The Link Between Neutrino Masses and Proton Decay in Supersymmetric Unification

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Fig. 1. Jogesh C. Pati delivering the B.M. Birla Science Centre Distinguished Lecture

Jogesh C. Pati was born in the Orissa State of India. After graduation with Honors from the famous Ravenshaw College of the Utkal University in 1955, Jogesh completed his Masters from the Delhi University in 1957. After obtaining his PhD from the University of Maryland in USA, he had a few very prestigious fellowships including the R.C. Tolman Post Doctoral position at Caltech from 1960 to 1962. Thereupon he joined as an Assistant Professor at the University of Maryland in 1963, becoming an Associate Professor in 1967 and a full Professor in 1973 at the same University. He was also the Chairman for the Centre for Theoretical Physics, University of Maryland between 1984 to 1987 and again 1993 to 1994.

Prof. Pati who is undoubtedly one of the foremost theoretical physicists of Indian origin has made pioneering contributions towards the goal of a unification of quarks and leptons, the fundamental particles, and of their gauge forces viz., strong interactions. His formulation done in collaboration

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with Prof. Abdus Salam, in original gauge theory of quark lepton unification and their resulting insight that violations of baryon and lepton numbers, particularly which would manifest in proton decay are likely to be consequences of such a unification, has been a cornerstone of modern particle physics. The Pati-Salam $SU(4)$-color, left right symmetry and the associated existence of the righthanded neutrinos provides some of the ingredients for understanding the recently discovered neutrino oscillations and masses. Much of this work was done in the 1970s at the International Center for Theoretical Physics.

Prof. Pati has been the recipient of several honors and awards and has also held several prestigious Visiting Professorships. He was a Member of the Institute of Advanced Study, Princeton, a Visiting Professor or Scientist at ICTP, Trieste, CERN, Geneva, SLAC at Stanford, University of Bonn, the Schrodinger Visiting Professor of the University of Vienna, the B.M. Birla Visiting Professor at the B.M. Birla Science Centre and so on. He has also been a Guggenheim Fellow, the distinguished Homi J. Bhabha Chaired Professor of the Government of India and received the prestigious Dirac Medal in 2000. In honor of his life time contributions to theoretical elementary particle physics the University of Maryland organized a Special Symposium. He has well over a hundred important publications.

My acquaintance with Prof. Pati goes back to nearly thirty years. Through this period we have been meeting off and on. He is a very soft spoken and thorough analytical scholar whose words are always well measured. Another of his striking characteristics is his utter simplicity.

Following recent joint works with K. Babu and F. Wilczek, I stress here that supersymmetric unification, based on symmetries like $SO(10)$ or a string-derived $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^C$ possesses some crucial features that are intimately linked to each other. They are: (a) gauge-coupling unification, (b) the masses and mixings of all fermions, including especially the neutrons, and last but not least (c) proton decay. In this context, it is noted that the value of $m(\nu_L) \sim 1/20eV$. suggested by the SuperK result, goes extremely well with the unification hypothesis, based on the ideas of (i) $SU(4)$ color, (ii) left-right symmetry and (iii) supersymmetry. A concrete proposal is presented within an economical $SO(10)$ framework that makes five successful predictions for the masses and mixings of the quarks and the charged leptons. The same framework explains why the $\nu_\mu - \nu_\tau$ oscillation angle is so large ($sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} \approx 0.82 - 0.96$) and yet V_{bc} is so small (≈ 0.04), both in accord with observation. The influence of the masses of the neutrinos and of the charged fermions on proton decay is discussed concretely, within the framework. The $\bar{\nu} K^+$ mode is expected to be dominant for SUSY $SO(10)$ as well as $SU(5)$. A distinctive feature of the $SO(10)$ model, however, is the likely prominence of the $\mu^+ K^\circ$ mode, which, for $SU(5)$, is highly suppressed. Our study shows that while current limits on the rate of proton decaying into $\bar{\nu} K^+$

[1] This is a technical talk.

is compatible with theoretical expectations, improvements in these limits by a factor of 5–10 should either turn up events, or else the $SO(10)$ framework described here, which is otherwise so successful, will be in jeopardy. Prominence of the $\mu^+ K^\circ$ mode, if observed, will be most significant in that it will reveal the intriguing link that exists between neutrino masses and proton decay in the context of supersymmetric unification.

1 Introduction

The SuperKamiokande (SK) result, convincingly showing the oscillation of ν_μ to ν_τ (or ν_X) with a value of $\delta m^2 \approx 10^{-2} - 10^{-3} eV^2$ and an almost maximal oscillation angle [1] $sin^2 2\Theta > 0.83$, clearly seems to require new physics beyond that of the standard model [2, 3]. This, as well as the other relatively firm result of solar neutrino-deficit [4] serve as important clues to physics at a deeper level. Understanding these neutrino anomalies as well as the bizarre pattern of masses and mixings of the quarks and the charged leptons is a major challenge that ought to be met within a fundamental unified theory.

It is of course known that the ideas of grand unification [5–8], as well as those of superstrings [9] call for gauge coupling unification at a high scale and for nucleon-instability. Furthermore, both these features are known to acquire a new perspective [10, 11] in the context of supersymmetry [12]. (For recent reviews on this topic and relevant references see e.g. [13] and [14]). While proton decay is yet to show, the clearest empirical support in favor of grand unification and supersymmetry has so far come from the dramatic meeting of the three gauge couplings of the standard model that is found to occur at a scale of $M_X \approx 2 \times 10^{16} GeV$, when these couplings are extrapolated from their measured values at LEP to high energies, in the context of supersymmetry [10].

One major goal of this talk will be to stress that supersymmetric unification based on symmetries like $SO(10)$ [15], or (for most purposes) a string-derived [6, 16] $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^C$, has implications not only for (i) gauge coupling unification and (ii) proton decay, but also for (iii) the masses and mixings of the charged fermions, as well as for (iv) those of the neutrinos. In fact, within a unified theory, all four features (i)–(iv) get intimately linked to each other, much more so than commonly thought. Each of these, including even charged fermion and neutrino-masses, provides some essential clue to the nature of higher unification. As regards the link between the four features, even neutrino masses turn out to have direct influence on proton decay. This is because the latter receives important contributions through a new set of $d = 5$ operators that depend directly on the Majorana masses of the right-handed neutrinos [17]. These new $d = 5$ operators, which were missed in the literature, contribute significantly to proton decay amplitudes, in addition of course to the “standard” $d = 5$ operators [11], which arise through the exchange of the color-triplet Higgsinos related to the electro-weak doublets. The standard and the new $d = 5$ operators, related to the charged fermion as well as the neutrino masses, together raise our expectation that proton decay should be observed in the near future [18].

I elucidate these remarks in the next four sections, covering the following topics:

(1) I first recall briefly the motivations for left-right symmetric unified theories, utilizing neutrino masses suggested by the SuperKamiokande result, as a guide. The support for supersymmetric unification in the light of the LEP data is noted. Further, the origin of such a unification in the context of superstrings as well as the potential problem of rapid proton decay that arises within supersymmetric theories are briefly reviewed. These discussions provide the background needed to cover the materials in the remaining sections.

(2) I then present arguments [2] to show that the SuperK result, especially the observed δm^2, interpreted as $m(\nu_\tau)^2$, receives a simple and natural explanation within the ideas of higher unification based on the symmetry group $G(224)$ [6], and thus $SO(10)$ or E_6. Such an explanation would not be possible within $SU(5)$.

(3) I present the first part of a recent work by Babu, Wilczek and myself [18], in which we attempt to understand, in the context of supersymmetric $SO(10)$, the masses and mixings of the neutrinos, suggested by the atmospheric and the solar neutrino anomalies, in conjunction with those of the quarks and the charged leptons. Adopting familiar ideas of generating hierarchical eigenvalues through off-diagonal mixings, and correspondingly cabibo-like mixing angles we find that the bizarre pattern of masses and mixings observed in the charged fermion sector, remarkably enough, can be adequately described (with $\sim 10\%$ accuracy) within an economical and thus predictive $SO(10)$ framework. A concrete proposal is presented involving a minimal Higgs system that provides five successful predictions for the masses and mixings of the quarks and the charged leptons in the three families. The same description goes extremely well with a value of $m(\nu_\tau) \sim (1/20)eV$ as well as with a large $\nu_\mu - \nu_\tau$ oscillation angle ($sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} \approx 0.82 - 0.96$), despite highly non-degenerate masses of the light neutrinos. Both these features are in good agreement with the SuperK result. Furthermore, this framework generically seems to support the small angle MSW explanation for the solar neutrino deficit [19].

I next present the second part of the work by Babu, Wilczek and myself [18] in which we link the rather successful supersymmetric $SO(10)$ framework describing fermion masses (noted above), with expectations for proton decay. We find that, given the SuperK result that suggests $m(\nu_\tau) \sim (1/20)eV$ and a large oscillation angle, the contribution from the new $d = 5$ operators mentioned above, and to some extent that from the standard operators as well, are significantly enhanced. As a result, in spite of generous allowance for uncertainties in the matrix elements and the SUSY spectrum, the inverse decay rate for the dominant $\bar{\nu} K^+$ mode is found to be bounded from above by about 7×10^{33} years. Typically, the lifetime should of course be lower than this bound. Furthermore, the $\mu^+ K^\circ$ mode is found to be prominent, with a branching ratio typically in the range of 10–50%, entirely because of contribution from the new operators. For comparison, minimal SUSY $SU(5)$, which has only the standard operators, typically leads to branching ratios $\leq 10^{-3}$ for this mode. Thus, our study of proton decay, correlated with fermion masses, strongly suggest that at least the candidate events for proton decay should be observed in the very near future, already at SuperK. The $\mu^+ K^\circ$ mode, if observed, would be specially important in exhibiting the link between neutrino masses and proton decay that exists within the $G(224)/SO(10)$ route to supersymmetric unification [18].

2 Learning from Neutrino Masses About Higher Unification

2.1 Motivations for $SU(4)$ Color Left-Right Symmetric Theories

If one assumes a hierarchical pattern of masses for the light neutrinos (with $m_{\nu_c} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$), which goes well within a quark-lepton unified theory, the SuperK result interpreted as $\nu_\mu - \nu_\tau$ oscillation, suggests a value for the ν_τ mass: $m_{\nu_\tau} \approx 1/20 eV((1/2)$ to $2)$. One can argue, as shown later in this section (see also [2]), that a ν_τ mass of this order can be understood simply within supersymmetric unified theories which are forced to introduce the existence of right-handed (RH) neutrino, accompanying the observed left-handed ones. Postponing an estimate of the ν_τ mass for a moment, if one asks the question: What symmetry on the one hand dictates the existence of the RH neutrinos, and on the other hand also ensures quantization of electric charge, together with quark-lepton unification, one is led to two very beautiful conclusions:

(i) Quarks and leptons must be unified minimally within the symmetry $SU(4)$ color, and that,
(ii) deep down, the fundamental theory should possess a left-right symmetric gauge structure: $SU(2)_L \times SU(2)_R$.

In short, the standard model symmetry must be extended minimally to the gauge symmetry [5, 6],

$$G(224) = SU(2)_L \times SU(2)_R \times SU(4)^C \tag{1}$$

With respect to $G(224)$, all members of the electron family fall into the neat pattern:

$$F^c_{LR} = \begin{bmatrix} u_x & u_y & u_b & \nu_v \\ d_x & d_y & d_b & c^- \end{bmatrix}_{LR} \tag{2}$$

The left-right conjugate multiplets F^c_L and F^c_R transform as (2,1,4) and (1,2,4) respectively, with respect to $G(224)$; likewise for the mu and the tau families.

Viewed against the background of the standard model, the symmetry structure $G(224)$ brought some attractive features to particle physics which include:

(i) Organization of all members of a family $(8_L + 8_R)$ within one left-right self-conjugate multiplet, with their peculiar hypercharges fully explained.
(ii) Quantization of electric charge, explaining why $Q_{\mathrm{electron}} = -Q_{\mathrm{proton}}$
(iii) Quark-lepton unification through $SU(4)$ color.
(iv) Left-right (i.e. parity) and particle-antiparticle symmetries in the fundamental laws which are violated only spontaneously [6, 20]. Thus, within the symmetry structure $G(224)$, quark-lepton distinction and parity violation may be viewed as low energy phenomena which should disappear at sufficiently high energies.
(v) Existence of right-handed neutrinos: Within $G(224)$, there must exist a right-handed (RH) neutrino (ν_R) accompanying the left-handed one (ν_L) for each family because ν_R is the fourth color partner of the corresponding RH up-quarks. It is also the $SU(2)_R$-doublet partner of the associated RH charged lepton (see eq. (2)). The RH neutrinos seem to be essential now (see later discussions) for understanding the non-vanishing light masses of the neutrinos, as suggested by the recent observations of neutrino oscillations.
(vi) B-L as a local gauge symmetry: $SU(4)$ color introduces B-L as a local gauge symmetry. Thus following the limits from Eotvos experiments, one can argue that B-L must be violated spontaneously. It has been realized, in the light of recent works, that to implement baryogenesis in spite of electro-weak sphaleron effects, such spontaneous violation of B-L at high temperatures may well be needed [21].

2.2 Route to Higher Unification: $SU(5)$ versus $G(224)/SO(10)$

To realize the idea of a single gauge coupling governing the three forces [5, 6], one must embed the standard model symmetry or $G(224)$, into a simple (or effectively simple, like $SU(N) \times SU(N)$) gauge group. The smallest such group is $SU(5)$ [7] which contains the standard model symmetry but not $G(224)$. As a result, $SU(5)$ does not possess some of the main advantages of $G(224)$ listed above. In particular, $SU(5)$ splits members of a family into two multiplets: $5 + 10$, whereas $G(224)$, subject to L-R symmetry, groups them into just one multiplet. $SU(5)$ violates parity explicitly. It does not possess $SU(4)$ color and therefore does not gauge B-L as a local symmetry. Further, $SU(5)$ does not contain the RH neutrinos as an integral feature. As I will discuss below, these distinctions between $SU(5)$ versus $G(224)$, or its extensions (see below), turn out to be especially relevant to considerations of neutrino as well as charged fermion masses, and thereby to those of proton decay.

Since $G(224)$ is isomorphic to $SO(4) \times SO(6)$, the smallest simple group to which it can be embedded is $SO(10)$ [15]. Historically, by the time $SO(10)$ was proposed, all the advantages of $G(224)$ [(i)–(vi), listed above] and the ideas of higher unification were in place. Since $SO(10)$ contains $G(224)$, the features (i)–(vi) are of course retained by $SO(10)$. In addition, the 16-fold left-right conjugate set $(F_L^c + F_R^c)$ of $G(224)$ corresponds to the spinorial 16 of $SO(10)$. Thus, $SO(10)$ preserves even the 16-plet family-structure of $G(224)$, without a need for any extension. If one extends $G(224)$ to the still higher symmetry [6] E_6, the advantages (i)–(vi) are retained, as in $SO(10)$, but in this case, one must extend the family structure from a 16 to a 27-plet.

Comparing $G(224)$ with $SO(10)$ as mentioned above, $SO(10)$ possesses all features (i)–(vi) of $G(224)$; in addition it offers gauge coupling unification. I should, however, mention at this point that the perspective on coupling unification and proton decay has changed considerably in the context of supersymmetry and superstrings. In balance, a string-derived $G(224)$ offers some advantages over a string-derived $SO(10)$, while the reverse is true as well. Thus, it seems that a definite choice of one over the other, as an effective theory below the string scale, is hard to make at this point. I will return to this point shortly.

2.3 Gauge Coupling Unification: Need for Supersymmetry

It has been known for some time that the precision measurements of the standard model coupling constants (in particular $sin^2\Theta_W$) at LEP put severe constraints on the idea of grand unification. Owing to these constraints, the non-supersymmetric minimal $SU(5)$, and for similar reasons, the one step breaking minimal non-supersymmetric $SO(10)$ model as well, are now excluded [23].

But the situation changes radically if one assumes that the standard model is replaced by the minimal supersymmetric standard model (MSSM), above a threshold of about $1TeV$. In this case, the three gauge couplings are found to meet [10], at least approximately, provided $\alpha_3(m_z)$ is not too low (see figures in [13, 23]). Their scale of meeting is given by

$$M_X \approx 2 \times 10^{16} GeV \quad (\text{MSSM} \quad \text{or} \quad \text{SUSYSU(5)}) \tag{3}$$

M_X may be interpreted as the scale where a supersymmetric grand unification symmetry (GUT) (like minimal SUSY $SU(5)$ or $SO(10)$) – breaks spontaneously into the supersymmetric standard model symmetry $SU(2)_L \times U(1) \times SU(3)^C$.

The dramatic meeting of the three gauge couplings thus provides a strong support for both grand unification and supersymmetry.

2.4 Compatibility Between MSSM and String-Unifications

The superstring theory [9], and now the M theory [24] provide the only known framework that seems capable of providing a good quantum theory of gravity as well as a unity of all forces, including gravity. It thus becomes imperative that the meeting of the gauge couplings of the three non-gravitational forces which occur by the extrapolation of the LEP data in the context of MSSM, be compatible with string unification.

Now, string theory does provide gauge coupling unification for the effective gauge symmetry, below the compactification scale. The new feature is that even if the effective symmetry is not simple, like $SU(5)$ or $SO(10)$, but instead is of the form $G(213)$ or $G(224)$ (say), the gauge couplings of $G(213)$ or $G(224)$ should still exhibit familiar unification at the string-scale, for compactification involving appropriate Kac-Moody levels (i.e. $k_2 = k_3 = 1, k_Y = \frac{5}{3}$ for $G(213)$), barring of course string threshold corrections [25]). And even more, the gauge couplings unify with the gravitational coupling $(8\pi G_N/\alpha')$ at the string scale, where G_N is the Newton's constant and α' is the Regge slope.

Thus one can realize coupling unification without having a GUT-like symmetry below the compactification scale. This is the new perspective brought forth by string theory. There is, however, an issue to be resolved. Whereas the MSSM unification scale, obtained by extrapolation of low energy data is given by $M_N \approx 2 \times 10^{16} GeV$, the expected one-loop level string unification scale [25] of $M_{st} \approx g_{st} \times (5.2 \times 10^{17} GeV) \approx 3.6 \times 10^{17} GeV$ is about twenty times higher. Here, one has used $\alpha_{st} \approx \alpha_G UT(MSSM) \approx 0.04$.

Possible resolutions of this mismatch between M_N and M_{st} by about a factor of 20 have been proposed (for a comprehensive review see e.g. [13] and [14]). These include:

(i) utilizing the idea of string duality that allows a lowering of M_{st} [26] compared to the value suggested by [25]; alternatively
(ii) the idea of a semi-perturbative unification that assumes the existence of two vector-like families at the TeV scale, $(16 + \overline{16})$ which raise α_{GUT} to about $0.25 - 0.3$, and thereby also M_X to a few $\times 10^{17} GeV$ [27]; or
(iii) the alternative of a string GUT solution, which would arise if superstrings yield an intact grand unification symmetry like $SU(5)$ or $SO(10)$, together with supersymmetry and the right spectrum – i.e. three chiral families and a suitable Higgs system – at M_{st}, and if the symmetry would break spontaneously at $M_X \sim 1/20 M_{st}$ to the standard model symmetry. In this last case, the gauge couplings would run together between M_X and M_{st} and thus the question of a mismatch between the two scales would not even arise. However, as yet, there does not seem to be even a semi-realistic string-derived GUT model [28]. Further, to-date, no string GUT solution exists with a resolution of the well-known doublet splitting problem, without which one would face the problem of rapid proton decay through the $d = 5$ operators [11] (see discussions below). This does not necessarily mean that a realistic GUT solution exhibiting doublet-triplet splitting cannot ultimately emerge from the string or the M theory.

While each of the solutions mentioned above possesses a certain degree of plausibility (see [13] for some additional possibilities), it is not clear, which, if any is utilized by the true string vacuum. This is related to the fact that, as yet, there is unfortunately no insight as to how the true vacuum is selected in the string or in the M theory.

2.5 A GUT or a Non-GUT String Solution?

Comparing string-derived GUT solutions with non-GUT solutions, where the former yield symmetries like $SU(5)$ or $(SO(10)$, while the latter lead to symmetries like $G(213)$ or $G(224)$ at the string scale, we see from the discussions above that each class has a certain advantage and possible disadvantages as well, compared to the other. In particular, a string GUT solution has the positive feature, explained above, that the issue of a mismatch between M_{st} and M_X does not arise for such a solution. For a non-GUT solution, however, although plausible mechanisms of the type mentioned above could remove the mismatch, a priori it is not clear whether any such mechanism is realized.

On the other hand, for a string-derived GUT solution [28], achieving doublet-triplet splitting so as to avoid rapid proton decay, is still a major burden. In this regard, the non-GUT solutions possess a distinct advantage because the dangerous color triplets are often naturally projected out [29,30]. Furthermore, these solutions invariably possess new "flavor" gauge symmetries, which are not available in GUTs. The flavor symmetries turn out to be immensely helpful in (a) providing the desired protection against gravity induced rapid proton decay [31], (b) resolving certain naturalness problems of supersymmetry such as those pertaining to the issues of squark-degeneracy, neutrino-Higgsino mixing and CP violation [32]–[34], and (c) explaining qualitatively the observed fermion mass hierarchy [29].

Weighing the advantages and possible disadvantages of both, it seems hard at present to make a clear choice between a GUT versus a non-GUT string solution. We will therefore keep our options open and look for other means, for example certain features of proton decay and neutrino masses, to provide a distinction. We will thus proceed by assuming that for a GUT solution, string theory will somehow provide a resolution of the problem of the doublet-triplet splitting, while for a non-GUT string solution, we will assume that one of the mechanisms mentioned above (for instance, that based on string-duality [26]), does materialize removing the mismatch between M_X and M_{st}. In general, a combination of the two mechanisms [26,27] may also play a role.

It turns out that there are many similarities between the predictions of $SO(10)$ and of a string-derived $G(224)$, especially as regards neutrino and charged fermion masses, primarily because both contain $SU(4)$ color.

With these discussions on higher unification, including the ideas of supersymmetry and superstrings to serve as a background, I proceed to discuss more concretely, firstly the masses and mixings of all fermions, and finally, their link to proton decay. An estimate of m_{ν_τ}, is presented next.

3 Mass of ν_τ: An Evidence in Favor of the $G(224)$ Route

One can now obtain an estimate for the mass ν_L^τ in the context of $G(224)$ or $SO(10)$ by using the following three steps [2]:

(i) First, assume that B-L and I_3R, contained in a string-derived $G(224)$ or $SO(10)$, break near the unification scale:

$$M_X \sim 2 \times 10^{16} GeV, \tag{4}$$

through VEVs of Higgs multiplets of the type suggested by string solutions [35]– i.e. $<(1,2,4)_H>$ for $G(224)$ or $<16_H>$ for $SO(10)$, as opposed to 126_H. In the process, the RH neutrinos $(\nu_R^\imath)$, which are singlets of the standard model, can and generically will acquire superheavy Majorana masses of the type $M_R^{\imath j} \nu_R^{\imath T} C^{-1} \nu_R^j$, by utilizing the VEV of $<16_H>$ and effective couplings of the form:

$$L_M(SO(10)) = \int_R^{\imath j} 16_\imath, 16_j 16_H, 16_H/M + hc \tag{5}$$

A similar expression holds for $G(224)$. Here $\imath, j = 1, 2, 3$, correspond respectively to e, μ and τ families. Such gauge-invariant non-renormalizable couplings might be expected to be induced by Planck-scale physics involving quantum gravity or string effects and/or tree-level exchange of superheavy states, such as those in the string tower. With $f_{\imath j}$ (at least the largest among them) being of order unity, we would thus expect M to lie between $M_{\text{Planck}} \approx 2 \times 10^{18} GeV$ and $M_{\text{string}} \approx 4 \times 10^7 GeV$. Ignoring for the present off-diagonal mixing (for simplicity), one thus obtains:

$$M_{3R} \approx \frac{f_{33} < 16_H >^2}{M} \approx f_{33}(2 \times 10^{14} GeV)\eta^2 (M_{\text{Planck}}/M) \tag{6}$$

This is the Majorana mass of the RH tau neutrino. Guided by the value of M_X, we have substituted $<16_H> = (2 \times 10^{16} GeV)\eta$ where $\eta \approx 1/2$ to 2, for this estimate.

(ii) Second, assume that the effective gauge symmetry below the string scale contains $SU(4)$ color. Now using $SU(4)$ color and the Higgs multiplet $(2,2,1)_H$ of $G(224)$ or equivalently 10_H of $SO(10)$, one obtains the relation $m_\tau(M_X) = m_b(M_X)$, which is known to be successful. Thus, there is a good reason to believe that the third family gets its masses primarily from the 10_H or equivalently $(2,2,1)_H$. In turn, this implies:

$$m(\nu^\tau_{\text{Dirac}}) \approx m_{\text{top}}(M_X) \approx (100 - 120) GeV \tag{7}$$

Note that this relationship between the Dirac mass of the tau neutrino and the top mass is special to $SU(4)$ color. It does not emerge in $SU(5)$.

(iii) given the superheavy Majorana masses of the RH neutrinos as well as the Dirac masses as above, the see-saw mechanism [36] yields naturally light masses for the LH neutrinos. For ν_L^τ (irgnoring mixing), one thus obtains, using eqs. (6) and (7),

$$m(\nu_L^\tau) \approx \frac{m(\nu^\tau_{\text{Dirac}})}{M_3R} \approx [(1/20) eV (1 \,\text{to}\, 1.44)/f_{33}\eta^2](M/M_{\text{Planck}}) \tag{8}$$

Considering that on the basis of the see-saw mechanism, we naturally expect that $m(\nu_L^e \ll m(\nu_L^\mu) \ll m(\nu_L^\tau)$, and assuming that the SuperK observation represents $\nu_L^\mu - \nu_L^\tau$ (rather than $\nu_L^\mu - \nu_x$) oscillation, so that the observed $\delta m^2 \approx 1/2(10^{-2} - 10^{-3})eV^2$ corresponds to $m(\nu_L^\tau)_{obs} \approx (1/15\,\text{to}\,1/40)eV$, it seems truly remarkable that the expected magnitude of $m(\nu_L^\tau)$, given by eq. (8), is just about what is observed if $f_{33}\eta^2(M_{\text{Planck}}/M)$ seems most plausible and natural [2]. It should be stressed that the estimate (8) utilizes the ideas of both supersymmetric unification, which yields the scale of M_{3R} (eq. (6)), and of $SU(4)$ color that yields $m(\nu^\tau_{\text{Dirac}})$ (eq. (7)). The agreement between the expected and the SuperK result thus suggests that, at a deeper level, near the string or the coupling unification scale M_X, the symmetry group $G(224)$ and thus the ideas of $SU(4)$ color and left-right symmetry are likely to be relevant to nature.

By providing clear support for $G(224)$, the Super K result selects out $SO(10)$ or E_6 as the underlying grand unification symmetry, rather than $SU(5)$. Either $SO(10)$ or E_6 or both of these symmetries ought to be relevant at some scale, and in the string context, as discussed in Section 2, that may well be in higher dimensions, above the compactification scale, below which there need be no more than just the $G(224)$ symmetry. If, on the other hand, $SU(5)$ were regarded as a fundamental symmetry, first, there would be no compelling reason, based on symmetry alone, to introduce a ν_a because it is a singlet of $SU(5)$. Second, even if one did introduce $\nu_R^\imath$ by hand, their Dirac masses, arising from the coupling $h^\imath 5_\imath < 5_H > \nu_R^\imath$, would be unrelated to the up-flavor masses and thus rather arbitrary (contrast with eq. (7)). So also would be the Majorana masses of the $\nu_R^\imath S$, which are $SU(5)$ invariant and thus can even be of order Planck scale (contrast with eq. (6)). This would give $m(\nu_L^\tau)$ in gross conflict with the observed value. In this sense, the SuperK result appears to disfavour $SU(5)$ as a fundamental symmetry, with or without sypersymmetry.

4 Fermion Masses and Neutrino Oscillations in *SO*(10)

4.1 Preliminaries

I now discuss the masses and mixing of the quarks and charged leptons in conjunction with those of the neutrinos, to see first of all how well they can be understood together within the ideas of higher unification.

The most striking regularity in the masses of the fermions belonging to the three families (at least of the charged ones) is their inter-family hierarchy. This is reflected by the uniform pattern: $m_t \gg m_c \gg m_u; m_b \gg m_s \gg m_d$; and $m_\tau \gg m_\mu \gg m_e$. Apart from this gross feature however, if one examines the pattern in more detail, it looks rather bizarre, especially when one compares intra-family mass splittings of the three families. For instance, while $m_t^\circ/m_b^\circ \sim 60$, one finds that $m_c^\circ/m_s^\circ \sim 10\,\text{and}\,m_u^\circ/m_d^\circ \sim 1/2$. Here, the superscript $\circ$ denotes that the respective mass is evaluated at the unification scale. Note that the ratio of the up - and down-flavor masses within a family varies widely in going from the third to the second to the first family. Further, comparing quark versus lepton masses of the down-flavor within a family in contrast to $m_b^\circ \approx m_\tau^\circ$, that suggests $b - \tau$ unification for the third family, one finds: $m_s^\circ \sim m_\mu^\circ/3$ and $m_d^\circ \sim 3m_e^\circ$ [37]. In short, there does not seem to be any obvious regularity in the intra-family mass splittings. The question is: do these apparent irregularities still have a simple origin?

The pattern seems to be equally bizarre when one examines the mixing angles. While the parameter $V_{us} = \Theta_c$, representing the mixing between the electron and the muon families in the quark sector, is moderately large (≈ 0.21), the parameter V_{cb}, representing $\mu - \tau$ family mixing, also in the quark sector, is small (≈ 0.044). This feature seems even more strange, when one compares V_{cb} with the $\nu_\mu - \nu_\tau$ oscillation angle, which also represents $\mu - \tau$ family mixing, although in the leptonic sector. This angle seems to be almost maximal: $sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} > 0.83$. One might have been tempted to associate such a large mixing angle with near degeneracy of ν_μ and ν_τ, as has been attempted by several authors. But, then, such degeneracy does not go well with the see-saw formula, especially within a unified scheme in which the Dirac masses of the neutrinos are related to those of the quarks which exhibit a large inter-family hierarchy. Thus one major puzzle is: Why V_{bc} is so small and yet $\Theta^{osc}_{\nu_\mu \nu_t}$ so large? Could the smallness of one imply the largeness of the other within a quark-lepton unified theory? Further, are these peculiarities of the mixing angles related to the irregularities in the intra-family mass splittings mentioned above?

From a theoretical viewpoint, the goal is to resolve some of these puzzles within a unified predictive theory, in particular to understand the masses and mixing of the neutrinos in conjunction with those of the quarks and the charged leptons, rather than in isolation. It is however known that there is no obvious way to address any of these puzzles in the context of the standard model (SM), because, a priori, the SM allows for all the masses and mixings to be arbitrary parameters. Even ignoring CP violation for the present discussion, there are 12 such observables: $m_t, m_b, m_\tau, m_c, m_s, m_\mu, m_u, m_d, m_l$, V_{us}, V_{cb} and V_{ub}. The 3×3 mass matrices of the 3 sectors (up, down and charged lepton) would in general have as many as $9 \times 3 = 27$ real parameters, which represent, however, only 12 observables. The parameters would even increase if one introduces RH neutrinos and considers both the Dirac and the Majorana mass matrices of the three neutrinos.

To reduce the number of parameters, it thus seems that one may have to appeal to symmetries of two kinds: first like those in $G(224)$ or $SO(10)$, which relate quark versus lepton as well as up-versus down-Yukawa couplings, and second "flavor" symmetries which distinguish between the three families (c, μ and τ) and could account for inter-family mass hierarchy. Interestingly enough, these latter symmetries do seem to arise in string solutions [29, 30] though not in GUT's.

To proceed further, we will use the following guidelines.

(1) Hierarchy through off-diagonal mixings: Recall earlier attempts [38] that attribute hierarchy in the quark mass matrices of the first two families to matrices of the type:

$$M = \begin{pmatrix} 0 & \epsilon \\ \epsilon & l \end{pmatrix} m_s^{(0)}, \tag{9}$$

for the (d, s) quarks, and likewise for the (u, c) quarks. Here $\epsilon \sim 1/10$. Note the symmetric form of eq. (9) (i.e. $M_{12} = M_{21}$) and especially the hierarchical pattern: $(1,1) \ll (1,2) \ll (2,2)$, where $(1,1) \leq 0(\epsilon^3)$. The symmetric nature of eq. (9) is guaranteed by group theory if the relevant Higgs field is a **10** of $SO(10)$. The hierarchical entries in eq. (9) can be ensured by imposing a suitable flavor symmetry that distinguishes between the two families (origin of such symmetries must ultimately be attributed to, for example, string theory). The pattern (eq. (9)) has the virtues that (a) it generates a hierarchy larger than the input parameter $\epsilon : |m_d/m_s| \approx \epsilon^2 \ll \epsilon$, and (b) it leads to the rather successful expression for the Cabibo angle:

$$\Theta_C \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{\imath\phi} \sqrt{\frac{m_u}{m_c}} \right| \tag{10}$$

Using $\sqrt{m_d/m_s} \simeq 0.22$ and $\sqrt{m_u/m_c} \simeq 0.06$, we see that eq. (7) works within 30% for any value of the phase ϕ, and perfectly for a value of the phase parameter ϕ around $\pi/2$.

A generalization of the pattern (eq. (9)) to the case of three families would suggest that the first and the second families (i.e. the e and the μ families) receive their masses primarily through their mixings with the third family (τ); the $(3,3)$ – element in this case is then the leading one in each sector. One must also rely on flavor symmetries that distinguish between the e, μ and τ families so as to ensure that the (1, 3) and (1, 2) mixing elements are smaller than the (2, 3) – element. We will follow this guideline, except, however, for the modification noted below.

(2) The need for an antisymmetric component: Although the symmetric hierarchical mass matrix (9) works well for the first two families, a matrix of the same form fails altogether to reproduce V_{cb}, for which it would yield:

$$|V_{cb}| \simeq \left| \sqrt{\frac{m_c}{m_t}} - e^{\imath\chi} \sqrt{\frac{m_s}{m_b}} \right| \tag{11}$$

Given that $\sqrt{m_s/m_b} \simeq 0.17$ and $\sqrt{m_c/m_t} \simeq 0.06$, we see that eq. (11) would yield $|V_{cb}|$ varying between 0.11 and 0.23, depending upon the value of the phase χ. This is however too big compared to the observed value of $V_{cb} \approx 0.04 \pm 0.003$, by at least a factor of 3. We thus see that the simple square root formula for the mixing angle in each sector ($sin\Theta_{\imath j} \approx tan\Theta_{\imath j} = \sqrt{m_\imath/m_j}$;

(see eq. (10) or (11)), arising from a symmetric matrix of the form eq. (9), fails for V_{cb}. We would interpret this failure as a clue to the presence of antisymmetric contribution to off-diagonal mixing in the mass matrix together with a symmetric one, (thus $m_{ij} \neq m_{ji}$) which would modify the square-root formula for the mixing angle to $\sqrt{(m_i/m_j)}\sqrt{(m_{ij}/m_{ji})}$, where m_i and m_j denote the respective eigenvalues. We will note below a simple group theoretical origin of such an antisymmetric component in $SO(10)$, even for a minimal Higgs system, and point out its crucial role in resolving some of the puzzles alluded to, above. The resolution would depend, however, on an additional feature noted below.

(3) The need for a contribution proportional to B-L: The success of the relations $m_b^\circ \approx m_t^\circ$ and also $m_\tau^\circ \approx m(\nu_\tau)^\circ_{\text{Dirac}}$ suggests that the members of the third family receive their masses primarily from the VEV of a Higgs field, which is a singlet of $SU(4)$ color and thus independent of B-L. That is in fact the case for the Higgs transforming as (2,2,1) of $G(224)$ or 10 of $SO(10)$. However, the empirical observations of $m_s^\circ \sim m_\mu^\circ/3$ and $m_d^\circ \sim 3m_e^\circ$, as well as the suppression of V_{bc} (noted above) together with the enhancement of $\Theta^{osc}_{\nu_\mu\nu_\tau}$, (SuperK result) clearly calls for a contribution proportional to B-L as well. This would be the case for contributions from the VEV of a Higgs transforming as 15 of $SU(4)$ color. We note below how such a contribution can arise simply for a minimal Higgs system in $SO(10)$. The amusing thing is that such a contribution, while it is proportional to B-L, turns out to be anti-symmetric as well, in the family-space, fulfilling the need (2).

I now present, following [18], a simple and predictive mass matrix, based on $SO(10)$, which is constructed by using the guidelines (1)–(3). For simplicity, I first consider only the μ and the τ families. The discussion is extended later to include the electron family.

4.2 The Minimal Higgs System for $SO(10)$ Breaking and Fermion Masses

The minimal Higgs system, capable of breaking $SO(10)$ at the unification scale M_X into the SM symmetry $G(213)$ consists of a 45_H, a 16_H and (for supersymmetry) a $\bar{16}_H$. Of these $<45_H>\sim M_X$ breaks $SO(10)$ into $G(2213) = SU(2)_L \times SU(2)_R \times U(1)_{B-l} \times SU(3)^C$, while $< 16_H >=< \bar{16}_H > \sim M_X$ breaks $SU(2)_R$ and B-L and thus $G(2213)$ into $G(213)$. To break $G(213)$ into $U(1)_{em} \times Su(3)^C$ at the electro-weak scale, one minimally needs in addition the VEV of a 10_H. Thus the minimal Higgs system, that is needed for appropriate $SO(10)$ breaking, consists of the set:

$$H_{\text{minimal}} = \{45_H, 16_H, \bar{16}_H, 10_H\} \tag{12}$$

Of these, only 10_H can have Yukawa coupling with the fermions at the cubic level of the form $h_{ij}16_i16_j10_H$, which could be the dominant source of masses, especially for fermions belonging to the third family. But the first two families must have additional sources for their masses because $a < 10_H >$ by itself would lead to three undesirable results: (a) $V_{CKM} = 1$, (b) purely symmetric mass-matrices, and (c) (B-L)-independent masses. We have on the other hand argued above the antisymmetric and (B-L)-dependent contributions to mass matrices are needed.

Now, there exist large-dimensional tenrosial multilets of $SO(10)$, that is 126_H and 120_H, which can have cubic-level Yukawa couplings with the fermions and give (B-L)-dependent contributions. Further, $< 120_H >$ gives purely family-antisymmetric contributions, as needed. There are however, two a priori reasons why we prefer not to use these large-dimensional multiplets: (a) They seem to be hard, if not impossible, to emerge from string solutions [35], and (b) generically, such large-dimensional multiplets tend to give large threshold corrections (typically exceeding 20%) to $\alpha_3(m_Z)$, thereby rendering observed coupling unification fortuitous. By contrast, the multiplets in the minimal set can arise in string solutions leading to $SO(10)$ ((45_H) arises at Kac-Moody level ≥ 2, while $16_H, \bar{16}_H$ and 10_H arise at level 1), and their threshold corrections have been computed. They were found not only to be smaller in magnitude, but also to have the right sign to go well with observed coupling unification [18].

Given these advantages of the minimal Higgs system (compared to those containing large multiplets like 126_H and/or 120_H) for $SO(10)$ breaking, the question arises: can this minimal system meet the requirements arising from fermion masses and mixing – that is, (a) $V_{CKM} \neq 1$, (b) presence of antisymmetric, and (c) that of (B-L)-dependent contributions? It was noted in [18] that minimal Higgs system can indeed meet all three requirements quite simply, if one allows for not just cubic, but also (seemingly) non-renormalizable effective quartic couplings of this minimal set with the 16-plets of fermions. Such quartic couplings could well arise through exchanges of superheavy particles (for example those in the string-tower) involving renormalizable couplings, and/or through quantum gravity.

Allowing for such cubic and quartic couplings of the minimal Higgs system and adopting the guideline eq. (1) of family hierarchical couplings, we are led to suggest the following effective Lagrangian for generating masses and mixings of the μ and τ families [18]. (The same consideration is extended later to include the electron family. For a related but different pattern, see [39]).

$$L_{\text{Yukawa}} = h_{33} 16_3 16_3 10_H + \frac{a_{23}}{M} 16_2 16_3 10_H 45_H$$
$$+ \frac{g_{23}}{M} 16_2 16_3 16_H + h_{23} 16_2 16_3 10_H \tag{13}$$

Note that a mass matrix of the type shown in eq. (9) (barring its symmetric form) results if the first term $h_{33} < 10_H >$ is dominant. This ensures $m_b^\circ \approx m_\tau^\circ$ and $m_\tau^\circ \approx m(\nu_{\text{Dirac}}^\tau)^\circ$.

The smallness of the remaining terms responsible for all-diagonal mixings, by about an order of magnitude compared to the h_{33} term, may come about as follows. First, as mentioned before, the smallness of the $SO(10)$ invariant coupling $h_{22}16_2 10_H$ (not shown) compared to the h_{23} coupling and that of h_{23} compared to h_{33} (i.e. $h_{22} \ll h_{21} ll h_{33}$) may well have its origin in a flavor symmetry (or symmetries), which assigns different charges to the three different families, and also to the Higgs-like fields. In this case, assuming that the h_{33} term is allowed by the flavor symmetries and that the second and the third families have different flavor charges, the h_{23} term will not be allowed as a genuine cubic coupling. It can still arise effectively by utilizing an effective non-renormalizable coupling $h_{23}16_2 16_3 10_H < S > /M$ where S is an $SO(10)$-singlet carrying appropriate flavor charge(s), and acquires a $VEV \sim M_U$. In this case, $h_{23}(= h_{23} < S > /M)$ can naturally be $O(1/10)h_{23}$, if $h_{23} \sim h_{33}$ and $< S > /M \sim M_U/M_{st} \sim 1/10$. The h_{22} term would then be suppressed by $(< S > /M)^2 \sim 10^{-2}$, compared to h_{23}, as desired. Now, as regards the effective non-renormalizable terms in eq. (13), assuming that they are generated by quantum gravity or stringy effects and/or by tree-level exchanges of superheavy states (see e.g. those in the string tower), the scale M is naturally expected to be of order $M_{st} \sim$ few $\times 10^{17} GeV$, while $<45_H> /M$ and $g_{23} <16_H> /m$ could quite plausibly be of order $h_{33}/10$.

It is interesting to observe the symmetry properties of the a_{23} and g_{23} terms. Although $10_H \times 45_H = 10 + 120 + 320$, given that $<45_H>$ is along B-L, which is needed to implement doublet-triplet splitting, only 120 in the decomposition contributes to the mass matrices. This contribution is however antisymmetric in the family index and, at the same time, proportional to B-L. Thus the a_{23} term fulfills the requirements of both (2) and (3) simultaneously. With only h_{ij} and a_{ij} terms however, the up and down quark mass matrices will be proportional to each other, which would yield $V_{CKM} = 1$. This is remedied by the g_{ij} coupling as follows. The 16_H has a VEV primarily along its SM singlet component transforming as to

$$U = \begin{pmatrix} 0 & \epsilon + \sigma \\ -\epsilon + \sigma & 1 \end{pmatrix} m_U, \quad D = \begin{pmatrix} 0 & \epsilon + \eta \\ -\epsilon + \sigma & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon + \sigma \\ 3\epsilon + \sigma & 1 \end{pmatrix} m_u, \quad L = \begin{pmatrix} 0 & -3\epsilon + \eta \\ 3\epsilon + \eta & 1 \end{pmatrix} m_D,$$

Here the matrices are multiplied by left-handed fermion fields from the left and by anti-fermion fields from the right. (U, D) stand for the mass matrices of up and down quarks, while (N, L) are the Dirac mass matrices of the neutrinos and the charged leptons.

The entries $(1, \epsilon, \sigma)$ arise respectively from the h_{33}, a_{23} and h_{23} terms in eq. (13), while η entering into D and L receives contributions from both g_{23} and h_{23}; thus $\eta \neq \sigma$. Note the quark-lepton correlations between (U, N) as well as (D, L), and the up-down correlation between (U, D) as well as (N, L). These correlations arise because of the symmetry structure of $G(224)$. The relative factor of -3 between quarks and leptons involving the ϵ entry reflects the fact that $(45_H) \propto (B-L)$, while the antisymmetry in this entry arises from the $SO(10)$ structure as explained above.

Assuming $\epsilon, \eta, \sigma \ll 1$, we obtain at the unification scale:

$$\left|\frac{m_c}{m_t}\right| \simeq |\epsilon^2 - \sigma^2|, \left|\frac{m_s}{m_b}\right| \simeq |\epsilon^2 - \eta^2|,$$

$$\left|\frac{m_\mu}{m_\tau}\right| \simeq |9\epsilon^2 - \eta^2|, |m_b| \simeq |m_\tau||1 - 8\epsilon^2|, \tag{14}$$

$$|V_{cb}| \simeq |\sigma - \eta| \approx \left|\sqrt{m_s/m_b}\left(\frac{\eta+\epsilon}{\eta-\epsilon}\right)^{1/2} - \sqrt{m_c/m_t}\left(\frac{\sigma+\epsilon}{\sigma-\epsilon}\right)^{1/2}\right|, \tag{15}$$

$$\Theta^l_{\mu\tau} \approx -3\epsilon + \eta \approx \sqrt{m_\mu/m_\tau}\left(\frac{-3\epsilon+\eta}{3\epsilon+\eta}\right)^{1/2} \tag{16}$$

The relations in eqs. (15) and (16) lead to two sum rules:

$$\left|\frac{m_b}{m_\tau}\right| \simeq \left|1 - 8\left\{\left|\frac{m_\mu}{m_\tau}\right| - \left|\frac{m_s}{m_b}\right|\right\}\right|,$$

$$\frac{m_s}{m_b} \simeq \frac{m_c}{m_t} - \frac{5}{4}V_{cb}^2 \pm V_{cb}\left[\frac{9}{16}V_{cb}^2 + \frac{1}{2}\frac{m_\mu}{m_\tau} - \frac{9}{2}\frac{m_c}{m_t}\right]^{1/2} \tag{17}$$

The superscript zero, meaning unification scale values, is not exhibited, but should be understood in all the relations in eqs. (15)–(18).

The mass matrices in eq. (14) contain 5 parameters $\epsilon, \sigma, \eta, m_D = h_{23} < 10_d >$ and $m_U = h_{33} < 10_U >$. These may be determined by using, for example, the following input values: $M_t^{phys} = 174GeV, m_c(m_c) = 1.37GeV, m_s (1GeV) = 110 - 116MeV$ and the observed masses of μ and τ. While the input value of m_s is somewhat lower than that advocated in [40], it is in good agreement with recent lattice calculations [41]. With these input values, the parameters are found to be:

$$\sigma \simeq -0.110\eta_{cb},\ \eta \simeq -0.151\eta_{cb},\ \epsilon \simeq 0.095\eta_\epsilon,$$

$$m_U \simeq m_t(M_U) \simeq (100 - 120)Gev,$$

$$m_D \simeq m_b(M_U) \simeq 1.5GeV \tag{18}$$

Here η_ϵ and η_{cb} denote the phases of ϵ and V_{0cb} respectively (i.e. $\epsilon = \eta_\epsilon|\epsilon|$ etc.). We assume for simplicity that they are real (barring phase angles of $\pm 10°$). Thus, $\eta_\epsilon = \pm 1$ and $\eta_{cb} = \pm 1$. The relative signs of σ, η and ϵ get fixed by ensuring that the results are optimized as regards their agreement with observation. This yields $\eta_{cb} = \eta_\epsilon$. Note that in accord with our general expectations discussed above, each of these parameters are found to be of order 1/10, as opposed to being $O(1)$ or $O(10^{-2})$, compared to the leading

(3,3) element. Having determined these parameters, one can now obtain the following predictions:

$$m_b(m_b) \simeq (4.6 - 4.9) GeV; V_{cb} \simeq 0.045, \tag{19}$$

$$m^D_{\nu\tau}(M_U) \simeq m_t(M_U) \simeq 100 - 120 GeV,$$
$$m^D_{\nu\mu}(M_U) \simeq (9\epsilon^2 - \sigma^2) m_U \simeq 8 GeV,$$
$$\Theta^l_{\mu\tau} \simeq -3\epsilon + \eta \simeq -0.437 \eta_\epsilon (\text{for}\, \eta_{cb}/\eta_\epsilon = +1) \tag{20}$$

In quoting the numbers in eq. (20), we have extrapolated the GUT scale values down to low energies using the beta functions of the minimal supersymmetric extension of the Standard Model (MSSM), assuming $\alpha_s(M_Z) = 0.118$, an effective SUSY threshold of $500\, GeV$ and $tan\beta = 5$. Our results depend only weakly on these input choices, so long as $tan\beta$ is neither too large (≥ 30) not too small (≤ 2). The first two of the predictions listed above (eq. (20)) correspond to directly observed entities. The last three (eq. (21)) cannot be observed directly, but they are important because they need to be combined with the Majorana masses of the RH neutrinos to yield observable entitles (see below).

Given the bizarre pattern of quark and lepton masses and mixings, it seems remarkable that the simple pattern of fermion mass matrices, motivated by the group theory of $G(224)/SO(10)$ gives an overall fit to all of them which is good to within 10%. This includes the two successful predictions on m_b and V_{cb} (eq. (20)). It is worth noting that, in supersymmetric unified theories, the "observed" value of $m_b(m_b)$ and renormalization group studies suggest that for a wide range of the parameter $tan\beta, m_b^\circ$ should in fact be about $10-20\%$ lower than m_τ° [42]. This is neatly explained by the relation: $m_b^\circ \approx m_\tau^\circ(1-8\epsilon^2)$ (eq. (15)), where exact equality holds in the limit $\epsilon \to 0$ (due to $SU(4)$ color), while the decrease by $8\epsilon^2 \sim 10\%$ is precisely because the off-diagonal ϵ entry is proportional to B-L (see eq. (14)).

Specially intriguing is the result on $V_{cb} \approx 0.045$ which compares well with the observed value of $\simeq 0.04$. The suppression of V_{cb}, compared to the value of 0.17 ± 0.06 obtained from eq. (6), is now possible because the mass matrices (eq. (14)) contain an antisymmetric component $\propto \epsilon$. Such a component corrects the square-root mixing angle formula $\Theta_{sb} = \sqrt{m_s/m_b}$ (appropriate for symmetric matrices of the type given by eq. (9)) by the asymmetry factor $|(\eta+\epsilon)/(\eta-\epsilon)|^{1/2}$ (see eq. (15)), and similarly for the angle Θ_{ct}. This factor suppresses V_{cb} if η and ϵ have opposite signs. The interesting point is that, the same feature necessarily enhances the corresponding mixing angle $\Theta^l_{\mu\tau}$ in the leptonic sector, since the asymmetry factor in this case is given by $[(-3\epsilon+\eta)/(3\epsilon+\eta)]^{1/2}$ (see eq. (1)). This enhancement of $\Theta^l_{\mu\tau}$ also seems to be borne out by observation in the sense that that is a key factor in accounting for the nearly maximal oscillation angle observed at SuperK (see discussion below). Note that this intriguing correlation between the mixing angles in the quark versus leptonic sectors – that is, suppression of one implying enhancement of the other – has become possible because the ϵ-contribution is simultaneously antisymmetric and is proportional to B-L. As a result, it changes sign as one goes from the quarks to the leptons.

Taking stock, we see an overwhelming set of evidences in favor of B-L and in fact for the full $SU(4)$ color-symmetry. These include: (1) the suppression of V_{cb}, together with the enhancement of $\Theta^l_{\mu\tau}$ just mentioned above, (ii) the successful relation $m_b^\circ \approx m_\tau^\circ(1-8\epsilon^2)$, where the near equality follows from $SU(4)$ color, while the decrease of m_b° relative to m_τ° by $8\epsilon^2 \sim 10\%$ is a consequence of the (B-L)-dependence of the off-diagonal ϵ-entry, (iii) the usefulness again of the $SU(4)$ color-relation $m(\nu^\tau_{\text{Dirac}})^\circ \approx m_t^\circ$ in accounting for $m(\nu^\tau_L)$, as discussed, and (iv) the agreement of the relation $|m_s^\circ/m_\mu^\circ| = |(\epsilon^2-\eta^2)/(9\epsilon^2-\eta^2)|$ with the data, in that the ratio is naturally less than 1, if $\eta \sim \epsilon$. The presence of $9\epsilon^2$ in the denominator as opposed to ϵ^2 in the numerator is again a consequence of the off-diagonal entry being proportional to B-L. Finally, a spontaneously broken (B-L) local symmetry may well be needed to ensure preservation of baryon excess in the presence of electro-weak sphaleron effects [21].

Although all the entries for the Dirac mass matrix are now fixed, to obtain the parameters for the light neutrinos one needs to specify the Majorana mass matrix of the RH neutrinos (ν_R^μ and ν_R^τ). For concreteness, we assume that this too has the hierarchical form of eq. (9):

$$M_\nu^R = \begin{pmatrix} 0 & y \\ y & 1 \end{pmatrix} M_R \tag{21}$$

In the spirit of our discussion that flavour symmetries are the origin of hierarchical masses, we will assume that $10^{-2} \ll |y| \leq 1/10$ as opposed to $|y|$ being ≥ 0.3 (say). A priori, $y = \eta_y|y|$ can have either sign, i.e., $\eta_y = \pm 1$. Note that Majorana mass matrices are constrained to be symmetric by Lorentz invariance. The see-saw mass matrix $(-N(M_\nu^R)^{-1}N^T)$ for the light $(\nu_\mu - \nu_\tau)$ system is then

$$M_\nu^{\text{light}} = \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} \frac{m_U^2}{M_R}, \tag{22}$$

where $A \simeq (\sigma^2 - 9\epsilon^2)/y$ and $B \simeq -(\sigma + e\epsilon)(\sigma + 3\epsilon - 2y)/y^2$. With $A \ll B$, this yields

$$m_{\nu_3} \simeq B\frac{m_U^2}{M_R}; \frac{m_{\nu_2}}{m_{\nu_3}} \simeq -\frac{A^2}{B^2}; tan\Theta^\nu_{\mu\tau} = \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}, \tag{23}$$

For a given choice of the sign of y relative to that of ϵ, and for a given mass ratio m_{ν_2}/m_{ν_3}, we can now determine y using eqs. (23) and (24), and the values of ϵ and σ obtained in eq. (19). Taking $m_{\nu_2}/m_{\nu_3} = (1/10, 1/15, 1/20, 1/30)$, the requirement of hierarchy mentioned above – i.e. $10^{-2} \ll |y| \leq 0.2$ (say) – can be satisfied only provided y is positive relative to ϵ, i.e., $\eta_y = \eta_\epsilon$; corresponding values for y are: $y = (0.0543, 0.0500, 0.0468, 0.0444, 0.0424)\eta_\epsilon$. With $\eta_y = \eta_\epsilon = \pm 1$, we obtain for the neutrino oscillation angle:

$$\Theta^{osc}_{\nu_\mu\nu_\tau} \simeq \Theta^{l}_{\mu\tau} - \Theta^{\nu}_{\mu\tau} \simeq \left(0.437 + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}\right)(-\eta_\epsilon) \tag{24}$$

$$\begin{aligned} sin^2 2\Theta^{osc}_{\nu_\mu\nu_\tau} &= (0.96, 0.91, 0.86, 0.83, 0.81) \\ \text{for} \quad m_{\nu_2}/m_{\nu_3} &= (1/10, 1/15, 1/20, 1/25, 1/30) \end{aligned} \tag{25}$$

Note the interesting point that just the requirement that $|y|$ should have a natural hierarchical value leads to $\eta_y = \eta_\epsilon$, and that in turn implies that the two contributions in eq. (25) must add rather than subtract, leading to an almost maximal oscillation angle. The other factor contributing to the enhancement of $\Theta^{osc}_{\nu_\mu\nu_\tau}$ is, of course, also the asymmetry-ratio which increased $\Theta^{l}_{\mu\tau}|$ from 0.25 to 0.437 (see eqs. (17) and (21)). We see that one can derive rather plausibly a large $\nu_\mu - \nu_\tau$ oscillation angle $sin^2 2\Theta^{osc}_{\nu_\mu\nu_\tau} \geq 0.8$, together with an understanding of hierarchical masses and mixings of the quarks and the charged leptons, while maintaining a large hierarchy in the see-saw derived masses ($m_{\nu_2}/m_{nu_3} = 1/10 - 1/30$) of ν_μ and ν_τ, all within a unified framework including both quarks and leptons. In the example exhibited here, the mixing angles for the mass eigenstates of neither the neutrinos nor the charged leptons are really large, $\Theta^{l}_{\mu\tau} \simeq 0.437 \simeq 23^\circ$ and $\Theta^{\nu}_{\mu\tau} \simeq (0.18 - 0.31) \approx (10 - 18)^\circ$, yet the oscillation angle obtained by combining the two is near-maximal. This contrasts with most previous work, in which a large oscillation angle is obtained either entirely from the neutrino sector (with nearly degenerate neutrinos) or almost entirely from the charged lepton sector.

It is worth noting that the interplay due to the mixing in the Dirac and the Majorana mass matrices via the see-saw mechanism has the net effect of enhancing $M_R \approx B(m_{\nu_2}/m_{\nu_e})$ for a given m_{ν_3} precisely by a factor of $|B| \approx 5$ (see eq. (23)), compared to what it would be without mixing. Using $m_U \approx 100GeV$ (see eq. (7) or (19)) $m_{\nu_3} \approx (1/10 - 1/30)eV$ (SuperK result) and $|B| \approx 5$, one gets:

$$M_R \approx (5 - 15) \times 10^{14} GeV \tag{26}$$

Compare this with its counterpart, estimated in eq. (6), which yields $M_{3R} \approx$ few $\times 10^{14} GeV$, for $f_{33}\eta^2 \approx 1$, if $M \approx M_{\text{Planck}}$. It is interesting that the larger value of $M_R \approx 10^{15} GeV$ goes well with the theoretical estimate of eq. (6) if the characteristic mass M is chosen (perhaps more appropriately) to be $M_{\text{string}} \approx 4 \times 10^{17} GeV$ rather than M_{Planck}. Further, this larger value of M_R also goes well with the observed m_{ν_3}, once one includes the effect of mixing.

Inclusion of the first family: The first family may now be included following the spirit of the hierarchical structure shown in eqs. (9) and (14). As mentioned before, this may have its origin in flavor symmetries of a deeper theory. In the absence of such a deeper understanding, however, the theoretical uncertainties in dealing with the masses and mixings of the first family are much greater than for the heavier families, simply because the masses of the first family are so small that relatively small perturbations can significantly affect their values.

Assuming that flavor symmetries and $SO(10)$ permit the (3, 3) coupling at a genuine cubic level, but the (2, 3) couplings only at the quartic level, which are thus effectively suppressed by about an order of magnitude compared to the (3, 3) element (see discussion following eq. (13)), we would naturally expect that the (1, 2) and (1, 3) couplings (e.g., a_{12} and g_{12}, see below) would be suppressed compared to the corresponding (23) couplings. This in turn would account for the observed inter-family mass hierarchy.

Following this as a guide, and in the interest of economy, we add only two effective quartic couplings to eq. (13) to include the first family: $a_{12}16_1 16_2$ $45_H 10_H/M$ and $g_{12}16_1 16_2 16_H 16_H/M$. The first coupling introduces an ϵ' term in the (1, 2) entry, which is antisymmetric and proportional to B-L (analog of ϵ); the second introduces an η' term in the (1, 2) entry of only D and L, which is symmetric. The resulting 3×3 Dirac mass matrices are:

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon + \sigma \\ 0 & -\epsilon + \sigma & 1 \end{pmatrix} m_U,$$

$$D = \begin{pmatrix} OY & \epsilon' + \eta' & 0 \\ -\epsilon' + \eta' & 0 & \epsilon + \eta \\ 0 & -\epsilon + \eta & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon + \sigma \\ 0 & 3\epsilon + \sigma & 1 \end{pmatrix} m_U,$$

$$L = \begin{pmatrix} 0 & -3\epsilon' + \eta' & 0 \\ 3\epsilon' + \eta' & 0 & -3\epsilon + \eta \\ 0 & 3\epsilon + \eta & 1 \end{pmatrix} m_D \quad (27)$$

With $\epsilon, \sigma, \eta, m_U$ and m_D determined essentially be considerations of the second and the third families (eq. (19)), we now have just two new parameters in eq. (28), i.e., ϵ' and η' which describe five new observables in the quark and charged lepton sector: m_u, m_d, e_e, Θ_C and V_{ub}. Thus with $m_u \approx 1.5 MeV$ (at M_U) and m_e/m_μ taken as inputs one obtains: $e^1 \simeq \sqrt{m_{mu}/m_c}(m_c/m_t) \approx 2 \times 10^{-4}$ and $|\eta'| \simeq \sqrt{m_e/m_\mu}(m_\mu/m_\tau) \simeq 4.4 \times 10^{-3}$. We can now calculate m_d, Θ_c and V_{ub}. Combining the two predictions for the second and the third families obtained before (see eq. (19)), we are thus led to a total of five predictions for the observable parameters of the quarks and charged leptons belonging to the three families.

$$m_b(m_b) \simeq (4.6 - 4.9) GeV$$

$$V_{cb} \simeq 0.045$$

$$\begin{aligned} m_d(1GeV) &\simeq 8MeV \\ \Theta_C &\simeq |\sqrt{m_d/m_s} - c^{\imath\phi}\sqrt{m_u/m_c}| \\ |V_{ub}/V_{cb}| &\simeq \sqrt{m_\mu/m_c} \simeq 0.07 \end{aligned} \tag{28}$$

Further, the Dirac masses and mixing of the neutrinos and the mixings of the charged leptons also get determined. Including those for the $\mu - \tau$ families listed in eq. (21), we obtain:

$$M^D_{\nu\tau} \approx 100 - 120GeV' m^D_{\nu\mu}(M_U) \simeq 8Gev, \Theta^l_{\mu\tau} \simeq -0.437\eta_\epsilon,$$

$$m^D_{\nu\epsilon} \simeq [9\epsilon'^2/(9\epsilon^2 - \sigma^2)]m_U \simeq 0.4MeV,$$

$$\Theta^l_{\epsilon\mu} \simeq \left[\frac{\eta' - e\epsilon'}{\eta' + e\epsilon'}\right]^{1/2} \sqrt{m_e/m^\mu} \simeq 0.85\sqrt{m_e/m_\mu} \simeq 0.06,$$

$$\Theta^l_{\epsilon\tau} \simeq \frac{1}{0.85}\sqrt{m_e/m_\tau}(m_u/m_\tau) \simeq 0.0012. \tag{29}$$

In evaluating $\Theta^l_{\epsilon\mu}$, we have assumed ϵ' and η' to be relatively positive.

Note that the first five predictions in eq. (29) pertaining to observed parameters in the quark system are fairly successful. Considering the bizarre pattern of the masses and mixings of the fermions in the three families (recall comments on $V_{cb}, m_b/m_\tau, m_s/m_\mu$ and m_d/m_e), we feel that the success of the mass pattern exhibited by eq. (28) is rather remarkable. This is one reason for taking patterns like eq. (28) seriously as a guide for considerations on proton decay. A particularly interesting variant is obtained in the limit $\epsilon' \to 0$, as I will mention later.

To obtain some guidelines for the neutrino system involving ν_e, we need to extend the Majorana mass matrix of eq. (22), by including entries for ν^e_R. Guided by economy and the assumption of hierarchy, as in eq. (9), we consider the following pattern:

$$M^R_\nu = \begin{pmatrix} x & 0 & 1 \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R \tag{30}$$

Equation (30) introduces four effective parameters: x, y, z and M_R. The magnitude of $M_R \approx (5 - 50) \times 10^{14} GeV$ can quite plausibly be justified in the context of supersymmetric unification (see estimate given in eq. (6) and discussion following eq. (27)). And, to the same extent, the magnitude of $m(\nu_\tau) \approx (1/10 - 1/30)eV$, which is consistent with the SuperK value, can also be anticipated. Since all the Dirac parameters are determined, there are, effectively, three new parameters: x, y, and z. However, there are six observables in the light three neutrino system: the three masses and the three oscillation angles. Thus one can expect three predictions for the light neutrinos. These may be taken to be $\Theta^{osc}_{\nu_\mu\nu_\tau}$ (eq. (25)), m_ν, (see eqs. (8) and (24)), and for example, $\Theta^{osc}_{\nu_e\nu_\mu}$.

Recall that the parameter y was determined above by assuming that the MSW (small or large angle) solution for the solar neutrino-deficit corresponds to $\nu_e - \nu_\mu$ oscillation, with $(\delta m^2)MSW \approx m(\nu_\mu)^2 \sim 10^{-5}eV^2$. This gave a value of $|y| \approx 1/20$, in full accord with our general expectation of a hierarchy of order (1/10) for the (2, 3) entry compared to the (3, 3). We do not, however, have much experimental information at present, to determine the other two parameters x and y, reliably, because very little is known about the observable parameters involving ν_e. To have a feel, consistent with our presumption that the inter-family hierarchical masses arise through successively smaller off-diagonal mixing elements, we will assume that $y \approx 1/20$ (as above), $z \leq y/10$ and $x \sim z^2$. Thus, in addition to $M_R \approx (5-15) \times 10^{14} GeV$ and $y \approx 1/20$, which as mentioned above are better determined, we take as a guide: $z \sim (1-5) \times 10^{-3}$ and $x \sim$ (1 to few) $(10^{-6} - 10^{-5})$. Including the three predictions mentioned above, the mass eigenvalues and the oscillation angles are then:

$$
\begin{aligned}
m_{\nu\tau} &\approx (1/10 - 1/30)eV \\
m_{\nu\mu} &\simeq 10^{-3}(5\,\text{to}\,1)eV \\
m_{\nu e} &\simeq (10^{-5} - 10^{-4})(1\,\text{to}\quad \text{few})eV \\
\Theta^{osc}_{\mu\tau} &\simeq 0.437 + \sqrt{m_{\nu_2}/m_{\nu_3}} \\
\Theta^{osc}_{e\mu} &\simeq \Theta^{l}_{e\mu} - \Theta^{\nu}_{e\mu} \simeq 0.06 \pm 0.015 \\
\Theta^{osc}_{e\tau} &\simeq \Theta^{l}_{e\tau} - \Theta^{\nu}_{e\tau} \simeq 10^{-3} \pm 0.03
\end{aligned}
\tag{31}
$$

We see that the masses of ν_e and ν_μ and the oscillation angle $\Theta^{osc}_{e\mu}$ goes well with the small angle MSW explanation of the solar neutrino-deficit.

Although, the superheavy Majorana masses of the RH neutrinos cannot be observed directly, they can be of cosmological significance. The pattern given earlier and in this section suggests that $M(\nu^\tau_R) \approx (5-15)\times 10^{14} GeV, M(\nu^\mu_R) \approx (1-4) \times 10^{12} GeV (\text{for}\, y \approx 1/20)$; and $M(\nu^e_R) \sim (1/2 - 10) \times 10^9 GeV$ (for $x \sim (1/2 - 10)10^{-6} > z^2$). A mass of $\nu^e_R \sim 10^9 GeV$ is of the right magnitude for producing ν^e_R following reheating and inducing lepton asymmetry in ν^e_R decay into $H^\circ + \nu^\imath_L$, that is subsequently converted into baryon asymmetry by the electro-weak sphalerons [21].

We have demonstrated that a rather simple pattern for the four Dirac mass matrices, motivated and constrained by the group structure of $SO(10)$, is consistent within 10% with the observed masses and mixing of all the quarks and the charged leptons. This fit is significantly over constrained, leading to five predictions, which are successful. The same pattern, supplemented with a similar structure for the Majorana mass matrix, quite plausibly accounts for the SuperKamiokande result with the large $\nu_\mu - \nu_\tau$ oscillation angle required for the atmospheric neutrinos, and accommodates a small $\nu_e - \nu_\mu$ oscillation angle elevant for theories of the solar neutrino deficit.

Before turning to proton decay, it is work noting that much of our discussion of fermion masses and mixings, including those of the neutrinos, is essentially unaltered if we go to the limit $\epsilon' \to 0$ of eq. (28). This limit clearly involves:

$$m_u = 0, \Theta_C \simeq \sqrt{m_d/m_s}$$

$$|V_{ub}| \simeq \sqrt{\frac{\eta - \epsilon}{\eta + \epsilon}} \sqrt{m_d/m_b}(m_s/m_b) \simeq (2.1)(0.039)(0.023) \simeq 0.0019$$

$$m_{\nu_e} = 0, \Theta^{\nu}_{e\mu} = \Theta^{\nu}_{e\tau} = 0$$

All other predictions will remain unaltered. Now, among the observed quantities in the list above, $\Theta_C \approx \sqrt{m_d/m_s}$ is indeed a good result. Considering that $m_\mu/m_t \approx 10^{-5}, m_u = 0$ is also a pretty good result. There are of course, plausible small corrections (arising from higher dimensional operators for example), involving Planck scale physics which could induce a small value for m_u through the (1, 1) entry $\delta \approx 10^{-5}$. For considerations of proton decay, it is worth distinguishing between these two variants, which we will refer to as cases I and II respectively.

$$\text{Case I}: \quad \epsilon' \approx 2 \times 10^{-4}, \delta = 0$$

$$\text{Case II}: \quad \delta \approx 10^{-5}, \epsilon' = 0 \tag{32}$$

5 Link Between Fermion Masses and Proton Decay in Supersymmetric $SO(10)$

5.1 Preliminaries

I present now the results of a recent study [18] of proton decay in SUSY $SO(10)$, which was carried out by paying attention specially to the link that exists in SUSY $SO(10)$ between proton decay and the masses and mixings of all fermions, including especially the neutrinos.

It is well known that in supersymmetric unified theories (GUTs), with $M_X \sim 2 \times 10^{16} GeV$, the gauge-boson mediated $d = 6$ proton decay operators, for which $e^+\pi^0$ would have been the dominant mode, are strongly suppressed. The dominant mechanism for proton decay in these theories is given by effective $d = 5$ operators of the form $Q_\imath Q_j Q_k L_l/M$ in the superpotential, which arise through the exchange of color triplet Higgsions that are the GUT partners of the standard Higgs doublets such as those in $5_H + \bar{5}_H$ of $SU(5)$ or the 10_H of $SO(10)$. Subject to a doublet-triplet splitting mechanism which makes these color triplets acquire heavy GUT-scale masses, while the doublets remain light, these standard $d = 5$ operators, suppressed by just one power of the heavy mass and the small Yukawa couplings, lead to proton decay, with a lifetime $\tau_p \sim 10^{30} - 10^{34} yrs$ [43]–[46]. Note that these standard $d = 5$ operators are proportional to the product of two Yukawa couplings, which are related to the masses and mixing of the charged fermions. Further, for these operators to induce proton decay, they must be dressed by wino (or gluino)-exchange so as to convert a pair of suqrks to quarks. Owing to (a) Bose symmetry of the superfields in QQQL/M, (b) color antisymmetry, and especially (c) the hierarchical Yukawa couplings of the standard Higgs doublets, it turns out that these operators exhibit a strong preference for the decay of a proton into channels involving $\bar{\nu}$ rather than e^+ or (even) μ^+ and those involving an $\bar{s}$ rather than a $\bar{d}$. Thus the standard operators lead to dominant $\bar{\nu}K^+$ and comparable $\bar{\nu}K^+$ modes, but in all cases to highly suppressed $e^+\pi^o, e^+K^o, e^+K^o$ and even μ^+K^o modes. For instance, for SUSY $SU(5)$ one obtains (for $tan\beta \leq 15$, say):

$$[\Gamma(\mu^+K^o)/\Gamma(\bar{\nu}_\mu K^+)]_{std} \sim [m_u/m_c sin^2\Theta_C]^2 R \approx 10^{-3}$$

where $R \approx 0.1$ is the ratio of the products of the relevant $|$ matrix element $|^2\times$ (phase space) for the two modes.

Now, it was recently realized that in left-right symmetric unified theories possing super-symmetry, such as those based on $G(224)$ or $SO(10)$, there is very likely a new source of $d = 5$ proton decay operators, which are related to the Majorana masses of the right-handed neutrinos [17]. For instance, in the context of the minimal set of Higgs multiplets $\{45_H, 16_H, \bar{16}_H$ and $10_H\}$, which have been utilized earlier to break $SO(10)$ and generate fermion masses, these new $d = 5$ operators arise by combining three effective couplings, i.e., (a) the couplings $f_{ij}16_i16_j\bar{16}_H\bar{16}_H/M$ (see eq. (5)) which are essential to assign Majorana masses to the right-handed neutrinos, (b) the couplings $g_{ij}16_i16_j16_H16_H/M$, which are needed to generate non-trivial CKM mixing and (c) the mass term $M_{16}16_H\bar{16}_H$. In the presence of these three (unavoidable) effective couplings and the VEVs $<16_H> = <\bar{16}_H> \sim M_x$, the color triplet Higgsinos in 16_H and $\bar{16}_H$ of mass M_{16} cab be exchanged between $\bar{q}_iq_j$ and $\bar{q}_kl_l$ pairs. This exchange gives rise to a new set of effective $d = 5$ couplings of the form:

$$L^{d=5}_{\rm new}[f_{ij}gkl(16_i16_j)(16_k16_t)/M_{16}]\frac{<\bar{16}_H><16_H>}{M^2} \tag{33}$$

which induce proton decay, just as the standard operators do. Note that these new $d = 5$ operators depend, through the couplings f_{ij} and g_{kl}, both on the Majorana and on the Dirac masses of the respective fermions. This is why within SUSY $G(224)$ or $SO(10)$, proton decay gets intimately linked to the masses and mixings of all fermions, including neutrinos.

Specifically, it is found that the SuperK result on atmospheric neutrinos, that suggests $m(\nu^\tau_L) \sim 1/20eV$ and a large $\nu_\mu - \nu_\tau$ oscillation angle leads to a significant enhancement especially in the new $d = 5$ operators, compared to previous estimate which were based on guesses of much larger values of $m(\nu^\tau_L) \sim (2-4)eV$ [17]. Curiously enough, the net effect of including the enhancement of f_{33} (due to a lowering of $m(\nu^\tau_L)$) and the suppression of the relevant CKM mixings is such that the strength of the new $d = 5$ operators is found to be comparable to that of the standard ones [18]. The flavor structure of the new operators are, however, very different from those of the standard ones, in part because the former depend on the Majorana masses of the RH neutrinos, and the latter do not. As a result, the new operators lead to some characteristic differences in the proton decay pattern (that is, branching ratios of different decay modes) compared to the standard ones (see below).

5.2 Framework for Calculating Proton Decay Rate

To establish notations, consider the case of minimal SUSY $SU(5)$ and, as an example, the process $\bar{c}\bar{d} \to \bar{s}\bar{\nu}_\mu$, which induces $p \to \bar{\nu}_\mu K^+$. Let the strength of the corresponding $d = 5$ operator, multiplied by the product of the CKM mixing elements entering into wino-exchange vertices, (which in this case is $sin\Theta_C cos\Theta_C$) be denoted by $\bar{A}$. Thus, putting $cos\Theta_C = 1$, one obtains:

$$\bar{A}_{\bar{c}\bar{d}}(SU(5)) = (h^{\mu}_{22}h^{d}_{12}/M_{H_c})sin\Theta_c \simeq (m_c m_s sin^2\Theta_C/v_u^2)(tan\beta/M_{H_c}$$
$$\simeq (1.9 \times 10^{-8})(tan\beta/M_{H_c}, \tag{34}$$

where $tan\beta \equiv v_u/v_d$, and we have put $u_u = 174GeV$ and the fermion masses extrapolated to the unification scale, i.e., $m_c \simeq 300MeV$ and $m_s \simeq 40MeV$. The amplitude for the associated four fermion process $dus \to \bar{\nu}$ is given by:

$$A_5(dus \to \bar{\nu}_\mu) = (\bar{A}_{\bar{c}\bar{d}}) \times (2f), \tag{35}$$

where f is the loop factor associated with wino-dressing. Assuming $m_{\bar{w}} \ll m_{\bar{q}} \sim m_j$ one gets; $f \simeq (m_{\bar{w}}/m^2_{\bar{q}})(\alpha_2/4\pi)$. Using the amplitude for $(du)(s\nu_l)$, as in eq. (35), ($l = \mu\,\mathrm{or}\,\tau$), one then obtains [44]–[46], [18]:

$$\Gamma^{-1}(p \to \bar{\nu}_\tau K^+) \simeq (2.2 \times 10^{31})yrs \times \left[\frac{0.67}{A_S}\right]^2 \left[\frac{0.006GeV^3}{\beta_H}\right]^2$$
$$\left[\frac{(1/6)}{(m_{\bar{w}}/m_{\bar{q}})}\right]^2 \left[\frac{m_{\bar{q}}}{1TeV}\right]^2 \left[\frac{2 \times 10^{-24}GeV^{-1}}{\hat{A}(\bar{\nu})}\right]^2 \tag{36}$$

Here β_H denotes the hadronic matrix element defined by $\beta_H u_L(\boldsymbol{k})(\boldsymbol{t} \equiv \epsilon_{\alpha\beta\gamma}$ $\langle 0|(d^\alpha_L u^\beta_L)u^\gamma_L|p, \boldsymbol{k}\rangle$. While the range $\beta_H = (0.003 - 0.03)GeV^3$ has been used in the past [45], given that one lattice calculations yield [50] $\beta_H = (5.6 \pm 0.5) \times 10^{-3}GeV^3$, we will take as a plausible range: $\beta_H = (0.006GeV^3)(1/2$ to 2). $A_s \approx 0.67$ stands for the short distance renormalization factor of the $d = 5$ operator. Note that the familiar factors that appear in the expression for proton lifetime – i.e., M_{H_C}, $(1+y_t K)$ representing the interference between the $\bar{t}$ and $\bar{c}$ contributions and tan β – are all effectively contained in $\hat{A}(\bar{\nu})$. Allowing for plausible and rather generous uncertainties in the matrix element and the spectrum we take:

$$\beta_H = (0.0006GeV^3)(1/2\,\mathrm{to}\,2),$$

$$(m_{\bar{w}}/m_{\bar{q}}) = 1/6(12\,\text{to}\,2), m_{\bar{q}} \approx m_{\bar{l}} \approx 1TeV(1/\sqrt{2}\,\text{to}\,\sqrt{2}) \tag{37}$$

Using eqs. (36) and (37), we get:

$$\Gamma^{-1}(p \to \boldsymbol{\nu}_\tau K^+) \approx (2.2 \times 10^{31})yrs$$
$$\times [2.2 \times 10^{-24} GeV^{-1}/\hat{A}(\bar{\nu}_l)]^2 [32\,\text{to}\,/32] \tag{38}$$

This relation is general, depend only on $\hat{A}(\bar{\nu}_l)$ and on the range of parameters given in eq. (38). It can thus be used for both $SU(5)$ and $SO(10)$.

The experimental lower limit on the inverse rate for the $\bar{\nu}K^+$ modes is given by [47],

$$[\sum_l \Gamma(p \to \bar{\nu}_l K^+)]^{-1}_{expt} > 7 \times 10^{32} yrs. \tag{39}$$

Allowing for all the uncertainties to stretch in the same direction (in this case, the square bracket = 45), and assuming that just one neutrino flavor (e.g. ν_μ for $SU(5)$) dominates, the observed limit (eq. (40)) provides an upper bound on the amplitude:

$$\hat{A}(\bar{\nu}_l) \leq 2 \times 10^{-24} GeV^{-1}, \tag{40}$$

which holds for both $SU(5)$ and $SO(10)$. For minimal $SU(5)$, using eq. (35) and $tan\beta \geq 2$ (which is suggested on several grounds), one obtains a lower limit on M_{HC} given by:

$$M_{HC} \geq 2 \times 10^{16} GeV(SU(5)) \tag{41}$$

At the same time, higher values of $M_{HC} > 3 \times 10^{16} GeV$ do not go very well with gauge coupling unification [48]. Thus, keeping $M_{HC} \leq 3 \times 10^{16}$ and $tan\beta \leq 2$, we obtain from eq. (35):

$$\hat{A}(SU(5)) \geq (4/3) \times 10^{-24} GeV^{-1}$$

Using eq. (39), this in turn implies that

$$\Gamma^{-1}(p \to \bar{\nu}K^+) \leq 1.5 \times 10^{33} yrs(SU(5)) \tag{42}$$

This a conservative upper limit. In practice, it is unlikely that all the uncertainties, including that in M_{HC}, would stretch in the same direction to nearly extreme values so as to prolong proton lifetime. A more reasonable upper limit, for minimal $SU(5)$, thus seems to be:

$$\Gamma^{-1}(p \to \bar{\nu}K^+)(SU(5)) \leq (0.7) \times 10^{33} yrs.$$

Given the experimental lower limit (eq. (40)), we see that minimal SUSY $SU(5)$ is almost on the verge of being excluded by proton decay searches. We have of course noted earlier that SUSY $SU(5)$ does not go well with the neutrino oscillations observed at SuperK.

Now, to discuss proton decay in the context of supersymmetric $SO(10)$, it is necessary to discuss first the mechanism for doublet-triplet splitting. Details of this discussion may be found in [18]. Here, I present only a synopsis.

5.3 A Natural Doublet-Triplet Splitting Mechanism in $SO(10)$

In supersymmetric $SO(10)$, a natural doublet-triplet splitting can be achieved by coupling the adjoint Higgs 45_H to a 10_H and a $10'_H$ with 45_H acquiring a unification scale VEV in the B-L direction [49]: $\langle 45_H \rangle = (a, a, a, 0, 0) \times \tau_2$ with $a \sim M_U$. As discussed already, to generate CKM mixing for fermions, we require an $\langle 16_H \rangle_d$ to acquire an electro-weak scale vacuum expectation value. To insure accurate gauge coupling unification, the effective low energy theory should not contain split multiplets beyond those of MSSM. Thus the MSSM Higgs doublets must be linear combinations of the $SU(2)_L$ doublets in 10_H and 16_H. A simple set of superpotential terms that ensures this and incorporates doublets in 10_H and 16_H. A simple set of superpotential terms that ensures this and incorporates doublet-triplet splitting is:

$$W_H = \lambda 10_H 45_H 10'_H + M_{10} 10'^2_H + \lambda' \overline{16_H 16_H 10_H} + M_{10} 16_H \bar{16}_H \qquad (43)$$

A complete superpotential for $45_H, 16_H, \bar{16}_H, 10_H, 10'_H$ and possibly other fields which ensure that $45_H, 16_H$, and $\bar{16}_H$ acquire unification scale VEVs with $\langle 45_H \rangle$ being along the (B-L) direction, that exactly two Higgs doublets (H_u, H_d) remain light with H_d being a linear combination of $(10_H)_d$ and $(16_H)_d$, and that there are no unwanted pseudoGoldstone bosons, can be constructed with the vacuum expectation value $\langle 45_H \rangle$ in the B-L direction. It does not contribute to the doublet matrix, so one pair of Higgs doublet remains light, while all triplets acquire unification scale masses. The light MSSM Higgs doublets are

$$H_u = 10_u, H_d = cos_\gamma 10_d + sin_\gamma 16_d, \qquad (44)$$

with $tan\gamma \equiv \lambda' \langle \bar{16}_H \rangle / M_{16}$. Consequently, $\langle 10 \rangle_d = cos\gamma v_d, \langle 16_d \rangle = sin\gamma \nu_d$ with $\langle H_d \rangle = v_d$ and $\langle 16_d \rangle$ and $\langle 10_d \rangle$ denoting the electro-weak VEVs of those multiplets. Note that the H_u is purely in 10_H and that $\langle 10_d \rangle^2 + \langle 16_d \rangle^2 = v_d^2$. This mechanism of doublet-triplet (DT) splitting is rather unique for the minimal Higgs systems in that it meets the requirements of both D-T splitting and CKM mixing. In turn, it has three important consequences:

(i) It modifies the familiar $SO(10)$ relation $tan\beta \equiv v_u/v_d = m_t/m_b \approx 60$ to

$$tan\beta / cos\gamma \approx m_t/m_b \approx 60 \qquad (45)$$

As a result, even low to moderate values of $tan\beta \approx 3$ to 10 (say), are perfectly allowed in $SO(10)$ (corresponding to $cos\gamma \approx 1/20$ to $1/6$).
(ii) In contrast to $SU(5)$, for which the strengths of the standard $d = 5$ operators are proportional to $(M_{H_c}^{-1}, M_{H_c} \sim M_U \sim$ few $\times 10^{16} GeV$ (see eq. (35)), for the $SO(10)$ model, with DT splitting given as above, they become proportional to M_{eff}^{-1}, where $M_{eff} = (\lambda a)^2 / M_{10'} \sim M_U^2 / M_{10'}.M_{10'}$ can be naturally smaller than M_U, and thus M_{eff} is correspondingly larger (than M_U) by one or two orders of magnitude [18]. Now, the proton decay amplitudes for $SO(10)$

in fact possess an intrinsic enhancement compared to those for $SU(5)$, owing primarily due to differences in their Yukawa couplings for the up sector (see Appendix C of [18]). As a result, these larger values of $M_{eff} \sim 10^{18} GeV$ are found to lead to expected proton decay lifetimes that are on the one hand compatible with observed limits, but on the other hand allow optimism as regards future observation of proton decay (see below).
(iii) M_{eff} gets bounded above by considerations of coupling unificatioin and GUT scale threshold effects. Owing to mixing between 10_d and 16_d (see eq. (45)), the correction to $\alpha_3(m_z)$ due to doublet-triplet splitting becomes proportional to $ln(M_{eff}/cos\gamma)$. Inclusion of this correction and those due to splittings within the gauge multiplets (i.e. 45_H, and $\bar{16}_H$), together with the observed degree of coupling unification allows us to obtain a conservative upper limit on $M_{eff} \leq 3 \times 10^{18} GeV$ (see [18]). This in turn helps provide an upper limit on the expected proton decay lifetime (see below):

The calculation of the amplitudes $\hat{A}_{std}$ and $\hat{A}_{new}$ for the standard and the new operators for the $SO(10)$ model are given in detail in [18]. Here, I will present only the results. It is found that the four amplitudes $\hat{A}_{std}(\bar{\nu}_\tau K^+)$, $\hat{A}_{std}(\bar{\nu}_\mu K^+)$, $\hat{A}_{new}(\bar{\nu}+)$ and $\hat{A}_{new}(\bar{\nu}_\nu K^+)$ are in fact very comparable to each other, within about a factor of two, either way. Since there is no reason to expect a near cancellation between the standard and the new operators, especially for both $\bar{\nu}_\tau K^+$ and $\bar{\nu}_\mu K^+$ modes, we expect the net amplitude (standard + new) to be in the range exhibited by either one. Following [18], I therefore present the contributions from the standard and the new operators separately. Using the upper limit on $M_{eff} \leq 3 \times 10^{18} GeV$, we obtain a lower limit for the standard proton decay amplitude given by

$$\hat{A}(\bar{\nu}_\tau K^+)_{std} \geq \begin{bmatrix} (7 \times 10^{-24} GeV^{-1})(1/6 \,\text{to}\, 1/4) \\ (3 \times 10^{-24} GeV^{-1})(1/6 \,\text{to}\, 1/2) \end{bmatrix} \tag{46}$$

Substituting into eq. (39) and adding the contribution from the second competing mode, $\bar{\nu}_\mu K^+$ with a typical branching ratio $R \approx 0.3$, we obtain

$$\Gamma^{-1}(\bar{\nu} K^+)_{std} \leq \begin{bmatrix} (3 \times 10^{31} yrs.)(1.6 \,\text{to}\, 0.7) \\ (6.8 \times 10^{31} yrs.)(4 \,\text{to}\, 0.44) \end{bmatrix} (32 \,\text{to}\, 1/32) \tag{47}$$

The upper and lower entries in eqs. (47) and (48) henceforth correspond to the cases I and II of the fermion mass matrix (i.e., $\epsilon' \nu 0$ and $\epsilon' = 0$, respectively, see eq. (33)). The uncertainty shown inside the square brackets correspond to that in the relative phases of the different contributions. The uncertainty (32 to 1/32) corresponds to the uncertainty in $\beta_H, (m_{\bar{W}}/m_{\bar{q}})$ and $m_{\bar{q}}$, by factors of 2, 2, and $\sqrt{2}$ respectively, either way, around the "central" values reflected in eq. (38). Thus, we find that for MSSM embedded in $SO(10)$, the inverse partical proton decay rate should satisfy:

$$\Gamma^{-1}(p \to \bar{\nu} K^+)_{std} \leq \begin{bmatrix} 3 \times 10^{31} \pm 1.7 yrs. \\ 6.8 \times 10^{31 +2.1}_{-1.5} yrs. \end{bmatrix}$$

$$\leq \begin{bmatrix} 1.5 \times 10^{33} yrs. \\ 7 \times 10^{33} yrs. \end{bmatrix} (SO(10)) \tag{48}$$

The central value of the upper limit in eq. (49) essentially reflects the upper limit on M_{eff}, while the remaining uncertainties of matrix elements and spectrum are reflected in the exponents.

Evaluating similarly the contribution from the new operator, we obtain:

$$\hat{A}(\bar{\nu}_\mu K^+)_{new} \approx (1.5 \times 10^{-24} GeV^{-1})(1/4 \,\text{to}\, 1.3) \tag{49}$$

$$\Gamma^{-1}(\bar{\nu} K^+)_{new} \approx (3 \times 10^{31} yrs)[16 \,\text{to}\, 1/1,7]\{32 \,\text{to}\, 1/32\} \tag{50}$$

In this estimate we have included the contribution of the $\bar{\nu}_\tau K^+$ mode with a typical branching ratio $R \approx 0.4$. Here the second factor, inside the square bracket, reflects the uncertainties in the amplitude, while the last factor corresponds to varying $\beta_H, (m_{\bar{W}}/m_{\bar{d}}$ and $m_{\bar{q}}$ around the central values reflected in eq. (38). With a net factor of even 20 to 100 arising jointly from the square and the curly brackets, i.e. without going to extreme ends of all parameters, the new operators related to neutrino masses lead by themselves to proton decay lifetimes

$$\Gamma^{-1}(\bar{\nu} K^+)^{\text{expected}}_{\text{new}} \approx (0.6 - 3) \times 10^{33} yrs.(SO(10)) \tag{51}$$

The Charged Lepton Decay Mode ($\mathbf{p \to \mu^+ K^\circ}$): I now discuss a special feature of the $SO(10)$ model pertaining to the possible prominence of te charged lepton decay mode: $p \to \mu^+ K^\circ$, which is not permissible in SUSY $SU(5)$. Allowing for uncertainties in the way the standard and the new operators can combine with each other for the three leading modes, i.e., $\bar{\nu}_\tau K^+, \bar{\nu}_\mu K^+$ and $\mu^+ K^\circ$, we obtain [18]

$$B(\mu^+ K^\circ)_{std} + \text{new} \approx [1 \,\text{to}\, (50 - 60)\%] \rho (SO(10)) \tag{52}$$

where ρ denote the ratio of the squares of relevant matrix elements for the $\mu^+ K^\circ$ and $\bar{\nu} K^+$ modes.

In the absence – presumably temporary – of a reliable lattice calculation, which is presently missing for the $\bar{\nu} K^+$ mode [50], one should remain open to the possibility of $\rho \approx 1/2$ to 1 (say). Using eq. (53), we find that for a large range of parameters, the branching ratio $B(\mu^+ K^\circ)$ can lie int he range of 20–30% (if $\rho \approx 1$). Thus we see that the $\mu^+ K^\circ$ mode is likely to be prominent in the $SO(10)$ model presented here, and if $\rho \approx 1$, it can even become a dominant mode. This contrasts sharply with the minimal $SU(5)$ model in which the $\mu^+ K^\circ$ is expected to have a branching ratio of only about 10^{-3}. In the $SO(10)$ model, the standard operator by itself gives a branching ratio for this mode of (1–10)% while the potential prominence of the $\mu^+ K^\circ$ mode arises only through the new operator related to neutrino masses.

6 Some Crucial Observations Pertaining to Unification: A Summary

The preceding discussion can be best summarized by listing the implications of some crucial findings which bear on unification.

A. The family multiplet structure: The observed multiplet structure in each family consisting of either sixteen members (including the ν_R) or fifteen members (without ν_R) is the first empirical hint in favor of an underlying gauge symmetry like $G(224), SO(10)$ or $SU(5)$. While the standard model organizes the 15 members of a family into five multiplets, $SU(5)$ groups them into two, and $G(224)$ with L-R discrete symmetry, $SO(10)$ places all sixteen members within just one multiplet. Further, each of these higher symmetries $(G(224), SO(10)$ or $SU(5))$ explain precisely the $SU(3)^C \times SU(2)_D$-representations and the weak hypercharge (Y_W), quantum numbers of all the members in a family. This feature as well as the need to explain the observed quantization of electric charge, have been two of the primary motivations for proposing the idea of grand unification [5]–[7].

B. Meeting of the gauge couplings: The meetings of the gauge couplings, which is found to occur when their measured values at LEP are extrapolated to higher energies in the context of supersymmetry, clearly supports the ideas of:

- An underlying unity of forces, as well as of supersymmetry
- The relevance of effective gauge symmetries like $SU(5)$, or $SO(10)$, or a string-derived $G(224)$, or $[SU(3)]^3$ at the underlying level
- Unification at a scale $M_x \sim 2 \times 10^{16} GeV$ (assuming MSSM spectrum below M_x)

C. Neutrino masses, especially $\mathbf{m(\nu_\tau) \sim 1/20eV}$: This single piece of information, suggested by the SuperK result, brings to light the existence of the RH neutrinos accompanying the left-handed ones, and reinforces the ideas of:

- SU(4) color
- Left-right symmetry
- Supersymmetric unification
- See-saw

In short, the SuperK result, suggesting $m(\nu_\tau) \sim (1/20)eV$, slects out the route to higher unification based on a string-derived $G(224)$ or $SO(10)$, as opposed to $SU(5)$. Further, it suggests that B-L breaking occurs at the unification scale, $M_{B-L} \sim M_X \sim 2 \times 10^{10} GeV$ rather than at an intermediate scale.

D. Masses and mixings of all fermions $(\mathbf{q}, \mathbf{l}, \nu)$: Adopting familiar ideas of generating lighter eigenvalues through off-diagonal mixings and using the group theory of $SO(10)$ for the effective Yukawa couplings of the minimal Higgs system, it was found in [18], that, remarkably enough, the bizarre pattern of the masses and mixings of the charged fermions as well as of the neutrinos can be adequately described (with $\sim 10\%$ accuracy) within an economical and predictive $SO(10)$ framework. In particular, the framework provides five successful predictions for the masses and mixings of the quarks and the charged leptons. The same description goes extremely well with a value of $m_\nu, \sim 1/20 eV$ as well as with a large $\nu_\mu - \nu_\tau$ oscillation angle ($sin^2 2\Theta_{\nu_\mu \nu_\tau} \approx 0.82 - 0.96$), despite highly non-degenerate masses for the light neutrinos. Both these features are in good agreement with the SuperK results on atmospheric neutrinos. The same framework also typically leads to the small angle MSW solution for the solar neutrino puzzle, with $m_{\nu_e} \sim 3 \times 10^{-3} eV \gg m_{\nu_e}$.

One intriguing feature of the $SO(10)$ framework presented is that the largeness of the $\nu_\mu - \nu_\tau$ oscillation angle emerges naturally together with the smallness of the analogous mixing parameter in the quark-sector: $V_{bc} \approx 0.04$. This remarkable correlation between the leptonic versus the quark mixing angles clearly points to the presence of a contribution of the mass matrices, which is proportional to B-L, and its antisymmetric in the family space. The minimal Higgs system together with the group theory of $SO(10)$ precisely yields such a contribution.

E. Proton decay: The hall-mark of quark-lepton unification: Proton decay, if seen, would directly verify the idea of quark-lepton unification. Note that this crucial aspect of grand unification is not probed directly by the other three observations listed above: B, C, and D.

We have argued that three different sets of observations, i.e. (a) the observed meeting of the three gauge couplings, (b) the SuperK result on atmospheric neutrino oscillations, and (c) fermion masses and mixings – go extremely well with the idea of supersymmetric unification, based on symmetry structures such as $SO(10)$. Babu, Wilczek and I have studied proton decay in this context, paying attention to its correlation with fermion masses and mixings [18]. We found that the proton decay amplitudes receive a major contribution from a set of new $d = 5$ operators which are directly related to the Majorana masses of the RH neutrinos and to the CKM mixing [17, 18]. This is in addition to the contribution from the standard $d = 5$ operators, which are related to the Dirac masses of the charged fermions. The study shows that the mass of $m_{\nu_\tau} \sim 1/20 eV$ (as opposed to previously considered values of a few eV) and the large oscillation angle suggested by the SuperK result, in fact imply a net enhancement in the rates of proton decay into the $\bar{\nu} K^+$ and especially in the $\mu^+ K^\circ$-modes [18], relative to previous estimates.

There are of course uncertainties in the prediction for proton decay rates owing to those in the SUSY spectrum, the hadronic matrix elements and the relative phases of the different contributions. Allowing for rather generous uncertainties in this regard, we expect proton to decay dominantly into the $\bar{\nu} K^+$ and very likely to the $\mu^+ K^\circ$-mode as well, with a lifetime:

$$\tau_{\text{proton}} \leq 7 \times 10^{33} yrs(SO(10)) \tag{53}$$

This is a conservative upper limit which is obtained only if all the uncertainties are stretched in the same direction to nearly their extreme values, so as to extend proton longevity. Since the likelihood of this happening is small, we expect that within either a string-derived $G(224)$ or the $SO(10)$ model of the sort presented here, proton should decay with a lifetime shorter than the limit shown above. With the current experimental lower limit already at 7×10^{32} years, we conclude that improvement in the present limit for $p \to \bar{\nu}K^+$ and $p \to \mu^+ K^\circ$ modes by a factor of 2 to at most 10 should either turn up events, or else the remarkably successful $SO(10)$ framework described here will be called into question seriously. On the basis of our study, we expect that the SuperK detector should in fact see a few proton decay events in the $\bar{\nu}K^+$ and quite possibly in the $\mu^+ K^\circ$ channel in the near future. To establish the reality of this important process firmly and also to study efficiently the branching ratios of some crucial modes, like the $\mu^+ K^\circ$, next generation detectors with sensitivity of at least 5×10^{34} and perhaps 10^{35} years are essential.

We have stressed that observation of proton decay into $\mu^+ K^\circ$ with a branching ratio exceeding 20% (say) would provide a clear signature in favor of (a) supersymmetric unification based on symmetry structures such as a string-derived $G(224)$ or $SO(10)$, as well as (b) the mechanism described here of generating the masses and mixings of all fermions including especially the neutrinos [18].

To conclude, proton decay has been anticipated for quite some time as a hallmark of grand unification. With coupling unification and neutrino masses revealed, proton decay is the missing link. While its discovery, with dominance of the $\bar{\nu}K^+$ mode, would confirm supersymmetric unification, prominence of the $\mu^+ K^\circ$ mode establish the beautiful link that exists between the neutrino masses and proton decay within the $G(224)/SO(10)$-route to unification.

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