Chapter 4

MHD and the Solar Dynamo

In this chapter we will explain the basic MHD equations which are needed to understand solar active phenomena such as spots, prominences, flares etc. The solar dynamo is needed to maintain the solar activity cycle[1].

We sometimes mix the unit system. This was done by intention because in some cases it is easier and more intuitive to deviate from the SI system.

Let us give a simple example. The Coulomb law is defined as[2]:

$$F = k\frac{q_1 q_2}{r^2} \tag{4.1}$$

In the SI system $k = 1/4\pi\epsilon_e; \epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}$. In the cgs system k=1.

4.1 Solar Magnetohydrodynamics

4.1.1 Basic Equations

To understand the surface activity of the Sun and the solar cycle it is necessary to briefly outline the principles of MHD. The properties of electromagnetic fields are described by Maxwell's equations:

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial \mathbf{t}} \tag{4.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4.3}$$

$$\mathrm{div}\mathbf{B} = 0 \tag{4.4}$$

$$\mathrm{div}\mathbf{D} = \rho_\mathrm{E} \tag{4.5}$$

Here $\mathbf{H}, \mathbf{B}, \mathbf{D}, \mathbf{E}, \mathbf{j}, \rho_\mathrm{E}$ are the magnetic field, magnetic induction, electric displacement, electric field, electric current density and electric charge density.

[1]See also e.g. Advances in Solar System Magnetohydrodynamics, E.R. Priest, A. W. Hood, Cambridge, 1991

[2]it defines the force acting between two charges q_1, q_2

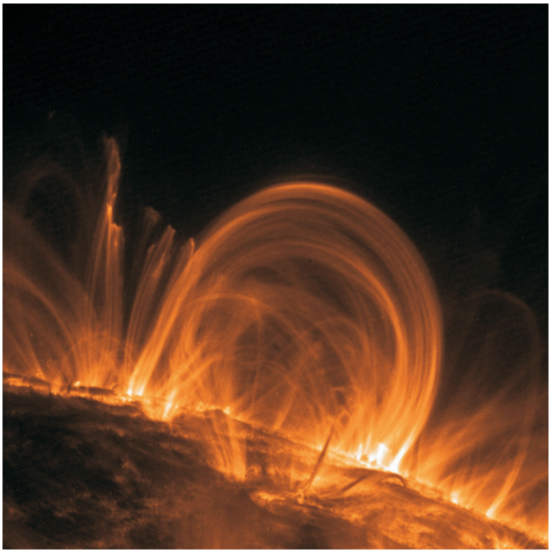

Figure 4.1: Looped magnetic field lines in the solar chromosphere and corona. Photo: TRACE mission, NASA

Equation 4.2 is the Ampere law which states that a spatially varying magnetic field (given by $\nabla \times \mathbf{H}$) produces currents- in MHD often the variation of the $\mathbf{E}$ fields is neglected, thus the term $\frac{\partial \mathbf{D}}{\partial t} \rightarrow 0$. We can also state that a current $\mathbf{j}$ induces a magnetic field that is in a direction opposite to it.

Equation 4.3 is the Faraday law and states that a time varying magnetic field produces an electric field.

If μ_0, ϵ_0 are the permeability and permittivity of free space, then for most gaseous media in the universe:

$$\mathbf{B} = \mu_\mathbf{0}\mathbf{H} \qquad \mathbf{D} = \epsilon_\mathbf{0}\mathbf{E} \tag{4.6}$$

The following equation relates the electric current density to the fields producing it (generalized Ohm's law):

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{4.7}$$

σ is the electrical conductivity and $\mathbf{u}$ is the bulk velocity of the matter. The final equations depend on the state of matter; if it consists of electrons and one type of ion:

$$\mathbf{j} = n_i Z_i e \mathbf{u_i} - n_e e \mathbf{u_e} \qquad \rho_\mathrm{E} = n_i Z_i e - n_e e \tag{4.8}$$

$n_i, \mathbf{u_i}, n_e, \mathbf{u_e}$ are the number density and velocity of the ions and electrons respectively and $Z_i e, -e$ are the charges on the ion and the electron.

In astrophysics two simplifications are applied:

- magnetic fields are treated as permanent,
- electric fields are regarded as transient.

The third Maxwell equation (4.4) states that there are no magnetic monopoles[3]. Electric fields can be produced by separating positive and negative charges through the fourth Maxwell equation (4.5) however the attraction between these charges is so strong that charge separation is usually cancelled out very quickly. Through the second Maxwell equation electric fields can be produced by time varying magnetic fields. Such fields are only significant, if there are rapid changes by time varying magnetic fields. Magnetic fields produced by the displacement current $\partial\mathbf{D}/\partial t$ are usually insignificant in astrophysical problems because electric fields are unimportant; however they can be produced by a conduction current $\mathbf{j}$, if the electrical conductivity is high enough. Such magnetic fields may be slowly variable in time and space.

We therefore neglect $\partial\mathbf{D}/\partial\mathbf{t}$, combine the equations

$$\nabla\times\mathbf{H}=\mathbf{j} \qquad \nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial\mathbf{t} \qquad \mathbf{B}=\mu_{\mathbf{0}}\mathbf{H} \qquad \mathbf{j}=\sigma\mathbf{E}, \tag{4.9}$$

obtaining

$$\frac{\partial\mathbf{B}}{\partial t}+\frac{1}{\mu_o\sigma}\nabla\times\nabla\times\mathbf{B}=\mathbf{0} \tag{4.10}$$

and using $\nabla\times\nabla\times\mathbf{B}=\mathrm{grad}\,\mathrm{div}\mathbf{B}-\nabla^{\mathbf{2}}\mathbf{B}$ and $\mathrm{div}\mathbf{B}=\mathbf{0}$:

$$\frac{\partial\mathbf{B}}{\partial t}=\frac{1}{\mu_0\sigma}\nabla^2\mathbf{B} \tag{4.11}$$

This is also called the *induction equation*- for the static case. In cartesian coordinates this equation for the x coordinate is:

$$\frac{\partial B_x}{\partial t}=\frac{1}{\mu_0\sigma}\left[\frac{\partial^2 B_x}{\partial x^2}+\frac{\partial^2 B_y}{\partial y^2}+\frac{\partial^2 B_z}{\partial z^2}\right] \tag{4.12}$$

The solution of these equations shows that magnetic fields decay together with the current producing them. We can derive an approximate decay time: let us assume that currents vary significantly in distance L, then from (4.11) the decay time becomes

$$\tau_{\mathrm{D}}=\mu_0\sigma L^2 \tag{4.13}$$

If at time $t=0$ there exists a sinusoidal field

$$B_x=B_0\exp(iky) \tag{4.14}$$

the solution at a later time t is:

$$B_x=B_0\exp(iky)\exp(-k^2t/\mu_0\sigma) \tag{4.15}$$

[3]This is a common experience: a division of a permanent magnet into two does not separate north and south poles.

The wavelength λ of the spatial variation of the field is $2\pi k$, the original field decays by a factor e in the time $\mu_0\sigma\lambda^2/4\pi^2$.

Let us consider typical fields of stars: the dimension of the star L and the electrical conductivity are both high (if the gas is fully ionized). Therefore, the lifetime of a magnetic field could exceed the main sequence lifetime, such a field is called a *fossil field*.

The same is not true for the Earth. Its field is produced by currents in a liquid conducting core and continuously regenerated by a *dynamo mechanism*.

The electrical conductivity of an ionized gas is $\sim T^{3/2}$. That means that the characteristic time for decay of currents in the outer layers of the Sun is much less than the solar lifetime, whereas the decay near the center exceeds the lifetime (since the temperature near the surface is about 6 000 K and near the center about 1.5×10^7 K). If the field in the solar interior were a fossil field extending throughout the Sun, the field in the outer layers would now be current free - similar to the field of a dipole. However we don't observe this. The surface field is very complex and therefore it must be also regenerated by a dynamo. It is conceivable that a fossil field of the Sun was destroyed at the very early evolution of the Sun, when it was fully convective before reaching the main sequence. Also helioseismology argues against a strong field.

4.1.2 Some Important MHD Effects

A magnetic field in a conducting fluid exerts a force per unit volume which is

$$\mathbf{F}_{\text{mag}} = \mathbf{j} \times \mathbf{B} = (\text{curl}\mathbf{B} \times \mathbf{B})/\mu_0 = -\text{grad}(B^2/2\mu_0) + \mathbf{B}.\nabla\mathbf{B}/\mu_0 \tag{4.16}$$

This can be interpreted as:

- $\text{grad}(B^2/2\mu_0)$ isotropic pressure,
- $\mathbf{B}.\nabla\mathbf{B}/\boldsymbol{\mu_0}$ tension along the lines of magnetic induction.

The isotropic pressure must be added to the gas pressure: let us assume we have a tube of magnetic flux, and P_{out} denotes the pressure outside and P_{in} the pressure inside the tube, then for equilibrium:

$$P_{\text{out}} = P_{\text{in}} + B^2/2\mu_0 \tag{4.17}$$

The gas pressure can be written as $\Re\rho T/\mu$, where $\Re$ is the gas constant and μ the mean molecular weight. With $T_{\text{in}} = T_{\text{out}}$ we must have:

$$\rho_{\text{in}} < \rho_{\text{out}} \tag{4.18}$$

A tube of magnetic flux is lighter than its surroundings and will start to rise which is called *magnetic buoyancy*.

To a good approximation, the fluid is tied to the magnetic field. For the Sun two extremes occur:

- photosphere: the fluid motions drag the magnetic field lines around (the magnetic field is frozen in);

- corona: the magnetic force is so strong that it constrains the motion of the fluid.

The tying of the fluid to the magnetic field lines also permits the propagation of MHD waves which have some similarity to sound waves but a characteristic speed (Alfvén speed):

$$c_{\mathrm{H}} = \sqrt{B^2/\mu_0\rho} \tag{4.19}$$

The sound speed is given by

$$c_{\mathrm{s}} = \sqrt{\gamma P/\rho} \tag{4.20}$$

This can also be seen from the induction equation. Let us consider again the Maxwell equations. From $\mathbf{j} = \sigma\mathbf{E} + \mathbf{u}\times\mathbf{B}$ we can extract $\mathbf{E}$:

$$\mathbf{E} = \frac{\mathbf{j}}{\sigma} - \mathbf{u}\times\mathbf{B} \tag{4.21}$$

This is substituted into the Maxwell equation (4.3) yielding:

$$\nabla\times\left(\frac{\mathbf{j}}{\sigma} - \mathbf{u}\times\mathbf{B}\right) = -\frac{\partial\mathbf{B}}{\partial t} \tag{4.22}$$

We have already argued that the displacement current can be neglected in the first Maxwell equation and therefore $\nabla\times\mathbf{B} = \mu\mathbf{j}$, from which $\mathbf{j} = \mathbf{1}/\mu\nabla\times\mathbf{B}$ and

$$\nabla\times\left(\frac{1}{\mu\sigma}\nabla\times\mathbf{B} - \mathbf{u}\times\mathbf{B}\right) = -\frac{\partial\mathbf{B}}{\partial t} \tag{4.23}$$

using the formula

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\mathbf{A}) - \nabla^{\mathbf{2}}\mathbf{A} \tag{4.24}$$

from vectoranalysis, we get:

$$\nabla\times\left(\frac{1}{\mu\sigma}\nabla\times\mathbf{B}\right) = \frac{1}{\mu\sigma}\left[\nabla(\nabla\mathbf{B}) - \nabla^{\mathbf{2}}\mathbf{B}\right]$$

This gives us the final form of the so called *induction equation*:

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \eta\nabla^{\mathbf{2}}\mathbf{B} \tag{4.25}$$

Here $\eta = 1/\mu\sigma$ is the magnetic diffusivity. The case where the plasma is stationary was already discussed above ($\mathbf{u} = \mathbf{0}$), the field decays in the ohmic decay time $\tau = L^2/\eta$. Let us discuss the case when $\eta = 0$. Then, the field $\mathbf{B}$ is completely determined by the plasma motions $\mathbf{u}$ and the induction equation is the equivalent to the vorticity equation for an inviscid fluid. The magnetic flux Φ through a material surface S which is a surface that moves with the field, is:

$$\Phi = \int_S \mathbf{B}.\mathbf{dS} \tag{4.26}$$

If G is the material closed curve bounding S, the total rate of change of Φ is (see also eq. 4.34):

$$\frac{D\Phi}{Dt} = \int_S \frac{\partial \mathbf{B}}{\partial t}.d\mathbf{S} + \oint_{\mathbf{G}} \mathbf{B}.(\mathbf{u} \times \mathbf{dl}) \tag{4.27}$$

$$= \int_S \frac{\partial B}{\partial t}.d\mathbf{S} + \oint_{\mathbf{G}} (\mathbf{B} \times \mathbf{u}).\mathbf{dl} \tag{4.28}$$

$$= \int_S \left[\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u} \times \mathbf{B})\right].d\mathbf{S} \tag{4.29}$$

In the last equation we have used the Stokes Theorem:

$$\int_L (A)\mathbf{Cdl} = \int_{\mathbf{A}} (\nabla \times \mathbf{C})\mathbf{dA}' \tag{4.30}$$

If $\eta = 0$ the total flux across any arbitrary surface moving with the fluid remains constant, the magnetic field lines are said to be *frozen* in to the flow.

If v_0, l_0 are typical velocity and length-scale values for our system, then the ratio of the two terms on the right hand side of the induction equation gives the *Magnetic Reynolds Number*

$$R_m = l_0 v_0 / \eta_0 \tag{4.31}$$

In an active solar surface region one has $\eta_0 = 1\,\mathrm{m}^{-2}\mathrm{s}^{-1}$, $l_0 = 700\,\mathrm{km} \sim 1\,\mathrm{arcsec}$ and $v_0 = 10^4\,\mathrm{m/s}$ we find $R_m = 7 \times 10^9 >> 1$. Thus the field is frozen to the plasma and the electric field does not drive the plasma but is simply $\mathbf{E} = -\mathbf{u} \times \mathbf{B}$. However, if the length-scales of the system are reduced the diffusion term $\eta \nabla^2 \mathbf{B}$ becomes important. Then the field lines are allowed to diffuse through the plasma and this yields to magnetic braking and changing the global topology of the field (magnetic reconnection).

4.1.3 Magnetic Reconnection

Magnetic reconnection is the process by which lines of magnetic force break and rejoin in a lower energy state. The excess energy appears as kinetic energy of the plasma at the point of reconnection. In Fig. 4.2 single arrow lines denote magnetic field and double line arrows the magnetofluid velocity. As it can be seen, the merging of two magnetofluids with oppositely oriented magnetic fields causes the field to annihilate. The excess energy accelerates the plasma out of the reconnection region in the direction of the full double line arrows. Note the characteristic X-point, where the topology changes for the field lines.

The plasma, where the field is annihilated is accelerated outwards to Alfvén speed v_A:

$$v_A = B_0 / \sqrt{4\pi M n_B} \tag{4.32}$$

n_B... density inside the current sheet, M the plasma average molecular weight.

A similar process occurs in coronal loops that were observed in hard and soft x-rays by Yohkoh and SOHO instruments. Such a coronal loop (see right drawing in Fig. 4.2) is stretched out by pressure which is provided by buoyancy. A magnetic

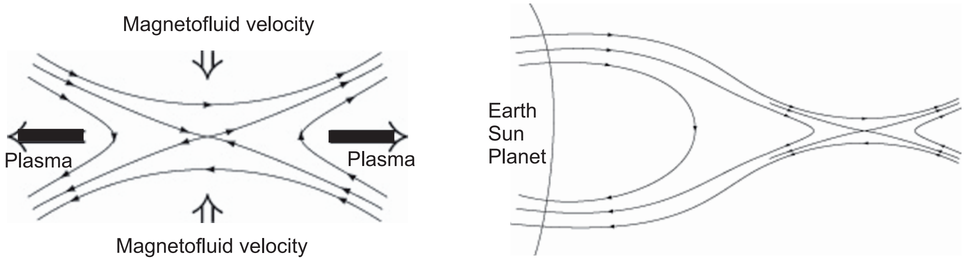

Figure 4.2: Principle of magnetic reconnection.

structure is buoyant because the particle density is lower there since it contains larger magnetic energy density (see magnetic buoyancy). Thus the external pressure is balanced by a lower gas pressure in conjunction with a magnetic pressure. The top of the loop distends and reconnection occurs. Particles in the reconnection region accelerate towards the surface of the sun and out away. Those particles that are accelerated towards the sun are confined within the loop's magnetic field lines and follow these lines to the footpoint of the loop where they collide with other particles and lose their energy through x-ray emissions. Such processes are the cause of solar flares and will be discussed in the next chapter.

Magnetic reconnection also provides a mechanism for energy to be transported into the solar corona.

A similar process occurs in the earth's magnetotail. The solar wind distends the Earth's dipole field so that the field extends far behind the Earth. Earthward flowing plasma streams with flow velocities up to 1000 km/s (which is close to the local Alfvén speed) have been observed (Birn *et al.* 1981 [37]).

A recent review on solar MHD was given by Walsh (2001) [329].

4.1.4 Fluid Equations

The continuity or mass equation for a fluid is:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla.\mathbf{u} = \mathbf{0} \tag{4.33}$$

and the total derivative means here:

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \mathbf{u}.\nabla \tag{4.34}$$

(See any textbook on fluid dynamics for a derivation of this formula). Now let us consider the equation of motion in a plasma with velocity $\mathbf{u}$: the momentum equation includes the Lorentz force term $\mathbf{j} \times \mathbf{B}$ and other forces $\mathbf{F}$, such as gravity and viscous forces:

$$\rho\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F} \tag{4.35}$$

Here p is the plasma pressure. Let us assume a Newtonian fluid with isotropic viscosity, then $\mathbf{F}$ may be written as:

$$\mathbf{F} = -\rho \mathbf{g}(\mathbf{r})\frac{\mathbf{r}}{\mathbf{r}} + \rho\nu\nabla^{\mathbf{2}}\mathbf{u} \tag{4.36}$$

$g(r)$ is the local gravity acting in the radial direction and ν the kinematic viscosity. Let us make thinks more complicated: Consider a frame of reference with angular velocity Ω at a displacement $\mathbf{r}$ from the rotation axis:

$$\rho\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \mathbf{j}\times\mathbf{B} + \mathbf{F} + \rho\left[2\mathbf{u}\times\mathbf{\Omega} + \mathbf{r}\times\frac{d\mathbf{\Omega}}{dt} + \frac{1}{2}\nabla|\mathbf{\Omega}\times\mathbf{r}|^2\right] \tag{4.37}$$

The three terms in [ ] denote: Coriolis force, change of rotation and centrifugal force. Stars rotate more rapidly when they are young. Under most circumstances the latter two terms are small compared with the Coriolis term $\mathbf{u}\times\mathbf{\Omega}$.

4.1.5 Equation of State

The perfect gas law

$$p = \frac{k\rho T}{m} = nkT \tag{4.38}$$

determines the constitution of stars, $k = 1.38\times 10^{-23}$ J/K being the Boltzmann constant, m is the mean particle mass and n the number of particles per unit volume. If s denotes the entropy per unit mass of the plasma, then the flux of energy (heat) through a star becomes:

$$\rho T\frac{\mathrm{D}s}{\mathrm{D}t} = -L \tag{4.39}$$

L is the energy loss function. This function describes the net effect of all the sinks and sources of energy. For MHD applications this becomes:

$$\frac{\rho^\gamma}{\gamma-1}\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{p}{\rho^\gamma}\right) = -\nabla.\mathbf{q} + \kappa_r\nabla^2 T + \frac{j^2}{\sigma} + H \tag{4.40}$$

In this equation we have:

- $\mathbf{q}$: heat flux due to conduction
- κ_r: coefficient of radiative conductivity
- T temperature
- j^2/σ ohmic dissipation (Joule heating)
- H represents all other sources.

4.1.6 Structured Magnetic Fields

If the plasma velocity is small compared with the sound speed ($\sqrt{\gamma p/\rho}$), the Alfvén speed ($\sqrt{B/\mu\rho}$) and the gravitational free fall speed ($\sqrt{2gl}$), the inertial and viscous terms in equation 4.35 may be neglected yielding:

$$0 = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F} \tag{4.41}$$

This equation must then be solved with $\nabla \times \mathbf{B} = ...$, $\nabla.\mathbf{B} = \mathbf{0}$ and the ideal gas law as well as a simplified form of the energy equation.

Let us introduce the concept of *scale height*. Let

$$0 = -\frac{dp}{dz} - \rho g \tag{4.42}$$

Substitute in the above equation $\rho = pm/kT$ (ideal gas) and integrate:

$$p = p_0 \exp\left[-\int_0^z \frac{dz}{H_p(z)}\right] \tag{4.43}$$

(p_0 is the pressure at $z = 0$). This defines the local pressure scale height H_p:

$$H_P = kT/mg = p/\rho g \tag{4.44}$$

At solar photospheric temperatures ($T \sim 5000\,\mathrm{K}$) we find $H_p = 0.150\,\mathrm{Mm}$, whereas at coronal temperatures $T \sim 10^6\,\mathrm{K}$ we find $H_p \sim 30\,\mathrm{Mm}$.

That concept can also be applied to MHD in the case of magnetostatic balance discussed above. Assume that gravity acts along the negative z direction and s measures the distance along the field lines inclined at angle θ to this direction, then the component of eq. (4.41) in the z-direction becomes:

$$0 = -\frac{dp}{ds} - \rho g \cos\theta \qquad dz = ds\cos\theta \tag{4.45}$$

Therefore, the pressure along a given field line decreases with height, the rate of decrease depends on the temperature structure (given by the energy equation).

If the height of a structure is much less than the pressure scale height, gravity may be neglected. The ratio β is given by gas pressure p_0 to magnetic pressure $B_0^2/2\mu$. If $\beta << 1$, any pressure gradient is dominated by the Lorentz force and (4.41) reduces to:

$$\mathbf{j} \times \mathbf{B} = \mathbf{0} \tag{4.46}$$

In this case the magnetic field is said to be *force free*. In order to satisfy (4.46) either the current must be parallel to $\mathbf{B}$ (Beltrami fields) or $\mathbf{j} = \nabla \times \mathbf{B} = \mathbf{0}$. In the latter case the field is a current free or potential field.

If β is not negligible and the field is strictly vertical of the form $\mathbf{B} = \mathbf{B}(\mathbf{x})\mathbf{k}$, then (4.41) becomes:

$$0 = \frac{\partial}{\partial x}\left[p + \frac{B^2}{2\mu}\right] \tag{4.47}$$

4.1.7 Potential Fields

Potential fields result when $\mathbf{B}$ vanishes. We can write $\mathbf{B} = \nabla \mathbf{A}$ so that $\nabla \times \mathbf{B} = \mathbf{0}$; with $\nabla . \mathbf{B} = \mathbf{0}$ one obtains Laplace's equation:

$$\nabla^2 \mathbf{A} = \mathbf{0} \tag{4.48}$$

If the normal field component B_n is imposed on the boundary S of a volume V, then the solution within V is unique. Also if B_n is imposed on the boundary S, then the potential field is the one with the minimum magnetic energy.

These two statements have many implications for the dynamics of the solar atmosphere. During a solar flare e.g. the normal field component through the photosphere remains unchanged. However, since enormous amounts of energy are released during the eruptive phase, the magnetic configuration cannot be potential. The excess magnetic energy could arise from a sheared force-free field.

Let us consider an example of a potential field in two dimensions: Consider the solutions $A(x, z) = X(x)Z(z)$ such that $\nabla^2 A = 0$ gives:

$$\frac{1}{X}\frac{d^2X}{dx^2} = -\frac{1}{Z}\frac{d^2Z}{z^2} = -n^2 \tag{4.49}$$

where $n = const$. A solution to (4.49) would be:

$$A = \left(\frac{B_0}{n}\right)\sin(nx)e^{-nz} \tag{4.50}$$

this gives for the field components:

$$B_x = \frac{\partial A}{\partial x} = B_0 \cos(nx)e^{-nz} \tag{4.51}$$

$$B_z = \frac{\partial A}{\partial z} = -B_0 \sin(nx)e^{-nz} \tag{4.52}$$

The result is a two dimensional model of a potential arcade.

4.1.8 3 D Reconstruction of Active Regions

If we look at an active region on the solar disk center we have no information about the 3 D structure of it, especially about the 3 D magnetic field configuration which is important for modelling such regions. Information about the height dependence of active regions can only be obtained when observing such features near the solar limb. Let us consider some simple model to reconstruct these features.

$$B_x = \frac{\partial A}{\partial z},\ B_y(x, z),\ B_z = -\frac{\partial A}{\partial x} \tag{4.53}$$

We see immediately that $\nabla . \mathbf{B} = \mathbf{0}$ Let us assume that the footpoints of the field are anchored down into the photosphere (z=0). Projecting the resulting field onto the xz plane gives:

$$\frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial z}dz = 0 \tag{4.54}$$

Therefore $dA = 0, A = const$. From $\nabla \times \mathbf{B} = \mu \mathbf{j}$ calculate the components of the current density:

$$j_x = -\frac{1}{\mu}\frac{\partial B_y}{\partial z} \tag{4.55}$$

$$j_y = \frac{1}{\mu}\left(\frac{\partial B_y}{\partial z} - \frac{\partial B_y}{\partial x}\right) \tag{4.56}$$

$$j_z = \frac{1}{\mu}\frac{\partial B_y}{\partial x} \tag{4.57}$$

And then the components of the Lorentz force ($\mathbf{j} \times \mathbf{B}$):

$$\nabla^2 A \frac{\partial A}{\partial x} + B_y \frac{\partial B_y}{\partial x} = 0 \tag{4.58}$$

$$\frac{\partial B_y}{\partial x}\frac{\partial A}{\partial z} - \frac{\partial A}{\partial x}\frac{\partial B_y}{\partial z} = 0 \tag{4.59}$$

$$\nabla^2 A \frac{\partial A}{\partial z} + B_y \frac{\partial B_y}{\partial z} = 0 \tag{4.60}$$

4.1.9 Charged Particles in Magnetic Fields

In this chapter we consider first the motion of a single charged particle in a given electromagnetic field. The particle has charge q and the equation of motion is:

$$m\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{4.61}$$

Let us write:

$$\begin{aligned} \mathbf{B_0} &= B_0 \mathbf{b} \\ \mathbf{E_0} &= E_{\|}\mathbf{b} + \mathbf{E}_{\perp} \\ \mathbf{v} &= v_{\|}\mathbf{b} + \mathbf{v}_{\perp} \end{aligned}$$

where $\mathbf{b}$ is a unit vector and $\mathbf{v} \times \mathbf{B_0} = \mathbf{B_0}(\mathbf{v}_{\perp} \times \mathbf{b})$ is perpendicular to $\mathbf{b}$. Equation 4.61 splits into a parallel and a perpendicular component (e.g. parallel means parallel to the magnetic field lines):

$$m\frac{\mathrm{d}v_{\|}}{\mathrm{d}t} = qE_{\|} \tag{4.62}$$

$$m\frac{dv_{\perp}}{dt} = q[E_{\perp} + B_0(v_{\perp} \times \mathbf{b})] \tag{4.63}$$

Equation 4.62 has the solution

$$v_{\|} = (qE_{\|}/m)t + v_{\|0} \tag{4.64}$$

Here $v_{\|0}$ is the velocity component at $t = 0$ in the direction of the magnetic field line. We see that particles of opposite sign of charge q move in opposite directions, they move along an electric field parallel to a magnetic field which destroys $E_{\|}$.

Now let us solve equation 4.63 by writing:

$$\mathbf{v}_\perp = \mathbf{v}'_\perp + \mathbf{E}_\perp \times \mathbf{b}/\mathbf{B_0} \tag{4.65}$$

and equation 4.63 becomes:

$$m\frac{\mathrm{d}\mathbf{v}'_\perp}{\mathrm{d}t} = qB_0(\mathbf{v}'_\perp \times \mathbf{b}) \tag{4.66}$$

Summarizing we arrive at:

- Motion in which acceleration is $\perp$ to the velocity,
- const. acceleration to the velocity.

This is a motion in a circle around the direction of **b** and the motion has frequency $|q|B_0/m$, the magnitude of the velocity is $v_{\perp 0}$; the radius of the orbit, the gyration radius r_g is

$$r_g = mv_{\perp 0}/|q|B \tag{4.67}$$

For an electron the gyration frequency is 1.8×10^{11}(B/Tesla)Hz. The corresponding gyration radius is $6 \times 10^{-9}(v_{\perp 0}/\text{ km/s})(B/\text{Tesla})$m.

The difference in mass between electrons and protons is about 1800. What follows for the radius and frequency of gyration?

The gyration follows also from the simple statement, that in the absence of other forces, the Lorentz force balances the centripetal force of the particle's motion around the field line. Let α be the pitch angle, which is the angle between the direction of motion and the local field line. Then for the gyro radius (Larmor radius :

$$R_L = \frac{cp_\perp}{qB} = \left|\frac{mc\mathbf{v} \times \mathbf{B}}{qB^2}\right| \tag{4.68}$$

The Larmor radius for a 100 keV electron (which is typical for electrons in the inner radiation belt of the Earth) is about 100 m.

Let us assume, that the magnetic flux through a particle's orbit is constant- this is certainly the case when changes of the magnetic field are small over the gyro radius and one gyro period. From the condition that $d\Phi_B/dt = 0$ the so called first adiabatic invariant follows:

$$\mu_B = \frac{p_\perp^2}{2mB} \tag{4.69}$$

or in terms of the particle's energy:

$$\mu_B = \frac{E\sin^2\alpha}{B} \tag{4.70}$$

From the conservation of the first adiabatic invariant it follows, that the pitch angle increases, when the particle moves to larger field strength, until $\alpha = 90^0$ at the mirror point.

Summarizing the motion of a particle:

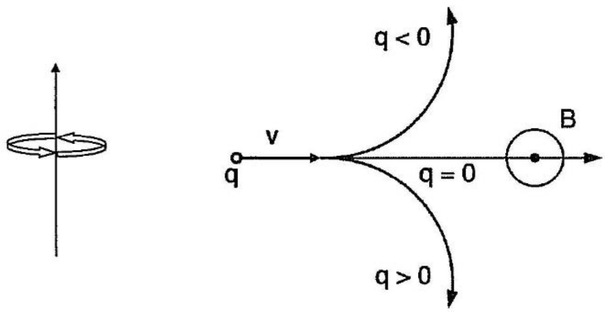

Figure 4.3: Left: Gyration of a charged particle around magnetic field lines, the sense depends on the + or - charge; right: charged particles are deflected by magnetic fields, depending on their charge. Here the magnetic fieldlines point vertically inwards, denoted by $\odot$

- accelerated motion along the field lines,
- circular motion around the field,
- drift velocity $\mathbf{E}_\perp \times \mathbf{b}/\mathbf{B_0}$ perpendicular to both electric and magnetic fields,
- the sense of the accelerated and the circular motions depends on the sign of the electric charge.
- the drift velocity is the same for all particles,
- in the absence of electric fields, a particle moves with a constant velocity in the direction of the magnetic field and with a velocity of constant magnitude around the field, thus it moves along a helical path.
- In all this discussion we have neglected one important effect. Accelerated charged particles radiate, for non relativistically moving particles this radiation is known as *cyclotron radiation* and for relativistic particles as *synchrotron radiation.*

Finally, if there is a constant non magnetic force $\mathbf{F}$ perpendicular to $\mathbf{B}$, there is a drift velocity:

$$\mathbf{v}_{\mathrm{DF}} = \frac{c\mathbf{F} \times \mathbf{B}}{qB^2} \tag{4.71}$$

Please note that again $\mathbf{v}_{\mathrm{DF}}$ is charge dependent.

Let us give some examples for drift motions:

- Gradient $\mathbf{B}$ drift: the field strength in planetary magnetospheres decreases with increasing distance from the planet and the gradient in the field strength induces a force which can be written as:

$$\mathbf{F} = -\mu_B \nabla B \tag{4.72}$$

and the drift velocity:

$$\mathbf{v_B} = \frac{\mu_B c\mathbf{B} \times \nabla B}{qB^2} \tag{4.73}$$

→ Particles move perpendicular to the lines of force and perpendicular to the magnetic gradient. Protons and electrons drift in opposite direction. In the Earth's magnetic field, electrons drift in the eastward direction. This drift motion causes a current system known as ring current. The ring current strengthens the field on its outside, helping expand the size of the magnetosphere but weakens the magnetic field in its interior. The ring current plasma population is enhanced in a magnetic storm. A southward oriented IMF (Bz negative) leads to reconnection at the side of the magnetosphere to the sun and the magnetopause is pushed closer to the Earth. Also injection of particles from the tail occurs. This enhances the number of particles. For a typical 100 keV electron or proton it takes about 5-6 hours to complete one orbit drift.

- field line curvature drift: particles move along curved field lines. The guiding center follows the curved field line and the resulting centripetal force is equal to:

$$\mathbf{F_c} = \frac{m\mathbf{v_\parallel^2}}{R_c}\mathbf{n} \tag{4.74}$$

where $\mathbf{n}$ is a unit vector outwards. The drift motion is perpendicular to the field line's radius of curvature and the field line itself.

- gravitational field drift. Let us assume that $\mathbf{F}$ is the gravitational force $\mathbf{F} = m\mathbf{g}$, then

$$\mathbf{v}_{\mathrm{DF}} = m\mathbf{g} \times \mathbf{B}/qB^2 \tag{4.75}$$

Thus the drift velocity depends on the mass/charge ratio, the ion drift is much larger than the electron drift; the particles drift in opposite directions, a current is produced.

- electric field drift: here the force is

$$\mathbf{F} = q\mathbf{E} \tag{4.76}$$

And the drift velocity

$$\mathbf{v}_E = \frac{c\mathbf{E} \times \mathbf{B}}{B^2} \tag{4.77}$$

Thus, charged particles move in a direction perpendicular to a) $\mathbf{E}$ and b) $\mathbf{B}$. Protons and electrons move in the same direction.

We stress here, that the $\nabla\mathbf{B}$ and $\mathbf{E} \times \mathbf{B}$ drift and curvature forces dominate the drift motions of particles in a magnetosphere.

Let us consider a large assembly of particles; these particles interact which is called collision. If τ_c is the characteristic time between collisions the collision frequency is $\nu_\mathrm{c} = 1/\tau_\mathrm{c}$. If ν_c is large, the particle motions will be disordered and decoupled from the magnetic field, the fluid will not be tied to the field. If collisions are relatively rare, not only individual particles but the whole fluid will be tied to the field. The collisions provide the electrical resistivity of matter; in a fully ionized gas a good approximation to the value of the electrical conductivity is:

$$\sigma = n_e e^2 \tau_c / m_e \tag{4.78}$$

and $\tau_c \sim T^{3/2}$.

4.1.10 MHD Waves

The equation that describes the connections between the force exerted by the magnetic field and the fluid motions is

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\mathrm{grad}P + \mathbf{j} \times \mathbf{B} + \rho \mathrm{grad}\phi \tag{4.79}$$

The forces on the gas are the gas pressure P, the gravitational potential ϕ and the magnetic force $\mathbf{j} \times \mathbf{B}$. For a full description of the system we write down two additional equations:

a) equation of continuity (conservation of mass)[4]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \mathrm{div}\mathbf{v} = \mathbf{0} \tag{4.80}$$

b) The relation between P and ρ e.g. in the adiabatic form

$$\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\gamma}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} \tag{4.81}$$

Consider the simplest case: a medium with uniform density ρ_0, pressure P_0, containing a uniform magnetic field $\mathbf{B_0}$. We ignore the influence of the gravitational field and assume that σ is so large that $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0}$. Now let us assume a perturbation for any variable in the form of:

$$f_1 \sim \exp i(\mathbf{k}.\mathbf{r} - \omega t) = \exp i(k_x x + k_y y + k_z z - \omega t) \tag{4.82}$$

$\mathbf{k}$ is the wave vector, ω the wave frequency. The dispersion relation between ω and $\mathbf{k}$ when in the absence of a magnetic field is:

$$\omega^2 = k^2 c_s^2 \tag{4.83}$$

Therefore, in that case only one type of waves can propagate – sound waves. The wave propagates through the fluid at the wave speed $c_s = \omega/k$, $k = |\mathbf{k}|$, which is called the phase velocity of the wave.

If there is a magnetic field, the force $\mathbf{j} \times \mathbf{B}$ couples to the equation and also the Maxwell equations must be taken into account. It is very important to note that the magnetic field introduces a preferred direction into the system. In a uniform medium, sound waves travel equally strongly in all directions from its source, this is not true for MHD waves. If we write the magnetic field again in the form $\mathbf{B_0} = \mathbf{B_0}\mathbf{b}$, then we find three types of MHD waves:

[4]Note that d/dt is the rate of change with time following a fluid element moving with velocity $\mathbf{v}$: $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathbf{v}.\mathrm{grad}$

- Alfvén waves : the dispersion relation is given by

$$\omega^2 = (\mathbf{k}.\mathbf{b})^2 c_{\mathrm{H}}^2 \tag{4.84}$$

- fast and slow magnetosonic waves; their dispersion relation is given by:

$$\omega^4 - \omega^2 k^2 (c_{\mathrm{s}}^2 + c_{\mathrm{H}}^2) + k^2 (\mathbf{k}.\mathbf{b})^2 c_{\mathrm{s}}^2 c_{\mathrm{H}}^2 = 0 \tag{4.85}$$

Let us consider two special cases: if the waves propagate along the field $\mathbf{k}.\mathbf{b} = \mathbf{k}$, there are two waves with $\omega^2 = k^2 c_{\mathrm{H}}^2$ and the sound wave $\omega^2 = k^2 c_{\mathrm{s}}^2$ unaffected by the field. For wave propagation perpendicular to the field only one wave survives with $\omega^2 = k^2(c_{\mathrm{s}}^2 + c_{\mathrm{H}}^2)$. When waves propagate anisotropically, it is necessary to introduce another wave velocity in addition to the phase velocity, the group velocity, given by $\partial\omega/\partial\mathbf{k}$ with which the wave carries energy or information. The group velocity for Alfvén waves is always $c_{\mathrm{H}}\mathbf{b}$. What does that mean? Regardless of the direction in which it propagates, energy always travels along the field lines with speed c_{H}.

4.1.11 Magnetic Fields and Convection

Let L be the length of a box, v a typical velocity. The magnetic diffusivity is $1/\mu_0\sigma$, the eddy turnover time L/v. The resistive decay time is then $\mu_0\sigma L^2$ and the resistive decay time/eddy turnover time is denoted as magnetic Reynolds number for the flow.

$$R_{\mathrm{m}} = Lv\mu_0\sigma \tag{4.86}$$

If the magnetic Reynolds number R_{m} is very low, the field is unaffected by the motions, if it is high, it is wound up many times before dissipation occurs. For an intermediate value of R_{m} the magnetic field is carried from the center of the eddy becoming concentrated in flux ropes at the edge. This buoyant flux ropes rise towards the surface and this leads to the appearance of sunspots. However we must also take into account that convection involves different length scales. Large eddies affect the overall structure of the magnetic field as it has been just described. Others may be influenced e.g. granulation. Granulation is suppressed in a sunspot. As it was shown earlier, in the absence of magnetic field convection occurs in a gas, if the ratio of the temperature gradient to the pressure gradient satisfies the relation:

$$\frac{P}{T}\frac{\mathrm{d}T}{\mathrm{d}P} > \frac{\gamma - 1}{\gamma} \tag{4.87}$$

If a vertical magnetic field of strength B threads the fluid, then this has to be modified to:

$$\frac{P}{T}\frac{\mathrm{d}T}{\mathrm{d}P} > \frac{\gamma - 1}{\gamma} + \frac{B^2}{B^2 + \gamma P} \tag{4.88}$$

Thus a strong magnetic field can prevent convection and a weaker field can interfere with convection. Note also that the magnetic field cannot prevent motions which are oscillatory up and down the field lines but these are likely to be less efficient at carrying energy.

4.2 The Solar Dynamo

So far we have discussed the different aspects of solar activity. In the section on MHD it was shown that due to dissipation, such recurrent phenomena on the solar surface and atmosphere cannot be explained by just assuming a fossil magnetic field of the Sun. Therefore, many attempts had been made in order to explain the recurrent solar activity phenomena such as sunspots, their migration toward the equator in the course of an activity cycle etc. In the first section of this paragraph we will give a general description of the basic dynamo mechanism, in the following chapter some formulas are given.

4.2.1 The Solar Dynamo and Observational Features

Let us briefly recall what are the observational facts that a successful model for the solar dynamo must explain:

- 11 year period of the sunspot cycle; not only the number of sunspots varies over that period but also other phenomena such as the occurrence of flares, prominences,.... etc.
- the equator-ward drift of active latitudes which is known as Spörers law and can be best seen in the butterfly diagram. At the beginning of a cycle active regions appear at high latitudes and toward the end they occur near the equator.
- Hale's law: as we have mentioned the leader and the follower spot have opposite polarities. This reverses after 11 years for each hemisphere so that the magnetic cycle is in fact 22 years.
- Sunspot groups have a tilt towards the equator (this is sometimes also called Joy's law).
- Reversal of the polar magnetic fields near the time of the cycle maximum.

As we know from fundamental physics, magnetic fields are produced by electric currents. How are these currents generated in the Sun? The solar plasma is ionized and it is not at rest. There are flows on the solar surface as well as in the solar interior producing magnetic fields which contribute to the solar dynamo.

4.2.2 The $\alpha - \omega$ Dynamo

The ω Effect

Let us consider magnetic fields inside the Sun. There the conditions require that the field lines are driven by the motion of the plasma. Therefore, magnetic fields within the Sun are stretched out and wound around the Sun by differential rotation (the Sun rotates faster at the equator than near the poles). Let us consider a north-south orientated magnetic field line. Such a field line will be wrapped once around the Sun in about 8 months because of the Sun's differential rotation (Fig. 4.4).

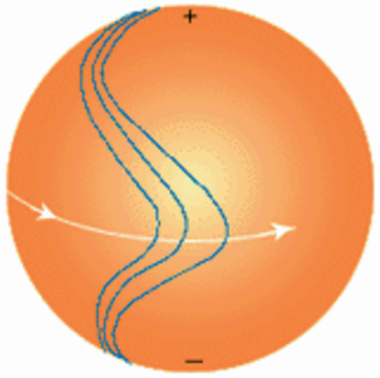

Figure 4.4: Illustration of the ω effect. The field lines are wrapped around because of the differential rotation of the Sun

The α Effect

However, the field lines are not only wrapped around the Sun but also twisted by the Sun's rotation. This effect is caused by the coriolis force. Because the field lines become twisted loops, this effect was called α effect. Early models of the dynamo assumed that the twisting is produced by the effects of the Sun's rotation on very large convective flows that transport heat to the Sun's surface. The main problem of that assumption was, that the expected twisting is too much and would produce magnetic cycles of only a couple of years. More recent dynamo models assume that the twisting is due to the effect of the Sun's rotation on rising flux tubes. These flux tubes are produced deep within the Sun.

The Interface between Radiation Zone and Convection Zone

If dynamo activity occurs throughout the entire convection zone the magnetic fields within that zone would rapidly rise to the surface and would not have enough time to experience either the alpha or the omega effect. This can be explained as follows: a magnetic field exerts a pressure on its surroundings ($\sim B^2$, proportional to its strength). Therefore, regions of magnetic fields will push aside the surrounding gas. This produces a bubble that rises continuously to the surface. However such a buoyancy is not produced in the radiation zone below the convection zone. Here, the magnetic bubble would rise only a short distance before it would find itself as dense as its surroundings. Consequently, it is assumed that magnetic fields are produced at this interface layer between the radiation zone and the convection zone.

Helioseismology has established the existence of a layer of strong gradients of angular velocity at the base of the solar convection zone. This layer, having a thickness of about 0.019 $R_\odot$, the *tachocline*, separates the convection zone exhibiting a strong latitudinal differential rotation from the radiative interior that rotates almost rigidly. Turbulence generated in the tachocline is likely to mix material in

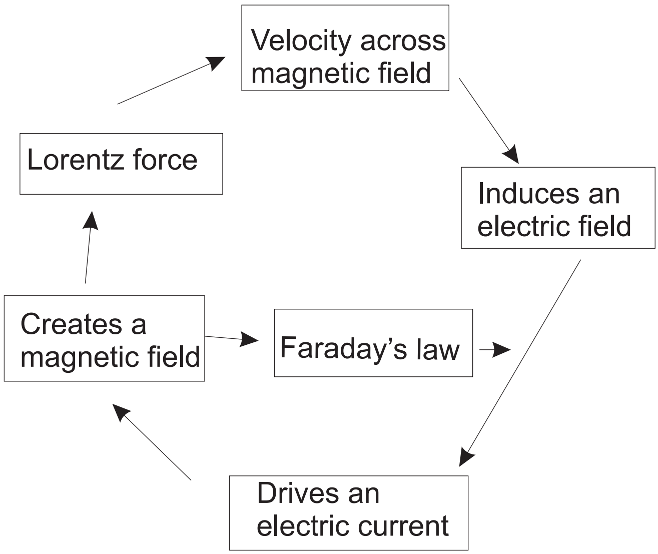

Figure 4.5: The MHD relation between flows and magnetic fields

the upper radiative zone resulting in the observed deficit of Li and Be. Gilman, 2005 [112] wrote a summary about the tachoclyne stressing its importance for in situ generation of poloidal fields as well as creating magnetic patterns that are seen on the surface.

The Meridional Flow

The solar meridional flow is a flow of material along meridional lines from the equator toward the poles at the surface and from the poles to the equator deep inside. At the surface this flow is in the order of 20 m/s, but the return flow toward the equator deep inside the Sun must be much slower since the density is much higher there- maybe between 1 and 2 m/s. This slow plasma flow carries material from the polar region to the equator in about 20 years.

Thus the energy that drives the solar dynamo comes from a) rotational kinetic energy, b) another part in the form of small-scale, turbulent fluid motions, pervading the outer 30% in radius of the solar interior (the convection zone).

4.2.3 Mathematical Description

Let us discuss some basic mathematics. In the magnetohydrodynamic limit the dynamo process is described by the induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta_{\mathbf{e}} \nabla \times \mathbf{B}) \tag{4.89}$$

The flow $\mathbf{u}$ is a turbulent flow. In the mean-field electrodynamics one makes the following assumptions: magnetic and flow fields are expressed in terms of a large-scale mean component and a small scale fluctuating (turbulent) component. If we average over a suitably chosen scale we obtain an equation that governs the evolution of the mean field. This is identical to the original induction equation but there appears a mean electromotive force term associated with the (averaged) correlation between the fluctuation velocity and magnetic field components. The basic principles of mean field electrodynamics were given by Krause and Rädler(1980) [174]. The velocity and the field are expressed as:

$$\mathbf{u} = <\mathbf{u}> + \mathbf{u}' \qquad \mathbf{B} = <\mathbf{B}> + \mathbf{B}' \tag{4.90}$$

$<\mathbf{u}>$, $<\mathbf{B}>$ represent slowly varying mean components and $\mathbf{u}'$, $\mathbf{B}'$ non axisymmetric fluctuating components. The turbulent motion $\mathbf{u}'$ is assumed to have a correlation time τ and a correlation length λ which are small compared to the scale time t_0 and scale length l_0 of the variations of $<\mathbf{u}>$ and $<\mathbf{B}>$. In other words, τ is a mean time after which the correlation between $\mathbf{u}'(\mathbf{t}=\tau)$ and $\mathbf{u}'(\mathbf{t}=\mathbf{0})$ is zero and λ is comparable to the mean eddy size. We assume that $<\mathbf{u}'>, <\mathbf{B}'> = \mathbf{0}$.

This is substituted into the induction equation and subtracted from the complete equation:

$$\frac{\partial \mathbf{B}'}{\partial t} = \nabla \times (<\mathbf{u}> \times \mathbf{B}' + \mathbf{u}' \times <\mathbf{B}> + \mathbf{G}) - \nabla \times (\eta \nabla \times \mathbf{B}') \tag{4.91}$$

where

$$\mathbf{E} = <\mathbf{u}' \times \mathbf{B}'> \qquad \mathbf{G} = \mathbf{u}' \times \mathbf{B}' - <\mathbf{u}' \times \mathbf{B}'> \tag{4.92}$$

$\mathbf{E}$ is a mean electric field that arises from the interaction of the turbulent motion and the magnetic field. This field must be determined by solving the equation for $\mathbf{B}'$ and here several assumptions are made. First of all we stressed that $<\mathbf{v}'> = \mathbf{0}$. This may be a good assumption when considering a fully turbulent velocity field. However in the Sun we are dealing with a sufficiently ordered convective field where the Coriolis force plays an important role. The other approximation is a first order smoothing: $\mathbf{G} \sim \mathbf{0}$. That is valid only if $\mathbf{B}' << <\mathbf{B}>$. Then our equation reduces to:

$$\frac{\partial \mathbf{B}'}{\partial t} + \nabla \times (\eta \nabla \times \mathbf{B}') = \nabla \times (\mathbf{u}' \times <\mathbf{B}>) \tag{4.93}$$

We want to determine $\mathbf{E}$. Thus only $\mathbf{B}''$ the component of $\mathbf{B}'$ which is correlated with $\mathbf{u}'$ must be considered. By definition τ, $\mathbf{B}(\mathbf{t}+\tau)$ is not correlated with $\mathbf{B}(\mathbf{t})$ for any t. $\mathbf{B}''(\mathbf{t})$ may be determined by integration of the above equation from $t-\tau$ to t. Note also, that the order of the convective turn over time $\tau \sim \lambda/v$ and thus both $\mathbf{u}'$ and $<\mathbf{B}>$ may be regarded as independent of t. Thus the integration yields:

$$E_i = \alpha_{ij} < B_{ij} > + \beta_{ijk} \frac{\partial < B_j >}{\partial x_k} \tag{4.94}$$

where α_{ij}, β_{ijk} depend on the local structure of the velocity field and on τ. If the turbulent field is isotropic, then $\alpha_{ij} = \alpha\delta_{ij}$, $\beta_{ij} = \beta\epsilon_{ijk}$, and

$$\mathbf{E} = \alpha < \mathbf{B} > -\beta\nabla\times < \mathbf{B} > \tag{4.95}$$

If τ is small compared to the decay time λ^2/η, the diffusive term may be neglected and from 4.93 we get

$$\alpha = -\frac{1}{3}\tau < \mathbf{u}'.\nabla\times\mathbf{u}' >, \qquad \beta = \frac{\mathbf{1}}{\mathbf{3}}\tau\mathbf{v^2} \tag{4.96}$$

And finally:

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\alpha\mathbf{B} + \mathbf{u}\times\mathbf{B}) - \nabla\times[(\eta+\beta)\nabla\times\mathbf{B}] \tag{4.97}$$

Compared to the normal induction equation, this contains the term $\alpha\mathbf{B}$ and the eddy-diffusivity coefficient β. In the mean field dynamo, the magnetic diffusivity η is replaced by a total diffusivity $\eta` = \eta + \beta$ and the equation becomes:

$$\frac{\partial B}{\partial t} = \nabla\times(\alpha\mathbf{B} + \mathbf{u}\times\mathbf{B}) + \eta`\nabla^{\mathbf{2}}\mathbf{B} \tag{4.98}$$

Please note that most often the prime is dropped on η; however, in the presence of α it is implied to use the turbulent diffusivity. It is assumed that $\mathbf{B}$ is axisymmetric. Then it can be represented by its poloidal and toroidal components $A(x,z,t)$ and $B(x,z,t)$ and $\mathbf{u} = \mathbf{u}(\mathbf{x},\mathbf{z},\mathbf{t})\mathbf{j}$. Neglecting the advection terms:

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)B = [\nabla u\times\nabla A].\mathbf{j} - \alpha\nabla^{\mathbf{2}}\mathbf{A} \tag{4.99}$$

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)A = \alpha B \tag{4.100}$$

Note that the dynamo action is possible because we have a regeneration of both toroidal and poloidal fields. Let us consider the source term in the first of the two above equations. ∇u describes a non uniform rotation. It can be argued that this term is larger than the next term involving α. This set of equations then describes the so called $\alpha - \omega$-dynamo. The equations describe:

- ω effect: the poloidal field is sheared by non uniform rotation to generate the toroidal field.

- α effect: this is the essential feedback. The helicity $\mathbf{v_c}.\nabla\times\mathbf{v_c}$ of the non axisymmetric cyclonic convection generates an azimuthal electromotive force $\mathbf{E}$ which is proportional to the helicity and to B_ϕ.

Let us define a characteristic length scale l_0, a decay time $t_0 = l_0^2/\eta$ and $u = s_0\omega$, where s_0 is of the order of the local radius of rotation and ω the local angular velocity. We may rewrite the above equations in terms of the non dimensional variables $t' = t/t_0$ and $\mathbf{r}' = \mathbf{r}/\mathbf{l_0}$. By an elimination of B and neglecting the α^2 terms we arrive at

$$\left(\frac{\partial}{\partial t'} - \nabla'^2\right)^2 A = \frac{\alpha l_0^2 s_0}{\eta^2}[\nabla`\omega\times\nabla`A].\mathbf{j} \tag{4.101}$$

If α_0 and ω_0 are scale factors giving the orders of magnitudes of α and $|\nabla`\omega|$ then

$$\left(\frac{\partial}{\partial t'} - \nabla'^2\right)^2 A = D\frac{\alpha}{\alpha_0}\left[\frac{\nabla'\omega}{\omega_0} \times \nabla' A\right].\mathbf{j} \tag{4.102}$$

In that equation the non dimensional *dynamo number* D is

$$D = \frac{\alpha\omega_0 l_0^2 s_0}{2\eta^2} \tag{4.103}$$

It is extremely important to note that the onset of a dynamo action depends on D. If D for a given system exceeds some critical value than there will be dynamo action. Examining our set of equations we may also note that dynamo action is possible when ∇u is negligible compared to α. Such dynamos are called α^2 dynamos. If both terms of the source term are comparable then we speak of an $\alpha^2\omega$ dynamo.

Solar like stars have well developed and structured convection zones. Thus, the $\alpha - \omega$ dynamo is the most likely dynamo mode.

Reviews on the solar dynamo and the emergence of magnetic flux at the surface can be found in Ossendrijver, 2003 [239], Fisher *et al.* (2000) [97] and Moreno-Insertis (1994) [226].

So far we have discussed large dynamos which are invoked to explain the origin of the solar cycle and of the large scale component of the solar magnetic field. We should add here that the origin of small scale magnetic fields can also be understood in terms of dynamo processes. Recent advances in the theory of dynamo operating in fluids with high electrical conductivity – fast dynamos, indicate that most sufficiently complicated chaotic flows should act as dynamos (Cattaneo, 1999 [57]). The existence of a large scale dynamo is related to the breaking of symmetries in the underlying field of turbulence (Cattaneo, 1997 [56]).

Steiner and Ferriz-Mas, 2005 [300], showed how solar radiance variability might be connected to a deeply seated flux-tube dynamo.

Observations form SOHO

Near the base of the convection zone the analysis of solar oscillations (data from the SOHO/MDI) has shown that there exist variations in the rotation rate of the Sun. A successive acceleration and deceleration with a strange period of 1.3 years was found near the equator and 1.0 years at high latitudes. The largest temporal changes were found both above and below the 'tachocline', a layer of intense rotational shear at the interface between the convection zone and the radiation zone (see Spiegel and Zahn, 1992 [297]). The variations near the equator are strikingly out of phase above and below the tachocline, and involve changes in rotation rate of about 6 nHz, which is a substantial fraction of the 30 nHz difference in angular velocity with radius across the tachocline. The solar magnetic dynamo is thought to operate within the tachocline, with the differential rotation there having a crucial role in generating the strong magnetic fields involved in the cycles of solar activity. This is illustrated in Fig. 4.6.

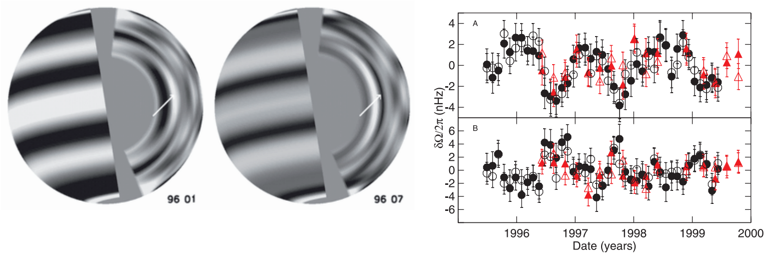

Figure 4.6: a) Cutaway images of solar rotation showing a peak and a trough of the 0.72R variation, with black indicating slow rotation, grey intermediate, and white fast. b) Variations with time of the difference of the rotation rate from the temporal mean at two radii deep within the Sun, with the site at 0.72 $R_\odot$ located above the tachocline and that at 0.63 $R_\odot$ below it, both sampling speeding up and slowing down in the equatorial region. Results obtained from GONG data for two different inversions are shown with black symbols, those from MDI with red symbols. (Image courtesy NSF's National Solar Observatory)

4.2.4 Solar Activity Prediction

Generally, prediction of solar activity is related to the problem of prediction of a given time series since solar activity parameters such as sunspot numbers are given as a function of time. Therefore, the problem can be examined on the basis of recent nonlinear dynamics theories. The solar cycle is very difficult to predict due to the intrinsic complexity of the related time behavior and to the lack of a successful quantitative theoretical model of the Sun's magnetic cycle. Sello (2001) [278] checked the reliability and accuracy of a forecasting model based on concepts of nonlinear dynamical systems applied to experimental time series, such as embedding phase space, Lyapunov spectrum, chaotic behavior. The model is based on a local hypothesis of the behavior on embedding space, utilizing an optimal number of neighbor vectors to predict the future evolution. The main task is to set up and to compare a promising numerical nonlinear prediction technique, essentially based on an inverse problem, with the most accurate prediction methods, like the so-called "precursor methods" which appear now reasonably accurate in predicting "long-term" Sun activity, with particular reference to "solar" and "geomagnetic" precursor methods based on a solar dynamo theory.

Snodgrass (2001) [289] studied azimuthal wind bands known as the *torsional oscillations* . These have been revealed primarily by studying the longitudinally averaged solar rotation over a period spanning several full solar rotations. This averaging yields what look like broad but slow, oppositely-moving (∼5 m/s) bands lying to either side of the centroid of the sunspot butterfly, making the activity band appear to be a zone of weakly enhanced shear. The torsional pattern tells us something about the cycle, and since it precedes the onset of activity, it might be useful as a predictor of the level of activity to come. For cycle 23, the torsional pattern did not emerge until just before solar minimum, whereas for cycles 21 and 22 it appeared several years earlier. This would have suggested by 1996 the cycle 23 would be weaker than the previous two.

Calvo *et al.* (1995) [53] used the neural network technique to analyze the time series of solar activity (given by the Wolf number).

Hernandez (1993) [132] also used neural nets to construct nonlinear models to forecast the AL index (auroral electrojet index) given solar wind and interplanetary magnetic field (IMF) data.

Gleisner and Lundstedt (2001) [113] used a neural network-based model for prediction of local geomagnetic disturbances. Boberg *et al.* (2000) [38] made real time Kp predictions from solar wind data using neural networks.

4.3 Stellar Activity

4.3.1 Detection and Observation of Stellar Activity

The Sun is the only star that permits a two-dimensional study of its activity[5]. However, it is only a single set of stellar parameters, since its mass, composition and evolutionary status are fixed[6] . Stars are one-dimensional objects when observed from the Earth but they cover a wide range of physical parameters. Thus the solar-stellar connection is essential for a better understanding of solar phenomena as well as for stellar phenomena. In the 40s e.g. the solar chromosphere was thought to be unique.

Spectral line indicators for stellar activity are:

- EUV lines,
- Hα, He λ1083.0 nm,
- H and K lines of Ca II,
- Mg II.

The first detection of stellar activity phenomena was made by the observation of magnetic fields. Field strengths in the range of 1-2 kGauss can only be measured by a comparison of magnetically sensitive lines with magnetically insensitive lines. It was surprising that these stars seem to be covered by such strong fields about 20-80% of the total surface (the Sun is only covered $\leq$1%). The problem is, that by these methods coverages lower than 20% cannot be detected. That means that the Sun's magnetic field would not have been detected if it were at the distance of these stars. More than 100 years ago Pickering suggested that luminosity fluctuations in stars of the order of 20% over periods of days or a few weeks might indicate that they are spotted. In the 1970 extensive investigations were performed to explain luminosity variations of e.g. the RSCVn stars or BY Draconis stars (having luminosities $< 1/2\,L_\odot$). The observed lightcurves required circular spots. The RSCVn stars occur in binary stars were tidal interactions play an important role,

[5] A good textbook is C. Schrijver, Solar and Stellar Magnetic Activity Cambridge Astrophysics, 2000

[6] See also the new edition of Solar and Stellar Activity Cycles, P. R. Wilson, A. King, Cambridge 2004

therefore their starspots are quite different from the sunspots. BY Dra stars are rapidly rotating young low massive stars characterized by intense chromospheric emission. Large spots on the Sun cause a variation of the integrated flux $< 1\%$, whereas up to 30 % for RSCVn and BY Dra stars.

The size and extent of chromospheric active regions varies dramatically over the course of the activity cycle. Thus by measuring the H and K lines of other stars we can infer on stellar activity cycles.

How can we measure stellar parameters like differential rotation that play a key role in the onset of stellar dynamos? Let us assume we have a rapidly rotating spotted cool star and that it is observed one week apart. By comparing brightness/magnetic images of that star over such time intervals one can measure the rotation rates of starspots at different latitudes over several rotation cycles (Barnes *et al.* 2001) [26].

Also flares were detected on stars. Here it is extremely important to have observations in the EUV/X ray window. Generally pre main sequence stars show high levels of magnetic activity and strong flares. FU Orionis stars may be in a phase between T Tauri and post T Tauri stars. More details about that topic can be found in the review of Haisch *et al.* (1991) [124]. So far we have considered only stars which have an activity level by orders of magnitude larger than the Sun.

4.3.2 Stellar Activity Cycles

One of the programs that is being carried out since a long time is the HK project where the H and K activity of a large sample of stars is recorded. Almost 100 stars have been observed continuously since 1966; at present the project is monitoring long-term changes in chromospheric activity for approximately 400 dwarf and giant stars. In order to compare the data with the Sun, observations of reflected sunlight from the Moon are done at Mt. Wilson and at Sac Peak and Kitt Peak National Observatory. The sampling of the stars occurs rapidly: usually less than 10 min per star. The accuracy of the instrument is between 1% and 2%. When plotting the HK index against the $B - V$ color index (which is a measure for temperature as explained in chapter 1) then a clear trend can be seen. The HK index increases as the stellar temperature decreases. At this point one must be careful with the interpretation. It is not meant an absolute increase but a relative increase because in cooler stars also the continuum decreases.

In 1972 Skumanich [288] stated the $t^{-1/2}$ law for the time of stellar rotation and stellar chromospheric decay; the rotational velocity and the strength of the CaII emission of a late type star vary inversely with the square root of the star's age - *Skumanich law.* However later it was found that except massive T Tauri stars the majority of low mass stars rotates slowly.

It was also found that there exists a granulation boundary in the HRD at F5 III. Stars of later spectral type begin to develop a convective envelope that grows for the rest of their evolution. At the boundary these envelopes are extremely thin (only 3% of the star's radius). Stars on the right hand side in the HRD of the granulation border have smaller rotation rates.

In hydrodynamics, by definition, the *Rossby Number* is a ratio of inertial forces to the Coriolis Force for a rotating fluid. In astrophysics it is the ratio of the rotation period to the turnover time of the largest convective eddy. In stars with low Rossby numbers the rotation rate dominates the convective turnover time. The low Rossby number correlates well with the strong MgII 1940 emission. A low value of the Rossby number indicates a greater influence of the Coriolis forces. That means that the α effect becomes more important.

Stars can only be observed as point sources since we have no spatial resolution. Some stars show two simultaneous cycle periods. Other stars either have variable activity, or long trends in activity - longer than our 30-year baseline, or appear to be very inactive. For further details on that topics the book of Schrijver and Zwaan (2000) [274] is recommended where you find further references.

Dravins *et al.* (1993a) [79] made a detailed comparison of the current Sun (G2 V) with the very old solar-type star Beta Hyi (G2 IV) in order to study the post main-sequence evolution of stellar activity and of non thermal processes in solar-type atmospheres. This star has an age of 9.5 +/- 0.8 Gyr. The relatively high lithium abundance may be a signature of the early sub giant stage, when lithium that once diffused to beneath the main-sequence convection zone is dredged up to the surface as the convection zone deepens. Numerical simulations of the 3D photospheric hydrodynamics show typical granules to be significantly larger (a factor of about 5) than solar ones. The emission of the Ca H and K profiles was found to be weaker than that of the Sun. The observations suggest continuous changes in the chromospheric structure, rather than the sudden emergence of growth of active regions (Dravins *et al.*, 1993b [80])

Since several extrasolar planets have been found one should rise the question whether some of them might be suitable for life. Climatic constraints on planetary habitability were investigated by Kasting (1997) [153]. They found such zones around main sequence stars with spectral types in the early F to the mid K-range. The large amount of UV radiation emitted by early type stars poses a problem for evolving life in their vicinity. But there is also a problem with late-type stars; they emit less radiation at wavelengths < 200 nm which is required to split O_2 and initiate ozone formation. The authors show that Earth-like planets orbiting F and K stars may well receive less harmful UV radiation at their surfaces than does the Earth itself.