LOW-ENERGY PHOTON SCATTERING ON A POLARIZABLE PARTICLE

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Abstract. In the present paper a covariant Lagrangian constructed on the bases of the correspondence principle between the relativistic moving medium electrodynamics and relativistic quantum field theory requirements is used. The contribution and physical interpretation of the Lagrangian coefficients to invariant Compton scattering structures have been obtained. We have analyzed the phenomenological tensor constant quantities as well.

1. Introduction

By virtue of the low-energy theorem in $\mathcal{O}(\omega^2)$ the amplitude for spin-0 and -1/2 hadrons depends on their internal degrees of freedom, which are determined by the fundamental parameters such as the electric (α) and magnetic (β) polarizabilities [1, 2]. In turn, at $\mathcal{O}(\omega^3)$ the effective Lagrangian describing the photon-nucleon interaction and, as a result the amplitude for Compton scattering are determined by spin polarizabilities [3]-[9]. These characteristics are immediately connected to the spin properties of hadrons as a composite particles.

The classical process for the investigation of these features of the photon-hadron interaction is Compton scattering of photons, whose energies are below the resonance region. Nevertheless, data about spin polarizabilities can be extracted from other electrodynamic processes (see, for example Ref. [10]).

The determination of the hadron polarizability contributions to the amplitudes of QED processes is sequentially carried out by the effective Lagrangians of interaction of the electromagnetic field with the hadron as a composite particle. In the nonrelativistic electrodynamics such a Lagrangian is rather well determined. On the other hand, in the relativistic QED when hadrons look like bound states, due to the kinematic relativistic effects, the interpretation of polarizabilities is ambiguous [2, 9].

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In the present report the correspondence principle between relativistically moving medium electrodynamics and relativistic quantum field theory will be sequentially used for the covariant Lagrangian construction of interaction of the electromagnetic field with polarized spin hadrons. Then we determine the contribution of the Lagrangian structures to the invariant Compton scattering amplitudes. The field-theoretical properties of the energy-momentum tensor and the Hamiltonian in the static limit have been determined.

2. The covariant Lagrangian

First we define the Lagrangian of the photon-hadron interaction taking into account the electric and magnetic polarizabilities.

In the nonrelativistic case, the interaction Hamiltonian of the isotropicly-gyrotropic medium is [11, 12]:

$$H_I = -2\pi \left(\mathbf{PE} + \mathbf{MH}\right), \tag{1}$$

The polarization vectors $\mathbf{P}$ and magnetization $\mathbf{M}$ look like [11]

$$\mathbf{P} = \hat{\alpha}\mathbf{E}, \quad \mathbf{M} = \hat{\beta}\mathbf{H}, \tag{2}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the tensors of electric and magnetic polarizabilities, $\mathbf{E}$ and $\mathbf{H}$ are the strengths vectors of electric and magnetic fields.

Let us now define the effective Lagrangian in the relativistic QED for spin-0 and -1/2 hadrons taking into consideration their usual electric and magnetic polarizabilities.

According to the relativistic electrodynamics of the moving mediums the effective Lagrange function is [12, 8]:

$$L_{eff}^{pol} = 2\pi \left\{ e^{\mu}\alpha_{\mu\nu}e^{\nu} + h^{\mu}\beta_{\mu\nu}h^{\nu} \right\}. \tag{3}$$

In this expression $e_{\mu} = F_{\mu\nu}U^{\nu}$, $h_{\mu} = \tilde{F}_{\mu\nu}U^{\nu}$, $\tilde{F}_{\mu\nu} = \frac{i}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, where $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are the tensors of the electromagnetic field, $\alpha_{\mu\nu}$ and $\beta_{\mu\nu}$ are tensors determined by the polarizabilities in a medium at rest, U_{μ} is the 4-dimensional velocity of the medium, $\varepsilon_{\mu\nu\rho\sigma}$ – Levi-Chevita antisymmetric tensor ($\varepsilon^{0123} = 1$).

If we move from the Lagrangian (3) to the field-theoretical Lagrangian on the basis of the correspondence principle, then we obtain [8, 13]:

$$L_{eff\,spin-0}^{pol} = \frac{\pi}{m^2}\left(\varphi^{*}\,\overleftrightarrow{\partial^{\mu}}\overleftrightarrow{\partial_{\nu}}\,\varphi\right)K^{\nu}_{\mu}, \tag{4}$$

$$L^{pol}_{eff spin-1/2} = -\frac{i\pi}{m}\left(\bar{\psi}\,\gamma^{\mu}\,\overleftrightarrow{\partial_{\nu}}\,\psi\right)K^{\nu}_{\mu}, \tag{5}$$

for spin-0 and spin-1/2 particles respectively, where $K^{\nu}_{\mu} = \alpha_0 F_{\mu\rho}F^{\rho\nu} + \beta_0 \tilde{F}_{\mu\rho}\tilde{F}^{\rho\nu}$, $\overleftrightarrow{\partial_{\nu}} = \overleftarrow{\partial_{\nu}} - \overrightarrow{\partial_{\nu}}$, φ and ψ are the wave functions of spin-0 and spin-1/2 particles.

In order to define the effective Lagrangian with account for the nucleon spin polarizabilities, Eq. (1) is used, where the polarization and magnetization vectors ($\mathbf{P}$ and $\mathbf{M}$) are determined as follows [11]

$$\mathbf{P} = \hat{\alpha}\mathbf{E} + \hat{\gamma}'_{\mathbf{E}}\,[\nabla\mathbf{E}]\,, \quad \mathbf{M} = \hat{\beta}\mathbf{H} + \hat{\gamma}'_{\mathbf{M}}\,[\nabla\mathbf{H}]\,. \tag{6}$$

Then the following Lagrangian of the electromagnetic field interacting with the moving medium is obtained [15] :

$$L_I{}^{eff} = 2\pi\{[e_{\mu}\alpha^{\mu\nu}e_{\nu} + h_{\mu}\beta^{\mu\nu}h_{\nu}] - [(\gamma'_E)_{\mu\nu}e^{\mu}(U\partial) \\ h^{\nu} + (\gamma'_M)_{\mu\nu}h^{\mu}(U\partial)e^{\nu}]\}, \tag{7}$$

where $(\gamma'_E)_{\mu\nu\rho}$ and $(\gamma'_M)_{\mu\nu\rho}$ are the gyration pseudotensors, $(U\partial) = U_{\rho}\partial^{\rho}$.

If the pseudotensors $(\gamma'_E)_{\mu\nu}$ and $(\gamma'_M)_{\mu\nu}$ are determined via $g_{\mu\nu}$ as $(\gamma'_{E,M})_{\mu\nu} = (\gamma'_{E,M})g_{\mu\nu}$, then it violates the spatial parity conservation low (*i.e.* spin of a composite particle is not taken into account).

From expression (7) it follows that the usual polarizabilities (α, β) in a rest medium and the gyration give non-zero contributions to the Lagrangian, starting with the second and the third order in powers of frequency of an external electromagnetic field respectively.

Due to the correspondence principles [8, 13] and the expression (7), the field theory effective Lagrangian of interaction of the electromagnetic field with spinless hadrons will not satisfy the parity conservation law when the components of pseudotensors $(\gamma'_E)_{\mu\nu}$ and $(\gamma'_M)_{\mu\nu}$ are not equal to zero. However, in the case of spin particles, it is easy to determine the dependence of γ-structures from $(\bar{\psi}\;(\overleftrightarrow{\partial_{\alpha}}\overleftrightarrow{\partial_{\beta}})\gamma^{\mu}\gamma^{5}\psi)$ and $(F^{\alpha\nu}\,\overleftrightarrow{\partial^{\nu}}\,\tilde{F}_{\sigma\mu})$.

Hence, the effective field Lagrangian describing the electromagnetic field and the spin-1/2 hadron interaction is defined as follows:

$$L_{eff} = L^{pol}_{eff} + L^{Sp}_{eff}, \tag{8}$$

where L^{pol}_{eff} is determined by the expression (5) and L^{Sp}_{eff} is [15] :

$$L^{Sp}_{eff} = -\frac{\pi}{2m^2}(\bar{\psi}\overleftrightarrow{\partial_{\alpha}}\overleftrightarrow{\partial_{\beta}}\,\gamma^{\mu}\gamma_5\psi)\{-\frac{1}{2}\,\gamma_{E_1}\cdot F^{\alpha\nu}\,\overleftrightarrow{\partial^{\beta}}\,\tilde{F}_{\mu\nu}\frac{1}{2}\,\gamma_{M_1}\cdot\tilde{F}^{\alpha\nu}\,\overleftrightarrow{\partial^{\beta}}\,F_{\mu\nu} \\ -\gamma_{E_2}\cdot(F^{\alpha\nu}\,\overleftarrow{\partial_{\mu}}\,\tilde{F}^{\beta}_{\nu} - \tilde{F}^{\alpha\nu}\,\overrightarrow{\partial_{\nu}}\,F_{\mu}{}^{\beta})\gamma_{M_2}\cdot(\tilde{F}^{\alpha\nu}\overleftarrow{\partial_{\mu}}\,F^{\beta}_{\nu} - F^{\alpha\nu}\overrightarrow{\partial_{\nu}}\,\tilde{F}_{\mu}{}^{\beta})\}. \tag{9}$$

3. Invariant amplitudes

The amplitude T_{fi} for Compton scattering on the nucleon is defined by

$$\langle f\,|S-1|\,i\rangle = i(2\pi)^4\delta^4(k+p-k^{'}-p^{'})T_{fi}. \qquad (10)$$

The photon momenta are denoted by $k,\ k'$ and the nucleon momenta by $p,\ p'$. Assuming invariance under parity, charge conjugation and time reversal symmetry the general amplitude for Compton scattering can be written in terms of six invariant amplitudes T_i as [1, 9, 14]

$$\begin{aligned} T_{fi} \;=\; & \overline{u}^{'}(p^{'})e^{'*\mu}\left\{-\frac{P^{'}_{\mu}P^{'}_{\nu}}{P'^2}(T_1+\gamma\cdot KT_2)-\frac{N_\mu N_\nu}{N^2}(T_3+\gamma\cdot KT_4)\right.\\ & \left.+i\frac{P^{'}_{\mu}N_\nu-P^{'}_{\nu}N_\mu}{P'^2K^2}\gamma_5T_5+i\frac{P^{'}_{\mu}N_\nu+P^{'}_{\nu}N_\mu}{P'^2K^2}\gamma_5\gamma\cdot KT_6\right\}e^{\nu}u(p), \qquad (11)\end{aligned}$$

where $\overline{u}^{'}(p^{'})$ and $u(p)$ are spin wave functions of particles, $\vec{e}$ and $\vec{e}^{\,'*}$ – polarization vectors of incident and scattered photons. The system of units $c=\hbar=1$ in this case is chosen. The orthogonal 4-vectors $P',\ K,\ N$ and Q are defined as $P^{'}_{\mu}=P_\mu-K_\mu\frac{P\cdot K}{K^2}$, $\quad P=\frac{1}{2}(p+p^{'})$, $\quad K=\frac{1}{2}(k+k^{'})$, $\quad N_\mu=\varepsilon_{\mu\nu\lambda\sigma}P^{'\nu}K^\lambda Q^\sigma$, $\quad Q=\frac{1}{2}(p-p^{'})=\frac{1}{2}(k^{'}-k)$.

The Lagrangian (8) allows us to get the covariant amplitude for Compton scattering on nucleons taking into account electric, magnetic and spin polarizabilities. By the decomposition of the covariant amplitude on invariant spin structures of expression (11) we obtain the relations

$$\begin{aligned} T_1 &= (-2\pi)\alpha_0\cdot(-4K^2)+\left(-\frac{2\pi}{m}\right)(\gamma_{E_1}-\gamma_{M_2})\cdot 4(PK)^2;\\ T_2 &= -\frac{2\pi}{m}(\alpha_0+\beta_0)\cdot 2(PK)-\frac{2\pi}{m^2}(-\gamma_{E_1}+\gamma_{M_2})\cdot 4(PK)(m^2+K^2);\\ T_3 &= (-2\pi)\beta_0\cdot(4K^2)+\left(-\frac{2\pi}{m}\right)(-\gamma_{M_1}+\gamma_{E_2})\cdot 4(PK)^2; \qquad (12)\\ T_4 &= -\frac{2\pi}{m}(-\alpha_0-\beta_0)\cdot 2(PK)-\frac{2\pi}{m^2}(\gamma_{M_1}-\gamma_{E_2})\cdot 4(PK)(m^2+K^2);\\ T_5 &= \left(-\frac{2\pi}{m}\right)\cdot\left[2(PK)^2(\gamma_{E_1}-\gamma_{M_1}-\gamma_{E_2}+\gamma_{M_2})+4P^{'2}K^2(-\gamma_{E_2}+\gamma_{M_2})\right];\\ T_6 &= \left(-\frac{2\pi}{m^2}\right)\cdot\left[2(PK)^2(\gamma_{E_1}+\gamma_{M_1}-\gamma_{E_2}-\gamma_{M_2})+4P^{'2}K^2(-\gamma_{E_2}-\gamma_{M_2})\right];\end{aligned}$$

where combinations of orthogonal momenta are expressed via generally accepted kinematic variables ν and t as follows $P^{'2}=\left(\frac{4m^2}{t}\right)\left(\nu^2+\frac{t}{4}-\frac{t^2}{16m^2}\right)$, $N^2=\left(\frac{m^2t}{4}\right)\left(\nu^2+\frac{t}{4}-\frac{t^2}{16m^2}\right)$, $K^2P^{'2}=-m^2\left(\nu^2+\frac{t}{4}-\frac{t^2}{16m^2}\right)$, $\quad 4K^2P^{'2}=-m^2\eta$, $\quad PK=m\nu$, $\quad -4K^2=t$.

If we realize amplitude (11) in the Lab frame, then the scalar functions A_i of spin structures [9] look like

$$
\begin{aligned}
A_1 &= (-2\pi)(\alpha_0 - \beta_0) + \left(-\frac{2\pi}{m}\right)\nu^2 \cdot (\gamma_{E_1} - \gamma_{M_1} + \gamma_{E_2} - \gamma_{M_2});\\
A_3 &= (-2\pi)\, m \cdot (\gamma_{E_1} + \gamma_{M_1} - \gamma_{E_2} - \gamma_{M_2});\\
A_5 &= \left(-\frac{2\pi}{m}\right)(m^2 - \frac{t}{4}) \cdot (-\gamma_{E_1} + \gamma_{M_1} - \gamma_{E_2} + \gamma_{M_2});\\
A_6 &= (-2\pi)(\alpha_0 + \beta_0) + \left(-\frac{2\pi}{m}\right)(m^2 - \frac{t}{4}) \cdot (-\gamma_{E_1} - \gamma_{M_1} + \gamma_{E_2} + \gamma_{M_2});\\
A_2 &= \left(-\frac{2\pi}{m}\right)\left[(-2m^2 + \frac{t}{2}) \cdot (-\gamma_{E_2} + \gamma_{M_2}) - \nu^2(-\gamma_{E_1} + \gamma_{M_1} - \gamma_{E_2} + \gamma_{M_2})\right];\\
A_4 &= (-2\pi)\, m \cdot (+\gamma_{E_1} + \gamma_{M_1} + \gamma_{E_2} + \gamma_{M_2}),
\end{aligned} \tag{13}
$$

where $\eta = \frac{1}{m^2}(m^4 - su) = 4\nu^2 + t - \frac{t^2}{4m^2}$ ($s,\ u,\ t$ are the usual Mandelstam variables). The invariant amplitudes A_i are even functions of ν and are free of both kinematic singularities and zeros.

In the Lab system the kinematic invariants $\nu,\ t$ and η read: $\nu = \frac{1}{2}(\omega + \omega^{'}),\quad t = -2\omega\omega^{'}(1-z),\quad \eta = 2\omega\omega^{'}(1+z)$.

In the scattering amplitude (11) within the second order in the photon energy we have got the contribution for electric and magnetic polarizabilities in the form

$$
T_{fi}^{pol} = \frac{8\pi m\omega\omega^{'}}{N(t)}\left[\mathbf{e}^{'*} \cdot \mathbf{e}\alpha_0 + \mathbf{s}^{'*} \cdot \mathbf{s}\beta_0\right], \tag{14}
$$

and within the third order – the contribution for spin polarizabilities as

$$
\begin{aligned}
T_{fi}^{Sp} = -\frac{8i\pi m}{N(t)}&\Big[-\nu\omega\omega^{'}\sigma \cdot \mathbf{e}^{'*} \times \mathbf{e}(\gamma_{E_1} - \gamma_{M_2}) + \nu\omega\omega^{'}\sigma \cdot \mathbf{s}^{'*} \times \mathbf{s}(-\gamma_{M_1} - \gamma_{E_2})\\
&+ \left(\sigma \cdot \widehat{\mathbf{k}}\mathbf{s}^{'*} \cdot \mathbf{e}\omega^2\omega^{'} - \sigma \cdot \widehat{\mathbf{k}}^{'}\mathbf{e}^{'*} \cdot \mathbf{s}\omega\omega^{'2}\right)(-\gamma_{E_2})\\
&+ \left(-\sigma \cdot \widehat{\mathbf{k}}\mathbf{e}^{'*} \cdot \mathbf{s}\omega^2\omega^{'} + \sigma \cdot \widehat{\mathbf{k}}^{'}\mathbf{s}^{'*} \cdot \mathbf{e}\omega\omega^{'2}\right)(-\gamma_{M_2})
\end{aligned} \tag{15}
$$

4. The energy-momentum tensor for interaction of the electromagnetic field with spinor particle

In the case of the spinor and electromagnetic fields the expression for energy-momentum tensor has the form [13]

$$
T_{\nu}^{\mu} = \frac{\partial L}{\partial(\partial_\mu\psi)}(\partial_\nu\psi) + (\partial_\nu\overline{\psi})\frac{\partial L}{\partial(\partial_\mu\overline{\psi})} + (\partial_\nu A_\rho)\frac{\partial L}{\partial(\partial_\mu A_\rho)} - L\delta_\nu^\mu. \tag{16}
$$

The total interaction Lagrangian for spin-1/2 particles with the electromagnetic field will consist of the Lagrangian for free electromagnetic field L_{e-m}, the spinor or Dirac's field L_D, the interaction Lagrangian of the free electromagnetic field with the Dirac's field L_{int-D}, the Pauli term L_P and the Lagrangian which considers electric and magnetic polarizabilities of particles $L_{\alpha_0\beta_0-D}$:

$$L_{total-D} = L_{e-m} + L_D + L_{int-D} + L_P + L_{\alpha_0\beta_0-D}. \tag{17}$$

All parts of the Lagrangian (17) are known. The last term $L_{\alpha_0\beta_0-D}$ will be equal to (5). So the total Lagrangian (17) has the form

$$\begin{aligned} L_{total-D} &= -\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - \overline{\psi}(\frac{1}{2}i\gamma_\alpha \overleftrightarrow{\partial^\alpha} + m)\psi - e(\overline{\psi}\gamma_\alpha\psi)A^\alpha - \\ &\quad -\mu\overline{\psi}\sigma^{\alpha\beta}\psi F_{\alpha\beta} - \frac{i\pi}{m}K^\nu_\mu(\overline{\psi}\gamma^\alpha \overleftrightarrow{\partial_\nu} \psi), \end{aligned} \tag{18}$$

Then we substitute the Lagrangian $L_{total} = L_{e-m} + L_D + L_{int-D} + L_P + L_{\alpha_0\beta_0-D}$ into the expression for the energy-momentum tensor (16). By simplifying the total energy-momentum tensor we have

$$\begin{aligned} T^\gamma_\xi = &-(\partial^\gamma A^\mu)\partial_\xi A_\mu - \left(2\mu\overline{\psi}\sigma^{\gamma\mu}\psi\right)\partial_\xi A_\mu + \frac{1}{2}i\overline{\psi}\gamma^\gamma(\partial_\xi\psi) - \frac{1}{2}i(\partial_\xi\overline{\psi})\gamma^\gamma\psi \\ &- \left(-\frac{1}{2}\partial_\alpha A_\beta\partial^\alpha A^\beta - \overline{\psi}\psi m\right)\delta^\gamma_\xi + \left\{\left(-\frac{i\pi}{m}\right)\overline{\psi}\left[(\alpha_0-\beta_0)\left(F^{\mu\nu}(\gamma^\gamma \overleftrightarrow{\partial_\nu})\right.\right.\right. \\ &\left.\left.\left.-F^{\gamma\nu}(\gamma^\mu \overleftrightarrow{\partial_\nu}) + F^{\alpha\gamma}(\gamma_\alpha \overleftrightarrow{\partial^\mu}) - F^{\alpha\mu}(\gamma_\alpha \overleftrightarrow{\partial^\gamma})\right) - 2\beta_0F^{\gamma\mu}(\gamma^\nu \overleftrightarrow{\partial_\nu})\right]\psi\right\}\partial_\xi A_\mu \\ &+ \left[(\partial_\xi\overline{\psi})(\delta^\gamma_\nu\gamma^\alpha\psi) - (\delta^\gamma_\nu\overline{\psi}\gamma^\alpha)(\partial_\xi\psi)\right]\left(-\frac{i\pi}{m}\right)K^\nu_\alpha. \end{aligned} \tag{19}$$

The energy-momentum conservation law is fulfilled for the expression (19), i.e. $\partial_\gamma T^\gamma_{\xi(I)} = 0$.

Now we determine $T^0_{0(\alpha_0,\beta_0-D)}$, i.e. the Hamiltonian. The following expression was obtained [9]

$$T^0_{0(\alpha_0,\beta_0-D)} = H_{(\alpha_0,\beta_0)} = -2\pi\left(\alpha_0E^2 + \beta_0H^2\right). \tag{20}$$

Thus, we have got the well known in the electrodynamic of continuous media [11] and elementary-particle physics [9] Hamiltonian for the case of interaction of the electromagnetic field with a polarizable particle.

5. Conclusion

The covariant Lagrangian of interaction of the electromagnetic field with a polarizable spin particles have been obtained. This Lagrangian satisfies the main relativistic quantum field theory requirements (cross-invariance, P-, T- and gauge invariance). The Lagrangian can be used for the description of spin polarizabilities in two-photon electromagnetic processes. Besides, the Lagrangian gives relevant contribution of the electric, magnetic and spin polarizabilities to scattering amplitude. This contribution is in a good agreement with spin structures of the scattering amplitude of paper [9] .The correlations between the covariant Lagrangian and canonical energy-momentum tensor have been determined. This fact on the basis of the correspondence principle has given a proper definition of the low-energy presentation of the Lagrangian function.

References

1 V.A. Petrun'kin, Phys At. Nucl. 12 (1981) 692

2 A.J. L'vov, FIAN preprint N344 (1987)

3 S. Ragusa, Phys. Rev. D47 (1993) 3757

4 S. Ragusa, Phys. Rev. D47 (1994) 3157

5 K.Y. Lin, Nuovo Cimento A2 (1971) 695

6 K.Y. Lin, Phys. Rev. D24 (1981) 1014

7 M.I. Levchuk, L.G. Moroz, Vesti AN BSSR Ser.: fiz.-mat. nauk 1. (1985)

8 N.V. Maksimenko, L.G. Moroz, In Proc. 11 Intern. School on High Energy Physics and Relativistic Nucl. Phys. Dubna JINR D2-11707 (1979) 533

9 D. Babusci, J. Jiordano, A.J. L'vov, J. Matone , A.N. Nathan, Phys. Rev. C58 (1998) 1013

10 S. Scherer, A. Yu Korchin, J.H. Koch, Phys. Rev. C54 (1996) 904

11 F.I. Fedorov Gyrotropy theory. Minsk: Nauka i tehnika, (1973)

12 B.V. Bokut', A.N. Serdyukov, Journal of Applied Spectroscopy, 11 (1969)

13 A.A. Bogush, L.G. Moroz, Introduction in classic fields theory.– Nauka i tehnika.– Minsk. (1968).

14 A.C. Hearn and E. Leader, Phys.Rev 126,789 (1962)

15 S.A. Belousova, N.V. Maksimenko Proc. Of "OFTHEP'2000". Tver, Russia, 2000 305-308, hep-ph/0009334