NONLINEAR TECHNIQUES FOR SHORT TERM PREDICTION OF THE GEOMAGNETIC FIELD AND ITS SECULAR VARIATION

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Abstract. Recent studies appeared in literature on the chaotic behavior of the dynamical system producing the geomagnetic field, i.e. the geodynamo. They analyzed the secular variation as deduced from observatories annual means (Barraclough and De Santis, 1997; De Santis et al., 2002), as well as the information content of global models for the last century (De Santis et al., 2004), showing some interesting nonlinear properties. Suitable nonlinear techniques can be applied for short term prediction of the geomagnetic field, i.e. to extrapolate the field 1-2 years into the future. Using these methods it is possible to update geomagnetic field maps for navigational purposes and to improve the prediction in heliports and airports of the magnetic declination which is important for the safety and security of all operations related to landing and take-off.

Keywords: Nonlinear prediction; geomagnetic field; chaos; declination

1. Introduction

The geomagnetic field surrounding the Earth protects us from most of the outer space radiation. With its space and time variations it reveals many features of the dynamics of the outer terrestrial core, where the field is generated by means of the electric currents produced by the fluid convection of conductive iron alloys, a process called the geodynamo mechanism. Compasses provide the simplest way to know orientation in the

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Earth reference frame. Exact knowledge of the compass pointing requires periodic monitoring of the magnetic declination, i.e. the angle between the direction of the magnetic field and true North. To this purpose national networks of magnetic stations are maintained and repeated every 3-5 years to collect magnetic measurements. Also, at airports and heliports the magnetic declination is measured periodically with particular attention for compass calibration.

Accurate measurements of the geomagnetic field are fundamental for the above mentioned reasons. But because of the unpredictable year by year change of the field, the secular variation becomes outdated as soon as a map is produced. To use the compiled maps some sort of short term prediction is needed, so prediction is not an option but a necessity. Usually prediction is simply linear extrapolation, that takes into account the mean change of the field over a certain period of time and then assumes the same change for the future. Commonly this linear prediction provides significant deviations from the real values, even after only one or two years. For this reason, a repetition of the magnetic measurement becomes necessary. Clearly, improving the prediction would allow prolonging the time interval between series of measurements.

The aim of this paper is to introduce and apply a new technique that should in principle improve the prediction results.

Some nonlinear techniques have been applied to magnetic data to find possible chaos or fractality of the geomagnetic field (Barraclough and De Santis, 1997; De Santis et al., 2002) with satisfying results. Thus, the idea here is to apply the same techniques to predict the geomagnetic field. After some generalities on the results obtained recently in terms of nonlinear features of the geomagnetic field, a nonlinear technique called the nonlinear forecasting approach (NFA) will be described and applied to make reasonable short term (1-2 years) predictions.

The NFA's most important points and possible future applications will be assessed as well.

2. Nonlinear features of the geomagnetic field

When a phenomenon shows that small or great changes of some initial conditions correspond to small or great changes of its evolution, respectively, its dynamics are said to be linear. Conversely, when small changes of some initial conditions involve unpredictable great changes in the future evolution, the dynamics are said to be nonlinear and this phenomenon is called sensitivity to initial conditions. In other words, in the latter case there is a nonlinear relation between the input (changes of initial conditions) and the output (future values of the signal under study). If this relation can be written in exponential form with positive exponent, the dynamics and the corresponding system are said to be chaotic. The chaotic nature of the geomagnetic field reflects the chaoticity of the system generating it, that is the chaoticity of the geodynamo.

In the recent years we have tried to find evidence of the chaoticity of the geomagnetic field. The vector power spectrum of the observatory annual means shows an almost power-law (linear) behaviour in the (log-log) plot for periods ranging from 6 years to around a century (De Santis et al., 2003). This can be explained as a consequence of the chaotic state of the magnetic field because when a phenomenon is chaotic it usually shows scaling spatial and temporal spectra with defined spectral exponents. Starting with simple assumptions about the spatial power spectra of the geomagnetic field and its secular variation, it was possible to predict the temporal power law spectrum with a specific scaling.

More recently (De Santis et al., 2004) some statistical concepts related to the Information Theory, such as the *information content* $I(t)$, have been applied to the last century (years 1900-2000) of IGRF (International Geomagnetic Reference Field) global models. $I(t)$ is a negative quantity which measures the knowledge of the state of the system when knowing only the probability distribution of all the possible states of the system. When a system is chaotic, $I(t)$ decreases linearly in time and the inverse of the (negative) slope defines a characteristic time of the dynamical system after which it is not possible to make any prediction at all. Linear plots with characteristic times of around 850 and 420 years were found when applying this concept to the geomagnetic field and its secular variation, respectively. From the application of L'Hôpital theorem to the definition of the probability used for the information content, the rough agreement of the two characteristic times was interpreted as a possible symptom of an impending geomagnetic reversal or excursion. The chaotic state of the geomagnetic field has then been considered a manifestation of this possible change of state of the field (De Santis et al., 2004).

The possible chaotic state of the geomagnetic field is also supported by the fractal magnetic potential at the core-mantle boundary as deduced from global models from around 1600 to present (De Santis and Barraclough, 1997).

The above considerations and results are indicative of nonlinear, possibly chaotic dynamics, of the geomagnetic field and suggest that a nonlinear technique is probably more reliable for making predictions than linear techniques.

In the following section, one of these nonlinear techniques will be introduced and some preliminary results will be shown.

3. Nonlinear forecasting

The application of nonlinear forecasting here described originates from some recent results supporting the idea that the geomagnetic field secular variation seems to be the result of a dynamical system (the fluid outer core of the Earth) possibly characterized by a chaotic behavior. Barraclough and De Santis (1997 and De Santis et al. (2002) apply a nonlinear forecasting technique (Sugihara and May, 1990) to discriminate deterministic chaos from randomness and periodicity in geomagnetic time series. These time series consist of the secular variation of the Cartesian components (X, Y and Z) of the Earth's magnetic field estimated as the first-differences from observatory annual means.

With random signals, the ability to predict future values is small and independent of the prediction interval, i.e. how far into the future the prediction is made. Periodic signals are characterized by high predictability and are independent of the prediction interval. With chaotic signals, predictability deteriorates as the prediction interval increases: it is good for short time predictions, but rapidly approaches zero after a certain characteristic time (related to the specific dynamics of the system generating the signal under study; Sugihara and May, 1990). The first step in nonlinear forecasting is to reconstruct the phase space starting with the time series and applying the Takens theorem (Takens, 1981). According to this theorem, the dynamics on each n-th axis of the space can be represented by the time series itself if shifted by (n-1) times a proper delay, τ. The second step is to evaluate the so-called Largest Lyapunov exponent of the chaotic system which is related to the way the prediction ability deteriorates by increasing the prediction interval (Wales, 1991). In fact, for a chaotic system, two initially close orbits in the phase space diverge along a certain axis as $e^{\lambda t}$, where t is time and λ the so-called Lyapunov exponent associated with that axis. A three-dimensional dynamical system has three Lyapunov exponents, and if the largest exponent is positive, we say that the system is chaotic, because there is the tendency for the orbits to diverge at least in one direction of the phase space. For the Eastward component, Y, of the geomagnetic field the found largest Lyapunov exponent was around 0.2 year^{-1} corresponding to a characteristic time of around 5 years, after which no reasonable prediction can be made. This value supports the practice of updating the International Geomagnetic Reference Field (IGRF) every 5 years.

This paper considers the nonlinear forecasting approach suggested by Fowler and Roach (1993). This approach allows us to determine the predicted value at a certain time t, termed the *predictee*, by comparing the (t-1) value with all past values. In fact, looking for the past numerical value, say the *n*-value, closest (thus termed the *similar*) to the (*t*-1) value, the *predictor* will be identified as the next value to the similar, i.e. the *n*+1 value. More generally in an *E*-dimensional phase space, the last known value (an *E*-dimensional point) is compared with all the other known values to find the *E*+1 nearest points (similar points) that are, therefore, characterized by the shortest distances from the last known point. Finally, the forecast is made by controlling the time evolution of these similar points after they have been inverse squared weighted.

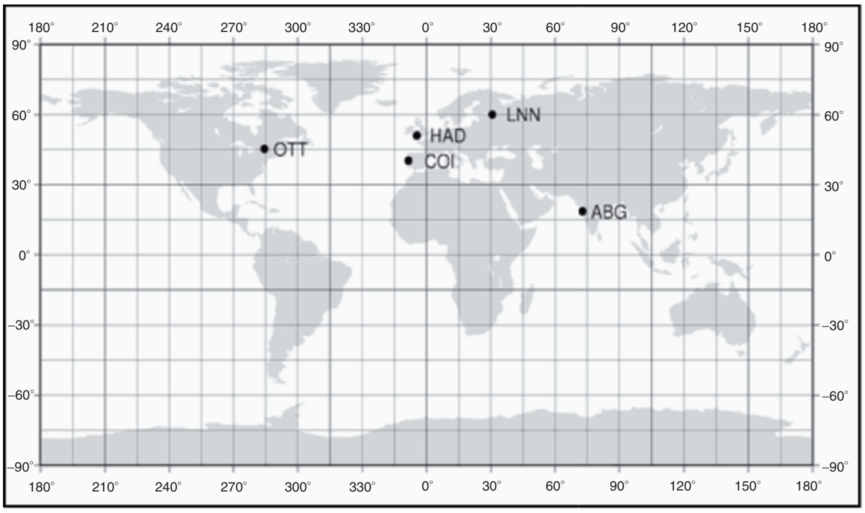

Figure 142. Geographical distribution of the five selected observatories indicated by their IAGA codes.

The number *n* of axes necessary to reconstruct the dynamics represents the degrees of freedom of the system, and *n*+1=*E* is also said the *embedding dimension* of the dynamics. For the secular variation of the geomagnetic field, a three-dimensional space (*n*=3 and *E*=4) is quite enough to get all the topological structure of the ideal phase space (Barraclough and De Santis, 1997). The appropriate delay τ can be estimated as the time when the autocorrelation function of the signal is close to zero. For observatory annual means, this value is about 1 year, that is the sampling itself of the time series.

Analyzed data come from five selected geomagnetic observatory time series of the Y component secular variation, whose geographical distribution is shown in Figure 142. The forecasting technique previously described was applied by averaging just the four points of the phase space closest to the most recent value in the time series. This technique was well able to predict the secular variation of the geomagnetic field 1-2 years into the future. In principle, a longer prediction interval would not be reliable because the ability of forecasting approaches zero after 5-6 years. Figure 143 shows the prediction (grey bold line) of the secular variation for the selected observatories together with the real values (black thin line). Since the technique is particularly suitable for prediction of the magnetic component Y, and therefore presumably, for the magnetic declination D, it can be used to update declination values at specific places, in particular at airports and heliports, where accurate measurements are critical to the safety of aircraft operations.

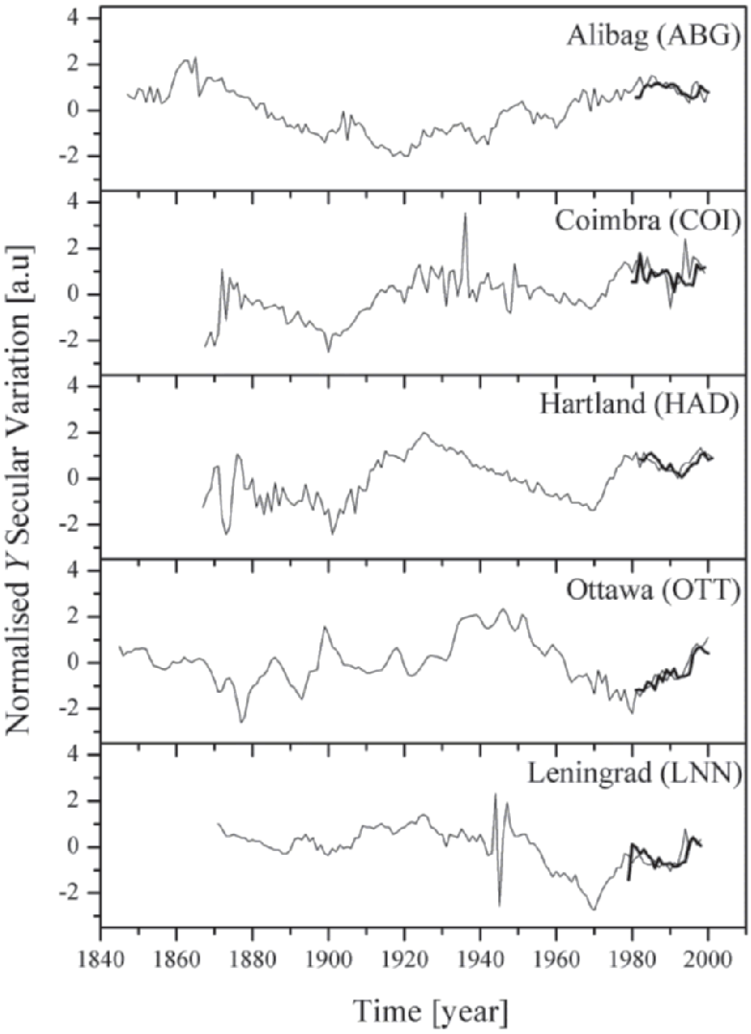

Figure 143. Normalized secular variation of the Eastward component of the geomagnetic field in arbitrary units: first-differences estimated from observatory annual means data (black thin line); secular variation predicted by means of nonlinear forecasting approach (grey bold line).

4. Conclusions

In this paper some nonlinear features of the geomagnetic field have been described. These, in turn, justify the application of a nonlinear technique to make reliable short-term predictions (1-2 years) of secular variation. Both the small number of time series to which the technique was applied, and the simple prediction scheme applied to the phase space, gave encouraging results and warranted further investigation. The application to a greater number of time series would be useful to search for a better scheme of phase space interpolation that would improve the final prediction. This kind of short term technique is potentially applicable to forecast future (1-2 years) values of the magnetic field elements at observatories. This would allow a better extrapolation of the geomagnetic secular variation of global models or at repeat stations. Therefore it would improve the regional maps of the geomagnetic field, in particular those of magnetic declination, which are so useful for navigation. Another application would be to make short term prediction of the declination at heliports and airports, where it is so important for the safety and security in all related operations.

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References

Barraclough, D.R., De Santis, A., 1997. Some possible evidence for a chaotic geomagnetic field from observational data. *Phys. Earth Planet. Inter*., **99**, 207-220.

De Santis, A., Barraclough, D.R., 1997. A fractal interpretation of the topography of the scalar geomagnetic scalar potential at the core-mantle boundary, *Pure and Applied Geophys*., **149**, No.4, 747-760.

De Santis, A., Barraclough, D.R., Tozzi, R., 2002. Nonlinear variability in the geomagnetic secular variation of the last 150 years, *Fractals*, **10**, 3, 297-303.

De Santis, A., Barraclough, D.R., Tozzi, R., 2003. Spectral and temporal spectra of the geomagnetic field and their scaling properties, *Phys. Earth Planet. Int*., **135**, 125-134.

De Santis, A., Tozzi, R., Gaya-Piqué, L., 2004. Information Content and K-entropy of the Present Earth Magnetic Field, *Earth and Planetary Science Letters*, **218**, 269-275.

Fowler, A.D., Roach, D.E., 1993. Dimensionality analysis of time-series data: nonlinear methods. *Comput. Geosc.*, **19**, 41-52.

Sugihara, G., May, R.M., 1990. Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series. *Nature*, **344**, 734-741.

Takens, F., 1981. Detecting strange attractors in turbulence. In: *Lecture notes in mathematics*, **898**. Ed.s: D.A. Rand, L.S. Young. Springer, Berlin, pp. 366.

Wales, D.J., 1991. Calculating the rate of loss information from chaotic time series by forecasting. *Nature*, **350**, 485-488.

DISCUSSION

Question (Jean Rasson): Can you give the exact nature of the axes in the 3D phase space you use to establish your prediction?

Answer (Angelo De Santis): In general, a phase space is composed of a certain number of axes, each axis is a generalized coordinate characterizing the dynamics of the system we are studying and the number of axes corresponds to the degrees of freedom of the represented dynamic system. If the dynamics of the system is characterized by n independent differential equations of n unknowns, then n is the degrees of freedom and at each axis we can place the variation of each unknown. When the differential equations of the dynamics are not known, following to Takens' theorem (1981), from a signal f(t) it is possible to reconstruct the phase space placing at each axis f(t), f(t+τ), f(t+2τ) …, with τ an appropriate delay time, usually corresponding to the first zero of the autocorrelation function of f(t). Each state of the dynamics of the system will visit one and only one site in the reconstructed phase space, and the topology of the 'shape' reconstructed by all orbits is specific of that system only, so that its study allows in principle to extract much information about the properties of the system and its dynamics.

What was said above is strictly valid when the signal characterizes a chaotic system. Necessary ingredients for a system to be chaotic are determinism, nonlinear differential equations of the dynamics, and initial condition sensitivity.

Question (Jürgen Matzka): How do data gaps affect NFA and bicoherence? Can the past be predicted (before the observatory was established)?

Answer (Angelo De Santis): Gaps have little effect on the NFA, since if it is the 'topology' that we are interested (to extract information such as degrees of freedom, divergence of orbits in the phase space, etc. or to infer some prediction) small gaps do not necessary change the gross properties of the phase-space in that sense. Also Bicoherence could be little affected, if we use some specific scheme of Fourier Transformation for irregularly distributed data, although it is probably more sensitive to gaps than the former technique. Of course, for obtaining positive results, in both cases gaps must be the exception in the time series and not the rule.

Regarding the second part of the question, in principle the answer is ‘yes’, however, and this is counterintuitive, our ability to predict the past of chaotic phenomena is worse than the ability to predict their future! This can be explained by the fact that total divergence of the orbits is larger when going back in time.

Question (Sanja Panovska): What is the semi-angle of SCHA for normal field for the territory of the Republic of Macedonia (within $\Delta\varphi=1^{o}31'$ and $\Delta\lambda=2^{o}35$ interval)

Answer (Angelo De Santis): In theory in your case a cap with half angle of around 1.5 degrees should be considered. However, in practice, such a small cap would imply basis functions having very high degrees n_k, entailing great difficulties in their computations. In my opinion, in the case of the Republic of Macedonia, it would be easier to apply some other technique for representing the geomagnetic field, for instance the rectangular harmonic analysis (Alldredge, 1981).

Question (Sanja Panovska): I know the theory for SCHA but I don’t know how to put the temporal factor in equation (if I have data from 2003 which value for <t> to use)?

Answer (Angelo De Santis): For data distributed in a short time as one year, I think you could consider just a linear time behaviour of the field, therefore t=1.