SPHERICAL CAP HARMONIC ANALYSIS OF THE GEOMAGNETIC FIELD WITH APPLICATION FOR AERONAUTICAL MAPPING

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Abstract. The Spherical Cap Harmonic Analysis (SCHA) is a regional modeling technique based on appropriate functions which are solutions of Laplace's equation over a constrained, cap-like region of the Earth. The concept was introduced in 1985 in the context of geomagnetism as a local or regional extension of the classic global spherical harmonic analysis. Starting from the basic principles in which the analysis method is founded, this paper describes the latest applications for the modeling of the main magnetic field and its secular variation. Although examples of applications over small areas will be given, it will be shown that, in general, the bigger the region the more appropriate the technique. Therefore, this paper focuses on the results and perspectives over continental areas, like Antarctica or Europe. The possible application to the derivation of isogonic charts for navigational purposes with suitable time predictions will be emphasized. At the same time, the limitations of the method will be examined. Although recent revisions of the technique seem to solve some of the problems, our present research focuses on the quest for solutions to the still unanswered questions.

Keywords: SCHA; geomagnetic field modeling; spherical harmonics; declination

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1. Introduction

The analytical representation of the geomagnetic field has been a topic of research for a long time, either for the aspects related with the definition of the origin of the field itself or for its variations. There is also great scientific interest in the phenomena that produce geomagnetic variations, like ionospheric and magnetospheric current systems, and those induced by them in the Earth's interior; along with the Earth's self-sustained dynamo, which is the origin of the main field and its secular variation. Modeling the sources that originate the crustal contribution, caused by differential magnetization of the rocks in the Earth's crust, are found in the same way.

From the beginning, such representations have been oriented towards global modeling or they have tended toward the representation of the phenomenon over a particular portion of the Earth's surface, because of a special interest in its study, or because of a denser distribution of the measurements in a particular region. A regional analysis tends to represent the field with better resolution, which is often a great advantage; but the mathematical algorithms that serve as a basis for such representations suffer frequently from restrictive constraints or impossible convergences. So, the algorithms have traditionally been better solved in the global case, given the quasi-spherical geometry of the Earth.

The Spherical Harmonic Analysis technique, introduced by Gauss in 1839, has resulted, by far, in the most popular method for modeling the main field and its secular variation at the global scale. Starting from Maxwell's equations, applied over the Earth's surface, it can be accepted with a good approximation that we are free from electric currents, so that the curl and the divergence of the field are null. The field can then be represented as the negative gradient of a magnetic potential V:

$$\mathbf{B} = -\nabla V \tag{1}$$

and such potential must then satisfy Laplace's equation:

$$\nabla^2 V = 0 \tag{2}$$

A solution for this equation in spherical coordinates may be obtained by the method of separation of variables (radial distance r from the Earth's center, colatitude θ and longitude ϕ) given as $V(r, \theta, \phi) = U(r)P(\theta)Q(\phi)$. Therefore, the problem is reduced to finding the solutions for these 3 differential equations, which depend on each of the variables:

$$\frac{d^2U}{dr^2} - \frac{n(n+1)}{r}U = 0 \tag{3}$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right)+\left[n(n+1)-\frac{m^2}{\sin^2\theta}\right]P=0 \tag{4}$$

$$\frac{1}{Q}\frac{d^2Q}{d\phi^2}=-m^2 \tag{5}$$

By adequately choosing the boundary conditions for the earth's sphere, the general solution of Laplace's equation can be expressed as a superposition of potential functions of this type:

$$\begin{aligned}V(r,\theta,\phi)=a\sum_{n=1}^{Ni}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}\left\{g_n^{m,i}\cos m\phi+h_n^{m,i}\sin m\phi\right\}P_n^m(\cos\theta)+\\+a\sum_{n=1}^{Ne}\sum_{m=0}^{n}\left(\frac{r}{a}\right)^{n}\left\{g_n^{m,e}\cos m\phi+h_n^{m,e}\sin m\phi\right\}P_n^m(\cos\theta)\end{aligned} \tag{6}$$

In this way, the represented potential consists of two parts: one produced by sources located within a sphere of radius a, and another by sources located outside this volume. The P functions are the associated Legendre functions of first kind of degree n and order m, which are integer parameters for this solution.

The product of the Legendre functions with the trigonometric functions in longitude forms the series of two-dimensional spherical harmonics. The g and h are the spherical harmonic coefficients, or Gauss coefficients. The general solution results in an infinite series of terms. In practice, it is truncated at finite indices N_i and N_e.

The potential V, however, is not observable. According to equation (1), the cartesian components of the geomagnetic field are obtained as the partial derivatives of V with respect to r, θ, and ϕ:

$$X\equiv -B_\theta=\frac{1}{r}\frac{\partial V}{\partial\theta} \tag{7}$$

$$Y\equiv B_\phi=\left(-\frac{1}{r\sin\theta}\right)\frac{\partial V}{\partial\phi} \tag{8}$$

$$Z\equiv -B_r=\frac{\partial V}{\partial r} \tag{9}$$

The most popular example of a global model for the main field (only internal long wavelength coefficients) is that known as the International Geomagnetic Reference Field (IGRF). The last up-to-date version of IGRF, known as the IGRF 10th generation (IAGA, 2005), includes models of the main field from 1900 to 2005 and a secular variation model for 2005-2010. The value of N_i for the fixed field is equal to 10 (that is, 120 coefficients) for all models prior to 1995, while for years 2000 and 2005 it includes coefficients up to degree n=13 (i.e., 195 coefficients). The secular variation model is expanded up to degree n=8.

The most ambitious recent effort to model fields, not only from the Earth's core but also from the lithosphere, the quiet-day ionospheric sources and the magnetosphere, along with the associated induced currents, and interhemispheric field-aligned currents, is known as the comprehensive model of the near-Earth magnetic field (Sabaka et al., 2004). It includes about 2,000,000 data entries from the POGO, MAGSAT, Ørsted, and CHAMP satellites (for which the ionospheric fields are internal), and magnetic observatory hourly and annual means from 1960 to 2000, resulting in more than 25,000 parameters.

2. Regional techniques

When geomagnetic observations are known only over a small portion of the Earth's surface or the analysis is only required over a particular area, the above functions for the spherical analysis are not the most appropriate anymore. The different techniques for obtaining regional models can be subdivided into graphical and analytical (Haines, 1990).

The oldest models of the geomagnetic field, for which the maps were drawn by hand, and those which have used algorithms to generate uniform grids from non-uniformly distributed data by numerical interpolation were derived graphically.

The simplest analytical method uses a polynomial expression in latitude and longitude (e.g. De Santis et al., 2003). However, this technique, as with graphical methods, does not account for altitude variations, permits the possibility of geometrical inconsistencies, and does not guarantee the conditions imposed by the electromagnetic theory which requires that in regions free from magnetic sources and electric currents, the magnetic potential satisfies Laplace's equation.

Another procedure sometimes used consists in the application of spherical analysis to data in a restricted region. However this can generate numerical instabilities in the determination of the coefficients because the functions are not orthogonal over the limited area in which the analysis is developed.

So, instead of using basis functions which are orthogonal over the whole sphere, it is more natural to use appropriate functions for such regions. Two techniques employed for smaller regions on the globe are the Rectangular

Harmonic Analysis (RHA) and the Spherical cap Harmonic Analysis (SCHA).

In the RHA, the general solution for Laplace's equation is given by an expansion in terms of the ordinary Cartesian or rectangular coordinates, with the origin usually taken at the centre of the region where the data are located:

$$
\begin{aligned}
V = Ax + By + Cz + \sum_{m=1}^{M_0} \{ a_0^m \cos(mx) + b_0^m \sin(mx)\} \exp\{-k_x mz\} + \\
+ \sum_{n=1}^{N_0} \{ a_n^0 \cos(ny) + c_n^0 \sin(ny)\} \exp\{-k_y nz\} + \\
+ \sum_{m=1}^{M} \sum_{n=1}^{N} \Big\{ a_n^m \cos(mx)\cos(ny) + b_n^m \sin(mx)\cos(ny) + \\
+ c_n^m \cos(mx)\sin(ny) + d_n^m \sin(mx)\sin(ny) \Big\} \exp\Big\{ -\sqrt{(k_x m)^2 + (k_y n)^2}\, z \Big\}
\end{aligned}
\tag{10}
$$

where $k_x=2\pi/L_x$ and $k_y=2\pi/L_y$, where L_x and L_y, are the dimensions of the rectangular region in the x and y directions, respectively. In this way the dimensions in the horizontal coordinates are normalized to 2π.

Although the components of the geomagnetic field obtained in this way are really derived from a potential that satisfies Laplace's equation, so that they suppose an analytical solution to the problem, the expansion does not converge uniformly over its interval of validity, but it is convergent only in mean square. This is because the functions used as a basis are periodic within such an interval; meanwhile the potential expanded in terms of such functions is not. In this way the termwise derivatives with respect to x or y are divergent. The effect can be appreciated by the exhibition of some ringing at the boundaries.

On the other hand, the terms Ax, By, and Cz violate the boundary conditions for a potential only due to internal sources, which impose that it must be zero when z tends to infinity. Their presence is explained by the fact that they tend to compensate the mentioned ringing, as well as the problems that appear (especially when the area is large) by the rectangular approximation of the spherical geometry.

3. Spherical Cap Harmonic Analysis

The Spherical Cap harmonic Analysis, or SCHA, developed by Haines (Haines, 1985), does not have the above mentioned problems and its basis functions give a convergent expansion both for the potential and for any of its derivatives.

In this case, when solving Laplace's equation, the boundary conditions are the same as those in the spherical case, except those in θ at the cap boundary. For a spherical cap the potential V at θ_0 and its derivative with respect to θ must satisfy the following boundary conditions, where f and g are arbitrary functions:

$$V(r,\theta_0,\phi) = f(r,\phi) \tag{11}$$

$$\frac{\partial V(r,\theta_0,\phi)}{\partial\theta} = g(r,\phi) \tag{12}$$

It has been demonstrated that the first condition is satisfied by choosing those values of n such that the derivative of the potential with respect to the colatitude is zero:

$$\frac{dP_{n_k}^{m}(\cos\theta_0)}{d\theta} = 0 \tag{13}$$

Meanwhile, the second is satisfied for other values of n such that the potential itself is zero:

$$P_{n_k}^{m}(\cos\theta_0) = 0 \tag{14}$$

These boundary conditions are satisfied by the associated Legendre functions with again a real, but not necessarily integer, degree.

Since the different real values of n depend on m, they are described by $n_k(m)$, where k is an integer index chosen to order the different roots n for a given m. Thus defined, the $n_k(m)$ for which k-m *are* even are the roots of equation (13), and those for which k-m are odd are the roots of equation (14), when these equations are considered as equations in n.

By superposition and assuming the finite expansion approximation, the general solution of Laplace's equation for the spherical cap is:

$$\begin{aligned} V = a \sum_{k=0}^{KINT} \sum_{m=0}^{k} \left(\frac{a}{r}\right)^{n_k(m)+1} P_{n_k(m)}^{m}(\cos\theta) \sum_{q=0}^{QINT} \left\{ g_{k,q}^{m,i} \cos(m\phi) + h_{k,q}^{m,i} \sin(m\phi) \right\} t^q + \\ + a \sum_{k=1}^{KEXT} \sum_{m=0}^{k} \left(\frac{r}{a}\right)^{n_k(m)} P_{n_k(m)}^{m}(\cos\theta) \sum_{q=0}^{QEXT} \left\{ g_{k,q}^{m,e} \cos(m\phi) + h_{k,q}^{m,e} \sin(m\phi) \right\} t^q \end{aligned} \tag{15}$$

As it can be seen, equation (15) includes the possibility of adding a polynomial temporal dependence for the potential. Here the 2-D functions given by the product of the Legendre functions in colatitude with the trigonometric functions in longitude are called spherical cap harmonics, in analogy with the spherical harmonics in the global context.

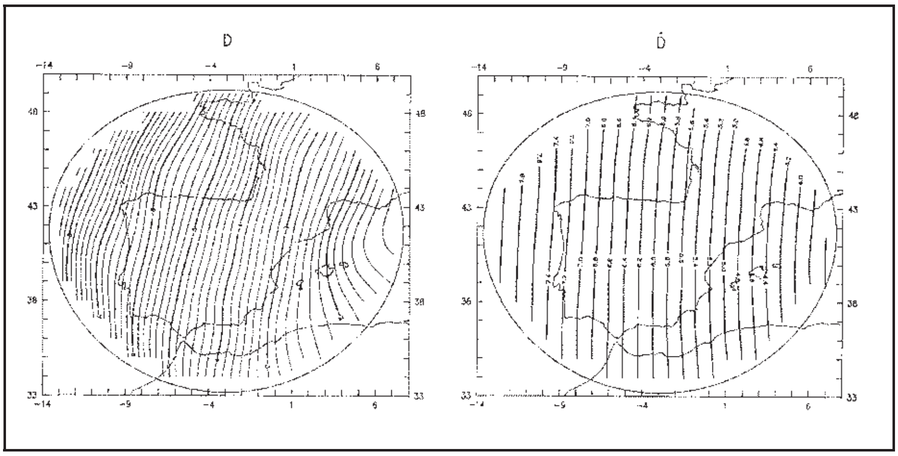

Figure 144. Contour maps of magnetic declination (left) and its annual change (right) for 1990 from a reference model for Spain (Torta et al., 1993).

The spatial wavelength over the Earth's surface represented by the model, for a harmonic of degree n_k, is simply given by the quotient between the Earth's perimeter and the harmonic degree. Table 29 shows the maximum (when K is equal to 4) and minimum degrees associated with an SCHA performed over different cap sizes, and the corresponding wavelengths involved. So, for a 16º cap, which is roughly the size of Europe, the harmonics start at n equals 6.1 and quickly reaches almost 25. When looking at the main field and its secular variation, which are characterized by degrees ranging from 1 to 10 or 12, the use of big caps is necessary; otherwise unrealistic detail is obtained in the maps. This did happen in a first attempt to apply the technique to the secular variation over Spain and adjacent areas (Torta et al., 1992), where, even though the data was kept over the original 16º cap, the model became more and more realistic as the size of the cap was increased. The boundary conditions (13) and (14) of the spherical cap harmonics were then defined in a realistic way at the border of the cap.

A similar procedure (Duka et al, 2004) was recently used for Albania and Southern Italy with data only restricted to a 3º cap but with the real area enlarged to a cap of 8º. Since this cap is still very small, the authors limited the expansion to a K equal to 2.

With the model coefficients, it is possible to obtain maps for any of the magnetic elements and for any epoch within the interval of validity of the model; for instance, for the magnetic declination, the relevant element for aeronautical navigation. Figure 144 shows an example of the magnetic declination obtained for the Iberian Peninsula for 1990, in degrees East, and the annual change, in minutes per year, for the same epoch.

Table 29. Maximum and minimum degrees and wavelengths (in Km) associated with spherical cap harmonic analyses over different cap sizes.

θ_0	n_{max} (K=4)	w_{min}	n_{min} (K=4)	w_{max}
16º	24.6277	1625	6.1481	6511
20º	19.6044	2042	4.8432	8265
25º	15.5864	2568	3.8056	10519
30º	12.9083	3101	3.1196	12832
35º	10.9958	3641	2.6347	15194
40º	9.5619	4187	2.2754	17593
45º	8.4471	4738	2.0000	20016

For all the above mentioned reasons the use of the SCHA technique has been growing among the geomagnetic field modeling community. Table 30 is a compendium of all the up-to-date English-written published papers and reports known to us about the technique and its applications.

Table 30. List of English language papers related to the SCHA as of May 2005.

ABOUT THE TECHNIQUE
Haines, J. Geophys. Res. 90, 2583, 1985
Haines, HHI-Rep. 21, 27, 1987
Haines, Comput. Geosc. 14, 413, 1988
Haines, J. Geomag. Geoelectr. 42, 1001, 1990
Haines, Phys. Earth Planet. Inter. 65, 231, 1991
Torta et al., Phys. Earth Planet. Inter. 74, 209, 1992
Haines, Geophys. J. Int. 114, 490, 1993
De Santis & Falcone, Proc. II Hot. Marus. Symp. 1994
Torta & De Santis, Geophys. J. Int. 127, 441, 1996
De Santis et al., Phys. Earth Planet. Inter. 97, 15, 1996
De Santis et al., J. Geomag. Geoelectr. 49, 359, 1997
De Santis & Torta, J. Geodesy 71, 526, 1997
De Santis et al., Annali di Geofisica 40, 1161, 1997
Lowes, Geophys. J. Int. 136, 781, 1999
De Santis et al., Phys. Chem. Earth A 24, 935, 1999
Düzgit & Malin, Geophys. J. Int. 141, 829, 2000
Korte & Holme, Geophys. J. Int. 153, 253, 2003
Thébault et al., Geophys. J. Int. 159, 83-103, 2004
VARIATIONS OF THE TECHNIQUE
De Santis, Geophys. J. Int. 106, 253. 1991
De Santis, Geophys. Res. Lett. 19, 1065, 1992
REFERENCE FIELD MODELS

Haines & Newitt, J. Geomag. Geoelectr. 38, 895, 1986
Nevanlinna et al., Deut. Hydro. Zeits. 41, 177, 1988
Newitt & Haines, J. Geomagn. Geoelectr. 41, 249, 1989
De Santis et al., J. Geomagn. Geoelectr. 42, 1019, 1990
Newitt & Haines, Curr. Res. E, G.S.C., 275, 1991
Nevanlinna & Rynö, HHI Rep. 22, 106, 1991
Torta et al., J. Geomag. Geoelectr. 45, 573, 1993
An et al., J. Geomag. Geoelectr. 46, 789, 1994
An et al., Geomagn. Aeron. 34, 581, 1995
Haines & Newitt, J. Geomag. Geoelectr. 49, 317, 1997
Kotzé & Barraclough, J. Geomag. Geoelectr. 49, 452, 1997
Chiappini et al., Phys. Chem. Earth 24, N.5, 433, 1999
Kotzé, Earth Planets Space 53, 357, 2001
De Santis et al., Geophys. Res. Lett. 29, N. 8, 33-1, 2002
Gaya-Piqué, PhD Thesis, URL, 162 pp., 2004
Duka et al., Ann. Geophys. 47, 1609-1615, 2004
Gaya-Piqué et al., Earth Obs. with CHAMP. Berlin: Springer, 317-322, 2005
SECULAR VARIATION MODELS
Haines, J. Geophys. Res. 90, 12563, 1985
García et al., Phys. Earth Planet. Inter. 68, 65, 1991
Miranda et al., J. Geomag. Geoelectr. 49, 373, 1997
Torta et al., Tectonophysics 347, 179, 2002
DETERMINATION OF THE NORTH MAGNETIC POLE
Newitt & Niblett, Can. J. Earth Sci. 23, 1062, 1986
Newitt & Barton, J. Geomag. Geoelectr. 48, 221, 1996
Newitt et al., EOS 83, 381, 2002
ANOMALY FIELD MODELS
Haines, J. Geophys. Res. 90, 2593, 1985
De Santis et al., NATO ASI Series C 261, 1, 1989
Torta et al., Cahi. Cent. Eur. Geod. Seis. 4, 179, 1991
An et al., J. Geomag. Geoelectr. 44, 243, 1992
Duka, Annali di Geofisica 41, 49, 1998
Zhen-chang et al., Chin. J. Geophys. 41, 42, 1998
Rotanova & Odintsov, Phys.Chem. Earth A 24, 455, 1999
Rotanova et al., Acta Geophys. Pol. 48, 223, 2000
Korte & Haak, Phys. Earth Planet. Inter. 122, 205, 2000
Kotzé, Geophys. Res. Lett. 29, N. 15, 5-1, 2002
GEOMAGNETIC VARIATIONS EXTERNAL ORIGIN
Walker, J. Atmos. Terr. Phys. 51, 67, 1989

Newitt & Walker, J. Geomag. Geoelectr. 42, 937, 1990
Haines & Torta, Geophys. J. Int. 118, 499, 1994
Torta et al., J. Geophys. Res. 102, 2483, 1997
Walker et al., J. Atmos. Sol. Terr. Phys. 59, 1435, 1997
Amm, Ann. Geophys. 16, 413, 1998
IONOSPHERIC PARAMETERS
De Santis et al., Ann. Geophys. 9, 401, 1991
De Santis et al., Adv. Space Res. 12, N. 6, 279, 1992
De Santis et al., Comput. Geosc. 20, 849, 1994
El Arini et al., Proc. Int. Beacon Sat. Symp. 358, 1994
Dremukhina et al., J. Atmos. Sol. Terr. Phys. 60, 1517, 1998
GEODESY
Jiancheng et al., Manuscr. Geod. 20, 265, 1995
Hwang & Chen, Geophys. J. Int. 129, 450, 1997

Spherical Harmonic Analysis is well founded, but the SCHA is still in the dawn of its existence, so it must be used with some precaution. For instance, one must be aware of the problems concerning external-internal separation. Torta and De Santis (1996) performed an analysis of the daily variation over a cap of 18º, corresponding to the area represented by the European continent. They showed that while the fit to the total variation for any of the points of that area is excellent, the external and internal parts of such variation are not exactly the real ones, so that the errors in the external and internal fields are equal and opposite. The situation improves substantially with a cap of 30º, and further with larger caps, as soon as the intrinsic spectral content of the phenomenon to analyze coincides with that of the models basis functions. In any case, as the real and modeled separated fields are approximately in phase, the information about the ionospheric current systems generated by the magnetic variations is still valid.

These problems appear because the external-internal separation in reality implies the comparison of separate analyses for the horizontal and vertical components (Matsushita and Campbell, 1967). And, when the region becomes small enough, one of the first things that we appreciated (García et al., 1991) is that the potential for the horizontal and radial components cannot be simultaneously exactly represented. In fact, analyzing all components at the same time provides an approximation that attempts to fit both, but it is not as precise as fitting Z separately from X and Y. In any case, this problem is not unique to the SCHA; it appears with any method that attempts to analyze fields with wavelengths much larger than the area covered by the data (Lowes, 1995, 1999).

Even though the resulting potential represents a good approximation within the cap where the data are located and in their altitude range, it irremediably diverges for the opposite pole (because of the non-integer degree of the Legendre functions). So that, it is little by little intrinsically different form the real potential out of that area, and it prejudices the vertical extrapolations as well (Figure 145).

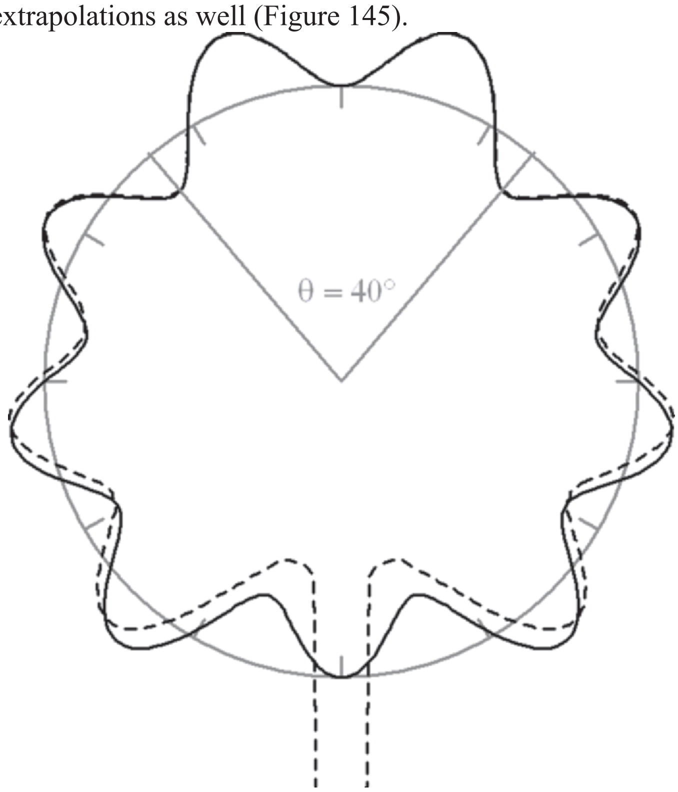

Figure 145. Solid line: associated Legendre function (n=9, *m*=2) at θ=35º represented at all longitudes around a sphere. The same function has been computed on a regular grid inside a 40º spherical cap and fitted with an SCHA. Dashed line: the result of such SCHA over the same circular path around the sphere.

A revision of the technique has been recently presented (Thébault et al., 2004) in which the potential expansion is expressed as complex Legendre (conical) functions in colatitude and log-trigonometric series in longitude. The local potential Vc expanded in the local basis (a spherical cap defined within r=a and r=b) is given as $Vc = V_1 + V_2$, where V_1 *is* the same potential as given by Haines (1985) for the even-set and V_2 is defined as:

$$V_2 = a\sum_{p\geq 1}\sum_{m>0} R_p(r)\left\{G_p^m \cos(m\phi) + H_p^m \sin(m\phi)\right\} K_p^m(\theta) + \\ + a\sum_{m>0} R_0\left\{G_0^m \cos(m\phi) + H_0^m \sin(m\phi)\right\} P_0^m(\theta) \quad (16)$$

where R_0 is the square root of $1/(e^S-1)$ (with $S = \ln(b/a)$),
and

$$R_p(r) = \frac{(2S)^{-1/2}}{\sqrt{\left(\frac{2\pi p}{S}\right)^2 + 1}}\sqrt{\frac{a}{r}}\left\{\frac{2\pi p}{S}\cos\left[\frac{\pi p}{S}\log\left(\frac{r}{a}\right)\right] + \sin\left[\frac{\pi p}{S}\log\left(\frac{r}{a}\right)\right]\right\} \quad (17)$$

where p is an integer belonging to the imaginary part of a complex harmonic degree n (see Thébault et al., 2004). $K^m{}_p$ are conical functions, i.e. Legendre functions with n = complex.

The advantage of this proposal is that it provides a better fit for the radial variation, even for the *X, Y*, and *Z* components, with respect to the classical SCHA. However, the internal-external separation is not made in *r*, but with respect to the cap region (i.e. internal or external to the cap).

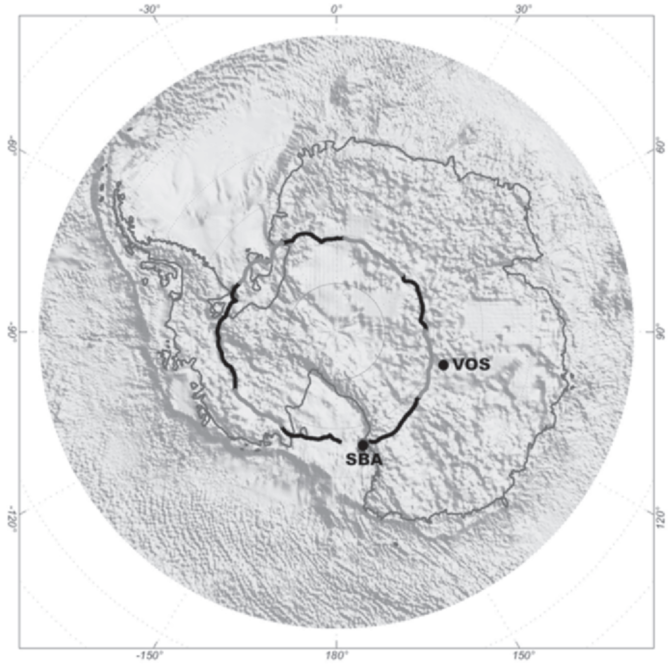

Figure 146. Path followed by the balloon (anticlockwise sense) carrying the magnetic instrumentation. Alternate colors mean different days.

4. Antarctic Reference Model

Assuming very carefully all the above mentioned limitations, the last application of the technique was revised (Gaya-Piqué, 2004; Gaya-Piqué *et al.*, 2006). It is called the Antarctic Reference Model (ARM) and to our knowledge it is the first full main field (i.e. main field plus its secular variation) geomagnetic model for the Antarctic. A model in this region is very important not only for scientific studies but especially for navigating with a compass there. It would be almost impossible without correct knowledge of large differences in declination typical of Antarctica.

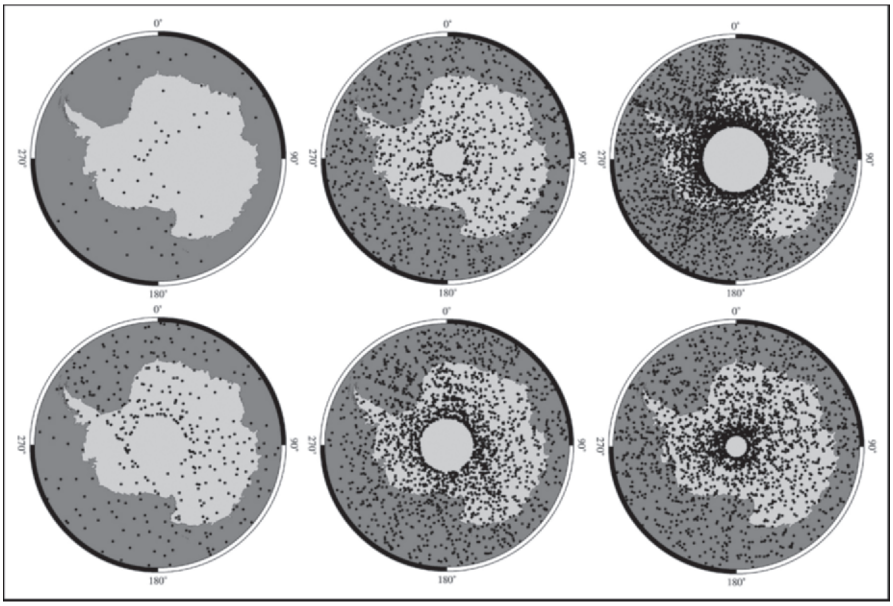

Figure 147. Spatial distribution of the satellite magnetic measurements used to develop the Antarctic Reference model. From left to right and from top to bottom: OGO-2, OGO-4, OGO-6, MAGSAT, ØRSTED and CHAMP.

The model has been developed using the most recent data sets available for the region. The annual means from 1960 on from all Antarctic magnetic observatories south of 60ºS were used. However, the number and the extent of gaps in the data are important to note, because the time derivatives simply taken as the first differences tend to provide non-realistic values of the secular variation. To overcome this problem we took differences relative to a fiducial observation, in particular with respect to the mean over all data at each observatory. In this way, both the main field and the crustal anomaly are removed, obtaining for the i^{th} measurement of the field at the v^{th} observatory at epoch t_{iv} (Haines, 1993):

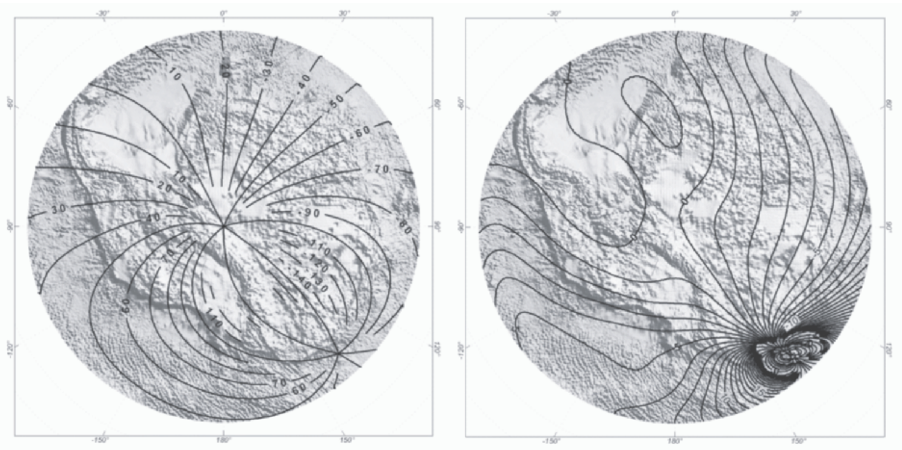

Figure 148. Contour maps of declination (left, units: degrees East) and its annual change (right, units: minutes/year) for 2005 from the Antarctic Reference model.

$$\vec{B}_{iv}(t) - \overline{\vec{B}}_v = \sum_{q=1}^{Q} \vec{b}_{qv} \left(t_{iv}^q - \overline{t_v^q}\right) + \vec{\varepsilon}_{iv} - \overline{\vec{\varepsilon}}_v \qquad (18)$$

where $\vec{b}_{qv}$ are spatial functions (expressed as series expansions in spherical cap harmonics) evaluated at the position of the v^{th} observation, and $\vec{\varepsilon}_{iv}$ is the measurement error. In equation (18) the temporal basis functions are expressed as power functions, but they can be chosen as Legendre, Fourier, or any other appropriate set of functions.

Secondly, a balloon mission was undertaken to obtain a data set of magnetic measurements from stratospheric altitude (Figure 146). Special attention was given to this mission since its magnetic measurements were used for the first time in this work.

Finally, magnetic data from six satellite missions were used, covering epochs over the 40 years of validity of the model. These data have been selected according to different criteria to model only values corresponding to magnetically quiet periods (Figure 147).The model parameters follow:

- A 30° half-angle Spherical Cap centered at the South Pole
- A maximum spatial degree expansion of K=8, which means $n \approx 25$, or a wavelength of approximately 1,600 km
- A variable maximum temporal degree expansion using cosine series

One hundred sixty three statistically significant coefficients were obtained by means of a stepwise regression procedure (see Haines and Torta, 1994 for details; an alternative to this procedure based on a regularized method has been recently presented by Korte and Holme, 2003). The regression procedure allows for the determination of the field for epochs between 1960 and 2005 (Figure 148). The fit to the secular variation of the observatory annual means is better than those of global models like IGRF or the Comprehensive model (Gaya-Piqué *et al*.., 2006), and in Figure 149 one can see its validity, with good fit of the magnetic elements at four different observatories being obtained.

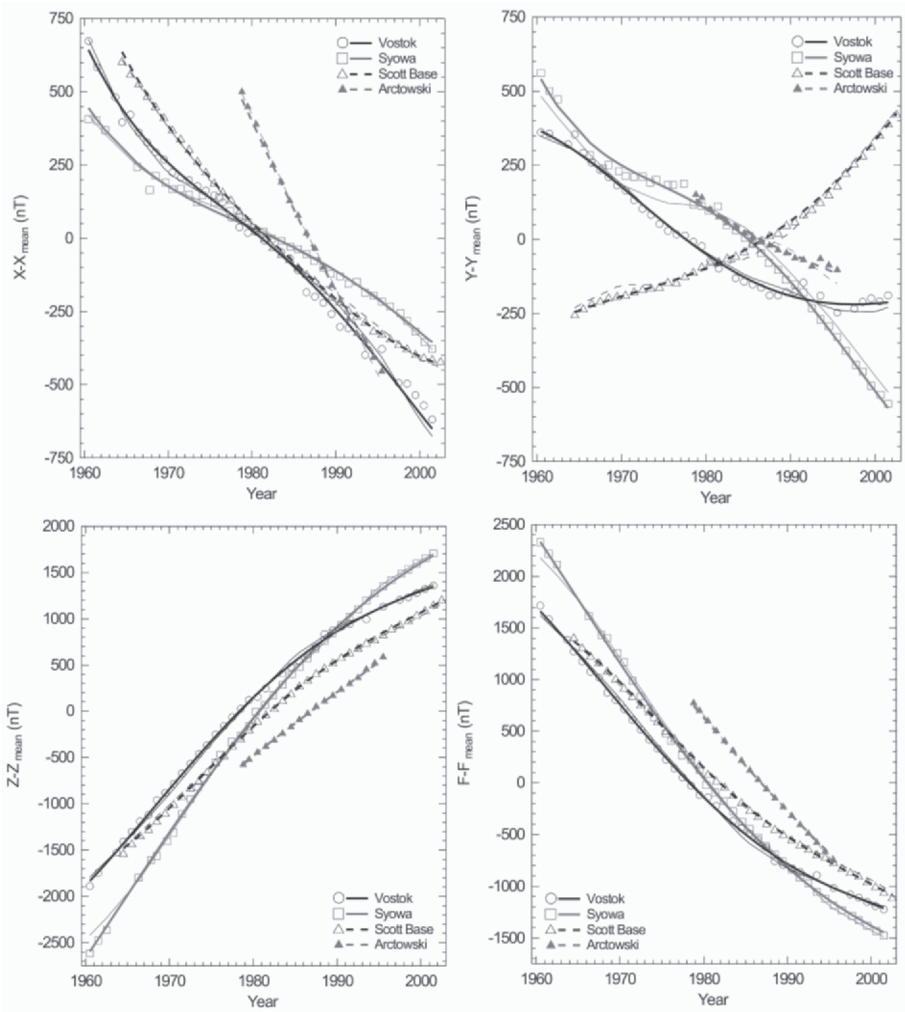

Figure 149. X (top left), Y (top right), Z (bottom left), and F (bottom right) annual means registered at ARC (solid triangles), SBA (open triangles), SYO (squares), and VOS (circles) observatories relative to their respective mean values over the time period. The thick lines show the fit given by ARM, and the thin lines that by IGRF 9th generation.

5. Conclusions

It has been shown how it is possible to develop models that reflect the spatial and temporal variations of the geomagnetic field in a restricted region with more detail and precision than is usually possible using the standard global models. Thus, regional models of secular variation allow for a better integration of disparate magnetic surveys and uniform digital anomaly charts can be obtained in this way. The spatial precision can be achieved using the Spherical Cap Harmonic Analysis technique. The temporal dependence of differences in the main field relative to the means at each observatory has been shown by analysis to be definitively more robust than the fit to variations obtained by numerical differentiation.

It was not the intent of this paper to present the SCHA method as the definitive technique for the analysis of the geomagnetic field in a restricted region of the Earth's surface. The intention has been to demonstrate how the SCHA can be of great value in some applications, once its drawbacks are analyzed, and whenever it is used conscientiously with its limitations recognized.

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DISCUSSION

Question (Spomenko J. Mihajlovic): For which area do you think that SCHA is an optimal method (How wide should the area be)?

Answer (Miquel Torta): The area where SCHA is optimal depends on what kind of field one wants to represent. The spatial wavelength over the Earth's surface represented by the model, for a harmonic of degree n_k, is simply given by the quotient between the Earth's perimeter and the harmonic degree. And the different values that take the n_k harmonics depend on the cap size, the bigger the cap the smaller the harmonics, and so the larger the wavelengths associated with each harmonic. Therefore, if one is interested in representing the main field and its secular variation, which we know are characterized by degrees going from 1 to 12 or 13, the use of big caps (say of continental size) are necessary; otherwise we will have unrealistic detail in our maps. If the model is aimed at representing smaller scale features (e.g. lithospheric anomalies), a small cap (e.g. of few degrees cap half-angle) can be suitable. As a rule of thumb, the area of the existence of the feature (or features) of the field that is going to be represented must be at least to some extent coincident with the cap-like region defining the analysis.