ON THE SUBCRITICAL AMPLIFIER OF NEUTRON FLUX BASED ON ENRICHED URANIUM

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Abstract. Further progress in nuclear power engineering to large extent will depend on the solution of two basic problems: improvement of the safety level and on the efficient transmutation of the long-lived radioactive waste. Possible solutions of these problems consist in the construction of subcritical systems capable to multiply neutrons from an external source.

In the present paper we examine the main aspects of the construction of such systems, with emphasis on the choice of the neutron source and optimization of the parameters of the subcritical system. We calculate the neutron flux amplification factor for a one-zone spherical systems made of enriched uranium, water solution of the enriched uranium, and for the spherical enriched uranium system with a reflector.

1. Introduction

Studies by different authors [1, 2] show that the construction of accelerator-driven subcritical systems is a promising way in nuclear power engineering. Such systems enable to improve the safety level and to work out efficient methods of transmutation of long-lived radioactive waste.

The theoretic background of the accelerator-driven subcritical systems can be found e.g. in Ref. [3]. Preliminary results of our studies were published (in Russian) in Ref. [3].

Here we investigate the amplification of the neutron flux by a number of subcritical systems, namely by a homogeneous spherical assembly made of enriched uranium, by a homogeneous spherical assembly made

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of water solution of enriched uranium, and by a homogeneous spherical assembly made of enriched uranium and containing a beryllium reflector.

2. Description of the assemblies

The main objective of our investigation is to establish the basic features in the behaviour of the amplification factors of neutron flux and energy depending on primary features of the assembly - such as the nuclide composition, energy of neutrons of the external source, effective multiplication factor of the system, ratio of nuclear concentrations. Studies of such model assemblies is of interest for a better understanding of amplification properties of more complicated systems and for the optimization of their parameters. Even such simple systems show a number of nontrivial properties concerning amplification of neutron flux. In particular, the amplification factors have nonmonotonic behaviour depending on the uranium enrichment and on the ratio of the nuclear concentrations (H/U). We choose the physical systems to be slightly subcritical, namely we fix the value of the neutron effective multiplication factor of each system to be equal $k_{eff} = 0.99$.

We consider a point-like isotropic source of neutrons with energy of $14MeV$ to be the "external" source, which is located at the centre of a spherical subcritical assembly. Although realistic neutron sources from $D - T$ reactions are neither isotropic nor monoenergetic, the simplified model we use is rather typical and it is frequently used. The geometric dimensions (radius) of the assembly are fixed by the requirement that the effective multiplication factor of the system be equal to some specified value, which is slightly less than unity, i.e. the assembly should be slightly subcritical. Calculations of the amplification factors and other physical characteristics of the systems under consideration were performed with the help of the neutron Monte Carlo transport code MCNP-4C [4], which employs the latest ENDF/B-VI nuclear data library.

Our choice of the neutron source energy, $14MeV$, was motivated by two factors. Firstly, there are two possibilities to obtain neutrons from the interaction of accelerated charged particles with matter, namely with the help of the $D - T$ reaction and by means of a spallation process. Spallation reactions produce neutrons from the interaction of fast charged particles (for example, protons with energy $\sim 1GeV$) with nuclei of heavy metals (for example, with a mixture of lead and bismuth) [5]. Fusion reaction may be accomplished with the help of deuteron accelerators working at the energy $100 - 200KeV$ and with a currents $\sim$ *Ampere*. Such a current is capable to yield a neutron flux equivalent to the neutron flux produced by proton accelerators with energy $\sim 1GeV$ and a current $\sim mA$. Any project based on $D-T$ reaction costs a few times less than the one based on spallation. Secondly, spallation neutrons have a rather wide energy spectrum with the maximum between $200-300MeV$. Neutron cross-sections in this region, however, are not known sufficiently well and they are not available for all nuclides needed. Since relevant cross-section libraries in this energy range are missing, calculation with the neutron source from spallation process is problematic. At the same time the neutron source with the energy of $14MeV$, resulting from a $D - T$ reaction, is available technically, the neutron cross-sections in this energy range being well known and collected in relevant libraries. Therefore we find important studies of the efficiency of transmutation with 14–MeV neutrons and the compare of the results with those, obtained with spallation neutrons.

3. Results

First we consider a one-zone homogeneous spherical assembly made of enriched uranium. We fix the value of the neutron effective multiplication factor of the system at $k_{eff} = 0.99$. Any variation of the uranium enrichment w in the isotope ^{235}U, at a fixed value of k_{eff}, results in a change of the assembly radius R. To be more specific, the decrease of the uranium enrichment results in the decrease of the multiplication factor of an infinite medium k_∞, and hence the radius of the system should increase in order to keep the value of $k_{eff} = 0.99$ constant.

As already mentioned, we consider the external source of neutrons to be point-like, isotropic and monoenergetic with the energy $E_0 = 14MeV$. It is located at the centre of a spherical assembly. We define the flux amplification factor q as the ratio of the total number of neutrons passing through the external boundary surface in a time unit N_S, to the intensity of the neutron source I_0, i.e. to the number of neutrons emitted by the source in a time unit $q = N_S/I_0$. It will be convenient to choose the intensity of the neutron source to be equal to unity, $I_0 = 1n/sec$, i.e. our results are normalized to be per starting single neutron from the source. The amplification factors, obviously, does not depend on the normalization. We define the energy amplification factor G as the ratio of the total energy deposition in the system to the source energy E_0.

We study the dependence of the flux amplification factor q and the energy amplification factor G on the uranium enrichment w_{U235}

for the system under consideration. The results of our calculations are presented in Table 1 together with the calculated values of the multiplication factor of an infinite medium k_{∞}, of the radius of the assembly R, of the mean neutron flux in the volume $\overline{\Phi}$, and of the neutron flux through the external boundary of the system Φ_S. The values of the fission density ρ_{fis} (which means the number of fissions per volume unit), and the total number of fissions in the system N_{fis} are also presented.

Table 1. Physical properties of a one-zone spherical amplifier of neutron flux made of enriched uranium, as functions of the uranium enrichment. The value of the neutron effective multiplication factor of the system is fixed at $k_{eff} = 0.99$. The external neutron source, which is pointlike, isotropic and monoenergetic, is located in the centre of the system. The energy of the source is $E_0 = 14 MeV$, and the intensity of the source is $I_0 = 1 n/sec$.

w_{U235}, %	k_{∞}	R, cm	$\overline{\Phi}$, $n/cm^2 \cdot sec$	Φ_S, $n/cm^2 \cdot sec$	ρ_{fis}, $1/cm^3 \cdot sec$	N_{fis}, $1/sec$	G	q
6	1.07	85.81	$4.99 \cdot 10^{-3}$	$2.29 \cdot 10^{-4}$	$2.89 \cdot 10^{-5}$	76.46	988.61	21.19
7	1.18	56.38	$1.59 \cdot 10^{-2}$	$1.23 \cdot 10^{-3}$	$1.04 \cdot 10^{-4}$	78.42	1013.94	49.03
8	1.26	45.34	$2.74 \cdot 10^{-2}$	$2.69 \cdot 10^{-3}$	$2.02 \cdot 10^{-4}$	78.72	1017.79	69.50
9	1.34	39.13	$3.83 \cdot 10^{-2}$	$4.38 \cdot 10^{-3}$	$3.11 \cdot 10^{-4}$	78.19	1010.89	84.38
10	1.41	35.05	$4.82 \cdot 10^{-2}$	$6.17 \cdot 10^{-3}$	$4.28 \cdot 10^{-4}$	78.33	996.54	95.12
20	1.82	20.81	$1.15 \cdot 10^{-1}$	$2.39 \cdot 10^{-2}$	$1.83 \cdot 10^{-3}$	77.08	892.29	130.34
30	2.01	16.38	$1.59 \cdot 10^{-1}$	$4.00 \cdot 10^{-2}$	$3.53 \cdot 10^{-3}$	64.85	838.16	135.01
40	2.12	13.93	$2.00 \cdot 10^{-1}$	$5.68 \cdot 10^{-2}$	$5.63 \cdot 10^{-3}$	63.81	824.67	138.53
50	2.20	12.32	$2.45 \cdot 10^{-1}$	$7.55 \cdot 10^{-2}$	$8.28 \cdot 10^{-3}$	64.77	837.09	143.81
60	2.25	11.13	$2.93 \cdot 10^{-1}$	$9.62 \cdot 10^{-2}$	$1.15 \cdot 10^{-2}$	66.60	860.60	149.86
70	2.29	10.22	$3.46 \cdot 10^{-1}$	$1.19 \cdot 10^{-1}$	$1.54 \cdot 10^{-2}$	68.96	891.14	156.54
80	2.32	9.49	$4.12 \cdot 10^{-1}$	$1.48 \cdot 10^{-1}$	$2.05 \cdot 10^{-2}$	73.30	947.15	167.36
90	2.35	8.88	$4.76 \cdot 10^{-1}$	$1.77 \cdot 10^{-1}$	$2.61 \cdot 10^{-2}$	76.42	987.39	175.19
100	2.37	8.36	$5.67 \cdot 10^{-1}$	$2.17 \cdot 10^{-1}$	$3.38 \cdot 10^{-2}$	82.92	1071.31	190.52

The main features of the system were found to be as follows. The flux amplification factor q depends monotonically on the enrichment and it increases with the increase of the enrichment. At the same time the energy amplification factor G, depending on enrichment, decreases, reaching a minimum, followed by a subsequent rise (see Table 1 and Figs. 1, 2). To scrutinize this phenomenon, we have calculated also the fission density and the total number of fissions in the system (see Table 1). The fission density monotonically increases with the growth of the enrichment, while the total number of fissions has the same dependence on the enrichment as the energy amplification factor G. Such a behaviour may be explained as follows. Inasmuch as we fix the neutron effective multiplication factor, we have a large volume of

Table 2. Total mean free path of neutrons in the medium and mean free path before the fission as functions of the uranium enrichment for a one-zone homogeneous spherical assembly made of the enriched uranium.

w_{U235}, %	R, cm	L_{tot}, cm	L_{fis}, cm	L_{fis}/R
6	85.81	2.04	172.71	2.01
7	56.38	2.07	151.75	2.69
8	45.34	2.09	135.65	2.99
9	39.13	2.12	122.88	3.14
10	35.05	2.14	112.49	3.21
20	20.81	2.27	63.00	3.03
30	16.37	2.35	44.99	2.75
40	13.93	2.40	35.48	2.55
50	12.31	2.45	29.54	2.40
60	11.13	2.48	25.44	2.29
70	10.22	2.50	22.43	2.19
80	9.49	2.53	20.12	2.12
90	8.88	2.55	18.27	2.06
100	8.36	2.56	16.76	2.00

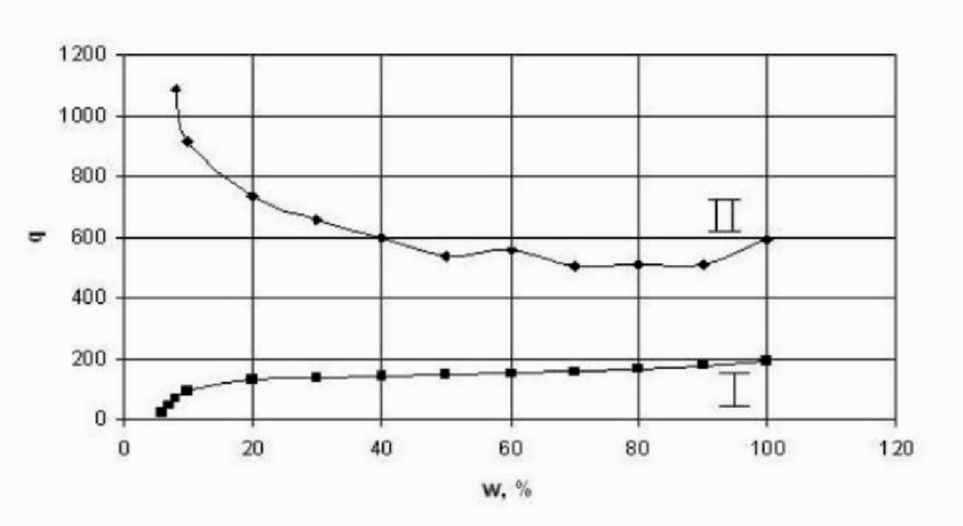

Figure 1. The neutron flux amplification factor versus the uranium enrichment. I — system without any reflector, II — system with beryllium reflector.

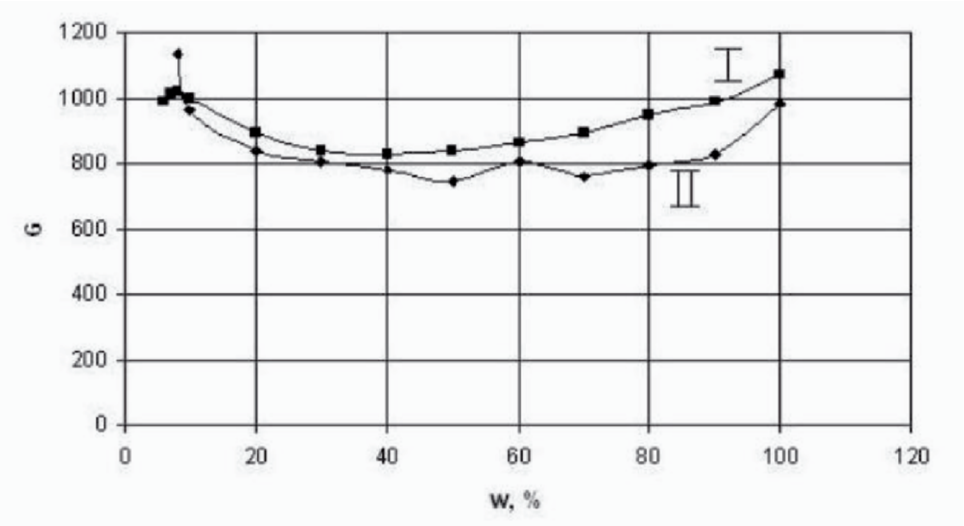

Figure 2. The energy amplification factor versus the uranium enrichment. I — system without any reflector, II — system with beryllium reflector.

the system at low enrichment and most of neutrons have time to be absorbed. By increasing the enrichment, the dimensions of the system decrease and part of the neutrons simply do not have time to interact with uranium (see Table 2 support this phenomenon). Here L_{tot} is a total mean free path of neutrons in the medium, L_{fis} is the mean free path before the fission. We thus conclude that the maximal amplification factor will appear in the system with dimensions exceeding the neutron mean free path before the fission ($1/\Sigma_{fis}$).

It should be noted that while the flux amplification factor q is falling slowly in the range of the enrichment between 100 and 20 percents, it starts decreasing drastically below $w < 20\%$. So, it is advantageous to choose the enrichment of the system to be slightly greater than 20%.

In a similar way, we consider a one-zone spherical assembly involving water solution of the enriched uranium. At fixed value of $k_{eff} = 0.99$, the radius of the assembly R will change, depending on the ratio of nuclear concentrations of the moderator (hydrogen) and the fuel (uranium) - H/U. The main parameters necessary for calculations were determined similarly to the first system.

We have calculated the flux amplification factor q and the energy amplification factor G as functions of the ratio of nuclear concentrations H/U, sometimes called "moderation parameter". The results of calculations are presented in Table 3, together with the values of the radius of the system R, the volume of the system V, the mean free path of the neutron L_{tot}, and the neutron mean free path before the fission L_{fis}.

The main features of this system were found to be the following. The dependence of the flux amplification factor q on the ratio of nuclear concentrations H/U shows a nonmonotonic behaviour (see Fig. 3). First, as the moderation parameter H/U increases, q starts increasing very slowly, but then it jumps at the value of the moderation parameter $H/U = 0.5$. Subsequently q increases and reaches a maximum at $H/U = 2 - 3$, whereupon it starts decreasing. This dependence also has a jump at values $H/U = 15$ and $H/U = 50$. Thus, the dependence of the flux amplification factor on the moderation parameter has a pronounced maximum at $H/U = 2 - 3$. This result can be used to construct feasible assemblies. The energy amplification factor G has a similar dependence on the moderation parameter H/U, the only difference being that there is a minimum at $H/U = 500$, followed by a slight increase.

We have also calculated the properties of the subcritical system with a reflector. The spherical system under consideration is made of a core involving enriched uranium and a beryllium reflector $50cm$ thick. The parameters of the system were chosen similarly to the previous cases to achieve the fixed value of neutron effective multiplication factor $k_{eff} = 0.99$.

Table 3. Amplification factors and other physical characteristics as functions of the moderation parameter for a one-zone spherical assembly involving water solution of the enriched uranium.

H/U	R, cm	V, cm^3	q	G	L_{fis}, cm	L_{tot}, cm
0.0001	8.377	2462.37	171.233	960.2714	16.84524	2.577012
0.0005	8.382	2466.79	171.744	962.7500	16.85188	2.576257
0.001	8.388	2472.09	168.829	947.2500	16.8506	2.576097
0.005	8.401	2483.60	176.630	991.0214	16.88639	2.572807
0.01	8.414	2495.15	172.063	967.3500	16.92226	2.567789
0.05	8.553	2620.86	180.124	1018.071	17.22229	2.531544
0.1	8.708	2765.95	192.501	1095.579	17.57097	2.487461
0.5	9.691	3812.36	275.874	1652.207	19.54832	2.197037
1	10.528	4887.94	348.202	2188.836	21.06218	1.972522
2	11.622	6575.54	419.654	2814.507	22.94609	1.735358
3	12.315	7823.33	348.351	2426.221	24.2008	1.612303
4	12.782	8747.52	223.486	1591.136	25.11427	1.535745
5	13.109	9436.20	172.750	1245.621	25.84373	1.482758
10	13.831	11082.8	89.8052	659.0771	27.98335	1.350692
15	14.043	11600.3	53.2601	385.8764	29.21852	1.293725
20	14.128	11812.2	46.2400	331.7121	30.09629	1.258828
25	14.174	11928.0	43.2266	308.2293	30.66048	1.231354
30	14.206	12008.9	31.5489	221.4721	31.41745	1.219358
35	14.236	12085.2	26.0852	180.8243	32.05519	1.206755
40	14.263	12154.1	27.0735	187.1864	32.44564	1.190629
45	14.293	12230.9	24.409	167.5779	32.91605	1.180125
50	14.328	12321.0	23.1536	158.4000	33.31658	1.169539
100	14.658	13192.1	12.9954	85.06857	37.25373	1.110468

200	15.545	15734.8	9.717	63.45700	43.02007	1.024013
300	16.465	18697.1	8.37102	55.90264	47.9047	0.963363
400	17.435	22200.1	7.24806	49.82293	52.84375	0.921062
500	18.455	26328.8	7.13538	51.27821	57.04997	0.881092
600	19.558	31337.3	7.00582	53.19929	61.16092	0.848517
700	20.718	37250.5	6.88028	55.348	65.15647	0.821667
800	22.008	44560.9	6.851	58.94607	69.03865	0.798449
900	23.375	53498.7	7.0863	66.07914	72.50314	0.776008
1000	24.925	64862.6	6.91704	69.80257	76.39028	0.759799

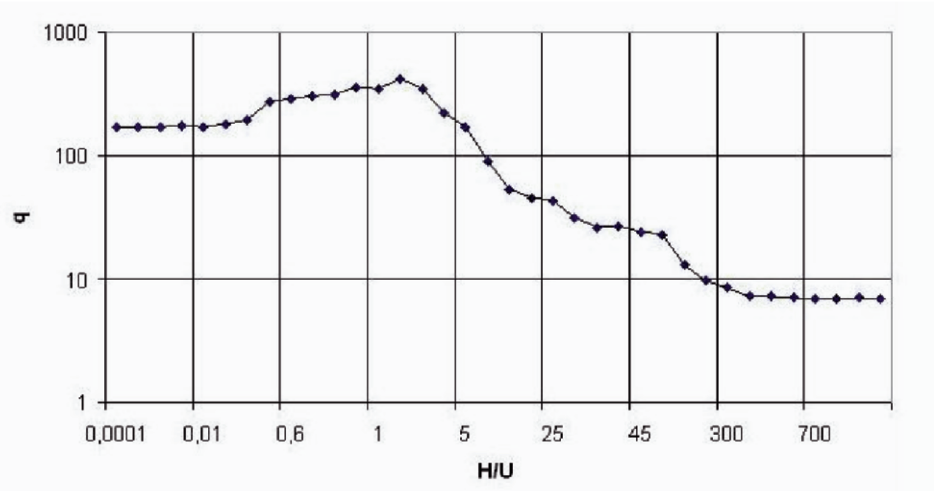

Figure 3. Neutron flux amplification factor versus the moderation parameter for a one-zone spherical assembly involving water solution of the enriched uranium.

The increase of the flux amplification factor q amounts to $3-5$ times for the system with beryllium reflector. At the same time, the nature of the q-dependence on the enrichment also changes (Fig. 1). As to the energy amplification factor G, it does not change neither by magnitude, nor by the character of its dependence (Fig. 2). The last fact is due to decrease of the fission volume of the system with a reflector at fixed value of $k_{eff} = 0.99$.

4. Conclusion

Thus, even a very simple one-zone subcritical system makes possible a neutron flux amplification by 1 to 2 orders of magnitude and energy amplification by 2 to 3 orders. Another interesting result is that relatively low enrichments ($10\% - 20\%$) yield energy amplifications close to those at high enrichment, which is very advantageous from the economical point of view. The flux amplification factor varies insignificantly in the interval of enrichments between 20% and 40%, but decreases drastically at smaller enrichments. Hence a system of a 20% enrichment in uranium-235 can serve as a reasonable amplifier both in flux and energy.

A subcritical system involving water solution of uranium-235 enables one to obtain better values of amplification factors as compared to a system made of metallic uranium. These values amount to $q = 420$ for the flux amplification factor, and $G = 2800$ for the energy amplification factor.

A system with a reflector enables one to increase the flux amplification factor $3 - 5$ times compared to a similar system without any reflector. The flux amplification factor reaches the value $q = 500 - 900$ in this case, and the energy amplification factor reaches the value $G = 800 - 1000$.

The results on the enhancement of the amplification factors of subcritical neutron-multiplying systems make only part of our research program. The characteristics of more advanced multi-zone systems, as well as the possibility to use alternative fissile materials will be studied in the future. In particular, two-zone systems are of great interest since the utilization of a booster involving a material with $k_\infty > 1$ makes it possible to increase the amplification factor significantly [6]. However, even the present stage of the study shows that subcritical neutron-multiplying assemblies are promising alternatives to the reactors as powerful sources of neutrons, suggesting various applications in nuclear physics and nuclear power engineering.

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