Chapter 4.3

STRUCTURAL STABILITY OF HISTORIC UNDERGROUND OPENINGS IN ROCK
Two Case Studies from Israel

Yossef H. Hatzor[1] , Michael Tsesarsky[2] and Roni C. Eimermacher[3]

[1] Dept. of Geological and Environmental Sciences, Ben-Gurion University of the Negev, Beer-Sheva, 84105, Israel; [2]Faculty of Civil and Environmental Engineering, The Technion – Israel Institute of Technology, Haifa 3200, Israel; [3]Dept. of Maritime Civilizations, Haifa University, Haifa 31905, Israel

Abstract: In this paper the structural stability of the roof in two historic monuments excavated underground in different dimensions and rock types is discussed. An emphasis is made on stability analysis of roofs excavated in bedded and jointed rocks, referred to as laminated Voussoir beams. It is shown that the significance of joint friction, spacing and orientation can not be overlooked because these factors dictate roof stability. For accurate modeling of such a problem numerical methods must be employed. In this work the power of numerical discontinuous deformation analysis (DDA) method is demonstrated.

Key words: tunneling; rock mechanics; voussoir; DDA; old monuments.

1.   THE TEL BEER SHEVA SITE

1.1   Introduction

The 3000 year old underground water storage system, excavated at Tel Beer-Sheva in southern Israel, is discussed first. The rock-mass into which the reservoir is excavated is a horizontally bedded and vertically jointed, low strength, upper Cretaceous marly chalk, of the Gareb formation. In such a rock mass, prismatic blocks are expected to slide out of the loosened zone in the roof, up to a level where arching stresses are sufficiently high to interlock any further block displacement. The relevant stability issue for this monument is therefore the analysis of the arching mechanism in the immediate roof, which may be modeled as a layered and jointed beam, referred to as

laminated *Voussoir*[1]. Specifically, the height of the loosening zone and the magnitude of maximum compressive stress must be evaluated. Accurate determination of these factors enables determination of factors of safety against failure by crushing, shear along abutments, or buckling. Consequently, the required length and capacity of active support elements (rock bolts) can be designed. The problem is first solved using a semi-analytical solution for a *Voussoir* beam[2], and then it is solved numerically using the Discontinuous Deformation Analysis (DDA) method[3]. It is found that the joint spacing (*Sj*) and friction angle (ϕ), while ignored in the standard solution procedure[2] including later modifications[4-6], are of paramount importance in the correct modeling of the roof response to excavation. It is shown that the function ϕ_{req} (required joint friction angle for equilibrium) vs. *Sj* presents a minimum at which the arching mechanism is most effective. With joint spacing smaller or larger than the optimum, the arching mechanism becomes less effective, and failure by shear along the abutments prevails. Since the mean joint spacing in the site, $Sj_{,mean}$=25 cm, is much smaller than the optimal joint spacing value, it is concluded that the arching mechanism was not sufficiently effective during construction, as confirmed by the collapse of the entire center of the roof in historic times.

1.2   **Structural setting**

In the archeological site of Tel Beer Sheva, an ancient city dated back to the Late Iron stage 1,200 - 700 B.C. was explored (Figure 1). Modern excavation of the underground water system revealed that the roof of the main reservoir chamber had collapsed, probably during time of construction, and that the ancient engineers have erected a massive support pillar in the center of the chamber in order to support the remaining roof. The same plaster coating which was explored on the opening side walls at ground level was also discovered higher, above the level of the original roof, indicating the proximity of the failure episode to the original time of excavation.

The reservoir was excavated in horizontally bedded chalk with three vertical joint sets. The two most abundant sets are orthogonal with mean spacing of Sj=20 to 25 cm, and the mean bed thickness t = 50 cm. The intersection of these three joint sets creates a dense network of cubic blocks that form the roof of the excavation.

The roof collapsed into the shape of a three dimensional dome with three distinguishable structural zones[7]: zone 1 - the original roof level delimited by a vertical step with *t* = 50 to 125 cm; zone 2 - a sub horizontal plane parallel to an existing bedding plane delimited by a vertical step similar to boundary between zones 1 and 2; and zone 3 - the uppermost failure level which like zone 2 is parallel to an existing bedding plane. A structural map of the roof is shown in Figure 2. It can be seen that the center of the roof is comprised of zone 3 with a circular boundary, and that the external sections of the roof

are comprised of zone 1 - the original roof level. The support pillar was erected directly below zone 3, and extensions were built in order to support the unstable transitions from zones 3 to 2.
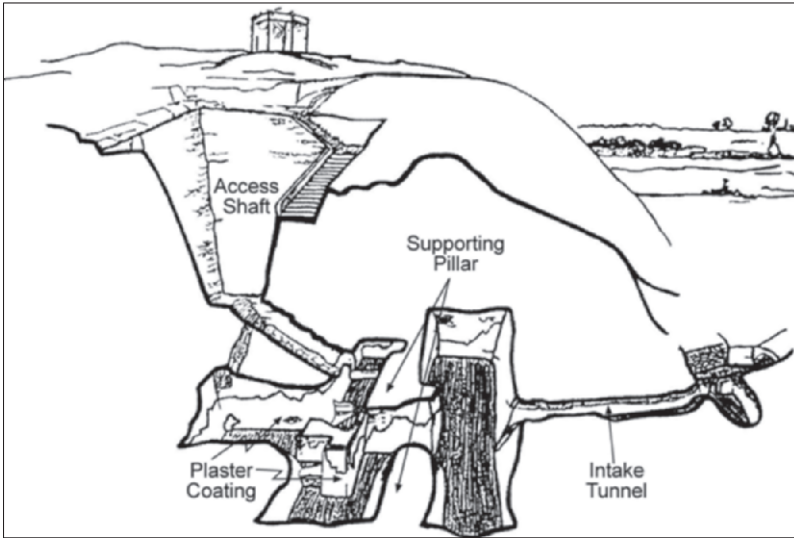

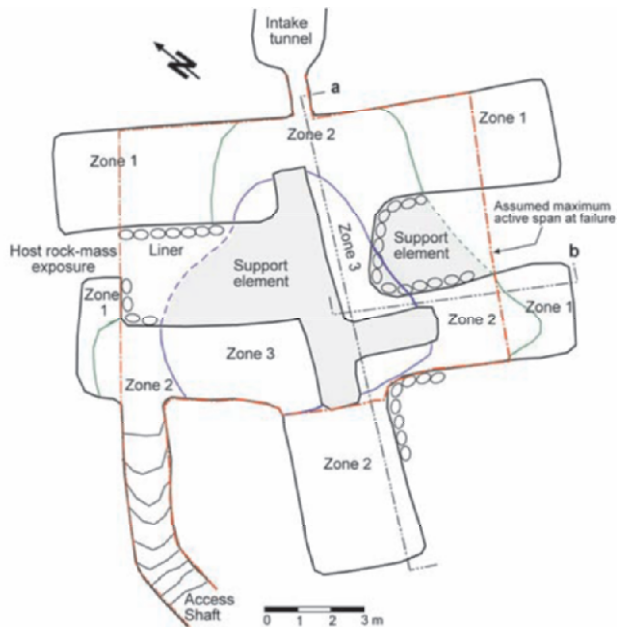*Figure 1*. Artist conception of underground water reservoir in Tel Beer Sheva.

Figure 2. Structural map of roof showing dome structure after the historic collapse (modified after[7]).

The mapped roof is considered here as failed laminated Voussoir beam. The failed beam arrived at a new equilibrium after the collapse, with the aid of support measures taken by the ancient engineers, primarily in the form of the central support pillar (Figure 2). The spherical dome shape of the current roof indicates that the problem, although axis-symmetric, is really three dimensional. The stability analysis presented below however is limited to two dimensional solutions, and its validity for the three dimensional case must be verified.

1.3   Mechanical properties of rock

The chalk exhibits homogeneous porosity, between n = 27 to 31% and unit weight between γ = 18.1 to 20.1 kN/m^3. Atterberg limits of interbedded marl layers indicate relatively low plasticity and low swelling potential. The mechanical behaviour of the chalk was determined from 4 triaxail and 5 uniaxial compression tests, performed under a constant strain rate of 10^{-5} s^{-1}, in the stiff, hydraulic, servo controlled triaxail load frame at Ben-Gurion University. The confining pressures values used were 2, 4, 6, and 10 MPa. A linear Coulomb-Mohr failure envelope fitted to the peak strength values yielded a cohesion of 3.1 MPa and internal friction angle of 32°.

The chalks exhibit transverse isotropy, with bedding planes being planes of isotropy in the material. The uniaxial compressive strength values vary from 7 to 34 MPa, for bed-normal and bed-parallel compression respectively (Figure 3). Similarly, Young's modulus varies from 1.9 to 31 GPa for bed-normal and bed-parallel compression respectively. Finally, Poisson's ratio ranges from 0.05 to 0.19 again for bed-normal and bed-parallel compression.

Rock discontinuities are persistent, clean and tight exhibiting relatively planar surfaces. In order to evaluate the influence of surface roughness on joint shear strength direct shear tests were performed using the stiff, hydraulic, servo-controlled, direct shear system at BGU rock mechanics laboratory. The tests were performed under an imposed constant normal stress condition. Sample TBS-1 was tested in two consecutive segment shear tests under a constant shear velocity of 0.00127 mm/sec. Results are shown in Figure 4 where the influence of normal stress on friction is also presented. The peak friction angle is ϕ_p=47° while the residual one is ϕ_r=24°. The decrease in friction with increasing normal stress represents the degradation of asperities during shearing cycles in the relatively weak joint surface material.

Polished joint surfaces were studied further, by cyclic direct shear tests performed at a constant cyclic shear displacement rate of ± 0.0254 mm/sec. Shear displacement–shear stress curves for sample TBS–4 are shown in Figure 5 together with the resulting shear stiffness and friction angle values. The shear stiffness values increase with increasing normal stress, as expected. The friction angle exhibits anisotropy: during forward shear the friction angle

reduces from $\phi=47^{o}$ at $\sigma_n=500$ kPa to $\phi=38^{o}$ at $\sigma_n=2000$ kPa, and during backward shear from $\phi=30^{o}$ at $\sigma_n=500$ kPa to $\phi=35^{o}$ at $\sigma_n=2000$ kPa.
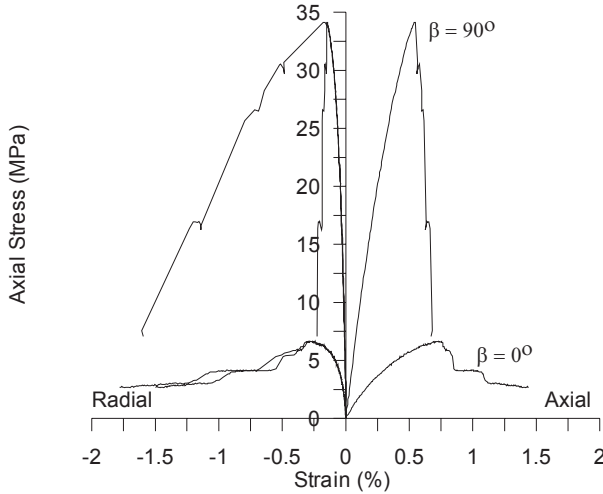

Figure 3. Comparison between bed parallel ($\beta = 90^{o}$) and bed normal ($\beta = 0^{o}$) uniaxial compression tests of Tel Beer Sheva marly chalk (Ghareb Fm.).
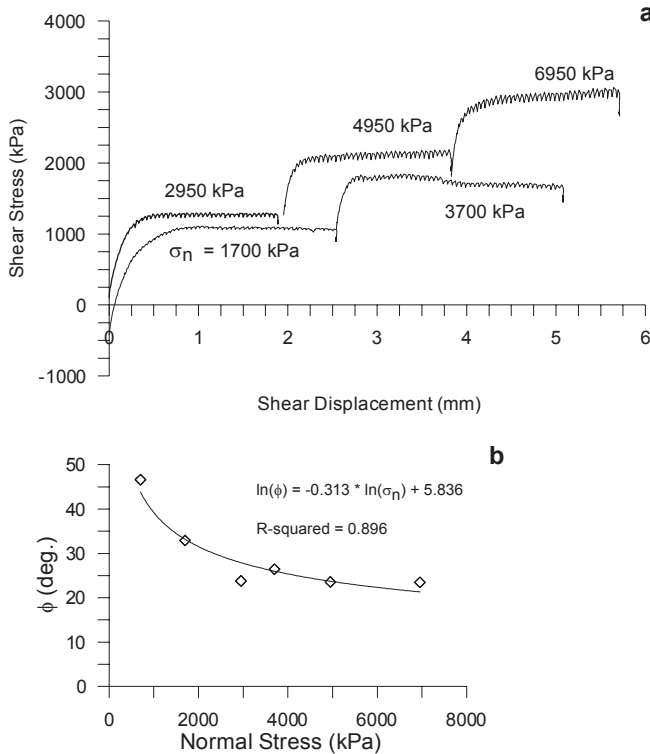

Figure 4. Direct shear test results for natural joints: (a) Segment direct shear test under constant normal stress, (b) Influence of normal stress on friction angle.
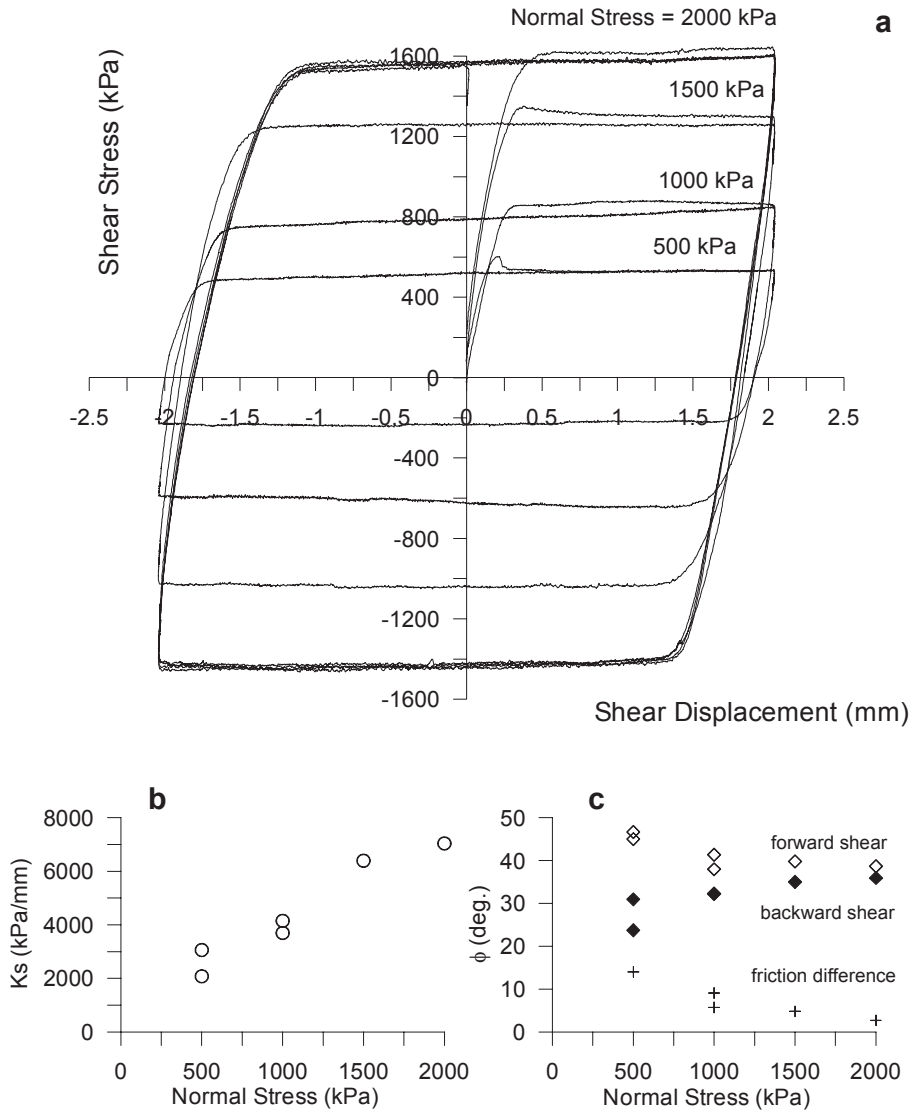
Figure 5. Results of cyclic shear tests on polished samples.

1.4   Stability analysis by the Voussoir beam analogue

The stability of the immediate roof at the site was first estimated using the iterative procedure for the static solution of a Voussoir beam, originally proposed by Evans[8] and later modified by Beer and Meek[2], Brady and Brown[4], Diederichs and Kaiser[5] and Sofianos[6]. The Voussoir model and the stress distribution assumed are shown in Figure 6 where S and t are the beam span and thickness respectively.
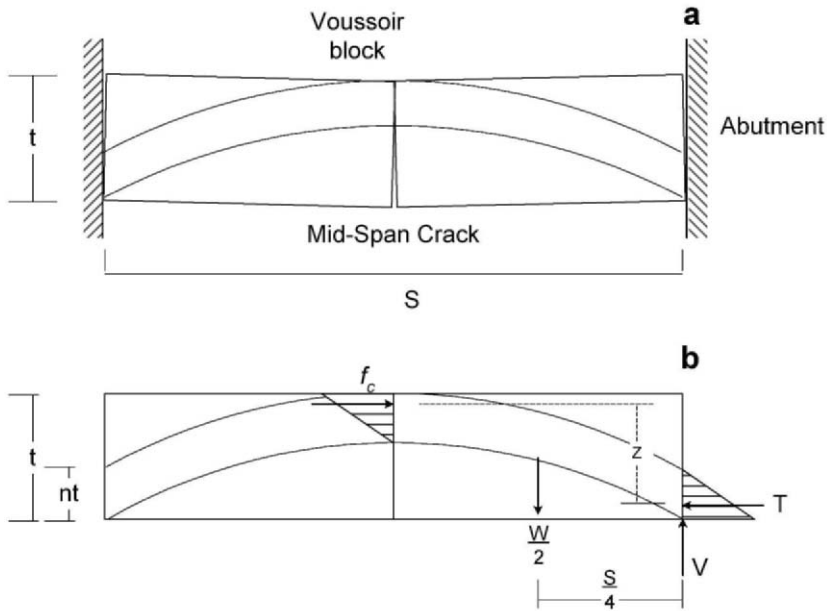
Figure 6. The Voussoir beam analogue (after Brady and Brown[4]).

The following mechanical parameters were used for the analysis: density ρ=1900 kg/m³, elastic modulus E=7840 MPa, Poisson's ratio ν=0.17, uni-axial compressive strength UCS=27.6 MPa, joint cohesion c=0 MPa and joint friction angle ϕ=47°. The factors of safety against failure in compression (axial crushing) or shear (sliding of beam along vertical abutments) were calculated for a range of beam spans (S=5 to 16 m) and beam thicknesses (t=0.25 to 5 m) accounting for all possible geometries at time of failure, while assuming solid roof beams with no layers and only one vertical discontinuity at the centerline that are free to displace along the abutments. The results are shown in Figure 7. Assuming an active beam span of 8 m and beam thickness of 2.5 m at time of failure, the beam is perfectly stable against failure in compression. Similarly, with a friction angle of 47° the factor of safety against failure in shear approaches 2.0. These results are in contrast to field observations and therefore reflect the limitations of the Voussoir approach when analyzing layered and jointed beams.

1.5   Stability analysis by the numerical DDA method

The Voussoir analogue discussed above is limited in the sense that it does not allow for layered, and multiply jointed beams. Rather, it considers the beam as a continuous solid with only one discontinuity at the centerline, which is free to slide along the abutments. Furthermore, the Voussoir analogue can not handle multiple joints, and completely ignores joint friction.
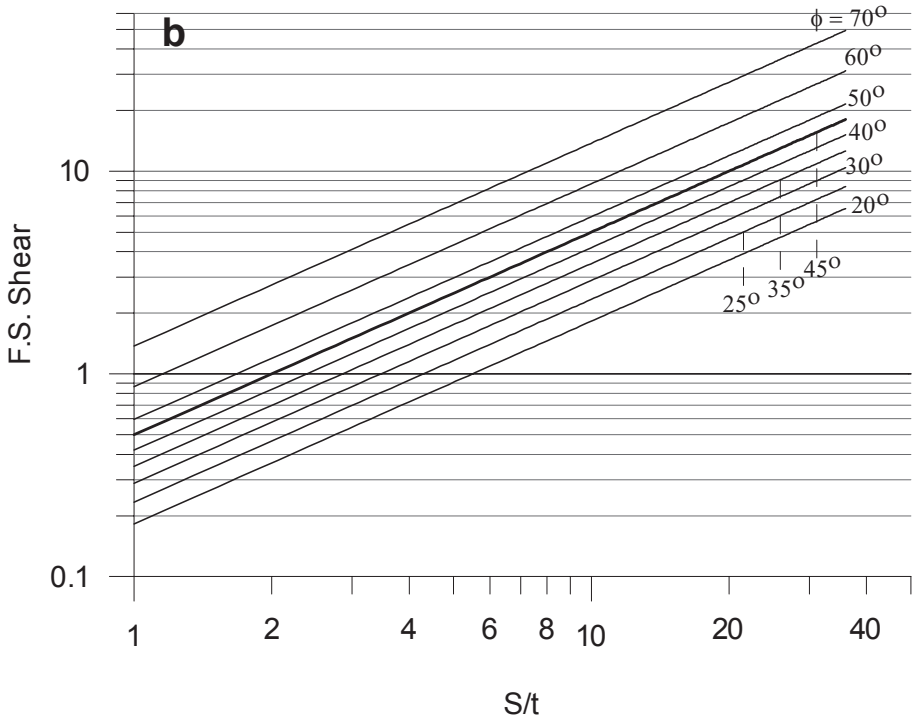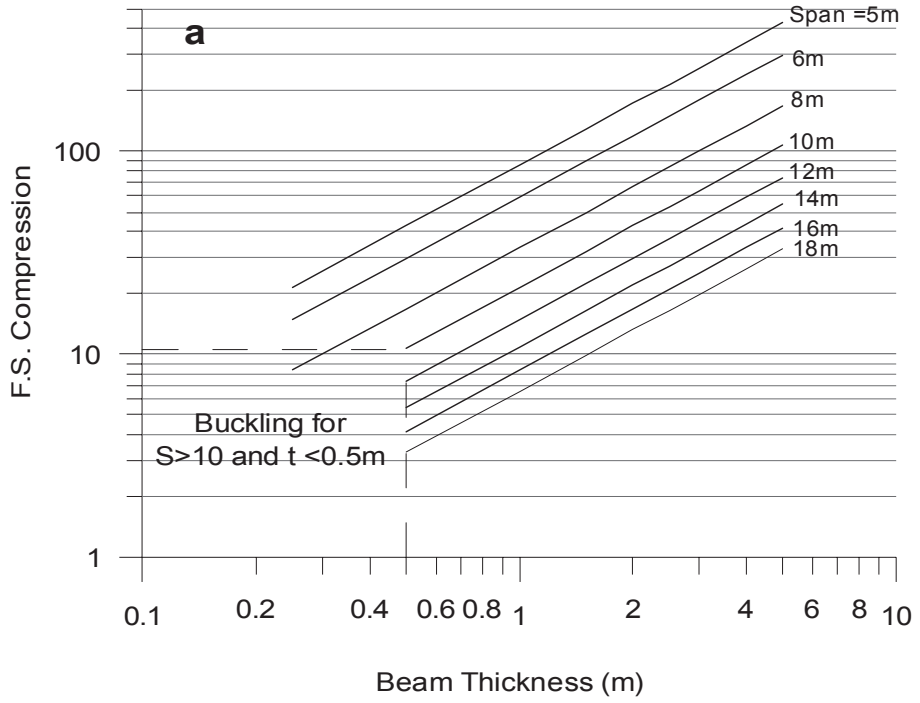
Figure 7. Results of Voussoir beam analogue for the Tel Beer-Sheva reservoir.

In order to account for more complex (and realistic) geometries numerical methods must be employed. In this paper the study is focused on the discontinuous deformation analysis (DDA) method of Shi[3]. Details on the DDA method are discussed by Jing[9], and a review of a decade of validations is presented by MacLaughlin and Doolin[10].

Both single and multiple layered beams are analysed with DDA, the geometry of which is shown in Figure 8.
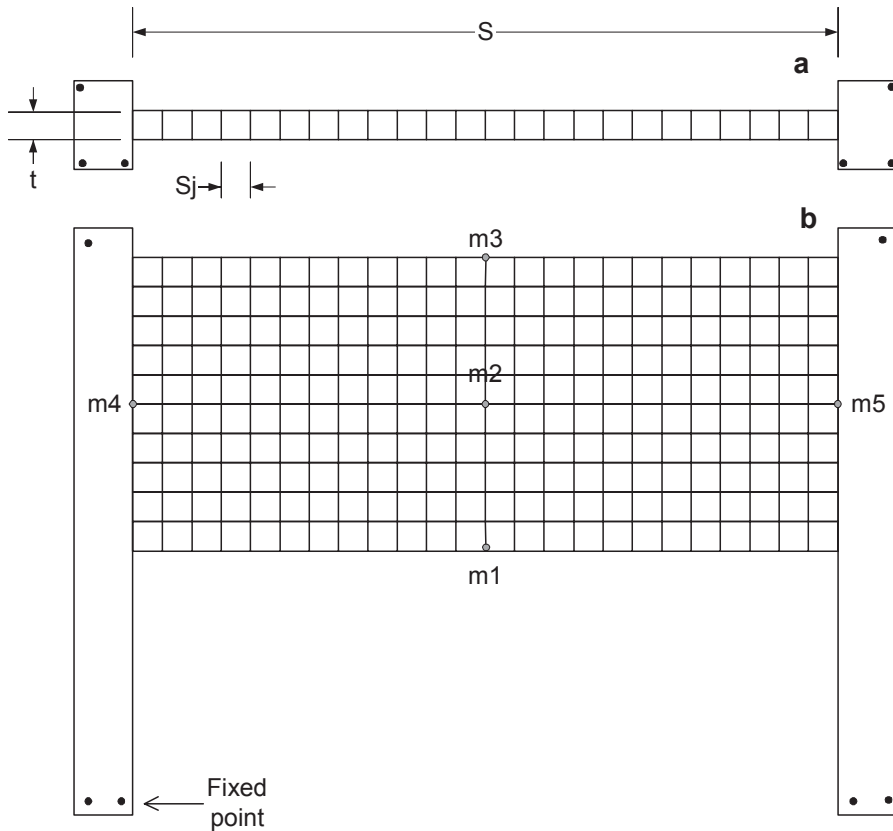

Figure 8. Single layered and multiple layered beam models studied in DDA.

Location of measurement points for analysis output are marked by circles in Figure 8 and are labelled mi (i = 1,2,...5). Two geometries are analyzed:

1) a single layer with t = 0.5 m modelling the immediate roof layer (Figure 8a);

2) a 5 m thick stack of horizontal layers each with t=0.5 m (Figure 8b), modeling the entire excavation roof. The active span in both configurations is S=8 m. The following input parameters are used in DDA (see Hatzor and Feintuch[11] for details): ρ=1900 kg/m^3, E=7840 MPa, ν=0.17, penalty stiffness

g(0)=1000 MN/m, time step size g(1)=0.00025 sec, penetration control parameter g(2) = 0.00025, and dynamic control parameter k01 = 1.

The effect of joint friction angle is demonstrated in Figure 9 for a single layered beam with thickness of t=0.5m and joint spacing of Sj=0.25m. The measured deformation variables (u, v, ω) of blocks in the single layered beam are shown in Figure 10 according to their position at the beam.
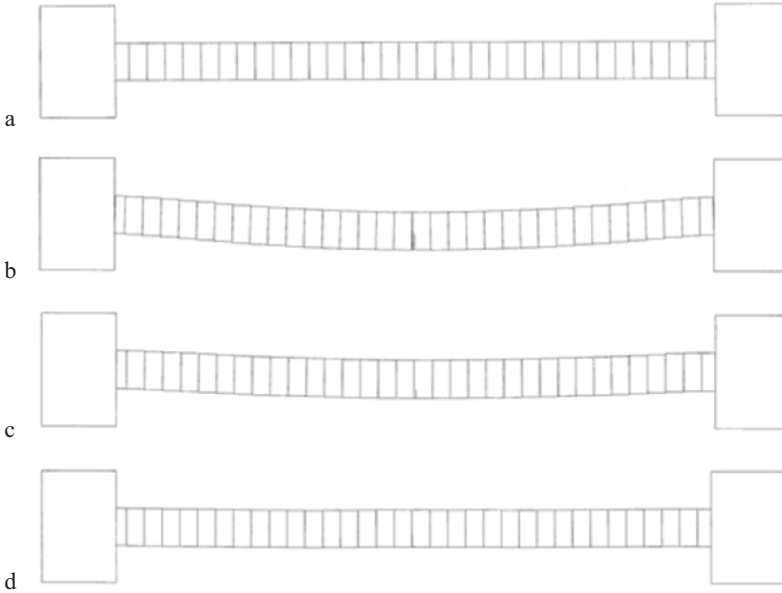

Figure 9. DDA graphic output of single layer deformation for different values of joint friction angle: (a) original geometry; (b) $\phi_{av} = 45^{\circ}$; (c) $\phi_{av} = 75^{\circ}$; (d) $\phi_{av} = 80^{\circ}$.

The results of the DDA single layer model indicate that the required friction angle for stability, with immediate roof layer thickness of 0.5m and vertical joint spacing of 0.2 5m (as found in the field) is $\phi_{req} \geq 78^{\circ}$. Since the available joint friction angle in the field is assumed to be $\phi_{av}=47^{\circ}$, the DDA results explain the observed failure of the immediate roof in the field, de-limited by the contour of zone 1 in the present-day structural map of the roof (Figure 2).

The response of a laminated Voussoir beam is shown in Figure 11. The laminated Voussoir is modelled by an 8 m span, 5 m stack of individual layers, each of 0.5 m thickness, simulating the general structure of the roof in the field. The computed deflections of measurement points m1, m2, and m3 (see Figure 8 for location) are plotted as a function of joint friction angle (open symbols in Figure 11) and of joint spacing (solid symbols in Figure 11). The influence of friction is analyzed for a case of constant joint thickness of Sj = 0.25 m, similar to the case in the field. The influence of joint spacing is ana-lyzed for constant friction of $\phi = 47^{\circ}$, as believed to be the case in the field.
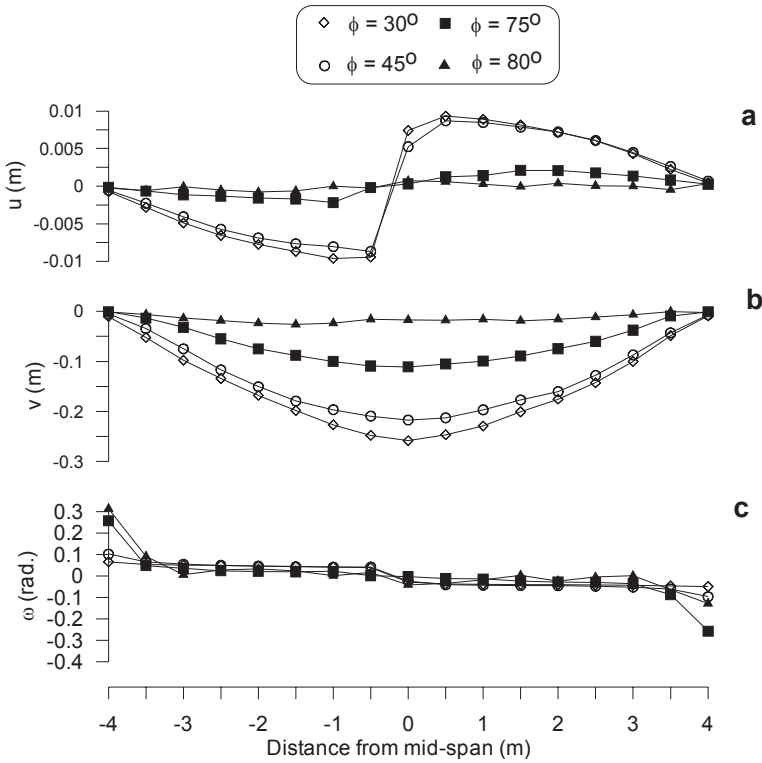
Figure 10. Deformation of the DDA single layer model, measured at the lowermost fiber of the beam: (a) horizontal displacement (u); (b) vertical displacement (v); and (c) rotation (ω).
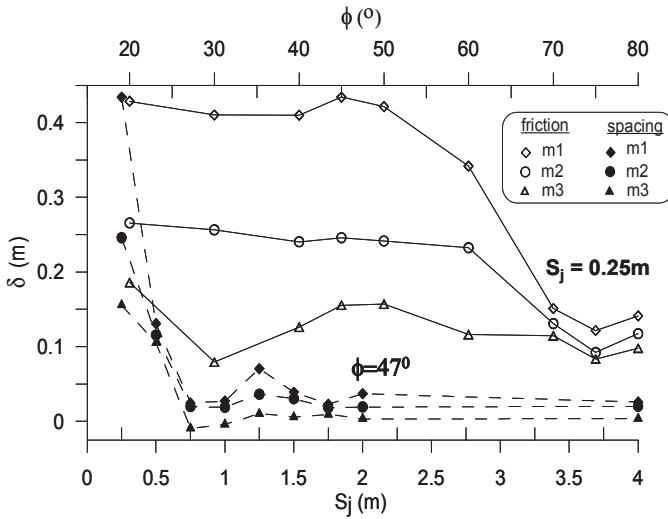

Figure 11. Response of a layered beam as computed by DDA (see text for details).

From Figure 11 it can be deduced that when friction angle is smaller than 70° vertical deflections in the immediate roof (m1) are excessive indicating instability, but with friction angles greater than 70° the composite beam stabilizes, and all measurement points deflect homogenously by about 10 cm.

The influence of spacing on layered beam stability is revealing as well. The layered beam, with a constant friction angle of $\phi = 47°$ seems to stabilize when joint spacing Sj≥0.75 m. For the analyzed friction angle, increase in joint spacing beyond Sj=0.75 m does not improve stability, indicating development of a stable arching mechanism as long as joint spacing is greater than the required value.

It would be intuitive to expect that the same optimal joint spacing value could be found for any value of interlayer friction. This analysis was conducted by Hatzor and Benary[1] with DDA for a slightly different beam configuration: Total beam thickness=2.5 m; S=7 m; t=0.5 m; γ=18.7 kN/m³, E=2 GPa, ν = 0.2. The results are shown in Figure 12.

The DDA results shown in Figure 12 clearly demonstrate that in a layered Voussoir beam of given geometrical and mechanical properties there is an optimal joint spacing which requires a minimal value of discontinuity shear strength in order to generate a stable arching mechanism. For the analyzed beam in Figure 12 the optimal joint spacing is Sj = 175 cm and the required friction angle for equilibrium is ϕ_{req}=24°. This result suggests that the optimal S/Sj ratio for equilibrium is (S/Sj)$_{eql}$=4.0.

The DDA results shown in Figure 12 also demonstrate that for every available discontinuity shear strength value, a corresponding joint spacing value necessary for equilibrium exists. For the analyzed case study with mean joint spacing of 0.25 m the required friction angle for stability is ϕ_{req}=80°, since ϕ_{av}=47° the layered roof could not remain stable once the opening was attempted. As shown in Figure 11 for a very similar beam configuration, the required joint spacing for equilibrium with ϕ_{av}=47° is 0.75 m, three times greater than the mean joint spacing in the field.
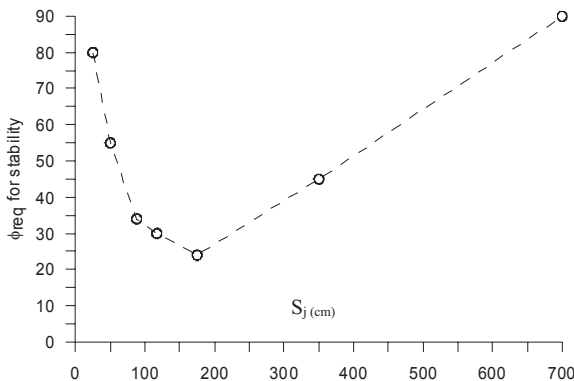

Figure 12. ϕ_{req} for stability vs. joint spacing, and optimal joint spacing for a given layered Voussoir beam[1].

2. THE FREEMASONS HALL AT ZEDEKIAH CAVE

2.1 Introduction

Zedekiah Cave has been used as an underground quarry below the city of Jerusalem from about 700 - 800 BC, and continuously until the end of the late Byzantine period, in order to extract high quality building stones for monumental constructions in Jerusalem and vicinity. The quarry is excavated underneath the old city of Jerusalem (Figure 13a) in a sub-horizontally bedded and moderately jointed, low strength, upper Cretaceous limestone of the Bina formation[14]. The underground quarry is 230 m long, with maximum width and height of 100 m and 15 m, respectively.

The most striking feature of the quarry is certainly the 30m span, unsupported central chamber, widely known as "Freemasons Hall", because of ritual ceremonies taken place at the chamber by Freemasons in recent times (Figure 13a). Site investigations revealed that large roof slabs in several side chambers have collapsed over the years, but that the roof of Freemasons hall remained intact.

In this section roof stability in Freemasons hall is analyzed and discussed. First a continuum mechanics approach is taken[12], followed by an attempt to apply the Voussoir beam analogue[2], and finally the numerical DDA [3] is used to study the interaction between joint friction, spacing and orientation.

2.2 Clamped beam model

The 30 m span, unsupported roof of Freemasons hall seems intact in field inspection and therefore calls for a preliminary analysis based on continuum mechanics principles. Obert and Duvall[12] review the elastic solution for a clamped beam which provides deflections, shear forces and bending moments across the beam. This solution may be applicable for the immediate roof of Freemasons hall provided that the limestone bed comprising the immediate roof material is completely continuous with no intersecting joints. An accurate cross section through Freemasons hall is shown in Figure 14c and its location in the underground monument is shown in Figure 13b (section A-A'). The concept of Obert and Duvall[12] for a clamped beam model as applied to underground openings excavated in rock masses containing planes of weakness parallel to the roof is illustrated in Figure 15 below. Using field mapping and laboratory tests the relevant input parameters for the case of Freemasons hall are listed in Table 1. The results of the clamped beam analysis for the roof of Freemasons hall are listed in Table 2.

Application of the analytical clamped beam model predicts that the axial stresses that will develop in the beam will be $\sigma = \pm 10.5$ MPa. While the upper fiber of the beam is safe against failure by crushing, the lowermost fiber
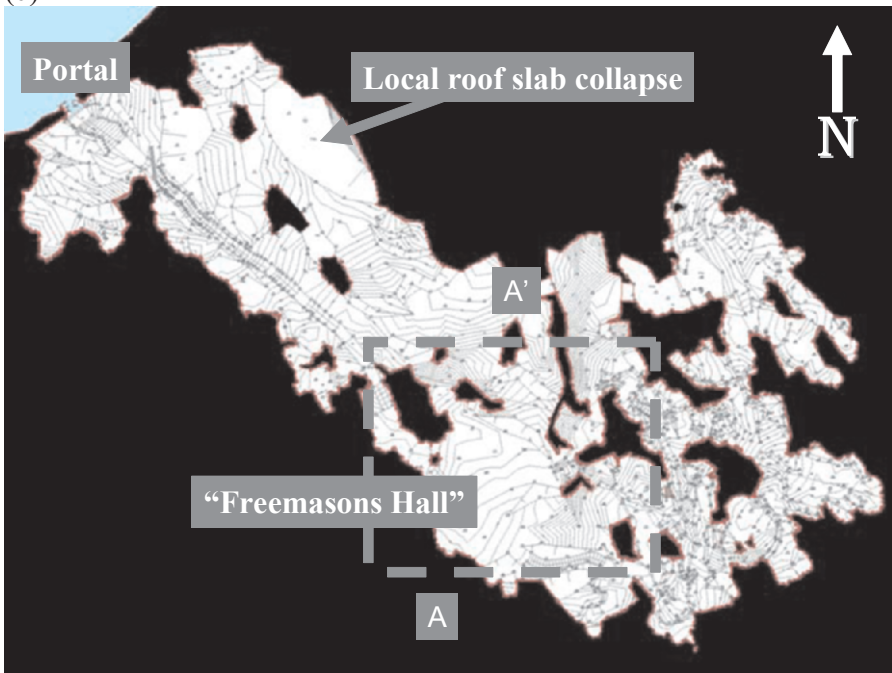
(a)

(b)

Figure 13: (a) Layout of Zedekiah cave superimposed on the old city of Jerusalem, (b) plan of Zedekiah cave, "Freemasons hall" delimited by dashed square.
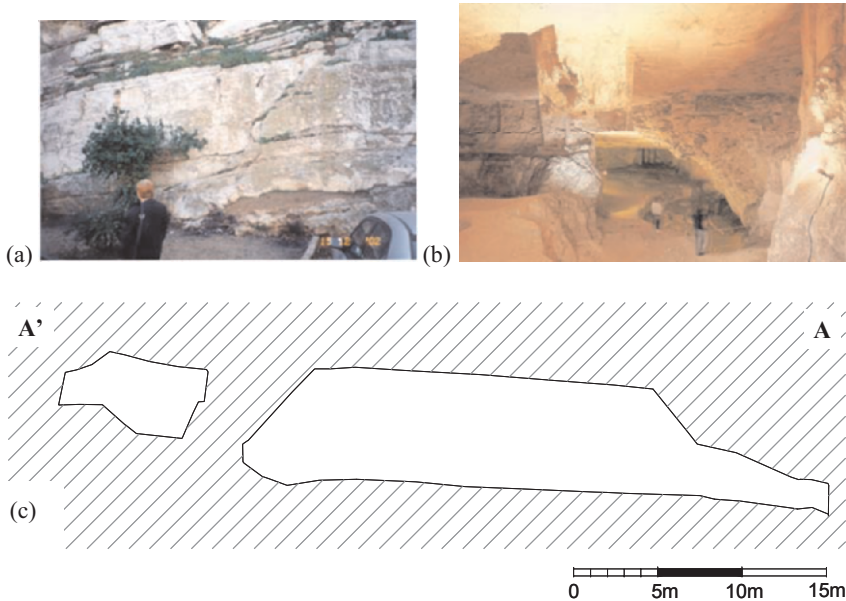
(a)   (b)

A'   A

(c)

0   5m   10m   15m

Figure 14. (a) The Bina formation outside Zedekiah cave, (b) A view towards Freemasons hall (Photograph by Y. Novitz [13]), (c) A cross section through Freemasons hall (for location see Figure 13b).

of the beam is unsafe against failure in tension; consequently, the clamped beam model predicts that a tensile fracture will initiate at the centerline and propagate upwards, disjointing the beam into two blocks, each of 15 m length. The stability of such a three – hinged beam can be estimated by application of the Voussoir beam analogue, discussed in section 1.4 above.

Table 1. Physical and mechanical properties of immediate roof in Freemasons hall.

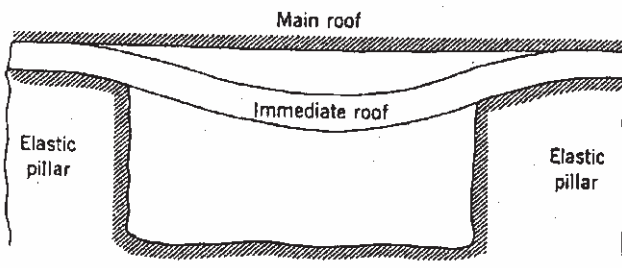|	
Free span (L)	30 m
Beam thickness (t)	0.85 m
Unit weight (γ)	19.8 kN/m
Elastic modulus (E')	$8*10^3$ MPa/m^2
Uniaxial compressive strength (σ_c)	16.4 MPa (bedding parallel)
Tensile strength (σ_t)	2.8 MPa

Figure 15. Deflection of a single layer on elastic pillars, after Obert and Duvall[12].

Table 2. Results of clamped beam analysis for immediate roof in Freemasons hall.

Maximum deflection at centerline (η)	8.67 cm
Maximum shear stress at abutments (τ_{max})	445.5 kPa
Maximum axial stress (σ)	10.5 MPa

2.3   **Voussoir beam analogue**

The Voussoir beam analogue can be applied to the immediate roof of Freemasons hall assuming that a tension crack formed at the centerline, as discussed in section 2.2 above. All required input parameters for such an analysis are listed in Table 1. Application of the iterative procedure[4] shows that the *n* value (see Figure 6) does not converge; after one iteration *n* becomes greater than 1.0. Using the modified approach suggested by Diederichs and Kaiser[5] by introducing incremental steps in *n*, reveals that when the system is supposed to attain equilibrium (when the extreme fiber stress is minimum, incidentally this always happens when $n = 0.75$[6]) the thickness of the compressive arch *Z* is negative. The meaning of this result is that under the given loads, geometry, and material properties, the beam will undergo buckling deformation leading to a "snap through" mechanism[4]. Indeed, in some roof sections in the site snap-through mechanism may be responsible for the observed failures (see Figure 13b). This is certainly not the case in the roof of Freemasons hall which still stands unsupported. Therefore, complete understanding of the mechanics of the roof in Freemasons hall requires a further, more robust, analysis which allows for interaction between blocks and incorporates friction laws for discontinuities. Such an analysis is presented in the next section using DDA.

2.4   **Discontinuous deformation analysis (DDA)**

The elastic solution for the roof predicts tensile fracture if modelled as a continuous beam. Application of the Voussoir analogue for the roof predicts a snap-through mechanism. Both scenarios did not materialize in Freemasons hall during its 2500 years history; field inspections suggest that the current immediate roof is the original one. These findings suggest that interactions between distinct blocks in the rock mass above the immediate roof may stabilize, rather than weaken, the roof. To explore this possibility the rock mass

Table 3. Rock mass structure at Zedekiah cave

Discontinuity Set	Genetic Type	Mean Orientation	Mean Spacing (m)	Joint friction angle
1	Bedding	08/091	0.85	41°
2	Shears	71/061	0.79	41°
3	Shears	67/231	1.48	41°
4	Joints	75/155	1.39	41°

around Freemasons hall is modeled as a mesh of distinct elements, using discontinuous deformation analysis[3]. The rock mass structure in Zedekiah cave consists of one set of sub-horizontal beds and three sets of inclined joints (Table 3).

The numerical analysis is limited to two dimensions because of the large number of blocks in the rock mass around Freemasons hall. Because of the inherent limitations of the two-dimensional solution, the complex rock mass structure is simplified and represented by two extreme end members: 1) A rock mass consisting of horizontal beds and vertical joints; 2) A rock mass consisting of horizontal beds and inclined joints. Furthermore, the actual spacing value for each joint set is doubled in the generation of the DDA mesh in order to reduce the total number of blocks. Even so, the total number of blocks in scenarios 1 and 2 are 368 and 735 respectively. Figure 17 displays these two end members as represented in a DDA mesh. The measurement point location for displacement and stress output are marked in Figure 17b for reference.

The two mesh configurations are modeled for response under gravitational loading for duration of 5 sec, an equivalent of 20,000 DDA time steps. A friction angle value of 41° is assumed for all discontinuities based on tilt tests of saw cut planes and measured surface profiles in the field. The deformation configuration is shown in Figure 18, where principal stress trajectories at the end of the computation are marked as well.

The response of the two mesh configurations is strikingly different. The vertical joint configuration (Figure 18a) exhibits some shear displacement along vertical joints that extend upwards from the abutments, but ultimately downward displacement is restrained, most probably due to the development of an effective arching mechanism in the roof. The inclined joints configuration in contrast exhibits ongoing downward displacement with no indications of stabilization. The downward displacement output data for the four measurement points are plotted in Figure 19. The onset of arching and displacement arrest is clearly demonstrated in the vertical joints configuration, where stabilization is indicated after 2000 time steps (Figure 19a). In the case of the inclined joints however all measurement points exhibit downward displacement at a constant velocity with the immediate roof undergoing downward displacement of 50 cm after 5 seconds of loading (Figure 19b). This result suggests that with an inclined joints configuration the development of an effective arching mechanism in the roof is hampered. This can also be detected by the erratic orientation of the principal stress trajectories in Figure 18b. The magnitude of the horizontal stress in the immediate roof (mp1) in the two structural configurations is shown in Figure 20. Clearly, the vertical joint configuration stabilizes under a constant horizontal stress of 650 kPa. The inclined joints configuration however displays an erratic stress behaviour that never attains equilibrium.
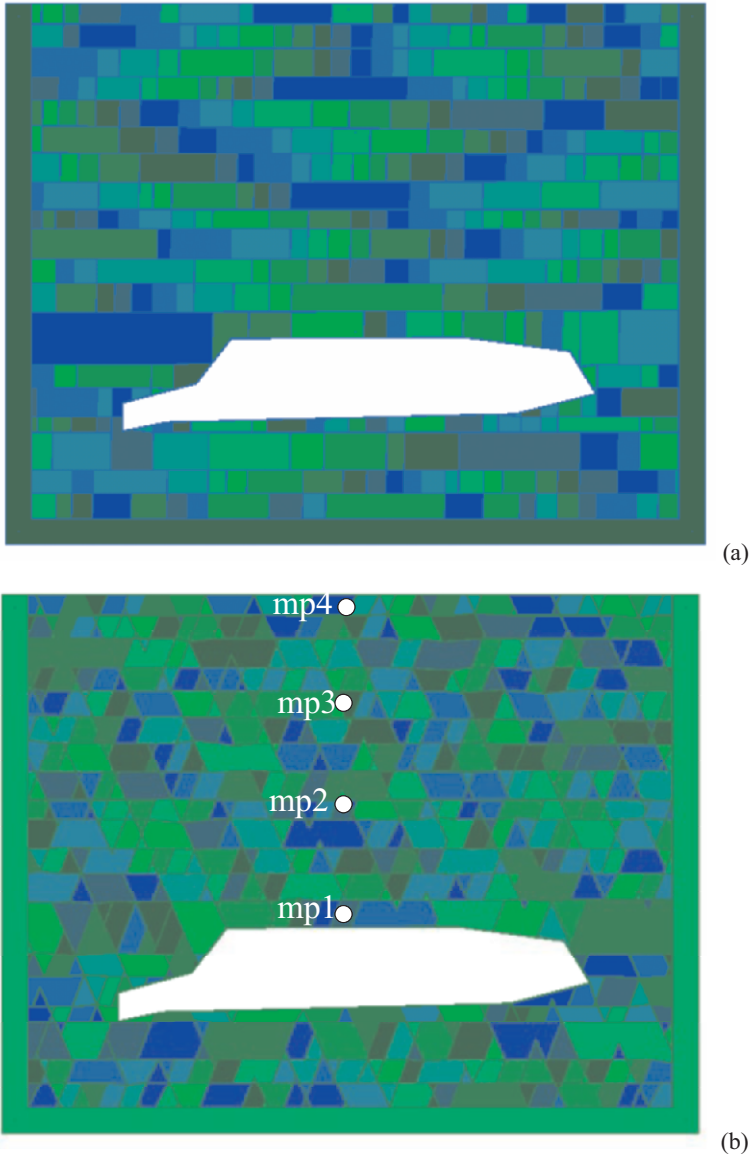
(a)

(b)

Figure 17. Two rock mass structures to be modelled in DDA: (a) horizontal beds with vertical joints, (b) horizontal beds with inclined joints.

3.   SUMMARY AND CONCLUSIONS

Two historic monuments, excavated in bedded and jointed rock masses are discussed in this paper: 1) the 3000 year old water storage system at Tel Beer - Sheva, 2) the 2500 year old quarry at Zedekiah cave in Jerusalem. In Tel Beer-Sheva the immediate roof of the reservoir collapsed during construc-
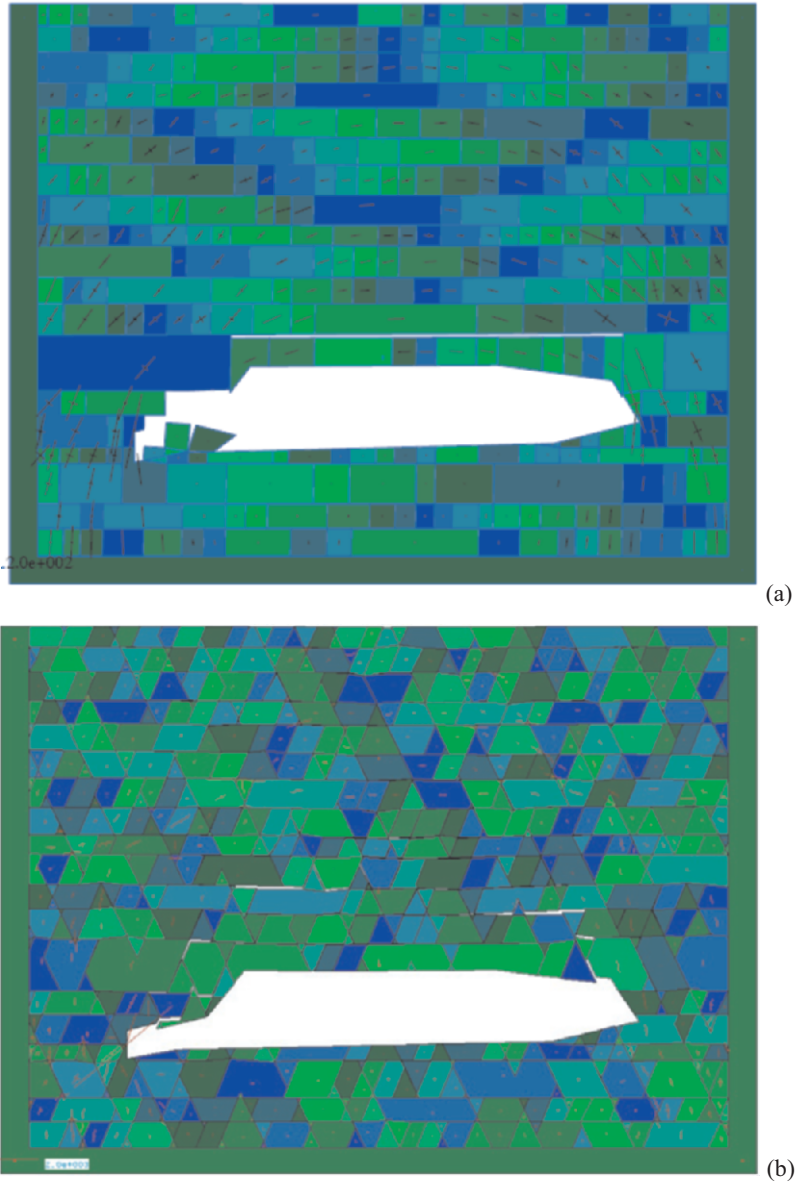
Figure 18. Deformed configurations after 5 seconds of gravitational loading: a) Vertical joints, b) Inclined joints.

struction while at Zedekiah cave the magnificent roof of the central 30 m span chamber, known as Freemasons hall, remained intact over the years. Both cases are used to test existing analytical methods (elastic theory and the Voussoir beam analogue) and to verify validity of numerical codes (the discontinuous deformation analysis method).
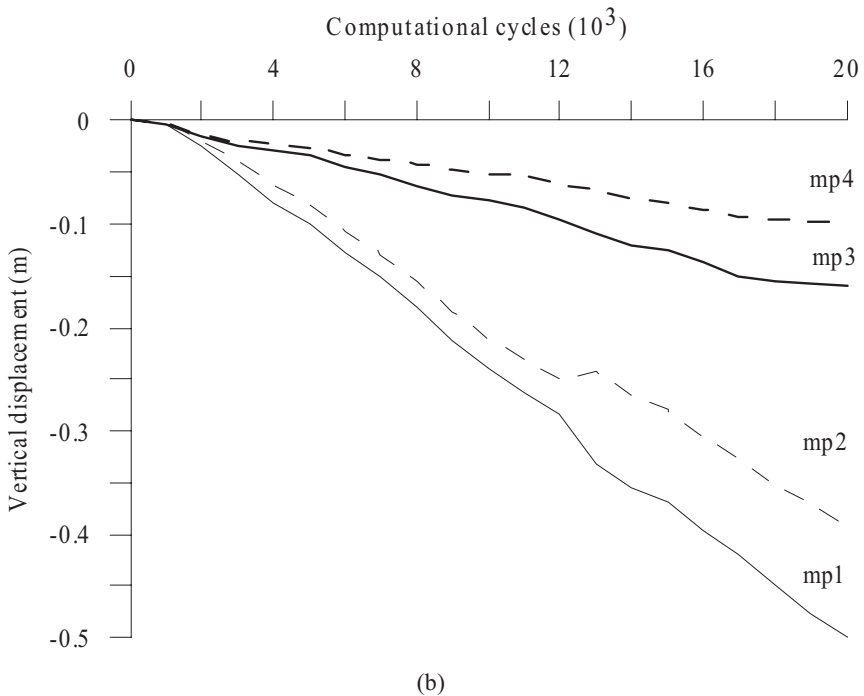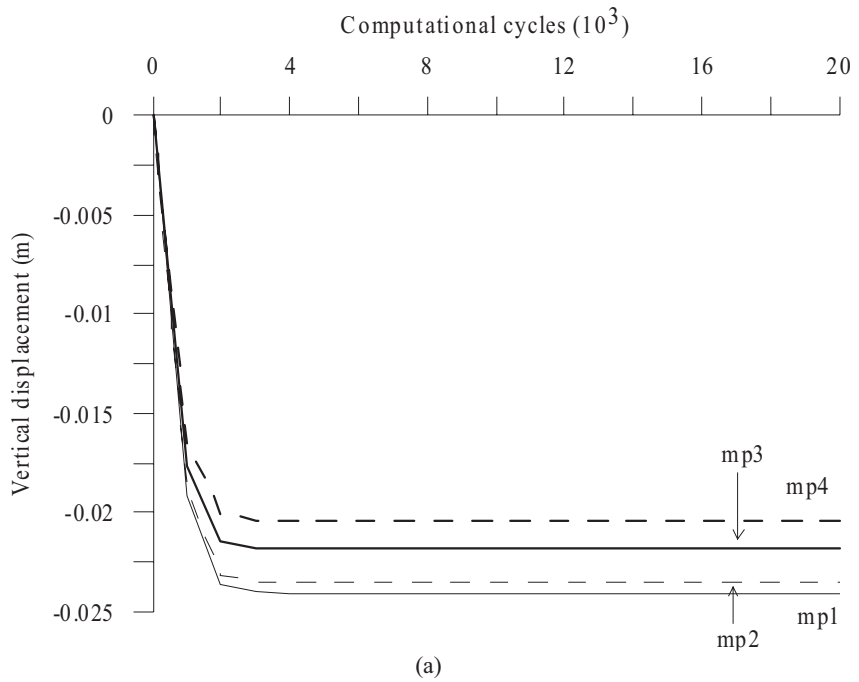
Computational cycles (10^3)

(a)

Computational cycles (10^3)

(b)

Figure 19. DDA measurement point output (for location see Figure 17b). (a) vertical joints configuration, (b) inclined joints configuration.
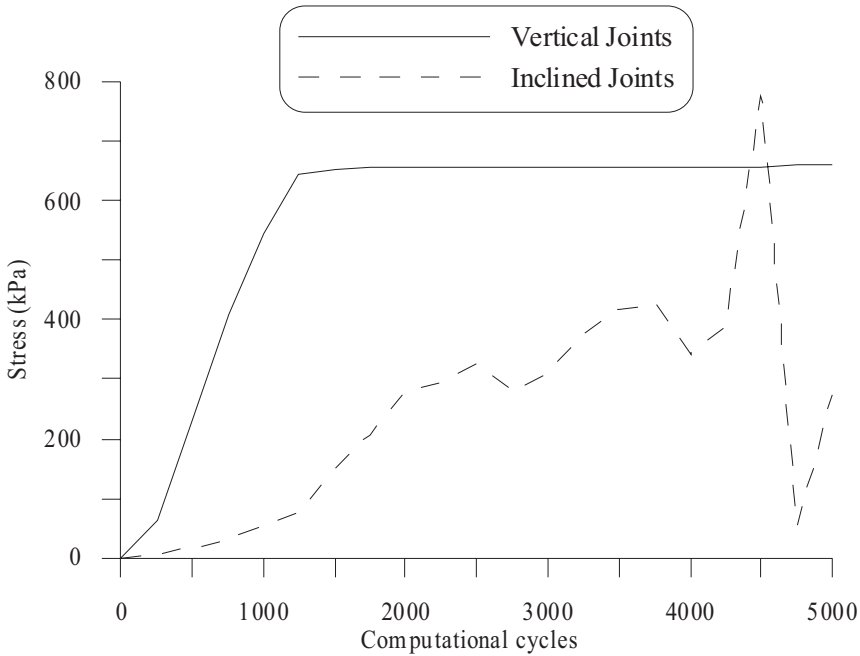
Figure 20. Horizontal stress development in vertical (solid line) and inclined (dashed line) joint configuration in the roof.

Back analysis of the immediate roof failure at Tel Beer-Sheva suggests that the Voussoir beam analogue may be un-conservative: it predicts stability against failure by shear or axial crushing, while in reality the roof has collapsed. Application of the DDA method which considers joint spacing and friction reveals that with the joint spacing value in the field the roof could not have remained stable against failure in shear along the abutments.

DDA further provides the following general conclusions:

- The ratio between free span (*S*) and joint spacing (Sj) is critical for immediate roof stability.
- Both joint spacing and friction must be considered in evaluation of immediate roof stability.
- For a given horizontal beam transected by vertical joints, there exists a unique function which relates beam equilibrium to joint spacing and friction. The unique function exhibits a minimum, which defines the optimal spacing-friction combination for the analyzed beam.

Back analysis of the immediate roof of Freemasons hall in Zedekiah cave suggests that application of the elastic solution for a clamped beam to the actual roof may be misleading. The predicted extreme fibre stresses in the immediate roof layer never materialize in reality because the material does not behave as a continuous solid but as a disjoint beam, the deflections in the field are therefore much smaller than predicted by either the elastic solution

or the Voussoir beam analogue. Application of a distinct element numerical method, DDA, confirms this result and further helps explore the significance of a third parameter: joint orientation. It is shown that vertical joints are much more preferable for immediate roof stability than inclined joints. This could be because of better arching mechanism due to longer moment arms, and because of better interlocking between parallel joints due to inherent surface roughness.

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