *an ontological concept*." [420] But that means it is a *philosophical* concept, not merely a *logical* one.

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[418] FTL 144; Hua XVII 150:7–14.

[419] FTL 45; Hua XVII 150:27–32.

[420] FTL 145; Hua XVII 151:11–13. "*According to its final sense* such a logic is therefore *not a pure formal apophantic logic but a formal-ontological logic*." FTL 145; Hua XVII 150:38–151:1. To be sure, it can remain as an apophantics, a thematic *Einstellung auf Urteile*, and this in fact is its historical tendency. "But the deep sense of *formal analytics, the sense that measures up to its task as theory of science, is that of being the science of the possible categorial forms in which substrate objectivities can truly exist*." FTL 145; Hua XVII 151:5–10.

How, then, do we situate Husserl in the formalism intuitionism debate? His sympathies are certainly with Brouwer, with his insistence on the clarification of foundational structures in mathematics in accordance with the standards of evidence provided by intuition. But Husserl was philosophically sophisticated enough to realize that even finitistic mathematics is saturated with the accomplishments of intuitive lived experience, that its apparent nominalist focus on language and symbolisms does not ipso facto isolate it from the resources of intuitivity. And more: he recognized that the failure to recognize the inherent intuitivity of all thinking, even the most arid logicisms far from ordinary language, leads to the possibility of failing to recognize the treasures that modern mathematics offers for the tasks of thinking.

CHAPTER 10

THE TWO-SIDEDNESS AND THE RATIONALISTIC IDEAL OF FORMAL LOGIC: HUSSERL AND GÖDEL[421]

Pierre Cassou-Noguès

Abstract. This paper aims at bringing together the architecture that Gödel attributes to mathematics and logic, and that which Husserl describes in his essay of 1929, *Formal and Transcendental Logic*. In the first part, we recall some elements of Husserl's analysis. In the second part, we discuss Gödel's distinction between a theory of sets and a theory of concepts, and compare it with Husserl's formal ontology and formal apophantic. In the third part, we discuss the reference in Gödel and in Husserl to a rationalistic Ideal that would lead mathematics, and the role of the phenomenological reflection in mathematical progress.

Introduction

The aim of this paper is to compare Husserl's logic with that of Gödel and, in this manner, to revivify Husserl's logic. Starting with his technical results and, of course, his incompleteness theorem, Gödel develops an epistemological reflection on the status of logic and mathematics. We will try to show that Gödel rediscovers the main features of Husserl's logic as it is described in the essay of 1929, *Formal and transcendental logic*. As does Husserl, Gödel distinguishes two orientations in formal logic, and attributes to mathematics some kind of completeness. The incompleteness theorem, which Gödel proves in 1931, shows that Husserl's description of mathematical theories and of their "Euclidean ideal" must be revised. However, Gödel does believe in a rationalistic ideal: in mathematics, every question admits a definite answer, and every proposition that

[421] This paper is partly based on the study of Gödel's Papers, at Princeton University Library, which I could visit in February 2004 with a Fellowship from the Society of the Friends of Princeton University Library and with the help of the program Preuve from the MSH Nord-Pas de Calais. I also wish to thank all the staff of the Library for their kind help during my stay. This paper was written in Spring 2004. It slightly ovelaps in part II a paper recently published by G. Crocco (Crocco, 2006).

L. Boi et al. (eds.), Rediscovering Phenomenology, 309–337.

can be formulated, should be either demonstrable or refutable. He also separates logic from mathematics by their orientation, and, in this way, attributes two sides to formal science. Thus, Gödel seems to give to formal science the same architecture. This would make it possible to take up again Husserl's analysis. Indeed, according to Gödel, Husserl, in his analysis of logic or in his intended reform of logic, opens a new direction for the development of logic, and it seems that Gödel is ready to take the same direction. As he writes to G. C. Rota:

"I do have a high opinion of Husserl, especially of his introspective analysis of mental phenomena, of his discussion of the concepts involved and, most of all, of his idea to use this kind of investigations as a scientific method for building philosophy.

"*As far as Husserl's reform of logic is concerned, I don't think he aimed at the rejection of anything in today's mathematical logic, but rather at supplementing it and laying its foundations deeper.*"[422]

D. Føllesdal and others have brought to light the close relationship of Gödel's Platonism with the Husserl's conception of mathematical objectivity.[423] After 1959, Gödel starts reading Husserl's works of which he possesses copies and which he abundantly annotates. However, Gödel traces his Platonism back to his student years in Vienna, long before his reading of Husserl. It is then following an independent path that Gödel meets with Husserl. Afterwards, Gödel refers explicitly to phenomenology in a text of 1961 and in his conversations with Hao Wang. At least two points bring together Gödel and Husserl. First, Gödel admits that mathematical objects exist independently of our cognitive acts but are given to us in a specific intuition, different from sense perception: "These concepts have an objective reality of their own, which we can not create or change but merely perceive and describe."[424] The position of a mathematical reality, independent of our cognitive acts and given in a perception that we can reflect upon, brings Gödel close to the *Logical Researches* of 1901. Second, Gödel establishes an analogy between mathematical and sense objects, analogy which concerns both their reality and their perception: "[...] despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory [...] I don't see any reason why we should have less confidence in this kind of perception,

[422] Gödel's Papers, Box 2c, folder 141, item 012030, Letter to G.C. Rota, 7 September 1972. The emphasis is mine.

[423] We refer to "Principally to Føllesdal, 1995". See also, Atten and Kennedy, 2003.

[424] Gödel 1951, in 1986–2003, III, p. 320.

i.e., in mathematical intuition, than in sense perception [...] The question of the objective existence of the objects of mathematical intuition [...] is an exact replica of the question of the objective existence of the outer world."[425] The same analogy can be found in particular in the *Ideas Pertaining to a Pure Phenomenology*, of 1913.

In this paper, we will simply take for granted this proximity between Husserl and Gödel, on the existence and the intuition of mathematical objects.[426] We will not discuss what would be properly speaking Gödel's phenomenology. We will rather compare the architectures, which Husserl, on one hand, and Gödel, on the other, attribute to formal science. We will start by recalling some aspects of Husserl's logic. We will then try to show how Gödel rediscovers at least three features of the Husserl's conception: the rationalistic ideal that governs mathematics, the two-sidedness of formal science, the idea of founding mathematics in a phenomenological investigation of our mental acts. We will make use of Gödel's unpublished papers, from Princeton University Library, and of the conversations transcribed by Wang, in *A Logical Journey: From Gödel to Philosophy*.

1. Husserl's Logic

We will simply follow the essay of 1929, *Formal and Transcendental Logic*, where Husserl gives his theory its final shape. At first, we will stay as close as possible to Husserl's text, though its vocabulary is rather uncongenial to modern logic.

The aim of the essay of 1929 is an elucidation of the meaning of logic. The first section describes the structures of formal logic. However, it leads to the discovery of hidden presuppositions, which can only be fully justified by a reflexive analysis of the intentional acts that produce logical objectivity. The second section describes the constitution of logical objects in the transcendental subjectivity. This transcendental logic is necessary to obtain an ultimate "foundation" for formal logic.[427] We will only delineate the main features of formal logic and will not discuss the "transcendental logic."

[425] Gödel 1964, in 1986–2003, II, p. 268.
[426] We discuss Gödel's Platonism in connection with Husserl's phenomenology in another paper, Cassou-Noguès 2005.
[427] Husserl, 1929, Introduction, p. 13 et §71, p. 181.

In 1929, Husserl identifies the object of logic as the "forms of judgement." A judgement is the meaning of an assertion and, more precisely, of an assertion which can be brought to distinction or whose articulations can be clearly distinguished. The judgement-form *S is p* is then obtained from the concrete judgement "The paper is white" through a substitution analogue to that of arbitrary letters to definite words.

Now it will be useful for our comparison with Gödel to investigate Husserl's conception of judgement. In the classical texts, such as the *Ideas Pertaining to a Pure Phenomenology* of 1913, Husserl distinguishes the same four terms for any mental act, such as perceiving and judging. There is (i) the act itself, (ii) the *noema*, which is the full correlate of the act, (iii) the meaning, i.e., the noema deprived of its modalities, and (iv) the object of the act. Seeing a bird in the garden is an act of perception. This act has a correlate, the *noema*, the bird as it is seen and considered as real. But we could see the bird as a mere imagination or remember seeing a bird this way. The *noemata* would be different but they would have a common core, their meaning, the bird as it is seen.[428] The bird, itself, which we may see again tomorrow, is the object of our perception. In the same way, when we say, "this paper is white," we make an act of judging. This act has a correlate, the "judgement." This judgement is asserted. If we only made a conjecture, we would have a different noema, but these noemata would have a common core, their meaning or the "proposition."[429] The object of our judgement is the "state of affair," that the paper is white. Now, the proposition is the meaning of the judgement, i.e., the judgement deprived of its modality (just as the bird as it is seen is the meaning of our perception). The proposition is also defined in the *Lectures of 1908* as the "content" of the judgement.[430] In accordance with the common usage, the proposition can be considered as the object of logic. Indeed, the essay of 1929, *Formal and transcendental logic*, simply introduces a change of terminology. The "judgement" in 1929 is the "proposition" of the previous texts. Husserl, in the essay of 1929, does not take into account the modalities of the noema. He may then identify the noema with the

[428] Husserl, 1913, §130.
[429] Husserl, 1913, §94.
[430] Husserl, 1987, §35.

meaning and the judgement with the proposition.[431] The "judgement," in that sense, becomes the object of logic.

Husserl distinguishes three logical disciplines. The first one is the grammar of pure logic. It was already mentioned in the *Logical Researches*. It has first to distinguish the different categories of theoretical thinking: concept, judgement, connexion between judgements. Its aim is then to investigate the laws according to which complex judgments are produced from simple judgements. Its task is simple. One will make an inventory of the fundamental forms of judgement, such as *S is p* or *S has p* (i.e., p is a part of S) and of the different "operations"[432] which produce complex judgements from simpler ones, such as conjunction, which from *S is p* and *R is q*, makes *S is p and R is q*, or "nominalization," which turns the form of judgement *S is p* into a logical subject for another judgement: *"S is p" is q*. In that manner, this logical grammar will circumscribe the forms of judgement that possess a distinct meaning and can appear in a given theory. Now, taken in this sense, it corresponds to the definition of formulas, in our logical calculus: it lists the atomic formulas and the rules according to which new formulas can be produced from simpler ones. However, it is absolute in that there is only one such grammar, defining all the judgements (i.e., all the propositions) that can be formulated in any possible theory.

The second discipline, consequence-logic or logic of non-contradiction, defines the laws of deduction. These laws should permit one to decide whether a judgement is a consequence of another, or whether a judgement is contradictory with another. It seems then that the logic of consequence corresponds to the setting of our logical calculus, say predicate calculus. However, it is absolute in that there can be only one set of rules for all deductions. The same rules should apply in any theory.

The third discipline, the theory of deductive systems, investigates the possible forms of theories. A theory, "in the pregnant sense,"[433] is defined by a system of axioms from which the mathematician deduces new propositions according to the rules given in the consequence-logic. Since the rules of deduction are permanently set, one could investigate the possible

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[431] Husserl, 1929, §45.

[432] Husserl, 1929, §13.c., p. 52

[433] Husserl, 1929, §28, p.90.

theories, that these rules authorize, and their relations. In particular, possible theories are distributed in a sort of hierarchy. We only consider formal theories or forms of theories, made of forms of judgements. But theories, which have common axioms, can be considered as different specifications of a general form. This general form, when compared with other theories, appears itself as a specification of a still more general form: "[…] thus the possible forms of theory constitute an empire which is hierarchized in different species. The lower species are so to speak the individuals of this empire."[434] The third logical discipline is the study of the theories and the rules of this "empire." It should start from a set of axioms and be systematic so as to constitute itself a deductive system. Thus logic tends to a "theory of possible forms of theories."[435] In fact, according to Husserl, the "new mathematics," which has emerged during the nineteenth century, is guided by this ideal of a theory of theories. The example of this trend is Riemann's geometry, which defines different geometries by certain specifications in a unique system of laws.

However, one must distinguish a special kind of theory. The rationalistic ideal or, in Husserl's words, the "Euclidean ideal," which guides the scientist, is such that one should be able to explain a series of phenomena or to describe a certain domain from a restricted number of fundamental laws, stated as axioms. This reduction is only achieved when every possible true proposition is a consequence of the axioms. One should then be able to decide from the axioms, either demonstrate or refute, any proposition formulated in the system. Such theories are, in Husserl's words, "definite" or "nomological": "any proposition (proposition-form, naturally) that can be constructed, in accordance with the grammar of pure logic, out of the concepts (concept-forms) occurring in that system, is either "true" – i.e., to say: an analytic (purely deducible) consequence of the axioms – or false – i.e., to say: an analytic contradiction – ; *tertium non datur*."[436] Nomological theories would be, in our vocabulary, syntactically complete. Husserl does not say that the theories that mathematicians study are syntactically complete, or nomological. Nevertheless, he maintains that the theories *par excellence*, in the theory of the possible theories, are nomological, and that mathematicians, in the construction of a theory, are guided by the ideal of

[434] Husserl, 1994, p. 534.
[435] Husserl, 1929, §28, p. 90.
[436] Husserl, 1929, §31, p. 96.

a "*tertium non datur*." One could argue that these convictions were shared by Hilbert, and that his motto, there is *no ignorabimus* in mathematics, had in the papers of the twenties the same implications.

However, this encounter of logic with mathematics leads Husserl to another problem. If the higher task of formal science can be described as a theory of theories, the mathematician seems to be more interested in the domains of objects, which are correlated to the theories, than in the theories themselves. After all, Riemann makes the theory of different spaces, of different curvature, rather than the theory of different geometries. The question, in fact, concerns the relationship between the theories and their domains. Very early and previously to his meeting with the mathematician of Göttingen, Husserl describes the aim of axiomatization in terms similar to those of Hilbert and, in particular, stresses the abstraction that goes with the axiomatization of a theory.[437] The domain of a theory, a system of axioms, is a multiplicity of objects unspecified and determined only by this clause that such and such axioms must be true in this domain. The objects, in a formal theory, are pure "Something,"[438] or what Husserl calls derivative modes of the notion of "Something," such as sets (made of a collection of "Something").[439] Formal mathematics appears to be a theory of mere objects describing the possible multiplicities that these objects can constitute. In that sense, mathematics represents a formal ontology rather than a logic of judgement. However, Husserl admits that, in a nomological theory, the axioms suffice to specify without ambiguity the domain of objects. In other words, theories syntactically complete are also supposed categorical. To a "nomological" theory corresponds one, and only one, multiplicity of objects. This seems to make the theory of possible theories and the theories of possible multiplicities exactly correlative and, in a sense, equivalent. It is the same to investigate possible theories, on one hand, and possible multiplicities, on the other hand, since, for every theory, there is one and only one corresponding multiplicity. So mathematics is more particularly the theory of possible multiplicities, and logic is more particularly a theory of possible theories. They are merely distinguished by their different orientations. Mathematics follows the natural orientation

[437] Husserl moved to Göttingen in 1901, where he regularly met Hilbert. See Cassou-Noguès, 1999 and 2004.
[438] Huserrl, 1929, §38, p. 107.
[439] Husserl, 1929, §39, p. 107.

of knowledge. It turns towards the objects of knowledge. On the other hand, logic takes a “critical” attitude. The logician distinguishes the judgement itself and the state of affairs. He turns towards the judgment and tries to investigate its specific laws. Therefore, formal science or logic, broadly speaking, may have two different orientations, towards judgements and towards objects. It divides itself in two correlative theories: on one hand, a formal apophantics, a theory of judgements, or a logic, in a strict sense, which tends towards a theory of forms of theories; on the other hand, a formal ontology, a theory of the “Something” and the possible multiplicities that these “Something” can constitute. This formal ontology is the true goal of mathematics.

The distinction between the two orientations of logic and of mathematics closes the first section of the essay of 1929. The second section is devoted to the task of showing that formal science is built upon hidden presuppositions (such as the identity of the objects of different judgements, the assumption that two judgements can have the same objects). These presuppositions must be clarified and founded. This requires an analysis of the constitution of mathematical objects in the transcendental subjectivity. We had to mention this second section of the essay of 1929, but we will not go further into it. As Gödel says to Wang:

“Husserl speaks of constituting mathematical objects but what is contained in his published work on this matter is merely programmatic. [However] phenomenological investigations of the constitution of mathematical objects are of fundamental importance for the foundations of mathematics.”[440]

2. The Paradoxes and the Two-Sidedness of Logic

We have followed the exposition of *Formal and Transcendental Logic.* Husserl’s vocabulary is rather foreign to modern logic. We may overlook some aspects of Husserl’s logic, such as the tri-partition of logic, in a logical grammar, a logic of consequence, a theory of theories. We could simply take this tri-partition to mean that, in defining a logical

[440] Gödel’s Papers, Box 3c, folder 207, item 013167. This is a text from Wang entitled “Quotations from Gödel” and corrected by Gödel. The quotation is reproduced in Wang, 1996, p. 256.

system, one must give the rules for the formation of formulas, the rules for deduction and that, eventually, the aim is to build a meta-logic where one can study the properties and relationships of the theories, with different axioms, using this logic. But we will distinguish three prominent features: the distinction between two orientations, one which defines logic as a theory of judgements or propositions, and the other which defines mathematics as a theory of objects, undetermined objects (objects with an "empty core"); the idea that there is in mathematics a rationalistic ideal and that the mathematician works with the idea of a "tertium non datur," or under the assumption that every proposition, which he can formulate, is either refutable and demonstrable; the perspective of giving a foundation to mathematics through a reflexive analysis, an analysis of the mental acts involved in the mathematical work. Our claim is now that Gödel rediscovers these three features. We will start with the two-sidedness of formal science and compare it with Gödel's distinction between logic as a theory of concepts and mathematics as a theory of sets.

Gödel does not seem to always use the word "concept" in a technical sense. But it is sometimes the case, as we will see (e.g., the expression "theory of concepts"). It seems then that the notion of "concept" has three ingredients. The first and most obvious is Russell's propositional function. A propositional function is a property or a relation, which, when applied an argument, yields a proposition. "Being green" is a propositional function: applied to the object designated as "this," it yields a proposition either true or false, "this is green." In his article, "Russell's mathematical logic," 1944, Gödel understands by a "concept," "a propositional function [...] as a separable entity [...] something separable from the argument [...] and also something distinct from the combination of symbols."[441] Gödel requires that a concept be a genuine entity, which should not be reduced to an assemblage of symbols nor a class of objects. But, this granted, one may identify Gödel's concepts with Russell's propositional functions.[442]

[441] Gödel, 1944.

[442] There are other texts with a Russellian accent. For example, the following note from Wang corrected by Gödel: "[Gödel crosses out the first sentence: Primitive concepts vary with people]. Concepts are generalities which applies to many things. [Here Gödel adds: "They form the realm of the 'general' as opposed to the realm of the 'particular'].

However, Gödel seems to determine the status of these concepts through a distinction, which may be inherited from Frege but is similar to that of Husserl, between the act itself, its content, or meaning, and its object:

"Husserl. Note that concerning abstract entities/concepts one has to distinguish thoughts and their content (obtained by psychological and logical reflection respectively). The former (to which intuitionists try to confine themselves) are occurrences in the real world and therefore are in a sense just as concrete as combinations of symbols."[443]

"Thoughts: mental images containing conceptual meaning. Content of thought $\neq$ objects of thought."[444]

In these notes, Gödel separates the act, i.e., the thought itself, its content and its object. He seems to associate the concept with the content of the thought. These distinctions are the same as Husserl's, and the concept seems to be at the same place as Husserl's proposition, or judgement, in the content of the act. One may then take the concept, in Gödel's sense, to be the meaning of a thought correlated to an object. The object, here, is the class which is the extension of the concept. Applied a term, the concept would be a part of a proposition, in Husserl's sense, considered as the meaning of the thought and opposed to the state of affairs that makes the proposition true. The following remark is another example of Gödel's proximity with Husserl:

"There are more similarities than differences between sense perceptions and the perception of concepts. The analogue of perceiving sense objects from different angles is the perception of different logically equivalent concepts."[445]

Taken literally, this analogy with sense perception evokes Husserl's analysis. In Husserl's terminology, an object, perceived from different angles, is presented in two different noemata and with two different meanings. In the same way, equivalent concepts, which have the same extension, would be different presentations of the same object, the class that is the

They correspond to abstract parts of things." (Gödel's Papers, Box 3c, Folder 209, item 013182.5).

[443] Gödel's Papers, Box 9b, folder 148.5, item 0440498.60, note for the revision of Gödel, 1972.

[444] Gödel's Papers, Box 9b, folder 148.5, item 0440498.26, note for the revision of Gödel, 1972.

[445] Gödel's Papers, Box 8c, Folder 117, item 040403.3 (draft from Gödel for a note in Wang, 1974).

extension of the concepts. The concepts seem then to have, in the sphere of the thought, the function and the status of the perceptual meanings. The concept, as a meaning, expresses a way in which an object, a class, can be apprehended.

A last ingredient, but less important, in Gödel's notion of concept, comes from Kant. As Kant, Gödel distinguishes between concepts, which have definite objects, and ideas, whose objects are not given and which represent something like a principle, ruling a certain trend of thinking. "Beyond concepts, Gödel also envisages ideas. [...] Ideas are inexhaustible: we can never describe an idea in words exhaustively or completely clearly. We can see an idea more and more clearly, and this process may be uniquely determined."[446]

We may disregard this last ingredient. The point, on which we want to put emphasis, is that the concept, in Gödel's sense, seems to be a property or, more generally, a propositional function considered as the meaning associated with a thought and opposed to an object. It is at the same level as Husserl's proposition. Gödel's distinction between the theory of concepts and the theory of sets could then be compared with the two orientations of Husserl's logic. It remains to see why Gödel distinguishes these two sides of mathematical logic. We will follow the article of 1944, "Russell's mathematical logic." This article is, for the greater part, a critical examination of Russell's solution to the paradoxes. We will discuss some elements of Russell's solution, before following Gödel's criticism.

Russell starts with an analysis, which he applies to all paradoxes since the ancient liar.[447] The Cretan says: "I lie." If he lies, he tells the truth. If he tells the truth, he lies. The antinomy comes from the fact that the proposition, expressed in "I lie," tells something about itself: it states its own falsehood. It is this "self reference" that Russell sees at the bottom of all the paradoxes. In particular, this "reflexivity" reappears in the paradox that Russell himself has found in Frege's logic. The paradox concerns either classes or concepts.

Let's consider the class of classes that do not belong to themselves. Does this class belong to itself? If it does, it is one of its own elements, a class

[446] Gödel's papers, Box 8c, Folder 117, item 040403.3 (draft in Gödel's handwriting for a note in Wang, 1974). See also Wang, 1996, pp. 268–269.
[447] We rely on Russell, 1908.

that does not belong to itself. But if it does not belong to itself, it is then one of these classes that do not belong to themselves, it is one of its own elements and it does belong to itself. On the other hand, let's consider the concept, say "heterology," of concepts that do not apply to themselves. Does it apply to itself? The same line of reasoning shows that, again, both sides of the alternative lead to a contradiction. However the paradox, in its two forms, supposes that we recognize that a class may belong to itself and a concept may apply to itself. But a class seems to presuppose its elements so that a class that belongs to itself would presuppose itself. Symmetrically, a concept seems to presuppose its domain of application so that a concept which applies to itself would presuppose itself. Just as a proposition that says something of itself, a class that belongs to itself and a concept that applies to itself have a kind a "reflexivity" and, in fact, they are built on a vicious circle.

The logical paradoxes are based on a vicious circle, which amounts to constituting an object from a collection in which it then reappears. This object seems then to presuppose itself. In order to eliminate the paradoxes, one should avoid this circle. The following principle should serve as a guide: "What involves all of a collection must not be one of the collection; or, conversely, if, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total."[448] The first part of the principle recalls Poincaré's rejection of impredicative definitions. The second part outlines what will be Russell's own solution. It concerns the quantification, which is used to make a "total" of a collection so as to create new objects that will then reappear as members of the collection. Such a quantification on undue totalities, as *all concepts*, *all classes* must be forbidden. One must delimitate the domain of a quantification.

In Russell's exposition, the theory of types involves an analysis of quantification. A universal proposition, such as *all men are mortal*, which, according to Russell, would still be true if there were no men, hides an implication. *All men are mortal* means that it is always true that, if x is a man, x is mortal. But the rules, which should define the range of a quantification, cannot come as explicit premises, which would be added to the

[448] Russell, 1908, p. 63.

proposition. One would get: it is always true that, if x belongs to the class, say I, if x is a man, then x is mortal. The range of the first quantification would stay undetermined. The only solution is to admit that the complex concept *if x is a man, x is mortal* has a domain of signification, in which it is either true or false, depending on x, and outside of which it has no meaning. Applied to an individual, such as Socrates, this propositional function has a meaning. Applied to another propositional function or to itself, it has no meaning, and one doesn't have to consider whether it be true or false. Finally, the proposition *all men are mortal* means that the function *if x is a man, x is mortal* is true wherever it has meaning. The difficulty will be to define the range of signification of propositional functions. This is the purpose of the theory of types.

A type is "the range of significance of a propositional function, i.e., the collection of arguments for which the said function has values."[449] We start from elementary propositions, which contain relations and terms but no quantificators. We call "individuals" the terms appearing in the elementary propositions. The individuals make the type 1. They represent the range of significance of proposition functions, obtained from elementary propositions through the substitution of a variable for an individual. These propositional functions and all those that have as domain the type 1, form the type 2. The type 2 itself is a domain for propositional functions, which form the type 3, and so on. The propositional functions are distributed in a hierarchy of types and, inside each type, in a hierarchy of orders (which we will not discuss). A propositional function is of a definite type, and it has for domain of significance the previous type. In that hierarchy, no propositional function can be applied to itself. In other words, it is meaningless to apply a concept to itself. If classes are identified with the extensions of concepts, it is meaningless to ask whether a class belongs to itself. The (simple) theory of types already excludes the logical paradoxes. We will not speak here of the theory of order.

Gödel first accepted Russell's solution. In 1933 and still in 1939, he puts aside all objections against the theory of types.[450] However, in the article

[449] Russell, op. cit., p. 75.

[450] See Gödel, 1933o, in Gödel, 1985–2003, III, pp. 48–49. See also the course of logic at Notre Dame, in Gödel's Papers, Box 8a, folder 65, notebook V. Gödel remarks that a type free theory would suit better "natural thinking. But it is "one of the most interesting

of 1944, "Russell's mathematical logic," Gödel directs a series of criticisms against Russell. Though the point is not made in the paper, Gödel distinguishes different kinds of paradoxes, contrary to Russell who applies the same analysis to all paradoxes. There are semantical, intensional, and extensional paradoxes. They do not lead to the same problems. They do not require the same kind of solutions.[451]

The semantical paradoxes are only a matter of language. They have been solved by a precise analysis of the language of mathematical theories. The liar paradox is eliminated as soon as one recognizes that the truth attributed to the formulas of a language cannot be expressed in the same language but only in a meta-language.[452] This distinction between language and meta-language renders impossible the formulation of a proposition stating its own falsehood. In fact, semantical paradoxes, such as the liar but also Richard's, are only apparent and their solution only requires a closer attention to the mathematical languages.

Intensional and extensional paradoxes lead to genuine problems. Their domains distinguish them. Extensional paradoxes concern the objects of mathematics, classes or sets, whereas intensional paradoxes concern the concepts of mathematics. Russell's paradox has both an intensional and an extensional form. It can be stated with the class of classes that do not belong to themselves, or with the concept of concepts, which do not apply to themselves. However, the implications of the paradox, concerning classes and concerning concepts, are not the same. Before all, the discussion requires a more precise analysis of the vicious circle principle. Gödel remarks, and it can be seen in the statement quoted above, that Russell uses alternatively the expressions "being definable in terms of," "involving," "presupposing": no totality can contain members definable

facts in modern logic […] that the evidences of natural thinking are not consistent with themselves."

[451] Gödel's Papers, Box 3c, folder 207, item 013167, note written by Wang, corrected by Gödel. See also Wang 1996, pp. 271–272.

[452] This was only established with Tarski's definition of truth, published in Polish in 1933 and translated in German in 1935. However, Gödel seems to have anticipated the undefinability of truth in the language to which it applies. See, notably, Gödel's letter to Balas, in Gödel, 1986–2003, IV.

only in terms of the totality/involving the totality/presupposing the totality.[453] There are then three different forms of the vicious circle principle. For all practical purposes, we may identify the two last ("involving" and "presupposing"). However, we must distinguish the first one from the other two.

The vicious circle principle in the first form (stated with "definable") amounts to forbidding impredicative definitions, definitions that introduce an object in reference to a set to which it belongs. However, according to Gödel, impredicative definitions are indispensable for classical mathematics. Classical mathematics does not then satisfy the vicious circle principle in its first form.[454] And this would rather prove that this first form is unacceptable.[455] There is no doubt for Gödel that classical mathematics is correct and that the vicious circle principle in its first form is false. Quite generally, Gödel seems to take the course opposite to that of Poincaré.[456] Poincaré had shown that the impredicative definitions and their apparent circle are related to a Platonistic conception of mathematical objects. However, they are inadmissible in a constructive context. If mathematical objects do not exist in themselves but are created by us, an impredicative definition introduces an object on the basis of a collection to which it belongs, and, therefore, presupposes the object itself in the collection: it is circular. However, if the objects exist independently of our acts, there is no absurdity in designating an object using a collection to which it belongs: the oldest tree in the forest, the tallest building in town.[457] Poincaré who believed that the circle is the source of the paradoxes, rejected impredicative definitions and pleaded for a constructive conception of mathematical existence, whereas Gödel, starting from a

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[453] Gödel, 1944, in 1986–2003, II, p. 127.

[454] "It is demonstrable that the formalism of classical mathematics does not satisfy the vicious circle principle in its first form, since the axioms imply the existence of real numbers definable in this formalism only by reference to all real numbers." Gödel, 1944, in 1986–2003, II, p. 127.

[455] Gödel, 1944, in 1986–2003, p. 127. See also Gödel, 1939b (in 1986–2003, III, 126).

[456] Henri Poincaré, Les mathématiques et la logique III, 1906, in 1908, p. 167.

[457] The example is given in Gödel, 1933o, in 1986–2003, III, p. 50.

Platonistic point of view, accepts impredicative definitions and considers the circle as benign.[458]

However, though he refuses the first formulation of the Russell's principle, Gödel accepts the second formulation if it concerns sets.[459] Mathematical objects can be distributed on a hierarchy, which respects the principle of the vicious circle in its second or third formulation (with "involving" or "presupposing"). This is achieved either with Russell's theory of types or with Zermelo's cumulative hierarchy. In Zermelo's universe, no set can belong to itself. In that sense, no set "presupposes" itself. Nevertheless, one can still introduce impredicative definitions, through the axiom of separation. The universe of sets, as it is described by Zermelo, does not respect the principle of the vicious circle in its first formulation but it illustrates its second or third formulation.

If, concerning set theory, Gödel accepts the vicious circle principle in its second or third form, it is on the basis of his own conception of sets. In Gödel's view, a set is a whole or a unity made from the multiplicity of its elements: "A set is one object which contains its many elements as constituents, i.e., a whole consisting of them."[460] A set depends on a "synthesis" of its elements, which are then "thought together." The elements are constituents of the set, in the same way as different parts in a physical object: "Sets are a limiting case of spatio-temporal objects, either as an analogue of construing a whole physical body as determined entirely by its parts (so that the interconnection of the parts play no role) or as an analogue of synthesizing aspects to get one object (with the difference that interrelations of the aspects are disregarded)."[461] This conception of sets, as wholes constituted from their elements, may have its shortcomings. In particular, the null set, the unit set have to be considered as fictions, introduced to simplify the theory, in the same way as points at infinity

[458] Gödel, 1944, in 1986–2003, II, pp. 128–129. See also Cassou-Noguès, to be published.
[459] Gödel, 1944, in 1986–2003, II, p. 131.
[460] Gödel's Papers, Box, 3c, folder 207, item 013167; Box, 3c, folder 207, item 013161; Box 8c, folder 117, item 040398.
[461] Gödel's Papers, Box 3c, folder 207, item 013167. See also Wang, 1996, where sets are described as "quasi-spatial" or "quasi-physical."

in geometry.[462] Nevertheless, this conception excludes that a set could belong to itself. A set, no more than a physical object, can be a part of itself:

"The concept of set contains the component that sets are [Wang wrote: extensional objects; Gödel corrects:] wholes with the cl[assical] aspect, and therefore rules out the possibility of a set belonging to itself [in another note with the same text, Gödel adds: because it would be its own constituent]."[463]

No set can belong to itself. Therefore, the vicious circle principle, in its second form, is correct in set theory. However, another analysis is required on the side of concepts. To the note quoted above, Gödel adds: "Gödel declares such a state of affair [i.e., a set belonging to itself or being a constituent of itself] to be possible for concepts (in opp[osition] to objects)."[464] The same idea is developed in other notes from Wang: "Sets and concepts are so different, their connection is only outwardly. In particular, no set can belong to itself but some concepts can apply to themselves, e.g., the concept of concept (is a concept)."[465] In fact, there is no absurdity in a concept applying to itself. This is the case of the concept of concept, which is a concept. This explains why, in "Russell's mathematical logic," Gödel distinguishes the theory of sets, where the vicious circle principle in its second form applies, and the theory of concepts: "speaking of concepts, the situation is changed completely," "the vicious circle principle does not hold even in its second form for concepts and propositions."[466] The theory of concepts does not verify the vicious circle principle in any of its forms.

The consequence is also that the connection between sets and concepts is problematic. One can admit, though it is not evident, that every set is the extension of some concept.[467] However, it is not true that, to each concept,

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[462] Gödel's Papers, Box 3c, folder 207, item 013167.

[463] Gödel's Papers, Box, 3c, folder 207, item 013161; Box 20.

[464] Box 20. It is a note in Gödel's handwriting added in a text from Wang.

[465] Gödel's Papers, Box 3c, folder 207, item 013164 and 013167. The texts are entitled "Quotations from Gödel." See also, Wang, 1996, p. 278.

[466] Gödel, 1944, in 1986–2003, II, p. 130.

[467] "Such a conclusion may be provable once we have a developed theory of concepts and, more crucially, a more completely developed theory of sets. We shall assume the conjecture: every set is the extension of some concept." Gödel's Papers, Box 3c, folder

there corresponds a set.[468] The extension of a concept, which applies to itself, cannot be a set. One can call it a class. But, according to Gödel, a class, in this sense, is merely a "*façon de parler*." It has no reality. It is not properly an object. It seems that the exclusion of classes from the realm of mathematical objects comes from the fact, that classes may not have the unity that belongs to sets. Their elements cannot be thought together as a whole. Classes are "pluralities and therefore not objects."[469] Sets, which are "quasi-spatial" and cannot belong to themselves, no more than a sense-object can be a part of itself, are the true objects of mathematics. It follows that some concepts, though they may be considered as meanings, content associated with thoughts, will have no object properly speaking. There is a dissymmetry between the theory of extensions, which is the theory of sets, and the theory of intensions, which is the theory of concepts. The theory of concepts is in a way richer than the theory of sets.

It is true that mathematicians have not developed such a theory of concepts. According to Gödel, the difficulty, for the development of a theory of concepts, comes from the intensional paradoxes.[470] The semantical paradoxes have been solved by a precise delimitation of the language of mathematical theories. The extensional paradoxes have been solved by the definition of universes, whose objects are distributed in a hierarchy, and which verify the vicious circle principle in its second form. But, since concepts can apply to themselves, the corresponding hierarchies are

207, 013167. The text from Wang is entitled "Quotations from Gödel." See also, Wang, 1996, p. 272 sqq.

[468] "Sets are extensions and concepts are intensions. Frege erroneously thought that to each concept there corresponds a set, but there are concepts which correspond to no set" (Gödel's Papers, Box 3c, folder 207, item 013164 and 013167, the texts are entitled "Quotations from Gödel"). See also Wang, 1996, p. 272, sqq.

[469] Gödel's Papers, Box 3c, folder 207, 013167. The texts are entitled "Quotations from Gödel." See also Wang, 1996, pp. 274–275.

[470] "Mathematicians are primarily interested in extensions and we have a systematic study of extensions in set theory, which remains a mathematical subject except in its foundations. Mathematicians form and use concepts but they do not investigate generally how concepts are formed. We do not have an equally well developed theory of concepts comparable to set theory. […] Gödel believes that the unsolved difficulties are mainly in connection with the intensional paradoxes" (Gödel's Papers, Box 3c, folder 207, item 013164).

illegitimate on the side of concepts. In fact, Gödel seems to look for a solution to the intensional paradoxes in what could be called a logical grammar. In the article of 1944, Gödel refers to Russell's definition of types: a type is the range of significance of a propositional function. It seems that this definition contains an idea independent from the vicious circle principle:

> "The theory of types brings in a new idea for the solution of the paradoxes, especially suited to their intensional form. It consists in blaming the paradoxes [...] on the assumption that every concept gives a meaningful proposition, if asserted for any arbitrary object or objects as argument."[471]

The conclusion that Gödel draws is that a concept has a range of significance, where it can be meaningfully applied, and that the paradoxes result from the application of concepts outside their range of significance. The task would then be to determine the range of significance of concepts or, in other words, to determine to which terms a certain concept can apply. Of course, the Russell's types are inadequate, since they would not allow the application of a concept to itself. Gödel's conjecture is rather that one would have to isolate "singular points" where a concept has no meaning: "It might even turn out that it is possible to assume every concept to be significant everywhere except for certain "singular points" or "limiting points," so that the paradoxes would appear as something analogous to dividing by zero."[472] The theory of concepts is still to be developed. But one can see that it would have different features from those of set theory.

It is now clear why Gödel has to distinguish a theory of sets and a theory of concepts. In his view, logic has two orientations: "Logic is the theory of the formal. It consists of set theory and the theory of concepts."[473] Mathematics is identified to set theory, and the theory of concepts is logic, in a restricted sense.[474] Now, if Gödel does adopt the same distinction as

[471] Gödel, 1944, in 1986–2003, II, p. 137.
[472] Gödel, 1944, in 1986–2003, II, p. 138.
[473] Quoted from Wang, 1996, p. 268.
[474] For example: "From this viewpoint, logic contains mathematics as a proper part under the conjecture [that every set is the extension of some concept]. There has been over a long time a confusion between logic and mathematics. *Once we realize the sharp distinction between sets and concepts*, we have made several advances. We have a reasonably

Husserl, between the thought, its meaning, and its object, concepts can be conceived as meanings, and sets as objects correlated with concepts. We obtain a distinction between logic and mathematics comparable to the one that Husserl advocated. We have, on one hand, a theory concerning the meanings, Gödel's theory of concepts and Husserl's apophantics, and, on the other hand, a theory concerning the objects, which could be set theory. However, the theory of concepts is not simply a reduplication of the theory of sets. Concepts can be applied to themselves whereas sets cannot belong to themselves. Consequently, there are concepts that have no object in the theory of sets. The principles of the theory of concepts will be different from those of the theory of sets. Husserl's logic would have to be reformed. Nevertheless, his distinction between a logic concerned with meanings and a mathematics concerned with objects can be maintained.

3. The Rationalistic Ideal and the Phenomenological Foundations of Mathematics

As we have seen, Husserl believes that mathematicians are guided by the Euclidean ideal, which stipulates that, in a theory "in the pregnant sense," every proposition should be either demonstrable or refutable from the axioms. Of course, after the theorem of incompleteness, such an ideal can hardly be maintained. However, Gödel does hold that, *in mathematics*, every proposition that can be formulated is either demonstrable or refutable. This requires that mathematical theories may be indefinitely extended by new axioms. But, according to Gödel, a mathematical theory, such as set theory, can indeed be extended by plausible axioms, and these axioms can be justified by a phenomenological reflection. Again, the features that Husserl gives to logic must be revised. Nevertheless, Gödel reintroduces a rationalistic ideal and a foundation for mathematics similar to those that Husserl advocated.

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convincing foundation for ordinary mathematics according to the iterative concept of set [in the cumulative hierarchy]. Going beyond sets becomes an understandable and, in fact, a necessary step for a comprehensive conception of logic. We come back to the program of developing a grand logic except that we are no longer troubled by consequences of the confusion between sets and concepts." (The first sentence is not the item 013164. In that item, the emphasis is Gödel's.)

The incompleteness result, which Gödel gives in 1931, is an answer to Hilbert's program. The main goal of Hilbert's program was to prove the consistency of the formal systems, which represent mathematical theories, through finitary reasoning. In a formal system, a proof is a combination of symbols built according to rules. A contradiction would be a proof which would end with a line $0 \neq 0$. Hilbert's idea was to introduce a new discipline, meta-mathematics, whose objects were formal proofs and which would establish that no such proof could be contradictory. The meta-mathematics was to use only simple inferences, which in no way involved considerations on the infinite. It would prove the consistency of mathematical theories with indisputable inferences and, in that manner, give a foundation to mathematics. The foundation was then becoming a (meta)-mathematical exercise. Hilbert could claim to have eliminated philosophical discussions from the foundation of mathematics.[475] Besides, if the intuitionists were denying the reality of the infinite, Hilbert could also claim to give a foundation to mathematics independently of the reality of the infinite. Mathematical theories, which involve considerations on the infinite, were reduced to a game of symbols. Their consistency was to be proven through finitary inferences. Hilbert could admit that the infinite is nowhere realized,[476] and, still, aim at a foundation of classical mathematics. However, the proof of consistency was not the only task of meta-mathematics. Another task was to prove that, in a formal system, every formula is either demonstrable or refutable. Though he does not use the term, Hilbert shared Husserl's belief in the Euclidean Ideal.[477] And, with his meta-mathematics, he could give a precise formulation and hope to prove what had been an early conviction: "in mathematics, there is no *ignorabimus*."[478] The mathematicians will never be reduced to admit that they do not know. They would know at least that, for any proposition formulated in a particular theory, there exist either a proof or a refutation.

The theorem of incompleteness, which Gödel gives in 1931, has two parts. First, in any consistent system, in which elementary arithmetic can

[475] Hilbert, 1927, p. 233; Hilbert, 1931, p. 273
[476] Hilbert, 1926, pp. 371–372; Hilbert, 1927, p. 269.
[477] Hilbert, 1926, p. 384.
[478] Hilbert, 1900.

be expressed, one can formulate propositions that are neither demonstrable nor refutable from the axioms of the system. After Turing's paper of 1936, this can be restated: a machine, deducing theorems, will always leave undecided arithmetical propositions. Second, if the system is consistent, no reasoning that can be expressed in the system can prove its consistency.[479]

We will first discuss the second theorem. A proof of the consistency of a mathematical theory, e.g., elementary arithmetic, must use inferences that cannot be expressed in the theory. The finitary inferences that Hilbert and his school used are expressible in elementary arithmetic. Therefore, they cannot prove its consistency. After Gödel's result, Hilbert's program is suspended to the question whether there may be other finitary methods, which could not be expressed in arithmetic. At least until 1933, Gödel leaves the question open.[480] However, in his later writings, Gödel upholds that a proof of consistency cannot be based on inferences that would be in any sense weaker than those that are formalized in the theory. The most one can do is to replace classical inferences, if they are considered as doubtful, by different inferences, which are not expressible in the theory because they are equally powerful but use different concepts, and which can be justified by an epistemological analysis. The concepts, the methods, which are used in classical theories, cannot be eliminated. They can only be reformulated. Gödel advocates a general principle, "which could perhaps be called the *non-eliminability of the mathematical content of an axiomatic system*."[481] This principle has two consequences. First, one cannot eliminate the infinite. The inferences used in the proof of the consistency will not be finitary. They will make use of mathematical concepts which involve the infinite and which must be considered as given. One has then to recognize the reality of the infinite. This will lead Gödel to a kind of Platonism. Second, one cannot eliminate the epistemological analysis from the foundation of mathematics. The foundation of mathematics has two aspects: there is the technical problem of translating

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[479] To be more precise, in 1931, Gödel only proves these theorems with the hypothesis that the system is ω-consistent. ω-consistency was replaced by simple consistency in 1936 by Rosser.

[480] See Gödel, 1933o, in Gödel, 1986–2003, III, p. 53.

[481] Gödel, 1953/9, 1985–2003, III, p. 345.

classical inferences, and there is the epistemological problem of justifying the new methods, which now replace classical inferences. The problem of foundation is modified by the results of 1931. It is no longer purely technical, but it still exists, and Gödel can take it up again: "[...] the certainty of mathematics is to be secured not by proving certain properties by a projection onto material systems – namely, the manipulation of physical symbols – but rather by cultivating (deepening) knowledge of the abstract concepts."[482] After this remark, Gödel evokes Husserl's phenomenology. As we will see, this "deepening" of knowledge, required by the foundation of mathematics, is to be attained by a phenomenological reflection.

The first theorem of 1931 establishes that, any theory, if it is consistent and expresses elementary arithmetic, contains propositions that are undecidable and can neither be demonstrated nor refuted from the axioms. But, already in 1931, Gödel notes that this theorem does not establish the existence of absolutely undecidable propositions, propositions undecidable for the human mind, but only the existence of propositions undecidable in the theory considered. A proposition undecidable in this sense may become decidable in an extension of the theory with suitable axioms. Therefore, one must abandon the "Euclidean Ideal" as Husserl and, in his last papers, Hilbert had formulated it. But one does not have to abandon the belief that every mathematical proposition, every proposition that can be formulated in a mathematical theory, is either demonstrable or refutable on the basis of plausible axioms. Gödel does not reject Hilbert's early motto, before the formalist program: there is no *ignorabimus* in mathematics. Indeed, "it is not at all excluded by the negative results [the theorems of 1931] that nevertheless every clearly posed mathematical yes-or-no question is solvable in this way."[483] Gödel accepts the postulate that every mathematical question is decidable, in what he calls a "rationalistic optimism."[484] It implies that mathematical theories can be extended indefinitely and in a non-recursive way, or according to a procedure that no machine can follow. The task is then to define methods for the extension of mathematical theories and for the justification of new axioms. Again, Gödel is

[482] Gödel, 1961?, in 1986–2003, III, p. 383.
[483] Gödel, 1961?, in 1986–2003, III, p. 385.
[484] See Wang, 1974, p. 325 and Wang, 1996, p. 207 and p. 317.

convinced that the phenomenological reflection has a role to play in the discovery and the justification of mathematical axioms.

Despite his results of 1931, Gödel takes up two objectives of Hilbert's program, the foundation of mathematics and the defence of a rationalistic ideal, "these aspects whose fulfilment would in any case be very desirable and which have much to recommend themselves: namely, on the one hand, to safeguard mathematics for the certainty of its knowledge, and, on the other, to uphold the belief that for clear questions posed by reason, reason can also find clear answers."[485] What Gödel rejects and what his results of 1931 seem to exclude are the philosophical bias of Hilbert's program: the refusal of the infinite and the exclusion of the epistemological analysis from the foundation of mathematics.[486]

Gödel's reflections on the extension of mathematical theories are also related to his work on the continuum hypothesis. In 1938, Gödel proves the relative consistency of the continuum hypothesis and the axiom of choice. This result leads him to conjecture that the continuum hypothesis is undecidable from the usual axioms of set theory. However, Gödel believes that the continuum hypothesis is false, and that it can be refuted on this basis of new axioms, concerning "large" sets. He then distinguishes two ways of justifying new axioms.

First, there is in mathematics something like an "induction." From this point of view, axioms are justified by their "success" or by their "fruitfulness."[487] In its own field, the axiom might establish a given conjecture, or simplify the proofs of a group of theorems already known. But, since elementary arithmetic is incomplete, an axiom in set theory may also be necessary to prove arithmetical propositions, which were expected or which can be, in a way, confirmed by the computation of several instances.

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[485] Gödel, 1961?, in 1986–2003, III, p. 381.

[486] See Gödel's letter to C. Reid, in 1986–2003, p. 187: "What has been proved is only that the *specific epistemological* objectives which Hilbert had in mind cannot be obtained." In the draft of the letter, Gödel mentions "Hilbert's general basic philosophical assertions, in particular as far as the nature of the infinite and its role in mathematical reality are concerned" (Gödel's Papers, Box, 1c, folder 129, item 011852).

[487] Gödel, 1947, in 1986–2003, II, p. 182; 1964, in 1986–2003, II, p. 261, p. 269; 1951, in 1986–2003, II, p. 313.

Cavaillès, in a different context, once wrote "fecundity is the authority before which refusal in the name of evidence becomes prejudice."[488] But, in Gödel's view, the justification of an axiom by its fruitfulness is analogous to induction in physics. An axiom, which has consequences in number theory, is similar to a hypothesis in physics which has observable consequences and which leads to make empirical predictions. The computations that enable to check different instances of an arithmetical proposition have in mathematics the function of experience in physics. They constitute indisputable evidences, and give ground for the admission of the laws of physics or the axioms of mathematics.[489]

However, the axioms that are introduced inductively may need to be revised in the same manner as physical laws. They do not yet have an "intrinsic necessity." Their truth is only "probable."[490] Therefore, the ultimate justification for an axiom can only be its own evidence. Mathematical objects are given to us in an intuition, and this intuition leads us to axioms that are then evident by themselves. In fact, the evidence of axioms is, for Gödel, an argument for the existence of a mathematical intuition: "despite their remoteness from sense perception, we do have something like a perception also of the objects of set theory, as is seen from the fact the axioms force themselves upon us as being true."[491] The inductive process is not sufficient. The truth of an axiom can only be established by the intuition of the objects and the concepts involved. Nevertheless, our intuition needs to be educated and developed. In particular, in order to introduce new axioms in set theory, we need to make more precise our concept of set. For this, the problem is to educate, to refine, our intuition. The solution, according to Gödel, is in the phenomenological method. It

[488] Cavaillès, 1994, p. 54, p. 361–362.

[489] Arithmetic is "the domain of the kind of elementary indisputable evidence that may be most fittingly compared with sense perception" (Gödel, 1944, in 1986–2003, II, p. 121). Also, "the mathematical character of the axioms, in spite of their inductive foundation, appears in the circumstance that they have consequences in that part of mathematics [...] whose primitive terms have an immediately understandable clear meaning (e.g., the axioms of infinity [...] have number-theoretical consequences)" (Gödel, 1953/9, in 1986–2003, III, p. 347).

[490] Gödel, 1947 and 1964, in 1986–2003, II, p. 182 and p. 261.

[491] Gödel, 1964, in 1986–2003, II, p. 268.

is by a reflection on ourselves, on mathematical acts that we can sharpen our intuition and clarify our concepts.

"In what manner, however, is it possible to extend our knowledge of these abstract concepts [...]? The procedure must thus consist, at least to a large extent, in a clarification of meaning [...]. Now in fact, there exists today the beginning of a science which claims to possess a systematic method for such a clarification of meaning, and that is the phenomenology founded by Husserl. Here clarification of meaning consists in focusing more sharply on the concepts concerned by directing our attention in a certain way, namely, onto our own acts in the use of these concepts, onto our powers in carrying out our acts, etc."[492]

It is also to be noted that the induction and the evidence, which determine the admission of new axioms, also secure their foundation: "For these axioms there exists no other rational (and not merely practical) foundation except either that they [...] can be directly perceived to be true [...], or that they are assumed (like physical hypotheses) on the grounds of inductive arguments, e.g., their success in the applications."[493]

Now, Gödel's rationalist optimism, according to which mathematical questions admit definite answers, requires that mathematical theories may be indefinitely extended by the addition of axioms. New axioms may be based, provisionally, on an induction. But, eventually, they depend on our intuition of mathematical objects. The phenomenological reflection, if it is an education of intuition, is then an essential tool for the development of mathematics.

However, Gödel also refers to the phenomenological method in another perspective, which seems more controversial. As we have seen, a recursive theory, in which a Turing machine can write down the axioms, is incomplete and contains undecidable propositions. Mathematical theories are

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[492] Gödel, 1961?, in 1986–2003, III, p. 383. Also this draft letter to Pr. Tillich: "I said that in mathematical reasoning the non comput.[ational] (i.e., intuition) elements consist in intuitions of higher and higher infinities. This is quite true. However, these intuitions are not uneducable. The situation can be further analysed, and then it turns out, as becomes perfectly clear when these things are carried out in details, that they result from a deeper and deeper self-knowledge of reason (to be more precise from a more and more complete rational knowledge of the essence of reason). [...] It seems to me that this is a verification (in the field of mathematics) of some tenets of idealistic philosophy" (Gödel's Papers, Box 3b, folder 188). See also, Atten, 2006.

[493] Gödel, 1953/9, in 1986–2003, III, p. 347.

incomplete but, in Gödel's view, they may be indefinitely extended, so that one can hope that no proposition remains undecided. However, this extension, if it is based on an induction or on the intuition of the object, proceeds step by step. There is, then, no way to embrace all the axioms of the theory. At least, if a theory is to be complete, it must be impossible for a machine to list its axioms. Nevertheless, if the axioms were determined by a non-recursive procedure, a procedure that no machine can follow, the theorem of 1931 would not apply, and the theory, thus defined, could be complete. Now, Gödel seems to have been looking for such a non-recursive procedure. Already in 1946, Gödel does not exclude that, in the definition of a theory or in the list of its axioms, "all the steps (or at least all of them which give something new for the domain of propositions in which you are interested) could be described and collected together in a non constructive way."[494] Non-recursive, or non-mechanical, procedures would be procedures that a machine can not follow and, admitting that Turing machines can do any manipulations of symbols, procedures which depend on the meaning of the terms: "procedures, such as involve the use of abstract terms on the basis of their meaning [and which are] not equivalent to any algorithm;" "a procedure into which some elements of meaning essentially enter (as an element which cannot be eliminated) and which is successful for all problems of a class for which no Turing procedure exists."[495] Now, in Gödel's view, it may possible to discover such procedures. They simply require a better understanding of the working of the mind, such as can be attained by the phenomenological method:

"In my opinion, there are no sufficient reasons for expecting this concept [computability by thought procedures] to have the same extension [as mechanical computability] in spite of what Turing says [... in his paper of 1936–37]. However, it must be admitted that, even in classical mathematics, the construction of a well defined thought procedure which actually could be carried out and would yield a number theoretic function requires a substantial advance in our understanding of the basic concepts of logic and mathematics and of our manner of conceiving them."

"The method for clarifying questions on thought procedures evidently is [the] phenomenological analysis of mathematical thinking, which is a very undeveloped field.

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[494] Gödel, 1946, in 1986–2003, II, p. 151. Here non-constructive is obviously to be understood as non-recursive.

[495] Gödel, 1934, in 1986–2003, I, p. 370 (Postscript of 1964). The second formulation is on the draft (Gödel's Papers, Box 8c, folder 106, item 040332).

The fruitfulness of psychological reflection for the foundations of mathematics is clearly shown by the concept of computable function defined in the prec[eding ?] paper and its applications."[496]

Gödel then seems to think that the phenomenological investigation of mathematical thinking might lead to the formulation of a well-defined but non-recursive procedure. Leaving aside this perspective, the phenomenological method is, in Gödel's view, a tool for the clarification our concepts and, eventually, the formulation of new axioms. As we tried to show elsewhere, it seems that, in Gödel's view, a complete analysis of our mental acts is impossible.[497] The acts, the thoughts involved in the intuition of mathematical objects keep a kind of opacity. The phenomenological investigation is always unfinished. Nevertheless, it can always be carried further and, in this process, it appears as an essential ingredient for the development and for the foundation of mathematics. Considering that mathematics can be indefinitely developed enables Gödel to take up, in a revised form, the rationalist ideal advocated by Hilbert and by Husserl. At least, on three points, Gödel rediscovers the features of Husserl's logic: the distinction between two orientations, a theory of meaning and a theory of objects; the rationalist ideal which guides mathematicians; and the role of the reflexive method in mathematics. One may not accept the phenomenological method, the perspective of an analysis of the mental acts involved in the mathematical work. Nevertheless, Gödel's epistemological analyses seem to lead to a renewal of Husserl's logic.

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[496] Gödel's Papers, Box 9b, folder 147, items 040468 and 040488. These are notes for Gödel 1972. It is not completely clear, in the second quotation, to which "prec. paper" Gödel refers.

[497] See Cassou-Noguès, 2005. See also the letter to Pr. Tillich above, note 492.

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CHAPTER 11

METTRE LES STRUCTURES EN MOUVEMENT: LA PHÉNOMÉNOLOGIE ET LA DYNAMIQUE DE L'INTUITION CONCEPTUELLE*. SUR LA PERTINENCE PHÉNOMÉNOLOGIQUE DE LA THÉORIE DES CATÉGORIES

Jocelyn Benoist

Abstract. Despite its permanent tension between forms and intuitions, Husserl's thought has many structural features. These are shared with the german mathematics of the 1920s and the 1930s – also known as the school of *moderne Algebra*, that would give rise later to Bourbaki and the theory of mathematical structures. However, whereas mathematical structures are known to produce a very rigid conception of knowledge, phenomenology is characterized by its emphasis on the dynamics of thinking and of intuition. We advocate that the modern theory of categories provides phenomenology with a conception of mathematics that suits its deep insights on the dynamical features of the process of knowledge.

Un siècle après les travaux décisifs de Husserl, il y a lieu de s'étonner de l'obscurité qui régne sur son rapport avec les mathématiques. Que la phénoménologie soit d'abord l'invention d'un mathématicien, et que son développement soit profondément marqué par un style mathématique, au delà de l'intérêt privilégié du philosophe, à l'origine, pour les questions relevant *proprio sensu* de l'épistémologie des mathématiques, il n'y a pas lieu d'en douter. Mais dès qu'il s'agit de savoir de quelles mathématiques il s'agit, dans cet intérêt toujours manifesté par lui pour cette science dont il venait, et à quel type de position philosophique il est arrivé par rapport à elles, les choses paraissent beaucoup moins claires.

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*Cet essai n'aurait pas été possible pour moi sans les recherches de Frédéric Patras et Giuseppe Longo, qui, différemment mais de façon convergente, m'ont éclairé sur ce que pourrait être un véritable usage de la phénoménologie en épistémologie des mathématiques. Je les remercie ici de tout ce qu'ils m'ont apporté – sans qu'ils puissent porter la responsabilité de mes erreurs.

L. Boi et al. (eds.), Rediscovering Phenomenology, 339–355.

Un rapprochement semble assez naturel, et il a été tenté du vivant de Husserl: c'est le rapprochement avec l'intuitionnisme. Comme on le sait, Hermann Weyl a reconnu dans Husserl le grand philosophe de l'intuitionnisme. Mais, comme on le sait moins, Husserl ne lui a pas vraiment rendu la pareille, en le renvoyant en réponse aux travaux de Hilbert….

Si la question est de savoir quelle était exactement «la position de Husserl en philosophie des mathématiques», il semble que, toute sa vie, au moins à partir de sa rencontre avec Hilbert à Göttingen, il ait penché dans le sens d'un point de vue formaliste, gouverné par l'idée de «science formelle», telle qu'on la voit encore à l'œuvre dans *Logique formelle et logique transcendantale* (1929).

Mais évidemment, dans l'état des connaissances techniques qui étaient celles de Husserl, cet engagement n'est pas nécessairement aussi significatif qu'on pourrait être enclin à le penser aujourd'hui. Il est probable que, s'il traduit d'abord un sens fort de l'originalité du formel, comme sphère gnoséologique *et ontologique* spécifique, il exprime aussi et d'abord un souci que l'on pourrait paradoxalement qualifier en un certain sens (non technique[499]) de *constructiviste*, au sens d'un désir de *maîtrisabilité par la conscience*. La profession de foi formaliste, chez Husserl, une fois reconnu ce qu'on pourrait appeler un mode formel de l'intentionalité (ce qu'il appelle «abstraction formalisante»), ne signifie rien d'autre qu'une confiance, probablement abusive puisqu'*illimitée*, en les pouvoirs constituants de l'intentionalité: la conscience peut déployer librement son plan d'objets, *à la simple condition de rester en cohérence avec elle-même*. L'important est qu'elle demeure maîtresse de ses opérations et que celles-ci puissent lui être *intuitivement données*, cela dans le respect de leur caractère formel même, puisqu'il y a bien, chez Husserl, une sorte d'intuition formelle. En ce point, formalisme et constructivisme se rejoignent d'une certaine façon, avant de diverger sur leurs modalités d'application (respect ou non du tiers-exclu, etc.). Cela n'étonnera personne, tant le rapport de l'intuitionnisme au programme de Hilbert, dont il est issu dans la dissidence même, peut être complexe – l'idéal constructiviste aurait été proprement inconcevable sans le programme hilbertien. On pourrait dire que Husserl, dont on peut

[499] Dans l'esprit de Husserl, il ne semble pas comporter les restrictions que comporte habituellement le constructivisme (rejet de l'infini actuel, etc.), et c'est bien là le problème.

affirmer qu'il cesse de s'intéresser *techniquement* aux mathématiques (ce qui ne veut pas dire qu'il ne s'y intéresse pas autrement par après) avant 1914, situe très largement sa problématique *en-deçà* de ces divergences.

On comprendra dès lors – à défaut de l'approuver – la tentation qui semble récurrente d'attribuer à Husserl, ce formaliste déclaré, une philosophie des mathématiques intuitionniste. Evidemment il faudra souligner encore une fois que c'est littéralement faux, compte tenu de ce qu'il dit explicitement des mathématiques et compte tenu peut-être encore plus de ce que les intuitionnistes, quant à eux, généralement, ont dit de l'intuition – à savoir pas grand' chose, ou en tout cas des choses très en retrait par rapport à la détermination et la richesse qui est celle de la théorie phénoménologique de l'intuition.

Nous pensons que, dans ce rapprochement, il y a du vrai et du faux: un germe de vérité, mais qui ne prend toute sa valeur qu'une fois amendé par une nette détermination des concepts qui sont à sa base. Tout, ici, tourne autour du concept d'intuition.

En quoi la conception phénoménologique des mathématiques les renvoie-t-elle si ce n'est à un fondement, en tout cas à une forme d'épreuve «intuitive»? C'est probablement là le cliché le plus enraciné sur la philosophie des mathématiques de type phénoménologique. On attend d'elle qu'elle reconduise l'édifice mathématique à l'intuition, que ce soit pour l'en blâmer et affirmer qu'il n'y a pas de solution de ce côté-là quant à la question des fondements, ou au contraire pour souligner qu'on y récupère la dimension de pratique – pour ne pas dire de *praxis* – des mathématiques, et l'en féliciter.

Nous pensons qu'une telle vision, pour le moins schématique, de la philosophie des mathématiques phénoménologique (au moins chez Husserl) est vraie, mais qu'elle est aussi très égarante, et la source de tous les malentendus sur le rapport entre phénoménologie et mathématique aujourd'hui, parce que, quand on a dit cela, généralement, on n'a pas suffisamment déterminé le concept d'intuition.

Il y a certainement un malentendu qui réside dans l'opposition que nous aimerions qualifier de naïve entre deux conceptions des mathématiques:

- d'un côté il y aurait une conception formelle et conceptuelle, qui rechercherait une élucidation purement conceptuelle des fondements des mathématiques (par exemple du concept de nombre), et refuserait,

pour ce faire, de procéder par abstraction depuis l'expérience psychologique intuitive ordinaire, par exemple en partant de ce que Frege appelait, méprisant, «les nombres de jardin d'enfant».
– de l'autre, la perspective phénoménologique, qui supposerait au contraire que nous revenions à l'expérience intuitive, comme seul champ possible dans lequel les concepts mathématiques puissent prendre un sens (puissent s'illustrer, mais l'illustration, ici, serait condition du sens), et, peut-être, leur seule *source* possible. La géométrie, suivant les indications du dernier Husserl (*L'origine de la géométrie*), se rapprocherait de l'arpentage.

Or, si on s'en tient à ce genre d'alternative extrêmement simplifiée, il est clair que Husserl, même si l'on tient compte des remises en perspective opérées dans la dernière phase de sa pensée, se rattache à la *première* conception des mathématiques.

Dans les *Recherches Logiques*, les mathématiques sont, dans la lignée de Bolzano, tenues pour une science purement conceptuelle. Bien sûr, il y aurait une différence à faire, en leur sein, entre deux types de disciplines – mais ceci ne nous éloigne pas vraiment de Frege. D'un côté, il y a l'arithmétique, et l'analyse qui, dans la lignée de Weierstrass, est censée en dépendre. Cette partie des mathématiques est réputée «analytique», en un sens sur lequel il va falloir revenir. De l'autre, il y a la géométrie, dont on peut dire en un sens qui n'est plus vraiment kantien qu'elle est «synthétique *a priori*». Cette position n'est pas structurellement différente de celle de Frege. Il faut remarquer qu'en outre, chez Husserl, cette division va dans le sens d'une certaine subordination de la géométrie, puisque sa structure formelle (en tant que «théorie») lui est retirée pour être versée au compte de ce qui, comme «analytique», est proprement mathématique (relève de la *mathesis universalis*), ce qui la spécifie comme géométrie (et la fait tomber du côté du synthétique) étant ce qui la rapporte à ce qui n'est pas tenu par Husserl pour mathématique, à savoir la structure particulière de l'espace physique (*le fait* qu'il ait telle ou telle structure).

La proximité avec Frege, pour le premier domaine, est à prendre tout à fait au sérieux, puisque «l'analyticité» de la partie essentielle des mathématiques est présentée par Husserl comme strictement équivalente avec l'idée de *logicité*. Dire que les mathématiques (ou au moins l'arithmétique) sont analytiques, c'est dire que toute leur vérité est réductible (logiquement) à celle des lois logiques. Cette position – dont il faut remarquer au passage qu'elle n'était nullement celle de Bolzano,[500] auquel Husserl emprunte pourtant l'idée du caractère purement «conceptuel» des vérités des mathématiques – correspond très exactement ce qu'on appelle logicisme, qui est la doctrine de Frege. On est donc très éloigné du primat de la géométrie qu'on attribuerait parfois à la phénoménologie. Mais il faut s'arrêter un peu plus sur le véritable sens de ce «logicisme» du premier Husserl – que ne démentirait pas au fond *Logique formelle et logique transcendantale*.

Notre hypothèse personnelle est 1°) que ce «logicisme» ou «formalisme» (et c'est déjà une différence avec Frege, car chez Husserl les deux projets se rejoignent dans une certaine mesure, ce qui ne serait pas le cas chez Frege) est d'essence *structurale* 2°) qu'il ne s'oppose nullement à une certaine exigence de type intuitionniste. C'est dans cette coagulation de déterminations apparemment contradictoires – parce qu'issues de distinctions trop schématiques et impropres à capturer l'originalité du mathématique – que s'ancre la spécificité de la position phénoménologique.

Il faut premièrement bien comprendre ce que signifie, pour Husserl, la clause d'analyticité. La découverte par Husserl de ce qu'il croit être l'analyticité de la mathématique s'inscrit dans un contexte bien particulier. Le problème, tel qu'il est posé à la fin des *Prolégomènes*, est très clair: il s'agit de savoir, une fois de plus (la question traverse tout le XIXe siècle), si les nombres dits «imaginaires» sont à proprement parler des nombres, et si le *concept* de nombre supporte donc cette extension. La réponse l'est non moins, mais ne nous semble pas être prise habituellement dans toutes ses dimensions: non, les nombres «imaginaires» ne sont pas des nombres; tout au plus peut-on envisager un «passage par l'imaginaire», qui nous permet d'aller opérationnellement de nombres (c'est-à-dire de nombres entiers naturels) à des nombres, en manipulant au passage des entités fictives qui ne sont pas des nombres; MAIS, c'est là la position des *Prolégomènes*, et c'est ce qu'il y a de plus dans ces pages trop brèves qui les clôturent que dans les manuscrits de recherche consacrés au même sujet dans la première moitié des années 1890, *ce passage n'est en rien insignifiant*: on y expérimente quelque chose qui dépasse fondamentalement ce domaine

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[500] Voir notre livre *L'a priori conceptuel: Bolzano, Husserl, Schlick*, Paris, Vrin, 1999.

supposé ontologique particulier qui serait celui des «nombres», à savoir *des propriétés de structures qui n'ont rien spécialement à voir avec la notion de nombre*. En fait, la fin des *Prolégomènes* le dit très clairement, les mathématiques ne s'occupent pas spécialement des nombres – ce qu'il y a de mathématique dans la théorie des nombres, ce n'est de toute façon pas le nombre. Ce qui relève du mathématique comme tel, ce sont des propriétés de structure d'objets dont la particularité a été effacée et qu'on traite comme ce qu'ils sont, à savoir des nœuds de relations. Ces structures peuvent s'instancier («s'habiller») dans des théories de types différents, mais ce n'est pas là l'important. L'important, c'est leur aspect formel (le fait par exemple qu'à un certain niveau l'addition vectorielle puisse fonctionner comme l'addition numérique: à un certain niveau, elle *est* la même chose). En ce sens-là, la mathématique nous fait bien atteindre un niveau d'objectivité, mais d'objectivité *formelle*: elle traite les objets comme des «objets en général».

Il nous semble qu'il y a là une intuition très forte, à la source de tout ce qu'on pourrait appeler mathématique moderne, en tant que mathématique structurale, qui privilégie les relations sur les objets et la forme sur les contenus (détermine les contenus *par* la forme et les contenus *comme* formes). C'est ce que Husserl, quant à lui, appelle sans doute improprement analyticité, parce qu'il croit encore à l'harmonie préétablie du réel (fût-il formel) et du discours, et à la réflexion «directe» de la formalité de l'objet dans le système des énoncés. Le formalisme/logicisme (puisque les deux choses ici se confondent) n'est donc qu'un aboutissement naturel de cette espèce de «contrat phénoméno-logique» (clause d'harmonie entre les choses, ou en tout cas leur apparence, et le discours) qui traverse l'ensemble de la phénoménologie husserlienne.

Du point de vue de l'épistémologie des mathématiques, cela conduit à la réactivation d'une intuition frôlée dans certains textes par Leibniz: celle d'une mathématique (le singulier ici est important) soustraite à l'étude de la grandeur, pour être reversée à celle de l'objet en général, dans sa formalité. Husserl trouve cette intuition reformulée dans l'œuvre de jeunesse de Bolzano *Beiträge zu einer begründeteren Darstellung der Mathematik* (1810), qu'il cite dans ce sens dans *Logique formelle et logique transcendantale*. La mathématique, comme *mathesis universalis*, serait alors science des objets *possibles* (c'est-à-dire dans leur possibilité formelle), là où la métaphysique, selon une détermination d'ailleurs aussi reprise par

Husserl, serait vouée au réel (*Wirklichkeit*), comme ce qui est justiciable du principe de raison.

On lit parfois que Husserl, jusqu'aux *Recherches Logiques* comprises, serait resté indéfectiblement conservateur, et qu'il n'aurait «pas admis d'autres nombres que les entiers naturels» (ou à la rigueur les rationnels, positifs et négatifs). En un certain sens, c'est vrai. Mais l'important n'est pas là. L'important est que, derrière les structures apparentes de l'arithmétique se cachent d'autres structures, plus générales et plus puissantes, et libérées de toute référence à l'idée de nombre. A ce niveau-là, le problème de «l'extension» aux complexes, qui ne concerne que l'idée de nombre, ne se pose plus. Se pose celui de la *structure*, qui est le véritable objet de la mathématique – y compris là où, *prima facie*, on croit avoir affaire à des nombres.

Ce point de vue structural sur les mathématiques, qui se dit, sans doute maladroitement, dans la langue du logicisme dans les *Recherches Logiques*, peut tout aussi bien, du point de vue de Husserl, emprunter celle de cette première version du formalisme qu'est la proto-axiomatique hilbertienne (celle des *Fondements de la géométrie*, 1899). Dans la conférence faite en 1901 à la société mathématique de Göttingen ainsi que dans les études associées, Husserl défend l'idée de la possibilité, pour un système d'axiomes, de définir ce qu'il appelle des «objets formels» (*formale Gegenstände*). Ce sens formel de l'objet est bien celui qui était en question dans la séquence finale des *Prolégomènes*. Il y a mathématique dans la mesure où vient en question cette formalité de l'objet, dont le sens réside dans l'ensemble des opérations définies qu'on peut effectuer sur lui.

Or l'idée d'une telle «définition» (c'est-à-dire détermination) de l'objet par un système d'axiomes, si, du point de vue de Husserl, elle répond au réquisit d'«analyticité» avancé par les *Recherches Logiques* pour les mathématiques, ne va pas exactement dans le sens logiciste de Frege. Dans le même ensemble de documents préparatoires à sa conférence, ou liés à ses développements, Husserl a collationné une partie de la correspondance entre Frege et Hilbert, que Hilbert a mise à sa disposition, et qui porte justement sur ces questions. La grande différence entre Frege et Hilbert est que, s'ils souscrivent au même idéal d'architecture déductive des théories, le premier pense que cette architecture ne fait, ontologiquement, que capturer un donné qui lui préexiste et qu'elle ne peut en rien prétendre sécréter ou constituer. Des axiomes n'ont jamais constitué une preuve d'existence, et il y a lieu de prouver (ou d'admettre, car il revient autant à la logique de prouver ce qu'il y a à prouver que de reconnaître qu'on ne peut pas tout prouver) l'existence de ce dont ils parlent. Frege souscrit à une conception traditionnelle de l'axiomatique, selon laquelle celle-ci ne fait que formaliser des vérités indépendantes d'elle et qui valent en elles-mêmes. Au contraire, à la suite de Hilbert, Husserl, dans ces textes, semble accorder une sorte de portée ontologique à l'axiomatique, reconnaissant aux axiomes, dans leur diversité possible, la capacité de déployer et de constituer dans une certaine mesure des «domaines» – ce qu'il appelle dans les *Prolégomènes* «multiplicités» (*Mannigfaltigkeiten*), d'un terme emprunté à la géométrie riemanienne, faite alors modèle du passage à une axiomatique au sens moderne du terme d'une discipline (la géométrie) axiomatique au sens traditionnel. Ici Husserl est beaucoup plus proche de Hilbert – au moins du premier Hilbert, pas du Hilbert «métamathématique», qu'il a ignoré ou du moins auquel il n'a pas vraiment réagi – que de Frege.

Il y a toutefois une nuance forte à apporter à ce «formalisme» de la philosophie des mathématiques qui résulte de la première pensée de Husserl et encore dans une large mesure du Husserl plus tardif. Le formalisme conduit jusqu'au bout semble précisément mener à une conception «formelle» de l'objet, dans laquelle celui-ci ne pourrait pas être «donné», mais pourrait juste être «constitué»[501] par le jeu des axiomes, et perdrait toute teneur phénoménologique. Ce qu'on donne à l'axiomatique, c'est un thème récurrent, on le retire à l'intuition, et c'est au défaut supposé de celle-ci que prend naissance la nécessité de l'axiomatique.

Tel n'est pas le cas chez Husserl, et cela dès les *Recherches Logiques*. Le sens aigu de la «forme» qui est à l'œuvre dans la philosophie de Husserl n'exclut nullement, y compris, dans le cas du formel, le recours à l'intuition.

Il est important de détacher le point de vue phénoménologique sur les mathématiques de toute perspective empiriste ou naïvement abstractive. Il n'y a aucun moyen de faire dériver «directement» les objets mathématiques de l'intuition au sens usuel du terme, comme des constructions qui seraient échafaudées «à partir» de l'intuition.

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[501] En un sens plus carnapien qu'husserlien de la constitution.

Toute la perspective husserlienne sur les mathématiques, on l'a vu, est dominée par un sens du «formel» comme élément caractéristique de la mathématique moderne. De ce point de vue, il est extrêmement important que Husserl fasse une différence entre «abstraction» au sens traditionnel du terme (c'est-à-dire ce geste qui consiste à retirer d'un complexe intuitif une de ses parties et à l'isoler) et ce qu'il appelle «abstraction formalisante».[502] L' abstraction formalisante consiste à traiter l'objet comme un *objet quelconque*, support de substitutions indéfinies, et à le transformer ainsi en variable, en revenant à ses pures propriétés de forme, qui résident dans son seul rapport aux autres objets, comme possibilités combinatoires. Dans cette démarche baptisée par Husserl lui-même «formalisation» – il semble qu'il intronise philosophiquement le terme – c'est le sens même de la mathématique moderne qui est censé reposer. En tout cas on est très loin de l'idée d'une mathématique intuitive, à l'objectualité prélevée sur l'intuition. L' objectité logico-mathématique – puisque Husserl soutient la continuité et l'indistinction de ces deux plans – est résolument *formelle*, forme commune des objets et non profil de quelque champ d'objectité particulier: elle ne peut dès lors, semble-t-il, qu'être soustraite à toute intuition.

Or la pointe du logicisme ou tout au moins du formalisme phénoménologique, tel qu'il est présenté dans les *Recherches Logiques*, est qu'il n'en est rien. Toutes les *Recherches Logiques* sont construites pour culminer dans la thèse selon laquelle il y aurait une certaine *intuition du formel* (c'est-à-dire du formel comme tel), ce que Husserl nomme «intuition catégoriale». Cette intuition paraît être sur mesure pour supporter la prise en charge par l'intuition elle-même de structures syntaxiquement formées telles que semblent l'être, à en croire le Husserl des *Recherches Logiques* (et encore de *Logique formelle et logique transcendantale*), les structures mathématiques (en tant que structures *logiques*).

Y a-t-il à proprement parler une «intuition catégoriale»? C'est une question philosophique qu'il n'est pas facile de trancher, d'autant plus que les analyses husserliennes de la VIe *Recherche Logique*, qui y sont

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[502] Nous avons étudié en détail cette différence dans «Husserl et Frege sur le concept», in Robert Brisart (éd.), *Husserl et Frege. Les ambiguïtés de l'antipsychologisme*, Paris, Vrin, 2002, p. 203–224.

consacrées, sont extrêmement difficiles et ont un parfum d'inachevé. Il faut cependant rappeler que, du point de vue de Husserl, c'était là «la pointe» des *Recherches Logiques* et ce qui en constituait la «percée» (*Durchbruch*) proprement phénoménologique. L'invention même de la phénoménologie, contre toute forme de restriction et d' «interprétation éliminative» du champ de ce qui peut être intuitivement donné, devait tenir dans l'introduction d'une telle intuition, que l'on pourrait caractériser schématiquement en la qualifiant d'intuition de *ce qu'on fait sur les objets* (et non des seuls objets, dans leur nudité de départ). D'autre part, compte tenu de ce qui nous intéresse ici, il faudra remarquer que, quelle que soit la recevabilité de la doctrine husserlienne de l'intuition catégoriale elle-même, dans son authenticité, il nous semble que, au moins dans ce désir d'élargissement du champ de la donnée intuitive en direction d'une intuition *formelle* (celle des *opérations* que l'on effectue sur les objets, ou plus exactement de ces objets en tant que purs objets d'opérations, pour ainsi dire anonymés), il y a quelque chose qui ne peut que parler au mathématicien, et relève bien de ce qu'on appellera une *phénoménologie de l'expérience mathématique*.

La *formalité* de l'objet, c'est-à-dire les propriétés qui lui échoient en vertu de ce qu'il est le point d'application d'opérations (au sens le plus large du terme: disons de *transformations*, qui laissent – ou révèlent – des zones d'invariance), n'est-ce pas là, en effet, ce à quoi a affaire en propre le mathématicien?

De ce point de vue, il y a, dans les *Recherches Logiques* comme dans les textes plus tardifs de Husserl, quelque chose, comme un style général, également en dehors du champ de l'épistémologie des mathématiques *stricto sensu*, qui semble extrêmement congru à l'expérience du mathématicien.

Il nous semble cependant qu'au moins deux difficultés restent en suspens qui retiennent Husserl sur le seuil d'une véritable épistémologie des mathématiques telles qu'elles sont devenues (après lui, il faut le souligner), et telles que nous les connaissons. Il importe aujourd'hui de les résoudre afin d'accéder à une *épistémologie phénoménologique des mathématiques*, qui paraît à plus d'un titre souhaitable.

La solution de ces difficultés nous paraît passer par la prise en compte de ce que Husserl ne pouvait pas connaître, à savoir la *théorie mathématique* *des catégories*, telle qu'elle résulte des travaux de Saunders Mac Lane dans les années 40, puis d'Alexandre Grothendieck dans les années 60 du XXe siècle.[503]

La dimension clairement structurale de la mathématique husserlienne la fait venir à la rencontre de la mathématique moderne qui, durablement, nous semble-t-il, est une mathématique de la structure. Mais restait à accomplir un pas de plus, qui, aussi bien, est le pas que la mathématique structurale, à partir d'un certain moment (l'après-guerre et les années 60), a été conduite à faire au delà d'elle-même.

La phénoménologie husserlienne des mathématiques est structurale en ce qu'elle se fixe sur les invariances (donc *ce qui apparaît par variation*), dont elle fait le cœur de l'objectité mathématique, en tant qu'objectité *formelle*. Elle est aussi structurale, dans un seul et même engagement, par le fait de *privilégier l'opération sur l'objet*, ou plus exactement de définir le type d'objectité propre à laquelle a affaire la mathématique par les opérations qu'elle peut supporter, et qui en assigne les limites – formelles, encore une fois.

Mais, dans ce «structuralisme» acharné à faire ressortir la forme même de l'objet des *gestes* mêmes qui sont ceux de la mathématique, comme pratique opératoire, il y a encore quelque chose de *statique*. Même si le formalisme mathématique de Husserl est très éloigné de l'espèce de platonisme naïf des essences qu'on lui impute souvent, comme si celles-ci constituaient un en soi dissociable précisément des opérations qu'on peut effectuer sur lui, il n'en reste pas moins que, précisément, dans l'hypostase du formel qui traverse toutes les *Recherches Logiques* et en un sens encore l'œuvre plus tardive (au moins *Logique formelle et logique transcendantale*), il y a comme une forme de *réification* de ces opérations, figées précisément en *structures*.

Ce qui manque à la phénoménologie éidétique puis transcendantale des mathématiques, c'est le sens d'un *mouvement*. Cela sans doute, parce que, en tant que pensée de l'intentionalité, elle est toujours trop pressée de faire *objet*, de rabattre la pensée sur un objet de visée.

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[503] Ce lien nous est apparu à la lumière des travaux de Frédéric Patras: voir son article «Phénoménologie et théorie des catégories» in *Geometries of Nature, living Systems and human Cognition,* ed. L. Boi. World Scientific. 2005, pp. 401–419.

Ce qui atteste cette limite, qui est aussi bien limite d'époque, dans la pensée de Husserl, c'est sa fixation, depuis *La philosophie de l'arithmétique*, sur la forme *ensembliste* comme première forme, et privilégiée, du catégorial. L'ensemble est le marchepied de la structure, comme dans toutes les théories structuralistes mathématiques «classiques», qui sont des structuralismes statiques.

A cela, la théorie des catégories, sous la forme développée dans les années 60 et au delà, a apporté un correctif décisif. En effet, elle a, pour ainsi dire, *mis les structures en mouvement*, les pensant elles-mêmes comme mouvements et non plus comme point de départ ou en soi statique.

Ce qui est caractéristique de la théorie des catégories, c'est la radicalisation et l'auto-dépassement de l'intuition structuraliste suivant laquelle on ne peut séparer l'objet de ce qu'on fait sur lui. En ce sens, le point de départ de la théorie des catégories est particulièrement intéressant, puisqu'*il ne s'agit plus d'objets*, mais *d'objets munis de flèches*, sans qu'on puisse séparer les uns des autres – les objets «eux-mêmes» (mais c'est cette notion qui est par là-même ébranlée) étant pensés comme un cas particulier dans ce cadre général, puisque comme les objets munis de la flèche identique.

Mais ceci a des conséquences très profondes sur le «matériel» même de la mathématique et sur son fondement. Celui-ci, à suivre les théoriciens les plus engagés des catégories (comme Lawvere), ne peut plus être ensembliste. En effet, les structures «statiques» comme le sont les ensembles (soubassement infrastructural ou disons de l'ordre de la structure «pauvre» de l'édifice structural traditionnel), ne peuvent plus ici être au départ. Tout au plus sont-elles des résultats ou des projections d'une forme d'organisation qui est plus fondamentale qu'elles: celle des catégories et des foncteurs précisément, qui semble mettre d'abord quelque chose de plus riche (des objets munis de quelque chose comme des fonctions, puis des fonctions qui relient ces collections d'objets munies de fonctions en tant que munies de fonctions mêmes), mais qui en réalité met surtout en avant quelque chose de plus *fondamental – ce qu'on fait sur les objets* en tant précisément que cela produit des structures ou quelque chose comme des structures. Ainsi par exemple une relation aussi fondamentale que l'inclusion ensembliste est-elle interprétée en termes de flèche d'une collection vers une autre, qui «isole» une sous-collection dans celle-ci.[504] A une relation statique est substitué un mouvement, une mise en rapport active. En un certain sens, les opérations ici *engendrent* les structures d'objet, loin de se contenter d'en mesurer les limites, comme si celles-ci les précédaient comme un donné intangible – comme s'il y avait un *a priori* de la structure. Ce qui prime, c'est ce qu'on peut faire sur les objets, et c'est de là que dérivent les structures.

Ce caractère productif, et dynamique, de la théorie des catégories s'atteste au mieux dans sa capacité à prendre en charge la formalisation des systèmes dynamiques.[505] Tout système d'éléments dans lequel l'état du système à tel moment de son développement est pensable comme résultant par des transformations réglées de son état au moment précédent est justiciable d'une approche catégoriale, en termes d'*endomap*. Ce que capture alors la mise en forme catégoriale, c'est le mouvement même, ce qu'on pourrait appeler la *processualité du formel*, qui n'est plus formel statique, mais formalité de la transformation même.

Au fond, ce que permet l'approche catégoriale, ce n'est pas tant l'abandon du structuralisme que l'adoption d'une nouvelle forme de structuralisme, centrée sur l'idée de transformation. Il ne s'agit plus tant de penser des structures qui seraient elles-mêmes des rapports (figés), que la mise en rapport de structures mêmes (préalable à elles et en quelque sorte dont ces structures découlent),[506] voire d'abord, en premier lieu, la mise en rapport de ces mises en rapport.[507]

Il nous semble que de ce point de vue la pensée catégoriale n'est pas du tout étrangère, dans ses fondements, au type de «structuralisme» qui est celui de la phénoménologie, simplement en en faisant, avec des moyens que la phénoménologie ne pouvait pas soupçonner pour des raisons tenant à l'avancement du savoir mathématique autour de 1900, un structuralisme

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[504] Voir Robert Goldblatt, *Topoi. The Categorial Analysis of Logic*, Amsterdam, North-Holland Publishing, 1984, p. 75sq. Il s'agit de l'interprétation de la notion de sous-ensemble en termes de *subobject*.

[505] Cf. F. William Lawvere et Stephen H. Schanuel, *Conceptual Mathematics. A First Introduction to Categories*, Cambridge, Cambridge University Press, 1997, p. 161*sq*.

[506] Puisque ce qu'il y a au départ, ce sont des flèches.

[507] Si, comme le pensent les inventeurs des mathématiques catégoriales, Eilenberg et Mac Lane, la notion essentielle de ladite théorie des catégories n'est pas celle de catégorie, mais celle de foncteur.

dynamique qui, par certains côtés, *est beaucoup plus phénoménologique* (parce que beaucoup plus voué au *mouvement*, qui constitue la chair même du phénomène) *que celui-là même que la phénoménologie pouvait proposer*. En un certain sens, la pointe du structuralisme, ce n'est pas la structure, mais *ce qu'on fait de la structure*.

Le sens «dynamique» de ce structuralisme s'assortit d'un autre aspect qui là encore nous renvoie à la phénoménologie et nous éclaire peut-être mieux sur ce qu'elle pourrait dire sur les mathématiques qu'elle ne le fait elle-même: il s'agit de la dimension d'*intuitivité* de cette mathématique hyper-abstraite qu'est la théorie des catégories.

Lawvere et Schanuel intitulent à juste titre leur introduction à la théorie des catégories *Conceptual Mathematics*. Conceptuelle, la théorie des catégories l'est par sa portée, quasi-philosophique, et par sa nature, hyper-abstraite. Mais ce qui frappe dans l'exposé certes simplifié mais rigoureux de Lawvere et Schanuel, c'est le tour extraordinairement *intuitif* de cette théorie hyper-abstraite, qui peut s'illustrer dans des intuitions relativement simples, si fondamentales soient-elles – simples *parce que* fondamentales.

A ce niveau, il y a deux choses à remarquer. La première constitue un élément de critique de ce qu'on pourrait appeler le logicisme phénoménologique de Husserl à la lumière de la théorie des catégories. La seconde donne au contraire une confirmation, par la théorie des catégories, de ce qu'il y a quelque chose de profondément juste dans la façon que la phénoménologie a d'entendre son propre structuralisme.

L'aspect critique concerne le postulat, qui semble être à l'œuvre au moins dans les *Recherches Logiques* (et dans *Logique formelle et logique transcendantale*), suivant lequel il n'y aurait d'autre «forme» en mathématique que celle, logique, de la syntaxe, ou que, en tout cas, là où il y a une telle forme (comme en géométrie), elle est «allogène» – son intervention renvoie à l'immixtion d'un donné non mathématique dans la théorie mathématique.

La théorie des catégories conduit ce logicisme, de toute façon mis à mal depuis Gödel, à son point de rupture. En effet, au moins dans ses développements dans le sens de la théorie des *topoi*, elle présente les théories logiques comme des cas particuliers de théories plus englobantes, les reconduisant à un formalisme qui n'est plus essentiellement syntaxique au sens limitatif du terme (mise en séquences d'énoncés), mais qui est plutôt d'inspiration géométrique ou en tout cas topologique. En un certain sens, la théorie des *topoi*, c'est le triomphe de la topologie sur la logique. Plus généralement, la théorie des catégories semble rompre le privilège de la logique au sens étroit du terme au profit d'une conception plus générale de la forme mathématique.

Nous pensons que cet élargissement, loin d'être incompatible avec la phénoménologie, n'en représente que la conséquence la plus logique, et constitue le seul accès possible à une conception authentiquement phénoménologique du mathématique. Mais bien sûr, Husserl ne pouvait pas disposer des outils pour le penser. En un sens, on pourrait dire que *la phénoménologie attendait la théorie mathématique correspondante*: une théorie qui, au lieu de reconduire le sens de la «forme» mathématique à celui, restreint, de la forme logique, mette en évidence l'appartenance de ce dernier à un horizon plus général, proprement mathématique, de formalité.

Le point important – cela constitue le second élément de confrontation, cette fois positive, avec la phénoménologie husserlienne – est que cette formalité *n'exclut pas l'intuitivité et même, en un certain sens, est en son fond intuitive*.

Il nous semble que c'est là l'aspect de la théorie des catégories qui la caractérise, en son genre, comme phénoménologique – c'est-à-dire tout au moins comme appelant une épistémologie des mathématiques de type phénoménologique. Avec la théorie des catégories, le conceptuel n'exclut pas l'intuitif, bien au contraire – c'est inscrit dans la provenance topologique de la théorie, et dans le caractère en un sens «géométrique» de certains de ses débouchés.[508]

Le point intéressant, du point de vue phénoménologique, est que cela conduit nécessairement à une *réforme du sens de l'intuition*. En un certain sens, il nous semble que le point de vue catégorial en mathématique est ce qui est le plus à même de donner un sens opératoire à cette notion étrange qui constitue la pointe de la théorie husserlienne dans les *Recherches Logiques*, à savoir celle d'*intuition catégoriale*.

Dans leur livre d'introduction, Lawvere et Schanuel donnent un exemple extrêmement simple mais très parlant, qui nous semble peut-être le meilleur exemple qu'on puisse trouver de ce que Husserl aurait appelé

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[508] On a pu parler de «logique géométrique» à propos de la théorie des *topoi*: cf. Goldblatt, op. cit., p. 493*sq*.

une intuition catégoriale.[509] Il s'agit de l'exemple du restaurant chinois où le prix de chaque type de plat est matérialisé par la forme de l'assiette employée pour le servir, et où à la fin, le compte se fait en empilant les assiettes, sans qu'il ne soit plus besoin de savoir ce qu'on a mangé. Ce qui est en question ici, c'est une composition d'applications: des mets aux prix, et des prix aux assiettes, donc une structure abstraite. Mais l'intérêt est qu'elle est immédiatement visualisée: ce que «voit» le serveur, c'est un rapport, pour chaque assiette d'une forme déterminée, un prix qui y correspond. Il y a là une forme d'intuition catégoriale: comment voir des rapports abstraits – et se donner sur eux la maîtrisabilité d'un calcul – dans des formes concrètes, sensibles, en les faisant supports de quelque chose (d'applications qui portent sur elles). Il nous semble qu'il y a là quelque chose qui donne (enfin) un sens très concret, très immédiat, à la notion husserlienne d'intuition catégoriale.

Il est également intéressant que cette intuition abstraite, structurelle, ne soit plus, contrairement à l'exemple canonique donné par Husserl dans les *Recherches*, une intuition syntaxique (syntaxiquement formée, et du syntaxique comme tel, du style: l'intuition de ce *que* le couteau *est* sur la table), mais l'intuition d'une structure de type proprement mathématique, en deçà de tout «discours». Ce que «voit» le serveur dans l'assiette, c'est la projection du met sur un prix. L'intérêt est à la fois qu'il n'a pas besoin de se le «dire» (il le «voit»: c'est ce qui fait de cette expérience une intuition), et qu'en même temps ce qu'il voit là, bien qu'immédiatement donné, soit fondamentalement abstrait. Le sens de la forme mis en jeu ici est tout sauf syntaxique.

On aurait le même type d'expérience dans toute activité consistant à manipuler des diagrammes et à «raisonner» sur eux, activité qui est bien de l'ordre du raisonnement à proprement parler, et qui pourtant n'est pas nécessairement ni en tout cas exclusivement discursive *proprio sensu*. Parler et penser ne sont pas toujours des activités de même ordre, et il y a un niveau d'expérience de la forme qui renvoie à un autre type de formalité que la formalité discursive (autre que celle de la syntaxe).

En ce double sens, il nous semble que, aujourd'hui, la théorie mathématique des catégories fournit, peut-être pour la première fois,

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[509] Lawvere et Schanuel, op. cit., p. 76*sq*.

un cadre théorique à ce que pourrait être une véritable épistémologie *phénoménologique* des mathématiques,[510] ainsi que, du point de vue philosophique en général, un extraordinaire champ d'application à la phénoménologie. Elle nous donne enfin les moyens de remplir ce qui a toujours été le programme de la phénoménologie, *à savoir ne jamais séparer le concept de l'intuition*.

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[510] De cette épistémologie, rendue possible une fois dépassée la perspective d'un structuralisme trop *purement* conceptuel, nous voyons les ferments dans le livre de Frédéric Patras, *La pensée mathématique contemporaine*, Paris, PUF, 2001.

CHAPTER 12

POURQUOI LES NOMBRES SONT-ILS «NATURELS»?

Frédéric Patras

Abstract. Two ontological points of view dominate the current mathematical epistemology. According to the first, the existence of mathematical objects is always relative to a given formal axiomatic system. Each such system supports a universe. The mathematician who wants to give a meaning to the objects of this universe is free to do so by developing his own semantics, for example by appealing to the physical world. However, this semantic interpretation is not necessary and does not belong to the core of the corresponding mathematical theory. According to the second point of view, largely supported by mathematicians, the mathematical activity would rely on the existence of a transcendant domain of objects. Alain Connes is currently the most influential representative of this last conception. His "post-gödelian Platonism" can therefore be considered as representing one of the leading trends of the present understanding of ontological problems in the mathematical community. Starting from a philosophical analysis of Connes' thesis on elementary arithmetics, and confronting its hidden epistemological assumptions with Husserl's seminal *Philosophy of Arithmetics*, we develop the idea that current mathematics, and especially the modern theory of categories lead naturally to a renewal of Husserl's phenomenology of mathematics. The example of natural numbers is considered in detail.

Introduction

Depuis les travaux de Cantor[511] et Frege,[512] à la fin du dix-neuvième siècle, les mathématiciens pensent que le concept de nombre est fondé, logiquement et en raison, sur celui d'ensemble. Au-delà de l'arithmétique, c'est toute l'algèbre et toute l'analyse qui seraient reconductibles à la théorie moderne des ensembles, au sein de laquelle les paradoxes ont

[511] G. Cantor, *Sur les Fondements de la théorie des ensembles transfinis* (1895 et 1897). Trad. franç. F. Marotte, Sceaux, Jacques Gabay, 1989.
[512] G. Frege, *Les Fondements de l'arithmétique* (1884), Trad. franç. C. Imbert, Paris, Le Seuil, 1969.

L. Boi et al. (eds.), Rediscovering Phenomenology, 357–386.

été tant bien que mal éradiqués.[513] Les *Éléments de mathématiques*[514] du groupe de mathématiciens Bourbaki,[515] qui reflètent et mettent en œuvre une conception des mathématiques datée de la première moitié du vingtième siècle,[516] sont l'exemple abouti de cette vision «structuraliste» et «architecturale»[517] du corpus, où la théorie des ensembles fait office de fondation sur laquelle s'édifient les diverses structures: un magma, un monoïde ou un groupe est un ensemble muni d'opérations; un ensemble ordonné est un ensemble muni d'une relation binaire; un espace topologique est un ensemble auquel est associée une famille distinguée de sous-ensembles (les ouverts), chacune de ces diverses opérations, relations, familles, étant astreinte à satisfaire telle ou telle identité, telle ou telle propriété remarquable.

La logique mathématique, le plus souvent mal comprise ou ignorée des mathématiciens, est venue perturber l'imagerie correspondante d'un «paradis ensembliste cantorien».[518] Du point de vue de la logique contemporaine, le cadre au sein duquel travailler en mathématiques n'est pas fixé par avance et une fois pour toutes[519]: divers choix alternatifs au système axiomatique de la théorie classique des ensembles sont possibles, et c'est là l'essence des preuves de consistance et d'indépendance (Gödel, Cohen…). Ceux-ci ont eu longtemps peu d'incidence sur la pratique mathématique, tout en ayant des répercussions considérables sur les problèmes de fondements des mathématiques et, plus généralement, sur toute la philosophie des mathématiques. L'hypothèse du continu (ou l'axiome du choix) n'est

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[513] Par l'intermédiaire de constructions *ad hoc* comme l'axiomatique de Zermelo-Fraenkel.

[514] N. Bourbaki, *Éléments de mathématiques,* Paris, Hermann puis Masson, 1938–1984.

[515] Sur la «vie et la mort» de N. Bourbaki, voir la notice de P. Cartier, «Bourbaki» *in* D. Lecourt (éd.), *Dictionnaire d'histoire et philosophie des sciences*, Paris, P.U.F., 1999.

[516] Voir notre *La Pensée mathématique contemporaine* (2001), Paris, P.U.F., 2-ième éd., 2002.

[517] N. Bourbaki. «L'architecture des mathématiques» *in* F. Le Lionnais (éd.), *Les grands courants de la pensée mathématique,* (1946), Paris, Hermann, rééd. 1998.

[518] L'expression, emblématique de son opposition à l'intuitionnisme brouwérien, est due à D. Hilbert. Sur les relations complexes de la phénoménologie husserlienne à Hilbert et Brouwer, nous renvoyons à l'article de J. Dodd dans ce même recueil.

[519] Voir I. Moerdijk, «Sets, topoi and intuitionism», *Philosophia mathematica* (3) vol. 6 (1998), 169–177.

ni vraie ni fausse: elle est vraie dans certains univers mathématiques, fausse dans d'autres, et est souvent indécidable. De la même manière, il n'y a pas *une* logique mathématique privilégiée. La logique intuitionniste, dans bien des situations intrinsèquement mathématiques, est mieux adaptée, et est naturellement amenée à se substituer à la logique classique (celle du tiers-exclu).

Deux points de vue métaphysiques sur la pensée mathématique sont donc aujourd'hui concevables. Le premier, dans l'esprit des textes de Carnap sur la syntaxe logique et de son principe de tolérance, consiste à accepter un point de vue relativiste sur les contenus mathématiques: chaque système d'axiomes supporte un univers ou un horizon d'objets et de résultats, à charge pour le mathématicien qui se préoccuperait du sens de son activité ou s'intéresserait à l'applicabilité de ses résultats de développer une sémantique ad hoc. L'axiome du choix peut ainsi être accepté ou refusé sur le fondement de critères individuels, certains le jugeant évident, d'autres inacceptable dans le recours qu'il suppose à des procédés infinitaires. Selon l'autre point de vue, non entièrement exclusif du premier et probablement partagé implicitement par la plupart des mathématiciens, l'activité mathématique aurait pour support un réel (eidétique) primitif.

Connes est sans doute, en France, le représentant le plus en vue de ce platonisme renouvelé. Dans son livre d'entretiens[520] avec Lichnerowicz et Schützenberger,[521] il aborde la question de la nature des objets mathématiques à la lumière des acquis de la logique mathématique post-gödelienne. Cette confrontation à trois rend bien compte de l'état actuel du débat au sein de la communauté mathématique et sera pour nous l'occasion de montrer comment le point de vue de la phénoménologie husserlienne permet, aujourd'hui encore, d'aller au-delà des constats et des déclarations

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[520] A. Connes, A. Lichnerowicz et M.-P. Schützenberger. *Triangle de pensées.* Paris, Odile Jacob, 2000. Cité *TP* dans la suite.

[521] L'attention accordée ici à un ouvrage constitué de discussions entre mathématiciens peut surprendre dans un texte consacré prioritairement à une relecture de la phénoménologie husserlienne. Elle étonnera moins si l'on précise que cette relecture se fera aux termes de la perception contemporaine des problèmes de fondements des mathématiques et, plus particulièrement, du statut des nombres. L'arithmétique se trouve jouer un rôle central dans ce texte, la référence aux propriétés des nombres étant l'un des points clés de l'argumentation de Connes et l'un des principaux supports de ses thèses «platoniciennes».

d'intention pour élaborer une nouvelle ontologie mathématique. Nous insisterons tout particulièrement sur le cas paradigmatique des nombres et de leurs relations aussi bien aux théories ensemblistes qu'à leurs avatars dans les mathématiques modernes (les théories catégoriques et topossiques).

1. La Thèse Structuraliste

Triangle de pensées oppose, un peu à la manière des dialogues galiléens, deux thèses sur les mathématiques: outre le platonisme revu et corrigé défendu par Connes, un courant plus traditionaliste (le structuralisme de tradition bourbakiste) est représenté par Lichnerowicz. Malgré ses succès passés, la thèse fondamentale du structuralisme, d'ailleurs très bien illustrée et défendue par Lichnerowicz, est une thèse ontologiquement bancale et, à tout dire, épistémologiquement intenable. Il est rétrospectivement étonnant qu'elle ait pu occuper le devant de la scène mathématique des années 50 aux années 80. Elle a pourtant été en adéquation parfaite avec un certain état d'esprit de la communauté mathématique, qui, après la seconde guerre mondiale, souhaitait se concentrer de manière assez pragmatique sur le développement de sa discipline et, par là même, décidait de se désintéresser des querelles sur les fondements et l'épistémologie des mathématiques qui avaient marqué la première moitié du siècle.

Le constat husserlien de «crise des sciences européennes» dressé dans les années 30[522] était, de fait, assez désuet après la victoire militaire, technologique et scientifique des alliés. Le structuralisme, et avec lui l'entreprise bourbakiste d'écriture d'un traité regroupant les notions mathématiques fondamentales, allait dans le sens de cette recherche d'efficacité tout en tirant parti des potentialités offertes par les axiomatiques ouvertes.[523] En dépit de ses lacunes, sur lesquelles nous allons revenir, la thèse structuraliste indique très clairement la nature du problème fondamental

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[522] E. Husserl. *La crise des sciences européennes et la phénoménologie transcendantale* (cité *Krisis*), Trad. G. Granel. Paris, Gallimard, 1976.

[523] Les axiomatiques ouvertes sont, dans l'esprit de l'école algébrique allemande (Hilbert, E. Noether *et al.*), celles pour lesquelles le système d'axiome considéré admet plusieurs modèles non isomorphes, et pour lesquelles les théorèmes obtenus sont donc susceptibles d'applications aussi diverses qu'il y a de modèles: les théories algébriques – groupes, anneaux, corps…- en sont un bon exemple.

auquel ont été confrontées les mathématiques depuis les développements de la méthode axiomatique, à partir des débuts du vingtième siècle, difficulté sur laquelle a butté la phénoménologie husserlienne dans son rapport aux mathématiques[524]:

> «Nous avons appris que la première opération mathématique consiste à mettre des ensembles en dictionnaires parfaits les uns avec les autres [...]. La notion de structure est alors apparue et leur transfert par ces dictionnaires parfaits a donné la notion d'isomorphisme. C'est là où les mathématiques recommencent pour ainsi dire, car un des obstacles au développement des mathématiques grecques – l'existence «d'êtres» mathématiques ou «d'objets» – s'en est trouvé surmonté [...]. Les mathématiciens ont donc appris que «l'être des choses» sur lequel ils raisonnent n'était d'aucune importance pour eux [...]. C'est cette abstraction, mais une abstraction radicale, qui fait à la fois la puissance théorique des mathématiques et la fécondité de leurs rapports avec le réel.»[525]

Première thèse structuraliste, donc: un «décrochage ontologique» s'est opéré dans la pensée mathématique. Puisque dans le processus abstractif les mathématiques se séparent du réel et du domaine ontique, l'idée d'un Être des choses mathématiques doit être délaissée au profit d'une pure forme, la structure. La notion de structure est ici, et dans toute la tradition structuraliste en mathématiques, entendue en un sens ensembliste, ce qui ne va pas d'emblée de soi, car elle pourrait aussi bien être comprise plus génériquement comme le type de forme des objets mathématiques. Ce n'est pas le cas: la terminologie des structures est résolument technique, et il y a une tendance anti-métaphysique assez radicale dans les textes du structuralisme mathématique. Là encore, tout cela ne va pas sans rappeler les positions d'un Carnap, auquel il est très tentant de rattacher *a posteriori* toute la tradition structuraliste et en particulier Bourbaki, bien que ce dernier, qui connaissait Russell, ait sans doute ignoré le corpus carnapien.

Pour ce qui est des nombres, il est révélateur pour le débat qui nous occupe que Carnap ait adopté dans *La construction logique du monde*,[526] les thèses logicistes du premier Russell, avec en particulier l'idée d'une fondation du corpus mathématique sur la théorie des ensembles (dans sa variante typée). Ces thèses trouvent un développement philosophique

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[524] Nous renvoyons aux articles de J. Benoist et J. Dodd dans ce même recueil.
[525] A. Lichnerowicz, *TP*, p. 36.
[526] R. Carnap, *Der logische Aufbau der Welt*, Berlin, 1928.

dans l'idée carnapienne ultérieure,[527] d'inspiration wittgensteinienne, d'une partition du langage entre une composante proprement logique et une composante signifiante. Le noyau logique de la langue est formé d'une théorie logico-mathématique de type axiomatique, dont la propriété essentielle et constitutive est la complétude logique. L'arithmétique devrait y figurer de plein droit, mais tombe sous le coup du premier théorème de Gödel, qui interdit la propriété de complétude aux théories mathématiques non élémentaires (comme l'arithmétique avec quantificateurs). Il est particulièrement significatif que ces traits et ces apories caractéristiques du positivisme logique soient implicitement au cœur des analyses anti-structuralistes[528] de Connes, le représentant, dans *Triangle de pensée*, des mathématiques contemporaines et du nouveau rapport qui est en voie d'instauration, en mathématiques, aux problèmes logiques (théories de fondements, portée de la méthode axiomatique…) et ontologiques (rapports de la pensée mathématique aux sciences de la nature, nature des objets mathématiques...).

En tous les cas, indépendamment même de toute référence à Carnap, pour le structuralisme les choses semblent claires: toute référence à l'ontologie est proscrite; les mathématiques sont renvoyées au domaine formel; la charge sémantique du discours mathématique et l'analyse de son efficacité sont confiées aux sciences physiques et naturelles qui mettent en équation les phénomènes et les font participer des méthodes formelles: «On a eu la possibilité de construire des modèles, de comprendre qu'en faisant de l'hydraulique et en faisant de l'électrostatique on pouvait dire exactement les mêmes choses parce qu'on retrouvait les mêmes équations.»[529]

Pourtant, ce discours, d'origine hilbertienne, n'a jamais vraiment satisfait les mathématiciens. Il impliquerait, pour être soutenu avec cohérence, que ceux-ci renoncent à toute forme de sémantique et de référence ontologique *au sein même de la pratique mathématique*, renoncement qu'aucun mathématicien de profession ne peut accepter: il sait bien que son activité a un sens et se construit selon des règles eidétiques qui ne sont pas celles des systèmes formels. Le structuralisme mathématique, dont

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[527] R. Carnap, *Logische Syntaxe des Sprache*, Vienne, 1934.
[528] Cf. *Le platonisme post-gödelien,* dans la suite de cet article.
[529] A. Lichnerowicz, op. cit.

l'élaboration sous forme de corps de doctrine est le fait de mathématiciens, n'a jamais complètement perdu de vue, à ce titre, la réalité et la complexité du travail mathématique. Paradoxalement, en marge du discours formalisé et de leurs positions de principe anti-métaphysiques, les tenants du structuralisme ont donc ménagé *de facto,* sans en assumer l'existence sur un mode théorique, la possibilité d'un domaine ontologique pour les «êtres» et les «objets» mathématiques. Cette contradiction flagrante est très nette dans «L'architecture des mathématiques».[530] Après avoir rappelé les ressorts de la méthode axiomatique, Bourbaki s'y fait un devoir de ne pas accréditer les thèses formalistes:

«Le mathématicien ne travaille pas machinalement, comme l'ouvrier à la chaîne; on ne saurait trop insister sur le rôle fondamental que joue, dans ses recherches, une *intuition* particulière, qui n'est pas l'intuition sensible vulgaire, mais plutôt une sorte de divination directe (antérieure à tout raisonnement) du comportement normal qu'il semble en droit d'attendre, de la part d'êtres mathématiques qu'une longue fréquentation lui a rendus presque aussi familiers que les êtres du monde réel.»[531]

Lichnerowicz va dans le même sens:

«Toutefois ce n'est pas pour l'abstraction radicale, ce n'est pas pour faire des démonstrations rigoureuses et contraignantes qu'on devient mathématicien. Tout mathématicien se tient à lui-même, voire à quelques complices, un discours qui n'a rien à faire avec celui-là. Il faut bien distinguer le discours de communication universelle et le discours de création des mathématiciens. »[532]

Cette tendance schizophrénique dans les positions de principe de Bourbaki ou dans le discours de Lichnerowicz est l'indice d'une crise et traduit le rapport le rapport ambigu et souvent malsain de la science contemporaine au rôle créatif de la pensée. Les voies de sortie de la crise restent inchangées: il faut réconcilier discours de communication universelle et discours de création; formes pures de théories et représentations, intuitions vivantes. Or, le discours de création – celui qui a lieu avant que la découverte proprement dite ait lieu et se fige en énoncé et démonstration – est un discours en bonne partie descriptif: il explicite la nature du rapport de la conscience du mathématicien aux objets qu'elle appréhende. En d'autres mots, il cherche à dire la tension noético-noématique qui est celle d'une

[530] op. cit.
[531] op. cit. p. 42.
[532] A. Lichnerowicz, op. cit.

pensée dont les représentations sont incertaines (tel objet mathématique *devrait* avoir telle propriété, telle théorie *devrait être* en rapport analogique avec telle autre, etc.). Cette pensée progresse, entre autres, en analysant la structure interne de son rapport aux objets: si un mathématicien *croit* à l'existence d'une analogie entre corps de nombres et corps de fonctions ou entre polyèdres et fibrés en droites,[533] avant même que cette analogie ne soit rendue explicite par une construction mathématique effective, c'est-à-dire avant que le problème ne soit résolu, c'est parce qu'il perçoit que ces différents objets mathématiques ont des comportements similaires. Et une telle similitude, lorsqu'elle est vraiment profonde et novatrice, n'est pas exclusivement formelle: un mathématicien ne remarque pas incidemment que deux théories ont des systèmes d'axiomes voisins, il se rend plutôt compte que «la même chose se passe» lorsqu'il étudie telle classe d'objets et telle autre.

Pour le dire autrement, le «lieu» où l'analogie se manifeste d'abord est, dans les cas les plus intéressants, la conscience du mathématicien. L'analogie prend corps et se déploie au travers des modalités de saisie intentionnelle des objets: ainsi, deux objets mathématiques distincts qui se prêtent aux mêmes transformations ont sans doute des symétries (ou, plus généralement, des groupes d'automorphismes ou des semi-groupes d'endomorphismes) analogues. Il n'est d'ailleurs pas gratuit que le vocabulaire employé par Lichnerowicz pour décrire le rapport du mathématicien aux objets qu'il appréhende soit immédiatement susceptible d'une interprétation phénoménologique (une analyse descriptive des structures intentionnelles du rapport à l'objet):

> «Si un mathématicien travaille (c'est le témoignage de tous), en fait, il réfléchit à un certain champ où, là, il y a des êtres mathématiques, là il finit par jouer avec eux, suffisamment pour les rendre familiers [...]. Il y a deux types d'activité par conséquent. Si l'on devient mathématicien, c'est pour le discours de création, le jeu de l'intuition, pas du tout pour le pensum de la publication [l'aspect formel du travail]».[534]

Cette aporie fondamentale du structuralisme mathématique, tendu entre ses exigences formelles et le refus de ses partisans d'abandonner toute référence au travail créatif et au mouvement de la pensée, est aussi

[533] Ce sont deux analogies classiques, bien établies, qui ont suscité de nombreuses recherches.
[534] A. Lichnerowicz, op. cit., p. 37.

celle de la phénoménologie husserlienne des mathématiques, partagée entre ses racines kantiennes et hilbertiennes, entre le transcendantal et le formel.

2. Le Platonisme Post-Gödelien

Le platonisme de Connes fait pendant au structuralisme de Lichnerowicz. Il se veut une tentative pour penser les mathématiques après les théorèmes de Gödel et la remise en cause du programme formaliste hilbertien qui en a découlé.[535] Les théorèmes de Gödel ont montré qu'il existe des théorèmes arithmétiques vrais (sémantiquement ou à un niveau métathéorique) mais non démontrables (indécidables) au sein de l'arithmétique formalisée. Plus généralement, pour toute théorie mathématique contenant l'arithmétique, il existe des résultats vrais et pourtant indécidables au sein de la théorie même. La thèse de Connes est qu'il existe donc, au moins pour l'arithmétique, une «réalité archaïque» (celle des résultats vrais sur les nombres, mais indémontrables), qui serait antérieure à toute mise en forme logique et axiomatique de ses propriétés.

Il s'agirait là d'une simple reformulation des théorèmes de Gödel (tout n'est pas décidable dans une théorie formalisée et, plus radicalement, le domaine du vrai est plus étendu que celui du démontrable), si elle ne s'accompagnait de thèses résolument ontologiques et métaphysiques:

> «Les mathématiques ont de mon point de vue deux aspects, l'un que j'appellerais «linguistique» et qu'André [Lichnerowicz] décrit de manière précise. Il est utilisable dans d'autres sciences, et reflète le fonctionnement de la logique. Je voudrais mettre en évidence un deuxième aspect des mathématiques: je maintiens qu'elles ont un objet, tout aussi réel que celui des sciences dont je parlais plus haut [les sciences de la nature], mais qui n'est pas matériel, et n'est localisé ni dans l'espace, ni dans le temps. Il a cependant une existence tout aussi ferme que la réalité extérieure et les mathématiciens s'y heurtent un peu comme on se heurte à un objet matériel dans la réalité extérieure. Cette réalité dont je parle, du fait qu'elle n'est localisable ni dans l'espace ni dans le temps, donne, lorsqu'on a la chance d'en dévoiler une infime partie, une sensation de jouissance extraordinaire par le sentiment d'intemporalité qui s'en dégage.»

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[535] Sur Hilbert, la méthode axiomatique et le programme formaliste, nous renvoyons à P. Cassou-Noguès, *Hilbert*, Paris, Les Belles Lettres, 2001.

À y regarder de près, l'argumentation est de type néo-platonicien et assez proche des thèses d'un Plotin quant aux Êtres mathématiques[536]: n'y trouve-t-on pas jusqu'à la mystique de la participation aux vérités intemporelles ! Sans doute faut-il voir là un indice de la difficulté qu'il y a à renouveler l'ontologie des Êtres mathématiques: alors même qu'il essaie de s'affranchir des contradictions du structuralisme, de la logique mathématique post-gödelienne et, indirectement, de dépasser l'antinomie phénoménologique entre logique formelle et logique transcendantale, A. Connes se voit contraint de recourir au vieux système, obsolète et confus, de l'ontologie plotinienne.

Pour autant, et malgré toute la naïveté de sa formulation, l'idée est intéressante, et mérite d'être développée en suivant d'autres voies. Connes, pour soutenir ses thèses, fait essentiellement appel aux résultats de Gödel, et en particulier à la distinction entre vérité et prouvabilité: le domaine du vrai ne se restreint pas au domaine d'inférence des axiomes d'une théorie. Techniquement, les logiciens et les géomètres pensent aujourd'hui ce type de distinctions en termes de logique intuitionniste: une structure topologique sous-tend les opérations logiques et permet de comprendre les phénomènes d'insaturation du domaine du démontrable. Laissons pour l'instant de côté les références à l'intuitionnisme et à son explicitation topologique; il reste cette idée fondamentale que la crise des fondements et les techniques d'analyse logique développées à la suite de Gödel ont mis en évidence un déficit ontologique dans la pensée axiomatique et structuraliste classique.

Le domaine des nombres et de l'arithmétique élémentaire (auquel la thèse de Connes se limite d'ailleurs, celle-ci ne prétendant pas porter sur les théories du continu, techniquement et ontologiquement plus complexes) se prête particulièrement bien à ces analyses. Il y va, en effet, d'êtres dont l'existence est fermement assurée, de par la simplicité de leur constitution logique (on peut penser à l'axiomatique de Peano) et de par leur évidence intuitive, les nombres étant l'exemple paradigmatique d'êtres abstraits présentables dans l'intuition, au sens des premières recherches

[536] Plotin, *Traité sur les nombres*. Ennéade VI 6 [34]. Trad. J. Bertier et al., Paris, Vrin, 1980.

husserliennes sur les rapports entre intuitions et représentations.[537] Par ailleurs, la plupart des phénomènes logiques les plus significatifs se jouent dans l'arithmétique élémentaire, au travers des rapports entre finitude des méthodes axiomatiques et algorithmiques et «infinité en acte» des nombres ou «infinité en puissance» de leurs propriétés.

Si l'intrusion du thème de l'infini dans l'analyse logique et ontologique de l'arithmétique élémentaire est classique,[538] la logique contemporaine a enrichi cette thématique en insistant sur les aspects dynamiques et géométriques des techniques et intuitions à l'œuvre dans cette analyse. L'interprétation qu'en donne Connes, si elle peut être inscrite dans ce courant, s'en démarque par sa tonalité:

> «Je veux essayer de clarifier ce que j'entends par réalité. Je peux en donner un certain nombre de caractères, mais je ne suis pas certain d'être exhaustif. Le premier, c'est le fait que cette réalité résiste, c'est clair, on en a déjà parlé. Le deuxième qui, à mon avis, est extrêmement important, est qu'elle est source inépuisable d'informations. C'est le côté inépuisable qui est crucial [...]. Je vois là un des attributs essentiels de la réalité extérieure. Ce qui est fascinant dans la réalité extérieure, dans la réalité physique, c'est précisément que l'on soit incapable d'en épuiser la quantité d'informations.»[539]

L'infinité potentielle des vérités arithmétiques serait donc l'élément clé d'une thèse ontologique forte: les êtres mathématiques sont réels; leur réalité excède le domaine axiomatique-formel, qui ne traite que des méthodes d'investigation finitaires que l'esprit humain est capable de mettre en œuvre pour appréhender et démontrer les vérités arithmétiques; l'analogie entre réalité des êtres mathématiques et réalité du monde physique est établie par leur commune infinité (l'infinité en puissance des vérités ou informations qu'elles recèlent).

Philosophiquement, ces thèses soulèvent sans doute plus de difficultés qu'elles ne peuvent prétendent en résoudre. Des notions comme celle d'information, qui font référence à des théories physiques (thermodynamique, mesures de l'entropie des systèmes, etc.) ont un contenu

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[537] E. Husserl. Études psychologiques de 1894, *in Articles sur la logique*, Trad. J. English, Paris, P.U.F., 1975.

[538] C'est même le fondement de toutes les méthodes d'analyse fine de l'arithmétique formalisée et de son extension: ainsi de l'induction transfinie.

[539] A. Connes, op. cit. p. 51.

technique précis, et leur utilisation pour appréhender le réel phénoménal doit donc être soumise aux réserves usuelles concernant la transposition des théories scientifiques dans le domaine philosophique. Aussi, de toutes celles qui viennent d'être évoquées, l'idée qui semble la plus intéressante, et la plus à même de conduire à un dépassement des apories de l'ontologie mathématique de tradition structuraliste, est-elle celle d'infinité potentielle du domaine des vérités arithmétiques. Elle sera développée ici en suivant une autre approche que celle, inspirée par la physique, adoptée par Connes, tout en se démarquant résolument des tendances néo-platoniciennes d'un discours «réaliste» sur les êtres mathématiques.

3. L'être-en-puissance des Mathématiques

Pour l'ontologie classique des mathématiques, et pour ses avatars modernes, l'objet mathématique est bien défini. Plus encore, il est défini *une fois pour toutes*: que ce soit dans les théories axiomatiques modernes ou même aux différents niveaux des traditions antérieures (Euclide, Descartes, Leibniz, etc.), l'objet mathématique a toujours eu des traits fondamentaux: détermination, invariance, traits qui lui permettent de participer de l'Être, voire du concept de substance, dont la caractéristique fondamentale, depuis Aristote, est la permanence.[540]

Lorsque les mathématiciens, à l'encontre de tout relativisme ontologique, font valoir que les objets mathématiques existent bien, qu'ils *sont là*, devant nous, s'affirment dans leur fréquentation quotidienne, «résistent» même, pour reprendre l'expression de Connes, ils ont indubitablement raison. En termes philosophiques: une ontologie mathématique est légitime, et il faut attribuer aux objets et théorèmes mathématiques un mode d'existence propre. Qu'il faille en revenir à un «réalisme», un idéalisme naïf ou à toute autre forme d'ontologie analogue est par contre discutable.

De son côté, la logique mathématique du vingtième siècle montre, d'abord et avant tout, que la méthode axiomatique, dès lors qu'elle est résolument finitaire, s'avère, par nature, par essence, incapable

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[540] P. Aubenque. *Le problème de l'être chez Aristote.* Paris, P.U.F., 1962.

d'appréhender l'infini des *possibles* mathématiques. Cependant, cette inaptitude ne signifie en rien que la vérité puisse être dissociée de ses conditions d'apparition formelles (de la logique mathématique, en particulier). Il n'y a pas de vérité mathématique indémontrable, et c'est un non-sens que d'interpréter ainsi les résultats de Gödel; par contre, il est des résultats «vrais» (dans un métasystème qui correspond assez fidèlement à notre intuition, si bien que l'identification, dans ce cas, des notions de vérité et de prouvabilité est naturelle) mais qui ne sont pas démontrables (par les méthodes autorisées dans la théorie support).

Une solution épistémologiquement satisfaisante pour résoudre ces différentes apories, consisterait à remettre en cause, de manière radicale, la thèse «réaliste» en réévaluant le rôle constitutif de la pensée et de l'intentionnalité du mathématicien, solution en accord avec l'évolution récente des mathématiques. S'ils participent de l'Être selon des modalités à préciser, les objets mathématiques ont toutefois une existence, sinon précaire, du moins évolutive. Ce qui ne signifie en rien que les mathématiques soient soumises à une forme brutale de relativité ontologique: comme on l'a dit, leurs objets «existent» bien, et connaissent une permanence indéniable, mais leur mode d'existence est indissociable de l'environnement mathématique au sein duquel ils apparaissent, environnement susceptible de connaître mutations et transformations.

L'apparition des théories axiomatiques n'a modifié en rien la validité des théorèmes euclidiens ou de la géométrie antérieure. Pourtant, une notion comme celle de triangle n'a pas la même portée ni la même signification pour nous qui connaissons les géométries non euclidiennes que pour Euclide. Et, lorsque Connes parle de l'arithmétique, il a en vue une notion de nombre (et de propriétés des nombres) qui déborde l'axiomatique de Peano (et d'autres systèmes axiomatiques plus riches). Ces deux exemples peuvent sembler dissemblables, voire antithétiques. Dans un cas, la notion de triangle a été infléchie et étendue par le développement historique des mathématiques, dans l'autre, au contraire, Connes en appelle à un système de vérités arithmétiques «archaïques», dont la légitimité serait intangible, et susceptible de tenir tête à toute remise en cause par les développements de l'analyse logique. Pourtant, malgré leur caractère antagoniste, les deux exemples témoignent d'un même phénomène: la capacité d'intuitions fondamentales, primitives, à résister à une formalisation définitive. Notre intuition de l'espace, du nombre, et même de la vérité, déborde tout système formel prétendant en épuiser la portée et la signification: leçon, s'il en est, à retirer des développements des mathématiques au cours du vingtième siècle. Cette leçon était déjà prédictible, au niveau des principes, à l'école du Criticisme, puisque ce sont ces intuitions, antérieures à l'édification des systèmes formels, qui nous permettent de leur donner un sens, et donc d'opérer avec eux sur un mode signifiant plutôt que sur ce mode purement formel et au potentiel créatif limité qui est le mode opératoire des ordinateurs.

Plutôt que de parler de vérités archaïques, à la manière de Connes, il est dont plus correct de renvoyer à un système de notions ou d'intuitions fondamentales. L'être des objets mathématiques associés à ces intuitions fondamentales est un être-en-puissance tout autant qu'un être-en-acte, puisque la permanence de ces objets est seulement relative (à un environnement donné), et puisque leur définition mathématique formelle est toujours susceptible de variations ou de réalisations diverses (par exemple dans des théories axiomatiques concurrentes). La puissance devient acte à chaque fois que ces objets sont mobilisés dans telle ou telle théorie. Le plus souvent les mathématiciens s'y trompent, croyant à chaque fois au pouvoir coercitif des définitions et à leur capacité à fixer et rigidifier définitivement des concepts pourtant aussi ouverts et énigmatiques que peuvent l'être les concepts géométriques et topologiques fondamentaux: espace, continuité, symétrie. . .

Un concept aussi primitif que celui de point, et dont le caractère d'évidence rivalise avec celui du nombre, a ainsi vu au cours du siècle dernier la perception et l'intellection que les mathématiciens en avaient évoluer radicalement. Sous l'influence de la géométrie algébrique en particulier, les géomètres ont commencé à penser le point comme «lieu d'annulation des fonctions s'annulant en ce point». L'ensemble de ces fonctions ayant la structure d'un idéal maximal (un concept né en algèbre, avec les travaux de Kummer, Dedekind, puis ceux de l'école algébrique allemande, avec en particulier E. Noether), le concept d'idéal maximal s'est peu à peu identifié, pour la géométrie moderne, à celui de point, avec toutes les conséquences que l'on peut tirer de cette évolution (élargissement de la notion même de point et, concurremment, des notions géométriques qui lui sont associées: voisinage infinitésimal, existence de nouvelles topologies adaptées au nouveau point de vue algébrique sur la géométrie – topologie de Zariski, et autres topologies exotiques à la racine des résultats les plus récents et les plus spectaculaires aux confins de la topologie algébrique et de la géométrie arithmétique). Ces idées nouvelles rétroagissent sur la perception *intuitive* que les mathématiciens ont d'un point, et le point de vue selon lequel le concept aurait été figé une fois pour toutes avec le choix d'une axiomatique privilégiée apparaît donc dépourvu de toute légitimité historique. De ce fait, la célèbre formule hilbertienne selon laquelle on pourrait remplacer, dans les axiomes de la géométrie euclidienne les mots de point, droite ou plan par chaise, table ou tasse[541] montre bien les limites du point de vue axiomatique formel, qui ne spécifie, parmi *toutes les relations possibles* qu'un concept mathématique peut entretenir aux autres concepts et objets mathématiques, qu'un nombre restreint de relations. Il néglige donc la prise en compte de cet élément fondamental pour la pratique et le développement de la science qu'est la nécessaire extension et évolution au cours du temps du nombre voire de la nature même de ces relations.

La conviction que procure l'exercice des mathématiques (exercice permanent, sous la forme d'un travail de recherche), est que l'essentiel du travail créatif réside précisément dans cette aptitude à transgresser les frontières assignées aux objets par leurs définitions.[542] Un phénomène vraiment nouveau et intéressant, lorsqu'il se manifeste, entre peu souvent dans un cadre prescrit: il faut inventer les structures au sein desquelles il trouvera une expression correcte et libèrera son potentiel de résultats originaux. L'analyse du phénomène, au sens classique du mot (par exemple au travers de la recherche d'équations qui le caractérisent), est bien entendu un moment essentiel de ce travail, mais la connaissance de la *phénoménologie*[543] des êtres mathématiques y joue un rôle tout aussi essentiel.

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[541] C. Reid. *Hilbert*, Berlin, Springer, 1970.

[542] Nous suivons ici A. Grothendieck, *Récoltes et semailles. Réflexions sur un passé de mathématicien.* Montpellier, 1985. Voir aussi *La Pensée mathématique contemporaine,* op. cit.: chap. 7, «Les demeures de la pensée».

[543] Il faut entendre ici *phénoménologie* au sens que le mot a chez Hegel, ou en dehors du champ philosophique: un ensemble de modes et règles de comportement et d'évolution. Il devrait être clair dans la suite quel usage du mot «phénoménologie» il est fait en fonction du contexte.

Lorsque nous pensons aux nombres, nous avons à l'esprit non pas une définition formelle (comme la définition monstrueuse, d'origine frégéenne,[544] proposée par la théorie des ensembles, sous la forme d'hypothétiques et problématiques classes d'équivalences d'ensembles sous la relation d'équivalence donnée par les correspondances biunivoques, définition qui ne peut se comprendre que comme l'aboutissement d'un long processus historique et technique, avec ce qu'il comporte d'évidences trompeuses sur la «naturalité» des constructions mises en œuvre…), mais des règles d'usage (comment procéder pour additionner deux nombres, etc.) et la conscience, plus ou moins nette d'un champ d'application du concept: les nombres servent à mesurer, à compter, à ordonner, etc.. L'idée directrice des résultats de Gödel a consisté précisément à détourner (ou à révolutionner) l'usage de la numération: puisque les théories axiomatiques sont finitaires (leurs démonstrations procèdent par un nombre fini d'étapes à partir des axiomes), on peut coder leurs résultats et leurs démonstrations *dans l'arithmétique élémentaire*. C'est en particulier vrai pour l'arithmétique (axiomatisée) elle-même, dont les théorèmes et schémas de preuve peuvent être codés par des nombres ou des suites de nombres (procédé dit de gödelification).

Si les objets mathématiques résistent donc à notre esprit (établir une preuve est toujours entreprise difficile), ils résistent également aux tentatives de réduction. L'art mathématique est un art complexe, fait tout à la fois de rigueur et d'intuition, où par intuition il faut comprendre ici l'aptitude à des représentations dont l'objet est effectivement présent à la conscience et est donc dans un rapport d'immédiateté tel à celle-ci qu'elle peut jouer avec cette représentation et la manipuler, de manière à en libérer les diverses potentialités (formelles, structurelles, opératoires…). En ce sens, Lichnerowicz a raison: il y a deux niveaux mathématiques, celui de la forme logique et celui de la praxis. L'erreur du structuralisme a été de ne pas comprendre que cette dichotomie n'est pas accidentelle ou indépassable. Elle renvoie à une structure unitaire de la pensée mathématique et est ancrée dans les modalités mêmes du connaître. Le corrélat pratique de la thèse structuraliste (l'idée d'une fixité et d'une rigidité des fondements et des concepts) cesse de valoir du même coup: il n'y a pas

[544] G. Frege, *Die Grundlagen der Arithmetik* (*Les fondements de l'arithmétique*, abrégé: GA), Breslau, W. Koebner, 1884.

d'architecture des mathématiques; les mathématiques ne sont pas une science cumulative, mais évolutive. Il y a une véritable phénoménologie (au sens hégélien, à nouveau) de la pensée mathématique, marquée par un dépassement progressif des contradictions qui l'habitent: le nombre versus l'espace, le discret versus le continu, l'intuition versus le formel, etc.

4. L'Exemple des Nombres Naturels

Un des traits de la pensée mathématique les plus déstabilisants pour un regard extérieur, et à l'origine de nombre de malentendus, est que ce dépassement de ses contradictions s'opère toujours de manière interne. Les motivations qui en sont à l'origine peuvent pourtant être extérieures, en particulier lorsqu'il y va des fondations. Le cas de Frege est ainsi exemplaire d'une entreprise aux ambitions résolument épistémologiques (dépasser le point de vue kantien et l'idée du synthétique a priori lorsqu'il y va de l'arithmétique), entreprise ayant débouché sur l'une des transformations les plus radicales du corpus et de la pensée mathématique jamais accomplie. Toutefois, en conclusion, les résultats de cette entreprise se sont traduits en termes mathématiques et se sont fondus dans le corpus avec la formalisation de la théorie des ensembles et la résolution au coup par coup des paradoxes logiques.

Le travail du philosophe est donc rendu difficile. Il semble qu'il ne soit jamais possible que de narrer ou commenter *a posteriori* l'advenu. La synthèse dialectique des contradictions ou des difficultés méthodologiques s'opérant à l'intérieur même du corpus, il ne resterait qu'à en prendre acte et à renvoyer l'observateur érudit aux derniers traités mathématiques pour en comprendre les ressorts et la teneur véritable. Il faut sans doute voir là les racines de l'atonie de la philosophie mathématique contemporaine et du peu d'intérêt qu'elle suscite chez les mathématiciens.

Le rôle du phénoménologue sera, dans ces conditions, non pas de commenter les résultats obtenus, mais de comprendre les processus eidétiques à l'œuvre dans leur genèse et leur constitution. Pour le dire autrement, la philosophie mathématique est aujourd'hui condamnée, pour être pertinente, à être phénoménologie transcendantale. Elle doit saisir le caractère noétique de la création mathématique et rendre compte du fait que les structures de l'intentionnalité du mathématicien sont, tout autant que la structure interne des objets mathématiques, à l'origine du savoir et des idéalités. Et puisque la conscience du mathématicien est structurée de manière à pouvoir produire des idéalités, toute invention mathématique est à même de nous apprendre quelque chose sur les structures internes de l'intentionnalité, sur la possibilité du passage d'un système de représentations, du divers de l'intuition, à des contenus conceptuels sous lesquelles ces représentations vont tomber.

La notion de nombre naturel (ou, plus précisément, d'objet-nombres-naturels, *natural number object*) telle que la conçoit la mathématique contemporaine, est ainsi à même de préciser notre entente des processus à l'œuvre dans la constitution des idéalités arithmétiques – l'occasion aussi d'approfondir les idées husserliennes développées dans la *Philosophie de l'arithmétique*,[545] dont on sait le caractère inabouti et insatisfaisant. Nous nous proposons ici d'en entamer l'étude, en indiquant les moments-clés de l'analyse et leur pertinence au regard du projet qui est celui de la phénoménologie transcendantale.

L'une des grandes idées des mathématiques modernes, souvent méconnue ou mal comprise, faute d'un intérêt spécifique, a été le principe selon lequel un objet mathématique peut être défini explicitement (par exemple comme un ensemble déterminé sur lequel des relations sont définies: ainsi des groupes de transformations ou des objets de la géométrie classique), mais peut être également défini par ses propriétés. La distinction n'est pas toujours facile à comprendre car les deux points de vue sont intimement liés, comme deux facettes d'un même processus. En règle générale, un objet mathématique est, de fait, construit explicitement de manière à satisfaire un certain nombre de propriétés: la célèbre formule de Poincaré selon laquelle les axiomes seraient des définitions déguisées va dans ce sens. Pour autant, la distinction doit être maintenue et est d'importance, le premier point de vue correspondant schématiquement à celui des mathématiques constituées en corps de doctrine, le second étant assez naturellement associé aux moments de constitution (et donc à la dimension transcendantale) du savoir.

Qu'est ce qu'un nombre «naturel»? Pour Connes, mais également pour la majorité écrasante des mathématiciens, un nombre est un objet à part entière, quand bien même ils se défendent d'un tel jugement. Tout l'effort

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[545] E. Husserl, *Philosophie de l'arithmétique* (abrégé: PA), trad. franç. J. English, Paris, P.U.F., 1972.

de Frege a été dirigé vers la construction explicite de ces nombres-objets selon les lois pures de la pensée. La tentative frégéenne a conduit à un consensus, encore majoritairement accepté, auquel il a déjà été fait allusion: les nombres naturels seraient des classes d'équivalences, bien définies à l'intérieur de théories des ensembles convenables. Il s'agit là d'une découverte considérable, qui révèle des pans entiers de la structure interne de notre compréhension des nombres, c'est-à-dire de la manière dont notre pensée les conçoit en suivant des lois et des règles universelles, comme celles définies par les relations d'équivalence, lois et règles susceptibles de s'appliquer dans d'autres champs mathématiques, voire dans d'autres domaines du savoir.[546]

La voie frégéenne, qui définit les nombres naturels dans le contexte de la théorie des ensembles, n'est pourtant pas la seule possible. Ainsi, la théorie des catégories[547] suit-elle, pour définir la notion de nombre naturel, la voie des propriétés: les nombres naturels doivent répondre à des questions précises et bien formulées, liées aux notions de succession et d'itération, notions codifiables par des équations dans des théories mathématiques bien formulées. Ces questions, c'est là le point important, peuvent être formalisées dans d'autres théories que la théorie des ensembles. Par voie de conséquence, la notion de nombre est-elle, en droit et en raison, dissociable de celle d'ensemble.

Le contenu technique de ces idées sera précisé ultérieurement. Il faut en retenir, pour l'instant, cette idée que la notion de nombre, si elle peut être conçue sur un mode réaliste (les nombres sont des objets bien déterminés, qui peuvent être définis), peut également être envisagée comme la réponse

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[546] Voir F. Patras, «Le Fondement de l'arithmétique», *Husserl et Frege,* R. Brisart éd., Paris, Vrin, 2002.

[547] Une catégorie est définie par la donnée d'une collection d'objets et de transformations (dits morphismes ou flèches) entre ces objets, transformations que l'on peut composer selon les règles habituelles (loi d'associativité). Pour plus de références sur la théorie des catégories et un aperçu de sa signification épistémologique, on pourra lire F. Patras, «Catégories et foncteurs», *Dictionnaire d'histoire et philosophie des sciences,* D. Lecourt éd., Paris, P.U.F., 1999. Les ensembles, les groupes, les espaces topologiques, les nombres entiers sont associés à autant de catégories. C'est là, du point de vue des questions de fondements, l'un des aspects essentiels de la théorie: la théorie des ensembles apparaît, du point de vue catégoriel, comme un exemple parmi d'autres d'univers mathématiques de référence – exemple qu'il n'est pas toujours naturel de privilégier.

à un problème susceptible d'une formulation très générale, et dont la formulation dans la théorie des ensembles n'est que l'une des nombreuses formulations possibles. Ces problèmes mathématiques «très généraux» ont un nom pour les mathématiques modernes, en théorie des catégories: ce sont des problèmes dits «universels». Ce qui signifie que leur solution a des propriétés d'universalité: lorsqu'on les résout, on ne résout pas un problème spécifique, mais simultanément toute une classe des problèmes, en règle générale formulables en termes de diagrammes. Ils sont donc codés par des figures géométriques d'un type particulier, mais très élémentaires et susceptibles d'être intuitionnées: la mathématique catégorielle est souvent une mathématique très intuitive, même si les intuitions qu'elle met en œuvre sont des intuitions qui portent sur les méthodes de la théories plutôt que sur les objets considérés.

De manière plus précise, ces problèmes universels correspondent à des phénomènes d'adjonction.[548] Une indication permettra de comprendre l'essence formelle de ces phénomènes et de montrer qu'ils se rattachent à des structures élémentaires de la pensée. Des analyses plus précises montreraient qu'ils s'inscrivent naturellement dans le domaine de l'ontologie formelle, cet ensemble de structures formelles qui gouvernent notre entente de l'Être, y compris dans notre rapport quotidien et pragmatique au monde de la vie.

La notion de borne supérieure, qui servira ici d'illustration élémentaire, peut être envisagée d'un point de vue physique comme celle de supremum pour une famille de barrières de potentiel.[549] La notion est éminemment opératoire et, du point de vue de la praxis, a été acquise par l'humanité bien antérieurement à toute mathématisation de la physique. Considérons un exemple simple: fixons deux nombres, 3 et 5. Alors, pour tout nombre n supérieur à la fois à 3 et à 5, il existe un nombre k qui est tout à la fois supérieur à 3 et à 5 et inférieur à n. Ce nombre n'est en général pas

[548] La question de la signification de l'adjonction en théorie des catégories du point de vue de la phénoménologie est abordée de façon systématique dans F. Patras, «Phénoménologie et théorie des catégories», in *Geometries of Nature, living Systems and human Cognition,* ed. L. Boi. World Scientific. 2005, 401–419.

[549] C'est très probablement ainsi qu'un mathématicien comme R. Thom la concevrait. Voir, par exemple, R. Thom, *Paraboles et catastrophes*, Paris, Flammarion, 1983 et *Prédire n'est pas expliquer*, Paris, Eshel, 1991.

unique (si $n = 8$, $k = 5, 6, 7, 8$ sont autant de solutions du problème). Toutefois ce problème a une formulation *universelle*: le nombre n peut varier librement dans l'ensemble des nombres supérieurs à 3 et à 5. Le seul nombre k qui soit solution du problème universel (c'est à dire le seul nombre qui soit solution du problème pour toutes ces valeurs de n) est la borne supérieure de l'ensemble $\{3, 5\}$ (5, en l'occurrence). Dans le langage des catégories, une inégalité comme $3 \leq 8$ s'interprète comme un morphisme de 3 à 8. La propriété d'universalité de la borne supérieure s de deux nombres a et b se traduit par l'énoncé: «pour tout morphisme de a à n et tout morphisme de b à n, il existe un morphisme de s à n factorisant les morphismes précédents» (au sens où: $a \leq s \leq n$ et $b \leq s \leq n$). On dit alors que «s est le coproduit de a et b». Le même schéma de construction permet de définir des notions aussi disparates en apparence que les connecteurs logiques (et, ou), les produits cartésiens ou les sommes d'ensembles: il suffit de remplacer, dans la construction qui vient d'être effectuée, les nombres par les objets et les inégalités par les morphismes de la catégorie considérée.[550]

Qu'en est-il des nombres de ce point de vue? Quel est le problème universel que résout le concept même de nombre? Depuis les origines grecques, deux grandes définitions s'affrontent. L'une, statique, attribuée à Thalès, voit dans le nombre une «collection d'unités». La définition de Frege en est une variante. L'autre, dynamique, voit dans le nombre la «mesure d'une progression». Elle est attribuée à certains pythagoriciens (le nombre serait la «progression d'une multiplicité commençant avec une unité et une régression finissant en elle»). Elle est à l'origine de la conception kantienne de l'arithmétique (liée aux formes pures de la sensibilité, qui se manifesteraient dans l'intuition du temps). C'est cette deuxième conception, dynamique, qu'a retenu la théorie des catégories pour définir la notion générale de nombre.

Fixons un univers mathématique (une catégorie d'un type particulier: un topos). Cela signifie que l'on se donne des objets et des relations ou des règles de transformations (morphismes) entre ces objets. Ces données doivent satisfaire quelques axiomes qui, lorsque l'univers mathématique est celui de la théorie des ensembles, correspondent à des propriétés

550 Voir «Phénoménologie et théorie des catégories», op. cit.

ensemblistes élémentaires.[551] Les nombres apparaissent avec l'idée d'une règle ou d'une transformation *interne* à un objet. L'idée essentielle dans cette approche est qu'une transformation interne à un objet peut être itérée. Cet objet peut être un ensemble ou une entité plus abstraite: ainsi, toute application continue d'un espace topologique dans lui-même peut être itérée. La théorie moderne du chaos repose en bonne partie sur de telles idées.[552] Qu'est ce alors qu'un objet-nombres-naturels *au sein de l'univers mathématique considéré*? Rien d'autre qu'un objet de la théorie (un ensemble, l'ensemble des entiers naturels ordinaires, si l'univers est celui de la théorie classique des ensembles) vérifiant une propriété universelle: en termes du langage ordinaire, celle de servir de *marqueur* pour *toutes* les itérations *possibles* de transformations internes aux objets (à tous les objets, pour toutes les transformations: il s'agit d'un problème universel) de la théorie considérée.[553]

5. La Théorie Husserlienne du Nombre

La *Philosophie de l'arithmétique*[554] est un texte difficile à appréhender au sein du corpus husserlien. Texte de 1891 et texte de jeunesse, Husserl n'y dispose pas encore des outils de la phénoménologie transcendantale, ni même des acquis des *Recherches logiques*,[555] ou encore de la familiarité

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[551] Un topos (élémentaire) est une catégorie munie de produits fibrés, d'un objet terminal et d'une opération de puissance. La notion de produit fibré est définie par un schéma universel dont le principe est analogue à celui qui a été décrit à propos de la notion de borne supérieure. La notion d'objet terminal et celle de puissance généralisent la notion de singleton (ensemble à un élément) et d'ensemble des parties en théorie des ensembles. Voir I. Moerdijk, «Sets, Topoi and Intuitionism», op. cit.

[552] F. Lurçat, *Le chaos*, Paris, P.U.F., 1999.

[553] Voir I. Moerdijk, op. cit., pour une présentation technique. Du point de vue de l'histoire de la philosophie, il n'est pas inintéressant de noter que l'idée d'itération des morphismes sous-jacente à la définition topossique est précisément celle que Wittgenstein a spontanément retenu pour sa présentation des nombres naturels dans le *Tractatus* (L. Wittgenstein, *Tractatus logico-philosophicus*, 1921: 6.021, «Die Zahl ist der Exponent einer Operation»; 6.031, «Die Theorie der Klassen ist in der Mathematik ganz überflüssig»).

[554] op. cit.

[555] E. Husserl, *Logische Untersuchungen*, Halle, 1900–1901, 2° édition remaniée, 1913.

avec les idées hilbertiennes et les problèmes liés à l'émergence de la méthode axiomatique, familiarité qui ne sera conquise que dans les dernières années du dix-neuvième siècle. Et cependant, la plupart des grands enjeux de la théorie husserlienne de la connaissance y sont déjà bien délimités: rapport de l'intuition à la connaissance symbolique; autonomie du formel (du domaine des signes, plus exactement) à l'égard des activités de la conscience; passage des représentations et de leur structure interne (noétique) aux concepts associés (objets de la connaissance mathématique, noèmes).

La découverte de la pensée hilbertienne devait ensuite engager Husserl résolument sur la voie du formalisme et vers la prise en compte des possibilités offertes par les axiomatiques ouvertes, engagement d'où naîtront des idées comme celle de «théorie des théories» mathématiques esquissée dans *Logique formelle et logique transcendantale*[556] à l'issue d'une réflexion sur l'héritage riemannien et d'où naîtront des notions comme celle de *Mannigfaltigkeit*.[557] En ce qu'elle est antérieure à ce basculement de la pensée et des intérêts husserliens vers le domaine logique-formel, la *Philosophie de l'arithmétique* ouvre des voies sur lesquelles le Husserl de la maturité ne s'engagera plus, et qui ont pourtant un intérêt propre, aussi bien du point de vue mathématique, que d'un point de vue phénoménologique.

Pour ce qui est des études husserliennes, les développements qui suivent nous semblent indiquer un point précis d'achoppement pour toute la pensée husserlienne, dans les difficultés qu'elle a pu avoir à opérer la synthèse du formel et du transcendantal. Les voies, complexes, d'une telle synthèse nous semblent pouvoir passer aujourd'hui, outre par une meilleure prise en compte du «structuralisme» caché de la pensée husserlienne,[558] par une réflexion sur le métastructuralisme dont il va être question. Par «métastructuralisme», il est ici entendu un dévoilement des structures *mathématiques* à l'œuvre dans l'intentionnalité et dans les moments noétiques de la connaissance mathématique. Il s'agit donc de dépasser, conformément à toute la stratégie de la phénoménologie transcendantale,

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[556] E. Husserl, *Formale und transzendentale Logik*, Halle, 1929.

[557] Le mot est traduit dans les mathématiques contemporaines par «variété», mais il a un sens plus étendu pour Husserl qui envisage une *Mannigfaltigkeit* comme un domaine d'objets.

[558] Voir, dans ce recueil, les articles de J. Dodd et J. Benoist.

l'idée de processus et de structures qui seraient à l'œuvre seulement dans le domaine (noématique) des objets de la connaissance, et cela qu'il s'agisse d'objets en un sens très classique, comme les cercles ou les coniques, ou d'objets en un sens plus moderne: groupes, corps, variétés.

La structure même de la *Philosophie de l'arithmétique* est bien connue. Dans la première partie, Husserl essaie de dégager les mécanismes de formation des concepts de nombre. Il y suit l'approche statique inaugurée par Thalès. Les nombres proviennent de l'appréhension des multiplicités, au sein desquelles les objets perdent leur individuation, leurs caractères distinctifs pour devenir autant d'unités. L'analyse husserlienne a de nombreux traits originaux et met en évidence des phénomènes importants: ainsi du concept de «quelque chose» qu'il faut rapprocher du concept d'objet transcendantal en un sens kantien,[559] ou encore celui de liaison collective, qui est le fondement de la possibilité même de la notion, trop souvent considérée comme évidente, d'ensemble. Pour autant, se rattachant à la conception statique des nombres, elle est beaucoup moins éloignée de la conception de Frege qu'il n'y paraît de prime abord.[560]

Cette voie conduit cependant à une impasse épistémologique, et c'est l'objectif de la deuxième partie du texte que d'engager la pensée husserlienne sur des voies nouvelles. Le constat dressé au terme de la première partie tient en quelques mots: si la genèse des concepts de nombre passe par des moments intentionnels, par des modes de représentations privilégiés ou des phénomènes typiques des représentations de collections, tous ces moments ne permettent en rien de justifier la puissance de l'arithmétique, son autonomie à l'égard de nos représentations de collectivités dans la pratique quotidienne du calcul, ni même la manière dont nous opérons avec de grands nombres. Bien plus, il semble que les représentations impropres de nombres[561] soient la règle. La voie statique ne peut donc suffire à rendre compte de l'arithmétique et de la numération telles que nous les connaissons.

Husserl s'engage donc sur la voie d'une étude systématique des représentations impropres de nombres, qu'il qualifie d'emblée de représentations

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[559] Voir «Phénoménologie et théorie des catégories», op. cit.

[560] Voir «Le fondement de l'arithmétique», op. cit.

[561] Celles où nous opérons avec des nombres sans avoir une conscience distincte d'une multiplicité au sein de laquelle seraient individuées des «unités».

symboliques:

«Nous allons d'abord expliquer en quelques mots la différence entre représentations *symboliques* et représentations propres, qui est fondamentale pour les exposés qui vont suivre.

Une représentation *symbolique* ou impropre est, comme le mot le dit déjà, une représentation par des signes. Si un contenu ne nous est pas donné directement comme ce qu'il est, mais seulement indirectement *par des signes qui le caractérisent univoquement*, alors, au lieu d'une représentation propre, nous avons de lui une représentation symbolique [...] Toute description d'un objet intuitif tend à remplacer la représentation effective de cet objet par une représentation de signes qui la supplée.»[562]

Le domaine du symbolique n'est pas identique au domaine formel, où les signes ne jouent qu'un rôle accessoire, toute référence à des représentations étant anecdotiques d'un point de vue formel strict, quand bien même ces représentations seraient indirectes (l'existence d'êtres ou d'objets mathématiques y est «surmontée», selon la formule de Lichnerowicz). Il se distingue tout autant du domaine axiomatique, pour des raisons analogues. Les représentations symboliques, dont fait état Husserl, et sur lesquelles il fonde en fin de compte sa théorie de la numération, ont donc un contenu original au regard des pensées post-hilbertiennes. Elles autorisent le développement d'une sémantique et médiatisent les rapports entre le domaine du réel et celui des concepts, des idéalités. Quant aux notions mathématiques complexes, comme celles qui apparaissent dans l'algèbre moderne ou le structuralisme, elles opèrent sur des entités et des concepts déjà constitués, déjà abstraits, et ne peuvent exister (au moins chronologiquement) sans cette médiation préalable et sans un certain nombre de mécanismes, dont la nature reste à élucider et qui permettent l'édification d'un domaine formel à partir du système de nos intuitions matérielles, spatiales ou intellectuelles.

Pour autant, est-il bien justifié, à propos de l'arithmétique, de se contenter d'un renvoi aux représentations symboliques, et celles-ci sont-elles vraiment l'archétype des représentations impropres, comme l'affirme Husserl? Ou bien, ne faudrait-il pas distinguer dans nos représentations impropres divers modes d'existence, dont les systèmes de renvois signitifs ne seraient qu'un exemple privilégié? Suivons Husserl dans son étude des

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[562] *P.A.*, pp. 236–237 [215–216].

représentations de multiplicités qui ne se réalisent pas de manière immédiate (elles seraient propres), mais reposent sur l'appréhension successive de tous les membres de la multiplicité. Il s'agit d'une situation simple, où tous ces membres peuvent être appréhendés effectivement, ce qui ne serait pas le cas de multiplicités comme les grains d'un tas de sable ou les étoiles du ciel;

> «On peut donc s'attendre à ce que ces représentations symboliques, en tant qu'elles sont les plus proches des représentations propres correspondantes, forment pour ainsi dire le pont entre celles-ci et les symbolisations plus éloignées».[563]
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> «Assurément nous ne pouvons plus alors maintenir ensemble dans un seul acte les appréhensions successives des membres de la multiplicité. Il n'y en a à chaque fois qu'un petit nombre qui demeure dans un état de distinction bien tranchée sur le domaine de l'activité de collection. Tandis que continuellement des membres nouveaux sont appréhendés et rattachés, il s'en échappe en revanche d'autres parmi ceux qui ont été séparés auparavant; les actes qui les représentent pour eux-mêmes s'estompent toujours davantage dans l'arrière-fond de la conscience et s'évanouissent tout à fait.»
>
> «Pourtant, nous possédons de l'unité du processus *entier* un concept déterminé. Même s'il n'y a que le dernier morceau, très limité, à être effectivement présent devant nous, nous avons cependant connaissance du fait que ce morceau n'est pas le processus entier [...]. Avec tout cela, nous pouvons construire la représentation symbolique d'un processus complet, qui, dans n'importe quelle succession conduit à appréhender tous les membres possibles du tout intuitif.»[564]

Première remarque: sans en être pleinement conscient, Husserl a abandonné la conception statique du nombre pour adopter la conception pythagoricienne/dynamique. Plus radicalement, c'est désormais le *processus* qui est intuitionable et constitue le fondement de l'activité de numération, et non plus la totalité appréhendée comme telle des contenus d'une collection. Mais que signifie ici la terminologie «représentation symbolique»? Comment s'opère le passage de moments isolés d'un processus à la représentation d'une totalité de membres possibles d'un tout intuitif? N'est-ce pas que les moments de conscience impliqués dans les différents instants du processus de numération ont une structure invariable, que pourrait détecter une analyse phénoménologique bien conduite? Ou encore, n'est-ce pas parce que l'intentionnalité à l'œuvre dans ces

[563] *P.A.*, p. 243 [222].
[564] *P.A.*, pp. 243–244 [222].

instants du processus de numération obéit à des règles, des structures universelles, et ces structures ne peuvent-elles pas être analysées au travers d'une étude des moments noétiques de l'activité de dénombrement?

Ce sera la thèse que nous défendrons: les phénomènes décrits par Husserl, dans ces pages-clé de la *Philosophie de l'arithmétique*, ne relèvent pas de l'activité symbolique, mais d'un mode de pensée original, dont la meilleure description mathématique à ce jour est sans doute la théorie des catégories et ses avatars, comme la théorie des topos. Reprenons donc pas à pas les termes de l'analyse husserlienne, en nous autorisant une analyse technique de ses différents moments qui prenne en compte les acquis de la pensée catégoriale.

Lorsque nous sommes confrontés à une totalité trop grande pour être appréhendée par une intuition directe, voire une totalité assez mal définie (comme «les étoiles visibles dans le ciel»), comment se forme le schéma de numération qui permettra de parler de nombres en l'absence de représentations adéquates sur un mode naïf (la représentation globale, immédiate ou simplement ramassée dans une durée assez brève pour autoriser à parler de subsumption sous une totalité bien définie, d'individus dépouillés de leurs traits propres et identifiés à autant d'unités)?

La réponse donnée par Husserl est édifiante: le concept de nombre apparaît, dans de telles situations, par la seule considération du schéma itératif consistant à passer d'un élément de la totalité à l'autre. Ce schéma est lui-même essentiellement idéal: ce qui importe est la possibilité d'existence d'un tel schéma plutôt que sa mise en œuvre effective. Selon les mots de Husserl: «Et ainsi, avec tout cela, nous pouvons construire la représentation symbolique d'un processus complet, qui, dans *n'importe quelle* succession (cette succession nous est même indifférente), conduit à appréhender tous les membres possibles du tout intuitif».[565]

En termes catégoriels, la possibilité de la numération repose donc sur l'existence d'un schéma universel fondé sur la notion de «succession» dans une totalité déterminée. C'est, pour l'essentiel, une construction due historiquement à Dedekind, qui appelle «chaînes» les suites formées d'éléments consécutifs pour l'opération de succession. Dedekind manquait toutefois des outils théoriques qui lui auraient permis d'asseoir cette théorie

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[565] P. A., p. 244 [222].

sur des fondements convaincants, et c'est la théorie des ensembles cantorienne et frégéenne qui a, dans ces conditions, emporté l'adhésion de la majorité des mathématiciens. Cette notion de chaîne revit dans la théorie moderne des catégories au travers de la notion d' «objet-nombre-naturel».

Les nombres sont alors «naturels», non pas en ce que leur existence témoignerait de la présence problématique d'objets idéaux dans le ciel des Idées des différents néo-platonismes, mais en ce que leur construction, leur existence et leur unicité découlent de toute nécessité de l'existence d'un «problème universel» (consistant, en termes non techniques, à modéliser mathématiquement les phénomènes de succession, la dynamique pythagoricienne et les chaînes de Dedekind).

Épilogue

Que conclure de ce parcours dans les mathématiques modernes et la pensée husserlienne? Tout d'abord que les premières offrent des outils pour l'étude des textes husserliens. Les descriptions phénoménologiques (préphénoménologiques et psychologiques dans le cas de la *Philosophie de l'Arithmétique*) nous donnent à voir les opérations de la pensée mathématique antérieurement à toute formalisation. Affirmer que nous n'avons pas de représentation intuitive pleine, parfaite, des objets mathématiques, comme le fait Husserl dans ses *Recherches psychologiques de 1894*,[566] n'est certainement pas une assertion valable spécifiquement pour la théorie des ensembles. De même, le concept de totalité en est-il en bonne partie indépendant (en ce sens que la notion classique d'ensemble n'est qu'une modalité de codification mathématique du concept de totalité). En ce sens, et malgré l'ancrage évident des problématiques husserliennes dans les débats de son époque, relire ses textes dans la perspective d'une actualisation de leurs aspects et sous-entendus techniques paraît être une des tâches importantes qui attendent aujourd'hui la pensée phénoménologique.

En direction opposée, la pensée mathématique contemporaine souffre indubitablement d'un déficit d'intelligibilité. Non que ses constructions soient particulièrement sujettes à caution, ou qu'il y ait quelque raison que ce soit de douter de la validité de ses résultats, mais bien parce que la pratique scientifique se nourrit et s'enrichit des représentations,

[566] op. cit.

débats et autres apports discursifs qui portent sur elle. Malgré les limites qui lui sont actuellement reconnues, le discours structuraliste porté par l'équipée bourbakiste a, de ce point de vue, beaucoup contribué à la vivacité des mathématiques françaises, des années 50 jusqu'à la fin des années 70. Cette influence du bourbakisme, à y regarder de près, échappait en bonne partie à la technique mathématique proprement dite, et tenait à un certain état d'esprit quant au rôle, aux méthodes et à la portée du discours mathématique.

L'analyse catégoriale, conjuguée aux techniques phénoménologiques,[567] enseigne la possibilité d'une entente nouvelle du fonctionnement des mathématiques. Celui-ci, au lieu d'être fondé sur la contemplation et la découverte des essences, reposerait sur la capacité de l'esprit à *résoudre des problèmes* qui se posent *de manière universelle*. C'est-à-dire encore, à trouver, pour chaque type de phénomène mathématique susceptible de se présenter de manière récursive dans la pratique mathématique, un modèle «universel» et en cela «naturel» pour cette classe de phénomènes. Ainsi les nombres régulent-ils la notion de succession, de même que les points régulent (sur un autre mode) la notion d'espace, ou encore que les polynômes régulent la notion d'algèbre commutative sur un corps de base. Tous ces phénomènes sont en effet liés à l'existence de «problèmes universels» dans les catégories correspondantes, et relèvent de techniques catégorielles standard.[568]

Ces techniques présentent un double intérêt, pour l'épistémologie, la logique et, plus généralement, toute la philosophie de la connaissance. Tout d'abord, il n'est aucune raison d'en restreindre la portée au seul domaine mathématique. La notion de limite ou de processus itératif en sont deux bons exemples: leur extension conceptuelle va bien au-delà des concepts mathématiques correspondants. Toutes deux relèvent, *de facto*, de l'ontologie formelle – la possibilité de dégager des mécanismes universels dans la structuration (eidétique, conceptuelle, objectivante) de

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[567] En particulier aux techniques propres à la phénoménologie transcendantale, à même de décrire le fonctionnement de l'intentionnalité, les structures noétiques de la conscience dans son rapport aux domaines d'objets mathématiques, ou encore les mécanismes putatifs de création d'idéalités et les moments synthétiques correspondants de la connaissance théorique.

[568] Liées à la notion de limite, voir «Phénoménologie et théorie des catégories», op. cit.

«ce qui est». Par ailleurs, elles laissent supposer que l'activité synthétique, au cœur de tous les débats épistémologiques, en particulier sous l'influence de la critique exercée par le Cercle de Vienne sur la notion kantienne de «synthétique a priori», pourrait bien ne pas être une activité créatrice spontanée, ou une simple modalité des mécanismes d'association comme l'analogie ou l'induction, mais bien une activité structurée par des schémas complexes, susceptibles d'être décrits, au moins partiellement et en première approximation, au moyen de la méthodologie catégorielle et d'outils comme l'adjonction ou la notion catégorielle de limite, deux notions qui rendent compte de la manière de passer d'un «problème universel» à un énoncé d'existence pour un objet résolvant ce problème.

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