PSEUDO-PLANAR MOTION GENERATORS

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Abstract In this paper, a particular type of motion is called pseudo-planar motion and termed *Y* motion for brevity. A set of *Y*-motions having the same plane direction and the same pitch is endowed with the algebraic structure of a three-dimensional (3D) Lie group. The possible architectures of the mechanical generators of a *Y* subgroup are disclosed. All singular postures of *Y*-motion generators are described and their embodiments are graphically displayed.

Keywords: Pseudo-planar motion, Lie group, mechanical generator, singularity

1. Introduction

A special 3-dof-motion type, which includes any translation parallel to a given plane and any helical motion with a given pitch about any axis provided that the axis is perpendicular to the foregoing plane can be called pseudo-planar motion and is denoted *Y*-motion. A set of *Y* motions with a given plane direction and a given pitch is endowed with the algebraic structure of a 3D Lie group and, in other words, is a 3D displacement Lie subgroup. Hervé (1978) defined this kind of subgroup and some essential properties of the *Y*-motion generators together with some examples of applications were disclosed more recently (Hervé, 2004). In Angeles (2004), a subgroup of *Y*-motions is named a "translating-screw" group. In this paper, *Y*-motion generators including hinged parallelograms are introduced and the singular postures of the generators are comprehensively derived.

One can discriminate two types of singularity, namely permanent singularity and local singularity. The permanent singularity yields an inadequate chain that does never generate the desired *Y* motion. Local

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singularities are specific of particular poses of the chain that generally generates the Y motion.

In any singular pose of the chain, intermediate bodies between the distal bodies of the open chain can undergo motions that are passive with respect to the relative motion between the distal bodies. Such passive motions may have finite (or full-cycle) amplitude or only infinitesimal amplitude. For brevity, these two types of singularity will be designated by the shortened locutions, infinitesimal singularity and finite singularity, respectively.

After recalling some properties of the Y motion, we enumerate all generators of Y-motion with serial arrays of 1-dof Reuleaux pairs or hinged parallelograms. The article will emphasize the singularity in the previous generators.

2. Pseudo-Planar Motion Generators

A 3D Lie group of Y displacements is denoted $\{\boldsymbol{Y}(\mathbf{w}, p)\}$ where curly brackets indicate a set, $\mathbf{w}$ is a given unit vector perpendicular to the plane Pl and p is the given pitch of the feasible helical displacements. The pitch can be any real number and, therefore, the planar displacements are the special case $p = 0$. This is reflected by the notation: $\{\boldsymbol{Y}(\mathbf{w}, 0)\} = \{\boldsymbol{G}(\mathbf{w})\}$. Furthermore, any set of helical displacements with pitch p around any axis parallel to $\mathbf{w}$ is included in $\{\boldsymbol{Y}(\mathbf{w}, p)\}$. It is straightforward to verify that any translation parallel to Pl belongs to $\{\boldsymbol{Y}(\mathbf{w}, p)\}$. Hence, $\{\boldsymbol{Y}(\mathbf{w}, p)\}$ has 3 categories of proper Lie subgroups. They are: (a) $\{\boldsymbol{H}(N, \mathbf{w}, p)\}$: any set of helical movements of axis $(N, \mathbf{w})$ with the given pitch p, $\forall N$; (b) $\{\boldsymbol{T}(\mathbf{s})\}$: any set of rectilinear translations parallel to any given vector $\mathbf{s}$ that must be perpendicular to $\mathbf{w}$, $\forall \mathbf{s} \perp \mathbf{w}$; (c) $\{\boldsymbol{T}(Pl)\} = \{\boldsymbol{T}(\perp\mathbf{w})\}$: set of planar translations parallel to the Pl-plane (or perpendicular to $\mathbf{w}$). The improper subgroups of $\{\boldsymbol{Y}(\mathbf{w}, p)\}$ are $\{\boldsymbol{E}\}$, which contains only one element, namely the identity E, and $\{\boldsymbol{Y}(\mathbf{w}, p)\}$ itself.

The set of feasible displacements of a rigid body with respect to another body of the same kinematic chain is called kinematic bond between the bodies. The serial layout of kinematic pairs generates a bond between the distal bodies, which bond is the product of the bonds generated by the serial pairs. The chains producing a given bond are called its mechanical generators. Generally, a given bond has several mechanical generators.

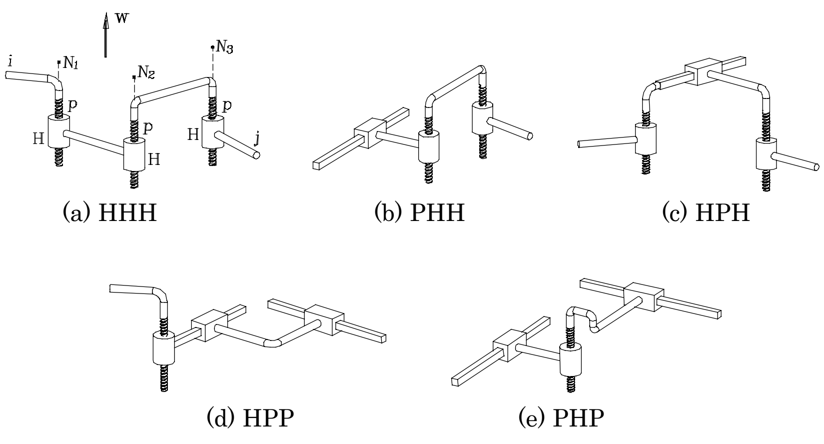
Because of the closure of the product in a 3D subgroup $\{\boldsymbol{Y}(\mathbf{w}, p)\}$, the product of three 1D manifolds included in $\{\boldsymbol{Y}(\mathbf{w}, p)\}$ can be equated to

$\{\boldsymbol{Y}(\mathbf{w},\ p)\}$ in a neighborhood of E provided that this product is 3-dimensional. The following equalities express the 3D subgroup $\{\boldsymbol{Y}(\mathbf{w},\ p)\}$ as products of three 1D subgroups, which are associated to the 1-dof Reuleaux pairs (Reuleaux, 1875), as shown in Figure 1.

$$\{\boldsymbol{Y}(\mathbf{w}, p)\} = \{\boldsymbol{H}(N_1, \mathbf{w}, p)\}\ \{\boldsymbol{H}(N_2, \mathbf{w}, p)\}\ \{\boldsymbol{H}(N_3, \mathbf{w}, p)\} \tag{1a}$$
$$= \{\boldsymbol{T}(\mathbf{u})\}\ \{\boldsymbol{H}(N_2, \mathbf{w}, p)\}\ \{\boldsymbol{H}(N_3, \mathbf{w}, p)\} \qquad (\forall\ \mathbf{u} \perp \mathbf{w}) \tag{1b}$$
$$= \{\boldsymbol{H}(N_1, \mathbf{w}, p)\}\ \{\boldsymbol{T}(\mathbf{u})\}\ \{\boldsymbol{H}(N_3, \mathbf{w}, p)\} \qquad (\forall\ \mathbf{u} \perp \mathbf{w}) \tag{1c}$$
$$= \{\boldsymbol{H}(N_1, \mathbf{w}\ \ p)\}\ \{\boldsymbol{H}(N_2, \mathbf{w}, p)\}\ \{\boldsymbol{T}(\mathbf{u})\} \qquad (\forall\ \mathbf{u} \perp \mathbf{w}) \tag{1d}$$
$$= \{\boldsymbol{T}(\mathbf{u})\}\ \{\boldsymbol{H}(N, \mathbf{w}, p)\}\ \{\boldsymbol{T}(\mathbf{v})\} \qquad (\forall\ \mathbf{u} \perp \mathbf{w},\ \forall\ \mathbf{v} \perp \mathbf{w},\ \mathbf{u} \neq \mathbf{v}) \tag{1e}$$
$$= \{\boldsymbol{H}(N, \mathbf{w}, p)\}\ \{\boldsymbol{T}(\mathbf{u})\}\ \boldsymbol{T}(\mathbf{v})\} \qquad (\forall\ \mathbf{u} \perp \mathbf{w},\ \forall\ \mathbf{v} \perp \mathbf{w},\ \mathbf{u} \neq \mathbf{v}) \tag{1f}$$
$$= \{\boldsymbol{T}(\mathbf{u})\}\ \{\boldsymbol{T}(\mathbf{v})\}\ \{\boldsymbol{H}(N, \mathbf{w}, p)\} \qquad (\forall\ \mathbf{u} \perp \mathbf{w},\ \forall\ \mathbf{v} \perp \mathbf{w},\ \mathbf{u} \neq \mathbf{v}) \tag{1g}$$

The serial Y-motion generators with 1-dof Reuleaux pairs are HHH, PHH, HPH, PHP and HPP. Reversing the order of joints also yields a mechanical generator of $\{\boldsymbol{Y}(\mathbf{w}, p)\}$.

The coupling of two opposite bars in a hinged parallelogram can also be used for the structural synthesis of Y motion generators. As a matter of fact, the two bars remain parallel and the motion of one bar with respect to the other bar is 1-dof translation along a circle. Replacing one or two P pairs in the generators of Figure 1 by one or two hinged parallelograms, we obtain seven Y-motion generators with hinged parallelograms, Figure 2. The planes of the parallelograms must be perpendicular to $\mathbf{w}$. Flattened parallelograms are singular and must be avoided.



(a) HHH (b) PHH (c) HPH

(d) HPP (e) PHP

Figure 1. Y-motion generators.

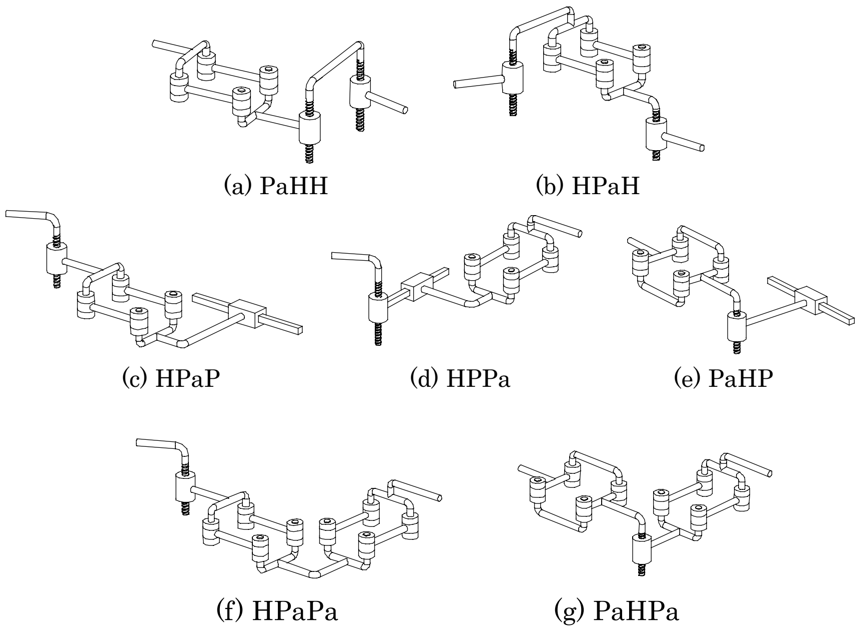


Figure 2. Y-motion generators with hinged parallelograms.

3. Finite Singularity of the Y-Motion Generators

In this section, the singularity rises from an undesired finite (or full-cycle) motion of an intermediate link in the chain aiming to generate Y motion. If the distal bodies of the chain are rigidly connected, then the resulting closed chain is movable with one or more degrees of freedom. Such a type of singularity may be permanent or local. The former does not correspond properly to a generator singularity but actually characterizes inadequate chains, which do never generate Y motion. The latter may happen in particular poses of a chain that generally generates the Y motion.

From the Delassus contribution [Delassus, 1922, Waldron, 1969, Lee, 1998], there are only "ordinary" types of movable three-bar linkages. Using Hervé's approach, the "ordinary" mobility can be explained via the group algebraic properties of the displacement set. It is straightforward to derive all the possible cases of group dependency between the subgroups of $\{\boldsymbol{Y}(\mathbf{w}, p)\}$. It is worth recalling that two subgroups are dependent iff their set intersection is not $\{\boldsymbol{E}\}$. The 2D subgroup of planar translations $\{\boldsymbol{T}(Pl)\}$ can be dependent of a subgroup of rectilinear translation parallel to Pl. A 1D subgroup of rectilinear translation or a 1D subgroup of helical displacements can be dependent only of itself.

Obviously, if the chain includes three P pairs parallel to the plane Pl, the closed chain PPP is a trivial 1-dof chain associated to the 2D subgroup of planar translations $\{\boldsymbol{T}(Pl)\} = \{\boldsymbol{T}(\perp\mathbf{w})\}$ parallel to the plane Pl

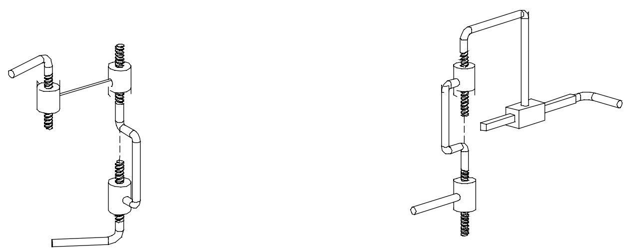
that is perpendicular to **w**. Its corresponding open chain PPP generates $\{\boldsymbol{T}(\perp \mathbf{w})\}\subset\{\boldsymbol{Y}(\mathbf{w}, p)\}$ with 1-dof of passive mobility. Hence, the generators of *Y*-motions necessarily include one or more H pair.

In a chain with two P pairs and one H pair, an undesired motion may happen if the P pairs are parallel. Such a geometric arrangement of the P pairs may be permanent or transitory. If the P pairs are adjacent, then their common rigid link maintains the parallelism and the chain is a wrong generator of *Y*-motion. In a PHP array, the angle between the two P pairs can change and the chain generates *Y*-motion when the P pairs are not parallel and may become locally singular in a possible posture with transitory parallel P pairs. Both cases are shown in Figure 3.

(a) permanent finite singularity (b) locally finite singularity

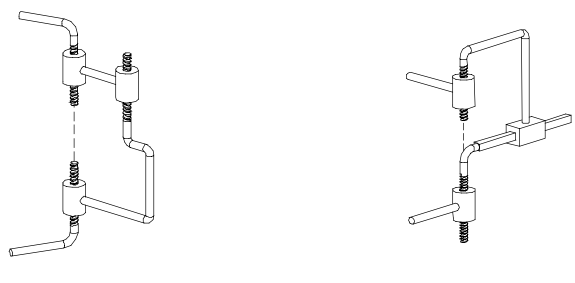
Figure 3. Finite Singularity of PPH and PHP *Y*-motion generators.

In a pseudo-planar chain with two H pairs, an undesired finite motion may happen when two H pairs are coaxial. It is an inadequate chain if the coaxial H pairs are adjacent as shown in Figure 4. Otherwise, it is a local (or transitory) singularity, Figure 5.



(a) permanent singularity (b) permanent singularity

Figure 4. Permanent finite singularity of chains with two coaxial H pairs.



(a) (b)

Figure 5. Finite local singularity of *Y*-motion generators with two coaxial H pairs.

4. Infinitesimal Singularity of HHH Generators

Through an infinitesimal displacement belonging to the $\{H(N, \mathbf{w}, p)\}$ subgroup, a generic point M of the Euclidean affine 3D space is transformed into another point $M' = M + \mathrm{d}\mathbf{M}$, that is,

$$M \longrightarrow M' = N + (\alpha\, p/2\pi)\, \mathbf{u} + \exp(\alpha\, \mathbf{w}\times)\, (\mathbf{NM}) \tag{4}$$

where is α an infinitesimal angle. The exponential series yields $\exp(\alpha\mathbf{w}\times)(\mathbf{NM}) = (\mathbf{NM}) + \alpha\mathbf{w} \times(\mathbf{NM})$. Hence,

$$M' = M + \alpha\, [(p/2\pi)\, \mathbf{w} + \mathbf{w} \times (\mathbf{NM})] \tag{5}$$

and, the point O being any origin,

$$(\mathbf{OM}') = (\mathbf{OM}) + \alpha\, [(p/2\pi)\, \mathbf{w} + (\mathbf{ON})\times\, \mathbf{w} + \mathbf{w} \times (\mathbf{OM})] \tag{6}$$

This further provides

$$\mathrm{d}\mathbf{M} = (\mathbf{OM}') - (\mathbf{OM}) = \alpha[\mathbf{w}\times (\mathbf{OM}) + (\mathbf{ON})\times \mathbf{w} + (p/2\pi)\mathbf{w}] \tag{7}$$

The expression in Eq.(7) is usually called the twist in the H pair. Actually, the twist is a mapping, $M \rightarrow \mathrm{d}\mathbf{M}$, of the 3D Euclidean affine space in the 3D Euclidean vector space. The expression of $\mathrm{d}\mathbf{M}/\alpha$ determines what is often called the “rate of twist”. The rate of twist is characterized at the origin O by the datum of two vectors, namely $\mathbf{w}$ and $\mathbf{t} = (\mathbf{ON}) \times \mathbf{w} + (p/2\pi)\, \mathbf{w}$. Conversely, the pitch p and the foot N of the perpendicular drawn from O to the screw axis can be derived from $\mathbf{w}$ and $\mathbf{t}$ by employing the two relations $k = p/2\pi = \mathbf{w}.\mathbf{t}$ and $(\mathbf{ON}) = \mathbf{w} \times \mathbf{t}$.

Consequently, the twist of any H pair, which has the axis $(N, \mathbf{w})$ and the pitch $p = 2\pi k$ can be expressed as

$$\alpha[\mathbf{w} \times (\mathbf{OM}) + (\mathbf{ON}) \times \mathbf{w} + k\mathbf{w}\,] = \alpha\, [\mathbf{w}\times(\mathbf{OM}) + \mathbf{t}] \tag{8}$$

In order to obtain simple expressions in what follows, without loss of the generality, the point N that can be any point belonging to the screw axis, is chosen in the plane that is perpendicular to the unit vector $\mathbf{w}$ and contains the origin O; this plane will be denoted $Pl\,(O, \perp\mathbf{w})$.

Any linear combination of two twists is a third twist, which is called resultant twist. The resultant twist is the twist of the serial array of two pairs characterized by the first two twists. Natively the resultant twist is linearly dependent on the first two twists and is an element of the linear span of the first two twists. In other words, the three twists belong to the same 2D vector subspace of the 6D vector space of all twists. If three kinematic pairs are characterized by three linearly dependent twists, then the serial array of these pairs is singular. Generally, this singularity is a local infinitesimal singularity.

Let us consider two H pairs with the same pitch $p = 2\pi k$ and parallel axes $(N_1, \mathbf{w})$ and $(N_2, \mathbf{w})$. We choose N_1 and N_2 in the plane $Pl\,(O, \perp\mathbf{w})$. Obviously, if $N_1 = N_2$, then the screw pairs are coaxial and the linear span of the two twists is made of the twists of all coaxial H pairs with the pitch p. In what follows, $N_1 \neq N_2$. The twist of the first H is $\mathrm{d}_1\mathbf{M} = \alpha[\mathbf{w} \times (\mathbf{OM}) + (\mathbf{ON}_1) \times \mathbf{w} + k\mathbf{w}]$ and the twist of the second H is $\mathrm{d}_2\mathbf{M} =$

$\beta[\mathbf{w}\times(\mathbf{OM}) + (\mathbf{ON}_2)\times\mathbf{w} + k\mathbf{w}]$; α and β denote infinitesimal angles in each of the two H pairs. The resultant twist $d_R\mathbf{M}$ of a serial array of two parallel H pairs having the same pitch p, can be expressed as

$$d_R\mathbf{M} = d_1\mathbf{M} + d_2\mathbf{M} = \alpha_R[\mathbf{w}\times(\mathbf{OM}) + (\mathbf{ON}_R)\times\mathbf{w} + k_R\mathbf{w}] = \alpha_R[\mathbf{w}\times(\mathbf{OM}) + \mathbf{t}_R] \quad (9)$$

in which

$\alpha_R = \alpha + \beta$

$\mathbf{t}_R = (\mathbf{ON}_R)\times\mathbf{w} + k_R\mathbf{w} = [\alpha(\mathbf{ON}_1) + \beta(\mathbf{ON}_2)]/(\alpha+\beta)\times\mathbf{w} + k_R\mathbf{w}$

$k_R = p_R/2\pi = (\alpha p + \beta p)/[2\pi(\alpha+\beta)] = p/2\pi = k$

N_R : a point belonging to the resultant screw axis, in $Pl(O,\perp\mathbf{w})$.

One can identify in $[\alpha(\mathbf{ON}_1) + \beta(\mathbf{ON}_2)]/(\alpha+\beta)$ the barycenter (center of mass) of the point N_1 and the point N_2 with the mass $\alpha/(\alpha+\beta)$ and the mass $\beta/(\alpha+\beta)$, respectively. Hence, N_R lies on the straight line N_1N_2. One can readily demonstrate

$$(\mathbf{N}_R\mathbf{N}_1) = (\mathbf{ON}_1) - (\mathbf{ON}_R) = (\mathbf{ON}_1) - [\alpha(\mathbf{ON}_1) + \beta(\mathbf{ON}_2)]/(\alpha+\beta)$$
$$= (\mathbf{ON}_1) - [\alpha(\mathbf{ON}_1) + \beta(\mathbf{ON}_1 + \mathbf{N}_1\mathbf{N}_2)]/(\alpha+\beta)$$
$$= \beta(\mathbf{N}_2\mathbf{N}_1)/(\alpha+\beta) \quad (10a)$$

By the same token,

$$(\mathbf{N}_R\mathbf{N}_2) = -\alpha(\mathbf{N}_2\mathbf{N}_1)/(\alpha+\beta) \quad (10b)$$

Equations (10a) and (10b) show that the coefficients $\alpha/(\alpha+\beta)$ and $\beta/(\alpha+\beta)$ can be expressed with ratios of vectors that are derived of the arrangement on a straight line of the three points N_1, N_2 and N_R, namely $\alpha/(\alpha+\beta) = (\mathbf{N}_2\mathbf{N}_R)/(\mathbf{N}_2\mathbf{N}_1)$ and $\beta/(\alpha+\beta) = (\mathbf{N}_1\mathbf{N}_R)/(\mathbf{N}_1\mathbf{N}_2)$.

The resultant twist can be identified as the twist of a H pair with the pitch p and the axis $(N_R, \mathbf{w})$. We find out the singular chain postures that are represented by the following product of three subgroups of helical displacements

$$\{\boldsymbol{H}(N_1, \mathbf{w}, p)\}\{\boldsymbol{H}(N_2, \mathbf{w}, p)\}\{\boldsymbol{H}(N_R, \mathbf{w}, p)\} \text{ with } N_R \in \text{line}(N_1N_2) \quad (11)$$

The position of N_R on the straight line N_1N_2 can be anyone. The three H pair axes lies in the same plane parallel to $\mathbf{w}$, Figure 6. Furthermore, the planar case is a special case ($p=0$) of the pseudo-planar case. Hence, a generator RRR of planar gliding motion becomes singular iff the three R axes lies in a plane as shown in Figure 7.

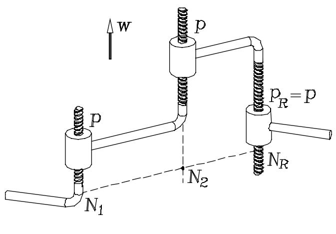


Figure 6. A HHH chain with a local infinitesimal singularity.

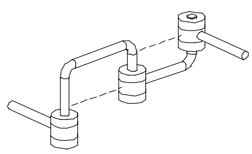


Figure 7. Infinitesimal singularity of RRR generator of planar motion.

5. Infinitesimal Singularity in Chains with One P

Let us consider a pseudo-planar chain including a P pair generating $\{\boldsymbol{T}(\mathbf{s})\}$. An infinitesimal displacement belonging to the subgroup $\{\boldsymbol{T}(\mathbf{s})\}$ is the point transformation $M \to M' = M + \beta\mathbf{s}$ where β is an infinitesimal real number. This transformation is an infinitesimal translation that has no screw axis $\mathrm{d}\mathbf{M} = \beta\mathbf{s}$. Now, let us consider one H pair having the axis $(N, \mathbf{w})$ and the pitch $p = 2\pi k$ whose twist is $\mathrm{d}_1\mathbf{M} = \alpha[\mathbf{w}\times(\mathbf{OM}) + (\mathbf{ON})\times\mathbf{w} + k\mathbf{w}]$ and one prismatic P pair parallel to the unit vector $\mathbf{s}$ whose twist is $\mathrm{d}_2\mathbf{M} = \beta\mathbf{s}$. The resultant twist of the serial array HP can be expressed as

$$\mathrm{d}_R\mathbf{M} = \mathrm{d}_1\mathbf{M} + \mathrm{d}_2\mathbf{M} = \alpha[\mathbf{w}\times(\mathbf{OM}) + (\mathbf{ON})\times\mathbf{w} + (k\mathbf{w} + \beta/\alpha\mathbf{s})] = \alpha_R[\mathbf{w}\times(\mathbf{OM}) + \mathbf{t}_R] \quad (15)$$

in which $\alpha_R (=\alpha)$ denotes the infinitesimal angle in the resultant twist. One can readily identify the twist of a H pair. The twist axis is parallel to $\mathbf{w}$. The pitch is $p_R = 2\pi\mathbf{w}.\mathbf{t}_R = 2\pi[k + \beta/\alpha(\mathbf{w}.\mathbf{s})] = 2\pi k = p$ because $\mathbf{w}.\mathbf{s} = 0$, $\mathbf{s}$ being perpendicular to $\mathbf{w}$. The axis position is determined by a point N_R belonging to the resultant screw axis. The point N_R is chosen in the plane $Pl\,(O, \perp\mathbf{w})$. $(\mathbf{ON}_R) = \mathbf{w}\times\mathbf{t}_R = \mathbf{ON} + (\beta/\alpha)\mathbf{w}\times\mathbf{s}$. The vector $\mathbf{w}\times\mathbf{s}$ is parallel to the plane $Pl\,(O, \perp\mathbf{w})$ and is perpendicular to $\mathbf{s}$. Moreover, one can verify that $\mathbf{w}\times\mathbf{s}$ is a unit vector. The number $a = \beta/\alpha$ is the abscissa of point N_R in the frame of reference $(N, \mathbf{w}\times\mathbf{s})$. The abscissa a can be any real number.

We find out the singular postures of a HPH generator of Y-motion. The P pair is perpendicular to the plane of the two parallel H axes as shown in Figure 8.

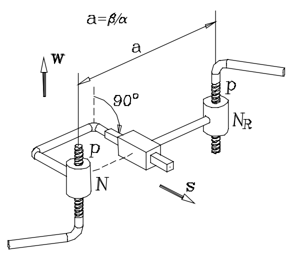


Figure 8. Local infinitesimal singularity of HPH Y-motion generator.

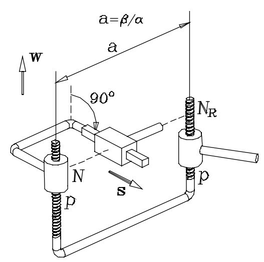


Figure 9. Local infinitesimal singularity of PHH Y-motion generator.

The previous resultant twist of a HP array is also the resultant twist of a PH array. Therefore, Figure 9 shows a singular pose of a PHH generator. Furthermore, Figure 10 and Figure 11 depict the singular poses in the generators RPR and PRR (or RRP) of the subgroup $\{\boldsymbol{G}(\mathbf{w})\}$ of planar gliding motions perpendicular to $\mathbf{w}$.

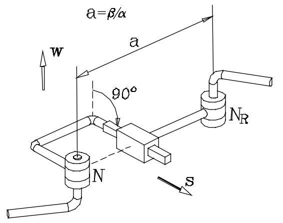


Figure 10. Infinitesimal singularity of RPR *G*-motion generator.

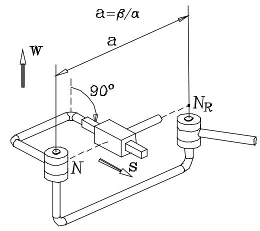
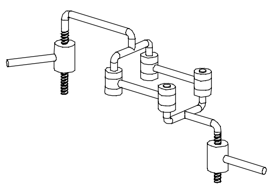


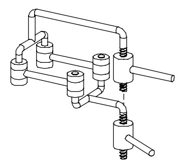
Figure 11. Infinitesimal local singularity of PRR *G*-motion generator.

To sum up, a HPH, PHH or HHP open chain aiming to be a generator of *Y*-motion has a local infinitesimal singularity iff the P pair is perpendicular to the plane of the two parallel H axes.

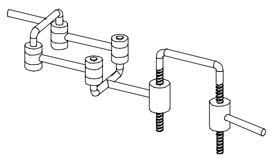
What is more, based on the above findings, the possible local singular postures of *Y* motion generators with hinged parallelograms are readily deduced and are displayed in Figure 12.



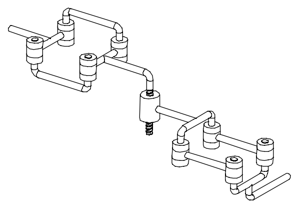
(a) deduced from Figure 8



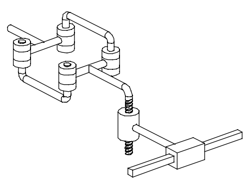
(b) deduced from Figure 5(b)



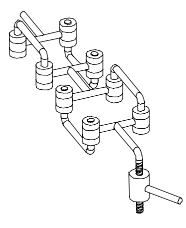
(c) deduced from Figure 9



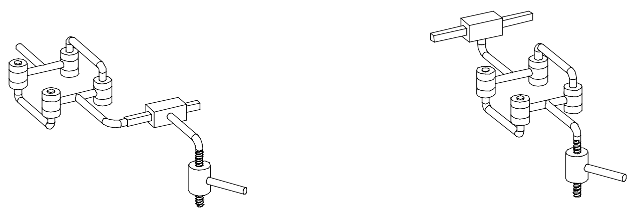
(d) deduced from Figure 3(b)



(e) also deduced from Figure 3(b)



(f) deduced from Figure 3(a)



(g) also deduced from Figure 3(a) (h) also deduced from Figure 3(a)

Figure 12. Singular postures of *Y*-motion generators with hinged parallelograms.

6. Conclusion

Five distinct architectures of *Y* motion generators with Reuleaux pairs and seven ones with hinged parallelograms are enumerated and their kinematic chains are graphically displayed in order to be a tool for the selection of limb structures in the synthesis of parallel manipulators. The singular postures resulting of undesired motion with finite and infinitesimal amplitude are disclosed through the study of group dependency of displacement subsets and linear dependency of twists, respectively. It is quite clear that several novel 3-dof parallel manipulators with pseudo-planar limbs can be envisioned but this lies outside of the scope of this paper.

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