THE QCD ANALYTIC PERTURBATION THEORY DESCRIPTION OF HADRONIC CONTRIBUTIONS INTO FEW IMPORTANT EFFECTS

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Abstract. A method based on analytic approach in QCD, involving a summation of threshold singularities and taking into account a nonperturbative character of the light quark masses, is applied to find hadronic contributions to different physical quantities.

1. Introduction

A comparison of QCD theoretical results with experimental data is often based on the conception of the quark-hadron duality [1]. The idea of quark-hadron duality formulated in [2] is as follows: inclusive hadronic cross sections, being appropriately average over an energy interval, had to approximately coincide with the corresponding quantities derived by using quark-gluon description. For many physical quantities the corresponding interval of integration involves a nonperturbative region and nonperturbative effects may play an important role for their description. We consider the following quantities and functions, which cannot be calculated reliably within the framework of perturbative QCD:

the ratio of hadronic to leptonic τ-decay widths in the non-strange vector channel

$$R_\tau^V = R^{(0)} \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) R(s); \qquad (1)$$

T. Čechák et al. (eds.), Nuclear Science and Safety in Europe, 161–167.

the Adler function [3], which can be constructed from τ-decays data [4],

$$D_V(Q^2) = Q^2 \int_0^\infty ds \, \frac{R(s)}{(s+Q^2)^2}; \tag{2}$$

the smeared R_Δ function [2]

$$R_\Delta(s) = \frac{\Delta}{\pi} \int_0^\infty ds' \frac{R(s')}{(s-s')^2 + \Delta^2}; \tag{3}$$

the hadronic contribution to the anomalous magnetic moment of the muon

$$a_\mu^{\rm had} = \frac{1}{3} \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s} \, K(s) \, R(s); \tag{4}$$

the hadronic contribution to the electromagnetic coupling

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = -\frac{\alpha(0)}{3\pi} \, M_Z^2 \, \mathcal{P} \int_0^\infty \frac{ds}{s} \, \frac{R(s)}{s - M_Z^2} \, . \tag{5}$$

A method, which we use here for description of these quantities and functions, is based on the analytic approach in QCD [5, 6]. The analytic approach allows us to describe self-consistently the timelike region [7, 8] that is used in the integrals in Eqs. (1) - (5). This method also involves into consideration a summation of threshold singularities and takes into account nonperturbative character of the light quark masses.

2. R-function

To incorporate the quark mass effects one usually uses the approximate expression proposed in [2, 9] above the quark-antiquark threshold

$$R(s) = T(v) \; [1 + g(v) r(s)] \; , \tag{6}$$

where

$$T(v) = v \frac{3 - v^2}{2} \, , \quad g(v) = \frac{4\pi}{3} \left[\frac{\pi}{2v} - \frac{3+v}{4} \left(\frac{\pi}{2} - \frac{3}{4\pi} \right) \right] . \tag{7}$$

In Eq. (6) one cannot directly use the perturbative expression for $r(s)$, which contains unphysical singularities, to calculate, say, the Adler

D-function (2). Instead of that one can use the analytic perturbation theory for $r(s)$. The explicit three-loop form for $r_{\rm an}(s)$ is given in [4].

In describing a charged particle-antiparticle system near threshold, it is well known from QED that the so-called Coulomb resummation factor plays an important role. For a systematic relativistic analysis of quark-antiquark systems, it is essential from the very beginning to have a relativistic generalization of the S-factor. A new form for this relativistic factor in the case of QCD has been proposed in [10] [1]

$$S(\chi) = \frac{X(\chi)}{1-\exp\left[-X(\chi)\right]}\,, \qquad X(\chi) = \frac{4}{3}\frac{\pi\,\alpha_s}{\sinh\chi}\,, \tag{8}$$

where χ is the rapidity which related to s by $2m\cosh\chi = \sqrt{s}$. The relativistic resummation factor (8) reproduces both the expected non-relativistic and ultrarelativistic limits and corresponds to a QCD-like Coulomb potential.

The modified expression for R-function is [4, 12]

$$\begin{aligned} R(s) \quad & = \left[R_0(s)+R_1(s)\right]\Theta(s-4m^2), \\ R_0(s) & = T(v)\,S(\chi), \qquad R_1(s) = T(v)\left[\, r_{\rm an}(s)\,g(v) - \tfrac{1}{2}X(\chi)\right]. \end{aligned} \tag{9}$$

The function $r_{\rm an}(s)$ is taken within the analytic approach as in [4]. To avoid a double counting the function R_1 contains subtracted term with $X(\chi)$. The term R_0 gives a principle contribution to $R(s)$, the correction R_1 is less than twenty percent for whole energy interval.

3. Quark masses

A solution of the Schwinger-Dyson equations performed in [13–17] demonstrates a fined infrared behavior of the invariant charge and quark mass function. This behavior can be understood by using a conception of the dynamical quark mass. This mass has an essentially nonperturbative nature. Its connection with the quark condensate has been established in [18]. By using the analysis based on the Schwinger-Dyson equations the similar relation has been found in [19]. It has been demonstrated in [20] that on the mass-shall one has a gauge-independent result: $m^3 = -4/3\pi\alpha_s < 0|\bar{q}\,q|0 >$. An analysis performed in [21] leads to a step-like behavior of the quark mass function.

According to these results one can assume that at small p^2 the function $m(p^2)$ is rather smooth (nearly constant). In the region $p^2 >$

[1] Here we consider the vector channel for which a threshold resummation S-factor for the s-wave states is used. For the axial-vector channel the P-factor is required. The corresponding relativistic factor has recently been found in [11].

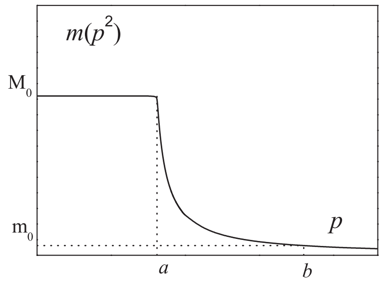

Figure 1. Effective quark mass.

$1 \div 2$ GeV the principle behavior of the function $m(p^2)$ is defined by perturbation theory.

We perform our analysis by using constant quark masses and the mass function $m(p^2)$ which is shown in Fig. 1. We take the line that connects the points a and b in the form $A^3/(p^2 - B^2)$. The parameter m_0 are taken from the known value of the running (current) mass at $b = 2.0$ GeV [22], the parameter a is taken as $a = 0.8$ GeV. The quantities considering here are not too sensitive to parameters of heavy quarks and we take for c, b and t quarks $m^f(p^2) = m_0^f = M_0^f$. We have a consistent description of all mentioned quantities, if the light quark parameter $M_0^{u,d} = 260 \pm 10$ MeV and the mass parameters M_0^s is varied in the limits from 400 to 550 MeV. The masses of the heavy quarks are $M_0^c = 1.3$ GeV, $M_0^b = 4.4$ GeV and $M_0^b = 174.0$ GeV.

4. Physical quantities and functions

Let us apply the formulated model to describe mentioned in Introduction physical quantities and functions connected with $R(s)$.

Inclusive decay of the τ-lepton. The experimental data presented by the ALEPH, $R_{\tau,V}^{\text{expt}} = 1.775 \pm 0.017$ [23], and the OPAL, $R_{\tau,V}^{\text{expt}} = 1.764 \pm 0.016$ [24], collaborations. In our analysis we use the non-strange vector channel spectral function obtained by the ALEPH collaboration and keep in all further calculations the value $R_\tau^V = 1.78$ as the normalization point.

D_V-function. In order to construct the Euclidean D-function we use for $R(s)$ the following expression $R(s) = R^{\text{expt}}(s)\,\theta(s_0 - s) + R^{\text{theor}}(s)\,\theta(s - s_0)$. The continuum threshold $s_0 \simeq 1.6$ GeV^2 has been found from the duality relation [25].

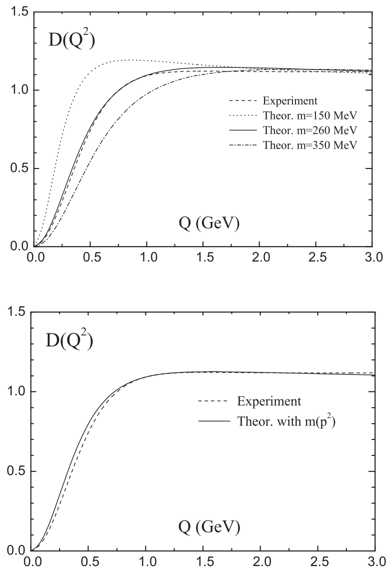

Figure 3. D-function for $m = m(p^2)$.

In Fig. 2 we plot curves corresponding to different values of the quark masses. A result for the D-function that obtained by using the mass function $m(p^2)$ with $M_0^{u,d} = 260$ MeV is shown in Fig. 3. Thus we obtained the result that is rather close to the result obtained for $m(p^2) = const$.

'Light' smeared R_Δ-function. By using the ALEPH data [23], we construct the 'light' experimental function $R_\Delta(s)$.

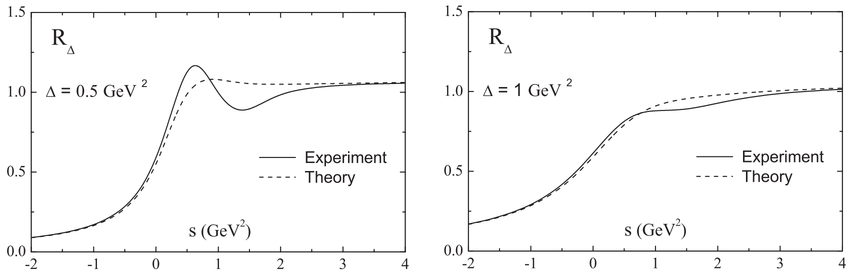

Figure 5. Smeared function for $\Delta = 1.0$ GeV^2.

Figures 4 and 5 demonstrate behavior of experimental and theoretical smeared functions for $\Delta = 0.5$ GeV^2 and $\Delta = 1.0$ GeV^2. Let us emphasize, in the spacelike region ($s < 0$) there is a good agreement between data and theory starting from $s = 0$.

Hadronic contribution to a_μ. The hadronic contribution to the anomalous magnetic moment of the muon in the leading order of the electromagnetic coupling constant is defined by Eq. (4). In our calculations we take into account the matching conditions at quark thresholds according to procedure described in [8]. The strong interaction contribution to a_μ we estimate as

$$a_\mu^{\rm had} = (693 \pm 34) \times 10^{-10}. \qquad (10)$$

The experimental value of $a_\mu^{\rm had}$ is extracted from e^+e^- annihilation and τ decay data: $(696.3 \pm 6.2_{\rm exp} \pm 3.6_{\rm rad}) \times 10^{-10}$ (e^+e^- - based), $(711.0 \pm 5.0_{\rm exp} \pm 0.8_{\rm rad} \pm 2.8_{SU(2)}) \times 10^{-10}$ (τ - based) [26].

Hadronic contributions to $\Delta\alpha$. Consider the hadronic correction to electromagnetic fine structure constant α at the Z-boson scale defined by Eq. (5). For $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ we obtain

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = (278.2 \pm 3.5) \times 10^{-4}. \qquad (11)$$

The experimental average value is $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = (275.5 \pm 1.9_{\rm expt} \pm 1.3_{\rm rad}) \times 10^{-4}$ [27].

5. Conclusions

Nonperturbative method of performing QCD calculations has been developed. The method based on analytic approach, takes into account a summation of infinite numbers of threshold singularities and involves non-perturbative light quark masses. The following quantities and functions have been analysed: R_τ^V, $D_V(Q^2)$, $R_\Delta(s)$, $a_\mu^{\rm had}$, and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$. It has been demonstrated that the method proposed allows us to describe well these objects.

Acknowledgments

It is a pleasure to thank Academician D.V. Shirkov for interest in the work, support and useful discussion. We also express our gratitude to A.E. Dorokhov, S.B. Gerasimov, A.V. Efremov, O.V. Teryaev, and A.V. Nesterenko for valuable discussions and helpful remarks.

This work was supported in part by the International Program of Cooperation between Republic of Belarus and JINR, the Belarus State

Program of Basic Research “Physics of Interactions”, the RFBR grant No. 05-01-00992, and NSh-2339.2003.2.

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