Financial Market Dynamics: A Synergetic Perspective

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Article Outline

Glossary

Econophysics An interdisciplinary research field where theories and methods originally developed by physicists are used to model financial markets and economic systems.

Market panic A state in which correlations among stock returns are very high together with highly elevated levels of the VIX Index.

Market volatility Market volatility is the uncertainty of price moves of a given market (rather than a single stock), such as the US stock market, which is well represented by the S&P 500 Index.

Stock returns The relative change in value of the price of a stock over a particular time horizon (e.g., 1 day or 1 year).

Stylized facts Statistical characteristics of financial time series that appear to be somewhat universal across asset classes and geographies. These include volatility clustering, long-range memory in absolute price returns, and the fat-tailed distribution of price returns that persist over horizons ranging from intraday to weeks.

The VIX Index Also known as the "fear" index, this represents a forward view of volatility or uncertainty in the market. It is computed from stock index option prices.

Volatility The risk or uncertainty of the magnitude of a stock's returns. Realized volatility can be calculated from the historical time series of stock returns over some past window, most commonly as the standard deviation of returns, but other proxies can be used such as the mean absolute value of returns.

Introduction

Over the past decades, the field of econophysics has become established as a subject area that connects concepts, ideas, and models stemming from physics to explain the underlying dynamics driving financial markets. In particular physicists have made advances in applying the fields of statistical physics and nonlinear dynamics to create models that explain some of the statistical and dynamic properties of financial markets. Underlying to many of these models are notions common to synergetics (Haken 1977), ranging from interacting agents and nonlinear feedback to predator-prey dynamics and spontaneous self-organization.

In reality, market researchers have access to historical price and volume time series for a collection of stocks. They analyze this data and try to understand the relationships and dynamics of this joint stochastic system either for the purpose of predicting future price changes or analyzing and predicting future risks. Traders and portfolio managers will use their inferences to construct

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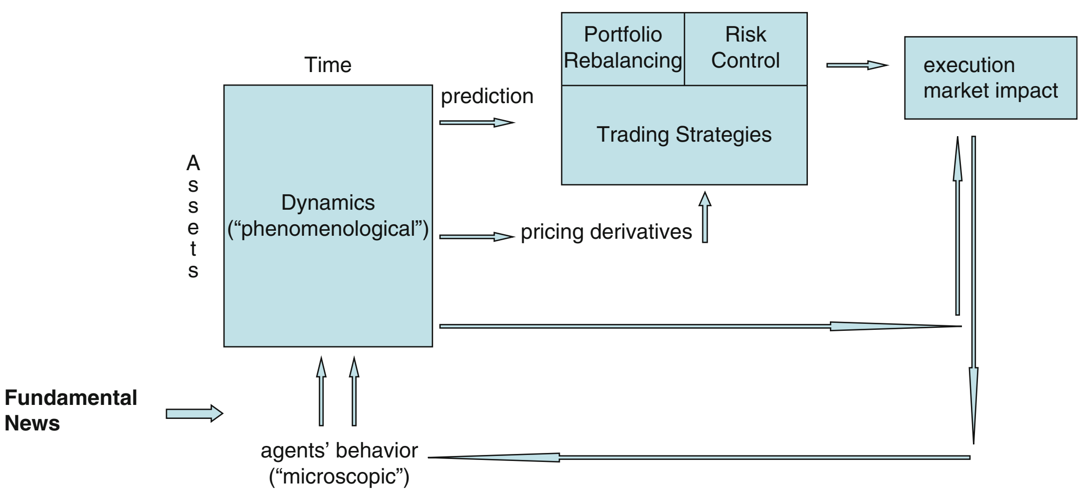
desirable portfolios which they must construct by executing buys and sells in the market. But this in turn affects the market itself which feeds back into the historical data that they continue to analyze over time. Furthermore, at any instant, there is not just one trader interacting with the market but many, many thousands, across the globe and with their own unique objective. Their actions also feed back into the price formation process affecting the data that they then continue to analyze. In addition, news, external events, fundamental properties, and macroeconomic phenomena also get incorporated into the price. These interactions are sketched in Fig. 1, and it is clear to see that there are multiple feedback loops at play.

In this entry we'll provide an overview of financial market dynamics and some of the models that have been developed to describe them and conclude by going in depth for one example that stems directly from synergetics.

Price Dynamics

In my view the price of a stock is the macroscopic observable that emerges as a result of the microscopic interactions of many agents in an extremely complex system. At any given time, there is no such thing as a well-defined price of a stock. It is not as simple as when you go to the grocery store, and prices are clearly marked so a transaction can be planned and executed exactly. Instead, in financial markets, all across the globe at any moment in time, traders are submitting orders to buy and sell a certain amount of a stock; furthermore, their orders are not all sent to the same place but rather to one of several exchanges.

In addition each trader acts with their own view and utility on a spectrum of timescales; they base their buy or sell decisions on their individual information set for their own individual intent. Traders' views may depend on fundamental properties of the company whose stock is being traded, as well as general trends in the particular industry in question. Stock-specific events, such as mergers and acquisitions, have a big impact, as do world events, such as wars, terrorist attacks, and natural disasters. It is ultimately the collection and interaction of this supply and demand that drive the microscopic dynamics of price formation in conjunction of course with the rules of individual exchanges and regulations, which have evolved over the years and continue to do so. For example, in the United States in the mid-1990s, the NYSE and the Nasdaq were the only two exchanges where transactions could occur. Typical market participants were large institutions, and typical order sizes were in the thousands of shares. How quickly you got your order into the market was not



Financial Market Dynamics: A Synergetic Perspective, Fig. 1 The big picture: observed asset prices affect traders' decisions, which feed back into observed asset prices an important factor. Today just some 20 years later, the market structure is quite different. The market is fragmented, consisting of several “dark pools” (where traders cannot see the orders of others) and 12 “lit” exchanges where traders can place their orders to buy or sell a given quantity of a stock at a given price. The orders on lit exchanges can be seen by market participants (hence the name). Typical order sizes are hundreds of shares, and institutional investors constitute a much smaller percentage since electronic trading has become accessible to anyone.

Trades can be organized into a so-called limit order book according to the price-time priority. An order has a time, price, size, and action (e.g., buy, sell, cancel, among others) associated with it. If two orders to buy come in with the same price, the one that came in first gets placed ahead in the order book queue. However if an order comes in to buy at a higher price, it goes ahead of the other orders regardless of when it entered the book. Orders to sell are handled in the same fashion. Size doesn’t affect priority, and the different amounts available to be bought or sold at different prices constitute the full order book. At any given moment, at the top of the book, there is a best price to buy a certain amount and a best price to sell a certain amount. The difference between them is called the spread. If an order comes in that crosses the spread, a transaction will occur. As long as there is enough volume at the top of the book, then that is the price that the transaction will occur at; otherwise some of the orders will be filled at worse prices as the liquidity at deeper levels of the book gets consumed.

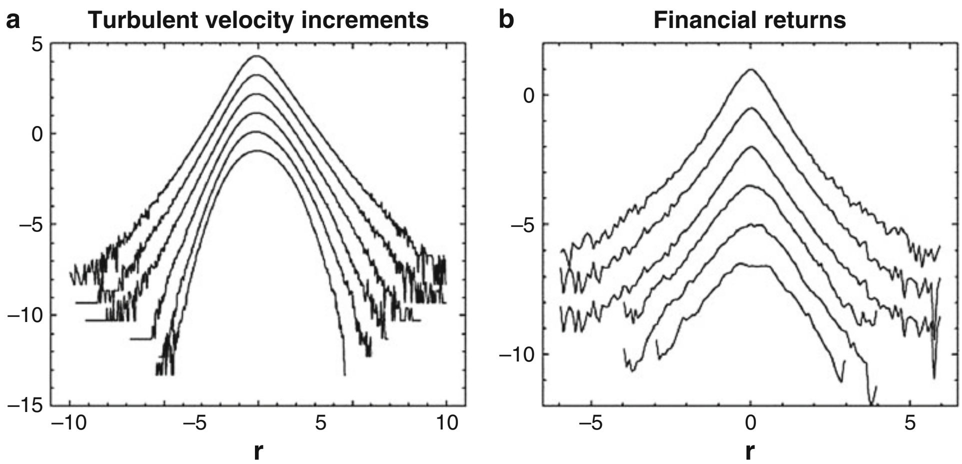
This illuminates the fact that there is no well-defined price. Is it the price to sell or the price to buy? Is it the mid of those? Is it some kind of volume-weighted mid, depending on how many orders to buy or sell at a given price? In practice, the last recorded transaction price is what is used here, but it is easy to see how this could be problematic if a stock doesn’t trade very frequently.

Data

Over the past 20 years, the amount of financial data that is recorded has literally exploded. This is largely due to the explosion of electronic trading and its easy access to the community. Apart from the increased volume of algorithmic and electronic trading and the increased number of mainly electronic exchanges, other factors such as increased regulations for risk management and audit trails, decreased latencies, and higher time resolution also contribute to the exponential boom in data. There are also more traded instruments, new types of ETFs (exchange-traded funds which are essentially investable funds that themselves trade like stocks) and new derivative instruments. As an example of increased time stamp granularity, the TAQ (Trade and Quote) database which has been one of the main sources of transaction data for the NYSE, AMEX, and Nasdaq exchanges started out recording trades and quotes with times stamps marked at the second precision from 1993 to 2003. After that the data was collected at millisecond precision until 2015 and is currently marked at nanoseconds after a brief microsecond era. This is just one example showing the evolution of the importance of speed and the actual timescales that are now relevant. While in the past, only the price and volume at the top of the limit order book was available, the entire depth of the book can be constructed because every order on each of the many electronic exchanges are collected and consolidated.

Stylized Facts of Markets

The huge amounts of data started attracting the attention of physicists around the mid-1990s, and many of the early seminal papers dealt with uncovering and understanding properties of the empirical distribution of returns (or relative price changes). Returns of stocks can be calculated over different timescales τ, and when the distributions of these are plotted out, it is clear that they are far from Gaussian, but rather are well fit with power-law tails in such a way that the power-law behavior persists from timescales ranging from intraday to the order of a few weeks. On daily timescales, the exponent of the power law is about 3, often referred to as the cubic law of finance (Gopikrishnan et al. 1999; Gabaix et al. 2003). The kurtosis of these distributions decays in a regular fashion, roughly as $\tau^{-0.2}$ where τ is the



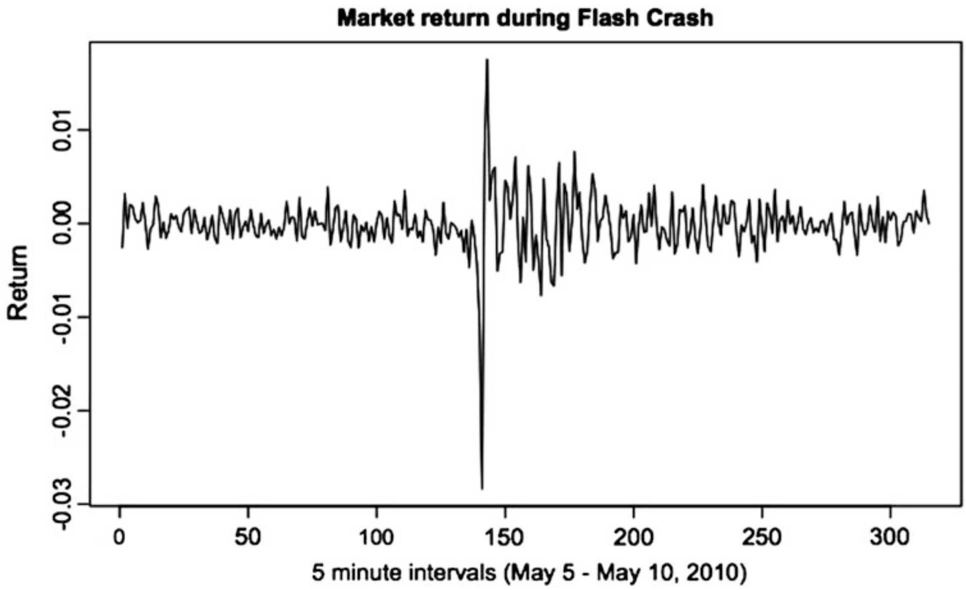
Financial Market Dynamics: A Synergetic Perspective, Fig. 2 The distribution of financial returns bear similarity to the distribution of velocity in turbulent systems. Small scales (top), larger scales (bottom). The y-axis is on a logarithmic scale, and curves are shifted for display purposes. In (**a**) r = velocity increments, and in (**b**) r = returns, in units of standard deviations

timescale over which returns are calculated (Bouchaud and Potters 2004). These distributions bear many similarities to those of turbulent systems (see Fig. 2).

Volatility (defined either as the standard deviation of returns over some past window or simply by a proxy such as the absolute value of returns) exhibits clustering behavior such that there appears to be regimes of higher or lower volatility, on all timescales (see Fig. 3). Furthermore, there is memory in volatility in the sense that the autocorrelation of absolute returns is very strong, decaying slowly as a power law. More subtle statistical features are inherent, such as a behavior analogous to the Omori law for earthquakes in that a large volatility shock will be followed by aftershocks at a certain rate. Furthermore, large negative returns are indicative of higher volatility, an effect known as the leverage effect (Bouchaud et al. 2001). Figure 3 beautifully illustrates some of these effects. It shows intraday prices for a few days surrounding May 6, 2010, the day of the so-called Flash Crash. After a large negative return, volatility increases and decays only slowly. Another interesting property of stock return time series is the presence of time-reversal asymmetry in the sense that future volatility conditioned on past observations is not symmetric (Lynch and Zumbach 2003). Finally, financial time series exhibits multi-fractal scaling of moments (see, e.g., Borland and Bouchaud 2012). All of these so-called stylized facts are not only observed for stock returns but also for other financial instruments such as commodities and currencies, and they are observed across geographies. Realistic market models should ideally capture the basics of these features for stock returns and volatility across time.

Financial Market Modeling

Modeling the intricate dynamics and microstructure of the limit order book is a field of study which has gotten some traction over the past decades. One of the most insightful and detailed studies attempting to understand the dynamics of price formation on this level, as well as the market impact of trading, has been done within the physics community, for example, by Bouchaud, Farmer, and Lillo (Bouchaud et al. 2004, 2009; Lillo et al. 2003). They reveal that the processing of supply and demand in markets has long-range memory and is also related to the origin of market fluctuations among many other interesting findings. Another interesting and rather intuitive model of the order book was developed by Cont et al. (2010) who formulated a stochastic equation for the mid-price based on the order book dynamics. More recently, some authors model the order



Financial Market Dynamics: A Synergetic Perspective, Fig. 3 A time series of market returns around the Flash Crash of May 6, 2010

flow dynamics of bids and ask via self-exciting Hawkes processes (Bacry and Muzy 2014; Alfonsi and Blanc 2015), leading to a nice framework where questions such as optimal trade execution, for example, can be studied. In spirit and in analogy to physics, these models eluded to above can be seen as microscopic models, based on underlying empirical observations of the actual order placement and execution process (viz., the order book). Ultimately though, the price once formed evolves as a stochastic process, and it is often more tractable to use a mesoscopic description which aims at describing the price process as a stochastic Langevin equation where the key feature is how to capture the volatility, or noise, that drives the process. This is the most important effect since stock price changes (or returns) from moment to moment are essentially unpredictable, so the deterministic part of the equation is less interesting. (Though of course, if you can predict it ever so slightly, you are in luck!)

For many years and in a large body of the financial literature, the random nature of price time series was modeled by most as a simple Brownian motion. The first to propose such a model was Bachelier in his thesis in 1900, which lays largely undiscovered until much later when Black and Scholes wrote their famous paper in 1973 based on a very similar model. They made important contributions in particular to the pricing of options, for which they received the Nobel Prize (Black and Scholes 1973) in 1997. Options are traded instruments that give the right, not the obligation, to buy a stock at a later date at a certain price, called the strike price. In Black and Scholes' work, the log price is assumed to follow a Gaussian distribution, and even today many trading assumptions and risk control notions are based off of that prior.

However, as we have seen, the Gaussian model of Black and Scholes is insufficient to describe the statistical properties of real financial time series data. Several alternative models have been proposed, and here we review some that fit well into the spirit of synergetics.

Predator-Prey, Many Interacting Agents, and Spin Models

The Lotka-Volterra equation is used to describe the positive and negative feedback loops between interacting species, where one preys on the other. It is also one of the first successful models for describing the fact that wealth among members of the society follows a Pareto power-law distribution (and hence also that fluctuations in financial markets follow a power law) (Levy et al. 2000; Levy and Solomon 1997; Solomon 1998). That model also recovers realistic features of financial markets such as bubbles and crashes as well as volatility clustering.

The basic ingredients of the model are to introduce feedback between individual and collective wealth fluctuations of a collective set of traders. The central feedback loop consists in computing the market price of the stock as the sum of the individual wealths w_i invested in the stock by the traders and then determining fluctuations of a given trader's wealth as their previous wealth multiplied by the stock return. The basic idea of the model is that wealth at time $t + 1$ is proportional to wealth at time t multiplied by the random factor which corresponds to relative gains or losses over the last period. There is also a coupling of the individual's wealth to the global wealth of the society (e.g., things like social services). In addition, there is also competition between each individual and the other members of society, which plays the part of limiting growth of the average wealth to values that are sustainable for the current conditions and resources. The Solomon-Levy model leads to a power law for the distribution of individual wealth, namely:

$$P(w) \propto w^{-1-\eta} \tag{1}$$

where η is typically between -1 and -2. In Solomon (1998) the very interesting conclusion is drawn that any quantity which is a sum of random increments proportional to the wealths w_i will have fluctuations described by a Levy distribution of index β equal to the exponent η of the wealth power distribution. Since the individual investments are stochastically proportional to the investors' wealth, the stock market fluctuations will be described by a truncated-Levy distribution of index equal to the measured exponent $\eta = 1.4$ (which results in a tail index close to 3 as eluded to above when we talked about the cubic law of finance). This is an amazing and nontrivial result: based on simple notions of competition, local and global feedback of the wealth of members of society, a mechanism for describing the distribution of stock market fluctuations is designed.

Solomon and Levy's wealth equation is a mesoscopic description of the stock market which was written down by the authors as a description of the outcome of many simulation runs of their microscopic model (Levy et al. 2000). The microscopic model looks at individual investors with various ways of deciding how much stock to buy or sell at a given time. Simulations of that collective group of investors then gave rise to the dynamics of wealth fluctuations as described by their Lotka-Volterra equation.

Another approach of interacting agents was proposed by Lux and Marchesi (2000). Individual trading agents are simulated, including an explicit price formation process. Agents are modeled as different types of traders interacting in a speculative market: "noise traders" and "fundamentalists." Fundamentalists base their action on fundamental valuation of the stock. The noise traders base their trading decisions on price data and flows, which leads to herding behavior. They react to the recent past of the market and can have either positive or negative expectations of the future based on that past. The dynamic of the model is that traders compare profits gained by the noise traders and fundamentalists and then switch their own strategy to that which was more profitable in the recent past. Depending on whether traders want to buy or sell, supply or demand will be infused into the market, and the price will be adjusted according to the excess demand. In addition, the dynamics of the fundamental value of the stock follows a standard lognormal Brownian motion with uncorrelated Gaussian noise.

Lux and Marchesi formulate the state-dependent transition probabilities that describe, for each group of traders, the probability of switching to the other group. For noise traders there is also the internal switching between a pessimistic and optimistic view of the market. The price gets adjusted up or down based on supply and demand according to excess demand being either on the buy or sell side.

Based on these simple yet realistic dynamics, a theoretical analysis and simulations show that the most important features of real financial markets emerge as a consequence. They find that the market is on average efficient in the sense that the price on average reflects a fundamental equilibrium. The amount of pessimistic and optimistic traders is roughly even, and in equilibrium both the noise traders and the fundamentalists do equally well. Though the system always tends toward a stable equilibrium, it exhibits autocorrelated fluctuations around that fundamental equilibrium and simulations of the model show on-off intermittency of fluctuations. This is very similar to the properties of real market fluctuations, where volatility shows memory and clustering.

Other classes of models are spin-based models in which analogies are made between the interactions and dynamics of spin systems and financial markets. Some interesting spin models have been proposed, for example, Cont and Bouchaud (2000), Chowdhury and Stauffer (1999), and Bornholdt (2001). These models can reproduce (under certain parameter settings) features such as the volatility clustering of financial markets.

Statistical Feedback Models

The above models are useful as frameworks to think about the dynamics of market participants and the emergence of the stylized facts one observes. For certain applications (e.g., risk management or the pricing of options), a slightly higher level view of price formation can be useful, namely, in terms of modeling the price itself as a stochastic process.

As mentioned above most of traditional mathematical finance is based on the Black-Scholes model which assumes a Brownian equation of motion for stock prices:

$$dS(t) = \mu S dt + S\sigma d\omega \tag{2}$$

where ω is drawn from a Gaussian distribution with zero mean and variance one. This type of model is very useful because it allows for the analytic calculation of many important quantities related to risk management and derivative instruments such as options. However, that model is too simple to capture all of the anomalous statistics observed in real financial time series. In an attempt to rectify that, several modifications to the standard Black-Scholes model of price returns have been proposed in the literature, and they all have in common that they somehow extend either the assumption of a constant volatility term σ in Eq. 2 or the source of the noise term ω. For example, there is the stochastic volatility model of Heston (1993) where σ itself is modeled as a mean reverting stochastic process and the Levy models where the noise ω is assumed to be drawn form a fat-tailed Levy distribution. Those models are a little more realistic than the standard model, but both have the shortcoming that they convolve too quickly to a Gaussian distribution, meaning that they do not capture the persistence of fat tails of return distributions over the timescales observed in reality, where returns over timescales ranging from seconds up to about 2 weeks or longer all still exhibit tails. One model which does really well in this sense (and also earned a Nobel Prize for Engle (Bollerslev et al. 1994)) is the GARCH model, which incorporates memory into σ. In fact, the memory of volatility is a key feature that reproduces many known stylized facts of financial price series.

Motivated among other things by this, we proposed a model (Borland 2002a, b; Borland and Bouchaud 2004) within the framework of nonextensive statistical physics (Tsallis 1988) in which the volatility term follows a statistical feedback process in the sense that it depends on the probability of past observations, explicitly:

$$dS = \mu S dt + \sigma S d\Omega \tag{3}$$

where

$$d\Omega = P(\Omega)\frac{1-q}{2} d\omega. \tag{4}$$

In this equation, P corresponds to the probability distribution of Ω, which simultaneously evolves according to the corresponding nonlinear Fokker-Planck equation (Tsallis and Bukman 1996; Borland 1998):

$$\frac{\partial P}{\partial t} = \frac{\partial P^{2-q}}{\partial \Omega^2}. \tag{5}$$

It can be solved exactly yielding:

$$P = \frac{1}{Z(t)} (1 - (1 - q)\beta(t)\Omega(t))^{\frac{1}{1-q}} \quad (6)$$

The exact form of the coefficients Z and β are given in Borland (2002a, b). Equation 6 recovers a Gaussian in the limit $q \rightarrow 1$ while exhibiting power law tails for $q > 1$. In that case, our model is exactly equivalent to the Black-Scholes model.

The statistical feedback term P can be seen as capturing the market sentiment. Intuitively, this means that if the market players observe unusually high deviations of $\Omega(t)$ (which is essentially equal to the detrended and normalized log stock price) from the reference value $\Omega(0)$, then the effective volatility will be high because in such cases $P(\Omega)$ is small, and the exponent $1 - q$ is a negative number. Conversely, traders will react more moderately if Ω is close to its more typical or less extreme values. As a result, the model exhibits intermittent behavior consistent with that observed in the effective volatility of markets. In practice, q can be obtained empirically from a fit to the data. Remarkably, $q = 1.4$–1.5 fits very well to return distributions of very many financial instruments, corresponding to a tail index of about 3.

This non-Gaussian statistical feedback model allowed us to derive closed-form option pricing formulae (Borland 2002a, b; Borland and Bouchaud 2004) that fit very well to real market prices over many time horizons, and we used the model in real-life trading situations. For further reading about this topic, summarized successes, applications, and shortcomings of the model, we refer to Borland (2008).

Multi-timescale Models

In spite of the success at pricing options and other derivatives such as credit default swaps, as a model of real returns, the statistical feedback formulation has the drawback that returns relative to a particular initial time constituting the memory in the volatility; instead we took inspiration from that model and proposed that the volatility depends on returns over multiple timescales (Borland and Bouchaud 2012). The intuition is that different traders pay attention to different timescales. For example, some only care about returns on an intraday or daily level; others are more focused on monthly or whatever the rebalancing frequency is of their trades. Explicitly, this multi-timescale model can be written as:

$$\Delta y = \sigma_t \, \Delta\omega \quad (7)$$

$$\sigma_t = \sigma_0 \sqrt{1 + \sum_{\tau=0}^{T} \frac{g}{\sigma_0^2 \tau_\alpha} \left(y_t - y_{t-\tau}\right)^2} \quad (8)$$

where y denotes the logarithm of the stock price; hence Δy represents the change in log stock price and corresponds to stock returns. The parameters g and α can be calibrated to fit empirical data ($g = 0.85$ and $\alpha = 1.15$) (Borland and Bouchaud 2012), and σ_0 corresponds to the baseline volatility. ω represents uncorrelated standard Gaussian noise. This model is motivated by the statistical feedback model that we presented in Borland (2002a, b) and Borland and Bouchaud (2004) and is very similar to the FIGARCH models (Lynch and Zumbach 2003; Bollerslev et al. 1994). In fact, without the summation and setting $t - \tau = 0$, this equation can be shown to be of the same form as our statistical feedback model. That model has the advantage of analytic tractability for options pricing, whereas Eq. 7 doesn't allow for that. However, it can be simulated and shown to fit real data remarkably well, reproducing a slew of known stylized facts (Borland and Bouchaud 2012), including volatility clustering, the fat-tailed distributions of returns persistent over increasing timescales, time-reversal asymmetry of volatility, and multi-fractal scaling properties.

Cross-Sectional Dynamics

The models we discussed up to now have focused on capturing the time series properties of stock returns. In order to fully understand the joint stochastic process, the cross-sectional dynamics of stock returns, i.e., the dynamics of the correlation structure of markets, is also important. Understanding how the distribution of returns as well as their correlations behaves during such times as the financial crisis in 2008 can be extremely important for managing financial risks. Several authors have studied these cross-sectional dynamics, such as Borland (2012), Preis et al. (2012), Kaizoji (2006), Lillo and Mantegna (2000), Munnix et al. (2012), Ferreira et al. (2015), Raffaelli and Marsili (2006), and Sornette (2002), but we shall delve into one model that fits right into the paradigm of synergetics (Borland 2012).

To set the stage, one should understand that financial markets go through different types of regimes (often referred to as “risk on” or “risk off”) or, as in this paper, “panic” times or “normal” times. The VIX Index captures forward-looking expected volatility. In “panic” times, the VIX is very high, whereas in “normal” times, it is more moderate. Based on the VIX, one can for US markets categorize the periods 2008–2009 (the financial crisis), 2010 (the foreclosure crisis), and 2011 (the debt ceiling crisis) as periods of high uncertainty or panic. As discussed in Borland (2012) we showed that, during panic times, the dispersion (standard deviation) of stock returns cross sectionally increases, as does the time series volatility. However, the kurtosis (corresponding to the tails of the cross-sectional distribution) tends to decrease. In addition, correlations approach 1.

To further explore the correlation structure, or co-movement of stocks at a given time, we define the variable s:

$$s = \frac{s_{\mathrm{up}} - s_{\mathrm{down}}}{s_{\mathrm{up}} + s_{\mathrm{down}}} \tag{9}$$

where s_{up} is the number of stocks that has positive returns over a given interval and s_{down} is the number of stocks that has negative moves on that same interval (e.g., a day). If $s = 0$ then roughly the same number of stocks moved up as down, and the assumption is that the stocks had little co-movement and so were uncorrelated. If all stocks move together either up or down, the value of s will be +1 or −1, and the stocks will have high correlation. So, the following picture emerges: if $s = 0$ there is no correlation, and we are in a disordered state. However if $s \neq 0$ then there is correlation, and we are in an ordered state; in other words, in the spirit of synergetics, s is the order parameter of the system. This behavior of s under different market conditions can be seen in histograms calculated for data during periods of panic versus more normal periods. In normal times, s is unimodal, and in panic times we obtain a bimodal distribution (see Fig. 4).

We refer to these phenomena as statistical signatures of market panic: high time series volatility, high cross-sectional dispersion, low cross-sectional kurtosis, and a bimodal distribution of s. In order to create a model that replicates these signatures, we proposed in Borland (2012) a synergetic model where the variable s plays the same role as the magnetic moment in a ferromagnetic spin system. In the same way that the magnetic system will either be in an ordered or disordered state depending on the value of the temperature, the financial system will be in a correlated or uncorrelated state depending on the value of volatility.

Explicitly, the stock returns for each instrument across time is modeled by Eq. 7 (inserting $\Delta\omega^i = \omega_t^i\sqrt{\Delta t}$ according to the assumptions of Brownian noise):

$$\Delta y^i = \sigma_t^i \omega_t^i \sqrt{\Delta t} \tag{10}$$

with

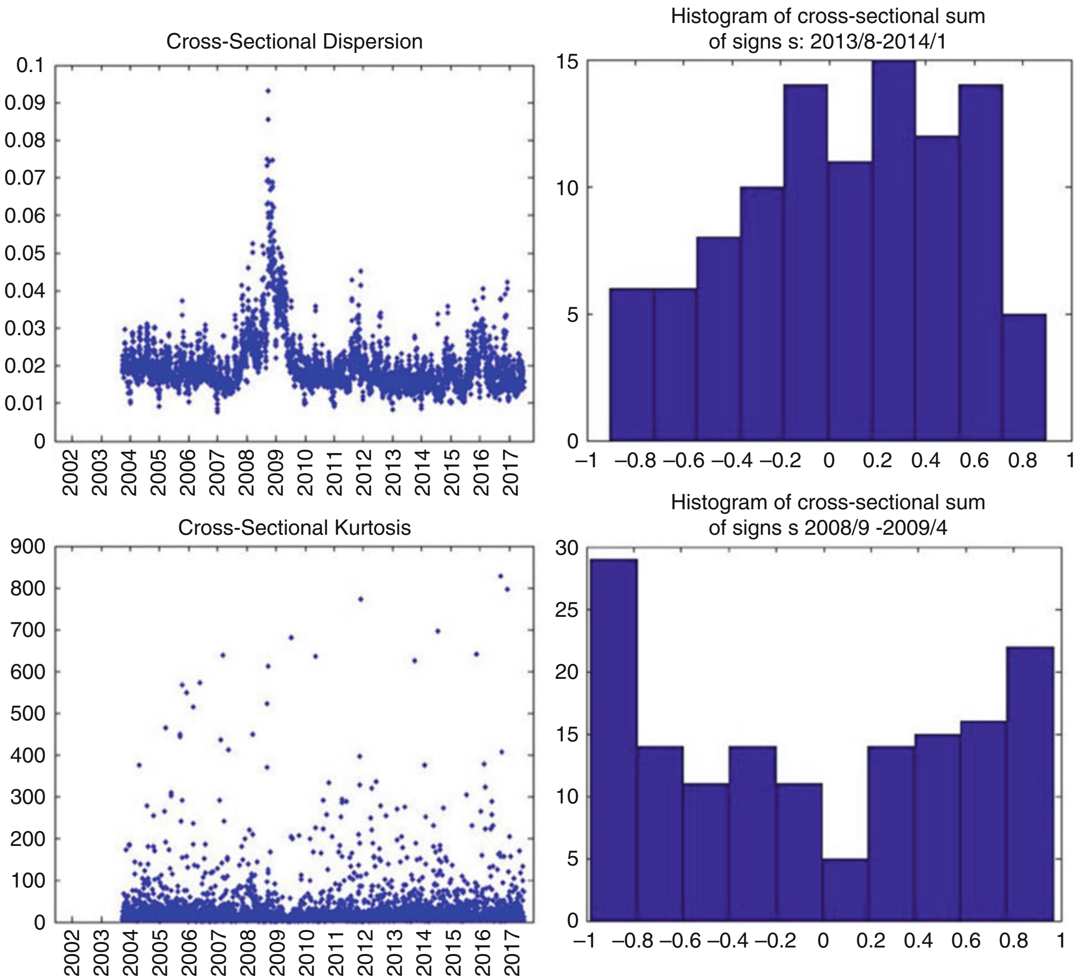
$$\sigma_t^i = \sigma_0^i \sqrt{1 + \sum_{\tau=0}^{T} \frac{g}{\left(\sigma_0^i\right)^2 \tau^\alpha} \left(y_t^i - y_{t-\tau}^i\right)^2} \tag{11}$$

where y_i is the log stock price of the i-th stock and ω_t^i is a zero mean Gaussian noise with unit variance so that $\left\langle \omega_t^i \omega_{t'}^i \right\rangle = \delta_{tt'}$. For an ensemble of N stocks, we assume that the ω_t^i, $i = 1, \cdots, N$ are correlated proportional to $|s|$ across stocks, so $\left\langle \omega_t^i \omega_t^j \right\rangle = |s|$ for $i \neq j$ and 1 for $i = j$.

We hypothesize that s can be described by the Langevin equation

$$\frac{ds}{dt} = -as - bs^3 + F_t \tag{12}$$

with $a = \gamma(\sigma_c - \sigma_M)$, b a scaling parameter, and σ_c a critical volatility level, and σ_M is the market



Financial Market Dynamics: A Synergetic Perspective, Fig. 4 Properties of cross-sectional stock returns in normal times, and times of panic (Borland 2012), calculated across the top 1000 stocks by market capitalization. "Panic" times are defined as periods of high uncertainty among investors, such as 2008–2009 (the financial crisis), 2010 (the foreclosure crisis), and 2011 (the debt-ceiling crisis)

volatility. In this model, s can be thought of as jiggling around in a potential well. The control parameter of the system is σ_M. When $\sigma_M < \sigma_c$, that well only has one minimum at 0, so $|s|$ fluctuates around that point. If $\sigma_M > \sigma_c$ the potential well attains two new minima at $\pm\sqrt{a/2b}$. $|s|$ becomes non-zero, and correlations are high: stocks tend to move up or down together in accordance, which is manifested in a bimodal distribution of s. We say that there is a phase transition as σ_M crosses above the critical value, since the collective behavior of the stocks is qualitatively very different. This type of phase transition model is based on the dynamics and theories of synergetic self-organizing systems. In particular, s can be seen analogous to the magnetic moment m in ferromagnetic systems, There, the system goes from the disordered to the ordered state as the temperature T (which is the control parameter) drops beneath a critical temperature T_c.

Hence, the parameter σ_M in the financial system plays a role similar to temperature in the magnetic system.

We model σ_t^i as in Eq. 11. Conditions for stability and values which calibrate to real returns are discussed extensively in Borland and Bouchaud (2012). The feedback in the model is controlled by g, and the power law memory in time is related to α.

The market volatility σ_M can increase to values larger than σ_c due to either (i) exogenous jumps (news, external fear) affecting all stocks so that σ_0 becomes $\sigma_0 + \sigma_{\text{shock}}$ or (ii) endogenous, idiosyncratic jumps which are more stock specific. In this paper we consider only exogenous jumps describing market-wide situations such as the Lehman Brothers collapse, although endogenous ones akin to the dynamics of the Flash Crash have been discussed in Borland and Hassid (2010).

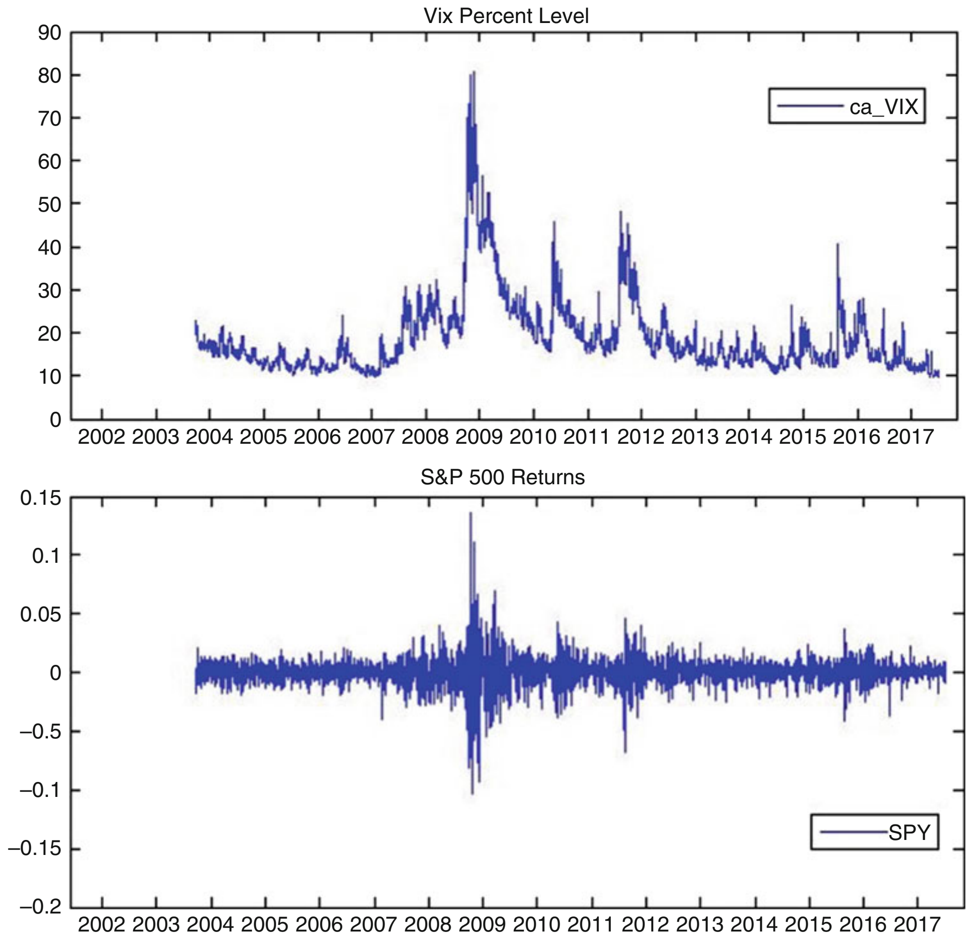
Simulations

To simulate the joint cross-sectional dynamics of the market, the individual dynamics of $N = 500$ stocks are generated using Eqs. 10 and 11. The "market" is then defined as the equal weighted average of those stocks' returns. This is akin to how the *S&P* 500, which represents the US market, is composed of 500 stock returns (albeit weighted by market capitalization). Furthermore, market-wide exogenous shocks which correspond to external fear factors or general sentiment are applied to the base volatility σ_0. In previous papers we have injected artificial shocks, but for this contribution, we let the actual jumps in the VIX drive the external exogenous volatility shocks in the model such that if the VIX corresponds to a volatility greater than 0.25, it will induce a shock σ_{shock} to the system such that σ_0 becomes $\sigma_0 + \sigma_{\text{shock}}$.

The dynamics of each individual stock were generated using Eqs. 10 and 11. The parameters used were those determined in Borland and Bouchaud (2012), namely, the feedback parameter $g = 0.85$ and $\alpha = 1.15$. The base volatility was chosen at $\sigma_0 = 0.20$ which is the typical annualized volatility of a stock, and we allow $\sigma_{t0}^i = \sigma_0(1 + \varepsilon * \eta(t))$ where η is a zero mean white noise, and $\varepsilon = 0.2$ was chosen. We included $T = 300$ terms in the volatility feedback term. The time step for the simulations was chosen to $\Delta t = 1/252$, corresponding to 1 trading day (there are 252 trading days in a year). The noise σ_t^i driving each stock is drawn from a normal Gaussian distribution uncorrelated in time with 0 mean and unit standard deviation yet with a correlation across stocks driven by Eq. 12.

More explicitly, the correlations across stocks at a given time point were modeled according to the phenomenological equation (12) using $b = 0.01$ and $F_t = 0.1\nu$ where ν is a standard Brownian noise. Given a value of s from this equation, the noise across stocks was then drawn from a correlated set of noises, with correlation equal to $\tanh |s|$. The mechanism for attaining that noise utilized the Cholesky decomposition technique. The critical volatility σ_c was chosen to be $\sigma_c = 0.4$ which is roughly twice the annualized standard deviation of market returns. At each time point in the simulation, individual stock paths are generated, and their mean is taken to represent the value of the market as a whole at that time point. The control parameter $\sigma_M = \sigma_0 + \sigma_{\text{shock}}$ feeds back into Eq. 12 thus affecting the correlation dynamics that defines the cross-sectional behavior of the 500 stocks in the simulation.

Figure 5 shows actual market returns in the time period 2003–2017, together with the VIX. Figure 6 shows simulated market returns, together with a plot of market volatility defined as the recent standard deviation of market returns. It is clear that the features follow the general profile of actual returns quite closely, and the simulated market volatility reflects the main features of the VIX (although remember these will not match exactly since the VIX is an index calculated from the implied volatility of options on the *S&P* 500). The simulation does have volatility spikes in 2008–2009, 2010, and 2011, as does the VIX. We also show a time series

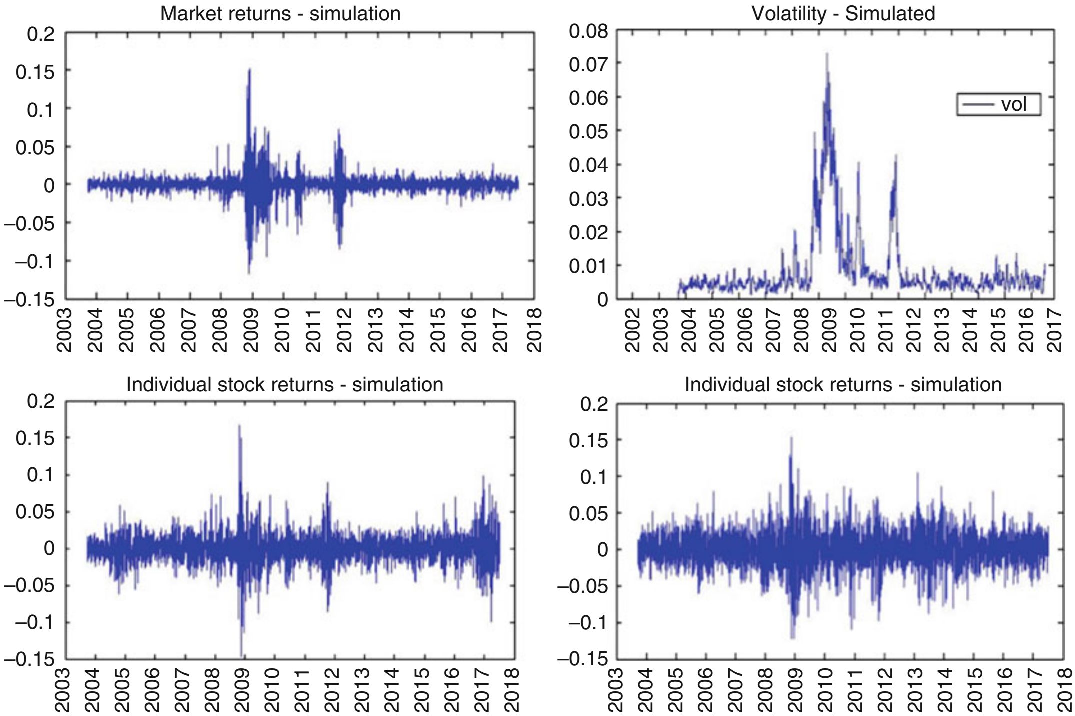


Financial Market Dynamics: A Synergetic Perspective, Fig. 5 The VIX volatility index and daily S&P 500 market returns

of 2 of the 500 simulated stock returns that constitute the market in aggregate. Individual stocks' price paths were generated by Eq. 11 and exhibit idiosyncratic periods of volatility clustering not apparent in the market, due to the multi timescale feedback aspect of the dynamics which is stock specific. In addition, cross-sectional dispersion of the simulated stock market increases in times of panic, while the kurtosis appears to dip in those periods. Finally, histograms of s are bimodal during the times of panic, and unimodal in normal times, just as in the real data. These cross-sectional properties are shown in Fig. 7. It is apparent that the model captures a similar behavior of cross-sectional features as in the real market data.

Final Comments

The multi-timescale collective feedback model we described for the joint stochastic process of a set of stocks over time agrees qualitatively with actual features seen in real markets, both across time and across stocks. Elements of synergetics enter the

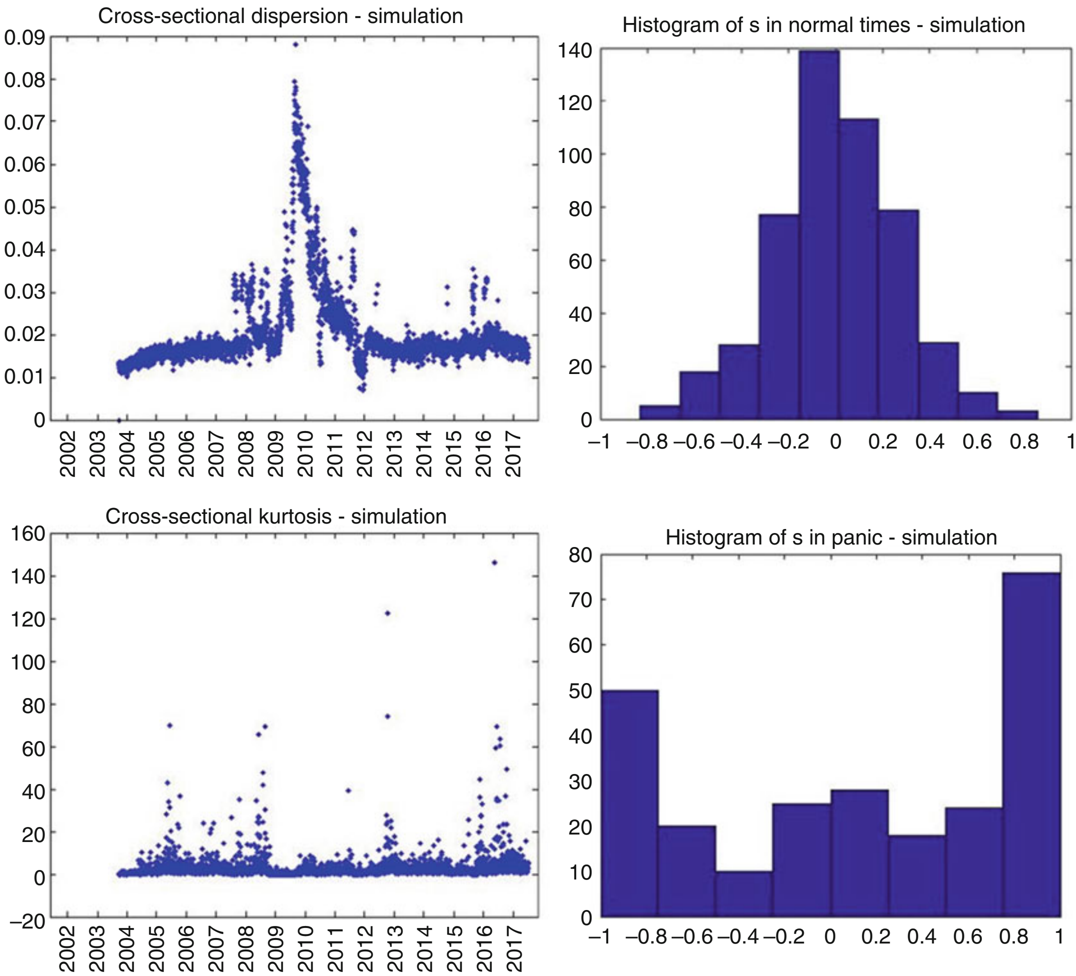


Financial Market Dynamics: A Synergetic Perspective, Fig. 6 The simulated market returns, together with two individual stocks' return time series. We also show simulated market volatility which can be compared to the VIX

model in various aspects. The correlation between stocks acts as the order parameter of the system, with the volatility, which is related to the collected external perception of panic or fear acting as the control parameter. Furthermore, the dynamics of each individual stock follows a feedback process that aims to capture the collective behavior of individual agents acting on different timescales. In addition, we have reviewed other more agent-based models that have been proposed within the physics and econophysics community that all have similar notions embedded, namely, that it is the interaction of individual market participants and the feedback with the overall macroscopic level of the financial system (be it via price, wealth, or volatility) that appear to lead to systems that reproduce many of the interesting anomalous statistics observed in real financial markets.

Future Directions

Financial markets are constantly evolving, producing ever-increasing sets of data, so the task of modeling the complexities driving their behavior will certainly remain a fruitful area of research for some time to come. Additionally, in spite of the success of the models we have discussed here when it comes to reproducing realistic market features and stylized facts, many financial practitioners still use the simpler notions consistent with a Black-Scholes-type Gaussian model of financial time series. One reason for this could be that many of the models have been developed within the field of econophysics, using concepts from physics, and are not yet incorporated in more traditional mathematical finance programs. An important area of further work would therefore



Financial Market Dynamics: A Synergetic Perspective, Fig. 7 Cross-sectional properties of the simulated 500 stock return paths comprising the simulated market

be to try and integrate some of the new insights that these models lend us into practical applications which can be used where real money is managed, in order to have better and more proactive tools both for price modeling and risk management.

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