Chapter 4.4

SEISMIC RESPONSE OF CLASSICAL MONUMENTS WITH FRACTURED STRUCTURAL ELEMENTS

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- Abstract: Classical monuments, although made by stone or marble pieces placed one on top of the other without mortar, are stable against earthquakes. Their good seismic behaviour can be attributed to the sliding and rocking of the structural blocks during the strong ground shaking. Unfortunately, damages, which usually exist in such structures, significantly decrease this stability. Previous investigations on the dynamic response of classical columns showed that an initial inclination and/or corner cut-offs of drums may lead to collapse during a medium-size earthquake in spite of the fact that the structure has survived much stronger seismic events in the past. One type of damage, which is common in monuments, concerns fractures at the structural elements due to imperfections of the original material. During a strong earthquake, existing cracks open threatening the stability of the structure. In this paper, an investigation of the seismic response of monuments with fractured structural elements is presented. The distinct element method was used for the analysis and the model employed concerns a part of the Olympieion in Athens, Greece. The results show that the degree of the crack opening during an earthquake increases almost linearly with the peak velocity of the ground motion and the number of repetitions of the excitation. If significant shear and tensile strength exist at the crack interface, a stronger seismic excitation is required, in general, to cause failure. Cracks at column drums do not endanger the stability of the structure, unless they produce wedge-type pieces, which may slide during the earthquake.
- Key words: earthquake response; classical monuments; cracks; fractures; distinct element method.

1. INTRODUCTION

1.1 Dynamic response of classical monuments

Classical monuments are made of carefully fitted stones (drums in the case of columns), which lie on top of each other without mortar. The dynamic response is dominated by the spinal form of the construction and is governed by the sliding and the rocking of the individual stones, independently or in groups. Therefore, it is quite different than the response of 'typical' structures.

The overall behaviour is nonlinear and sensitive. These characteristics are evident even in the simplest case of a rocking rigid block. The latter, in spite of its apparent simplicity, is a complicated problem, which attracted the attention of researchers since the end of the 19th century. The first attempt for the analytical treatment of its dynamic response was presented by Housner¹ in 1963. In the following years, many investigators examined the problem analytically or experimentally producing an impressive amount of research on this subject, which continues up to date.

In the contrary, relatively few investigations have been presented on the dynamic response of stacks of rigid bodies, as it is the case of classical monuments (for a list see Papantonopoulos et al²). This is mainly due to the growing complexity of the behaviour as the number of blocks increases. In this case, analytical solutions can be obtained only in simple cases, as for example for two-block assemblies (e.g. Psycharis³). If many blocks are involved, it seems that the response can be calculated only by numerical approaches. In the present analysis, the distinct (or discrete) element method was employed.

The complexity of the seismic behaviour of classical monuments originates from the fact that the structure continuously moves from one 'mode' of vibration to another; different joints are opened and different poles of rotation apply for each mode. The term 'mode' is used here to denote different patterns of the rocking response (for an example see Figure 1) and does not refer to the eigenmodes of the system, since spinal structures do not possess natural modes in the classical sense and the period of free vibrations is amplitude dependent. Note that the number of the possible modes of vibration increases exponentially with the number of the individual stone elements. Although the motion can be approximated by linear equations during each mode (for small rotations), the transition from one mode to another makes the overall response nonlinear. One of the consequences of the nonlinearity is that a column may collapse under a certain earthquake motion and remain stable under the same excitation magnified by a value greater than one.

Another interesting characteristic of the response is its sensitivity even to trivial changes of the parameters of the system or the excitation. This sensitivity is apparent in both experimental and analytical results. For example, experiments on the seismic behaviour of a marble model of a column of the Parthenon (Mouzakis et al⁴) showed that "identical" experiments might pro-

duce significantly different results (Figure 2), due to uncontrolled perturbations in the initial geometry of the column and/or the shaking table motion. Another effect of the response sensitivity is the significant out-of-plane displacements recorded for purely planar excitations; in some cases, the deformation in the direction normal to the plane of the excitation was of the same order of magnitude with the principal deformation⁴.

During rocking, the pole of rotation of each block may move from one corner of the base to the other. This transition produces impact phenomena among adjacent structural elements and energy dissipation, causing a sudden decrease in the angular velocities. An equivalent coefficient of restitution can be determined, the value of which plays an important role to the response. Theoretical and experimental investigations (Aslam et al⁵) showed that the effect of this coefficient is not monotonic and that an increase in its value may decrease or increase the response in an unpredictable way. Note that in linear systems, increasing the coefficient of restitution always results in increasing the damping and decreasing the response.



Figure 1. Modes of rocking for two-block assemblies³.



Figure 2. Top displacement of a model of a classical column for two "identical" experiments (shaking table results, Mouzakis et al^4).

The vulnerability of classical monuments to earthquakes depends on two main parameters: the predominant period of the ground motion and the size of the structure. The former significantly affects the response and the possibility of collapse with low-frequency earthquakes being much more dangerous than high-frequency ones. In the first case, the response is characterised by intensive rocking; in the latter, significant sliding of the drums occurs, especially close to the upper part of the structure, while rocking is usually restricted to small values. This good seismic behaviour may be attributed to their large 'apparent' period, which increases with the amount of rocking. The size of the structure is another important parameter, with bulkier structures being much more stable than smaller ones of dimensions with the same aspect ratio.

1.2 Effect of existing damage

In spite of the lack of inter-connection among the stone elements, classical monuments in their intact condition are not, in general, vulnerable to 'usual' earthquake motions. As mentioned above, their large 'apparent' period and their large dimensions make them vulnerable only to long-period earthquakes. The energy dissipation, caused by rocking and sliding, has also a beneficial effect. This good seismic behaviour has been proved in practice, since many classical monuments are standing for more than 2000 years, although they are located in regions of extensive seismic activity, as Greece and Italy.

Unfortunately, imperfections are present in many monuments. They are caused by previous earthquakes, foundation failure, material deterioration and man interventions, as fire and vandalism. The most common imperfections are cut-off of drum corners, displaced drums, inclined columns and broken element stones. Previous analyses^{6,7} have shown that such imperfections reduce significantly the stability and can lead to collapse even for middle-size earthquakes. An example of the significant reduction of the stability, produced by imperfections, is shown in Figure 3.



Figure 3. Top displacement of a model of the Parthenon Pronaos column with and without imperfections, for the Aigion, Greece, 1995 earthquake, scaled to several values of PGA⁷.

In this paper, an investigation of the seismic response of a part of the Athens Olympicion is presented for several cases of fractures at the stone elements. Some of the cases examined are simplified representations of flaws that are displayed in the present state of the monument, while others are fictitious ones, aiming to the investigation of the effect of several parameters to the possibility of failure, as for example the position and the inclination of the crack.

2. NUMERICAL ANALYSIS

2.1 Model description

2.1.1 Geometrical data

In this paper, all the analyses were based on a numerical model of columns 7.5 and 7.6 of the SE corner of the Temple of Olympios Zeus (Olympieion) in Athens, Greece (see Figure 4a). The first digit of the column numbering refers to the row in the E-W direction, numbered from north to south, in which the column belonged in the original structure; the second one refers to the number of the column within the row, from east to west. The two columns considered here are linked with a three-beam marble architrave.

The total height of the columns is 16.81 m (one of the largest encountered in practice) including the base and the abacus. The main column is of varying diameter ranging from 1.92 m at the base to 1.57 m at the top. Under the column, a base drum of varying diameter from 2.51 to 1.92 m is placed and under it there is a square stone base. The capital is made from two pieces. The height of the architrave is 1.25 m, its width is 1.83 m and its length 5.50 m, equal to the axial distance of the columns.

The height of each drum is not constant, depending on the pieces of marble available at the site during the construction. Also, the number of drums varies from column to column. Thus, column 7.5 has 15 drums and column 7.6 has 14 drums. The drums are connected to each other by two steel dowels (randomly placed in the direction N-S or E-W) with a cross section varying from 9 to14 cm² and a length of about 12-14 cm (data supplied by M. Korres). Originally, connections also existed between the architrave beams which, in most cases, are missing today.

In the present condition of the monument, most drums of the columns are displaced by a few millimeters from their original position, especially close to the top. However, the most severe damage concerns a crack close to the middle of the span of the architrave and a clear almost vertical crack at drum #14 of column 7.6, right underneath the capital.



Figure 4. (a) View of the south-east corner of Olympieion of Athens (Toelle–Kastenbein⁸). Numerical analyses concern the two leftmost columns; (b) numerical model.

2.1.2 Numerical model

The numerical model was based on the actual geometry of the structure (Figure 4b). The drums were represented by polyhedral pyramidal segments of 22-sided cross section and varying diameter according to the original structure. The number of sides considered is equal to the number of flutes of the real columns. All structural elements (blocks) were considered rigid. The marble density was taken equal to 2700 kg/m^3 .

A Mohr-Coulomb constitutive model was adopted to describe the mechanical behaviour of the joints between adjacent structural elements. In the normal direction, the joint behaviour is governed by the normal stiffness coefficient, K_n , which relates the contact stress with the normal contact displacement. No tensile strength was considered, so this spring element is only active in compression. In the shear direction, an elasto-plastic stress-displacement law was assumed. The elastic range is characterised by the shear stiffness, K_s , while the shear strength is governed by the Coulomb friction coefficient, with no cohesive strength component.

The joint properties used are: $K_n=5.0\times10^9$ Pa/m, $K_s=1.0\times10^9$ Pa/m and friction angle $\varphi=36.87^\circ$ (equivalent friction coefficient, $\tan\varphi=0.75$). The values of the stiffness coefficients were proposed by Papantonopoulos et al.², based on the numerical reproduction of the model column experiments⁴. The value of the friction angle is typical for marble.

The steel dowels connecting adjacent drums were considered by special springs (two at each joint) with elasto-plastic behaviour. Since real dowels

do not offer any axial resistance and they practically act as shear connections, zero axial stiffness was assigned to these springs. The shear stiffness, K_s , at the elastic range was calculated assuming a cross section of 9 cm², shear modulus for steel $G=77\times10^6$ kPa and an active length of 6 cm, which lead to a value of $K_s=580000$ kN/m for each dowel. The shear strength was considered equal to 220 kN, which corresponds to a yield stress of 240 MPa.

2.2 Method of analysis

2.2.1 Discrete element modelling

As it was mentioned above, the deformation and failure of classical temples is governed by the relative movement of the blocks. For such structures, discontinuous models, in which the structure is considered as a block assemblage and the joints are represented explicitly, should be used.

The distinct (or discrete) element method was proposed by Cundall in the '70's in the context of rock mechanics and later extended to 3D problems^{9,10}, leading to the code 3DEC¹¹ used in the present study. This method provides the means to apply the conceptual model of a masonry structure as a system of blocks, either rigid or deformable. Block deformability may be taken into account by internal discretisation of blocks into finite elements. However, in the present study only rigid blocks were used, as they were found to provide a sufficient approximation and reduce substantially the run times. The system deformation, and thus the non-linear material behaviour, is concentrated at the joints, where frictional sliding or complete separation may take place. As discussed in more detail by Papantonopoulos et al.², the discrete element method employs an explicit algorithm for the solution of the equations.

The efficiency of the distinct element method and particularly of 3DEC to predict with satisfactory accuracy the seismic response of classical structures has been proved by comparison of numerical results with experimental data (Papantonopoulos et al.²). In that study, the experimental data were obtained from the shaking table response of a 1:3 scale model of a column of the Parthenon (Mouzakis et al.⁴). The experiments were reproduced numerically and it was proved that, in spite of the sensitivity of the phenomenon, the numerical analysis depicted with sufficient accuracy all the main features of the response, as the amplitude, the period and the residual displacements.

2.2.2 Damping

The comparison of the numerical results with the experimental data² showed that the introduction of damping in the numerical analysis reduces unreasonably the amplitude of the response during the strong shaking, leading

to an underestimation of the deformation. On the other hand, zero damping leads to a better estimation of the response during the earthquake, but it does not attenuate the vibrations fast enough after the end of the seismic motion. For this reason, in the present analyses zero damping was applied during the first 12 sec of the earthquake motion, but 10% of critical mass-proportional damping at 0.3 Hz was added after that time. The damping was further increased to 20% of critical for t>30 sec in order to stop the free vibrations and obtain the residual deformation accurately. Mass-proportional damping was chosen instead of stiffness-proportional one, because the latter required a smaller time step of integration and longer run-time.

2.2.3 Seismic input

The seismic action was applied to the base of the numerical model by prescribing the 2 horizontal components of the motion. The records used were based on the two horizontal components of the Kalamata, Greece, 1986 earthquake, which were normalized to several levels of peak ground velocity (PGV), the same in both directions. The ground velocity was chosen as a means of the normalization, because it gives a better representation of the ground motion characteristics than the peak acceleration (PGA), for the response of the structures under consideration.



Figure 5. Horizontal components of the Kalamata, Greece, 1986 earthquake.

The Kalamata earthquake was recorded on hard ground at a distance of about 9 km from the epicenter and its magnitude was Ms=6.2. The record samples the near field strong motion that caused considerable damage to the buildings of the city of Kalamata. The duration of the strong motion is about 6 sec and the maximum accelerations are 0.24 g in the N-S direction and 0.27 g in the E-W direction. The corresponding peak velocities are 32.0 and 23.5 cm/s, respectively. Figure 5 shows the two horizontal components of the earthquake.

3. ANALYSES FOR THE CRACK AT THE ARCHITRAVE

As mentioned above, a crack exists at the southern and the middle beam of the architrave of columns 7.5 and 7.6. The simplified representation of this crack, shown in Figure 6, was considered in the analysis. No crack was assumed at the northern beam. For this geometry, gravity loads produce a vertical displacement of about 6 mm at the crack interface. In the following, the term 'relative displacement' is used to denote the additional dislocation at the crack faces, caused by the earthquake excitation, excluding the initial one due to gravity. In most cases, only friction was assumed at the crack interface; the same value of friction angle, $\varphi=36.87^\circ$, which was used for the drum joints, was employed. In some cases, however, cohesion and tensile strength were also considered, in order to account for a semi-open crack.

An example of the results obtained is illustrated in Figure 7, in which the time-histories of the vertical displacement at the crack interface of the S beam, for the seismic action normalized to PGV=20 cm/s, are plotted. The value of the ground velocity considered corresponds to a medium-size earth-quake and the results show permanent displacements equal to 169 mm for the left piece and 63 mm for the right. The variation of the residual displacements with PGV is shown in Figure 8. In general, the stronger the seismic motion the larger is the dislocation at the crack. As shown in Figure 8, initially, the displacements increase almost linearly with PGV but then they start increasing exponentially and eventually the architrave collapses.



Figure 6. Geometry of the architrave crack considered in the analyses.



Figure 7. Absolute vertical displacement at the crack interface of the S beam of the architrave for the Kalamata earthquake normalized to PGV=20 cm/s.



Figure 8. Absolute residual vertical displacement at the crack interface of the S beam of the architrave versus the peak velocity of the seismic excitation.

Fractures at structural elements of classical monuments are due to imperfections of the original material and, thus, the adjacent pieces are bonded initially. However, the crack strength weakens with the time, due to material deterioration. In order to examine this phenomenon, runs were performed including shear (cohesion) and tensile strength at the crack faces. In all cases, the tensile strength was equal to 80% of the cohesion while the value of the latter was varying. Representative results are shown in Figure 9. These results were obtained for the seismic input normalized to PGV=10 and 20 cm/s. It is interesting to note that, up to a certain value of c, the permanent displacements are practically independent of the cohesion and the tensile strength. In such cases, the strength of the joint was exceeded during the earthquake, the crack opened and the two pieces behaved like they were not bonded. In this sense, weak zones or partially cracked elements will behave as fully cracked ones, if the crack breaks during the earthquake.

For larger values of c, the internal forces are not capable to break the crack. As illustrated in Figure 9, the limit value of c, for which the crack does not break, increases with the intensity of the seismic motion. Since failure is associated with the opening of the crack, collapse occurs under weaker ground shaking for smaller values of the cohesion and the tensile strength. This is shown in Figure 10, in which the minimum value of PGV, required to cause failure, is plotted versus the cohesion, c. For small values of c, the architrave collapses when PGV becomes equal to 30 cm/s, independently of the exact value of the cohesion. For values of c>1000 kPa, however, the larger the value of c the stronger is the earthquake which is required to cause failure.

If a second earthquake hits the monument, the existing damage increases. Thus, the architrave may collapse even for small earthquakes, if they are repeated a few times. In Figure 11, the time-histories of the vertical displacement of the two pieces of the S beam are shown for the seismic motion repeated five times. A rather small earthquake with PGV=10 cm/s was considered in this case. The cumulative effect of the repetition of the ground shaking to the residual vertical displacements at the crack is shown in Figure 12. It is seen that initially the displacements increase almost linearly with the times of repetition of the earthquake, but, after a few events they start to grow exponentially and eventually the architrave collapses during the 6th earthquake. Actually, as it is seen in Figure 13a, the S beam was already close to failure





Figure 9. Relative residual vertical displacement of the left piece of the S beam of the architrave versus the cohesion considered at the crack interface.

Figure 10. Variation of the minimum peak ground velocity, required to cause failure of the architrave, with the cohesion at the crack.



Figure 11. Absolute vertical displacement at the crack interface of the S beam of the architrave for the Kalamata earthquake normalized to PGV=10 cm/s, repeated 5 times.



Times of repetition of the earthquake motion

Figure 12. Relative residual vertical displacement at the crack interface of the S beam of the architrave versus the times of repetition of the seismic motion, for PGV=10 cm/s.



Figure 13. Residual displacements of the architrave: (a) after 5 repetitions and (b) after 10 repetitions of the Kalamata earthquake with PGV=10 cm/s.

after the 5th event. It is interesting to note that the rest of the structure remained stable even after 10 repetitions of the base motion (Figure 13b), although the middle beam of the architrave had the same crack with the S beam.

4. ANALYSES FOR CRACKS AT THE DRUMS

4.1 System with two columns and an architrave

As mentioned above, drum #14 of column 7.6 is split in two pieces by an almost vertical crack. In the present condition of the monument, the two pieces are dislocated showing an opening of the crack of about 106 mm in the N-S direction (data supplied by M. Korres). The effect of this crack to the stability of the structure is examined here. In the numerical model, the crack was considered vertical through the centre of the drum and forming an angle of 30° with the longitudinal axis of the structure (E-W direction).

Figure 14 shows the residual crack opening, caused by the seismic motion normalized to several values of PGV. For comparison, the existing crack opening is shown with a dashed line. The crack opening increases almost linearly with the peak velocity of the seismic motion; however, the existence of the crack does not seem to affect significantly the collapse of the structure, which occurs for PGV=110 cm/s, i.e. for a very strong earthquake.

The repetition of the seismic excitation increases the opening of the crack almost linearly. This is shown in Figure 15, in which the results for PGV=20 and 30 cm/s are shown. For PGV=20 cm/s, the structure collapses during the 6^{th} repetition of the earthquake. Failure starts from the architrave and extends to the E column while the W column with the broken drum remains standing. For PGV=30 cm/s, the structure remains stable even after 7 repetitions of the seismic action. This 'abnormal' behaviour is attributed to the non-linearity of the response, as discussed in the introduction.



Figure 14. Residual opening of the crack at drum #14 of column 7.6. The dashed line corresponds to the existing opening of the crack.



Times of repetition of the earthquake

Figure 15. Residual opening of the crack at drum #14 of column 7.6 versus the number of repetition of the seismic motion.

4.2 Single column

The analysis presented above showed that the vulnerability of the structure to earthquakes is not affected significantly by the presence of the crack. This happens because a vertical crack through the centre of the drum does not produce instability, even if it opens a few centimeters. In order to investigate the effect of the crack orientation to the stability of the structure, analyses were performed with various types of cracks.

For these analyses, only column 7.6 (left column in Figure 4b) was considered and the crack was placed at the bottom drum #1 of the column (instead of drum #14), in order its effect to be more pronounced. Three types of cracks were examined, as shown in Figure 16: (A) vertical crack through the centre of the drum; (B) inclined crack by 45° with the cut-off piece not forming a wedge; (C) inclined crack as in type B, but with the upper piece forming a wedge. The variation of the crack opening with the peak value of the ground velocity is shown in Figure 17 for the three types of cracks. It is evident that vertical cracks (type A) do not open significantly, even for strong earthquake motions. In this sense, they do not seem to be dangerous for the stability of the structure. Inclined cracks of type B also do not seem to increase the probability of collapse, although they may open a few centimeters if the structure



Figure 16. Types of cracks at drum #1 of the single column, considered in the analyses.

is exposed to strong seismic motions. It is interesting to note that the column does not collapse even for PGV=110 cm/s and shows a slightly better stability than crack type A, for which failure occurs for PGV=100 cm/s.

The danger of the crack increases exponentially if a wedge of type C is formed. As it is shown in Figure 17, in this case the column collapses at a much smaller earthquake with PGV=40 cm/s. It is obvious that the vulnerability increases with the size of the wedge, since the larger the sliding piece the larger is the loss of area of contact for the part of the column above the crack. In the present analysis, the loss of contact was almost equal to one half of the area of the drum (refer to Figure 16, type C).



Figure 17. Residual opening of the crack at drum #1 of the single column versus the PGV of the seismic motion.



Figure 18. Time history of the opening of the crack at drum #1 of the single column for the Kalamata earthquake with PGV=20 cm/s.

As it was expected, the repetition of the seismic motion produces a cumulative effect on the residual opening of the crack, increasing the failure danger. This is shown in Figure 18, in which the time-history of the crack opening for a sequence of five earthquakes with PGV=20 cm/s is shown for the crack types B and C.

5. CONCLUSIONS

In this paper, an investigation of the seismic behaviour of classical monuments with cracked structural elements is presented. The analyses were performed for the model of columns 7.5 and 7.6 of the Olympieion of Athens, Greece, which are connected with an architrave, using the distinct element method. The conclusions drawn can be summarized as follows:

Cracks at the architrave are dangerous, first to the architrave itself, which may collapse, and then to the whole structure, because the architrave beams can cause damage to the columns, if they hit them during their downfall. In case that the crack is not initially fully open and represents a weak zone of the material, the damage, which an earthquake will cause to the structure, depends on whether the crack will break or not; if it breaks, the damage will be similar to the one that would occur for an open crack. In general, the risk of collapse increases as the strength at the crack interface decreases.

Cracks at column drums do not seem to comprise an immediate threatening to the stability of the structure, as long as they do not form wedges, the sliding of which produces loss of contact for the upper part of the column. In the latter case, critical is the size of the piece that is in danger to slide.

The analyses show that cracks are expected to open during earthquakes. The residual opening increases almost linearly with the intensity of the ground motion. The repetition of the seismic excitation further increases the dislocation of the adjacent pieces in an almost linear way. In this sense, even cracks at drums, which do not form wedges and at first place are not alarming, may become dangerous if not repaired, since they may open extensively after a number of earthquakes.

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