CHAPTER 1

LET'S STICK TOGETHER

Peirce's Conception of Continuity

> The entelechy and soul of the work, from which every part of its contents manifestly flows, is the principle of continuity, which has been the guiding star of exact science from the beginning, but of which novel and unexpected applications are now made.
>
> Peirce, 1893[5]

The concept of continuity is so central and basic to Peirce that it is not too much to say that he built the whole final version of his philosophy around it. Thus, the mature version of his phenomenological categories, the detailed semiotics he developed in the years after 1900 – and, a fortiori, his doctrine of icons and of diagrams – rest on a philosophy of continuity.[6] This chapter seeks to elucidate the basic reasons for the importance accorded to continuity by the mature Peirce in the years around the turn of the century.

Already in the 1880s, Peirce took part in the mathematical development concerning the continuum. Dedekind's formalization of the real numbers by means of the so-called 'Dedekind cut' only later came to Peirce's knowledge, but Cantor's foundation of set theory during that period gained Peirce's enthusiastic support and prompted his attempts to improve that theory. Most of Peirce's efforts, despite letters to the two internationally known mathematicians, were not published, and what was published only appeared later, so the mathematical side of Peirce's struggle with continuity remained with little influence on the scientific development. Dedekind had discovered that infinite sets could be described as sets containing subsets which have the same size as the sets themselves (like the even numbers are as infinite as the whole numbers, even if constituting a mere part of them), and Cantor, in his set theory, constructed a hierarchy of different infinities with increasing size, giving rise to his 'transfinite numbers' measuring them. An important step in that research was his ability to prove that one such class may be represented by the integers, while a larger class may be represented by the real numbers, that is, all numbers on the arithmetic line, defined, each of them, by an infinite decimal expansion. Cantor's proof, the famous 'diagonal' proof, showed that the former class could be represented in a list (like 1, 2, 3, 4, . . .), while the latter class could not. Even if the latter forms a linear series to the extent that any two given numbers can be ordered after size, they may not be listed nor counted one by one. The latter class can be

described as containing all subsets of the former (each decimal expansion is written with integers and so can be seen as an infinite subset of integers). This provided a procedure for constructing ever higher such sets, by taking the set of subsets of the former set – the so-called power set. Seemingly large sets, like the set of all rational fractions, could be shown also to be contained in the former, countable class – which naturally gave rise to the question of whether there were any set size 'in between' these two sets. Cantor's immediate idea was that there were not – which is equivalent to the so-called continuum hypothesis (CH), claiming that the real numbers, the continuum, constitute the next transfinite number after that of the integers. As is well-known, it turned out not to be possible to prove the CH, and it was only in the twentieth century proved consistent with (Gödel) and independent of (Cohen) set theory, thus being undecidable. To our day, research continues, trying to enlarge set theory by new axioms making CH decidable.

This was the context in which Peirce intervened. Peirce subscribed to the CH to the extent that it claimed that the real numbers were in fact Aleph-1, the second transfinite number after the Aleph-0 of the integers. But he did not subscribe to the implicit addition that the real numbers, in turn, corresponded to the continuum. This forms the core of Peirce's repeated attempts to rearticulate Cantorian set theory: Peirce wanted a version of set theory which made evident that the continuum transgressed any attempt at formalizing it as a line built up from points, a set consisting of individual numbers. As Cantor's whole series of Alephs, each of them, are sets consisting of individual numbers, Peirce's claim was that the continuum must lie beyond the whole series of Alephs. This implied a distinction between the arithmetical line and the geometrical line. While the former is defined by being the linear arrangement of real numbers, the latter is most often, since Descartes' analytical geometry, identified with it. This caused Peirce's protest: the geometrical line was a true continuum, thus being larger than any point set, including the arithmetical line. This basic idea formed the reason behind Peirce's repeated attempts at defining the geometrical line and constructing a revised set theory which could prove that this real continuum exceeded all sets. Potter and Shields (1977) have given an overview over Peirce's development from a Pre-Cantorian through a Cantorian to a Kantistic and finally (maybe) a post-Kantistic period in his understanding of the continuum. In the former, he naively accepted one of Kant's attempts at describing continuity as infinite divisibility (but this also holds already for the rational numbers), but from the mid-eighties far into the nineties, he struggled with his reformulations of Cantor, adding to infinite divisibility (now nicknamed 'Kanticity') the notion that all infinite series in a continuum contain their limit ('Aristotelicity'). In the last years of the nineties, he embraced an alternative, recursive idea in Kant, the idea that the continuum was defined by having parts which had parts of the same kind. During the Cantorian and Kantistic periods, Peirce's set theoretical attempts pointed in one direction: to prove that the continuum, considered as a set, differed from all other sets in consisting of *indistinct* elements only. The parts of a continuum being 'welded' or 'merged' together into a homogeneous mass, they may not be treated as if made up of determinate individuals. True, such individuals may be *selected* in

any number from a continuum, but this does not mean it consists of them. Peirce over and over attempted to find this indistinctive 'merging' which he supposed to appear gradually already within Cantor's transfinite hierarchy of Alephs, probably because he felt the transition from discontinuous sets to continuity should itself be continuous, so that in large transfinite numbers, indistinctness should somehow gradually emerge.

A recurrent way of stating Peirce's point is that the geometrical line may *contain* as large multiplicities of points as one may want – but that it, in no way, *consists* of those points. The continuum is a primitive concept of its own, and if anything, it is rather composed by infinitesimal line segments (the problem with this idea, of course, being that such segments are harder to identify or locate than points). Peirce's conception immediately entails a series of differences from the ordinary set theoretical conception of a line. In that conception, if you take away the end point of a line, what remains is an open interval without its limit point. That can not ('Aristotelicity') be allowed in Peirce's account which is why he would say that the point removed does not diminish the continuum at all. Rather, as many points as you wish (even transfinite sets of points) may 'fly off' the end of the line, leaving the original continuum unaffected. The explanation behind this idea is that the continuum 'sticks together', that points are only potentially parts of the continuum, and, as potential points only, they are indistinct. Only when, by some procedure, a point is singled out within a continuum, it gains actual existence and now forms a discontinuity within the continuum. If the continuum is broken, is crossed by another continuum, is composed by two adjoined continua, or if points by some function or other are selected within it, point sets accessible for set theoretical descriptions are actualized within it. But those points or point sets are invariably 'small' in comparison to the continuum, to the extent that they tend to vanish, or in any case, lose importance, in the overall continuous object:

> A continuum cannot be disarranged except to an insignificant extent. An instant cannot be removed. You can no more, by any decree, shorten a legal holiday by transferring its last instant to the work-day that follows that feast, than you can take away intensity from light, and keep the intensity on exhibition while the light is thrown into the ash-barrel. A limited line AB may be cut into two, AC and C'B, and its ends joined, C' to A and C to B. That is to say, all this may be done in the imagination.
> ('The Logic of Mathematics: An Attempt to Develop my Categories From Within', c. 1896, 1.499)

Such an idea of the geometrical continuum obviously breaks with Dedekind's description of the real numbers by means of Dedekind cuts: if the real line is broken in two parts, one of the parts will constitute a closed interval containing its limit point, the other part will constitute an open interval because its limit point is left at the former part. Not so in Peirce's conception which satisfies a comment by Gödel claiming that the geometrical line, if broken in two, ought to give two symmetrical parts as a result. In Peirce's account, of course, the two parts form, both of them, closed intervals, each of them containing a limit point which before the breaking was one and the same point, and which, if the two parts were put together again, would merge to one point again. Ken Ketner and Hilary Putnam have (1992) attempted, in a preface to Peirce's Harvard lectures (RLOT), to articulate Peirce's

point of view within non-standard analysis as developed since the 1960s. In this account, the line consists of standard points, each of which spans an infinitesimal 'monad' of further non-standard points around itself. Using non-standard analysis vocabulary, Peirce's seemingly inconsistent talk of points having points as parts, gives more sense, even if Peirce would not agree in the distinction between standard points and points belonging to their monads. Yet, in Peirce's discussion of the continuity of time, he takes care to distinguish infinitesimal 'moments' as opposed to purely punctual 'instants' where the latter may form parts of the former, so the distinction between monads and points would not be alien to him. I refer to the Appendix for a deeper discussion of these issues and their relation to philosophy of mathematics.

Here, it is important to emphasize the roots of Peirce's constant interest in the mathematics of the continuum. It stems from Peirce's metaphysical concept of continuity, and his mathematical interest is an attempt to construe a consistent vocabulary in which to reason about metaphysical continuity. The need for continuity in metaphysics has a whole series of related reasons, and to begin with Peirce's theory of perception, the basic status of continuity is suitably summed up in Peirce's argument against the idea that continuity may be dissolved as an illusion (as he believed it would be in a Cantorian set theory):

> My notion is that we directly perceive the continuity of consciousness; and if anybody objects, that which is not really continuous may seem so, I reply, 'Aye, but it could not seem so, if there were not some consciousness that is so.' I should like to see a good criticism of that reply.
> ('A Sketch of Logical Critic,' c. 1911, 6.182)

Essentially the same argument has been made independently by René Thom, claiming that as part of our experience, the continuum has an objective existence which, like all other experiences, may be subject to illusions (cf. the 24 discontinuous pictures per second giving a continuous time flow illusion in cinema), but if no real continuity is possible within neural physiology, how then could such an illusion be explained? (Thom 1992, 140). This argument for the ineradicability of continuity in experience might be nicknamed the Peirce-Thom argument.

This argument, however, only involves the phenomenological aspects of a whole related bunch of issues. Continuity is deemed metaphysically necessary to explain: (1) the intensional meaning of general concepts (the inexhaustibility of continuous extension); (2) the embeddedness of actually existing objects and occurring events within a horizon continuum of potentiality; (3) realism as to general tendencies (as opposed to the powder of unconnected singular events without continuity); (4) continuity of research – from its infinitesimal beginnings long before science and to its converging end point in truth; (5) fallibilism as implied by the vagueness inherent in continuity; and finally: (6) diagram manipulation as basically continuous and hence able to mirror real continuity. Let us run through these main points of Peirce's continuum metaphysics.

(1) Continuity is first and foremost important, because it provides an account for the generality of concepts. Concepts with real reference are seen by Peirce as abbreviated propositions without any specific subject. 'Heavy' is an abbreviation of

'X is heavy' where X may be filled in with any subject that satisfies the proposition. The law of gravitation, one of Peirce's favourite examples of a real force functioning in Nature, may not be reduced to any finite number of heavy objects subject to attraction. This implies that the collection of subjects potentially referred to by the proposition 'X is heavy' exceeds any extension made up of recorded, actual, or even imagined single cases of gravitation attraction. It is continuous. Herein lies, Peirce argues, the reality of gravity: it has worked, and will continue to work, in a number of cases which are so vast as to be beyond the reach of any possible charting – that is, they remain vague and indistinct. The vagueness and indistinctness of extensional reference is thus, according to Peirce, a necessary flip side of that reference involving real, general forces, tendencies or patterns in reality:

> True generality is, in fact, nothing but a rudimentary form of true continuity. Continuity is nothing but perfect generality of a law of relationship. ('Synechism', Baldwin's Dictionary, 1902, 6.172)

Therefore, no positivist reduction of such laws or relationships to mutually similar aspects of single cases is possible. It might indeed be the way such laws are discovered or recorded by induction, but this does not imply that the relation at work may be reduced to such cases:

> At any rate, it is plain that no possible collection of single occasions of conduct can be, or adequately represent all conceivable occasions. For there is no collection of individuals of any general description which we could not conceive to receive the addition of other individuals of the same description aggregated to it. The generality of the possible, the only true generality, is distributive, not collective. ('Consequences of Critical Common-Sensism', 1902, 5.532)

The distinction made here between distributivity and collectivity corresponds to the normal terminology of intension and extension. The latter may give rise to probabilistic investigations, the former, however, to apodictic, universal results:

> To say that the probability that a calf will not have more than six legs is 1, is to say that in the long run, taking calves as they present themselves in experience, the ratio of the number of those with not more than six legs to the total number is 1. But this does not prevent there being any finite number of calves with more legs than six, provided that in the long run, that is, in an endless course of experience, their number remains finite, and does not increase indefinitely. A universal proposition, on the other hand, asserts, for example, that any calf which may exist, without exception, is a vertebrate animal. The universal proposition speaks of experience distributively; the probable, or statistical proposition, speaks of experience collectively. ('Predicate', Baldwin's Dictionary, 1902, 2.358)

(2) Due to the reality of (certain) concepts, these claims of semantics immediately lead into ontology. The finite number of cases recorded by any extensional investigation thus refers to actual events selected from a continuum of potential, interrrelated events. The rule, law, tendency, or pattern governing that continuum hence has the character of potentiality:

> Since Kant it has been a very wide-spread idea that it is time and space which introduce continuity into nature. But this is an anacoluthon. Time and space are continuous because they embody conditions of possibility, and the possible is general, and continuity and generality are two names for the same absence of distinction of individuals. ('Multitude and Number', 1897, 4.172)

Such possibilities do not possess the actual individuality of realized cases, but they are still real – from around 1897 Peirce begins to term them 'real possibilities' (to which we shall return in the next chapter on Peirce's 'extreme realism'):

> A true continuum is something whose possibilities of determination no multitude of individuals can exhaust. ('Synechism', Baldwin's Dictionary, 1902, 6.170)

(3) Continuity thus forms the central feature of Peirce's realism with respect to such 'real possibilities':

> That which is possible is in so far general and, as general, it ceases to be individual. Hence, remembering that the word 'potential' means indeterminate yet capable of determination in any special case, there may be a potential aggregate of all the possibilities that are consistent with certain general conditions; and this may be such that given any collection of distinct individuals whatsoever, out of that potential aggregate there may be actualized a more multitudinous collection than the given collection. Thus the potential aggregate is, with the strictest exactitude, greater in multitude than any possible multitude of individuals. But being a potential aggregate only, it does not contain any individuals at all. It only contains general conditions which permit the determination of individuals.
> ('The Logic of Continuity', 1898, 6.185)

Generality, real possibility, and indistinctness are thus connected in metaphysical continuity. It should be added that the continuity doctrine is also intimately connected to Peirce's evolutionism which extends Darwinism from biology to cover the whole of the physical evolution in a strange cosmology, taking its beginning in pure continuous, chaotic (Peirce does not quite agree with himself how ordered or unordered pure possibility is) possibility which, via a growing amount of actualization, lets still more laws and tendencies introduce in the actual world which is thus aimed at continuous perfection in a remote future . . . We shall not go into these cosmological areas of Peirce's metaphysics in this book which focuses upon the basic relation between continuous realism and diagrams.

(4) Given that law- or rule-governed aspects of reality are thus continuous, the very process of acquiring knowledge must – as it is in itself such a process – be continuous. Peirce's epistemology, in which diagrams play center stage, thus rests upon an ontology of knowledge acquisition. Knowledge creation being continuous, it can never really begin nor end – in both cases continuity replaces any discontinuous beginning or halting of knowledge:

> If it is objected that there must be a first thing learned, I reply that this is like saying that there must be a first rational fraction, in the order of magnitudes, greater than zero.
> ('Some Logical Prolegomena', undated, 7.536)

Science thus is continuous with everyday knowledge which is, in turn, continuous with animal cognition and so on indefinitely down the scale of evolution. Knowledge is always already in the process of being constructed – even if this idea causes trouble for Peirce's definition of logical thought as explicitly self-controlled: already perception, entrance gate of raw knowledge, is not thus controlled, and even less so biological forms of knowledge gathering. We shall return to this. In the other end, of course, continuity defines Peirce's famous pragmatist notion of truth as that to which the scientific community will converge in the long run:[7]

But a scientific proposition is merely something you take up provisionally as being the proper hypothesis to try first and endeavor to refute. The only belief you – as a purely scientific man – have about it is that it is adopted in accordance with a method which must lead to the truth in the long run.
('Logic of Events', 1898, 6.216)

As in many rule-bound, continuous processes, there are considerable fluctuations in science, but they even out in the continuity of the long run, making necessarily science an unending, collective process involving generation after generation of scientists:

As we go on drawing inference after inference of the given kind, during the first ten or hundred cases the ratio of successes may be expected to show considerable fluctuations; but when we come into the thousands and millions, these fluctuations become less and less; and if we continue long enough, the ratio will approximate toward a fixed limit. ('The Doctrine of Chances', 1878, EPI, 146; 2.650)

(5) The continuity of knowledge thus also implies the pragmatist's fallibilism due to the ineradicable imprecision inherent in continuity:

The principle of continuity is the idea of fallibilism objectified. For fallibilism is the doctrine that our knowledge is never absolute but always swims, as it were, in a continuum of uncertainty and of indeterminacy. Now the doctrine of continuity is that all things so swim in continua.
(Untitled manus., c. 1897, 1.171)

This does not, as might be expected, wed Peirce to any irrational skepticism against the possibilities of science, quite on the contrary. General, continuous processes occur with many degrees of fluctuations, and the only minimally fluctuating parts of those may be charted with a high degree of precision.
(6) As indicated in the Introduction, continuity plays a central role for the possibility of reading off general regularities of a diagram. Diagrams may chart ideal relationships – as in the Pythagoras example of the Introduction – or they may map idealized aspects of empirical states-of-affairs of many varied sorts. In all cases, however, they involve a moment of observation. To Peirce, observation is also a process necessarily infused with continuity. There is no such thing as an observation of a completely unique event or entity: already in ordinary perception, generality and continuity play a central role – e.g. in our spontaneous recognition that this or that aspect of perception is an instantiation of some general type or process. This reliance of perception and knowledge on continuous generality is now highlighted and made an issue of explicit control in diagrams. This is why the very continuity of the sheet upon which a diagram is drawn becomes a matter of central importance:

Let the clean blackboard be a sort of diagram of the original vague potentiality, or at any rate of some early stage of its determination. This is something more than a figure of speech; for after all continuity is generality. This blackboard is a continuum of two dimensions, while that which it stands for is a continuum of some indefinite multitude of dimensions. This blackboard is a continuum of possible points; while that is a continuum of possible dimensions of quality, or is a continuum of possible dimensions of a continuum of possible dimensions of quality, or something of that sort. There are no points on this blackboard. There are no dimensions in that continuum. I draw a chalk line on the board. This discontinuity is one of those brute acts by which alone the original vagueness could have made a step towards definiteness. There is a certain element of continuity in this line. Where did this continuity come from? It is nothing but the original continuity of the blackboard which makes everything upon it

> continuous. What I have really drawn there is an oval line. For this white chalk-mark is not a line, it is a plane figure in Euclid's sense – a surface, and the only line there, is the line which forms the limit between the black surface and the white surface. Thus the discontinuity can only be produced upon that blackboard by the reaction between two continuous surfaces into which it is separated, the white surface and the black surface. The whiteness is a Firstness – a springing up of something new. But the boundary between the black and white is neither black, nor white, nor neither, nor both. It is the pairedness of the two. It is for the white the active Secondness of the black; for the black the active Secondness of the white.
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> Now the clue, that I mentioned, consists in making our thought diagrammatic and mathematical, by treating generality from the point of view of geometrical continuity, and by experimenting upon the diagram. ('The Logic of Continuity', 1898, 6.203–4)

Diagrams may, of course, use discontinuity, but it is only possible to represent discontinuity as a break of a presented continuity – like the triangle drawn on the geometer's blank slate. The continuity of the sheet is crucial, furthermore, because it is what allows us to perform the thought (or real) experiments continuously varying the diagram so as to ensure that the state-of-affairs recorded therein hold not only for the one case drawn, but for a continuum of similar cases. Which continuum of cases are in fact envisioned of course depends on the (most often) written or tacitly implied instructions relevant for the specific diagram in question – but the important thing is that the diagram always inherits the spatiotemporal continuity of the sheet in some specified respects. So, the continuity of diagrams is no accidental feature of representation, rather, it is what makes thought representing real relations possible in diagrams. Thus, diagram continuity is intimately connected to the continuity tying together semantics, realism, epistemology, and fallibilism in Peirce's mature doctrine. It goes without saying that the sketches of a philosophical architectonics which we have here briefly introduced do not form a coherent doctrine. Many obscure points indeed remain, and we shall address some of them in the chapters to come. Still, the continuity doctrine forms the basis of the most impressive results of Peirce's final endeavor. It comes as little surprise, then, that the continuum also lies beneath Peirce's mature version of his categories after the turn of the century, now considered under the headline of 'phenomenology' or 'phaneroscopy' (see Chap. 6). Peirce's categories belong to the earliest of his preoccupations, already presented, of course, in 'A New List of Categories' (1867). The continuity metaphysics of the later Peirce, however, permits him to cast them in a new light, integrating them at the basis of his thought.

CONTINUITY AS METAPHYSICAL GLUE IN PEIRCE'S SYSTEM

Continuity has wide implications in the different parts of Peirce's architectonics of theories. Time and time again, Peirce refers to his 'principle of continuity' which has not immediately anything to do with Poncelet's famous such principle in geometry (Chap. 5).[8] It is, rather, a metaphysical implication taken to follow from fallibilism: if all more or less distinct phenomena swim in a vague sea of continuity then it is no wonder that fallibilism must be accepted. And if the world is basically continuous, we should not expect conceptual borders to be definitive but rather conceive of

terminological distinctions as relative to an underlying, monist continuity. We shall not go far into these ideas, but rather depict the role of continuity in Peirce's efforts to build a system. In this system, mathematics is first science. Thereafter follows philosophy which is distinguished form purely hypothetical mathematics by having an empirical basis.[9] Philosophy, in turn, has three parts, phenomenology, the normative sciences, and metaphysics. The first investigates solely 'the Phaneron' which is all what could be imagined to appear as an object for experience: '... by the word *phaneron* I mean the collective total of all that is in any way or in any sense present to the mind, quite regardless whether it corresponds to any real thing or not.' ('Adirondack Lectures', 1905, CP 1.284) As is evident, this definition of Peirce's 'phenomenology' is parallel to Husserl's phenomenological reduction in bracketing the issue of the existence of the phenomenon in question (see Chap. 6). Even if it thus is built on introspection and general experience, it is – analogous to Husserl and other Brentano disciples at the same time – conceived in a completely antipsychological manner: 'It religiously abstains from all speculation as to any relations between its categories and physiological facts, cerebral or other.' ('Logic viewed as Semeiotics', 1904, 1.287) and '... I abstain from psychology which has nothing to do with ideoscopy.' (Letter to Lady Welby, Oct 12, 1904, 8.330). The normative sciences fall in three: aesthetics, ethics, logic, in that order (and hence decreasing generality), among which Peirce does not spend very much time on the former two. Aesthetics is the investigation of which possible goals it is possible to aim at (Good, Truth, Beauty, etc.), and ethics how they may be reached. Logic is concerned with the grasping and conservation of Truth and takes up the larger part of Peirce's interest among the normative sciences. As it deals with how truth can be obtained by means of signs, it is also called *semiotics* ('logic is formal semiotics') which is thus coextensive with *theory of science* – logic in this broad sense contains all parts of philosophy of science, including contexts of discovery as well as contexts of justification. Semiotics has, in turn, three branches: *grammatica speculativa* (or stekheiotics), *critical logic*, and *methodeutic* (inspired by mediaeval trivium: grammar, logic, and rhetoric). The middle one of these three lies closest to our days' conception of logic; it is concerned with the formal conditions for truth in symbols – that is, propositions, arguments, their validity and how to calculate them, including Peirce's many developments of the logic of his time: quantifiers, logic of relations, ab-, de-, and induction, logic notation systems, etc. All of these, however, presuppose the existence of simple signs which are investigated by what is often seen as semiotics proper, the *grammatica speculativa*;[10] it may also be called formal grammar. It investigates the formal condition for symbols having meaning, and it is here we find Peirce's definition of signs and his trichotomies of different types of sign aspects. Methodeutic or formal rhetorics, on the other hand, concerns the pragmatical use of the former two branches, that is, the study of how to use logic in a fertile way in research, the formal conditions for the 'power' of symbols, that is, their reference to their interpretants; here can be found, e.g., Peirce's famous definitions of pragmati(ci)sm and his directions for scientific investigation. To phenomenology – again in analogy to Husserl – logic adds the

interest in signs and their truth. After logic, metaphysics follows in Peirce's system, concerning the inventarium of existing objects, conceived in general – and strongly influenced by logic in the Kantian tradition for seeing metaphysics mirroring logic. Also here, Peirce has several proposals for subtypologies, even if none of them seem stable, and under this headline classical metaphysical issues mix freely with generalizations of scientific results and cosmological speculations.

Peirce himself saw this classification in an almost sociological manner, so that the criteria of distinction do not stem directly from the implied objects' natural kinds, but after which groups of persons study which objects: '. . . the only natural lines of demarcation between nearly related sciences are the divisions between the social groups of devotees of those sciences . . .' (CP 8.342). Science collects scientists into bundles, because they are defined by their causa finalis, a teleologial intention demanding of them to solve a central problem.[11]

Measured on this definition, one has to say that Peirce himself was not modest, not only does he continuously transgress such boundaries in his production, he frequently does so even within the scope of single papers. There is always, in his writings, a brief distance only from mathematics to metaphysics – or between any other two issues in mathematics and philosophy, and this implies, first, that the investigation of continuity and generality in Peirce's system is more systematic than any actually existing exposition of these issues in Peirce's texts, second, that the discussion must constantly rely on cross-references. This has the structural motivation that as soon as you are below the level of mathematics in Peirce's system, inspired by the Comtean system, the single science receives determinations from three different directions, each science consisting of material and formal aspects alike. First, it receives formal directives 'from above', from those more general sciences which stand above it, providing the general frameworks in which it must unfold. Second, it receives material determinations from its own object, requiring it to make certain choices in its use of formal insights from the higher sciences. The cosmological issue of the character of empirical space, for instance, can take from mathematics the different (non-)Euclidean geometries and investigate which of these are fit to describe spatial aspects of our universe, but it does not, in itself, provide the formal tools. Finally, the single sciences receive in practice determinations 'from below', from more specific sciences, when their results by means of abstraction, prescission, induction, and other procedures provide insights on its more general, material level. Even if cosmology is, for instance, part of metaphysics, it receives influences from the empirical results of physics (or biology, from where Peirce takes the generalized principle of evolution). The distinction between formal and material is thus level specific: what is material on one level is a formal bundle of possibilities for the level below; what is formal on one level is material on the level above.[12]

For these reasons, the single step on the ladder of sciences is only partially independent in Peirce, hence also the tendency of his own investigations to zigzag between the levels. His architecture of theories thus forms a sort of phenomenological theory of aspects: the hierarchy of sciences is an architecture of more and less

general aspects of the phenomena, not completely independent domains. Finally, Peirce's realism has as a result a somewhat disturbing style of thinking: many of his central concepts receive many, often highly different determinations which has often led interpreters to assume inconsistencies or theoretical developments in Peirce where none necessarily exist (this does not imply, of course, that there are no developments or inconsistencies at all in Peirce; there are indeed many). When Peirce, for instance, determines the icon as the sign possessing a similarity to its object, and elsewhere determines it as the sign by the contemplation of which it is possible to learn more about its object, then they are not conflicting definitions. Peirce's determinations of concepts are rarely definitions at all in the sense that they provide necessary and sufficient conditions exhausting the phenomenon in question. His determinations should rather be seen as descriptions from different perspectives of a real (and maybe ideal) object – without these descriptions necessarily conflicting. This style of thinking can, however, be seen as motivated by metaphysical continuity. When continuous grading between concepts is the rule, definitions in terms of necessary and sufficient conditions should not be expected to be exhaustive.

A recurring skeleton on all levels, however, is provided by Peirce's famous 'triadomania', as he himself calls it, which lets most of his decisive distinctions appear in threes, following the tripartition of his list of categories, the famous triad of First, Second, and Third, or Quality, Reaction, Representation, or Possibility, Actuality, Reality – or any other of the manifold of descriptions of this triad which he gives through his work.

The probably most concise – but also very self-referential – description is found in one of the letters to Lady Welby (CP 8.327) from 1904:

> Firstness is the mode of being of that which is such as it is, positively and without reference to anything else.
>
> Secondness is the mode of being of that which is such as it is, with respect to a second but regardless of any third.
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> Thirdness is the mode of being of that which is such as it is, in bringing a second and third into relation to each other.

The justification for this triad of possible experience can be stated briefly as follows: Firstness constitutes the *quality* of experience: in order for something to appear at all, it must do so due to a certain constellation of qualitative properties. Peirce often uses sensory qualities as examples, but it is important for the understanding of his thought that the examples may refer to phenomena very far from our standard conception of 'sensory data', e.g. forms or the 'feeling' of a whole melody or of a whole mathematical proof, not to be taken in a subjective sense but as a concept for the continuity of melody or proof as a whole, apart from the analytical steps and sequences in which it may be, subsequently, subdivided. In short, all sorts of simple and complex Gestalt qualities also qualify as Firstnesses. Firstness tend to form continua of possibilities such as the continua of shape, color, tone, etc. These qualities, however, are, taken in themselves, pure possibilities and must necessarily be incarnated in phenomena in order to appear. Secondness is

the phenomenological category of 'incarnation' which makes this possible: it is the *insistency*, then, with which the individuated, actualized, existent phenomenon appears. Thus, Secondness necessarily forms discontinuous breaks in Firstness, allowing for particular qualities to enter into existence. The mind may imagine anything whatever in all sorts of quality combinations, but something appears with an irrefutable insisting power, reacting, actively, yielding resistance. Peirce's favorite example is the resistance of the closed door – which might be imagined reduced to the *quality* of resistance feeling and thus degenerate to pure Firstness so that his theory imploded into a Hume-like solipsism – but to Peirce this resistance, surprise, event, this thisness, 'haecceity' as he calls it with a Scotist term, remains irreducible in the description of the phenomenon (a Kantian idea, at bottom: existence is no predicate).[13] About Thirdness, Peirce may directly state that continuity represents it perfectly (1.337): '... continuity and generality are two names of the same absence of distinction of individuals' ('Multitude and Number', 1897, 4.173). As against Secondness, Thirdness is *general*; it *mediates* between First and Second. The events of Secondness are never completely unique, such an event would be inexperiencable, but *relates* (3) to other events (2) due to certain features (1) in them; Thirdness is thus what facilitates understanding as well as pragmatic action, due to its continuous generality. With a famous example ('Thirdness', c. 1895, 1.341): if you dream about an apple pie, then the very qualities of that dream (taste, smell, warmth, crustiness, etc.) are pure Firstnesses, while the act of baking is composed of a series of actual Secondnesses. But their coordination is governed by a Thirdness: the recipe, being general, can never specify all properties in the individual apple pie, it has a schematic frame-character and subsumes an indefinite series – a whole continuum – of possible apple pies. Thirdness is thus necessarily general and vague. Of course, the recipe may be more or less precise, but no recipe exists which is able to determine each and every property in the cake, including date, hour, place, which tree the apples stem from, etc. – any recipe is necessarily general. In this case, the recipe (3) mediates between dream (1) and fulfilment (2) – its generality, symbolicity, relationality and future orientation are all characteristic for Thirdness. An important aspect of Peirce's realism is that continuous generality may be experienced directly in perceptual judgments: 'Generality, Thirdness, pours in upon us in our very perceptual judgments ...' ('The Three Normative Sciences', 1902, EPII, 207; 5.150).[14]

All these determinations remain purely phenomenological, even if the later semiotic and metaphysical interpretations[15] clearly shine through. In a more general, non-Peircean terminology, his phenomenology can be seen as the description of minimum aspects inherent in any imaginable possible world – for this reason it is imaginability which is the main argument, and this might point in the direction that Peirce could be open to critique for subjectivism, so often aimed at Husserl's project, in some respects analogous. The concept of consciousness is invoked as the basis of imaginability: phenomenology is the study of invariant properties in any phenomenon appearing for a mind. Peirce's answer would here be, on the one hand, the research community which according to him defines reality – an

argument which structurally corresponds to Husserl's reference to intersubjectivity as a necessary ingredient in objectivity (an object is a phenomenon which is inter-subjectively accessible). Peirce, however, has a further argument here, namely his consequent refusal to delimit his concept of mind exclusively to human subjects (a category the use of which he obviously tries to minimize), mind-like processes may take place in nature without any subject being responsible. Peirce will, for continuity reasons, never accept any hard distinction between subject and object and remains extremely parsimonious in his use of such terms.[16] Without calling him naturalist through and through, we may at this stage claim that he tries to let the antinomy between naturalism and idealism form a circle[17] which hinders scepticist critiques of idealism as well as subjectivist critiques of naturalism to gain foothold.

The place of continuity in Peirce's category doctrine has already been hinted at by its appearance as a central predicates for Thirdness, but all has not been said here. Secondness evidently has discontinuity as one of its properties, but what about Firstness? The fact that sensory qualities – prototypical examples of Firsts – as a rule form continua, suggests that continuity is also a crucial property in Firstness. Peirce is not unanimous here: a basic idea in Firstness is that each quality appears independently of anything else, and so Firstness seems a powder of infinitesimal quality bits. But still they have a tendency to form continua, and the different quality continua may even meet in a higher dimension continuous space: 'At one end of the sequence all the qualities come together in a *zero*. But they are separate from one another as they separate from zero' ('Abstracts of 8 lectures', undated, NEM IV, 128[18]).

In the text quoted here, Peirce indulges in a larger investigation of whether the space of qualities is 'perissid' or 'artiad' – depending on whether a Hegelian-like transformation from a maximal intensity of one quality is allowed to pass directly into a maximal intensity of the opposed quality. This is refused by Peirce, and he consequently envisages the continuum of possible qualities mirror itself in a zero point where two half continua meet. The decisive idea here, however, is that qualities constitute a continuum of many dimensions, and that one type of qualities – cf. the point zero argument – may pass continuously into another. The existent sensory and other qualities of experience are thus, to Peirce, only remaining, actualized fragments of a original, basic continuum of possibility, uniting all possible qualia in one continuous manifold[19] (and thus making visual, auditory, olfactory, etc. qualities parts of the same continuum).

> But now to return to the continuum of possible quality. Every complexus of qualities is a quality, and as such, considered by itself, is all that it is in and for itself. Not only every complex of qualities but every *generalization* of such complexes is a possible quality. But in this way, the dimensions of the continuum ought to exceed every discrete multitude. In short, they should form a continuum of dimensions. It is impossible. Hence these dimensions of complex qualities are only abstractly possible. They cannot have simultaneous being in the world of potentialities. ('Abstract of 8 lectures', undated, NEM IV, 135)

It is not completely evident why a quality continuum with a continuous number of dimensions should not be possible – but the decisive issue here is the idea that

Firstness appears as a multidimensional continuum, in which the single quality yet '. . . in itself is absolutely severed from every other' (ibid. 133).[20]

This implies that the reaction events of Secondness may be conceived on the background of this vast space of possibilities:

> But just as the qualities, which as they are for themselves, are equally unrelated to one other, each being mere nothing for any other, yet form a continuum in which and because of their situation in which they acquire more or less resemblance and contrast with one another; and then this continuum is amplified in the continuum of possible feelings of quality, so the accidents of reaction, which are waking consciousnesses of pairs of qualities, may be expected to join themselves into a continuum. (137)

Secondness is now taken to actualize these quality possibilities based on an idea that any actual event involves a clash of qualities – in the ensuing argumentation Peirce underlines that the qualities involved in actualization need not be restrained to two but may be many, if they may only be 'dissolved' into pairs and hence do not break into the domain of Thirdness. This appearance of actuality, hence, has the property of singularities, spontaneously popping up in the space of possibilities and actualizing pairs of points in it:

> Since, then an accidental reaction is a combination or bringing into special connection of two qualities, and since further it is accidental and antigeneral or discontinuous, such an accidental reaction ought to be regarded as an adventitious singularity of the continuum of possible quality, just as two points of a sheet of paper might come into contact. (137)

This transition from First to Second is conceived of along Aristotelian lines: as an actualization of a possibility – and this is expressed in the picture of a discontinuous singularity in the quality continuum. The topological fact that singularities must in general be defined with respect to the neighborhood of the manifold in which they appear, now becomes the argument for the fact that Secondness can never be completely discontinuous but still 'inherits' a certain small measure of continuity from the continuum of Firstness:

> But although singularities are discontinuous, they may be continuous to a certain extent. Thus the sheet instead of touching itself in the union of two points may cut itself all along a line. Here there is a continuous line of singularity. In like manner, accidental reactions though they are breaches of generality may come to be generalized to a certain extent. (137)

Singularities, being discontinuous along certain dimensions, may be continuous in others, which provides the condition of possibility for Thirdness to exist as a tendency for Secondness to conform to a general law or regularity. As is evident, a completely pure Secondness is impossible in this continuous metaphysics – it remains a conceivable but unrealizable limit case, because a completely discontinuous event would amount to nothing. Thirdness already lies as a germ in the non-discontinuous aspects of the singularity. The occurrences of Secondness seem to be infinitesimal, then, rather than completely extensionless points.

The continuity of Thirdness is, in contrast to Firstness, real – the recipe does in fact refer to a continuum of apple pies which gradually, to greater or lesser extent, are actualized. We are now well into Peirce's metaphysics – while the quality continuum of Firstness is perfect, complete, but purely potential (and thus neither

universal nor particular), then that of Thirdness is imperfect and fractioned, but real – realized via the swarm of Secondness actualizations, but still transgressing these in scope, as a finite number of events may never exhaust the universal law that governs them. From the reservoir of possibility in First, still larger doses of continuity seek through the slits of actual events in Secondness out into the growing reality of Thirdness – to sum up the three categories' relation to continuity in a metaphor which Peirce does not himself use. First is potential, Second actual, and Third real, and Peirce occasionally hints at the idea that the reality of Thirdness constitutes a consistent conception of a third Aristotelian notion: entelechy.[21]

VAGUENESS, DETERMINATENESS, GENERALITY

Continuity thus appears in two different forms in Firstness and Thirdness, respectively:

> Perhaps a more scientific pair of definitions would be that anything is general in so far as the principle of excluded middle does not apply to it and is vague in so far as the principle of contradiction does not apply to it. ("Issues of Pragmaticism", 1905, EPII, 351; 5.448)

Elsewhere, Peirce connects vagueness and generality to Firstness and Thirdness, respectively. In the former, continuity thus appears as *vagueness*, implying that a quality in the Firstness quality continua can never be identified with full exactitude and constitutes an infinitesimal variation of the quality dimension in question. In the latter, continuity appears as *generality* which is underspecified in comparison to its incarnations in particular, actual events. Peirce's brief definition of these two modes of indeterminateness by means of the logical principles of excluded middle (PEM) and contradiction (PC) requires some clarifications.

When claiming that the vagueness of Firstness does not follow the principle of contradiction, Peirce's idea does not refer to the standard PC in modern propositional logic (the principle that for a proposition p, not both p and non-p hold).[22] Thus he does not mean that propositions relating to Firstness in general can have several truth values (as would be the case when referring to the normal PC being false. Peirce's non-standard PC focuses on properties in predicate logic – it is the claim that for all properties P, no subject has both of the properties P and non-P (not both "S is P" and "S is non-P") – and it explicitly applies for definite subjects only. But firstnesses are possibilities and hence not such subjects. Their metaphysical status is thus in a typical Peircean move defined by the an ontological interpretation of logical principles. They are ontologically may-bes, and a may-be does not exclude the correlated may-not-be: whether any single subject instantiates a given may-be or not can not be decided on the basis of the may-be. The fact that PC does not apply thus refers to the modal character of the entities of Firstness, and its logical expression is that "S may be P" and "S may be non-P" may both be true – Peirce's own example is that "It may rain tomorrow" and "It may not rain tomorrow" are both true.

The generality of Thirdness, similarly, is claimed not to follow the principle of the excluded middle (or, the excluded Third). This should not be understood as

referring to the modern standard PEM (that for a proposition p, either p or non-p hold). claiming that propositions referring to Thirdnesses (general necessities) admit a third truth-value such as would be the case if standard logical PEM does not apply. Peirce's non-standard PEM is the idea that for all properties P, any subject has either the property P or the property non-P (either "S is P" or "S is non-P"), and it explicitly applies to individuals only. Claiming that PEM does not apply to Thirdnesses thus refers to the ontological status of Thirdness would-bes – they are not such subjects as required by PEM. Here, the logical expression is that both "S must be P" and "S must be non-P" may both be false (in Peirce's example: "It must rain tomorrow" and "It must not rain tomorrow" may both be false.[23] And even if a specific would-be holds, the general failure of PEM to apply refers to the fact that the single objects of would-bes remain undetermined as to all aspects not covered by the would-be real possibility in question – they are thus objects with lots of indeterminate aspects.[24] The following exemplifies this idea:

> The *general* may be defined as that to which the principle of excluded middle does not apply. A triangle in general is not isosceles nor equilateral; nor is a triangle in general scalene. The *vague* might be defined as that to which the principle of contradiction does not apply. For it is false neither that an animal (in a vague sense) is male, nor that an animal is female. (Pragmaticism, Prag. [4], 1905; 5.505)

In generals, further equally possible specifications are indeterminate (different kinds of triangles), while in vaguenesses, contradictory properties may appear (genderless or hermaphroditic animals).

The actual individual existence of Secondness, by contrast, may then be defined by its adherence to both of the principles of contradiction and excluded middle, because individuality taken as complete determination of all properties must obey both principles. The determinateness of Secondness thus forms the third member of the triad vagueness-determinateness-generality. A completely determinate individual must possess the property P or its contradictory non-P (PEM), but not both (PC):

> Although the principles of contradiction and excluded middle may be regarded as together constituting the definition of the relation expressed by "not", yet they also imply that whatever exists consists of individuals. (Baldwin's Dictionary, 1911, 537–38; 3.612)

Peirce hesitates, though, to accept full determinacy of all properties as a definition of the existence mode of actual, individual objects or events. His continuist reason for hesitating is that this would make the relation between the possibilities of Firstness and its subsets, the real possibilities of Thirdness, on the one hand, and individual Secondness, on the other, too discontinuous and insurmountable – and he points to the large and vain efforts of the Scholastics to explain the relation between general concepts and individual objects in terms of "contraction" etc. Vagueness and generality are supposed to inhere also in individual existence, albeit only infinitesimally in contrast to the two continuous possibility categories. This is why an alternative definition by *reactivity* is proposed for individuality:

> Another definition which avoids the above difficulties is that an individual is something which reacts. That is to say, it does react against some things, and is of such a nature that it might react, or have reacted, against my will. (Baldwin's Dictionary, 1911, 537–38; 3.612)

Full adherence to both PC and PEM, according to this idea, is taken to pertain to the ideal of fully determinate objects or events only, an ideal which actual existing individuals can never completely satisfy. Individuals may be singled out on all levels of reality and discourse without the ontological requirement of full determinateness, if reactivity is what grants their basic haecceity (thisness, see next chapter).

Peirce also describes the difference between the two indeterminacies, vague continuity and general continuity, in semiotic terms, based upon a dialogical notion of semiotics nowadays identified as a version of game-theoretical semantics (Pietarinen 2006). In vague signs, the possible further selection and specification of the object is left to the same source as uttered the first sign (The fortune-teller: "I see a great event . . ." "What is it?" "Some tall, dark man . . ."). In generality, the possible further selection and specification of the object is handed over to the dialogue partner (The logician: "Any man is mortal" "Which man?" "Any man you like!").[25] The normal situation being that the utterer commands the scope of signification of signs, general signs form a special, restricted subset of vague signs – a special subset where the utterer permits the dialogue partner to select examples (real generals supposedly not admitting (but few) counterexamples).

While the vagueness of Firstness has its metaphysical expression in the reality of (relative) indeterminism, the generality of Thirdness has its metaphysical expression in the real existence of laws and tendencies. The relation between the three categories and the two principles in Peirce's version may be summed up as follows (with the proviso that Secondness as defined by adherence to both principles forms an ideal limit case only).

		Principle of Contradiction	Principle of Excluded Middle
1	vagueness, possibility	−	+
2	determinateness actuality	+	+
3	generality necessity	+	−

Metaphysically speaking, the continuities of Firstness and Thirdness thus refer to the real existence of indeterminateness and laws, respectively, in addition to the reactive actuality of Secondness. Logically speaking, Peirce initiates the development of logic formalisms covering different indeterminate cases, such as modal logic, intuitionistic logic and fuzzy logic[26] – in his unfinished Gamma graphs he attempted to extend his logic representation systems to cover such cases.

This presentation of the role of continuity in Peirce's phenomenology has until now been merely reconstruction of his position and does not in any way present an argument for its validity. What is the *motivation* for this crucial role of continuity

in his category doctrine? In the case of Firstness, continuity serves the aim of describing the infinite density of *possible* predicates: phenomenologically spoken it corresponds to the experience of the infinite and continuous variability of qualia. Furthermore, the 'composition' of the continuum from parts, infinitesimals, of which the single infinitesimal is but vague and escapes the law of contradiction, serves to underline a conception of the possible quality itself as vague, because only the discontinuity of Secondness makes the precise quality evident. But it is especially the rule of continuity as central to Thirdness which is controversial. Continuity here serves the phenomenological purpose of accounting for experienced, realistically conceived *regularities* in the phenomenon: the fact that processes are more or less rule-bound. Continuity in Thirdness thus supports the idea of 'real possibilities' or 'would-be's which Peirce introduces in his mature theory from around 1896–97.[27] Whether this is a necessary implication of the identification of Thirdness with real continuity is, however, an open question to which we shall return.

An even more basic critical question will ask for the legitimacy of the three categories. In our days, the legitimacy of Secondness and its ontology of particular events will probably give rise to least controversy; the actually existing world of particular entities appears to most metaphysicians as beyond any reasonable doubt. The fact that this actual world realizes certain possibilities we may conceive makes some version of Firstness easily digestible if bracketing Peirce's further metaphysical implications such as the continuity of all possible qualities. Thirdness is obviously the most problematic category, gathering in one bundle a whole set of philosophical issues in one grand solution proposal:

Real possibilities; reality of tendencies, relations, and patterns; rule-following; the iconic structure of propositions; intensional meanings

An obvious conclusion might seem the positivist reaction: regularities found in certain processes do not have any further substance and do not require their own basic phenomenological category – they are in the last account reducible to psychological or logical organizations of what is empirically given in atomist data. Such explanations, however, loosens a psychological or logical level from the phenomenon itself – in the former case with the result that phenomenology is re-psychologized and based on a preliminary subject-object distinction with all the well-known lamentable consequences – in the latter case with the result that the validity and role of logic remains unexplained, with the result that it may be conceived of as mere formal tautologies whose contribution to the organization of experience seems trivial. From a logical point of view, this argument against Thirdness (and its ensuing universalist realism) targets Peirce's so-called reduction thesis claiming that all many-sided relations may be reduced to a combination of three-sided relations, but that one- and two-sided relations may not, on the other hand, combine to form genuine three-sided relations which consequently gets a fundamental role.

THE THREE CATEGORIES IN DIAGRAMS

We already commented upon the continuous sheet as basic for diagrams. The three categories also, however, inform the signs further used as constituents of diagrams. Let us as an example take Peirce's 'existential graphs' of which the Alpha version charts propositional logic, the Beta part first order predicate logic, and the unfinished Gamma part different aspects of modal logic, temporal logic, speech act logic, etc. In the Beta Graphs, the continuous sheet admits the following sign types: predicates (typically in verbal form) may be directly written on the sheet. They form propositions by being attached to subjects which are indicated by identity lines, at the other end(s) of which an index for the subject involved may appear. The end of an identity line merely states that such a subject exists, and the line may branch and connect to various predicates. Predicates having up to three places may also connect to other identity lines. Propositions occurring side by side on the sheet are linked with a conjunction. The last sign type is the cut, a connecting line severing part of the sheet from the rest. The diagram parts inside that cut are negated.[28] Two cuts, one appearing inside the other, are called a Scroll which forms a material implication. Certain rules govern the graphs: it is allowed to write a true proposition everywhere on the blank sheet or in evenly enclosed cuts, while it is allowed to add any proposition in unevenly enclosed cuts. Graphs not transcending cuts may be iterated or deleted. Double cuts with nothing in between may be deleted or inserted.[29] The basics of the Beta Graphs are most easily understood from an example ('Prolegomena to an Apology for Pragmaticism', 1906, 4.569):

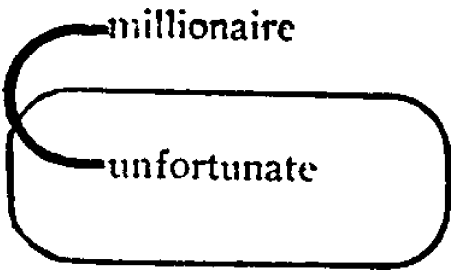

Figure 6.

This graph says that there exists a millionaire who is not unfortunate. The outermost end of the identity line functions as a quantifier claiming the existence of an object, the label identifies it as a millionaire. The innermost end of the identity line connects this subject to the predicate 'unfortunate', while the cut denies that predication. What interests us here is Peirce's category motivation for the inventory of signs used in the Graphs. In his 1898 lectures, not long after the invention of the Graphs, he says:

> In the system of graphs may be remarked three kinds of signs of very different natures. First, there are the verbs, of endless variety. Among these is the line signifying identity. But second the ends of the

line of identity (and *every* verb ought to [be] conceived as having such loose ends) are signs of a totally different kind. They are demonstrative pronouns, indicating existing objects, not necessarily material things, for they may be *events*, or even *qualities*, but still objects, merely designated as *this* or *that*. In the third place, the writing of verbs side by side, and the ovals enclosing graphs not asserted but subjects of assertion, which last is continually used in mathematics and makes one of the great difficulties of mathematics, constitute a third, entirely different kind of sign. Signs of the first kind represent objects in their Firstness, and give the significations of the terms. Signs of the second kind represent objects as existing, – and therefore as reacting, – and also in their reactions. They contribute the *assertive* character to the graph. Signs of the third kind represent objects as representative, that is in their Thirdness, and upon them turn all the inferential processes. In point of fact, it was considerations about the categories which taught me how to construct the system of graphs. ('Detached Ideas Continued', NEM IV, 339)

The three categories here permits Peirce to distinguish signs (1) referring to qualities and verbs, that is, polyvalent predicates, (2) referring to subjects related to those predicates, and (3) referring to logical relations like the operators of conjunction and negation, and, more implicitly, quantifiers which appear in the system due to the system of cuts modifying a basic existential quantification of the sheet. The three sign types given here relate to the term-proposition-argument triad in the way that the former are simply terms or rhemes, while the two latter are what may be added to rhemes in order to constitute propositions and arguments, respectively. We should thus expect these sign types to appear as typical instruments of different diagrams, representing qualities, subjects, and the relations between those, respectively.

To Peirce, it was an ideal to use signs representing these phenomena as iconically as possible.[30] He thus saw his logic graphs with their continuous depiction of continuous states-of-affairs as superior to his own earlier algebra of logic (which through Schröder and Peano developed into actual symbolic logic notation): all diagrams are based on the continuum, but still, the more continuous a diagram, the better.