12 Uncertainty and Risk

Abstract

This chapter shows how multiple realizations can be used to support the assessment of uncertainty and risk.

12.1 Models of Uncertainty

All estimates have some error or uncertainty. Predictions are always inaccurate, with errors stemming from widely spaced data, geological variability, lack of knowledge to determine the best parameters for estimation, approximations made in the estimation procedure, and limitations of the models used.

Although the error will never be known except at locations where data are collected in the future, traditional statistics and geostatistics provide models of uncertainty. Chapters 8–10 discussed estimation, estimation variances, and methods to obtain a conditional distribution of uncertainty for a random variable:

$$F(z; u \mid (n)) = \Pr ob\{Z(u) \leq z \mid (n)\} \quad (12.1)$$

Equation 12.1 is a complete description of uncertainty in the variable z based on our random function model. Obtaining reliable models for the conditional distributions denoted in Eq. 12.1 has proven difficult, particularly for small volumes (one block at a time), as opposed to large deposit-scale volumes.

Early attempts in geostatistics to characterize uncertainty relied on the kriging variance, typically in the form of confidence intervals attached to each estimated block grade:

$$(\mu_{x|(n)} - d) \leq \mu_{x|(n)} \leq (\mu_{x|(n)} + d) \quad (12.2)$$

where d is the difference from the average value that defines the confidence level. For example, $d=2^*\sigma$ (twice the standard deviation of the random variable) represents the 95 % confidence level if the distribution has a Gaussian shape (Chap. 2).

The minimized estimation variance or kriging variance can only be equated to a local estimation error if the error distribution is Gaussian and the estimation error does not depend on the actual sample values, a property called homoscedasticity, discussed in Chap. 8. In this case, the estimation variance could be associated to the variance of the error distribution. This is seldom found in practice because most grade distributions are positively skewed and the local uncertainty will depend on the local grades; more uncertainty will be expected in high grade areas. The estimation variance does not provide a reliable uncertainty model for small blocks.

The kriging variance may be used in instances where the distribution is likely to be Gaussian. This may apply if very large volumes of material are considered, since most spatial distributions will tend to become more symmetric, and therefore become more Gaussian-like as more small scale values are averaged together. The reasonable limits of application are not known ahead of time, see Davis (1997) among others.

Other, more recently developed techniques, have attempted to introduce local measures of uncertainty by making the kriging estimation variance data dependent. Most of these techniques have been applied in the context of resource classification (for example, Arik 1999).

Non-linear geoestatistical techniques rely on data transformation to obtain a probabilistic estimate that carries uncertainty (Chap. 9). Except for the case of the indicator transform, the uncertainty model is developed in the transformed space, most commonly Gaussian.

Conditional simulation provides a model of uncertainty at each location by a set of simulated realizations. The uncertainty is better described when a large number of realizations are available, but a relatively small number (say 100) is sufficient to provide a reasonable approximation.

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Simulation techniques and the resulting models of uncertainty rely heavily on stationarity; trends and departures from stationarity significantly affect the model of uncertainty and its quality and usefulness. A model of uncertainty based on simulation depends on the Random Function model used. In certain deposits, a Gaussian-based model may be appropriate, while for others a non-parametric technique such as indicator simulations may be preferable. The model of uncertainty will also depend on the number and statistical characteristics of the conditioning data. Therefore, the model of uncertainty is not unique, nor is there an objective or true model of uncertainty: uncertainty is model-dependent. This has been discussed in Journel and Kyriakidis (2004) and Goovaerts (1997) among others.

Typically, simulations models cannot capture all possible sources of uncertainty that exist in a resource model. In this sense, they are incomplete descriptions of the space of uncertainty, and thus it is relevant to discuss how appropriate the conditional simulation model is with respect to the problem at hand.

A practical consequence of the dependence on a model is that the simulation method should be simulated from the same RF model used to obtain the resource model. It is important that they both share the same basic assumptions and implementation parameters; otherwise, the base case resources could be different and the models incompatible.

Sources of Uncertainty Resource models will include uncertainty from many sources. There are several factors that contribute to the overall uncertainty, and they do not necessarily cancel each other out. The sample values themselves have a degree of uncertainty, partly coming from the intrinsic heterogeneity of the material being sampled; however, most sampling errors are due to the sampling process itself. Sampling theory deals with the development of procedures for minimizing sampling variances, although there will always be an error that cannot be fully eliminated. Sample collection, sample preparation, the chemical analysis itself, and the overall data handling are all sources of uncertainty.

The amount of drill hole information available depends on the geology and the project's development stage. Typically, when additional data is included in the model, the uncertainty will tend to decrease. Geologic models are also a major source of uncertainty. Based on sparse drilling, they are representations of mineralization controls but still carrying a degree of uncertainty stemming from mapping and logging; data handling; the interpretative process itself; and the development of the computerized model. Often, the geologic model's uncertainty has the most important impact on the resource model since it heavily conditions the estimated tonnages above cutoff (Fig. 12.1).

There is uncertainty related to the process of grade interpolation including data spacing, kriging method chosen, variogram model and kriging plan. In addition, a correct amount of dilution must be included in order to predict tonnages and grades available at the time of mining. The prediction of recoverable resources and reserves is another significant source of uncertainty for resource models.

The model of uncertainty can also change when different implementation parameters of the geostatistical models are used, as discussed in Chap. 11 and also Rossi (2003), among others. Seemingly minor decisions, such as whether a random path or a multiple grid search for simulating values is used, can impact the resulting uncertainty model. Other parameters typically considered are search radii, number of original data used, number of previously simulated data used, the number of simulations to be run, and the kriging method to be used, among others. One alternative is to assess the uncertainty related to implementation criteria by choosing bounds or "best" and "worst" cases, although the process is subjective and difficult to justify.

There is limited information with large, unsampled areas between data points. There is uncertainty in the statistical parameters such as the overall mean of the deposit. A model of parameter uncertainty is also subjective, but may lead to a more realistic assessment. Some possible approaches to quantify parameter uncertainty include using an analytical model, the conventional bootstrap method or the spatial bootstrap method.

Bootstrapping is a name generically applied to statistical resampling schemes that allow uncertainty in data statistical parameter to be assessed from the data used to calculate the same parameter in the first place. The basic procedure is to draw n values from the original data with replacement, calculate the statistic from the bootstrapped sample, and repeat a number of times to build up a distribution of uncertainty. It is assumed that the input distribution is representative of the overall distribution. If the drawing is done using Monte-Carlo simulation (MCS), then there is an additional assumption that the data are independent.

Assuming that the sample data are independent is not realistic when they are known to be correlated. The spatial bootstrap simulates at the data locations. The uncertainty generally decreases as the number of drawn values (n) increases. The spatial bootstrap requires a variogram for the data set, simulation, and then computation of the mean for each simulated set of data.

12.2 Assessment of Risk

An uncertainty model can be used to characterize risk. It is important to distinguish uncertainty and risk, since large uncertainties, in some cases, may not lead to significant risks. In other situations, small uncertainties may correspond to unacceptable risk.

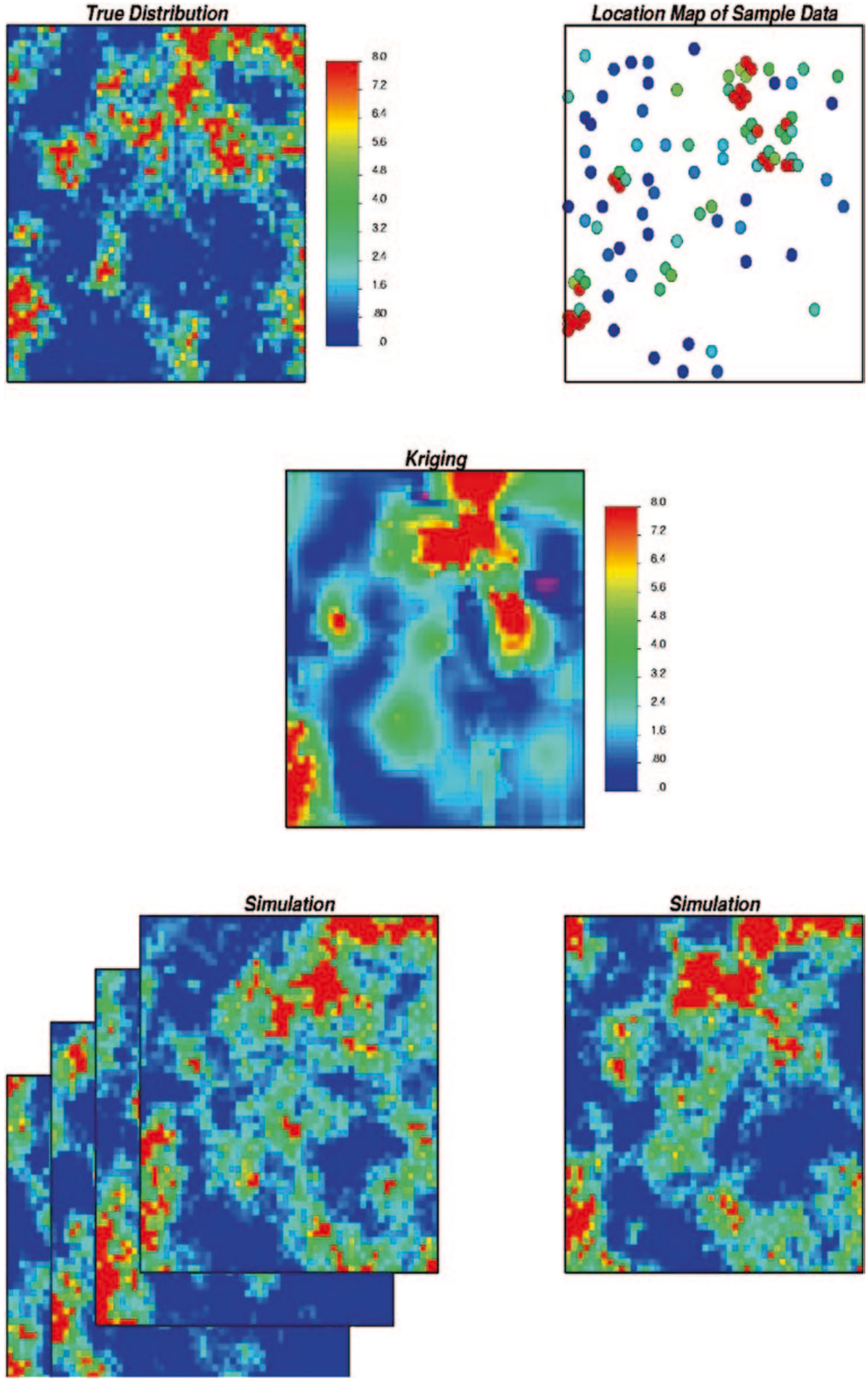


Fig. 12.1 Multiple realizations represent a model of uncertainty of the original variable, while an estimated map does not have an attached uncertainty model

Risk considers the impact of uncertainty on the application being assessed. The concept is summarized as a "Transfer Function" (TF, Matheron 1976), which conceptualizes all processes required to obtain the final product. For example, the TF can represent a pit or a stope optimizer and a production or mine scheduler, used to define mineable reserves. If the uncertainty model is carried through the TF, then the risk of not delivering to the mill the expected number of tons

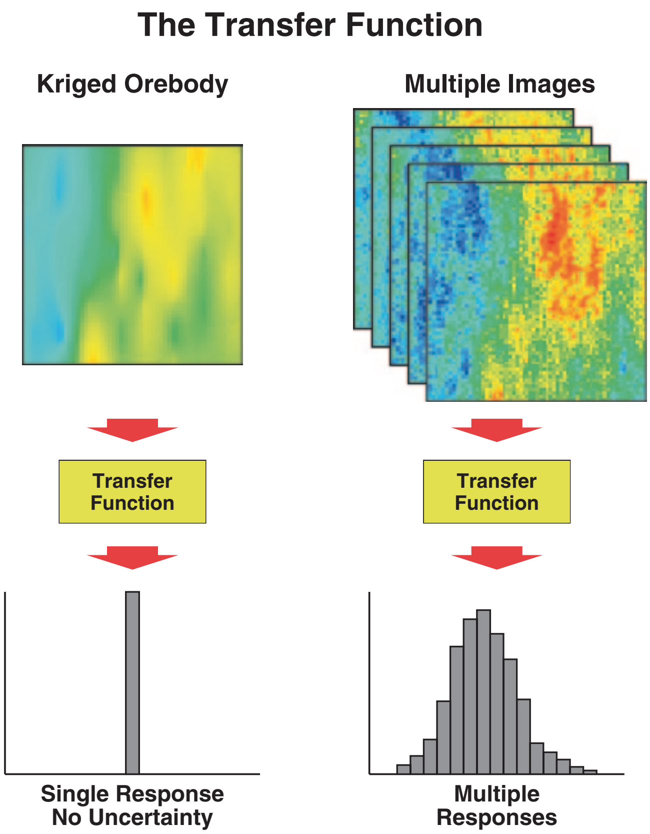


Fig. 12.2 The Transfer Function for estimated and simulated models

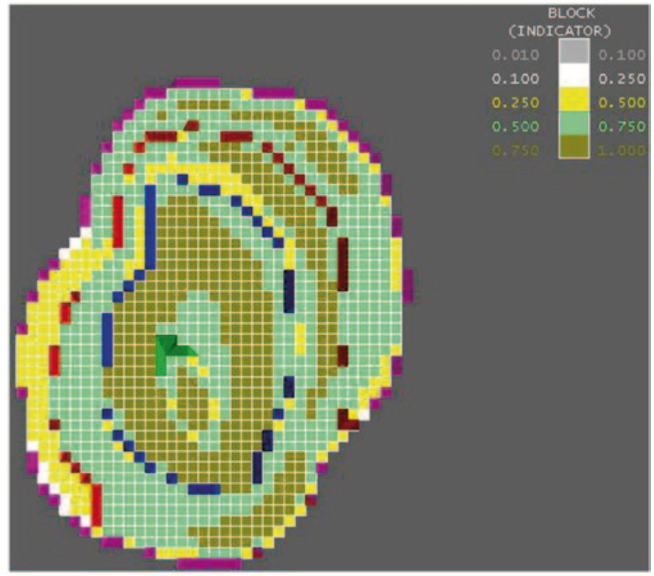


Fig. 12.3 Bench mining probability map. Blocks are coded by probability of being mined. *Magenta*, *blue*, and *maroon* colors indicate the position of the intermediate and final mining walls. (From Van Brunt and Rossi 1999)

at the appropriate grades can be assessed. From this assessment, risk mitigation measures can be developed. The concept is illustrated in Fig. 12.2.

Sensitivity analyses are commonly carried out by mining engineers. The impact of the commodity price or change in the estimated grades is assessed. If the different commodity price or grades result in significant changes in the designed pit walls, for example, then the material in question may be marginal. It is also important to identify areas of the pit that are extracted to contain few erratic or highly uncertain zones of mineralization. Engineers developing mine plans will usually consider simple sensitivity analyses, for example by adjusting the block model grades +10% and −10%. A similar approach is used to analyze the sensitivity of the project to metal prices, operating costs, and other relevant variables. But there are no standard procedures for this purpose.

A full risk assessment requires that the complete uncertainty model (all realizations) be processed through the transfer function; this may involve a full mine planning exercise, including scheduling of ore through the mill for certain periods of the mine life (Jewbali and Dimitrakopoulos 2009). In practice, certain shortcuts are possible, such as processing only the best, worst, and most likely scenarios. These shortcuts have their own pitfalls, including the criteria to rank the realizations.

Producing a detailed mine design from an optimized pit outline involves smoothing the outlines to provide minable shapes, while deviating as little as possible from the optimal outline. This process is manual, and the decisions made regarding the location and width of accesses, ramps, berms, and other geometric parameters required to make the mining operational can be significant. Probability maps by bench and by phases can be used as guides during the final smoothing and design of the pit and definition of the ramp positions. Figure 12.3 (taken from Van Brunt and Rossi 1999) shows a bench map of the probability of each block being mined according to the mine plan developed from the resource model. Developing conditional probability maps such as the one in Fig. 12.3 gives the mine planning engineer an advantage over conventional planning. Risks resulting from highly variable mineralization can be mitigated through the addition of intermediate phases and modifying the position of the pit walls. Also, these maps can be used to target additional infill drilling.

Grade control is an application where risk analysis is used directly to make an economic decision. In this case, the consequences of grade uncertainty are directly evaluated and the optimal choice is made based on the maximum profit or minimum loss choice.

The decision to recover and send to the mill or not a certain panel in the open pit is typically based directly or indirectly on grade estimates, $z^*(\underline{x})$. The loss function $L(e)$ (Journel 1988; Isaaks 1990; Rossi 1999) is a mathematical expression that attaches an economical value (impact or loss) to each possible error, measured in, for example, dollars. By applying a loss function to the conditional probability distribution (Eq. 12.1) derived from the realizations, the expected loss can be found by:

$$E\left\{L(z^* - Z)\middle|(n)\right\} = \int_{-\infty}^{+\infty} L(z^* - z)\cdot dF(z;x\,|\,(n))$$
$$\cong \frac{1}{N_{real}} \sum_{l=1}^{N_{real}} L(z^* - z_l) \qquad (12.2)$$

where N_{real} is the number of realizations and z^* is the retained estimate.

The minimum expected loss can then be found by simply calculating the conditional expected loss for all possible values of the estimates z^*, and retaining the estimate that minimizes the expected loss. As explained in Isaaks (1990), the expected conditional loss is a step function whose value depends on the assumed costs of each bad decision, and the relative costs of misclassification. This implies that the expected conditional loss depends only on the classification of the estimate $z^*(\underline{u})$, not on the estimated value itself, as long as all benefits and costs are constant with respect to grade.

The Loss Function thus quantifies the consequences of false positives and false negatives, weighs the probability and relative impact of each, and then provides the minimum cost solution under the loss model used. For example, the loss incurred when an ore grade panel is sent to the waste dump is a type of lost opportunity cost, measured by the profit that should have been realized. If the same panel is waste, but is sent to the mill, the loss is a combination of the loss incurred in processing material that does not produce the metal to pay for itself, plus the loss derived from the opportunity lost in processing payable material, if any.

Loss functions are in general asymmetrical, since the consequences of under- or overestimation have different costs. In metal mining, where small volumes of ore may have high value, it is typically costlier to send ore to the waste dump than to process waste. Precious and most base metals mines have this characteristic, which is more notable if high economic cutoffs are used. There are other cases where the opposite is true, such as high volume, direct-shipping iron ore mines, who prefer to avoid dilution in the shipment.

Optimal estimates can be derived for a Loss Function if the conditional distribution of the random variable is available. The uncertainty model as described by the realizations provides all the information required to optimize decision-making under uncertainty.

When assessing uncertainty and risk it is also important to consider the scale of interest, i.e., the volume of material being assessed. There are differences between a global, deposit-wide geologic confidence assessment and a more local, mine production-oriented risk assessment. A global confidence measure cannot be used for local, block-by-block risk assessments. A typical example is the resource classification scheme, often used by mining engineers as a measure of confidence on mine schedules, for example on a monthly basis. Resource classification, as discussed below, is generally meant to be a global guideline of confidence, meant mostly for the benefit of shareholders and investors, and should not be used as an uncertainty model to provide a detailed risk assessment of the mine schedule.

Figures 12.4 and 12.5 illustrate how risk may change as a function of the volumes considered. Figure 12.4 shows the monthly probability intervals of Cu grades for an operating copper mine. The graph shows the two values that correspond to the P_{90} (90th percentile) and P_{10} (10th percentile) of the conditional distribution derived from the conditional simulations. It also shows the resource model grade for the same period, as well as the Mine Plan grade, which is generally a lower value than the resource model grade. This is because the mine planner sometimes adds dilution and a safety factor to the grade predicted by the resource model, typically on a monthly basis, not block by block. Mine planners may consider the monthly average grade provided by the resource model as risky, thus penalizing in some fashion the estimate. But the practice is variable and no standard methodology exists. It is dependent on the experience and prejudices of the engineer that defines the budgeted grade.

Figure 12.5 shows a similar graph for yearly periods of a 5-Year Mine Plan. Note that Year 1 in Fig. 12.5 is obtained by simply averaging the grades of the 12 months shown in Fig. 12.4.

Note how Fig. 12.4 shows much more variability than Fig. 12.5. As expected, the smaller volumes represented by the 12 months in Fig. 12.4 are more variable than grades averaged over a yearly volume (Year 1, Fig. 12.5). Also, it is interesting to note that the grades predicted by the resource model and the mine plan do no necessarily fall within the interval defined by the P_{90} and P_{10} limits. This occurs both for monthly and yearly volumes, and more so when considering periods further away in time. This is to be expected, since periods further away in time are likely to have less drilling and thus be more uncertain.

The risk of not achieving the predicted production for each period can be mitigated through further infill drilling. The infill drilling can be directed to those areas with higher uncertainty. A global confidence measure as used on most resource classification schemes would not allow optimization of the infill drilling to that level of detail.

12.3 Resource Classification and Reporting Standards

Public disclosure of estimated resources requires that resource estimates be classified according to degrees of confidence and allocated as measured, indicated and inferred. Reserves must be classified as either proven or probable reserves, derived under certain rules from resource categories. Different resource classification standards are used in differ-

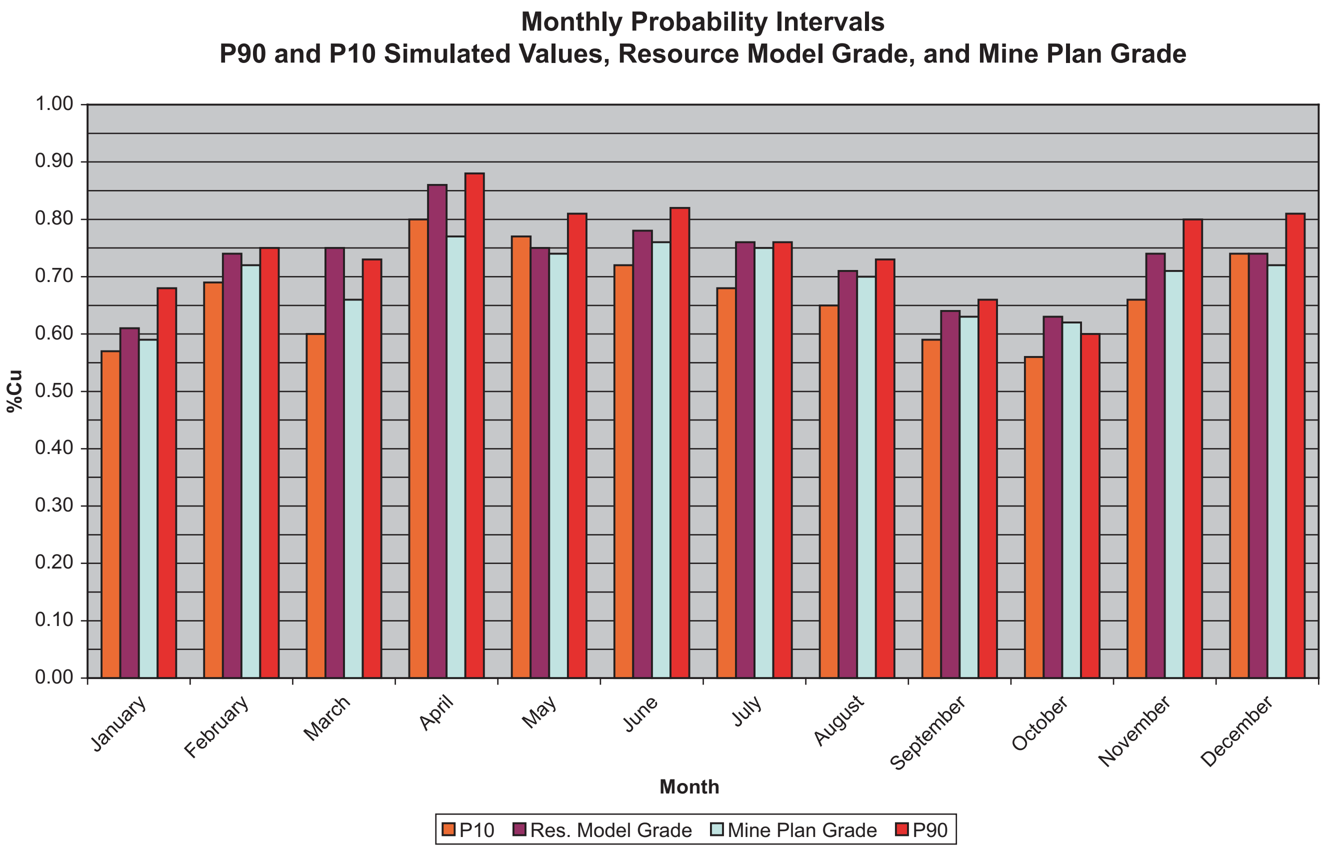
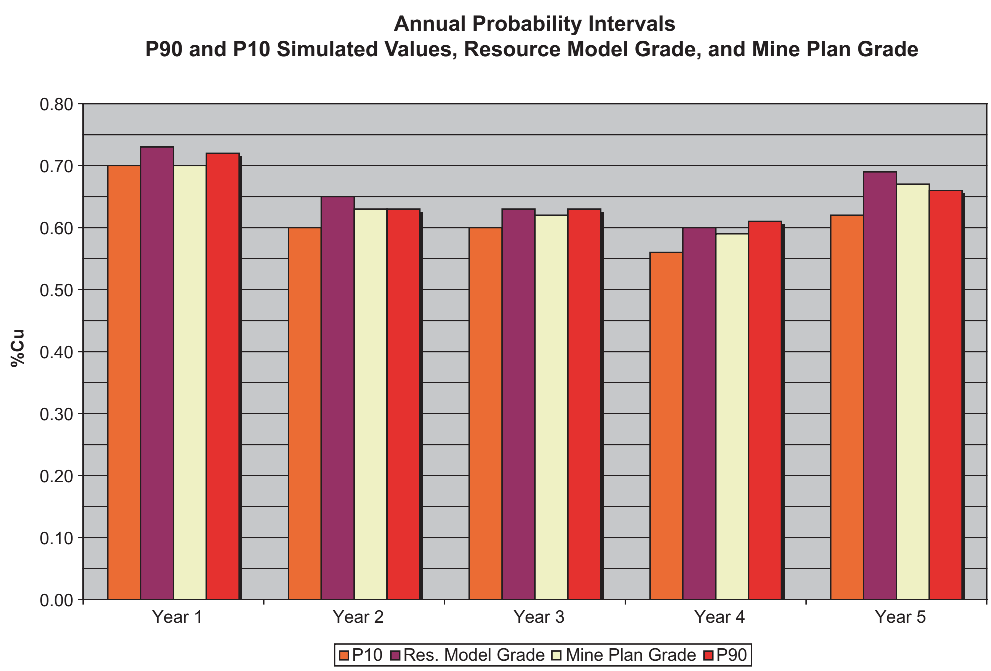


Fig. 12.4 Monthly Probability Intervals for year 1: P_{90} and P_{10} from Simulations, Resource Model Grade, and Mine Plan Grade

Fig. 12.5 Annual Probability Intervals for 5-year Mine Plan: P_{90} and P_{10} from Simulations, Resource Model Grade, and Mine Plan Grade



ent countries; while fairly similar in intent and form, each has its own particularities. The resource classification schemes are mostly intended to provide protection to the investor, and so are typically enforced by Securities Commissions or other appropriate government agency in each country.

Resource classification guidelines have been developed mostly as a response to the need for transparency in the disclosure of mineral resources. As such, resource classification is not necessarily a technical issue, but rather a self-regulated response of the mining industry for conveying investment risk, and also as a response to some notorious fraud cases. The codes have been developed according to specific needs for each jurisdiction, although all have a general common thread that makes them similar in spirit and in the application of its main concepts. Given the global nature of the mining industry, this commonality has led for a long-standing effort towards internationalization of the codes, unifying some of the details of application, to define a set of worldwide accepted set of definitions, namely the International Standards (Miskelly 2003).

Although the most commonly used codes have attached guidelines to them, they are non-prescriptive in all that relates to technical issues. Thus, the responsibility for the appropriateness of the disclosure is left to the technical competency of the individual(s) signing off on the resource calculations and classification, defined as the Competent or Qualified Person (CP or QP). In this context, the published Guidelines that accompany the different Codes are used to set minimum standards for practice, and are not intended to be used as enforcement tools.

The most widely used codes are the Joint Ore Reserves Committee (JORC, www.jorc.org); the CIM guidelines used in *National Instrument 43-101: Standards of Disclosure for Mineral Projects* (NI43-101) in Canada (www.cim.org); the Securities and Exchange Commision's Industry Guide 7 in the United States (www.sec.gov/about/forms/industryguides.pdf); the SAMREC code in South Africa (www.saimm.co.za/samrec.asp); and the Pan-European Union and United Kingdom's Reporting Code (www.crirsco.com/PERC_REPORTING_CODE_jan2009.pdf).

The JORC code has received broad international acceptance. In Canada, most Provincial Securities Commissions and the Toronto Stock Exchange (TSE) have adopted NI 43-101, which applies to all oral statements and written disclosure of scientific or technical information, including disclosure of a mineral resource or mineral reserve. NI 43-101 defers to the Canadian Institute of Mining, Metallurgy and Petroleum (CIM) for definitions and guidelines. The Council of Mining and Metallurgical Institutes (CMMI), of which CIM is a member, have developed a Resource/Reserve classification, definition and reporting system that is also widely accepted.

In recent years there has been an increased emphasis on the concept of a qualified (QP) or competent (CP) person. The professionals preparing resource models and statements are required to be experts in the field and also in the type of deposit being modeled. Typical requirements are that the individual(s) be members in good standing of recognized professional associations, which includes having approved a State or Provincial-sponsored professional exam, and have no less than 5 years experience modeling the same type of mineral deposits.

As an example, the 2010 CIM guidelines adopted in the National Instrument 43-101 of Canada allows classifying mineralization or other natural material of economic interest as a Measured Mineral Resource by the Qualified Person when the nature, quality, quantity and distribution of data are such that the tonnage and grade of the mineralization can be estimated to within close limits and that variation from the estimate would not significantly affect potential economic viability. This category requires a high level of confidence in, and understanding of, the geology and controls of the mineral deposit.

Mineralization may be classified as an Indicated Mineral Resource by the Qualified Person when the nature, quality, quantity and distribution of data are such as to allow confident interpretation of the geological framework and to reasonably assume the continuity of mineralization. The Qualified Person must recognize the importance of the Indicated Mineral Resource category to the advancement of the feasibility of the project. An Indicated Mineral Resource estimate is of sufficient quality to support a Preliminary Feasibility Study which can serve as the basis for major development decisions.

Mineralization is classified as Inferred Mineral Resource if the quantity and grade or quality can be reasonably assumed, but not necessarily verified. Due to the uncertainty that may be attached to Inferred Mineral Resources, it cannot be assumed that all or any part of an Inferred Mineral Resource will be upgraded to an Indicated or Measured Mineral Resource as a result of continued exploration. Confidence in the estimate is insufficient to allow the meaningful application of technical and economic parameters or to enable an evaluation of economic viability worthy of public disclosure. Inferred Mineral Resources must be excluded from estimates forming the basis of feasibility or other economic studies.

A Mineral Reserve is the economically mineable part of a Measured or Indicated Mineral Resource demonstrated by at least a Preliminary Feasibility Study. This Study must include adequate information on mining, processing, metallurgical, economic and other relevant factors that demonstrate, at the time of reporting, that economic extraction can be justified. A Mineral Reserve includes diluting materials and allowances for losses that may occur when the material is mined.

A Proven Mineral Reserve is the economically mineable part of a Measured Mineral Resource demonstrated by at least a Preliminary Feasibility Study. This Study must include adequate information on mining, processing, metallurgical, economic, and other relevant factors that demonstrate, at the time of reporting, that economic extraction is justified.

A Probable Mineral Reserve is the economically mineable part of an Indicated, and in some circumstances a Measured Mineral Resource demonstrated by at least a Preliminary Feasibility Study. This Study must include adequate information on mining, processing, metallurgical, economic, and other relevant factors that demonstrate, at the time of reporting, that economic extraction can be justified.

Reporting Codes and corresponding Guidelines use vague language in its definitions, as it is difficult to provide a general Guideline applicable to all different types of mineral deposits and resource estimation practices. There is a general tendency to suggest the use of some form of statistical description of uncertainty, if only as an accompanying tool that would clarify the degree of uncertainty.

All guidelines discuss geologic and grade continuity as key components of the classification criteria, sometimes adding modifying factors to adjust to local conditions. It is the QP's decision as to what an acceptable evidence of that continuity is, which may be partly dependent on the QP's prior experience with that type of deposits. In practice, resource classification is often reduced to deciding the criteria to be applied, including continuity, and then finding a method to classify the resources that best captures that basic criteria. A common misconception is that resource classification methods provide an objective assessment of confidence; in fact, the classification is an expression of a QP's opinion.

A common practice is to use some form of distance of drill holes to the estimated blocks. The choice of geometric criteria should be based on common practice for the deposit type, site-specific considerations and an expert judgment of other factors. The benefits of using simple distance measures are that the criteria can be simply stated, it is a transparent and easy-to-understand process, and leaves little room for mischief. Also, it does not depend on the estimation method chosen. Some of the most common concerns stated against these types of methods are that they are overly simplistic, as they fail to fully capture geologic confidence.

Geometric methods for classification generally do not give an actual measure of uncertainty, and if so, only for very large volumes, as with the kriging variance. There is an increasing interest in quantifying uncertainty at different volumes (block by block, if possible), which leads to relevant risk assessments.

Other alternatives encountered in practice include kriging variances, commonly applied early on in geostatistical resource estimation (Blackwell 1998; Diehl and David 1982; Froidevaux 1982; Royle 1977); a combination of distances to drill holes (in a certain pattern); the number of drill holes used to estimate each block; multiple-pass kriging estimation plans to account for density of information and other geologic factors; and possible combinations of these, as well as hand-contouring and smoothing, usually as a post-processing step to any of the above.

There has been a move toward systematic and standard methods to evaluate and present uncertainty (Dohm 2005). Common aspects of uncertainty reporting include specification of the population or sample being considered, measure

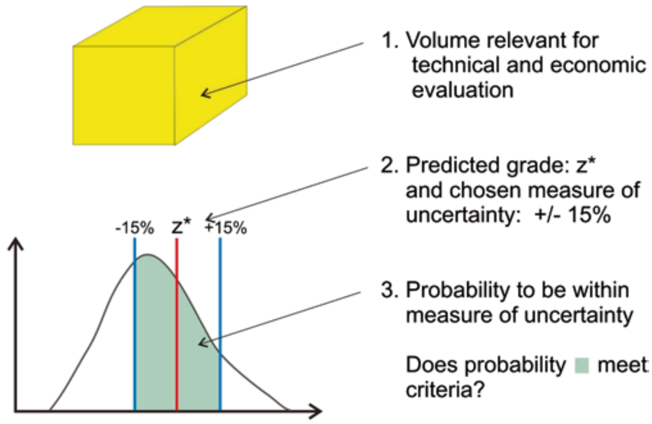


Fig. 12.6 Schematic illustration of the three parameters often used in probabilistic classification schemes: (1) volume related to a production period, (2) precision, and (3) probability to be within the specified precision

of the "+/-" uncertainty, probability to be withing the "+/-" measure of uncertainty, and a list of assumptions and components of uncertainty. There are three aspects to consider in resource classification. They are volume, measure of "+/-" uncertainty, and probability to be within the "+/-" measure of uncertainty. The format for uncertainty reporting is clear and understandable. For example, H.M. Parker (personal comminucation) proposes to classify as measured resources those monthly production volumes for which the true grade is predicted to be within 15 % of the estimated grade 90 % of the time. Quarterly production volumes where the true grade will be within 15 % of the predicted grade 90 % of the time are defined as indicated. There are no established rules or guidelines to decide on these three parameters; this remains in the hands of the qualified person.

Figure 12.6 highlights the three parameters often used in probabilistic classification schemes: (1) volume related to a production period, typically a month or a quarter, (2) the required precision, and (3) the probability to be within the specified precision. The volume need not be a contiguous block, but for simplicity it is often chosen as a simple volume. This can be a significant limitation, because production for any given period will generally come from different areas of the mine, areas that will likely present different geological characteristics, and have been estimated with uneven uncertainty. The second two parameters summarize uncertainty, which can be understood as proportions over a defined population. The probabilistic statement that *there is a 90 % probability that the grade of a monthly production volume be within 15 % of the estimated grade* means that 90 out of 100 true grades of similarly classified monthly production volumes will be within their estimate plus or minus 15 %.

Another alternative is to fix the volume of interest, for example a quarter's production, and then decrease the number of times the true value is expected to fall within the intervals, as shown in the schematic of Fig. 12.7. In this figure measured

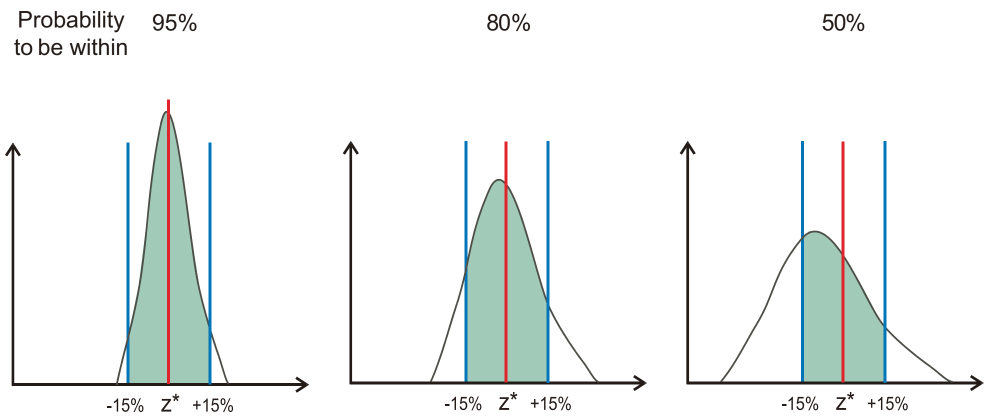


Fig. 12.7 Probability intervals for classification

resources are those for which the expected monthly production is within ±15% of the true value 95% of the time. Indicated resources are those for which the condition is relaxed to 80% of the time, while Inferred only requires that 50% of the time (or production months) the true value be within ±15%.

Uncertainty predictions can be from geostatistical or more traditional methods. If geostatistical procedures are used to construct probability distributions of uncertainty the parameters vary locally and within domains. There are a number of techniques that can be used, but conditional simulation is the best option, since the uncertainty of any parameter of interest can be predicted at different scales by simply averaging up the simulated values.

The uncertainty model can be checked by predicting the uncertainty at locations where there is information from drillholes or past production data. The probability intervals are constructed, counting the number of times that the true values fall within those intervals, thus determining if the predicted percentage is verified.

In any resource estimation work, the purpose of classifying the estimated resources should be clearly stated, and also a clear distinction between geologic confidence (i.e., resource classification) and mining risk assessment should be made. It is tempting to use resource categories as a means to obtain a mine production risk assessment, although they are intended for geologic confidence assessment in a very global sense.

There is no consistent scheme for resource classification for all deposits, although certain common practices can be identified.

12.3.1 Resource Classification based on Drill Hole Distances

Multiple variants of this concept have been used, but in its most simple form the resource is classified based on the distance from the centroid of the estimated block to be to the nearest sample used in the interpolation. Estimated blocks that have close samples nearby will have a higher confidence assigned to them. This is considered a very simplistic method.

Another alternative is to otain the average weighted distance of all samples used to estimate the block. This distance could be anisotropic, following the variogram model ellipsoid and/or the shape of the search neighborhood. It may appear as a reasonable option since *all* samples used in the estimation are considered. This could potentially avoid artifacts related to assigning high confidence to a block estimated with one very close sample and many others much further away. But there are drawbacks with this system, again related to the lack of uncertainty measures and the simple criteria used.

The actual classification of the resources should depend on the distances chosen to characterize confidence, which in turn should be based on geology, drilling density and variogram ranges. Commonly, different estimation domains will have different classification parameters applied to them. Also, a minimum number of samples and drilling density measures are sometimes used, as well as differences in the geologic characteristics in different areas of the deposit.

12.3.2 Resource Classification Based on Kriging Variances

The kriging variance is an index of data configuration. As such, it can be used to rank the resource model blocks based on how much information is used to estimate each block. It can be standardized, for example, to a local mean, such that the resulting relative kriging variance can be used across different grade mineralization zones.

The values for kriging variances that define resource categories are usually related to a pre-specified drill hole configuration, as exemplified in Fig. 12.8. This is an example taken from a porphyry copper deposit in northern Chile. After obtaining a variogram model for each of the three main copper mineralization types present in the deposit, two standard drill hole configurations were used as references to determine resource categories. The kriging variance values for the 5-composite configuration (Case B) defines the limit between measured and indicated for each mineralization type, while

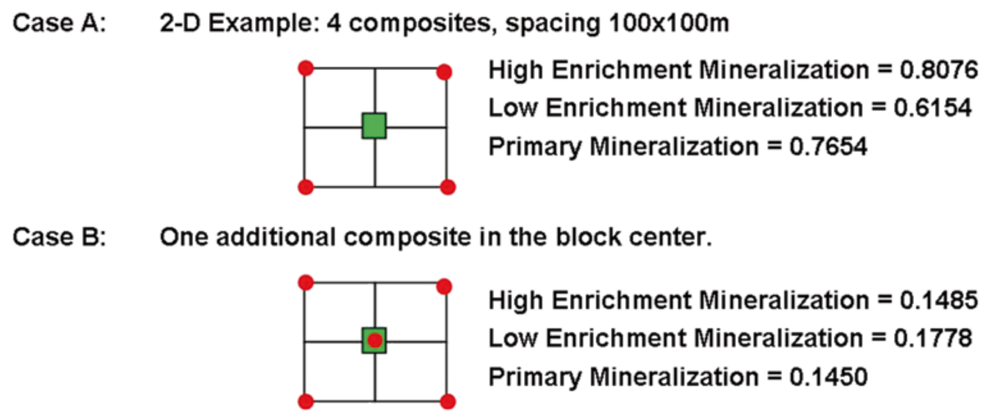


Fig. 12.8 Schematic example of resource classification through kriging variances for reference drill hole configurations

the corresponding kriging variances for the 4-composite configuration (Case A) define the limit between the indicated and the inferred categories. Note that the kriging variances are always used as relative thresholds, since the values themselves do not have any physical or geological meaning.

Other alternatives for defining resource categories can include visual inspection of the kriging variances, although rarely there will be a clear break or indication of kriging variances that can be related to resource classes. Therefore, it is highly dependent on the subjective criteria to define the thresholds for each category. Because of this, the method can be considered equivalent to the distance to the drill hole-based methods, just developed with in a more formal geostatistical framework.

12.3.3 Resource Classification Based on Multiple-Pass Kriging Plans

Another option is to derive the resource classification from multiple kriging passes. Several kriging iterations are done to estimate the model grades using different levels of restrictions, that is, from a more to a less constrained kriging.

The constraints are defined in terms of requisites for an estimate to occur; in the more constrained case, a higher minimum number of samples combined with a larger minimum number of drill holes, and shorter search radii may be used. A smaller number of blocks will be estimated in the more constrained pass, but they will be better informed than blocks estimated in later estimation passes. If the estimation passes are set based on geologic and geostatistical criteria, a flag for each block indicating in which pass it was estimated could be used as an initial indicator for resource classification.

12.3.4 Resource Classification Based on Uncertainty Models

Conditional simulation provides realizations that provide models of uncertainty in a global as well as local sense. These realizations are applicable to both resource classification and mine production risk analysis; however, the use of realizations from which probability intervals can be obtained and used for resource classification is not yet widespread. The resource classification codes, beginning with the JORC code, encourage quantification of uncertainty whenever possible, but they do not mandate it, nor do the corresponding Guidelines suggest specific methodology for such quantification.

Deutsch et al. (2006) argue that the uncertainty models derived from conditional simulations should only be used as a backup to other more simple, geometric methods, such as drill hole distance. Several reasons are given in the paper for this recommendation mostly because the probability intervals are shown to be sensitive to the definition of some of the parameters used to obtain them, as well as the overall model dependency. The uncertainty model is dependent on the specifics of the implementation parameters used in the simulations (Rossi 2003).

Probabilities can be checked using actual proportions, and, whenever possible, this check should be made. Operating mines will generally maintain sufficiently good production records to be able to check actual production tonnages and grades. If the modeled uncertainty can be verified by actual production, then there are several good reasons to rely on the uncertainty model for resource classification: (1) the magnitude of the grades and the local configuration of data are accounted for, (2) the mining volume is explicitly accounted for, and (3) uncertainty is perceived as more objective and transportable to different deposits.

The probability used to define measured, indicated, and inferred resources depends on the mining company's practice. Many will simplistically translate the kind of precision required of other engineering studies and cost estimates during pre-feasibility or feasibility studies into resource classification. Typically, a measured resource would be a quarter known within ±15 %, 90 % of the time; an indicated resource, within ±30 %, 90 % of the time; and inferred, within ±30 % and ±100 %, 90 % of the time. Material known within more than ±100 % will not qualify as resource, and may be flagged (but not publicly reported) as blue sky or potential mineralization.

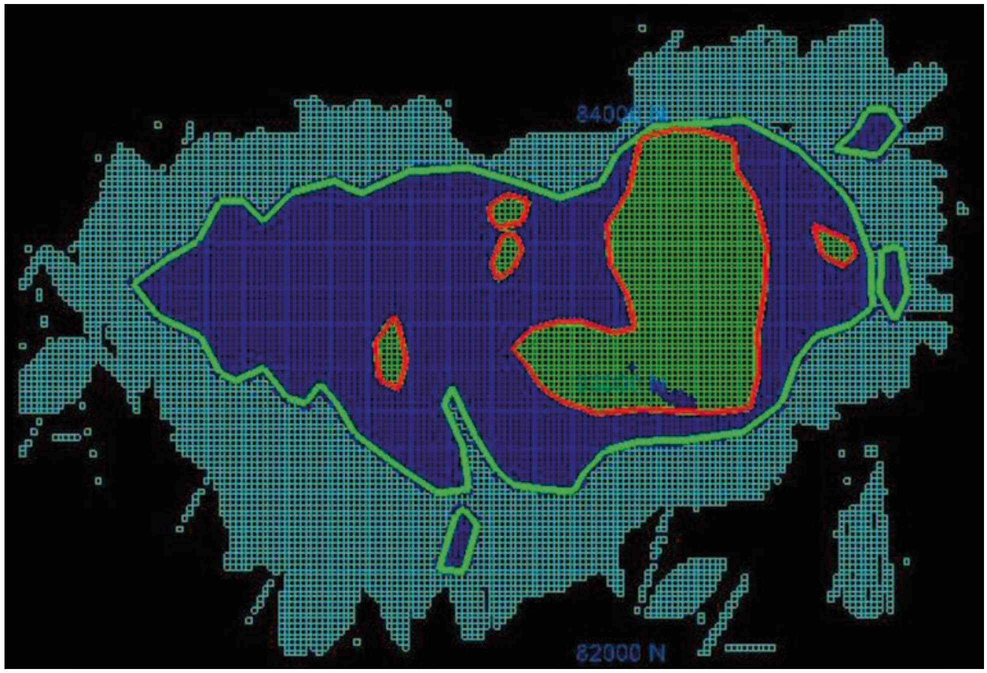


Fig. 12.9 Resource classification contours, Bench 2440m, Cerro Colorado 2003 Resource Model, Northern Chile. *Red* encloses measured material, *green* outline encloses indicated material. Courtesy of BHP Billiton

12.3.5 Smoothing and Manual Interpretation of Resource Classes

Since resource classification is usually performed on a block by block basis, most of the non-probabilisitic methods mentioned above will generally require a posterior smoothing of the resulting volumes, mostly because of the common accepted idea that the classified material should be fairly homogeneous, without intermixing of resource classes over short distances.

This is mostly an aesthetic issue, since classification schemes are meant to provide global indicators of confidence, and not necessarily smooth block-to-block images. Any of the methods described above will likely produce volumes for each resource class that are consistent with the criteria used to specify them. It is common to see in areas with heterogeneous drill hole spacings, variable geologic characteristics and abrupt transitions between the resource classes.

If smooth and contiguous volumes are desired, then manually interpreting the zones, based on the initial definition, is probably one of the most practical means to achieving this. Alternatives could include running a smoothing algorithm that would transform, based on windows of certain sizes, the resource classification of the blocks within to produce more homogeneous volumes. In any case, this should be done with care, not to bias or significantly alter the global volumes defined by the criteria established. There should only be minor corrections for consistency and what may be deemed inconsistent classification classes based on geologic or geostatistical knowledge. It is good practice to check the overall grade-tonnage curves by resource class before and after the smoothing process, to understand the degree of changes introduced.

Figure 12.9 shows an example of smoothing through hand-contouring done at Cerro Colorado, BHP Billiton's porphyry copper operation in Northern Chile. The smoothing was done by interpretating on benches and smoothing out the edges and, in some cases, the intermixing of resource classes. The red outline defines the measured volume, the bright green outline the indicated volume, and the remaining material is classified as inferred. Note how some of the material originally classified as indicated is inside the red outline (central-East portion of the bench), and thus finally classified as measured. Also, there is a small area in this bench to the Northeast of the picture where measured runs directly into inferred, due to a change in the geologic environment.

12.4 Summary of Minimum, Good and Best Practices

Minimum practice for the development of uncertainty models requires the application of simple and more traditional statistical techniques. The scope of application of these models is relatively small, and can only be attached to large volumes. The two most common examples include Resource Classification (for all the methods described, with the exception of conditional simulations), and global confidence intervals derived from the variance of averages for large volumes. Risk assessments are thus limited, and normally qualitative.

Good practice requires, in addition to the above, the development of conditional simulation to obtain realizations of an uncertainty model. This model should be reasonably comprehensive, in the sense of including as many sources of uncertainty as possible, but principally geologic and grade estimation uncertainties. Within these, issues related to dilution should be emphasized, as well as an assessment of the information effect. The resulting model of uncertainty should be checked against actual production, if available, or against some resource model taken as reference or base case. Risk assessments should be fully developed, validated, and documented, with clearly stated objectives.

Best practice consists of, in addition to the above, full modeling of all recognized and quantifiable uncertainties, including those attached to the data, to the sampling and assaying procedures, to the geologic model and simulation domain definition (as above), and the modeling of grade. Conditional simulations should thus be used to provide both global and local uncertainty measures, and a full description of the resource model. However, the exclusive use of probabilities for resource classification is not recommended. An arbitrary choice of probabilistic criteria will often lead to unreasonably large or small volumes in each category. It is however advisable to apply geometric criteria for resource classification, with or without smoothing out the zones with mixing of resource classes, and provide further support through a probabilistic analysis. The probabilistic analysis may cause the competent person to reconsider their geometric criteria, but the geometric criteria are used for disclosure.

If, however, the possibility exists of reliably validating the uncertainty model obtained from the conditional simulations through mine production, then it is reasonable to use the probabilistic intervals as basic definition for resource classification.

12.5 Exercises

The objective of this exercise is to review aspects of uncertainty and risk assessment together with loss functions and decision making. Some specific (geo)statistical software may be required. The functionality may be available in different public domain or commercial software. Please acquire the required software before beginning the exercise. The data files are available for download from the author's website—a search engine will reveal the location.

12.5.1 Part One: Sampling Uncertainty

The objective of this exercise is to experiment with different uncertainty sampling and sensitivity assessment approaches. Available methods for these two purposes can vary greatly depending on whether one is interested in sampling efficiency and/or realistic uncertainty assessment accounting for dependency structures. The set of tools we will explore in this exercise applies different methods that satisfy these two features in varying degrees. Consider a simple calculation of oil in place (OIP) that depends only on a few input parameters:

$$\mathrm{OIP} = 6.2898 * \mathrm{GRV} * \varphi * (1 - \mathrm{Sw}) / \mathrm{FVF}$$

where GRV is the gross rock volume, φ is the porosity, Sw is the water saturation, and FVF is the formation volume factor. The constant 6.2898 is a metric conversion factor to relate cubic metres to stock tank barrels. Suppose that each of the input variables can be described as a random variable: All variables are normally distributed with the following mean and variance values:

Variable	Mean	Variance
GRV	79 million cubic meters	5 million cubic meters
ϕ	17 %	5 %^{2}
S_W	11 %	9 %^{2}
FVF	1.3	0.2

Question 1: Using Monte Carlo simulation, draw 100 realizations for each input parameter and then calculate the corresponding OIP for each realization. Plot the distribution of uncertainty about OIP.

Question 2: Consider now partitioning each of the input distributions into ten different partitions (you can set the thresholds at the deciles). Apply latin hypercube sampling (LHS) and calculate OIP (you should only need to draw 10 realizations for each input and ensure that you only draw from each partition once). Plot and comment on this distribution of OIP.

Question 3: Suppose now that there is a relationship between φ and Sw, which can be described as bivariate Gaussian with correlation of 0.5. Given that there is no longer independence between all the input variables, describe how you would implement a Monte Carlo approach (similar to Question 1) to account for the impact this relationship has on uncertainty in OIP. If you have time, you may wish to implement this and compare against the distribution in Question 1.

Question 4: Perhaps the most common approach to sensitivity analysis is the vary one at a time approach. This requires keeping all the input variables at the base case value (usually the mean), and then for one input variable, choose say the p10 and p90 of that input variable and evaluate the impact on OIP. Plot this impact as a tornado chart by ordering the input variables in descending order of impact.

Question 5: Consider now varying each input variable (keep all other variables at the base case) by changing its value by ±5% increments from the base case value until say ±20%. For each case evaluate the change in OIP, and plot this as a spidergram.

Question 6: Rather than changing each input variable by a percentage difference from the base case, change each input by a set of percentages. For this, consider evaluating OIP as you change an input variable based on its deciles. Now plot this result in a similar format to a spidergram, and comment on any differences you notice from the spidergram in the previous question.

12.5.2 Part Two: Loss Functions

The consequences of over and under estimation are often not the same. The two common loss functions, however, are symmetric.

Question 1: Prove that the mean of a distribution always minimizes the mean squared error loss function, that is, a loss function where the loss increases as a square of the error for both over and under estimation.

Question 2: Prove that the median of a distribution always minimizes the mean absolute error loss function, that is, a loss function where the loss increases as the absolute value of the error for both over and under estimation.

Question 3: The L-optimal value is a specific quantile of the distribution of the penalty for over and under estimation is both linear with different slopes. The 0.5 quantile or median is optimal if the slopes are the same. What is the quantile for arbitrary (different) slopes for over and under estimation?

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