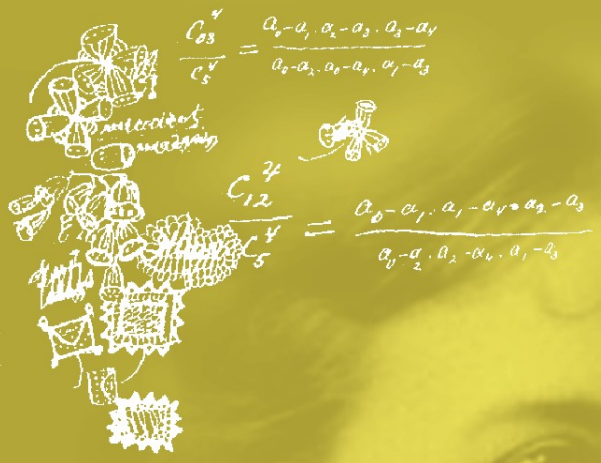
Michèle Audin

Remembering Sofya Kovalevskaya



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Oh! Happy time! Dominated by new ideas, we were persuaded that the existing social conditions could not long endure; that a glorious era of enfranchisement, universal civilization, all our dreams, seemed so close, so certain!

Sofya Kovalevskaya

For Sonya's friends, it was not her greatness that was precious to them. What made her unequaled in their eyes was that she possessed those rare qualities which Goethe said a human being should pursue:
Grosse Gedanken und ein gutes Herz

Ellen Key

Why then not represent science by the features of an attractive young woman, brash and self-assured, pursuing her way without caring what will be said about her, able to obtrude on a man's world, loving to please, happy?

Françoise Balibar

CONTENTS

INTRODUCTION

This is not a history book. Proof: I am not a historian. Nor is it a novel, since I am not a novelist either. And it is not even a mathematics book, although I am a mathematician. It is a personal book and it is a serious book. It deals with serious matters, with the work and life of a woman, of a serious woman, brilliant, professional, tenacious, and with the scientific reputation of this woman. I approach her story, sometimes with humor, often with jubilation, and always with pleasure. And with seriousness.

The first woman? Sofya Kovalevskaya was not the first woman to obtain a doctorate, even in mathematics: before her there was Maria Agnesi, in the 18^{th} century, in Bologna. She is perhaps not even the first woman to obtain a university position; the same Maria Agnesi undoubtedly preceded her (but it seems never took up her position and devoted herself to religion and charity). In this book we will even witness one of the most eminent Swedish mathematicians of our time confirm (although in a rather ambiguous way) that, yes, Sofya Kovalevskaya was indeed the first woman to obtain a professorial position ... in Sweden. She was not the first woman to receive a prize from the Académie des sciences: Sophie Germain, another Sophie, another mathematician, had received one in 1816. She is very likely the first woman to have served on the editorial committee of a scientific journal.

I know neither the form nor the content of Maria Agnesi's doctorate. It is not even completely clear that what was called a doctorate corresponded in this case to original mathematical work.

Why Sofya Kovalevskaya? She is without doubt the first woman to have had a professional university career in the way we understand it today: she proves original theorems that earn her the title of doctor, she gives courses, she concerns herself with politics, she believes in the responsibilities of scientists,

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_0,

she travels, she proves more theorems, she participates (without much enthusiasm) in committee meetings, she has a daughter, she is editor of an international journal (*Acta Mathematica*), she fights for women's rights, she attends and contributes to scientific meetings, she's up for promotion, she writes reports and letters of recommendation, she travels to meet with colleagues at other universities. As was done in the 20$^{\text{th}}$ century, as we continue to do in the 21$^{\text{st}}$. It is in this sense that she is close to us and it is why her life, her work and what she said touches us. Especially when we add that she led her professional life under very difficult conditions and in a wide variety of personal situations (married, separated from her husband, widowed, mother and head of the family).

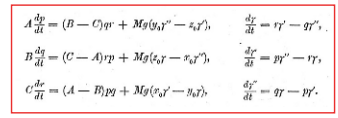
I also want to emphasize the profound unity of the various facets of Sofya Kovalevskaya's personality, brilliantly summarized under the nice title (*A convergence of lives*) of the biography that Ann Hibner Koblitz dedicated to her. The fact of her being a mathematician *and* a writer is inseparable from her political convictions. Sofya was a nihilist. Many people think of nihilists as dangerous anarchists (as they were called before the word terrorist was made fashionable by the Nazis). In reality, the nihilists wanted to reform society with the notion that they, men and women equally, should contribute to raising the level of knowledge of society, which, in 19$^{\text{th}}$–century Russia, was no small undertaking. In this regard, I refer the reader to the excellent preface of the book by Ann Hibner Koblitz [1993].

Where did this book come from? My association with Sofya's mathematical work (I explain on page 27 how and why I call her "Sofya") is long-standing, my association with her personality (see chapter XII) much less so: Sofya, her appearance, her life, her romances, her writings, the things she is thought to have experienced, Sofya, with all her facets, entered my life definitively at the end of 2004, for reasons both personal and mathematical, and actually took over my life after I got to know Jean-François Peyret and the cast of the theater production *The Case of Sophie K* in the Spring of 2005. It was in order to "go beyond" that I decided to write this book. Its title comes directly from having been present with that "troop", so many times did I hear Jean-François Peyret say that his play displayed "memories of someone who was never known".

What will you find in this book?

Three parts.

– First, what we should know about Sofya Kovalevskaya, the chronological benchmarks, her identity, some anecdotes about her that have often been repeated and that I too will tell, in my own way. These are chapters I, II and III.

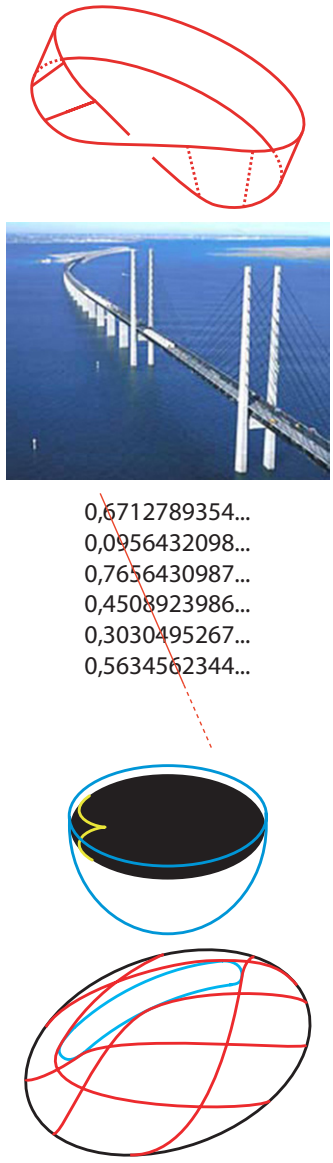


– Next, about her mathematics. I will come rather quickly in chapter IV to the three papers from her thesis. It is perfectly possible to skip the parts with formulas. We cannot pretend that there are not any mathematical formulas in this book. Often they are pretty by themselves, like those that decorated the nursery at Sofya's house (see chapter IV) and which turned out to be useful. Failing—or while trying—to understand them, we can attempt to appreciate their aesthetics. It is about mathematics (even though I know very well that it will not be accessible to everyone) but not just that. I talk extensively about "the solid" in chapter V: a subject in which I am truly competent even before beginning to think about Sofya, it is the subject that I most often have occasion to explain, more or less on all levels, but which I also explain there, once again, on all levels. I then discuss in chapter VI a letter of Sofya Kovalevskaya that contains mathematics related to the problem of the solid.

$$\iint_{\Sigma} F \cdot dS = \iiint_{V} \operatorname{div}(F)\, dx\, dy\, dz$$

$$\iint_{\Sigma} F \wedge dS = -\iiint_{V} \operatorname{rot}(F)\, dx\, dy\, dz$$

– I return to stories in chapters VII and IX, where I tell how Sofya Kovalevskaya obtained her various positions in Stockholm, the award of the Bordin prize and what I call Sofya's misfortunes, her posthumous misfortunes, her reputation. In chapter VIII, I interpose another letter of Sofya, addressed to Vollmar, which I think clarifies this whole story. I talk at length about Sofya's reputation in Paris, in particular because I cannot resist the temptation to mention the delightful letters from Hermite. We often read that Sofya did not have the reputation that she deserved, because of Bell, or Klein, or both of them, but it is never stated exactly why. I study Bell's text in detail in chapter X. We will find Klein's in chapter XI, taking its place among other opinions and memories of Sofya, which I attempt to arrange in a somewhat humorous and contrasting, although rigorous, manner. The "memories" of this chapter are (within translation) authentic, but their choice and the way they are juxtaposed are completely my own. In



Some of the digressions to be found in this book ...

the following and final chapter, I relate my encounter, my encounters, with Sofya.

It happened that Sofya Kovalevskaya would break off her mathematical work in order to write novels and plays. Here and there you will find, under the heading *Pause*, some tributes to her taste for literature. If I have based the arguments and certain details of these texts on facts and accounts given in this book, it is because I too am permitted all the fantasies, including a writer's license, creativity, embellishments of reality and anachronisms. These *pauses* have the nature of a pastiche and come from my own imagination. I have solicited four writers who have expressed an interest in science: George Eliot and Jules Verne (contemporaries of Sofya), Italo Calvino—a bit closer to us—and finally our contemporary A. S. Byatt.

The book was not conceived to be presented in a completely linear fashion. Perhaps you will find it difficult, but it seems to me that everyone should be able to find things they like—a bit like when we watch a top spin, with a rather naive pleasure depending on whether we are a child, Lagrange or again Sofya. It contains numerous internal cross-references and is provided with an index of persons and places to help us find our way.

Digressions. There are lots of digressions in this book, parenthetical remarks, footnotes and marginal commentaries. I haven't been able (or even tried) to resist the pleasure of providing a mathematical diagram or explanation, or a remark about the context.

On rigor. I have tried to apply a methodical rigor, a quality befitting both a historian and a mathematician. You will find enough examples of non-rigor quoted in this book to clarify what I mean. I have been very struck by reading all that I have read (the serious and above all the less serious, the abundantly less serious) by a number of authors, mostly contemporaries alas, mostly mathematicians alas, who practice the method "I didn't see it, I didn't read it, but I've heard it said that ..." which I find totally unacceptable. So I have given my sources systematically, occasionally having to cite a lack of sources. Perhaps you will find this policy a burden, but what can I do?

A caution, and sources. As I am not a historian, I haven't combed through any archive, I haven't discovered any new source (if the letter reproduced here in chapter VI was without doubt unpublished in French, it was published in Russian by

Pelageya Kochina [Yushkevich 1984] and excerpts have been quoted here and there, just as for the letter to Vollmar that is found in chapter VIII). I have of course used as much as possible available information and above all the two biographies of Sofya, those of Pelageya Kochina [1] [1985] and of Ann Hibner Koblitz [1993], the book of Roger Cooke [1984] and the letters of Weierstraß edited by Reinhard Bölling [1993], books that are well documented, serious and rigorous (in spite of the stupid title *Love and mathematics* for the English translation of that of Pelageya Kochina), [2] and which I recommend reading.

There are nonetheless two newcomers in this book. Nanny Lagerborg, whom I have never seen mentioned in any book on Sofya, and Dorothea Klumpke, who barely appears in that of Cooke [1984, p. 174]. It was in rereading the study [Gispert 1991] by Hélène Gispert that I had the curiosity to find out about this Finnish woman without profession (who had become a member of the French mathematical society in 1890), and this first woman doctor of mathematics in France, and I was surprised to find that both were scientifically tied to Sofya.

Pelageya Kochina (1899–1999)

As I was interested above all in Sofya's image, I have read many other books. In principle, all references to these "sources" should be found in the bibliography (at the end of the book).

Translations. I must confess first that I can read neither Russian nor Swedish. For this book we use the translation from Russian to English [Kovalevskaya 1898], which appears in a single volume together with [Leffler 1898], which is a translation from Swedish to English, together with the more modern translation of Beatrice Stillman [Kovalevskaya 1978].

I have attempted to avoid multiple translations to the greatest extent possible, so that in the typical instance a translation is made directly into English from the source language. The very few exceptions were unavoidable and do not threaten to damage the accuracy of our text.

We need to be on guard against nonsense due either to mistranslation or carelessness on the part of authors. E.g., in the English translation [Kozlov 2000, p. 1178] we find the quotation attributed to Felix Klein:

1. Note that the Russian mathematician and historian Pelageya Kochina appears here and elsewhere under the two names Kochina and Polubarinova-Kochina.

2. I have unfortunately only read Pelageya Kochina's book in the English translation [Kochina 1985].

During World War I, when I was a student

Speaking of inexact dates: you would not even imagine the number of authors who believe that the Paris Commune took place in 1870 or who think that it was possible to have

"spent several months in Paris during the Commune".

(which is, by the way, an accurate rendition of the Russian), whereas Klein in fact studied at the time of the war of 1870. In the book [Kennedy 1983] the same Klein is presented as a student of Weierstraß, the result of an error produced by double translation, as was pointed out to me by Roger Cooke.

In all instances I have attempted to specify the source of the texts that I have used. I also attempt, even when I just isolate a phrase or a part of a phrase, to give its context, so as not to bend it into just about anything one would like.

Acknowledgments to institutions. I thank

– The Mittag-Leffler Institute and its director Anders Björner for permission to reproduce the letter that is the subject of chapter VI: I point out that all letters addressed to Sofya or to Mittag-Leffler from which excerpts appear in this book belong to the Mittag-Leffler Institute; likewise for the original photographs of these two mathematicians and the original of Sofya's manuscript reproduced here.

– The *Internationaal Instituut voor Sociale Geschiednis* (International Institute for Social History) in Amsterdam and its archivist Mieke IJzermans for sending me a copy of the letter to Georg von Vollmar and giving permission to publish a translation (in chapter VIII).

– The archival service of the Academy of sciences of Paris, in the person of its Curator, Florence Greffe, for information on the Bordin prize.

I love books, the feel of books and I certainly would never have had the idea of writing a book having anything to do with history if I had not the possibility and the habit of frequenting the rich library of the IRMA (Mathematics Research Institute) in Strasbourg. It is certain that the volumes of *Acta Mathematica* removed to the coffers of the library have lost some of their dust, however tenacious, since I performed this work! But this is no reason for not thanking the institution.

It was at the library of the Henri Poincaré institute in Paris that I read some of the books and articles that I have used, especially the volume of the *Comptes rendus* of the Academy of sciences where there is an account of the award of the Bordin prize (see our chapter IX). The richness of this library is now so accessible that it is truly a pleasure to rush there whenever I find myself in Paris.

I thus thank these two institutions as well as Christine Disdier and Liliane Zweig and their staffs for their hospitality and their help. Thanks to her knowledge network and her confidence with Gallica (the French digital library), Christine succeeded in obtaining for me some of the extra-mathematical articles that I have used here.

Regarding this translation into English. I thank Lester Senechal for his enthusiasm in translating this book, and I thank, at Springer UK,

– Lauren Stoney, associate editor, for her help at every stage of this project

– Lyn Imeson for doing a wonderful job of copyediting the near-final maunscript.

Thanks to my friends and colleagues. Those who have read this text will not be surprised that I begin by thanking Jean-François Peyret, without whom it likely would not exist. In addition to everything else, Jean-François read and commented on a preliminary version, encouraged me to remove a few rather cumbersome passages and pointed out some connections and contexts of which I was unaware or had overlooked.

I also thank the writers I solicited without requiring their advice. And in truth it was also thanks to Jean-François Peyret that I read (and met) A. S. Byatt—and thanks to her that I read (and did not meet) George Eliot, whose novels make their appearance in this book. These encounters were not due to chance, for example Sofya Kovalevskaya is obviously—in the guise of Frederica Potter, the heroine of A. S. Byatt's tetralogy, a "whistling woman".

There are also more diffuse reminiscences of other texts in this book:

– writers that I have already quoted, of *Possession* [Byatt 1990] for example, in the story of the burned letters,

– or others, Perec's *Species of spaces* [Perec 2008] in the story of the asteroids or *A Void (la Disparition)* [2005], used appropriately on page 239 (not to mention [Perec 1978] or [Senon, Evero, Eben & Trovato 1982]),

– or the trilolgy by Vallès [2006] in the story of the Commune.

I thank the authors of these texts.

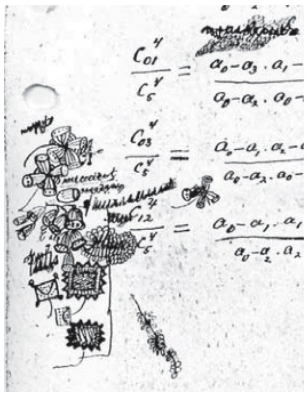
The coffers of the IRMA library do not contain any volume of the *Comptes rendus* prior to 1908; of course, in case of the impossibility of accessing the thick and dusty volumes, there is a digital version on the *Gallica* site:

http://gallica.bnf.fr

of the *Bibliothèque Nationale de France*.

Readers witnessing the upheaval of another investigator discovering other drafts, were able to read *Possession* the same year (1990) that Reinhard Bölling found and decrypted Sofya's burned letter.

The page of formulas, with its scrawls, some of which represent flowers, is from Sofya's hand. It belongs, as I have said, as does the accompanying photo, to the Mittag-Leffler Institute.



It is thanks to Reinhard Bölling that these two documents could be reproduced. In particular, he went to collect Sofya's flowers in the middle of a midsummer night. He should be thanked for that too!

I thank:

– Dimitri and Lidia Anosov for authorizing me to reproduce the photo of Andrei Bolibrukh,

– Marcel Bénabou, the definitively provisional and provisionally definitive secretary of the Oulipo, for giving me permission to use the photograph of Italo Calvino,

– Reinhard Bölling for the copy of the letter that he kindly sent me and the answers he provided to all my questions: without him chapter VI could not exist; I thank him too for the photo on page 63 and for his comments on (and for pointing out some errors in) a preliminary version of this book,

– Véronique Chauveau, for her commentary on a very preliminary (even embryonic) version of this text,

– Roger Cooke for all the details he sent me, his permission to reproduce some of these here, the documents he sent me, letters from Sofya to Gösta, from Weierstraß to Schwarz, poems by Sylvester and by Sofya and even the photo on page 181, for his enthusiasm and for his encouragement,

– René Cori for his careful reading, for the numerous kind remarks and the innumerable improvements that are due to him,

– Jacqueline Détraz, for her commentary on the entire first version of the text and for the photo on page 246,

– Jean-Denis Eiden for his reading and his suggestions,

– Catherine Goldstein for her availability, the criticisms that she provided, the information that she gave me, the references and the copies of letters of that "delightful letter writer", Hermite, that she communicated to me,

– Anna Helversen-Pasotto, for information and advice that she provided,

– Else Høyrup for having shared her knowledge about the Danish friends of Sofya Kovalevskaya and especially for the texts by Georg Brandes that she sought out and translated for me, as well as for her careful reading of a version of this book,

– Eero Hyry for her help regarding the Finnish student of Sofya Kovalevskaya,

– Ilia Itenberg for his help with Cyrillic fonts and with Russian names,

– Clio Lacroix for her interview [2006] of Jean-François Peyret,

– Christine Le Bœuf, one of the rare cordial contacts I have had with the publishing world, for her encouragement,
– Natalia Miasnikova for the first names and patronymics of Russian authors that she found for me,
– Ian Monk who deserves a special mention for his help with poetry,
– Mary-May Nielsen for translations of Brandes's texts,
– Ragni Piene for the articles she sent me and for the Norwegian and Finnish contacts,
– Agneta Rahikainen, producer at the *Svenska litteratursällskapet i Finland* (society of Swedish and Finnish literature) for information about Nanny Lagerborg,
– Bernard Randé for his reading and suggestions,
– Martin Raußen for sending me the article [Høyrup 2004] and for helping me contact its author,
– Peter Richter for the image of the Bremen top—I redrew the one that turns in his office, for the photo of the bust of Sofya Kovalevskaya in Bordeaux and for his friendly remarks and encouragement,
– Rebecca Rogers, an authentic historian, 19th-century expert and specialist in the education of young women, for her confidence and support,
– Norbert Schappacher for having lent me several books that I quote here, for his help with reading printed and manuscript German, especially that of the letter to Vollmar that appears here in chapter VIII, for his suggestions and for his enthusiasm and support,
– Robert Silhol for his cordial commentary and for the photo on page 244,
– Arild Stubhaug for Norwegian information,
– Cordula Tollmien for details she provided me on the letter from Sofya to Vollmar from which she quotes an abstract in her book [Tollmien 1995, p. 109] and of which there is a complete translation here in chapter VIII.

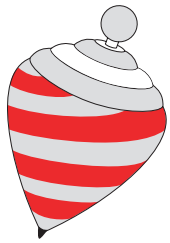
Finally, I thank Juliette Sabbah for her photo of the horse show jumping competition on page 173, for the photo of the Communards' wall that she took for me one morning in July 2006, and for her musical accompaniment, especially for the beautiful preludes by Scriabin that she played while I wrote this text—it even happened, in total innocence, that she interpreted the *Sonate Pathétique*.

The bust was made by Jan-Erik Björk and can be seen in the mathematical library at Bordeaux university. It was Sebastian Richter, Peter Richter's son, who took the photo that appears on page 222.

Many thanks to Rached Mneimné at Calvage & Mounet, who enthusiastically welcomed this book after I had despaired of finding a publisher, who patiently accepted my frustrations and who provided the counsel

"never lose your temper"

good advice that I have not always been able to follow.



Illustrations. The mathematical figures I drew by myself, with the exceptions of the top, which is due to Raymond Séroull, a top which has accompanied me since Raymond illustrated [Audin 1996], and the beautiful "landscape", in color, of the Weierstraß $\wp$-function that can be found in chapter V and which was realized by Olivier Elchinger. I thank both of them.

The photographs of mathematicians come from the Saint Andrews website http://www-groups.dcs.st-and.ac.uk/~history/.

Regarding LaTeX-nique, I once more thank Claude Sabbah who helped me achieve the rather intricate formatting that I wanted for this book.

IMAGES

Although mathematicians hardly know Sofya Kovalevskaya's work, they have all seen her portrait (generally the one reproduced here on page 168 or in the upper left of the above checkerboard) and they are all able to recognize her. And she is deemed

to be rather pretty. Other mathematicians are considered to be rather ugly. And it is discussed. The fact that one is pretty is a somewhat negative point for a scientific reputation because one cannot be simultaneously a woman, a real woman and a good scientist. The fact that someone else has been called ugly authorizes that person to be a good mathematician, by the same principle. Sofya Kovalevskaya is one of the victims of these prejudices. Emmy Noether is another. Readers who see the latter's portrait for the first time, for example the one placed at the right end of the fourth row (and shown again on page 242) will no doubt be astonished that mathematicians have learned to see an ugly and masculine woman. I have never seen anyone reproach Weierstraß for being ugly (which is disputable) nor that anyone described Mittag-Leffler as rather cute (which is however incontestable).

Here is an opinion of one of the protagonists of this book (Sofya Kovalevskaya) on the physique of another (the English writer George Eliot):

> I am absolutely unable to describe and explain what precisely constituted that peculiar, indisputable charm, to which everyone who came near to her had involuntarily to yield. [...]
>
> Turgenev, who is well known as a great admirer and connoisseur of female beauty, speaking once with me about George Eliot, expressed his opinion of her thus: "I know she is ugly, but when I am with her, I don't see it". He also said that George Eliot was the first woman to make him understand that one can fall madly in love with an unquestionably ugly woman. [Chapman & Gottlieb 1978]

It is for this reason that I have illustrated this book with numerous portraits of diverse and varied men and women, mathematicians, writers and revolutionaries, elderly and beautiful women, a very seductive young misogynist, a beautiful and serious young chemist with glasses, a mustached man whom age does not impede from remaining elegant, scientists, militants, writers, Sofya's friends and colleagues, young and ugly men, smiling and severe, Russians with beards, an Englishman with side whiskers, women of the 18^{th} century, woman in statues, gentrified or framed in postage stamps, the diverse humanity to which we belong, about which we ought to recall that we carry it wholly within ourselves and for which we would do well to feel solidarity.

⋆

The portrait of Sofya Kovalevskaya that appears on the cover of this book was produced by a Swedish photographer (who signed with a monogram inspired by the German artist Albrecht Dürer) in 1883 or 1884. A more precise dating can be found in the book by Reinhard Bölling [1991]—in homage to Sofya Kovalevskaya for the centenary of her death.

The one that begins our checkerboard and which can be found also on page 168 appears at the beginning of the issue of *Acta Mathematica* in which the biographical sketch [Mittag-Leffler 1892–93] is published and where Mittag-Leffler informs us that it is a

> photograph dating from the year 1887, a period in which Sofya Kovalevskaya was at the height of her career as mathematician, professor and scholar.

CHAPTER I

SOFYA'S CHRONOLOGY

In this chapter I say a few words about Sofya Kovalevskaya's origins (in the first section, which deals with her genealogy), then I make a chronological list of some of the highlights of her life.

Genealogy

Her mother is Елизавета Фёдоровна Шуберт (1820–1879), the great granddaughter of Johann Ernst Schubert, whose son Theodor (Фёдор Иванович), thus the grandfather of Elizaveta), emigrated to Russia and became an astronomer, later elected to the Academy of Sciences of Saint Petersburg. She is the daughter of Фёдор Фёдорович Шуберт, geodesist. (1)

Elizaveta Fyodorovna Schubert

Fyodor Ivanovich

Fyodor Fyodorovich Schubert

Her father is the artillery general Василий Васильевич Круковский, a descendant of the Hungarian king Matthias Corvinus. Starting in 1858, the family had rights to the name Корвин-Круковский. (2)

Vassili Vassilievich Krukovski

Korvin-Krukovski

They were married in 1843 and their first daughter Anna (Aniuta) was born in 1844.

1. Mentioned by Weierstraß [1861], to whom I will return.
2. The name Corvinus signifies "crow", so a crow appears in the family crests.

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_1,

Chronology

In order to place the events of Sofya Kovalevskaya's life in their historical context, I have drawn up a list of contemporary facts. This is a personal and thus very incomplete list. I have included the books that I love or that I find important or that relate to things that are important for Sofya, names of persons or events that seem to me useful for determining the chronology. For example, those that show the world becoming modern during Sofya's short life, the first automobile, the first airplane flight, bridges ... I clearly had to include some mathematical events and some of Sofya's friends (Dostoyevsky, George Eliot ...).

1850. Birth in Moscow on 15 January (3 January by the Julian Calendar) of Софья Васильевна Круковская (Sofya Vasilevna Krukovskaya).

Death of Balzac, of Gay-Lussac. Birth of Maupassant, of Stevenson. Karl Marx's *Class Struggles in France, 1848 to 1850.*

1851

End of the second republic in France. Death of Turner. Verdi's *Rigoletto*, Heine's *Romanzero*, Melville's *Moby Dick*, Bolzano's *Paradoxes of the Infinite.* Riemann's inaugural lecture.

1852

Death of Gogol. Harriet Beacher Stowe's *Uncle Tom's Cabin*, August Comte's *The Catechism of Positive Religion*, Karl Marx's *The Eighteenth Brumaire of Louis Napoleon.* Foucault invents the gyroscope.

1853

Crimean War begins, birth of Van Gogh, Hugo's *Les Châtiments*, Verdi's *Il trovatore* and *La traviata*, Liszt's *Sonata.*

Arthur Rimbaud, is born four years after Sofya, dies in the same year as she

1854

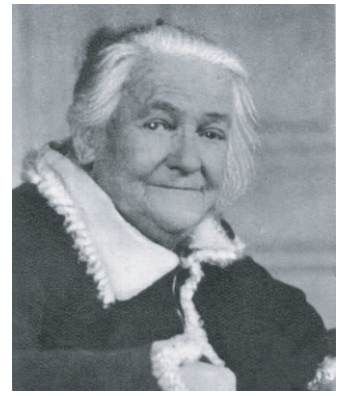
Birth of Arthur Rimbaud. Birth of Henri Poincaré. Work of Cayley on group theory.

1855. Birth of younger brother Фёдор (Fyodor).

Death of Nicolas I and accession of Alexander II. Death of Gauß and Nerval. Courbet's *Artist's Studio* is presented at the Exposition universelle. Менделеев's (Mendeleev's) thesis.

1856

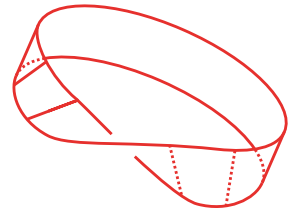
Death of Лобачевский (Lobachevsky), of Schumann and of Heine. Birth of Freud, Pétain and Picard. Hugo's *Contemplations*, Baudelaire's translation of Poe's tales (*Histoires Extraordinaires*). Discovery of Neanderthal Man. Weierstraß is appointed to a professorship at Berlin.



Clara Zetkin (1857–1933)

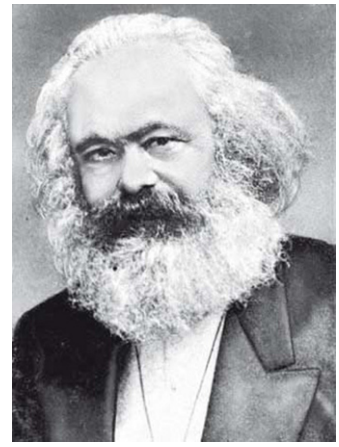
1857

Revolt of the Cipayes in India. Birth of Ляпунов (Lyapunov) and of Clara Zetkin. Death of Arthur Cayley, Cauchy and Auguste Comte. Flaubert's *Madame Bovary*, Baudelaire's *Les Fleurs du mal*. Riemann's *Theory of Abelian Functions*.



1858. Sofya's father enters retirement and realizes the ignorance of his daughters. English governess Margaret Smith and Polish tutor Joseph Malevich. Move to Palibino. Story of the wallpaper. Sofya's paternal uncle tells her about asymptotes and squaring the circle.

Birth of Selma Lagerlöf, of Puccini, of Peano, of Max Planck. Offenbach's *Orphée aux enfers*, Paul Féval's *Le Bossu*. Invention of crinoline and the Möbius band. Nordenskiöld's first polar expedition.



Karl Marx (1818–1883)

1859

Birth of Pierre Curie, of Conan Doyle, of Jaurès, of Alfred Dreyfus. Death of Dirichlet. Battle of Solferino. Goncharov's *Oblomov*, Darwin's *On the Origin of the Species*, Marx's *Contribution to the Critique of Political Economy*, Gounod's *Faust*, Liszt's *Years of Pilgrimage*. First oil wells.

	1860	Birth of Mahler, of Herzl, of Чехов (Chekhov), of Volterra, of Hugo Wolf. Death of Bolyai. George Eliot's *Mill on the Floss*, Тургенев's (Turgenev's) *Fathers and Sons*. Beginning of the American Civil War. First internal combustion engine.
	1861	Enfranchisement of the Russian serfs. Birth of Méliès. Dickens' *Great Expectations*, Dostoyevsky's *The Insulted and Humiliated*.
	1862	Birth of Debussy, of Hilbert. Hugo's *Les Misérables*, Dostoyevsky's *The House of the Dead*, Michelet's *Sorceress*.
	1863. Sofya is passionate about Poland and learns Polish with Malevich. The journal *Epoch* of Достоевский (Dostoyevsky) publishes two novellas by Aniuta.	Uprising in Poland, founding of the Red Cross. Birth of Edward Munch, of Painlevé. Gautier's *Captain Fracasse*, Renan's *The Life of Jesus*, Jules Verne's *Five Weeks in a Balloon*, Manet's *Olympia*.
	1864. The family passes the winter in Saint Petersburg. Sofya is in love with Dostoyevsky. Story of the *Pathetique sonata*.	Birth of Louis Lumière, Richard Strauß, Toulouse-Lautrec, Hermann Minkowski, of the First International. The right to strike in France. Offenbach's *Belle Hélène*, Jules Verne's *Voyage to the Center of the Earth*.
Here I ponder that Hadamard, who like Sofya was born in the days of stagecoaches, should live to learn of Yuri Gagarin's flight around our blue planet.	**1865.** A physicist neighbor, Н. Н. Тыртов (N. N. Tyrtov), is astonished that she has read a book on optics given her by her father and that she has invented a definition of the sine function.	Birth of Hadamard. Death of Hamilton, Lincoln. End of the American Civil War. Lewis Carroll's *Alice in Wonderland*, first performance of Wagner's *Tristan and Isolde*. Mendel's discovery of the laws of heredity.
	1866. Sofya and Aniuta travel with their mother to Germany and Switzerland.	Battle of Sadowa. Birth of Kandinsky. Death of Riemann. Dostoyevsky's *Crime and Punishment*, Verlaine's *Poèmes saturniens*.

1867. Sofya studies mathematics systematically with a new teacher, А. Н. Страннолюбский (A. N. Strannoliubski).

Death of Baudelaire, Ingres, Poncelet. Birth of Toscanini, Maria Skłodowska (Marie Curie). Marx's *Das Kapital*, Ibsen's *Peer Gynt*, Verdi's *Don Carlos*.

1868. "White" marriage on 27 September with Владимир Онуфриевич Ковалевский (Vladimir Onufrievich Kowalevski), born in 1842, a political radical interested in biology. Sofya meets Чебышёв (Chebyshev) in Saint Petersburg.

Death of Rossini. Birth of Gorki. Dostoyevsky's *Idiot*. Discovery of the first Cro-Magnon specimen.

1869. April. Departure with Aniuta and Vladimir for Vienna, then for Heidelberg. Study with Du Bois-Reymond, Königsberger. Bunsen story.
October. Visit to London, where Sofya and Vladimir meet Darwin, George Eliot, Huxley. Aniuta leaves Heidelberg for Paris.

Death of Lamartine. Birth of Gandhi, Élie Cartan, Gide, Matisse. Brahms' *German Requiem*, Tolstoy's *War and Peace*, Lautréamont *Maldoror*, Flaubert's *Sentimental Education*, Jules Verne's *Twenty Thousand Leagues under the Sea*. Mendeleev's periodic table. Suez Canal opens.

Oh severe mathematics, I have not forgotten you, ever since your living lessons, sweeter than honey, filtered into my heart, like a refreshing wave.

Lautréamont [1869].

1870. End of the summer. Berlin. Sofya begins study with Weierstraß. Hat story.

Franco-Prussian War, fall of the Second Empire. Death of Dumas, Dickens. Birth of Rosa Luxembourg, of Lenin. First performance of Wagner's *Die Walküre*, Hugo's *Les Châtiments*, Jordan's *Treatise on Substitutions and Algebraic Equations*.

Rosa Luxemburg (1870–1919)

1871. Sofya and Vladimir rejoin Aniuta during Paris Commune, leaving Paris a few days before the end of the massacres, returning there in June with Sofya's parents.

Paris Commune. Proclamation of IInd Reich and the IIIrd Republic. Death of Delescluze (and of many others). Zola's *The Fortune of the Rougons*, Rimbaud's *The Drunken Boat*.

Alexandra Kollontaï
(1872–1952)

There are many recordings of Toscanini and Chaliapin, both just a little younger than Sofya, which are thus essentially modern. As for Schönberg, he is often still spoken of as if he were a "contemporary". Because of this fact, their presence in this chronology testifies to our proximity to Sofya.

1872. Vladimir defends his thesis at Jena.
Sofya discloses the truth about her marriage to Weierstraß, who decides to have her do a thesis.

Oscar II becomes king of Sweden. Death of Delaunay. Birth of Léon Blum, Alexandra Kollontai, Paul Langevin, Scriabin, Blériot. Monet's *Impression, Sunrise*, Dostoyevsky's *Demons*, George Eliot's *Middlemarch*, Felix Klein's *Erlanger Program*.

1873. Sofya rejoins Aniuta in Zürich in spring. Summer trip to Switzerland with Vladimir.

Birth of Chaliapin, of Rachmaninov. Rimbaud's *Une saison en enfer*, Jules Verne's *Around the World in Eighty Days*. Hermite proves the transcendence of e.

1874. Göttingen Thesis *in absentia* in August.
Return to Russia. Beginning of a period devoid of mathematics.

Birth of Schönberg. Mussorgsky's *Boris Godunov*, Verdi's *Requiem*. Cantor's power of the continuum.

1875. Death of Sofya's father.

Birth of Lebesgue, of Thomas Mann, of Ravel. First performance of Bizet's *Carmen*.

Battle of Little Bighorn. Death of George Sand. Lewis Carroll's *The Hunting of the Snark*, Mark Twain's *Tom Sawyer*, Jules Verne's *Michel Strogoff*.

1876. First meeting with Mittag-Leffler.

1877. Sofya continues to write scientific and literary articles for journals.

Death of Courbet, of Thiers. Tolstoy's *Anna Karenina*, Zola's *L'Assommoir*.

1878. She begins again to write to Weierstraß. October. Birth of her daughter Софья Владимировна (Sofya Vladimirovna), called Fufa.

Death of Claude Bernard. Engels' *Anti-Dühring*, Poincaré's thesis.

1879. February. Death of her mother. Sofya resumes doing mathematics.

Death of Maxwell. Birth of Einstein, Paul Klee, Stalin, Trotsky. Brahms' Violin Concerto. Tchaikovsky's *Eugene Onegin*, Ibsen's *A Doll's House*, Picard's theorem. Altamira cave found.

1880. Sofya participates in a congress at Saint Petersburg where she gives a lecture. Mittag-Leffler, who is present, decides to help her obtain a position.
Vladimir takes up his publishing business again. Move to Moscow. Two months in Berlin.

Death of George Eliot, of Flaubert. Birth of Apollinaire. Dostoyevsky's *Brothers Karamazov*, Zola's *Nana*. Henry James' *Portrait of a Lady*.

1881. In March, Sofya leaves Vladimir and departs for Berlin with her daughter. Visit to Weierstraß at Marienbad during the summer. Sofya is elected to membership in the Moscow Mathematical Society.
She arrives in Paris at the end of the year.

Assassination of Alexander II. Death of Dostoyevsky, of Mussorgsky. Birth of Bartók, of Picasso. Renoir's *Luncheon of the Boating Party*, Flaubert's *Bouvard et Pécuchet*. Vaccination against anthrax by Pasteur.

1882. Paris. Sofya sends her daughter back to Russia. Connections with revolutionaries. She makes the acquaintance of the German social-democrat Vollmar and the Polish revolutionary Maria Jankowska. She becomes a member of the French mathematical society.

Death of Darwin, of Garibaldi, of Liouville. Birth of Joyce, of Virginia Woolf, of Emmy Noether. Wagner's *Parsifal*. Discovery of Bacillus bacteria by Koch and of the malaria parasite. Lindemann's proof of the transcendence of π.

The real numbers

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

and

$$\pi = 2 \sum_{n=0}^{\infty} \frac{2^n (n!)^2}{(2n+1)!}$$

are "transcendental", i.e. neither the one (e, Hermite) nor the other (π, Lindemann) is a solution of an algebraic equation $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$ whose coefficients $a_0, \ldots, a_n$ are integers.

1883. Bankruptcy and, end of April, suicide of Vladimir. At 33, Sofya is mature and respectable. She participates in a congress at Odessa. She is recruited as *Privatdozent* in Stockholm. Story of the burned letter.

Death of Marx. Birth of Kafka, of Mussolini. Hugo's *La Légende des siècles*, Stevenson's *Treasure Island*. Completion of the Brooklyn Bridge.

1884. Sofya gives her first course at Stockholm in German. She becomes an editor of *Acta Mathematica*. She gets a five-year position.

Division of Africa at the Berlin Conference. The Naquet (reinstatement of divorce) and Waldeck-Rousseau (legalization of unions) laws in France. The first four-wheel, four-stroke automobile, Eiffel's Garabit viaduct, Hilbert's thesis.

1885. Sofya visits Aniuta, who is suffering from cancer, in Russia.

Death of Victor Hugo, of Jules Vallès. Birth of Hermann Weyl. Van Gogh's *Potato Eaters*, Zola's *Germinal*. First anti-rabies vaccination by Pasteur. Weierstraß's polynomial approximation theorem.

1886. Sofya begins to get results on the solid. She writes an article on her reminiscences about George Eliot. Travel in Scandinavia with Anne Charlotte Leffler. Visit to Aniuta. Sophie brings her daughter Fufa to Stockholm.

Death of Liszt. Birth of Paul Lévy. Franck's Sonata for violin and piano.

I have permitted myself to dream of Sofya in Paris in 1888, at the solemn meeting of the Academy of sciences ...

We know from a letter quoted in [Leffler 1898] that she had gone to visit the Pasteur Institute, which had just been inaugurated, that she attended a hypnosis session by Charcot at Salpêtrière Hospital (see [Kochina 1985, p. 165]), as she reported in *La Gazette russe*. But did she go—could she not have gone—to see the site of the Eiffel Tower, now well along?

1887. Aniuta and her husband Victor Jaclard are expelled from Russia. Death of Aniuta in Paris following an operation. Sofya writes *The Struggle for Happiness* for theatre in collaboration with Anne Charlotte Leffler.

Birth of Ramanujan, of Schrödinger. Victor Hugo's *Choses vues*. Verdi's *Otello*.

1888. Contact with Максим Ковалевский (Maxim Kovalevski). Prix Bordin of the Académie des Sciences for her work on the solid.

Birth of Mordell. Van Gogh's *The Night Café*, Strindberg's *Miss Julie*, Lie's Theory of transformation groups. Inauguration of the Pasteur Institute.

1889. March to September in Paris. Permanent position in Stockholm. Elected corresponding member of the Russian Academy of Sciences. Prize of the Stockholm Academy. Publication of *Memories of Childhood* in Stockholm.	Birth of Hitler. Tolstoy's *Kreuzer Sonata*, Van Gogh's *Starry Night*, Jules Verne's *Topsy Turvy*. Birth of the IInd International in Paris. Eiffel Tower.	Remaining in Paris, Sofya attended the pre-opening of the Eiffel Tower, 4 May 1889 and wrote a newspaper story telling of her visit to l'Exposition Universelle.
1890. Sofya writes *Nihilist Girl*. End of the year, holidays with Maxim Kowalevski in Genova.	Death of Van Gogh. Ibsen's *Hedda Gabler*, Satie's *Gnossiennes*. Flight of Clément Ader.	So many questions remain: did she go to the Opera? Did she meet Jules Verne? I even have imagined her being moved by listening to the "little phrase" of Franck's sonata. But that doesn't fit, so I'll just stop ...
1891. Appearance of *Memories of Childhood* in Russian. Sofya's death from pneumonia.	Death of Melville, Rimbaud. Oscar Wilde's *Portrait of Dorian Gray*.	

CHAPTER II

SOFYA'S NAMES

We can discuss ad infinitum how to write this mathematician's name, and I believe that a name is important. So I dedicated this brief chapter to Sofya's names. I take it on principle that the person involved should be the one who chooses ... In present-day France women have a bit more choice than do men, the name of the father, of the husband [1] ... In the case of Sofya—very definitely a case—this choice has been multiplied by the passage from Cyrillic to Latin.

The name of the father. This is what she is called when she is born: Софья (Sofya, her first name) Васильевна (Vassilyevna, her patrinomic, which means simply that the first name

According to [Kochina 1985, p. 9], her father's name was originally Круковскои (Krukovskoy). This changes the Russian pronounciation considerably: Круковскои has the accent on the last syllable, Круковский has it on the second.

1. However, we note that one of the "heroes" of this book, the Swedish mathematician Gösta Mittag-Leffler, transformed his name by adding that of his mother.

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_2,

of her father is Vassili) Круковская (Krukovskaya, her surname, which is to say the feminine form of the surname of her father, Круковский, Krukovskiy). She relates, in her memories of childhood [Kovalevskaya 1898] that her very first recollection is precisely related to her name, to the name of her father.

> Upon leaving the church we see a friend of my nanny approach, a deacon or a subdeacon, judging from his long cassock: he offers us a godsend:
>
> "Eat it for your health, he says to her. And now tell me your name, my fine young lady." I remain silent and look at him wide-eyed.
>
> "What a shame not to know your name, miss!", he continues in order to tease me.
>
> "Answer, little mother", whispers my nanny; say: "my name is Sonya and my father is general Krukovskiy."
>
> I attempt to repeat these words, awkwardly I'm sure, for my nanny and her friend begin to laugh.
>
> My nanny's friend accompanies us to the house. I run and skip the whole way and force myself to repeat my nanny's words, arranging them in my own way. Evidently; the process is still new to me and I try to engrave the words in my memory. Approaching the house, the deacon accompanies me to the door.
>
> "Do you see this hook, 'kryuk' in Russian, on the door, young lady?" he says to me. 'When you forget your father's name, you can say to yourself: there's a 'kryuk' on the Krukovski house, and it will immediately come back to you."
>
> Well, I regret to say, the deacon's wicked pun was imprinted on my memory and became an era in my life [...]

The name of the king Corvin. Starting in 1858, Sofya's father had appended to his name that of his great ancestor, the king Corvin of Hungary, one of whose daughters had married a Krukovski, a Polish knight. The family name thus became Корвин-Круковский (Korvin-Krukovski).

Владимир Онуфриевич Ковалевский

The name of Vladimir. In 1868, Sofya married Vladimir Onufrievich Kovalevski. I don't know whether Russian women who married prior to the 19th century had the possibility of continuing to use their birth name. In Sofya's case the sole purpose of the marriage was to provide her with the status of a married woman, so that the question did not even arise. She thus called herself Sofya Vasilyevna Kovalevskaya: Софья (her

first name), Васильевна (her patrinomic), Ковалевская (the feminine form of the name of her husband).

In Latin characters. When she arrived in Germany, she decided to write her name "Kowalevski". She had to write it with Latin letters and make the best choice possible, the initial "в" of her name being pronounced like "w" in German (the "v" of French or English), whereas the second "в" sounds like the "v" of German (almost an "f" in French or English).

Kowalevski. Софья Ковалевская thus signed papers "Sophie Kowalevski", perhaps because she had drafted the papers in German or French. This is also the name she had engraved on her business cards (in Sweden) as professor in Stockholm (one of these is reproduced in [Koblitz 1984, p. 23]. So it is the way she thought she should spell her name, at least when writing in French or German (or in Swedish). The disadvantage, one will say, is that the Russian version is feminine. Feminine or masculine, it is always the name of a man. She chose to be married and bear the name of her husband, she chose to write it so, and that is enough for me! I respect Sofya's choice.

Kowalewska. None of the various versions of the spelling "Kowalewska", which is perhaps contaminated by Polish names such as Skłodowska, makes any sense nor has any justification.

The use of one of these spellings—which we will see appearing here and there, sometimes by the pen of a respected mathematician—merely shows ignorance of both the Russian language and Sofya's work, without proving any knowledge of Maria Skłodowska's achievements.

Sofya. The discussion of the surname can be extended to a discussion of the first name, Sophie, Sofie, Sofya, Sonia, and to their spellings (I admit to a small preference for Sonja, that is how Weierstraß spelled it). Roger Cooke comments on the usage of the diminutive Sonya in his book [1984] thus: for a Russian, "Sonya Kovalevskaya" sounds like "Fedya Dostoyevsky" or "Osya Stalin''—but may we imagine a book entitled "Winny Churchill", to draw an analogy in English?

I think of her by the name Sofya. I sometimes call Sofya Kovalevsky by her first name, to which we should not attribute a lack of respect. I of course have asked myself whether there is not a bit of sexism here, a question that is not at all irrelevant: in the book of Bell [1937] there are some forty mathematicians and but one woman (but one woman appearing in a chapter title, see our chapter X for particulars on the women in the book) and also but a single mathematician referred to by their first name, "Sonja" (except for the Bernoullis for distinguishing one from the other). More recently, in the book [James 2002],

sixty mathematicians, from Euler to von Neumann, are all designated only by their last names, except for Sophie Germain and Emmy Noether, designated by their first and last names, and our Sophie, whom the author manages to designate by her first name alone (and she is unique to this situation)—and guess how many women mathematicians appear in James's book!

As for the question of sexism, I reply that, no, I would not perhaps have the idea of calling Hilbert "our David", I would doubtless never write "our Paul" for Painlevé, but I would certainly write "our Évariste" if I were to write more than a few lines on Galois and you will see that I will call Mittag-Leffler "Gösta". On the other hand, it is more than certain that I would never write "Marie" for Marie Curie.

One will find in this book:

Sophie Kowalevski,
Mme Kowalewsky,
Kowalevskaya,
Kowalewski,
Kowalewskaja,
M^me de Kowalevski,
Kowaleski,
Sonja Kowalevsky,
Sonia Kowalewska,
Sonja Kowalewska,
Mme. Kovilevsky,
Sonya Kovalevski,
Sonja Kovalevsky,
Sophie Kovalevsky,
Mme. Kovalevskaia,
Frau v. Kovalevsky,
Kovalevskaia,
Sofia Kovalevskaia,
Kovalevskaïa,
Sofia Kovaleskaia,
Sofya Kovalevskaya,
Sonia Kovalevskaïa,
Sofja Kowalewskaja,
Kowalevskaya,
Sophie von Kowalevsky,
Sonja v. Kowalewsky
and perhaps still others that I have forgotten (see also the last but one marginal note on page 232).

From Sofya's name to those of others. In brief, I write Sophie Kowalevski or Sofya Kovalevskaya, but I respect the spelling of the writers I quote, it is a sort of game, they're usually consistent within an article but not always from one article to another. For the sake of readability I write Kowalevski for the name of Vladimir, Alexander and Maxim, the male Kowalevskis in this book. Likewise, I always write Gauß, Weierstraß and Saint Petersburg when it is me who is speaking.

I also write Russian names in such a way that they are pronounceable in English, as is customary in the great translations of Russian novels, e.g. I write Chebyshev for the name of the Russian mathematician Чебышёв, Aniuta for the diminutive by which Sofya refers to her sister Anna.

I write Sofya because the characters in Russian novels I have read are called Vanya, Mitya, Rodya and even Sonya, and I likewise write Kovalevskaya. I realize that all this is a bit incoherent and it is for this reason that I include the actual names, written in Russian, for the majority of the Russian persona in this book.

I don't understand the logic of retranscribing as "Shubert" the German name transcribed into Russian as "Шуберт", so I haven't done that. I have also preserved the usual English spellings of the names of well-known people, e.g. the writer Фёдор Достоевский, who is known here in the usual English form Fyodor Dostoyevsky. What can I do?

CHAPTER III

STORIES

Sofya's life, very well documented, is full of stories, of good stories, of sinister stories, of true stories and romances, of sad stories. There is an abundance of sources of all sorts, including the autobiographical: from *A Russian Childhood* [Kovalevskaya 1898] to a posthumously published autobiographical sketch, an English translation of which is found in the book [Kovalevskaya 1978], through an autobiography in Latin dating from the time of her thesis and reproduced in [Mittag-Leffler 1923, p. 48]. But, as was quite rightly noted by Roger Cooke [2002a, p. 14] in relating in detail a somewhat melodramatic story where all the elements are known without being really interesting, the sources are capricious, surviving by chance (we will see that the documents that would have been most useful for the history of mathematics and for Sofya's life have been destroyed).

The stories collected and related in this chapter in no way comprise a biography of Sofya Kovalevskaya, for which I refer once again to [Kochina 1985 ; Koblitz 1993]. One can refer to the chronology of chapter I as necessary.

Numerous people who knew her have composed their recollections of Sofya Kovalevskaya: these include
her brother Fyodor Vasilievitch Krukovski,
her cousin Sofya Adelung,
her friends Elizaveta Litvinova, Anne Charlotte Leffler, Julia Lermontova, Maria Jankowska,
her friends Gösta Mittag-Leffler, Georg von Vollmar,
her teachers Malevich and Strannoliubski.
Many of these recollections are published in Russian. For more details, see the bibliography of [Kochina 1985].

$$\int_\Gamma P\,dx + Q\,dy = \iint_\Sigma \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$\iint_\Sigma F \cdot dS = \iiint_V \operatorname{div}(F)\,dx\,dy\,dz$$

$$\iint_\Sigma F \wedge dS = -\iiint_V \operatorname{rot}(F)\,dx\,dy\,dz$$

The wallpaper story

Among these stories, the one that I have always preferred is the most classical, the best known, the one that she herself tells thus [Kovalevskaya 1898, p. 73]:

> When we transferred our abode to the country the whole house had to be done over afresh, and all the rooms were repapered. But as the rooms were many, there was not paper enough for one of the rooms belonging to us children; it was a great undertaking to order more from St.

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_3,

> Petersburg, and to order for a single room was decidedly not worth the while. They kept waiting for an opportunity, and in the interim this ill-treated room stood for many years with nothing but common paper on its walls. But by a happy accident the paper used for this first covering consisted of sheets of Ostrogradsky's lithographed lectures on the differential and integral calculus, bought by my father in his youth. These sheets, spotted over with strange, incomprehensible formulae, soon attracted my attention. I remember how, in my childhood, I passed whole hours before that mysterious wall, trying to decipher even a single phrase, and to discover the order in which the sheets ought to follow each other. By dint of prolonged and daily scrutiny, the external aspect of many among these formulae was fairly engraved on my memory, and even the text left a deep trace on my brain, although at the moment of reading it was incomprehensible to me.

Palibino was a country estate not far from the city of Velikiye Luki, in what is now the Pskov region of Russia, roughly 300 km south of Saint Petersburg and 430 km west of Moscow. Sofya's house was destroyed in WWII, then partially reconstructed and turned into a museum.

Regarding this passage, Beatrice Stillman notes that Sofya was, at that time, eleven [Kovalevskaya 1978].

That's a wonderful story. The differential and integral calculus is a subdiscipline of mathematics in which we write long and elegant formulas, each of which we can admire for its beauty. Here are some formulas not unlike those on the walls of Sofya's room in Palibino. Ostrogradsky has left his name on a formula that is taught in physics courses:

$$\iint_{\Sigma} F \cdot dS = \iiint_{V} \operatorname{div}(F)\, dx\, dy\, dz.$$

This initial formula, along with the formula of the rotational

$$\iint_{\Sigma} F \wedge dS = -\iiint_{V} \operatorname{rot}(F)\, dx\, dy\, dz$$

and the Green–Riemann formula

$$\int_{\Gamma} P\, dx + Q\, dy = \iint_{\Sigma} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\, dy$$

are all versions of the Stokes formula that is written nowadays as

$$\int_{V} d\omega = \int_{\partial V} \omega,$$

which is more compact but undoubtedly less pretty.

I do not know how the course in question was lithographed, perhaps starting with an attractive manuscript, as would be the case a few years later with the *Leçons de Stockholm* of Paul Painlevé [1897], to which I will return. For those who are interested in the way a mathematician's brain works, she continues:

> When, many years later, as a girl of fifteen, I took my first lesson in differential calculus from the famous teacher in mathematics in Petersburg, Alexander Nikolaevitch Strannoliubsky, he was astonished at the quickness with which I grasped and assimilated the conceptions of the terms and derivatives, "just as if I had known them before". I remember that this was precisely the way in which he expressed himself, and in truth the fact was that at the moment when he began to explain to me these conceptions, I immediately and vividly remembered that all this had stood on the pages of Ostrogradsky, so memorable to me, and the concept of limit seemed to have been familiar to me for a long time.

... which indicates that there were lots of other things on the wallpaper besides Ostrogradsky's formula and its variations.

Sofya was perhaps about eleven years old when she first contemplated the wallpaper. She may actually have been as old as seventeen when she took lessons from Strannoliubski, according to Beatrice Stillman [Kovalevskaya 1978].

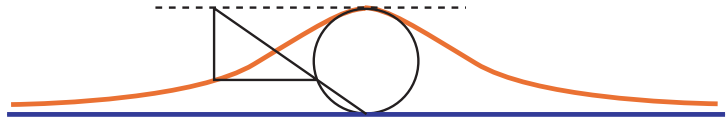
⋆

The asymptotes

I suppose that all humans remember how they began to grasp the notion of infinity, for example by contemplating one of those abysmal drawings, like the famous cheese box on which is represented a cow that wears as earrings boxes of cheese on which is represented a cow that wears as earrings boxes of cheese on which is represented ...

Those who become mathematicians have memories that are sometimes poetized by other appearances of infinity. Hadamard [1945] explains what he saw when he thought of the set of prime numbers (at a moment when he wanted to prove that this set is infinite): "I see a confused mass". More dazzling still is the encounter with the infinitely small, which is perhaps a rarer esthetic emotion. One of the first appearances of this infinitely small in the life of an apprentice mathematician is that which bears the name of asymptote to a curve: the curve approaches the asymptote indefinitely, which is like an ideal that we always seek and never attain, as here the curve approaches the horizontal line.

The notion of infinitely small is inseparable from the notion of infinitely large: in order for the curve to be infinitely close to its asymptote, we need to go infinitely far.



The witch of Agnesi, a curve (red) with an asymptote (blue). The black lines indicate the construction. We will find other curves with asymptotes here and there in this book, on pages 92 and 119 in particular.

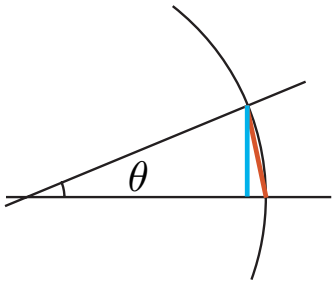
Here is Sofya's experience that she relates in her childhood reminiscences.

> Although he had never studied mathematics, he cherished the most profound respect for that science. He had gathered a certain amount of mathematical knowledge from various books, and loved to philosophize about them, on which occasions it frequently happened that he thought aloud in my presence. I heard from him for the first time, for example, about the quadrature of the circle, about the asymptotes which the curve always approaches without ever attaining them, and about many other things of the same sort—the sense of which I could not of course

> understand as yet; but which acted on my inspiration, imbuing me with a reverence for mathematics, as for a very lofty and mysterious science, which opened out to those who consecrated themselves to it a new and wonderful world not to be attained by simple mortals.

This uncle, whose reverence for a science he did not comprehend makes us think of a Jules Verne character, was the brother of Sofya's father, Piotr Vasilievitch Krukovski.

⋆



If the radius of the circle is 1, the length of the blue line is the sine of the angle θ. The length of the chord (in red) is Sofya's "small sine".

Story of the small sine

At Palibino they were part of high society. Tyrtov, one of the neighbors, was a physics professor and, parenthetically, was involved with courses for women in Saint Petersburg. One day, for one reason or another but undoubtedly for Sofya herself, he brought a physics book he had written to the Korvin-Krukovski house. Is it really necessary to say that Sofya took it and read it? In physics, however elementary it may be, we need to use trigonometry. Her good tutor Malevich didn't know a whole lot, so Sofya had to figure it out for herself. After all, a few years earlier she had managed to learn how to read on her own. But what could this strange, enigmatic "sine" be anyway, this $\sin\theta$? Could it be the chord subtended by an angle on a circle of radius 1? And so Sofya was able to continue with her reading. During his next visit, very proud of herself, the young trigonometrist began to speak about the book to its author.

"But you couldn't have understood, you don't know trigonometry, you don't even know what a sine is", he must have responded.

"But of course I understood, look, the sine is this", and she shows him the chord.

"But this little girl has just reinvented trigonometry! It's unbelievable! It's a new Pascal! General, you must be careful not to ruin the chances of your daughter."

And he convinced Sofya's father that he should give her real mathematical courses, given by a real mathematician. This would be Strannoliubski, who, along with his other professional and human attributes, was a "man of the 1860s", a nihilist!

If I like this story a lot, it is because it is not true that the sine and the chord are the same ... but it is almost true, and true enough to allow her to comprehend what was going on in Tyrtov's book: the length of the chord subtended by an angle θ

on a circle of radius 1 is not $\sin\theta$, but $2\sin(\theta/2)$ and we have

$$\begin{aligned}\sin\theta &= 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\\ &= 2\sin\left(\frac{\theta}{2}\right)(1+o(\theta^2)).\end{aligned}$$

It does not matter much if you do not understand this formula, just as it did not matter much that Sofya took the chord for the sine: the moral is that a small sine is about the same thing as a small chord.

According to Roger Cooke [1984, p. 10], the early trigonometric tables, those of Ptolemy for example, gave the chords rather than the sines: Sofya invented the sine as it had appeared historically.

A final remark, for mathematicians, who know that when θ is small $\sin\theta$ is about the same as θ. In the figure, θ (measured in radians) is the length of the arc of the circle subtended by Sofya's chord. It is not very natural if one does not know radians (for which it is close to being the definition), which was undoubtedly the case with Sofya.

$\star$

Ludwig van Beethoven
(1770–1827)

The pathetic story of the sonata

Sofya's sister Aniuta had written two novellas by the time she was scarcely twenty years old—not devoid of talent, says Pierre Pascal [1970, p. 138]—*The Dream* and *Mikhail*, and she sent them to Dostoyevsky, who published them in his journal and even paid her for them, which first provoked an outburst from the young ladies' father, who did not like the idea of a woman being a writer: "you start by selling your novellas, then it is yourself that you will be selling ..."). In truth, this was not a terribly severe father, everything was rather quickly arranged once he had read *The Dream*, and the family subsequently maintained friendly relations with the famous writer. The mother and her two daughters, who spent the winter in Saint Petersburg, received him frequently. Sophie was thirteen years old and she evidentally fell in love with the great man, who was forty-two. Here is what she relates [Kovalevskaya 1898]:

> One day he happened to say that of all musical compositions he loved most of all Beethoven's "Sonata Pathetique", and that this sonata always overwhelmed him with a whole world of forgotten sensations. Although the sonata was considerably more difficult than any of the pieces which I had hitherto played, I determined to learn it at any cost; and really, by expending a vast amount of labor on it, I got to the point where I could play it fairly well.

How well the young daughters of wealthy Russian families were brought up! Sofya spoke very well, in addition to Russian, French and English, and she played piano well enough in order to be able to execute (which is perhaps just the right word) what is undoubtedly Beethoven's most difficult sonata but which is by no means a work for beginners.

If it had not been for her nurse, Sofya's mother tongue would have been French, says Pelageya Kochina [1985, p. 18].

> All that I now waited for was a convenient opportunity when I might rejoice Dostoyevsky. This opportunity soon presented itself.
>
> Only five or six days remained before our departure. Mama and my aunts were invited to a grand dinner at the

> Swedish Embassy, the ambassador being an old friend of our family. Aniuta, who had already tired of balls and dinners, excused herself on the plea of a headache. We remained alone in the house. That evening Dostoyevsky came to us.
>
> Our approaching departure, the consciousness that none of the elders was at home, and that such an evening would not soon come again, put us in an agreeably excited frame of mind. Feodor Mikhailovitch, also, was in a rather strange, nervous mood—not irritable, as had often been the case with him of late, but, on the contrary, gentle, amiable.
>
> This was a capital moment to play his favourite sonata for him; I rejoiced in advance at the thought of the pleasure which it would cause him.
>
> I began to play. The difficulty of the piece, the necessity of looking well at every note, the fear of making mistakes, soon absorbed all my attention to such a degree that I was entirely taken out of my present surroundings, and did not observe what was going on around me. I finished with a self-satisfied consciousness that I had played well. I felt an agreeable weariness in my hands. Still quite under the influence of the music, and of that pleasant emotion which always lays hold of one after every bit of well-executed work, I awaited the well-merited applause. But silence reigned around me.
>
> I glanced around: there was no one in the room.
>
> My heart sank. Still suspecting nothing definite, but with a dull presentiment of something evil, I entered the adjoining room. That was empty also! At last, on raising the porière which masked the door into the small, corner drawing room, I beheld Aniuta and Feodor Mikhailovitch there. But heavens!? what did I behold?
>
> I perceived Dostoyevsky and Aniuta. And what I saw, my God!

Once more the quotation is taken from [Leffler 1898], translated from Swedish by Isabel Hapgood, who was born in the same year as Sofya, but in Boston. She knew Tolstoy and tranlated his books (and those of Turgenev) into English.

Well, he was simply in the process of disclosing his love for Aniuta. The sonata had been played uselessly, pathetically, it had not even been heard. Moreover, Aniuta had refused Dostoyevsky. But how could she have managed to do so?

To conclude this story, two remarks.

Aniuta. Sofya loved her older sister very much, as can be realized by reading her reminiscences of childhood [Kovalevskaya 1898], written immediately after Aniuta's death in 1887.

For those who know the biography of Dostoyevsky. It was in 1864 (year of the death of his first wife Maria Dmietrievna) that he received and published Aniuta's novellas. The scene of the sonata took place in 1865. The entire story takes place between the years 1862 (the year in which Dostoyevsky encountered Paulina Suslova, with whom he fell madly in love), 1863 (the year in which, on the way to Paris where he followed her, he stopped in Wiesbaden and forgot her while playing roulette, the year also when she left him) and 1866 (the year in which *Crime and Punishment* won great success and in which its author encountered another Anna, Anna Grigorievna Snitkina, who would become his wife and remain so for the rest of his life).

In verifying dates and places, I found myself reading the preface by Dominique Fernandez to the French *Folio* edition of *Le Joueur* (*The Gambler*) where he precisely quotes a letter from the writer to Mme Corvine-Krukovskaïa [Dostoïevski 2002, p. 9] whom we recognize as Sofya's mother, a letter from 1886 where he announces the writing of the novel.

Anna Grigorievna was a stenographer, the stenographer to whom he dictated *The Gambler*, a novel whose female protagonist is called Paulina, like the proud Paulina Suslova, a student twenty years younger than the writer, whose sister Nadezhda is famous for having been the first woman to obtain a medical degree in a European university, which was in 1867 and in Zürich.

Nadezhda was a model for Sofya and her friends. A very small world, the world of the Russian nihilists in the 1860s.

Sofya would remain friends with Dostoyevsky, whom she would see again and to whom she would write after she had returned to Saint Petersburg in the years after 1876.

⋆

Sofya aged around twenty

Story of the white marriage

A current practice among the young nihilists of the period was a "fictitious" marriage, that was called a "white" marriage. But really there was nothing fictitious about the fact of the marriage, it was legal and religious, undoubtedly in white. On her part, Sofya even bore a crown of myrtle and orange blossoms, and no one had ever seen such a pretty bride—as aunts say on such occasions—and as an aunt wrote who believed she read love's joy in Sofya's "radiant expression" (letter quoted in [Mittag-Leffler 1923, p. 134]). She undoubtedly should have noticed another kind of happiness. The sole purpose of the marriage was, in Sofya's eyes

> to open wide the doors of the science of numbers and space

as wrote, much later, in his metaphorical style, her friend Gösta Mittag-Leffler (in [Mittag-Leffler 1923]).

The function of the husband is to permit the wife to travel freely and to study abroad, for there were no possibilities for study in Russia. And that is all! The first thing to do in contracting a white marriage is to find a volunteer. The young man must be in revolt against society, but at the same time capable of "respecting" the young woman. Parenthetically, if the interest in the marriage for the young woman is clear, what it does for the young man is less evident. We remark that a married couple could travel with female friends or relatives of the wife. As one man suffices for chaperoning two other young ladies, a husband suffices for three—happily, for there were undoubtedly more young ladies than young men among the volunteers.

The three young ladies that interest us here, Sofya, her sister Aniuta and their friend Zhanna, cast their sights on a young professor whose loyalty to the cause foretold his acceptance. But he scarcely knew them and was at his worktable when they presented themselves. There was a short moment of embarrassed silence, after which Aniuta posed the question, the young man said quite simply no, they quite simply left and went to

find another volunteer. The next was a student of good family, but not very handsome—but what matter?—who wanted to continue his studies in Germany—what luck!—and he accepted. His name was Vladimir Onufrievich Kovalevski. He was eight years older than Sofya, had already traveled widely, published some books, had lost quite a bit of money, had acquired the reputation of being a revolutionary (and the surveillance that went with it) by publishing a book by Herzen, *Who is to blame?*, that was quickly banned and burned, and had participated in Garibaldi's 1866 campaign. When he had gotten to know Sofya, he had stimulated her intellectually and encouraged her to devote herself to science, states Pelageya Kochina [1985, p. 44]. He had already gotten to know Darwin, for whom he would one day be, in collaboration with Sofya, the translator into Russian, and he would become a distinguished "evolutionary paleontologist". Our young ladies thought of marrying Aniuta, but Vladimir would accept only under the condition that it was Sofya whom he married, for she seemed to be truly interested in science. She was only eighteen and her parents did not acceed to it, but she forced their hand. Quite simply, she went alone to Vladimir's house, thereby compromising her reputation, and then informed her father, who could do little but accept the marriage, a little scandal reminiscent of a scene from a Dostoyevsky novel.

However, this is how Sofya married, and could go to study in Germany.

Parenthetically, each time—and it will continue—that we see Sofya or Aniuta ask something of their father, we see that their father acceeds to their desire. Raging, protesting, threatening, but ending by doing what his daughters want. We might well ask ourselves whether he would not have allowed Sofya to go abroad anyway and whether the marriage to Vladimir was really necessary. Nonetheless, the fact that Sofya was married gave her a certain aura of respectability.

It seems that Sofya and Vladimir got on well with each other, at least at the beginning of their marriage. There is no doubt that he was in love—and she probably was too. The three-fold intimacy that included the chemist Julia Lermontova at Heidelberg was quite agreeable—up until the moment when Aniuta and Zhanna arrived, who took up the space, chased Vladimir away and spoiled the calm and pleasant working environment (see [Kochina 1985, p. 98]).

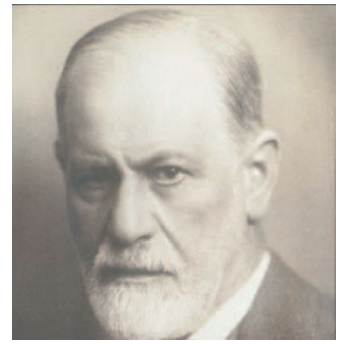
Later, Sofya's professor in Berlin, Weierstraß, and his sisters with whom he lived and who kept house for him, had trouble understanding Sofya's relationship with her husband, in particular the fact that they did not live together (Vladimir was then working on his thesis in Jena). Elizaveta Litvinova, one of Sofya's mathematical friends, describes the practice of white marriages thus: to comprehend what a white marriage is "it is not necessary to be a genius", which Weierstraß undoubtedly was, "but it is essential to be Russian", which Weierstraß certainly was not. In 1872, after Sofya had already worked with him for two years, she finally succeeded in explaining the situation. The result was that the master at last understood that she required a diploma in order to sanction her studies and perhaps even a position, so he decided to have her write a doctoral thesis. This will be related in chapter IV.

Elizaveta Litvinova had been Strannoliubski's pupil (as was Sofya), then she studied with Hermann Schwarz in Zürich. In her case it was not a marriage but a divorce that had made her study abroad possible! She wrote her recollections of Sofya in which she says notably that she was "a brilliant star who has guided young women who wanted to study". Among her pupils in Russia was Lenin's wife Nadezhda Krupskaya [Kochina 1985, p. 60]. Litvinova's influence on mathematics pedagogy in Russia and the Soviet Union are recalled in [Björk 2002].

Sofya's marriage was not to remain white forever. Here is what Anne Charlotte Leffler [1898] relates. The scene unfolds at the moment where a heart ailment suddenly takes away Sofya's father (in 1875):

> This blow was cruel for Sofya: she had always preferred her father to her mother, whose amiable and easygoing nature were less sympathetic to her; her father, on his side, had loved her fondly. His death left her sadly isolated. Aniuta could unburden her grief on her husband. Sofya remained alone: he who had asked nothing more than to console her had been repulsed until then, but the situation seemed more illogical to her than ever, the need for affection overwhelmed her and their veritable union was consecrated in the calm and silence of this house of mourning.

Sigmund Freud, six years younger than Sofya, was but nineteen in 1875 ... Scarcely twenty years later, Anne Charlotte was already one of his emulators, undoubtedly a bit precociously.



Sigmund Freud (1856–1939)

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The Bunsen story

Robert Wilhelm Bunsen (1811–1899) is the man who left his name on the apparatus, which those who have taken part in a chemistry laboratory will have used, the Bunsen gas burners that are always used in these laboratories. This German chemist has yet other claims to glory, for example his work with Kirchhoff on the spectroscope. He was professor in Heidelberg beginning in 1852.

Several months after their marriage, because in Russia it was not possible to study and because in Vienna the mathematics courses were not exceptional and living was too expensive, Sofya and Vladimir settled down in Heidelberg. There women could take university courses, provided the professors involved gave their consent. Precisely, a commission gave each one of the professors, individually, the right to accept them (or not) if they made the request. It was thus necessary to have Sofya accepted by Kirchhoff, Königsberger and Du Bois-Reymond. She took their courses, which made for a heavy schedule, twenty-two hours of courses per week. In particular, she took a course from the famous physicist Kirchhoff, who spoke of her in exceptional terms, which did much for her reputation. Heidelberg was then, and is today, a small town. Everyone knew who Sofya was and would stop to watch her go by. It is said that one day a woman with a child stopped and said to her child "Look, that's the woman who does such good work at the school".

Юлиа Лермонтова (Julia Lermontova) is a cousin of Zhanna, the friend of Aniuta who participated in the white marriage affair. She is also considered to be chaperoned by the Kowalevskis. She arrives in Heidelberg in the autumn of 1869, decides to study chemistry and moves in with Sofya and Vladimir. Sofya succeeds in getting Bunsen to accept her friend Julia, who will be one of the first women to earn a doctorate in chemistry. But he now has it in for Sofya and spreads some rumors about her. He will still be sufficiently angry five years later in speaking to Weierstraß, who came to spend a few days in Heidelberg. Weierstraß himself will explain, recount the

story in a letter to Sofya [Bölling 1993, letter of 21 September 1874]:

> There is something I need to relate. Bunsen, who I think did not know that you had become my student, called you "a dangerous woman". This is based on the following [...] He had sworn never to accept any woman, in particular any Russian woman, in his laboratory. He did not want to hear about Frl. Lermontof. You then went to see him and begged him so sweetly that he was no longer able to resist and had to change his mind.

This story might seem devoid of interest to innocent readers. It is, however, one of the sources for the insinuations by Bell and his successors (see chapter X).

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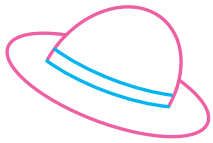
Julia Lermontova (1846–1919)



See chapter X for the crafty use Bell makes of this hat and its "loss".

Story of the ugly hat

One of the professors who taught mathematics at Heidelberg University, Leo Königsberger, urged Sofya to go study at Berlin. Here is our second encounter with the florid style of Gösta Mittag-Leffler [1923]:

> The lecture of the disciple—Königsberger being in fact one of the first disciples of Weierstraß—inspired in her the desire to go sit at the feet of the master himself, to receive knowledge from his own lips.

Weierstraß. He has already appeared in these stories and will be one of the main characters of this book. So let us dedicate a brief section to him here. A brief biography of this mathematician was written by Pelageya Polubarinova-Kochina [1966], our Russian specialist for Sofya, for the hundred fiftieth anniversary of his birth; see also her book on Sofya [Kochina 1985, pp. 61–68]. Strangely, it seems that there is no further-developed biography, in particular no book. See nonetheless the articles [Biermann 1966; Biermann & Schubring 1996] and their bibliographies. Everyone agrees that Weierstraß is an excellent professor, an excellent friend, sympathetic and friendly and kind, an excellent human being for whom nothing human is foreign (as it is expressed in [Kochina 1985, p. 67]), he loves nature, poetry, music. He is, moreover, reputed to have said:

> The highest point of our science is only accessible to the person who, in certain measure, is a poet, with a prophetic vision and a sense of beauty.

See the complete quotation in context on page 69.

He does not lack for humor, e.g. one day (19 November 1873 to be precise [Bölling 1993, letter 38]) he sent Sofya some material on minimal surfaces that contained nothing but formulas, which he called *Lieder ohne Worte*, songs without words. But we are getting ahead of ourselves and will return to 1870.

There is no ambiguity in Berlin, where the rules are stricter than in Heidelberg: whether accepted by a professor or not, it is impossible for a woman to enter the university. Here

I mean that not only was it impossible to enroll or take examinations, but it was also impossible even to physically enter it. Weierstraß, an established bachelor, himself opposed study for women.

Sofya is badly dressed, as always in this period (if we believe Anne Charlotte Leffler [1898]) and hides her face under an ugly hat that gives her the appearance of an old woman when, very affected, she goes to see Weierstraß for the first time. Anne Charlotte Leffler [1898] relates:

> Therefore the professor, as he himself told me later, had the slightest inkling of this lively and young physiognomy, who from the very first moment exercised such an attaction on everyone.

He is resistant. In addition to everything else, he has many administrative tasks and little time. But he will change his mind. Why?

– Sofya solves the problems (intended for his most advanced students) that he poses her. And in a subtle and ingenious manner that surprises him. According to Anne Charlotte [1898] it is at this moment that:

> Happy at being appreciated, Sophie briskly removed her hat; her curly hair escaped, her face flushed with pleasure, and the old professor was moved by a singular and paternal tenderness for this woman-child, whose abilities were the equal of those of his best students. From this very moment the great mathematician became the most faithful friend, the most benevolent, whose support for Sophie never failed; she was accepted into the Weierstraß family as a daughter or a sister.

– She is certainly very persuasive, even without using her physical charms (consider the hat): she is the young woman capable of debating for three-quarters of an hour with "old" Spencer about the scientific capabilities of women (see page 202).

– It is the

> fatal year 1870 which caused such mourning and tears for two great peoples, but which at the same time elevated and excited patriotic passions, [and which] upset

> the quiet habits of the great analyst of the banks of the Spree.

as expressed so aptly by Mittag-Leffler [1923]. [(1)] Rather than attend the university, young people went to war with France. Weierstraß thus had fewer students in his courses than in preceding years, about twenty instead of fifty.

– Sofya is the granddaughter of general Schubert, the geodesist whom Weierstraß quoted extensively in one his articles [1861].

For one or more or all of these reasons, we will never know. He also wrote to Königsberger to inquire whether the lady could offer any guarantees. In his defense, it must be stated that the tzarist police and their informants put out strongly compromising rumors about the morality of the Russian students. It was said, for example, that if they studied medecine it was for practicing abortion on their friends. In any case, he accepted Sofya, he taught her what he taught in his university courses and she would forever be his "most gifted" student. The celebrated analyst had accepted, but the great institution of the banks of the Spree refused

> The high council remained intransigent and it was not until years later when Sofya was already a professor at the university in Stockholm that she finally obtained, during the course of a visit at vacation time, permission to attend some of Weierstraß's lectures.

as Mittag-Leffler [1923] again relates, where he also gives us his version, soberer than that of his sister Anne Charlotte, of the renunciation of the hat:

> Mme Kowalevsky repeated her visits to Weierstraß, became less timid and gave up the brimmed hat. She had learned elliptic functions in Königsberger's course: Weierstraß gave her his lecture notes on hyperelliptic functions. He was so pleased with the ability she showed for penetrating the subject that he offered to give her, privately, the same course on which he gave lectures at the university.

⋆

I cannot resist the pleasure of inserting the results of general Schubert's *Essai d'une détermination de la véritable figure de la Terre*, published in Saint Petersburg in 1859, according to [Weierstraß 1861]. He gives the following values for the lengths of the three axes of Earth, regarded as an ellipsoid:

Grösseste Axe der Erde	3272671,5 Toisen
Mittlere	3272303,2
Kleinste	3261467,8

So that those who have always believed that Earth is a sphere that is flattened at the poles may rest assured: if the longest and middle axes have about the same length, then it is almost true that Earth is almost an ellipsoid of revolution.

⋆

See the letter of December 1884, presented in chapter VI, regarding the authorization given to Sofya for entering upon the grounds of the Berlin university.

1. I will always quote [Mittag-Leffler 1923] rather than from the brief version [Mittag-Leffler 1900] published on the occasion of the International Congress of Mathematicians in Paris. In the long version the letters of Weierstraß appear in the original German, whereas in the short version they are translated into French.

Story of the Paris Commune

> Qu'est-ce pour nous, mon cœur, que les nappes de sang
> Et de braise, et mille meurtres, et les longs cris
> De rage, sanglots de tout enfer renversant
> Tout ordre; et l'Aquilon encor sur les débris
> [...]
>
> Arthur Rimbaud [1972].

With associations such as theirs, those of Strannoliubski (who taught Sofya more than mathematics), and those of their friends, how can we be surprised that the two sisters, sharing the thirst for knowledge and the desire for emancipation with the young Russian nihilists of their time, should have become revolutionaries?

> In the first half of the 1860s nihilism in Russia was less an ideology than an attitude that we would now call protesting: rejecting the authority of parents, of the Church, of the State, it is materialistic and positivistic, priority is given to scientific studies (medicine, biology), there is negation of the romanticism and the idealism of the 1840s and even of amorous sentiments, there is a distinctive dress code (short hair, blue glasses, Garabaldi-style capes for young women, long hair and boots for young men—nihilism touched above all students, the majority of noble origins, of age 18 to 25). The budding feminism was expressed by demands for equality and for access for women to higher education,

wrote Michel Niqueux in his preface to the French version of the novel *Nihilist Girl* [Kovalevskaïa 2004].

This commitment did not always remain as the state of opinion of the Krukowskaya sisters, and above all for Aniuta. Sofya's sister did not remain very long in Heidelberg with Sofya and Vladimir but left alone for Paris. One will object that she no longer had a chaperon. This is the case. Therefore she sent her letters to her parents to Sofya, who sent them on, the postal stamp given witness that the older daughter was right where her father and her mother thought she was, i.e. in Heidelberg.

Pelageya Kochina quotes [1985, p. 33] a text from 1891 that indicates that the provincial government of Nizhni-Novgorod identifies as nihilists all women who

"wear round hats, blue glasses, short hair and do not wear crinolines"

and that it orders the police to arrest them and give up their belongings and, if they resist, exile them from the province.

It can be remarked that this "uniform" perhaps was not so well adapted for our young people gaining the confidence of the masses.

In 1871, she and her "friend" are very active in the ephemeral Paris Commune. I am here using an anachronistic terminology: in contrast to Sofya and Vladimir, Aniuta and her friend were not very legally married, at least not at first. They then contract a "republican marriage"; it is the labor delegate, the Internationalist Benoît Malon, who unites them on the 27th of March in the local town hall of the 17th (arondissement). In any case they lived as a married couple after 1870. Aniuta is a member, as is for example Louise Michel, of the "Women's vigilance committee of the 18th". Aniuta is one of five persons working on instruction, she is responsible for women's education, as Ann Hibner Koblitz [1993, p. 105] informs us. With André Léo (it is under this masculine pen name that the authoress Léodile Béra hides) she founds the journal *La Sociale*. Perhaps she is not the "young Russian of high birth, educated, beautiful, rich, who called herself Dimitrieff" of whom Lissagaray [1976] spoke, but perhaps one of her sisters: they were both among the very first members of the Russian Section of the International—it is said that it is Aniuta, who spoke as good French as her sister Sofya, who had translated into this language Marx's address to the first International.

According to Michel Niqueux, still in the preface to *Une nihiliste*, Elisabeth Dimitrieff was a former neighbor of Aniuta in Saint Petersburg. After the bloody week, she fled to Geneva. Having returned to Russia, it is said that she dedicated herself to family life.

When they learn of the Commune's proclamation of March 18th, without news of Aniuta, Sofya and her husband leave for Paris, passing the German lines (the romantic story of entering Paris under siege is recounted in Anne Charlotte Leffler's romantic biography [1898],: night scene, walls followed into the darkness, an abandoned boat, borne by the Seine that runs quiet and somber, we imagine oily water and the terrifying shadows that are, after all, just the smokestacks of the boats). They find the badly paved streets lined with the houses of the poor, full of ragged children, scruffy women, draggling old people, and who all become the bearers of red flags, pushers of cannonballs, barricaders. They have arrived on the 5th of April, the Versailles soldiers haven't lost time and had begun to bombard Paris on the 2nd, so there are wounded to care for and it is to this that Sofya dedicates her time, until the 12th of May, she who had always felt horror for misery, every sort of deformity, the violence of the wounds. Her husband visits the museums, the Parisian fossil collections are his joy, he meets those scientists who are still in town, while the bombs explode about him (and those who are familiar with the brief and bloody history of the Commune know that this expression is to be taken in the strict sense; the rest are invited to inform

Thanks to Jules Vallès for help in writing this passage.

themselves, for example by reading [Lissagaray 1976]), he decides to specialize in mammalian fossils. A cannon shot now and then that suddenly shakes windows and makes hearts jump, more fossils, another bomb, he leaves, an explosion! I have always admired the scientists who continued to work in Paris during the Commune and that Lissagaray evokes [1976]:

> The Academy of sciences always holds its sessions on Monday. It is not the workers who say: "The Republic has no need of scientists." M. Delaunay is in the President's chair. M. Élie de Beaumont goes through the correspondence and reads a note from his colleague, M. J. Bertrand, who has fled to Saint-Germain; this sterile mathematician is not for bold creativity, never having been able to have a natural theorem.

Lissagaray evokes here Joseph Bertrand, a powerful mathematician—linked to financial power—already an academician but not yet permanent secretary in 1871, which we will return to. See the notes on pages 164 and 165.

Sofya and Vladimir leave Paris on the 12th of May, shortly before the end of the massacres. Aniuta is sought, her friend André Léo is arrested. Here is what Anne Charlotte [1898] writes:

> Aniuta wrote to her sister to beg her to return and intervene with their father so as to secure her pardon and her safety in the desparate situation in which she finds herself. J has just been arrested and condemned to death!

Once again it involves a serious matter: several people have already been summarily executed who were taken for J, who was called Victor Jaclard. When he was in fact arrested, the government had eventually decided to organize trials. General Krukovski once again does what his daughters ask of him. And so he leaves for Paris with his wife and both arrive with Sofya and Vladimir. Let us return to the romance of Anne Charlotte Leffler [1898]:

> I cannot, alas! give the whole story of this troublesome time. General Krukovsky was acquainted with Thiers: he therefore turned to him to procure a pardon for his future son-in-law. Thiers answered that no one could obtain this favor; but one day, in the course of conversation, he mentioned, as if accidentally, that the band of prisoners among whom was Monsieur J would be moved the following day to another prison. They were to pass by a building in which there was an exhibition, and just at an hour when there would be a good many people about. Aniuta went to the spot and mixed with the crowd. The instant the prisoners appeared she slipped unnoticed among the soldiers who surrounded them, and, catching Monsieur J by

> the arm, disappeared with him through the crowd into the exhibition. From whence they escaped by one of the other doors, and reached the railway station in safety.
>
> This tale sounds wild and improbable, but I have only been able to write it down as I, and many of Sonya's friends, remember it.

We will have noted in passing that Anne Charlotte too remembers things that she had not known. It seems to be established that Aniuta was in London, where Marx himself helped her find lodging, and so it is perhaps Sofya or one of their friends who grasped the arm of J. And it does not seem to me that Thiers was the sort to facilitate the escape of a "communard", even a friend of the daughter of a Russian general. Aniuta was a "communarde", one of those "female arsonists" whom he hated so violently. The fact remains that Jacard left for Switzerland, which he entered thanks to Vladimir's passport and where he was joined in Zürich by Aniuta, the general and his wife. The "republican marriage" contracted by Jaclard and Aniuta was then duly legalized in an official Swiss marriage. The two young people were condemned in absentia to perpetual forced labor, then pardoned in 1879.

Before her death, Sofya had initiated the project of writing the history of these several weeks in Paris. Unfortunately she did not have the time to do it. We will likely never know how Jaclard managed to escape.

We return to Dostoyevsky. During 1874 and thereafter, Aniuta and Jaclard were neighbors of Dostoyevsky. It was perhaps Aniuta, or perhaps Sofya, who explained to the writer the logic behind the burning of the Tuileries during the Commune, so that Versilov will say, in *A raw youth* [Dostoyevsky 1875]:

> At that time especially it was as if a death knell could be heard over Europe. I'm not just speaking of the war, or of the Tuileries; I knew that even without that it would all pass away, the whole countenance of the old European world—sooner or later; but as a Russian European, I couldn't accept it. Yes, they had just burned the Tuileries then ... Oh!, don't worry, I know it was "logical" ...

I conclude this story with a remark. Jaclard was a former medical student and a revolutionary (Blanquist, then Marxist) but (?) it seems that he conducted himself like one of the worst "bourgeois" husbands and that he was a cruel fate for Aniuta, for the beautiful Aniuta, for the brilliant young writer, who had in the end a life that was rather bitter and quite unhappy.

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Notre histoire est noble et tragique
Comme le masque d'un tyran
Nul drame hasardeux ou magique
Aucun détail indifférent
Ne rend notre amour pathétique

(Our story is noble and tragic
As the mask of a tyrant
No perilous magic drama
Not a single indifferent detail
Renders our love pathetic)

Cors de chasse
Guillaume Apollinaire [1965].

Story of the return to Russia and the suicide

The life of the two young "doctors", who were really always three, when they returned to Saint Petersburg after the theses of Sofya and Julia in Göttingen in 1874 and that of Vladimir in Jena in 1872 (on "the Anchiterium and the paleontological history of the horse"), had been difficult. To teach in a Russian university it was necessary to pass the magister examination, which was not open to women (so much for Sofya!) and which Vladimir did not succeed in taking, undoubtedly because he had allowed himself to criticize the work of one of the examiners: no recognition of Sofya's work, no post either for Vladimir. We hear her evoke this return to Russia in the novel *Nihilist Girl* [Kovalevskaya 2001]:

> I was twenty-two years old when I went to live in Petersburg. Three months earlier I had finished my studies at one of the foreign universities and, my doctoral diploma in my pocket, I went back to Russia. After five years of solitary, almost secluded, life in a small university town, Petersburg life had right off grasped me, rather intoxicated me. Forgetting for the time questions about analytic functions, of space, of four-dimensions, which until recently composed my whole universe, I went wholeheartedly into new interests, made acquaintances right and left and tried to penetrate the most varied circles, I observed with avid curiosity this complex commotion, so vain basically but so attractive at first, that is called life in Petersburg. At this time I was interested and overjoyed by everything. I found amusement equally in the theater as in the charity balls or in the discussions of literary circles on all sorts of abstract subjects which never amounted to anything. The members of these groups were already weary of these debates, but for me they had a novel charm. I indulged myself with all the passion of which a young Russian can who is chatty by nature and has gone to spend five years on German soil in the sole company of two or three specialists, each absorbed in their own work and not comprehending how anyone could waste their time in idle talk. The pleasure that I took from these relationships extended to

Sofya was twenty-four when she returned to Russia. Her novel is not an autobiography ... and we will see her cheat on her age again—to rejuvenate herself (see page 112). The present fiction is roughly consistent with the other.

This turbulent period sharply separates Sofya's mathematical life into two periods:

before—Weierstraß's student

and after—autonomous mathematician.

> my entourage. My spirit breathed a new animation in the environment that I frequented. My reputation as a learned woman bestowed a certain aura on me; my friends expected something of me, two or three journals had already laid claim to me. This totally new role as a famous woman surely embarassed me a little, but pleased me much at first. In brief, I was in excellent spirits, on a honeymoon of fame somehow and wanted to shout: "All is best in the best of all possible worlds".

I haven't found any reference to articles published between July, 1877 and 1883. Birth of Fufa? Married life? Press of business?

She and Vladimir did not remain idle, they wrote, they translated. Sofya wrote, for Новое Время (*Novoe Vremya*, Modern Times) between 8 June 1876 and 21 July 1877, some ten theater reviews and four scientific essays about research on storing solar energy, important, she says, because the coal reserves will decrease (an old idea thus), on aeronautics, balloons and the flight of birds, on Bell's talking telegraph, the typewriter, finally on fermentation and the work of enzymes, the work of Pasteur.

Elizaveta Litvinova will a little later encounter the same difficulties as Sofya, will fight for some ten years to become the first women to give courses in the Russian higher educational system—but not at the same salary—as her male colleagues.

Let us pass quickly over these years that certainly were not years of quiet happiness and were also those during which Sofya stopped doing mathematics. She had proposed giving courses, even gratuitously, but had been refused. First she loses her beloved father, then she will further lose her inheritance: the general has left 50,000 rubles to Sofya, 20,000 of which would serve to pay off old debts of Vladimir's publishing house. Unhappily for the remaining 30,000 he becomes interested in business. These are the years where she and Vladimir begin to lead the life of a married couple, they have a daughter, and then all of a sudden he does not believe so much anymore in his scientific ability to the extent that he begins to lose confidence in it. He does business, in real estate, in commerce. Sofya helps out and the two of them are as bad at business as they are good scientifically. They also lead the bad life of a badly-married couple, but it is quite likely that they love each other, which seems to be completely the case when they are separated, but they are not capable of living together. Sofya loses her mother. She takes up contact with the mathematician Чебышёв (Chebyshev). She ends up leaving Vladimir and taking their daughter, first to see Weierstraß in Berlin and in Marienbad, from whence she again writes Vladimir an impassioned letter and then, at the end of 1881, goes to Paris to rejoin her sister and social-democratic friends and some Russian and Polish revolutionaries, participants in the Commune. It is at this moment when her political ideas are truly affirmed. Six months later Mittag-Leffler, who

See [Koblitz 1993, pp. 157–158]. This metamorphisis of husbands remains one of the social phenomena that I have the most trouble understanding (see a complementary remark on page 161). It certainly justifies Sofya's subsequent reluctance in facing another marriage.

from then on will be a sort of big brother for Sofya, has her meet the French mathematicians, Hermite and others.

During this time, back in Moscow, a catastrophic bankruptcy, prosecutions. Vladimir leaves a letter imploring Sofya's pardon for the way that he has wasted his life and his money. He has squandered her inheritance from her father, even sold her mother's jewels, and makes an end of it all, as one says, in a rather atrocious manner—he swallows chloroform (which happens on 27 April 1883). The story of Sofya and Vladimir as a couple certainly was not "noble and tragic", but ends pathetically.

In Paris, Sofya collapses when she hears the news. She spends five days crying, shut up in darkness without eating or drinking, despairing the death of her husband, perhaps feeling guilty for having left him alone. She refuses the physician that her friend Maria Jankowska has called, falls into a coma that the physician succeeds in bringing her out of by forcing her to swallow some medecine. On the sixth day she sits on her bed, begins to draw some symbols on the bedcover, then requests writing materials and takes to doing mathematics. A lovely and edifying story.

And significant: in fact, Vladimir's death will allow Sofya to (re-)start a career as mathematician. The suicide seems like an answer to a prayer to Mittag-Leffler and to Weierstraß, as Ann Hibner Koblitz [1984] does not hesitate to say, but in any case Sofya's widowhood is a relief for her mathematical friends: it is about the only way for a woman to be independent and respectable. She is as free as a woman could be.

And she is respectable, which will allow her friends, and above all Mittag-Leffler, to find her a position.

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For certain authors, if Sofya has left for Berlin, it is for fear of the repression against the nihilist suspects after the assassination of Tsar Alexander II on March 1st. Several hundred persons, in large part from the intelligentsia, were in fact arrested. See [Kochina 1985, p. 108].

In a long letter addressed to Sofya on 14 June 1882, Weierstraß writes [Bölling 1993, letter 106]:

"The several hours during which I had occasion to make Mr. K's acquaintance sufficed to convince me that your relationship has an internal rift that threatens to destroy it entirely. He has neither interest nor understanding of your ideas and your efforts and you must not remain in the turmoil of his life [?]"

After Vladimir's death, it is to Gösta that he writes [Bölling 1993, p. 415], 5 August 1883:

"Since Vladimir's death a very important obstacle in my eyes has disappeared."

Gösta Mittag-Leffler
(1846–1927)

We can appreciate the role of Mittag-Leffler throughout this book. See in particular page 128 for a list of his accomplishments.

Story of a friendship: Gösta, Karl and Sofya in Stockholm

I describe Sofya's terms of employment at the University of Stockholm and recall her life in this city in chapter VII. For the story that I now tell, the scene changes to 1884, it is the first of Sofya's Stockholm years and it is September, just after the summer holidays. We should also know that Mittag-Leffler is in the process of developing, indeed creating, mathematics in Sweden, that Sofya and he are just about the only two mathematicians in Stockholm and that they were trying to get others to come, whether Swedes or foreigners. Anne Charlotte Leffler [Leffler 1898, pp. 214–215] tell us:

> On her return to Sweden in September, Sofya went to Södertelje [Söderstälje] for a few weeks, in order to finish in peace the work commenced long before "The Transmission of light through a crystalline medium". Mittag-Leffler and a young German mathematician, whose acquaintance Sofya had made at Berlin during the summer, were with her at Södertelje, and the young mathematicians assisted her by correcting her German.
>
> [...]
>
> She was at this time occupied with preparing her lectures for the new term. These she read to the young German, saying jestingly that he must be her "pointer-dog", [*Versuchskaninchen* (experimentation rabbit), guinea pig] a role which had usually been filled by Mittag-Lefffler.

The guinea pig was Carl Runge (1856–1927), who had written his thesis in Berlin with Weierstraß and Kummer, and whose acquaintance Sofya had made in the summer of 1883, i.e. the preceding summer, according to [Kochina 1985] and [Bölling 1998]. They subsequently had a regular correspondence about mathematics and other subjects. Our two mathematicians perhaps had the idea of recruiting Runge for a position in Stockholm. It seems a happy work environment, friendly and relaxed. The posed photograph done in the photographer's shop that Reinhard Bölling published [1998] does not convey this atmosphere. In it we see, from left to right,

– a seated woman, black dress, black coat, black gloves, black (Astrakhan?) hat,
– a woman standing, black hat with a black veil, black dress, black coat, black gloves,
– a man standing with a beard, moustache and glasses, black coat, black (leather?) gloves,
– a woman standing, without hat, dress less somber, we do not see her hands,
– a man seated with glasses and moustache, his hat on his knees, somberly dressed, without gloves.

As Reinhard Bölling explains, that summer the father of Gösta and Anne Charlotte had died, which is why the two, like Gösta's wife, are in mourning. The persons shown in the photo are in fact Anne Charlotte, Signe (Gösta's young wife), Gösta himself (here with beard), Sofya and Carl Runge. As to what the photo reveals, I see there a strong and respectable man and his entourage, no one smiles, everyone looks sternly at the photographer, except for Sofya, who looks at Mittag-Leffler, which puts her almost in profile. Another striking thing for those interested in Sofya is that she is short. Even mentally removing Signe's hat, who is symmetrically placed (with respect to an axis that is Gösta), the latter is taller.

Let us retain in any case, in spite of the respectable Swedish coldness which permeates this photograph, taken in September 1884, that Sofya, Gösta and Runge were good friends. And, when they were not going to the photographer's shop, our friends were working, reading articles, recounting their mathematics, preparing their courses. For example, Runge read the paper on refraction that Sofya had just finished. Upon his return to Berlin, he even sent Sofya a list of misprints to be corrected.

In a letter to Sofya that Carl Runge sent from England on 21 August 1884 he tells her that he is in the process of reading *The Mill on the Floss* [Eliot 1860] by George Eliot ... before he goes on to discuss a theorem of Mittag-Leffler [Kochina 1985, p. 200].

The following year, things have changed. Gösta writes to Sofya in July 1885 to tell her that Runge has "discovered" that what we today call the Cauchy–Kovalevsky theorem (and that nobody at that time called the Kovalevskaya theorem) is a theorem of Cauchy, and he suggests to her that she write an article to explain the situation. We will see (in chapter IV) that Sofya, Weierstraß and the French mathematical community were already aware of Sofya's contribution for ten years. From Russia, Sofya thus responds to him rather firmly by reminding him that the priority dispute, which incidentally has not yet occurred, is closed, that all this was discussed by the Paris Academy of sciences after the publication of notes by Darboux who also

proved the theorem of Cauchy in question, in short, and that unless Runge had found an additional paper by Cauchy that contained her work, this theorem is her own and, besides, she is quite certain that she told all this to Gösta, and also to Runge, and that probably Gösta didn't listen.

We understand that she is not very happy. And, believe me, she is not wrong. A friend is someone who writes you to say that there is a theorem which looks a lot like yours in an old paper of Cauchy, his proof is not as good and he didn't think of the nice examples that you gave, but nonetheless, it is the theorem of your thesis, and then awaits your response; a friend does not start telling all your colleagues, during vacation, in your absence, too, hey fellows, Sofya's theorem, it was already proved by Cauchy before she was even born ...

And consequently Sofya is very angry with Runge.

That same summer (1885), Gösta brings up in a letter the question of a position for Runge in Stockholm; the young man seemed to have little chance in Germany because, says Gösta, of his independent personality, but if for one reason or another that's not alright with Sofya, she should tell him frankly and it won't be mentioned again. We don't need to be as good a politician as Gösta to understand that a mathematics department (as one did not say at the time) of three people could not function if these three people did not get on. She responded to him very frankly and very directly: Runge's vanity, as that of most Germans, is over-developed, she's not sure that she wants to work with him. It is Lindstedt who is recruited (as was not said either) and in a sense this facilitates Gösta's life because Lindstedt is Swedish. And don't worry, neither his independent personality nor his over-developed vanity—which are undoubtedly two ways of describing the same attitude, to each his own, to Gösta diplomacy, to Sofya vehemence—impeded Runge from being recruited to Hanover and beginning a brilliant career there.

I will conclude this story with two comments. First, what is today called "the" Mittag-Leffler theorem was proved independently by Runge around 1884 and Sofya suggested (in a letter to Mittag-Leffler on 7 June 1884 that is quoted in [Cooke 1984, p. 104]) publishing Runge's results in *Acta Mathematica* because she found the subject important and the two proofs very different. Next, we have seen Runge reading the article on refraction that Sofya finished in 1884 and even indicating some corrections to be made in it, but what we haven't seen, what he

unfortunately had not noticed, no more than had Weierstraß, is the error that Sofya made, committed previously by Lamé, and this article was published, with its error, in *Acta Mathematica*. But this is another story, one that will be taken up in chapter VII.

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Maxim Kowalevski (1851–1916)

Story of the mixed-up letters

This story is a bit of folklore. Sofya was living in Stockholm and teaching at the university. A famous Russian jurist and sociologist was invited to give a series of lectures in this town. Since they bore the same name it is no surprise that the post office confounded his name with that of Sofya, and mixed up their mail. We know that in that period people wrote a lot of letters—to the delight of historians. Perhaps this is how they got to know each other, the mathematician and the sociologist, at least if they had not already come across each other in Paris in 1882 where they had the same sort of associations (Russian, German and Polish revolutionaries and social democrats). His name is Maxim Kowalevski, probably a distant cousin of Vladimir, but in any case another socialist, friend of Marx, correspondent of Engels. He will become Sofya's last love, whom she will call, in a letter quoted in [Leffler 1898], "big M" because he took up so much room, not only on a sofa but also in her thoughts.

He is invited to give a series of lectures at the university of Stockholm in 1888 after having been expelled from the university of Moscow, a series in which he speaks of the transition of matriarchal structures in the modern family (a subject where Engels name in fact must arise); anyway, his book [Kowalevski 1890] is quoted several times in that of Engels [1884]. Sofya organizes a reception for him at her house, with Mittag-Leffler, the astronomer Hugo Gyldén, the writer Ellen Key and the social-democrat Karl Branting ... They subsequently meet in London in the summer, tour the country, visit the galleries and museums ... while the Paris Academy of sciences awaits the memoir for which Sofya will be awarded (at the end of the year) the Bordin prize. It perhaps is not a very easy love, both are so brilliant, both are so quick-tempered, that there must have been frictions. A story with sound and fury, some say. What do we know about it? That the letters of Maxim that were preserved in Mittag-Lefflers's archives are filled with love for Sofya (Sofya's letters to Maxim may exist somewhere, but no one knows

Maxim Kowalevski is said to have begun a course in Moscow by declaring to the students:

"I am supposed to give you a course on the state's laws, but since there is no law in this country, what is it I can say to you?"

and this is why he would be expelled. [Kochina 1985, p. 252]

where). That in 1890 Maxim dedicated his most important book to her. That again in her final moments Sofya will declare to her friends her love for Maxim. That it is Maxim who has the first edition of the novel Нигелиста (feminine diminutive form of nihilist) printed in Geneva in 1892.

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The novel *Nihilist Girl* was first called *Vera Vorontsova*. It was Ellen Key who determined that the title that Sofya had chosen was Нигелиста.

It was never printed in tsarist Russia. The first edition in the Soviet Union dates from 1928. It was recently published in English translation as [Kovalevskaya 2001].

The bridge over the Øresund in the 21st century and in summer ...

Story of the Danish isles and pneumonia

Winter 1890–91. Sofya and Maxim spend a moment in Nice, where Maxim has a villa, then in Genova. In January she returns to Stockholm. She has heard, or she has read in the papers, in any case she believes that in Copenhagen there is a flu epidemic, or smallpox or whatever, it doesn't really matter, but she chooses not to pass through it, and chooses a complicated itinerary that causes her to change trains frequently.

I try to imagine that I take the train from Genova to Stockholm, the French railway (SNCF) website dumps me because it is necessary to make more than three changes, the one for the *Deutsche Bahn* proposes Genova–Milan–Otten–Mainz–Hamburg–Copenhagen–Stockholm, or Genova–Milan–Basel–Hamburg–Copenhagen–Malmö–Stockholm, five changes, or even Genova–Milan–Brig–Basel–Frankfurt–Hamburg–Copenhagen–Malmö–Stockholm, eight changes.

In 1903 when the Curies received the Nobel prize for physics, they waited a lot before getting to Stockholm: the trip from Paris is very tiring and lasts forty-eight hours. [Curie 1937]

And there were all the borders to be crossed. And perhaps there were no bridges between the Danish isles, in any case no bridge over the Øresund, it was necessary to take a ferry, several ferries. It is also true that there may have been more direct trains in Sofya's era, the era of the great international trains, a Milan–Hamburg, for example, but just try to get around Copenhagen!

In reality, Sofya did not go directly from Genova to Stockholm, as a quick reading of the biography by Anne Charlotte Leffler might have us believe: we know that she first went to Paris where we know that she met with Hermite; we know that she subsequently spent several days in Berlin at the home of Georg von Vollmar and Julia Kjellberg, that she saw Weierstraß, and that it was from there that she reached the Swedish capital.

Julia Kjellberg was Sofya's student whom the German social-democrat Georg Vollmar (a close friend of Sofya, to whom we will return) had met in 1885 on a trip to Stockholm.
Regarding the last visits of Sofya to Paris and Berlin, see also pages 218 and 222.

For Berlin–Stockholm, the itineraries proposed by the Deutsche Bahn include either a change in Kolding or Hamburg and in all cases a change in Copenhagen ... It is less far than when talking about Genova, but not much easier.

Again, it's winter, it's Denmark, it's cold, it rains or it snows, there's wind on the railway platforms, on the ferries, on the way from one to the other. And then, surely, Sofya is sick when she arrives in Stockholm. Apparently not too much at first because she teaches her class, the first class of the semester, February 6, a Friday, she then goes to a gathering at the Gyldéns in the Observatory, which she leaves early because she feels feverish, she takes the wrong omnibus, it is cold ... She gets worse and she takes to her bed. On Monday she seems better, she speaks with Mittag-Leffler about her ideas on Euler's equations (those for a solid, see page 90). But her illness has turned into pneumonia, it's the 19$^{\text{th}}$ century, forty years before the discovery of penicillin ... you die of pneumonia, even if you are a brilliant scientist of forty-one, even if you have lots of scientific, personal and literary plans, as did Sofya, as she said, as she wrote to her friends before becoming sick, you die even if you are happy, as Sofya was at that time, and then, that's what Sofya does, she dies of it.

Sofya dies on Tuesday, 10 February 1891.

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Karl Weierstraß (1815–1897)

Story of the burned letters

We know that Weierstraß and Sofya wrote to each other frequently; we have already read extracts from their letters. They wrote at first during vacations or when illness prevented a meeting, then:

> After she had left Berlin in the autumn of 1874, the correspondence continued at more or less long intervals for the remainder of her days. The last letter to Weierstraß is dated 5 February 1890. This portion of their correspondence comprises 37 letters, a certain number of which have great scientific importance

as Gösta Mittag-Leffler tells us, but:

> As for Sofya Kowalewsky's letters to Weierstraß, he burned them all after her death, as well as most other letters he had received, and likely also more than one mathematical manuscript (which is more regrettable still).

The letters that Weierstraß wrote to Sofya and the beautiful mathematics they contain have been published in their entirety and edited [1993] by Reinhard Bölling, who will be one of the heros of this story.

We still have his letters to her. It is as if we were in Weierstraß's study while he is telephoning Sofya and we hear what he says without ever quite knowing how she responds or whether he is perhaps talking to her voice mail.

It is Mittag-Leffler himself who possesses what we are hearing, the letters from Weierstraß to Sofya; Weierstraß knew this and a new dramatic touch is added by our Gösta, who was not someone to let something vanish, whatever it was:

> I have however declared not wanting to read these letters—I already knew a part of the last period from Sonja herself—unless it should happen that I outlive Weierstraß.

If Mittag-Leffler is reputed not to be a person to let anything vanish, it is because he left gigantic archives to posterity ... but it is also said that he burned the letters Sofya had received from her revolutionary or social-democratic friends.

He published some large excerpts from them [1900], then [1923] twenty-three year later, still large excerpts only:

> For a long while however they could not be published in their entirety, there being more than one opinion and more than one judgement being formulated about persons who were still living.

What could be the end of the novel, a novel from the 19th century: after the death of the young heroine from a respiratory illness, the old mathematician sets fire to the letters from his favorite disciple who is lost forever ... but it is not over, adds a novel from the 20th century, the one about a historian discovering one of these letters that were desired but were forever vanished.

Reinhard Bölling

Let's say the end of the 20th century. January 1990, right after the destruction of the Berlin wall, a historian coming from what was still East Germany, member of the Karl Weierstraß Institute (oh yes) may finally make it to the Mittag-Leffler Institute. He is in the process of finishing his edition of the letters from Weierstraß to Sofya (as I have said, they have since been published in their entirety, we are no longer in 1923, it is the book [Bölling 1993]), his name is Bölling. He does not have much time, so he does not waste any, he reads, he sorts through, searches, copies. Let him recount [1992]:

> It was already late afternoon during the second day of my stay. I was just about to examine the contents of a box that contained material from Kovalevskaya's posthumous papers. At first glance, the documents appeared to be exclusively concerned with details surrounding the events of her death and the arrangements that had to be made afterward. I found various bills and receipts in connection to the funeral, others from purchases she had made shortly before her death, and a number of telegrams from friends and acquaintances expressing condolences. Between these, I found some photographs and several pages filled with handwriting. Staring at these, illuminated by the glimmer to the table lamp, I discovered to my surprise a text filled with crossed-out words and lines, and containing in places afterthoughts written in an almost illegible, scrawling had that looked like Sonya's own. Then I noticed that the text, whatever it was, had been written in German. At first I thought that perhaps this was a sketch for a literary work, but then I noticed some comments about Mittag-Leffler and his lectures. Thunderstruck, I strained to decifer a few more passages; the words *Dampfschiff* (steamboat) and Stockholm flashed by, and then: *Deine arme, kleine* [...] *Schülerin*. Unbelievable as it seemed, this appeared to be a page from the first draft of a letter that Kovalevskaya intended to send to Weierstraß.

What I like in this account is this "faint light", which adds an unquestionably romantic aspect ... but it is not a literary effect: we are in Sweden, in January and at the end of the afternoon!

When we think of the number of investigators, mathematicians and historians, including specialists on Sofya, who have

searched through these Mittag-Leffler archives, it is in fact scarcely believable. The collective despair caused to all the historians by the burning of those papers by the father of modern analysis becomes the joy of one of these historians who finds this unexpected evidence, probably the only survivor of the destruction.

More and more excited, Reinhard Bölling found other draft pages, deciphered them. The dead sheets buried in the posthumous box throbbed and became alive under his gaze. Sofya's style, her exquisite broken and faulty German, her raptures, her vivacity. Because, cherries on the cake, the letter recounted an episode unknown to her biographers. Sofya arrives in Stockholm—the draft gives no indication of date but, yes, it is her arrival in Stockholm in November 1883 to take up the position of *Privatdozent* that Mittag-Leffler helped her obtain that she recounts—by steamer, *Dampfshiff*, the boat arrives very early so that no one is waiting for her, she who was going to be received like a princess. She has not even brought Mittag-Leffler's address; there was no need since he would be there on the wharf upon her arrival; then another traveler, thinking that she is a young foreign governess, finds her a cheap hotel ...

Of course Reinhard Bölling then asks himself whether a definite version was mailed, whether Weierstraß received the letter whose draft lay before his eyes. A question for which he is one of the best positioned persons in the world for answering, he knows the letters from Weierstraß to Sofya thoroughly, and yes, Weierstraß answered this letter and, from his response, our historian is even able to guess, to reconstruct, what Sofya had written in the missing pages.

Why was this draft not destroyed? Why did it remain for more than seven years, the remainder of Sofya's life, among her papers? How did it end up with her posthumous papers, for which Sofya's untidiness therefore was not responsible? How is it that no one had opened this box? We'll probably never know.

A novelist (but as I have said, I am no novelist, in any case not here and now) would find something in this story to exercise the imagination, more in the style of Perec [1987] or Byatt [1990] than of Poe [1844]. For this chaper let us be content to dream of other boxes filled with other disorders.

Burned letters ... During the street battles during the *Semaine sanglante* (Bloody week), the house of Joseph Bertrand caught fire ... with many of his manuscripts. But Lissagaray

This was not Reinhard Bölling's first discovery. "The reader can easily imagine the joy of discovery that I felt", he wrote in 1989 in regard to the photograph album presented to Weierstraß for his seventieth birthday [Bölling 1989; 1994]. We have also seen that he would a little later publish the short article [Bölling 1998], the "unknown photo of Kovalevskaya" with the Mittag-Lefflers in high mourning and Runge that I described on page 55.

The "burned letter" appears in [Bölling 1993, p. 425].

reassured us in advance about Bertrand himself: he was not at home.

Some letters received by Charles Hermite too have disappeared, also burned, in a fire a little after his death, in a cabinet with mathematical notebooks, which burned as well, as was confirmed to me by Catherine Goldstein, who added that she was "very angry, especially over the notebooks".

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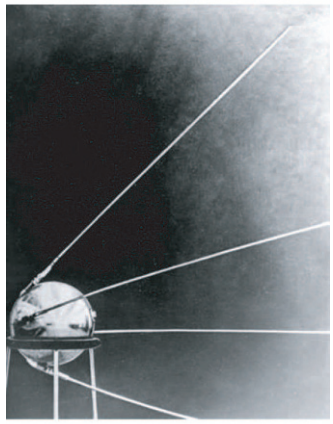

A sphere of radius 30 cm, four antennas, the first Sputnik

Saturn, Sputnik, the asteroids, the Moon

After Galileo who discovered their existence, Huygens who observed and described the most brilliant among them, Cassini who lent his name to a division (a gap) among them, Kant, yes Kant, the philosopher, who proposed a mathematical theory before Laplace, people have been much occupied with the rings of Saturn. Sofya had an astronomer great-grandfather, the father of general Schubert, who corresponded with Laplace; she herself wrote an article [Kowalevski 1885b] about the shape of the rings, about which little was known, where she refined some results of Laplace. We know further that she had worked on the motion of a solid body. In astronomy and mechanics numerous Russian mathematicians have followed her path, you cannot even count the mechanical systems to which they have lent their names. At that time astronomy and mechanics found a home inside of mathematics. The Russian descendants of Sofya launched the first Sputnik. Yuri Gagarin, then Valentina Tereshkova made their little trips in space ...

Saturn, with its rings and the Cassini division

In the now dusty volume of the *Astronomische Nachrichten* in which Sofya published her article on Saturn and following this article, we find the ephemeris of the comet 1884 Wolf. We may read a little further on that on 14 March 1885, the moment when the article [Kowalevski 1885b] appeared, the 247$^{\text{th}}$ "small planet", as asteroids were called in German, was discovered: 247 Eukrate.

Later, just a little later, a certain Dorothy Klumpke, to whom we will return, observed them, these little planets, with the telescope in the tower of the Paris Observatory.

Later, much later, Sofya's name was given to another asteroid, the 1859 Kovalevskaya, right after 1858 Lobachevsky, as well as to a lunar crater. Sofya's asteroid was discovered in Crimea on 4 September 1972. Before the astronomer who discovered it donated it to Sofya, it bore the poetic name 1972 RS2. And they continue to give names to bits of rock when they are discovered in space.

The two thousand eight hundred seventeenth among them, officially 1982 UJ, discovered 17 October 1982 in Flagstaff, is also called 2817 Perec. Meditative and concentrated, astronomers continue to observe space.

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And, if you look carefully ...
(see page 95).

Digressions, history, politics, books, bumps, degeneracy, Bryn Mawr, p-adic numbers and Fanny Mendelssohn, by way of conclusion

You will tell me that it has nothing to do with it, but ... just the same I can resist neither the measurement of Earth's axes in toises (fathoms) nor different versions of Stokes' formula, nor can I not mention here that Marie Curie, just as with Sofya, was only appointed professor after the death of her husband (in 1906) and that she was the second woman in the world (excluding Agnesi) to obtain a professorship, seventeen years after Sofya; I cannot resist going on a bit about history or about politics.

For example, I cannot help seeing or pointing out the sinister stamp *Mathematisches Institut der Reichsuniversität Straßburg* on the volume of [Mittag-Leffler 1923] on which I am working, a volume which was however in the library before the annexation of Alsace by the Third Reich, and which will be there quite a long time after I have finished reading it. Likewise you will understand that I am using the occasion given me by Sofya to give homage to the women of the Commune, likewise I cannot resist mentioning the political opinions of this or that mathematician from the 19^{th} century, likewise I will later evoke the negative (but anecdotal) influences of the political life in France on Sophie in this spring of 2006 (see chapter XII). Likewise, I do not see how I will be able to remain silent about the fact that the reunification of Germany caused the Karl Weierstraß Institut to disappear. Aren't we talking about the history of mathematics?

Yes, we are talking about it, about history. This excellent man who was Weierstraß had written, we have mentioned it already but we will say it again, that to be a mathematician you must be a bit of a poet. Here is a complete citation in context [Bölling 1993, letter 115] (a letter to Sofya, dated 27 August 1883):

> Among older mathematicians of advanced age, there are various sorts of people: a trivial proposition, but which

> explains much. My dear friend Kummer for example [...]. Kronecker is different, [...].
>
> In addition there a lack that occurs in many extremely intelligent people, notably in those of the semitic race; he [it is still about Kronecker] has not enough fantasy (I should rather say intuition), and it is true that a mathematician who is not a bit of a poet will never be a complete mathematician. Comparisons are instructive; the comprehensive view, directed to the highest, to the ideal, distinguishes Abel before Jacobi, Riemann before all his contemporaries (Eisenstein, Rosenhain), and Helmholtz before Kirchhoff (although in the last there is not a drop of semitic blood) in the most striking way.

And so the jews have a different manner, less good of course, for doing mathematics, likewise mathematical gifts are manifested by this or that bump or by the conformation of the left eye (see page 234), likewise women who do mathematics are the products of degeneracy (see page 229), commonplaces that were accepted, that were in fact quite ordinary, in Germany in 1883 ... but again in Paris in 1900 and in Sweden in 1923 when Mittag-Leffler [1923, p. 190] presented the above quotation of Weierstraß thus:

> One finds in this [...] letter [...] an interesting classification of diverse sorts of mathematicians.

Concerning ordinary and official antisemitism and again with regard to this passage of Weierstraß's letter, see [Bölling 1993, note 19, p. 295].

Not only can there be ideas about the superiority of this or that person to some other, not only can these ideas be argued on the basis of the dubious notion of "race" ... but beyond that one is willing to make up a list of who is a jew and who is not. How do we take it today? And how do we ignore it? Because in 1933, you know ...

Speaking of 1933. Let us return to the rigor, or the lack of it, but in a text written by a mathematician. In [Krantz 2002, p. 43] we read:

> It wasn't so long ago [...] that it was difficult for a woman mathematician to obtain a university position. Emmy Noether (1882–1935) came to Bryn Mawr in the United States because she wasn't able to obtain a professorship in Germany.

Nothing the author says is false. All the information he gives is perfectly exact. But it seems to me a strange conception of historic reality to talk about the departure of Emmy Noether from Germany by giving as the reason for her departure the fact that, as a woman, she was not able to obtain a position,

Emmy Noether (1882–1935)

The stamp does not make its infamous mark in gothic characters, but that does not keep me from reading it that way—a stupid prejudice, since it was Hitler's regime that decreed the abandonment of the gothic alphabet in 1941. Certain books in the library have been decorated with another stamp, with a swastika.

as the text quoted implies. The historic reality is much more sinister than the lamentable professional situation created by the German universities for one of its more brilliant products. The historical reality is that Emmy Noether was a jew and that it was in 1933 that she left her country, which changes the perspective a bit. Let us take account of historical context.

Another digression spun ... And, since I have mentioned the books, the book-objects of the IRMA library in Strasbourg, and will mention them again when I discusss the complete works of Weierstraß, be aware that the volumes that I have used belonged to Kurt Hensel and bear his nice ex-libris, a violin and some mathematical manuscripts before a window opening onto mountainous country. Kurt Hensel, a former student of Kronecker, which connects him to our account, is one of the mathematicians who invented p-adic numbers, a good claim to glory. He never taught at Strasbourg, but passed almost all his professional life in Marburg. In 1942, a year after his death, his mathematics books were acquired by the Reich for the new **Reichsuniversitäten**, those in Prague and Strasbourg, here for the Mathematisches Institut, yes, the very one I spoke about regarding the book-stamps, which still holds some hundred volumes that once belonged to Hensel. And that is how I could read Weierstraß by turning the pages that Hensel himself had touched, read and turned. And the violin, someone will ask? Well, undoubtedly Hensel played the violin. After all, he was a grandchild of Fanny Mendelssohn, which brings us closer to our subject ... Digression within digression, it is often said that there are more women composers alive than dead. This is certainly the case too for women mathematicians.

A nice digression, but like all nice parentheses, it has to close.

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CHAPTER IV

THE THESIS OF SOFYA, THE CAUCHY–KOVALEVSKAYA THEOREM

When in October 1872 Sofya explained to Weierstraß that her marriage was purely formal, he understood that she was not destined to remain an amateur mathematician supported by her husband but that she would have need of employment and thus of a diploma, and he decided to have her submit a thesis. There is general agreement, following Mittag-Leffler [1923], that this is what he wanted to convey to Sofya when he wrote on the morning of 26 October 1872 [Bölling 1993, letter 8]:

> I have been much preoccupied with you tonight—as it could not be otherwise—my thoughts have wandered in the most varied directions, have however each time returned to a single point, that I must discuss with you today. Don't be afraid that I will touch on things we have agreed not to talk about, at least for now. What I want to say to you is more closely tied to your scientific undertakings. But I am not sure that, with the admirable modesty with which you judge what you are capable of doing, you would want to agree to my plan. It is preferable to discuss this in person. Therefore permit me, although only a few hours have passed since our last meeting, which brought us so much closer, to visit you this morning for a little while (*ein Stündchen*) so you can hear me out.

And when he decided to have her submit this thesis, he took a few actions:

– He chose Göttingen University, perhaps because Göttingen had established a precedent by awarding a degree to a woman, Dorothea Schlözer Rodde, in the 18th century. I am not sure that this degree was a doctorate, nor do

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_4,

I know what a doctorate was in the 18th century. He certainly knew [1] that this university was prepared to award an honorary doctorate to Sophie Germain before she died. Göttingen seems to have been the least reactionary university in all Prussia [2] and by 1895 it was accepting women, whereas the others would wait a few years.

– He requested that the thesis be defended *in absentia* not wanting to expose timid little Sofya and her fluid but imperfect German to oral questions from a mob of old men. It seems to me that "little Sofya" would have defended herself just as courageously as she had done several years before to "old Spencer" (see page 202 ff.).

We note that Julia Lermontova will also defend at Göttingen in 1874, but that she will not be exempted from the oral examination.

– Above all, he sent three of her memoirs, any of which would have sufficed for a thesis.

The three memoirs of the thesis

The work that was the basis on which Göttingen University awarded the doctoral degree to Sofya on 29 August 1874 was comprised of three different texts, for which I give the publication references. The first appeared rather quickly, the two others later when Sofya was already in Stockholm:

– *Zur Theorie der partiallen Differentialgleichungen* (on partial differential equations), containing what is nowadays called the Cauchy–Kovalevskaya theorem [Kowalevski 1875].

The publication of one of the memoirs from her thesis in *Journal für die reine und angewandte Mathematik* is a great honor for a novice like Sofya.

– *Über die Reduction einer bestimmten Klasse abel'scher Integrale dritten Ranges auf ellitpisches Integrale* (on the reduction of a certain class of Abelian integrals to elliptic integrals) [Kowalevski 1884b].

– *Zusätze und Bemerkungen zu Laplace's Untersuchung über die Gestalt der Saturnringe* (additions and remarks on Laplace's investigations on the form of the Saturn rings) [Kowalevski 1885b].

Those who would now be called referees—whatever their titles then—were Lazarus Fuchs and Heinrich Weber [Cooke 1984, p. 21]. Weierstraß had really done his work well:

1. He mentions this in a letter to Fuchs from 27 June 1874 (see [Wentscher 1909]), to which I will have occasion to return.

2. On this subject see the commentaries of Grace Chisholm reproduced in [Cartwright 1944] and here on page 234.

> In my opinion there is no doubt at all but that *each* of these works suffices for a thesis

he wrote to Fuchs on 27 June 1874 in the letter in which he presented the works that he proposed for this thesis (this letter is published in [Wentscher 1909] [3]); but this was not at all like other theses.

I am going to talk at length in this chapter about the Cauchy–Kovalevskaya theorem because it is the most important result, based on two nice ideas (see page 239) that Sofya had in her short scientific life. But first a few words about the two other works.

Abelian functions. The second article required a good understanding of elliptic functions for studying what in the today's geometric language would probably be called a hyperelliptic curve whose Jacobian is isogenic to a product of elliptic curves (it is an anachronism!). A deep understanding, but perhaps no more original ideas than in a typical thesis (past or present) written under the influence of a research director. We note nonetheless that a deep understanding in the new domain was not just nothing. In any case, when she later presented this work, in 1880, to a scientific meeting in Saint Petersburg, Sofya greatly impressed her listeners by her great ease in this new subject, full of unexpected subtleties for non-specialists. In his analysis of his own work on Abelian functions, Poincaré [1921] speaks of his interest in Abelian integrals that are "capable of being reduced to elliptic integrals" and notes:

> My attention was attracted anew to this problem by a memoir of M^me^ Kovalevski, where two theorems of M. Weierstraß are mentioned [...]

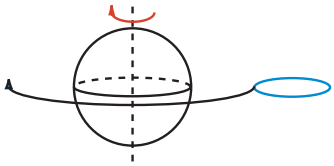
In this regard see the correspondence between Poincaré and Mittag-Leffler [Nabonnand 1999].

He gives proofs of the two Weierstraß theorems and extends Sofya's results in [Poincaré 1886].

This mastery of Abelian functions will later be useful to Sofya in her work on the solid. But let's not get ahead of ourselves.

The rings of Saturn. The third memoir was more a study in applied mathematics. Laplace had shown, under the assumption that the rings of Saturn are liquid, that their cross section has, in the first approximation, the form of an ellipse. In other

3. In the same volume this letter is followed by three others to Fuchs, all from summer 1874, all regarding this thesis and published in Schlesinger [1909].



Laplace's ellipse (in blue)

words, the rings form a torus, a sort of flattened air chamber, but flattened symmetrically. See the figure.

It has to do with determining the generating curve so that the liquid will be in equilibrium with the surface of the ring, under the influence of

– the attraction of the ring,
– that of Saturn,
– the centrifugal force.

I draw here from Tisserand's presentation in his book [1891], where chapters IX and XII are dedicated to the rings of Saturn (and the tenth to Sofya's memoir):

> M^me^ Sophie Kowalevski, already known by her good mathematical work, has brought a happy complement to Laplace's œuvre.

Tisserand's book appeared in 1891, written when Sofya was already famous. Recall that, if the results of the article in question were obtained in 1874 or before, the article only appeared in 1885.

Laplace's ellipse, a little deformed by Sofya, has become an ovoid (in red)

Sofya had calculated the next terms of the approximation and had shown that the section of the torus was rather an ovoid (in plain English, the rings of Saturn, viewed in cross section, have the form of an egg) and moreover that the exact curve cannot be symmetric with respect to an axis parallel to the axis of revolution (vertical, in my figures). It is a rather difficult problem in hydrodynamics that is relevant to a rotating fluid rather than to the actual rings of the actual planet Saturn because, as Maxwell had conjectured in 1857 (and as Sofya realizes since she mentions it in the article), as Poincaré would prove in 1885 and as Voyager's probes confirmed a century later, they are formed of solid particles.

It is not entirely finished. We see Poincaré attack the problem. The problem interests Tisserand enough that he had an additional term calculated in a thesis ... that of Dorothea Klumpke who would be, in 1893, the first woman to defend a thesis in the mathematical sciences in France. Here is what she writes in the introduction to this thesis [1895, p. C.3]:

> [...] Later M^me^ Sophie Kowalewski, taken from science prematurely, resumed this problem and added a happy complement. She shows that the ellipse found by Laplace as the equilibrium shape of the flow is transformed into an oval when terms of higher order are taken into account.
>
> Upon the invitation of M. F. Tisserand, professor in the *Faculté des Sciences* where I have taken courses, we have, following his suggestion, resumed the work of M^me^ Kowalewski and we have evaluated the corresponding terms to the third approximation.

> The expression for the speed, to which M^me Kowalewski's method leads, agree with that obtained by M. Poincaré for a fluid mass subject to a rotation.

We now cut to the chase. I state that I am not a specialist in the Cauchy–Kovalevskaya theorem, which is the basic theorem on partial differential equations over the complex numbers. I thus refer, for the history of this theorem, to a text by Roger Cooke [2002b] (more recent and detailed than the evaluation that appeared in his book [1984]), on which I have relied in writing this chapter and, for a shorter presentation of this theorem, to [Détraz 1993, p. 251].

We briefly mention here Sofya's last article [1891] (published posthumously), where there is concensus that it goes back to the period when she was working on the theorem of her thesis (see [Cooke 1984, p. 34 and Chapter 8]).

Although born in San Francisco, Dorothea Klumpke was French. The presence of Tisserand on her committee is no coincidence (see [Cooke 1984, p. 174])—it is he who has posed the problem—also not the fact that Darboux chaired the committee. It seems to me rather normal that the Dean of the Faculté des Sciences would preside over the first thesis in this institution to be defended by a woman, who moreover took his courses (see the report by Darboux on the defense on page 223).

A problem of Cauchy

The Cauchy–Kovalevskaya theorem is the principal existence and uniqueness result for solutions of partial differential equations.

For an ordinary differential equation. We first explain what such an equation is. We try to determine functions u on a variable t which satisfy a relation

$$\frac{d^m u}{dt^m} = \Phi(t, u, \dots)$$

where the dots represent the derivatives of u of order greater than 1 but less than m. We begin with a simple example (in which $m = 1$):

$$\frac{du}{dt} = u + e^t.$$

Suppose that v is a solution, which is to say a function that satisfies this relation. Then a simple calculation shows that for each real C, the function

$$u(t) = v(t) + Ce^t$$

is also a solution. We thus have lots of solutions (or none at all). If we want a unique solution, we need a supplementary constraint. Generally the problem arises in physics and we have information about the value of u at time $t = 0$, known

as an "initial condition". The so-called "Cauchy problem" is the differential equation plus an initial condition. In the example,

$$\begin{cases} \dfrac{du}{dt} = u + e^t \\ u(0) = u_0, \end{cases}$$

and in the general case

$$\begin{cases} \dfrac{d^m u}{dt^m} = \Phi(t, u, \dots) \\ \dfrac{d^k u}{dt^k}(0) = u_0^k \qquad\qquad \text{for } 0 \le k \le m-1. \end{cases}$$

And the so-called Cauchy theorem asserts, under suitable conditions, the existence and uniqueness of a solution in the neighborhood of time $t = 0$.

Cauchy–who? In the tradition of the French higher education, a "Cauchy–who theorem" is an assertion of existence (and when possible of uniqueness) of a solution having this or that regularity under a hypothesis of regularity of the function Φ depending on the mathematician "who" involved. For example, Cauchy–Lipschitz if it is Lipschitzian, Cauchy–Peano if it is continuous, etc.

Another important problem, which I cannot address in this margin for lack of space, deals with the question of "maximality" of solutions: are they defined whenever the coefficients of the equation are?

What Cauchy actually proved around 1835 is the "analytic" version of the theorem. The method he used consists of finding a formal solution in the form

$$u(t) = \sum a_n t^n$$

and showing that the series obtained is convergent (has a nonzero radius of convergence) by using the "majorant series principle", so that the solution is what is called an analytic function. Here is how this works in our example. The formal series satisfies the equation if and only if

$$\sum n a_n t^{n-1} = \sum a_n t^n + \sum \frac{t^n}{n!},$$

the equality of the constant terms yielding $a_1 = a_0 + 1$, that of the terms of degree 1 gives $2a_2 = a_1 + 1$ and so on, and we obtain generally

$$(n+1)a_{n+1} = a_n + \frac{1}{n!},$$

which allows us to determine the coefficients one after the other starting with a_0,

$$a_n = \frac{a_0 + n}{n!}$$

and there is a unique solution for each $a_0 = u(0)$. For example here, with $a_0 = 0$,

$$a_n = \frac{1}{(n-1)!} \text{ and thus } u(t) = t + t^2 + \frac{t^3}{2} + \frac{t^4}{6} + \cdots$$

We subsequently verify that the series obtained has a positive radius of convergence by comparing it with a geometric series. In the example considered the radius of convergence is infinite—and the solution is $u(t) = te^t$.

Cauchy (1789–1857)

The case of a partial differential equation. We wish to solve a partial differential equation of the form:

$$\frac{\partial^m u}{\partial t^m} = \Phi(t, x, u, \dots)$$

where now u is a function of several variables, a mapping from an open subset of $\mathbf{C} \times \mathbf{C}^n$ into an open subset of $\mathbf{C}^p$ and Φ is an analytic function of the time t, of the spatial variable x, of the unknown function u and of its partial derivatives of order with respect to t less than m. It is what is called a "Cauchy problem" when "initial" conditions are given, i.e. specification of the function u and its partial derivatives with respect to the "time" t at the instant $t = 0$:

$$\frac{\partial^k u}{\partial t^k}(0, x) = g_k(x) \text{ for } 0 \leq k \leq m - 1.$$

Kovalevskaya's counterexample. When Sofya attacked the problem for a partial differential equation, she began by finding a "counterexample" that much astonished Weierstraß. I will present this example and then explain why it is a counterexample. The equation is a very classical partial differential equation, the one that controls the propagation of heat and is simply called the "heat equation". It was already well known in Sofya's time because it was studied by Fourier (not a very elegant way of crediting Fourier's contribution).

Joseph Fourier (1768–1830)

The function $u(t, x)$ represents the temperature at time t at the point with abscissa x along a rod and satisfies

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

Sofya verified that the series

$$\sum_{k=0}^{+\infty} \frac{d^{2k} u_0}{dx^{2k}} \frac{(t-a)^k}{k!}$$

is a formal solution with, when $t = 0$, $u(0, x) = u_0(x)$. But she also observed that the initial data given by

$$u_0(x) = \frac{1}{1-x},$$

a function analytic for$|x| < 1$, yields

$$u(t, x) = \sum_{k=0}^{+\infty} \frac{(2k)!}{k!} \frac{t^k}{(1-x)^{2k+1}},$$

a series that converges only for $t = 0$. She furthermore showed that the formal solution diverges whenever the initial data u_0 has a singularity (which is to say a pole) somewhere, just like our $1/(1-x)$ has a pole at 1.

It is the example explained here, remarkable for its simplicity, that has passed into history. The first counterexample that Sofya found was a divergent formal solution to an equation that is much more complicated, as Weierstraß will confirm for us.

This result was unexpected and did not very well fit the notions of the time. In a letter to Fuchs from 27 June 1874 that I have already mentioned [Wentscher 1909], Weierstraß says that it is "an unexpected remark and which awakens suspicion". A young mathematician who surprises her research director (as was not said at the time) by finding something completely contrary to the intuition that the "old" director and his ("old") colleagues may have developed, does not present an unusual circumstance. This is even what should be expected. The young look at the problem with fresh eyes and the intuitions that I just mentioned can, from their point of view, be considered prejudices. When I mention the attribution problems for the theorem, it will be seen that Sofya certainly is not the only one to have proved it. But she is the only young mathematician in the race and also the only one to have begun her work by producing a surprising counterexample. And thus the only one to have insisted on the hypotheses of the theorem. Weierstraß writes to Du Bois-Reymond on 25 September 1874 (see [Kochina 1985]):

> Except for correcting her numerous grammatical mistakes, I did not do anything other than formulate the problem for the author of the dissertation in question. And in this connection I also have to remark that as a matter of fact, I did not expect any result different to what is known from the theory of ordinary differential equations. To stay with the simplest case, I had an opinion that a power series in many variables that formally satisfies a partial differential equation must always be convergent within a certain domain and must, therefore, represent a function that really satisfies the equation. This is not true, as you can see from the example of the equation $\partial\varphi/\partial t = \partial^2\varphi/\partial x^2$

> considered in the dissertation. This was discovered, to my great surprise, by my student completely independently, first for much more involved differential equations than the one cited, so that even she doubted that it would be possible to obtain a general result; the seemingly simple means she found to overcome the obstacle I value highly as proof of her mathematical flair.

The Cauchy–Kovalevskaya theorem. The theorem uses the hypothesis that all given functions are analytic. It affirms the existence of an analytic solution to the "Cauchy problem"

$$\frac{\partial^m u}{\partial t^m} = \Phi(t, x, u, \dots)$$

with

$$\frac{\partial^k u}{\partial t^k}(0, x) = g_k(x) \text{ for } 0 \leq k \leq m-1,$$

a solution that is unique in the neighborhood of each point $(t_0, 0)$.

We remark that the theorem does not apply in the case of the heat equation with the initial condition being the distribution of heat at time 0 since this equation is of order 2 but that the derivative with respect to time that appears in it is effectively of order 1. It applies if we interchange x and t, giving as initial condition the temperature (as a function of time) at the point $x = 0$ (which does not seem very interesting, not being very realistic).

We recall too that partial differential equations reach the desks of mathematicians because they model authentic physical problems and that they do not all come pre-packaged in the form

$$\frac{\partial^m u}{\partial t^m} = \Phi(t, x, u, \dots)$$

(what Sofya called a "normal form") but rather in an "implicit" form

$$\Psi(t, x, u, \dots) = 0.$$

After having considered her counterexamples, Sofya showed that *if* it is possible to put the equation into normal form, which is to say "to solve for $\partial^m u/\partial t^m$", then there will be a formal solution that converges.

Who proved the Cauchy–Kovalevskaya theorem?

Cauchy, Kovalevskaya, Darboux and the others. Cauchy in 1842, Sofya in 1873 (we have seen that the theorem served for her thesis in 1874 and that it appeared in 1875 in Crelle's journal [Kowalevski 1875]), but also Darboux who "announced"[(4)] a somewhat less general result in the *Comptes rendus* in 1875 and Méray, who in the same *Comptes rendus* and in the same year, announced "considerably more complete" results than those of Darboux ... Which proves that the problem was undoubtedly more in the air in 1874 than it was some thirty years before and above all that mathematicians had need of this result.

All this occurred amicably between these quality people. Weierstraß, who received his *Comptes rendus* a little late (he was late in renewing his subscription!), after considering whether it would be necessary to file a claim with the *Académie des sciences* (letter addressed to Sofya on 21 April 1875 [Bölling 1993, letter 78]), sent Sofya's article to Hermite and asked the editor of Crelle's journal, who was no longer Crelle but Borchardt, to write to Hermite to inform him that he had received Sofya's article in August 1874. Which parenthetically shows that Hermite did not know about Cauchy's article in 1875 any more than he and Weierstraß knew in 1874. The authors of another reference work on partial differential equations that was used at the time, Briot and Bouquet, do not mention it either.

Genocchi knew about Cauchy's article in 1875. It was Genocchi who, having seen Darboux's notes appear, wrote to the *Comptes rendus* to point out Cauchy's 1842 paper. Cauchy, who lived for sixty-eight years, wrote and published eight hundred articles (we can ask ourselves how he had the time to reflect upon and conjure up and prove the next article after he had finished one of them ...), which explains why some of his results have passed unnoticed, for example Hadamard's formula expressing the radius of convergence of a power series as a limit superior figured—it too—in a course of Cauchy. In the other direction, Weierstraß in 1841 proved a theorem that bears the name of Laurent ... and which the latter announced in 1843.

The name "Crelle's journal" is still used to designate the *Journal für die reine und angewandte Mathematik* (Journal of pure and applied mathematics), a journal founded by Crelle, to distinguish it from "Liouville's journal", whose official name is *Journal de mathématiques pures et appliquées*.

Carl Borchardt was also a very close friend of Weierstraß. Moreover, just as he did with Sofya—but not with most of his other colleagues or students—Weierstraß addressed him with the familiar *Du*.
Borchardt edited Crelle's journal, also at the time called "Borchardt's journal", from 1856 until his death in 1880.

Hadamard's formula, which is written nowadays

$$\frac{1}{R} = \limsup \sqrt[n]{|a_n|},$$

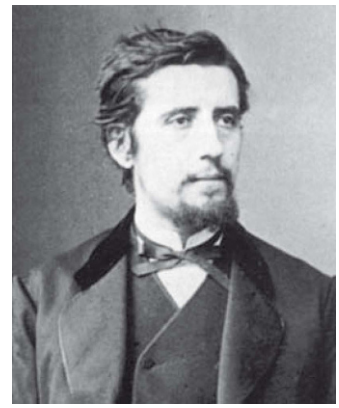
known to Cauchy in 1821 ... before Du Bois-Reymond invented the limit superior (lim sup), was proved in 1892 by Hadamard, who at the time knew about neither the work of Cauchy nor that of Du Bois-Reymond.

4. In the *Comptes rendus*, results are "announced", one says: I know how to prove that ... and in the best cases one adds: I use such and such a method. It serves to carve out territory. The property in question is not really established until the proof is published in a journal.

In any case, in 1875 and regarding what was not yet called the Cauchy–Kovalevskaya theorem, all the protagonists were aware of the parentage of the theorem in question.

Weierstraß pointed out to Hermite that neither Cauchy nor Darboux had made explicit the necessary condition for a partial differential equation to actually have solutions, so that neither Cauchy nor Darboux suspected that cases could exist where a formal solution did not converge.

We shall see that Sofya maintained amicable relations with the French mathematicians and notably with Darboux, who would be, thirteen years later, the referee (*rapporteur*) for the committee that awarded her the Bordin prize.



Gaston Darboux (1842–1917)

On rigor (continuation from page 4). Let us return to the story of Runge, which was told in chapter III. Here is a recent and somewhat odd version [Kozlov 2000]. After having explained the nature of the Cauchy–Kovalevskaya theorem, with the counterexample that we displayed above, the notes from Darboux in 1875 (the date is given), Kozlov writes in 2000 (translation from Russian):

> In fact, it is Carl Runge who pointed out the old results of Cauchy on the analytic solutions of differential equations; he was at the time *Dozent* at Berlin University. According to Mittag-Leffler, "Weierstraß was much astonished". For Kovalevskaya, it was especially disturbing that the information came from Runge, a young man with whom she was apparently on good terms.

One more example of the absence of rigor ... The discussion with Darboux and Hermite, the intervention of Genocchi pointing out Cauchy's article, the fact that the whole history was known and set since 1875, all have disappeared. And for what might be credible, there is no date; the fact that Runge, who was but nineteen in 1875, did not intervene until ten years later has also disappeared. To make way for an insinuation that has nothing to do with Cauchy's article.

Jacques Hadamard (1865–1963)

It was Hadamard who named the theorem. We have understood that, with its hypotheses, its proof, its conclusion and the examples showing the importance of the given hypotheses, this particular theorem is due to Sofya. The name of Cauchy–Kovalevskaya is well suited. First because Cauchy was the first person to study the problem, announce a result and give a proof. Next because we can understand it as a variant of "Cauchy–who" (see page 76).

The theorem in question is—further evidence that mathematicians needed it—quickly taught. For example to examinees for teaching positions in Toulouse in 1889, who are however—as students are of course everywhere—completely ignorant:

> And this year I explained to some of my students who, having obtained the teaching certificate, now aspire to the *agrégation* [competitive examination for a teaching position], the theorem of Cauchy and M^me^ Kovalewski on partial differential equations! Of course it was first necessary to give them supplementary lessons on the theory of functions.

wrote Stieltjes to Hermite on 22 March 1889 ([Baillaud & Bourguet 1905a, p. 376]). The hyphen is not there yet but the theorem has already been attributed.

According to Roger Cooke, in whom I have enduring confidence, the unnatural coupling (my term, Roger Cooke has not expressed it thus) of the name of the old reactionary Cauchy with that of a young (female) revolutionary (thanks to the hyphen) must be the work of that great progressive Hadamard in lectures he gave at the beginning of the 20^th^ century in New York (later published in [Hadamard 1923]).

The terminology "Cauchy problem" was already well established. In paging through the third volume of the Œuvres of Hadamard [1968], I have noted that the terminology "Cauchy–Kovalevskaya" (up to the spelling of Sofya's name) was also well established since the 1920s. Here are some of the ways in which the theorem appears, all the quotes come from [Hadamard 1968], the year designating the year of the appearance of the article from which the phrase was taken:

> The work of Cauchy and, in clearer and more easily accessible form, the famous proof of Sophie Kowalewski have established a fundamental existence theorem for partial differential equations (1926, p. 1457).

> The Cauchy–Kowalewsky theorem led to determination of a solution of this equation (1933, p. 1574).

> For classical analysis the problem was supposed to have a preliminary answer, simple and general, given by the theorem of Cauchy, for which we have Sophie KOWALEWSKI's celebrated and beautiful proof (1935, p. 1594).

> On the contrary, the conclusion of Cauchy–Kowalewski remains exact, without hypothesis of analyticity, for equations of hyperbolic type (1937, p. 1661).

It is known that this problem, always possible and determined (by virtue of Cauchy–Kowalewski) (1945, p. 1669).

On rigor (sequel)

Here is the entirety of the text that Gårding [1998, p. 93] dedicates to the Cauchy–Kovalevskaya theorem (as translated from Swedish into English by its author):

> The first fruit of Sonya Kovalevski's studies with Weierstraß in Berlin was the Cauchy–Kovalevski theorem, which is the basic proof of the existence of analytic solutions for analytic differential equations.
>
> Let $f(x)$ be a function of n variables $x = (x_1, \dots, x_n)$. The Cauchy initial data of order m of f on a surface $S : s(x) = 0$ are defined as the restriction to the surface of the function and its normal derivatives of orders $< m$. These Cauchy initial data are generically mutually independent and determine the derivatives of order $< m$ of the function restricted to the surface. For a general differential equation
>
> $$F(x, u, \partial u, \dots, \partial^m u) = 0$$
>
> of order m in several variables $x = (x_1, \dots, x_n)$, Cauchy formulated a boundary value problem that is called Cauchy's problem: to find a solution of the equation with Cauchy initial data of order $< m$ on a given surface. The problem makes sense only when the equation gives the normal derivative as a function of the others. If we introduce coordinates such that S is the plane $x_1 = 0$, which is to say that the equation can be written locally as
>
> $$\partial^m u / \partial x_1^m = G(x, u, \partial u, \dots, \partial^m u)$$
>
> where the term on the left does not appear among the derivatives $\partial^m u$ of the term on the right. In an equation of this form we can calculate—by differentiation—all the derivatives of a solution u restricted to S when the Cauchy data is known. Kovalevski shows that the formal solution, calculated in this way, is analytic at a point x_0 on $x_1 = 0$ if the Cauchy data is analytic and, in addition, the function G is analytic [(incomprehensible) in all the variables] for the values of the derivatives $u, \partial u, \dots, \partial^m u$ corresponding to the Cauchy data at the point x_0. The

> method, which is borrowed from Cauchy, consists of majorizing the coefficients of the Taylor series of u ["method of majorants"].
>
> The theorem extends to systems of differential equations for a certain number of unknown functions $u_1, \ldots, u_N$. The condition is that the system can be solved for the highest order derivatives of all the functions and that no derivative of the corresponding terms on the right has order greater than these normal derivatives. If this condition is not satisfied, for example in the case of the heat equation
>
> $$u_t = u_{xx}$$
>
> then the theorem does not apply. The solutions may be analytic in x without being analytic in t.

This scarcely comprehensible text seem to me more interesting for what it does not say than for what it does say. The sole contribution it attributes to Sofya is to have "borrowed" the method of majorant series from Cauchy to show that a formal solution is convergent. It is hard to understand that one's name would be given to a theorem for so little, especially so tardily as in 1874. It does not tell us the significance of the problem making sense, nor too that it was Sofya who brought forth the normal form condition (described here in a rather complicated way) and still less that it was she who showed that the theorem does not apply to the heat equation and who indeed had shown the necessity of the normal form hypothesis, and that it was this that was new and original in her work.

⋆

And now,
after so much mathematics
and before a chapter with still more mathematics
a literary pause.

⋆

Italo Calvino (1923–1985)

Pause:
The rings of Saturn

Laplace *showed that the cross sections of the rings of Saturn are elliptical. It was thought at the time that the rings were liquid. Today science allows that they satisfy the equilibrium conditions of a fluid, which conforms to the idea that Cassini had come up with in the seventeenth century about the rings of Saturn, that they are neither gaseous nor liquid, but that they are made up of solid particles of matter, discontinuous, separated by great distances, a multitude of little satellites linked only by their mutual attraction, very weak in comparison with that of the planet.*

Not at all, yelled old Qfwfq! I remember very well that it was a liquid, a very thick liquid, like a piece of mozzarella cheese, a thick soup, like milk, yes, milk, that's what it was. If you see pieces now, it's because the milk curdled. That's not very surprising after so much time ... Anyway, at that time, it was like that, the rings were liquid. As for the cross section, it was elliptical, it's very true! You can trust me. I know them well, these rings, by dint of having them around my head.

At first we use them only to create shade, like parasols. You can't imagine what they were like, these days on Saturn, always in bright sunlight without ever a cloud. Good that they didn't last too long and that nightfall came rather soon. But nonetheless, during the day, we placed ourselves so that the rings would shelter us a bit. We didn't yet know what they were made of. But of course they were liquid. Even the first imbecile who came along could tell that.

We all were there, my aunt M_i, who made us huge plates of tagliatelle, my deaf cousin, old captain XarlraX and his two sisters, with little $S_0Ph(i)$, an agreeable company if it hadn't been for that plague of a 0-b^{e}Ll, always turning around the plate and mostly about $S_0Ph(i)$.

How nice it was then to see little $S_0Ph(i)$ amusing herself with nothing, looking at these little pieces of wood turning,

absorbed in her thoughts, counting and recounting the moons, dreaming, her eyes on the rings, covering sheets of paper with mathematical symbols, of which this plague of a 0-b^{e}Ll never missed saying that it was not right for a cute girl to spend her time like that and that she'd do better by tramping around with him.

Except for watching little S_0Ph(i), for filling one's heart with her joy, there was not anything to do except to admire the round arms of my aunt M_i going back and forth over the big chunks of egg dough, her white arms smeared with oil right up to the elbows. Because, for making tagliatelle, for that, there was space on Saturn. Not only space for spreading out the dough, but space for gardens or ripening tomatoes, for fields for growing wheat, for mountains for the water to come down to irrigate them, and sun for ripening the wheat, for there was no lack of sun.

What was missing were flocks and, if there were, of prairies for them to frolic and graze. "And meat?", you're going to ask me. Well, no, there was not anyone to give us any meat. Oh, we had all sorts of birds, but we didn't eat them. On other, more advanced planets there were perhaps livestock, but we on Saturn didn't have any, none at all, so we savored tagliatelle with tomato sauce and were perfectly satisfied, except perhaps that this plague of a 0-b^{e}Ll was always complaining, the old grump. And it lasted forever like that until the evening, I recall that it was an evening, but of course on Saturn at that time the evenings didn't last so long, you couldn't call them long evenings, on that evening my aunt M_i exclaimed: "My children, if only I had a little milk or cream, how I would like to make you a Sicilian cassata!"

That's when little S_0Ph(i), although a modest and shy girl, had an idea. A brilliant idea I can tell you. And if this plague of a 0-b^{e}Ll tries to tell you that it was not she who had this idea, and that it was for example the old captain XarlraX, that would be plain meanness, don't believe it. Little S_0Ph(i)'s idea, it was the rings. Because, by having looked at them, she had understood, and she alone, that the rings were of milk. And because she was not lacking in practical sense, she also imagined a way of recovering some, some of this milk. "We're going to milk the rings", she said to us.

I have to tell you, the rings, they weren't so far away. They almost grazed us. So, that's how we proceeded. We climbed right to the top of the Zinc mountains, several of us went, old

captain XarlraX, my deaf cousin and myself, following little S_0Ph(i), who skipped along in front of us, holding her milkpot in her hands, sometimes on her head. Evidently this plague of a 0-b^{e}Ll walked behind us. How unpleasant it was to have that one on our heels! This is how little S_0Ph(i) proposed to realize her idea, this is how we would do the milking. We would bring a ladder, she would take off her shoes, climb it, fasten her tin milkpot to the left side, perched on the last rung, her left foot above the milkpot, yelling "I'm there!", she would manage to touch the bottom of the ring by reaching with her left arm, you can imagine that the whole thing was unstable and that our role was to hold the ladder so that she wouldn't fall. You should have seen her, little S_0Ph(i), a sense of balance, a competence, a tenacity, you wouldn't have believed it, in such a pretty little girl. And pretty she was, even if she was hidden by her big hat. She had to protect herself from the sun, you can't imagine what it was like, the sun, on Saturn, at that time. When, with her left forefinger, she would reach the ring, it, by a sort of capillarity phenomenon, would begin to run gently along her arm, along her body and her left leg right until her foot, and it would fill the milkpot. When it would be completely full, she would bring it gently down and we would go back home.

And my aunt M_i not only made us a Sicilian cassata, but also some *straciatella*, some Neapolitan bars, coffee and vanilla ice cream with tiramisu, chocolate, nougat, rum and raisins, and even one day in a vein of exoticism, a tutti frutti. So much so that the rings started to shrink.

One milking day, little S_0Ph(i) launched into a new calculation, you could see that she had been thinking about something for several days. Then she put down her pencil and said to my aunt M_i: I have to tell you, aunty, the rings have become ovoids. That is to say, she added, egg-shaped with a little part and a big part, and that my aunt M_i could understand, because we have some on Saturn, with all those birds. It must be said that for explaining something, little S_0Ph(i) was the champ. Aunty, we need to stop, concluded little S_0Ph(i).

Since that time, the rings have had this form. Ovoids, as little S_0Ph(i) said! And since that time, aunt M_i hasn't made ices for us. We have dispersed. Now when I feel like eating a Sicilian cassata, I go buy one at Nico's, on the *Zattere*. I've happened to run into that plague of a 0-b^{e}Ll there, but I pretend not to recognize him. It could be that one time or another I've run across little S_0Ph(i), but I've never seen her, whether because

This text is inspired by the *Cosmicomics* of Italo Calvino [1976], particularly by drawing of the moon (in *The Distance of the Moon*) and by the Big Bang (in *All at One Point*) which occurs, as is known, because a woman would have liked to have room for cooking a tagliatelle dish. You will undoubtedly find here too in the last phrase an echo of another Italian novel that I like a lot, *L'amante Senza Fissa Dimora* [Fruttero & Lucentini 1988].

I had bent down to tie one of my shoelaces or that I've turned my head to watch a pigeon fly off or that I've started running because my *vaporetto* has arrived.

⋆

CHAPTER V

THE SOLID

> In short, woman was a problem which, since Mr. Brooke's mind felt blank before it, could be hardly less complicated than the revolutions of an irregular solid.
>
> George Eliot [1872].

If the Cauchy–Kovalevsakaya theorem was a master stroke, the true bit of glory for Sofya is her work on the solid. It is also the subject that I understand best, through which I encountered Sofya. So I will dedicate a rather detailed chapter to it, with no small amount of mathematics.

It seems to me that it is possible today, in light of the work of these last thirty years, to do a much more positive evaluation of this work than can be found in books that are a bit older, [Cooke 1984 ; Détraz 1993] for example. This is what I want to try to show here by presenting the content of the articles [Kowalevski 1889 ; 1890–91]. I will attempt to separate the information from the commentary, first explaining what I consider to be an interesting mathematical work, then what's actually in these articles and wherein these articles are, in fact, remarkable.

Euler and his "case"

What is it that makes a mathematical work interesting? There are lots of possible answers to this question and I do not claim to give them all. In any case it is sure that a work that

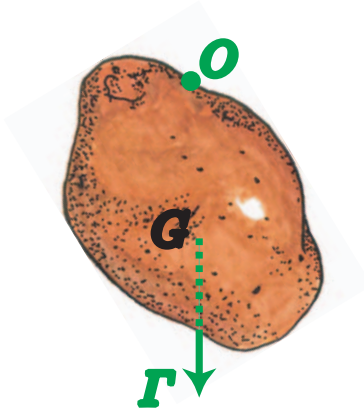
- advances a problem on which others have worked,
- offers a new approach to this problem,
- displays the possibility of applying new techniques,
- poses new problems on which others will want to work, and
- about which we start to talk again ninety years after its publication

is worthy of interest.

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_5,

V.1. What is in the articles

The two articles [Kowalevski 1889 ; 1890–91] contain the work proposed by Sofya for the Bordin prize.



A solid with a fixed point O in a constant gravitational field Γ

V.1.1. An already classic problem. It is a classic problem on which Euler, Lagrange and plenty of others had worked. It has to do with studying the movement of a solid "massive" body (in a constant gravitational field) with a fixed point. This movement is, of course, governed by differential equations, a rather complicated system of differential equations. Here is the form in which it is found of the first page of Sofya's article. According to Klein and Sommerfeld [1897, p. 142], it was the English mathematician R. B. Hayward [1854] who was the first to write the equations in this way. The unknowns are p, q, r, the coordinates of the instantaneous rotation vector and γ, γ', γ'' are the coordinates of the gravitational field Γ. They are written in a coordinate frame fixed to the body (and that is why the "constant" gravitational field seems to vary).

$$A\frac{dp}{dt} = (B - C)qr + Mg(y_0\gamma'' - z_0\gamma'), \qquad \frac{d\gamma}{dt} = r\gamma' - q\gamma'',$$

$$B\frac{dq}{dt} = (C - A)rp + Mg(z_0\gamma - x_0\gamma''), \qquad \frac{d\gamma'}{dt} = p\gamma'' - r\gamma,$$

$$C\frac{dr}{dt} = (A - B)pq + Mg(x_0\gamma' - y_0\gamma), \qquad \frac{d\gamma''}{dt} = q\gamma - p\gamma'.$$

To the elegance of the typography of *Acta Mathematica* (which is still one of the world's finest and most beautiful research journals) is added forever for me the sweetness, voice *off*, of the Russian actress Olga Kokorina reading, pronouncing these equations, in Russian, in *The Case of Sophie K* (see chapter XII).

The vector (x_0, y_0, z_0) relates the fixed point O to the center of gravity G (it is fixed ... on the moving frame), the positive numbers A, B and C are the elements of the inertia matrix,

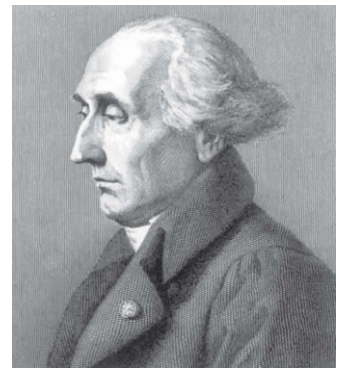
$$A = \iiint (y^2 + z^2)\rho\, dx\, dy\, dz, \text{ etc. (where } \rho \text{ is the density)}$$

which reflects the form of the solid (which too is constant, but possibly completely weird and irregular, like the potato-shaped object in the above picture).

The differential system is complicated. Moreover, at the moment Sofya attacked it, the problem was left over from the 18th century and the works of Euler and Lagrange. No one knew how to solve it, except in two cases:

– Euler's case, where the center of gravity of the body is the fixed point, i.e. the vector (x_0, y_0, z_0) is zero, as in

the figure accompanying our mathematician on the postage stamp presented here; in this case, the gravity does not influence the motion, just as in the case of a "free" body.
– Lagrange's case, where the line joining the center of gravity to the a fixed point is the axis of revolution of the body, i.e. the case where $A = B$ and $(x_0, y_0, z_0) = (0, 0, 1)$—as is the case for a top spinning on rough ground, the case where we can assume that the point of contact with the ground is fixed; it is why such solids are called "tops". The word "gyroscope" would also be appropriate.



Joseph Louis Lagrange
(1736–1813)

This is not totally trivial and really not even simple. Even in Euler's case the instantaneous axis of rotation is not constant. The solution requires the use of elliptic functions, so there was some progress during the 19th century in writing the solutions (due especially to Jacobi [1]). The Prussian academy of sciences initiated a 100 ducat prize in 1852, then closed it in 1858 without having received any contribution. The problem was known in Germany as *die mathematische Nixe* (the mathematical siren). The mention of the problem in 1871 in a novel by George Eliot, a writer who was surely interested in science, is an indication of its fame.

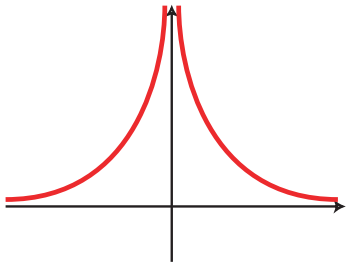
It is certain that Sofya thought about it in 1881, since she mentions it on 21 November of that year in a letter to Mittag-Leffler (published in [Cooke 1984, p. 95], see our page 116). In 1884 Poincaré asked Mittag-Leffler whether Sofya "had continued her research on the motion of a massive body about a fixed point" (see this letter in [Nabonnand 1999]). It is possible that she had thought about it earlier ... And whether it was she who spoke of it to George Eliot? This perhaps is a coincidence, says Roger Cooke [1984, p. 13] ...

A top whose fixed point is the point of contact with the (rough) ground

V.1.2. A new approach. We now come to what Sofya did. She notes that, in the Euler and Lagrange cases, the solutions of the system are meromorphic functions of time and asks whether this property holds in the general case.

Remarks.
– A meromorphic function is a function of a complex variable. It is thus that Sofya came to consider time as a complex variable, an idea that was surely more in the air

1. A brief history of the problem can be found in [Golubev 1960] (a mathematics book) and in [Cooke 1984] (a history book).



A pole: the curve presents a vertical asymptote

in the second half of the 19th century than it was in Euler's time.

– It is a function that has poles. A pole is a value of the time for which some of the functions that enter into the solution become infinite, as in the case of the function $t \mapsto 1/t^2$, whose graph is shown opposite.

It is therefore the case that there are values of time for which the solutions blow up, but these are values no one has ever seen (none of the variables describing the motion of a top ever goes to infinity), they are imaginary and not real ... Sofya observed the existence of invisible singularities for solutions that do not have any real singularities (but it will be seen below that these solutions must in fact have some poles).

It is implicit in [Kowalevski 1889] that the solutions of such a differential system generally have singularities that are much more complicated than simple poles (see § V.2.2). The way in which Sofya approaches the problem is then to ask: is it the case that this simplicity of the (invisible) singularities occurs for other forms of the solid? She asks: for what values of the intertia matrix (i.e. of the constants A, B and C) and which positions of the fixed point (i.e. values of the constants x_0, y_0 and z_0) do the solutions possess only poles?

V.1.3. Discovery of Sofya's case. She thus writes her unknown functions in the form

$$\begin{cases} p = (t-t_0)^{-m_1} P(t) & \gamma = (t-t_0)^{-n_1} \Gamma(t) \\ q = (t-t_0)^{-m_2} Q(t) & \gamma' = (t-t_0)^{-n_2} \Gamma'(t) \\ r = (t-t_0)^{-m_3} R(t) & \gamma'' = (t-t_0)^{-n_1} \Gamma''(t) \end{cases}$$

where the functions denoted by capital letters are power series that need to be determined and for which we need to prove convergence, and she requires that the p, q, r, γ, γ' and γ'' be solutons of the differential system. Since time does not appear explicitly on the right-hand side of the system (the system is thus said to be autonomous), we can suppose, as she does, that $t_0 = 0$. On the other hand, she wants all the solutions to be of this form. It is a system of order 6 and we have already chosen $t_0 = 0$, so there remain five constants of integration: we must thus find solutions of this form with five coefficients left arbitrary.

She first finds without difficulty that $m_1 = m_2 = m_3 = 1$ and $n_1 = n_2 = n_3 = 2$. She then verifies that, in general, the first coefficients in P, Q, R, Γ, Γ' and Γ'' are completely determined,

and that those that follow can be successively determined from them ... except in the two cases we already know about and yet another case (that is going to be called Kowalevski's top), where there is an axis of revolution (as in the case of the other top, Lagrange's), but this time orthogonal to the line joining the fixed point with the center of gravity and where, moreover, the solid has a special shape: $A = B = 2C$ in the inertia matrix. These are linear algebra calculations that do not appear in the main article [Kowalevski 1889] but that are the main point of the one that follows [Kowalevski 1890–91]. There is a small difficulty in proceeding in this way that I will explain by giving a completely analogous calculation in a simpler instance and in chapter VI.

In a sense, as was said in 1895 by Hadamard (see [1968, Vol.IV, p. 1719]), it is a negative result,

> tending to make us assume that the problem cannot be approached with the analytic tools presently at our disposal.

V.1.4. Solution, first phase. Sofya remarks that, in addition to the known invariant quantities (see below), another is easily found, i.e. the function

$$K = \left|(p+iq)^2 + (\gamma_1 + i\gamma_2)\right|^2$$

(a polynomial in p, q, γ_1 and γ_2, of degree 4), making it possible to eliminate certain unknowns and to put the system in a simpler form.

Remarks. In the Euler and Lagrange cases, it is known how to write the solutions explicitly because the system is "completely integrable", or "integrable in the Liouville sense", which is to say that there are plenty of "first integrals", quantities conserved over time. For a general solid, it is known that

– gravitation is constant, so that (within the choice of units),

$$\|\Gamma\|^2 = \gamma^2 + \gamma'^2 + \gamma''^2 \equiv 1,$$

– also constant: the moment with respect to the vertical (i.e. to the direction of the gravitational field Γ),

$$Ap\gamma + Bq\gamma' + Cr\gamma'' \equiv c.$$

– And, of course, the total energy H is conserved (and thus twice its value also):

$$Ap^2 + Bq^2 + Cr^2 - 2Mg(x_0\gamma + y_0\gamma' + z_0\gamma'') \equiv h.$$

There are six unknown functions, but the system is not a system with three degrees of freedom. For brevity I will say only that the first two integrals define a submanifold of $\mathbf{R}^6$ of dimension 4, on which the system is regarded as a system with two degrees of freedom. That is why we seek an additional first integral (in addition to the energy).

In the Lagrange and Euler cases, there is a fourth conserved quantity, and in each of these two cases the four first integrals are used to express the solutions as elliptic functions.

– In the Euler case, it is the square of the total kinetic moment, in the notation used here,

$$K = A^2p^2 + B^2q^2 + C^2r^2.$$

The solutions of the system

$$\begin{cases} \dfrac{dp}{dt} &= \dfrac{B-C}{A}\,qr \\ \dfrac{dq}{dt} &= \dfrac{C-A}{B}\,rp \\ \dfrac{dr}{dt} &= \dfrac{A-B}{C}\,pq \end{cases}$$

parametrize the elliptic curve that is the intersection of the two quadrics

$$\begin{cases} A^2p^2 + B^2q^2 + C^2r^2 = k \\ Ap^2 + Bq^2 + Cr^2 = h \end{cases}$$

(here h and k denote particular values of the functions $2H$ and K). The solutions are essentially Jacobi functions. The quantities γ, γ' and γ'' are subsequently expressed with the aid of these functions.

Of course, neither Euler nor Lagrange used elliptic functions like Jacobi and then Weierstraß have taught us to do—here I am giving a modern presentation.

– In the Lagrange case, it is the moment of the top with respect to its axis of rotation: $K = Cr$ simply. All unknowns are eliminated except for γ'', which geometrically is the height of the extremity of the axis of the top and which I will call x. We find that $x = \gamma''$ satisfies the differential equation

$$\left(\frac{dx}{dt}\right)^2 = (1-x^2)(\alpha - 2x) - (c - kx)^2$$

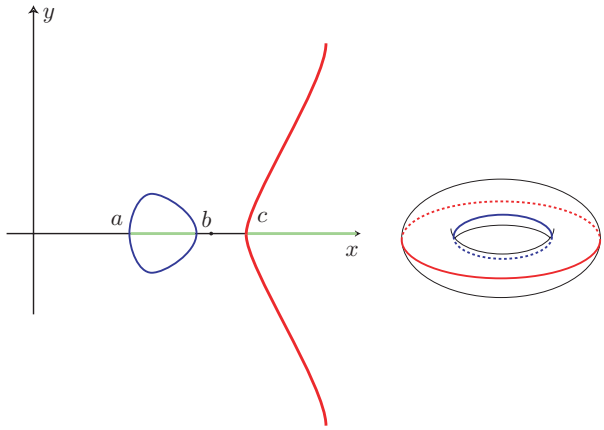
where c and k are the values of the first integrals indicated above and α is a function of h and k (which we do not need to write explicitly). A change of variable $X = ax+b$, where a and b depend only on this polynomial of degree 3 (and thus on the values of the first integrals) brings the polynomial into the form $4X^3 - g_2X - g_3$, so that the general

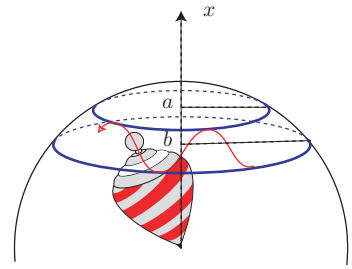
solution of our differential equation is then

$$x = \frac{1}{a}(\wp(at - D) - b)$$

where a and b have already been defined, D is a constant of integration and $\wp$ denotes the *Weierstrass $\wp$-function* associated with the lattice corresponding to the polynomial $4X^3 - g_2X - g_3$. We remark that this function has double poles, in agreement with Sofya's calculation that n_3, the order of the pole of γ'', must be equal to 2.

If you look carefully at the bookplate on page 68, you will see a curve quite similar to the one here, an elliptic curve.





Nutation of the top's axis

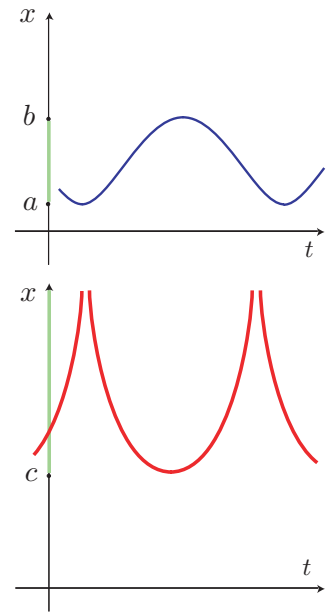
For simplicity we let P denote the polynomial

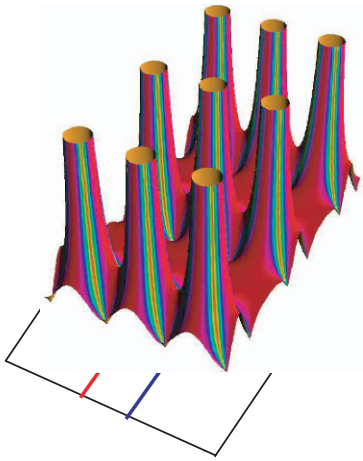
$$P(x) = (1 - x^2)(\alpha - 2x) - (c - kx)^2.$$

The figures show the curve with equation $y^2 = P(x)$, more precisely its real part in the first figure along with the complex curve itself (with a point "at infinity" appended to the red curve), which is isomorphic to the quotient of $\mathbf{C}$ by the lattice defining the function $\wp$; as for the second, it is an elliptic curve, a curve of genus 1.

The small blue oval accurately models the real movements of the top: on the small oval, the abscissa $x = \gamma''$ is less than or equal to 1, as it must be. We note that x is trapped between the two roots of the polynomial P that are smaller than one, a phenomenon that we actually see when we watch a top spin: it is the *nutation* motion of the axis.

The next two figures represent the variable x as a function of time, each showing a piece of the curve (note the poles on the red curve), which is to say a portion of the





values of the function $\wp$. The third figure describes the function $\wp$ itself[2]: it represents the graph of the function $|\wp|$, sometimes called the "analytical landscape" of the function $\wp$, which might lead us to suppose that we are not seeing "everything" about this function, which is not however true since the colors represent the otherwise missing argument.

In the case represented, the polynomial P has, as in the case of the top, three real roots (two between -1 and 1 and one greater than 1), whence the particular shape of the curve with its two components. The complex curve itself always has the shape of a torus. It is what is called an elliptic curve, a curve of genus 1. To continue exploiting this example, observe that the function $\wp$ has a double pole at each lattice point, and thus that our solutions also have double poles, but that the placement of these depends on the solution considered, via the constant of integration D. These are what are called *movable* poles (see § V.2.2).

And, to finish with the top, its poles are not real, which can be clearly seen in the figures. The component of the real curve that gives the real solutions is the blue oval, i.e. the bounded component ($x = \gamma''$ must be less than or equal to 1 since $\gamma^2 + \gamma'^2 + \gamma''^2 = 1$), the point at infinity (which corresponds, via $\wp$, to the points of the lattice) being on the other component. There are also "invisible" poles, as described above.

Strangely, in his evaluation of Sofya's scientific work [Cooke 1984], Roger Cooke does not notice the surprising nature of the appearance of this first integral, which is however noted by Голубев (Golubev) in the book [Golubev 1960], a book dating from the beginning of the 1950s, translated into English in 1960 and that Roger Cooke himself cites. The relation between "meromorphic solutions" and the existence of first integral is still mysterious today.

Which begs the question: how we find a first integral? In the case of the top, as we have seen, it is physics (or the geometry of the problem) that provides the answer. On the other hand, once an integral is written, it is not difficult to ascertain (by differentiating with respect to time) that it is in fact an invariant quantity. Note again that the supplemental first integral is of degree 2 in the Euler case, of degree 1 in the (Lagrange) case of the top, whereas in Sofya's case it is of degree 4, which is going to contribute to the difficulty of the computations ... and make its physical interpretation difficult.

This above all poses the question of the relation between the fact that the solutions are meromorphic—something that Sofya assumed—and the fact that there is a supplemental first

2. Olivier Eichinger, a student in mathematics at Strasbourg, obtained this figure using the program *Mathematica*.

integral, which appears in the article in a rather abrupt and astonishing way. I will come back to this.

V.1.5. Solution, second phase. What Sofya wants to do for the Bordin prize, perhaps because she is of the opinion, as is Weierstraß, that it is necessary to pay lots of attention to the details in the exposition, is to *solve* the system, if only to verify that the solutions really are meromorphic. She reduces the differential system to two equations

$$0 = \frac{ds_1}{\sqrt{R(s_1)}} + \frac{ds_2}{\sqrt{R(s_2)}}$$

$$dt = \frac{s_1\, ds_1}{\sqrt{R(s_1)}} + \frac{s_2\, ds_2}{\sqrt{R(s_2)}}$$

where R is a polynomial of degree 5. It is completely analogous to

$$\left(\frac{dx}{dt}\right)^2 = P(x)$$

for the case of a top, which can also be written as

$$dt = \frac{dx}{\sqrt{P(x)}}.$$

That is where she is in 1886 and it is at that very moment that the Bordin prize is announced. It still remains to "invert the integrals": she would like to express the dynamic variables as direct functions of time and not inversely.

She still has some fifty pages of changes of variable, proofs and calculations before she actually achieves expressing the solutions in terms of ϑ-functions [theta-functions] in two variables. Moreover, that cost Sofya no little effort, occupied as she was with her courses and other scholarly tasks (*Acta Mathematica*, for example), the sickness and death of her sister Aniuta at the end of 1887, her daughter Fufa, her literary work with Anne Charlotte, her affair with Maxim Kowalevski—but she managed to send a version of her memoir before the 1 June 1888 deadline, and the definitive version at the end of the summer.

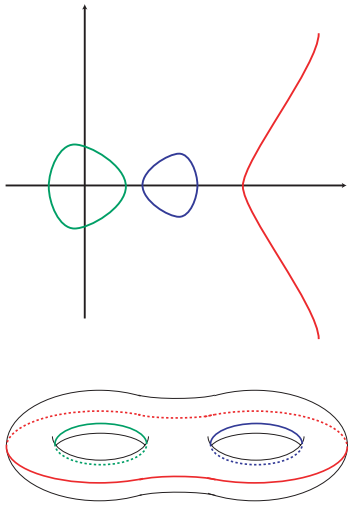
The ϑ-functions in two variables are analogues in dimension 2 of the function of the Weierstraß $\wp$-function: instead of a degree 3 polynomial and an equation, as for Lagrange, Sofya has a polynomial of degree 5 and two equations.

Remarks.

(1) One of the changes from the original differential system to the system that I have just copied from [Kowalevski 1889] is often described in mathematical literature as a "mysterious change of variables". We quote Roger Cooke [1984, p. 156 ff]:

> This chain of transformations is certainly formidable and proof of either extraordinary computational ability or of prodigious patience, or both of these together.

I admit that there are "mysteries" that I find deeper and more fundamental than Sofya's singular "computational ability". This ability seems to me not very remote from an equally dubious "feminine intuition", not to mention the no less dubious feminine patience—which in any case was likely not one of Sofya's cardinal virtues.



A curve of genus 2

(2) This time there is a hyperelliptic curve of genus 2, the one with equation $y^2 = R(x)$, for that polynomial R of degree 5; the figures show the curve and its real part.

(3) When we have attained a system in the form given above, we then know that theoretically we have the result. The solutions are expressed in terms of ϑ-functions (the curve is of genus 2, these are thus ϑ-functions in two variables, just as the elliptic functions in the Euler and Lagrange cases are ϑ-functions of one variable). But, as I have said, Sofya wanted a full solution, a complete solution. And as I said with regard to her thesis in chapter IV, Abelian functions were something that she knew well.

(4) I will mention below a natural method of achieving this system and its integrability, for which we will see a curve of genus 3 come into play. And this evocation will conjure up a new mystery, the relationships between this curve and Sofya's.

V.2. Topicality and modernity of this work

Let us begin with what old and new commentators on this work bring up.

(1) The Commission that awarded the prize brought up first and foremost the use of ϑ-functions in two variables, for it was the first time that these functions had been used to solve a problem outside of function theory. The title of the prize problem was:

> Perfecting an important point in the theory of motion of a solid body.

Not only had the problem as posed been advanced and the theory of motion of a solid body "perfected", but Sofya had done so using these new functions that the analysts were happy to see applied (see the quote from the report on page 169).

(2) I already mentioned the famous "mysterious" change of variables.

(3) The "strangeness" of Sofya's case is also noted, no one has ever seen a "Kowalevski top"; it is possible to draw one, there is for example a figure in [Polubarinova-Kochina 1978], but it is really artificial. Moreover, Roger Cooke mentions in [1984] and confirmed for me by sending me the text of a letter to Schwarz, that Weierstraß asked his other "favorite student" to make one for Sofya (whom he calls "Frau K." in this letter); what luck to have a jack-of-all-trades as one's student!

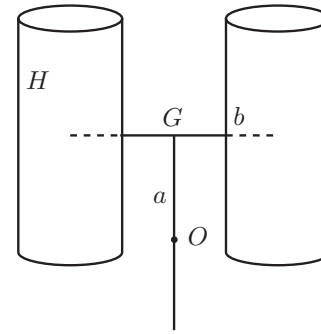
(4) The movement itself, ϑ-functions or not, is very complicated. I quote Roger Cooke [1984, p. 159]:

> The Kovalevskaya case is so complicated that no global description of the motion is possible.
>
> The case seems not to have any practical value; moreover it is seen that nowadays physics texts rarely mention and never study the result.

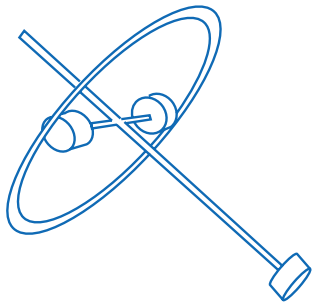
Disputing the "practical value" of the case in question seems about as legitimate as disputing the "practical value" of studying the solvability by radicals that one knows how to solve, at least after Newton, as precisely as one wishes. It cannot be denied that this problem without practical value has resolutely furthered advancement in mathematics (and its applications) since Galois and Abel.

It seems to me that with the perspective offered by the some hundred twenty years that have passed, we can make a more detailed analysis.

V.2.1. The motion is too complicated. This does not seem to me to be the most important point, so that I will begin with it. With a computer, life is easier and complicated solutions become simple. I return to work of Richter, Dullin and their collaborators. A "Kowalevski top" turns permanently on the website of the working group for nonlinear physics of Bremen



According to [Kochina 1985, p. 309], the "top" realized by Schwarz should have this form for appropriate values of the height $2H$ and the radius R of the cylinders, of their separation $2b$ and the distance a from the fixed point O to the center of gravity G.



The Bremen version of the Kowalevski top

university [3] and in the DVD *Kowalewskaja Kreisel* of Richter, Dullin and Wittek [1997].

It is the Bremen version of the Kowalevski top from the DVD, and which I also saw "in person" in Peter Richter's office, that I sketch here.

V.2.2. The Painlevé property. First, Sofya launched the Painlevé property. Painlevé would study more general differential equations (over the complex numbers) for which the singularities of the solutions are rather simple. For a nonlinear differential system, the singularities of the solutions can be poles, ramification points with possibly infinitely many branches, etc. Moreover, contrary to what happens with a linear system, the placement of these singular points can depend on the solution considered (on its initial conditions) and not just on the coefficients of the equation. These are what are called *moveable singularities*. Here are two somewhat silly examples (the prime $'$ represents differentiation with respect to time t):

– The equation $y' = -y^2$. The functions that appear in this differential equation (just one in this case: the polynomial y^2) do not have poles, as was also the case for the differential system for the Lagrange top. The nonzero solutions are the functions $y = 1/(t-c)$. They thus have a pole (at c) which, as was the case for Lagrange, depends on the solution chosen (see above for remarks on mobile poles, page 96).

– The equation $yy' = \frac{1}{2}$, whose solutions are the functions $\sqrt{t-c}$, each of which has a ramification point of order 2, also mobile.

Somewhat less silly is the example

$$y'' = y'^2 \frac{2y-1}{y^2+1}$$

given by Painlevé [1902]. The general solution,,

$$y = \tan(\log(At - B))$$

has a mobile ramification point at B/A and an essential singularity (Painlevé proved that, in order to have behavior of this type, an equation must be of at least order 2).

Nowadays we say that a differential equation has the Painlevé property if the only mobile singularities of its solutions are poles

3. http://www-nonlinear.physik.uni-bremen.de/.

(the first of the two silly equations above has the Painlevé property, but not the second). It is a small extension of the property that Sofya used, which requires that all singularities be poles. It seems to me that what Sofya did was to commit *plagiary by anticipation* (as I described it in [Audin 2002]), moreover specialists in integrable systems (see for example the article [Adler & van Moerbeke 1989]) don't hesitate to refer to the Kowalevski or to the Kowalevski–Painlevé property. The difference between the property required by Painlevé and that required by Sofya is described by Painlevé [1902] (at the very end of his article), where what she sought were

> the cases where the movement of the solid is defined by meromorphic functions of t that actually possess some poles. Her procedure allows the case to escape where these functions are uniform without have poles, whether they are holomorphic or whether their singularities are transcendental.

We can refer to the quotation from the article of Adler and van Moerbeke [1982], reproduced here on page 104. These authors consider that she wanted to use Abelian varieties (hidden behind the ϑ-functions). Now these are algebraic (we will not concern ourselves with transcendental singularities) and projective, so that there will necessarily be poles. In other words the problem that Sofya posed is that of algebraic integrability, that which Adler and van Moerbeke would define in 1982 as being algebraic integrability.

To conclude with the Painlevé property, two remarks.

First, for Sofya, it had to do with a method, applicable to other systems. In a letter addressed to Gösta on 28 December 1884, she in fact exhibits the method she is going to use for the solid in another, simpler, example. See this letter and my remarks in chapter VI.

Secondly, Painlevé himself would also have the Bordin prize, six years after Sofya, in 1894. The topic of the prize that year would be:

> The study of problems of analytical mechanics admitting algebraic integrals with respect to velocities and especially quadratic integrals

and the report of the Commission will note

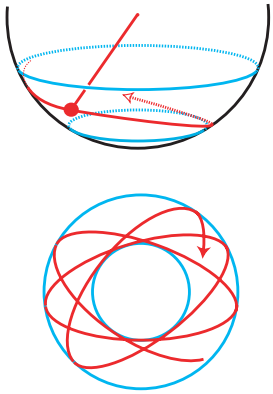
> finally an application of ordinary differential equations whose solutions have fixed singularities.

(in other words for which all the movable singularities are poles). We note the proximity to Sofya's approach: there will be only

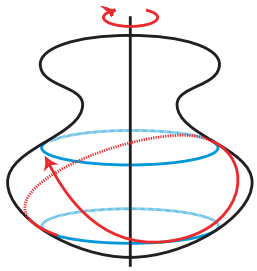
Paul Painlevé (1863–1933)

I find the tone at the end of this article by Painlevé to be a bit condescending. Painlevé in fact says a little further on,
"But, interesting as the way followed by M^e^ Kowalevski may be, it was desirable to resume the problem in a more rational way."
Oh, those women! Irrational! But hardly transcendental ...

I cannot resist the pleasure of remarking too that, at the time of the award of the Bordin prize to Sofya the Commission welcomed the fact that the ϑ-functions in two variables had at last been *applied* to a problem in mechanics (see below), likewise for Painlevé, it welcomed that his theory of differential equations was applied ...



A spherical pendulum and one of its trajectories, as seen from above



Geodesic of a surface of revolution

poles ... and an algebraic first integral. One is really in the same place, which brings me to the discussion of integrability that follows.

V.2.3. Liouville integrability. A system is *integrable in the sense of Liouville* if it possesses "enough" first integrals, a vague enough definition that can however be made precise (see for example [Audin 2002]). We furthermore need these integrals to commute (in the Poisson bracket sense), something that I will not attempt to explain here. All these integrals allow for solving the system by quadratures, at least in principle. The sudden appearance of the integral K in Sofya's case can also be expressed as the statement of a theorem:

***Theorem* (Kowalevski [1889]).** *When the equations of the solid have the Kowalevski–Painlevé property, the system is Liouville integrable.*

Moreover, in the work mentioned above and that earned him the Bordin prize, Painlevé used Sofya's property in (re-)proving that the *three-body problem* is not integrable with algebraic first integrals (see also § V.2.6).

A system that is Liouville integrable displays a long-term behavior that is very regular, what is called quasi-periodic, which has a precise mathematical sense but which may be understood as almost periodic, i.e. you eventually end up in almost the same state as you began. It is fashionable nowadays to make this property opposite to some vague notion of chaos. Indeed, let us leave it vague. Many well-known mechanical systems are integrable. It is the case, for instance, for the movement of a spherical pendulum or for that of a *free* particle (and thus tracing out a geodesic) on a surface of revolution: in these two cases, as in that of a top, because of the axis of revolution there is a supplementary first integral.

V.2.4. From whence all these curves? There is the curve of genus 2 stemming from the mysterious change of variables. There are the spectral curves of [Bobenko et al. 1989] and elliptic curves too. It is again a problem for which a bit of ink will subsequently flow (see especially [Markushevich 2001]).

V.2.5. What is the strange case? The problem of the massive solid is natural, and it is also natural to study the motions of a solid for which the center of gravity is fixed (Euler's case) or of a top, but then what in fact is Sofya's top when we can't even

draw it, some people say ... I have however already represented two! See the concluding phrase of Roger Cooke's analysis [1984, p. 164]

> [...] but if the case to which [Kovalevskaya's analysis] is applied is rather special, it not necessary to spend any more time on her arguments.

It is true that rather soon there was a loss of interest in integrable systems, since Poincaré proved that there are very few of these ... Until representation theory, the use of ϑ-functions for the study of nonlinear partial differential equations of the Korteweg–de Vries type and of affine Lie algebras, bringing the subject back into fashion for some thirty years now.

We take up these terms one by one, in a more or less precise manner:

– For representation theory, see [Kirillov 1976 ; Kostant 1970].

– For the use of ϑ-functions for the study of nonlinear partial differential equations of the Korteweg–de Vries type

$$\frac{\partial u}{\partial t} = 6u\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial x^3},$$

which can also be written

$$u_t = 6uu_x - u_{xxx},$$

I refer readers to [Dubrovin et al. 1976]. In the introduction to this article, Dubrovin, Matveev and Novikov reproduce an extract from a letter from December 1886 quoted in [Golubev 1960] in which Sofya writes about having said to Picard that the functions of the form

$$y = \frac{\vartheta(cx + a, c_1x + a_1)}{\vartheta_1(cx + a, c_1x + a_1)}$$

can be used to integrate certain differential equations and that the latter showed a certain skepticism, on which they comment thus:

> the analysis of the authors shows that Picard's doubts were only justified for the ninety years that separated Kowalevskaya's article and the work of 1974 on the KdV equations. They are no longer.

Clearly Sofya had no luck, for the English translation of this article written in Russian made a misinterpretation in this regard, as pointed out by Ann Hibner Koblitz [1984].

We further note that this work is one of the first, perhaps even the first, in which these functions appear in a differential system since [Kowalevski 1889]. But this is not finished ...

– For affine Lie algebras I refer readers to the papers of Adler and van Moerbeke (see [1982 ; 1989 ; 2004]) and to those of Reyman and Semenov–Tian–Shanski; and I am not just thinking of the remarkable article that they wrote with Bobenko [1989] and which is the source for [Audin & Silhol 1993]. The second paragraph of the introduction to [Adler & van Moerbeke 1982] states:

> This paper deals with a criterion for algebraic integrability, inspired by work of Kowalewski. In celebrated papers [...], she has shown that the only algebraically completely integrable systems among the rigid body motions are Euler's rigid body, Lagrange's top and the famous Kowalewski top. Her method is based on the idea that if the system is to be algebraically completely integrable, and if the phase variables of the problem are to be algebraic (abelian) functions, then the phase variables of the problem must be meromorphic in time. In addition, the trajectories which blow up (as they must) are nicely parametrized by a codimension one family of parameters. This implies the existence of enough codimension one parameter families of (complex) pole solutions of the system so that all the (abelian) phase variables get a chance to blow up (not necessarily simultaneously). The sufficiency of this criterion has not been established.

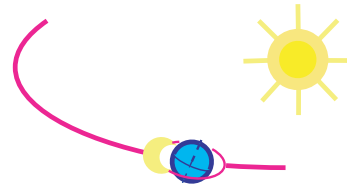
The Kowalevski case remained mysterious for about fifteen more years (a century after "our" article), when a very beautiful paper [Reyman & Semenov-Tian-Shanski 1994] revealed that the Kowalevski top is not so strange, since it is the manifestation in our dimension 3 of a family of tops that appear in a perfectly natural way in higher dimensions, at the very heart of relations between integrability and affine Lie algebras. And for which it is thus known exactly why they are (Liouville) integrable. It is, by the way, thanks to this description that the beautiful Lax

pair [Bobenko et al. 1989] which was discovered that Robert Silhol and I used in our article [1993].

V.2.6. Other integrable cases? The relations between the Kowalevski–Painlevé property (a form of regularity of solutions) and Liouville integrability (a form of geometric regularity) are not generally so clear. See [Zakharov 1991].

There also exist very respectable systems that are not integrable. This is notably the case, as I have said, with the *three-body problem* under gravitational interaction. It is mainly to this problem that the first volume of *Méthodes nouvelles de la mécanique céleste*, which appeared in 1892, is dedicated. But Poincaré [1987, p. 255] also speaks there of the solid problem:

> Let us pass to another problem; that of the motion of a massive body about a fixed point.
>
> This problem has been integrated in three distinct particular cases by Euler, by Lagrange and by M^me de Kowalevski (see *Acta Mathematica*, **12**). I believe that M^me de Kowalevski has discovered yet other new cases of integrability.



According to Kepler's laws, two celestial bodies (Sun and Earth, for example) form an integrable system. A third player, here the Moon, makes the situation more complicated, indeed chaotic: the three-body problem is not an integrable system.

He then proves, using his perturbation method, that it is necessary, in order that an algebraic first integral exist, that the ellipsoid of inertia be of revolution (which is to say, with the notations used here, that two of the moments A, B, C be equal). He concludes this study thus [1987, p. 259]:

> The conditions enunciated in this chapter are necessary, but not sufficient, nothing proves that this third integral exists; before deciding we need to await the complete publication of M^me de Kowalevski's results.

Sadly, this conclusion is accompanied by a note at the bottom of the page:

> Since these lines were written, the world has had to mourn the premature death of M^me de Kowalevski. Her notes that have been found are unfortunately insufficient to reconstitute her proofs and her calculations.

Apparently Sofya had ideas for working further on this question of integrability. She told Poincaré that she had written a letter to Hermite describing these new results [Cooke 1984, p. 118]. Another burned letter (see page 65)? We have also seen (page 61) that she spoke about it with Mittag-Leffler on the eve of her death, but Gösta, who has nonetheless stated that it was what she would have done best, was not able to reconstruct

what she said to him. Some time after Sofya's death, he discussed this by letter with Poincaré, each thinking that the other knew:

> I see that you do not know much more that I about M^me^ Kowalevski's last research on the rotation problem. It is that she did this work at a time when I was absent from Stockholm and we never found occasion to discuss this research together. (4)

We will undoubtedly never know what it was she thought of proving. There was still a bit of activity on the question subsequent to her death. After the pages of Poincaré that I have just evoked, we need to mention an article by Roger Liouville (no relation of Joseph Liouville), who competed for the Bordin prize of 1894 (that won by Painlevé), in which numerous integrability cases are exhibited. Not only did this work (with quite wrong results ...) gain an honorable mention for the Bordin prize, not only was it published in *Acta Mathematica* [Liouville 1897], but it even was made the object of a footnote in the redaction [1897] of Painlevé's inaugural lecture in Stockholm:

> Besides the results contained in this memoir [dealing with the article on the solid], Madame KOVALEWSKI announced verbally the existence of another case of the integration of the motion of a solid. Her results have been re-established by Monsieur R. LIOUVILLE in a memoir equally recognized by the Paris academy of sciences, that will form, in the Acta, a natural sequel to Madame KOVALEWSKI's work.

It was Husson who would show, in 1905, that the system is not integrable, with *algebraic* integrals, except for the three cases we know [Husson 1906]. We quote here the report by Appell on Husson's thesis [Gispert 1991, p. 394] (contrary to Painlevé in 1895, Appell knew, ten years later, that the article of R. Liouville was wrong):

> In summary M. Husson has taken an important step in the problem of the motion of a massive body suspended by a point on its axis [which axis?], a problem that attracted the attention of the greatest geometers, Euler, of Lagrange, of M. Poincaré, and which has been the object of numerous memoirs, notably of a famous memoir of Madame Kowalevski. He has shown, by a detailed and

4. Letter from Mittag-Leffler to Poincaré on 13 March 1891 [Nabonnand 1999].

> rigorous analysis, testifying to a deep knowledge of the latest methods, that Monsieur Roger Liouville committed grave errors in a singularly obscure memoir, to which the Academy of sciences awarded an honorable mention in the Bordin prize competition of 1894. He has succeeded in particular in resolving the difficult problem of determining all cases in which there exists a fourth algebraic integral.
>
> It is an excellent work which exceeds the average completely and which will do honor to French science.

Much later, Ziglin [1983] showed the same result as Husson, but for *meromorphic* integrals.

The three cases we have just spoken about rather lengthily here are the only ones that are integrable for all initial conditions. If we specialize these, for example in a way so that the moment $Ap\gamma + Bq\gamma' + Cr\gamma''$ has value 0 ($c = 0$, in the notation of page 93), there is yet one more integrable case, that of Goryachev–Chaplygin. This is perhaps what Sofya was thinking about before her death.

The question implicitly posed here is: how do we prove that a system is not completely integrable? We have seen that we can find first integrals, thanks to physical or geometric arguments (symmetry considerations) as with the cases of Euler and Lagrange in the problem of the solid or again in the examples of the spherical pendulum and geodesics on a surface of revolution. We have also observed Sofya finding her least obvious first integral K. But what happens when we do not succeed in finding one?

The recent theorem of Morales and Ramis [Morales-Ruiz 1999] (see also [Audin 2002]) gives an important role in detection to some symmetry groups that are a bit hidden, the differential Galois groups. Briefly: we look at the differential system with a sort of microscope along one of its solutions, precisely the solutions infinitely close to the given solution. More technically, we replace the original differential system by a linear system—the variational equation already used by Poincaré (always in the *Méthodes nouvelles de la mécanique céleste* [Poincaré 1987]) and by Liapunov (see our page 122).

There is a group that takes account of these nearby solutions (to the first order). It is a rather large group (an algebraic group). The theorem of Morales and Ramis asserts that if the system is completely integrable, this group is almost commutative: the commutativity (in the sense of the Poisson bracket)

and the abundance of first integrals is translated into a commutativity property of the group. One of the applications is a new proof of the Ziglin theorem mentioned above.

***Theorem* (Ziglin [1983], Maciejewski and Przybylska [2005])**
If the equations of the solid form an integrable system in the Liouville sense (with meromorphic first integrals) then we have one of the three cases of Euler, Lagrange or Kowalevski.

This is the converse of Sofya's theorem stated on page 102!

Besides applications of the theorem of Morales and Ramis, we remark that this theorem relates a regularity property of solutions (like that of Kowalevski), specifically the fact that the Galois group is not terribly complicated with Liouville integrability. See also the article of Juan Morales [2000] for a more detailed discussion. Incidentally, Juan Morales concludes his article by remarking that the differential Galois theory (or the Picard–Vessiot theory) was born simultaneously with the Kowalevski top.

Now that the differential Galois theory [Morales-Ruiz 1999] allows for better grasping the different notions of integrability, it is undoubtedly time to re-evaluate the innovative and revolutionary qualities of the work of the mathematician Sofya Kowalevski.

CHAPTER VI

A LETTER TO MITTAG-LEFFLER

What is this about? In December 1884 Sofya, who went to spend Christmas in Berlin, writes to Mittag-Leffler, who is in Paris, a rather long letter. Like all Sofya's letters to Gösta, this one belongs to the Mittag-Leffler Institute. Reinhard Bölling was kind enough to send me a copy, which I had requested because I had read an allusion to the mathematics it contains in [Kozlov 2000] (there is a translation into Russian of Sofya's correspondence with Mittag-Leffler [Yushkevich 1984]); I found right off that it is a very beautiful letter, and I am very happy to have authorization to publish it here: it contains personal and social apects that will clarify other chapters of this book. Above all it contains very interesting mathematical remarks. Its mathematical content was reproduced by Pelageya Kochina [1985, p. 293], and it is the mathematical formulas of this letter that appear on the cover of her book. The English translation here is directly from the original French.

Despite some personal considerations connected to Aniuta's health, the content of the letter is not very intimate—thus no voyeuristic aspect to its publication.

I describe the letter, then present it in its entirety with some explanatory marginal notes; I then come to a more technical discussion of the mathematics it contains.

Description of the letter

The letter is not dated. However, it is clear that it was written between Christmas and New Year's Day. Reinhard Bölling tells me that the postal stamping bears the date 28 or perhaps 29 December, with a further stamping in Paris 30 December; at that time a letter sent from Berlin one day would arrive in Paris on the next, he adds. The letter is written in French. All

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_6,

commentators note Sofya's imperfect German; we have already seen this done by Reinhard Bölling and Weierstraß and we have yet to see Cordula Tollmien, Norbert Schappacher and perhaps others do it. Oh, well! No one can say the same about her French: the letter shows clearly how well she must have spoken this language. A few errors with homophones, a few past participals used with *to have* and not suitably agreeing with the direct object, a few errors with accents, nothing that would be heard in any case. It is all the more regrettable that with the exception of the brief study [Détraz 1993], there was previously no modern book in French on Sofya.

We now come to the material aspect of the letter. There are four pages on each side of each sheet, which Sofya folds in half. The stationery bears her elegant monogram.

There are a few deletions that I will not reproduce, words and phrases crossed out and some additions as in the letter that we saw Reinhard Bölling discover and describe (page 63). There are words written smaller and smaller toward the end of a line because there is not enough space, these line ends are curved by the ends of these words that descend into the margin. The postscript is written upside down at the top of the last page, again because there was not enough space. Sofya's handwriting is perfectly clear and there is no problem in reading it. I have kept all the capitals. The two underlinings of words were made by Sofya. All the notes are clearly of my creation.

The letter

In this letter, Sofya calls Gösta *Monsieur*, she addresses him formally, and she even uses capitals. The German influence?

Berlin
Schellingstrasse 16.
(W) Bei Frau Alm

Dear Monsieur!

The child in question is "Fränzchen", Franz Weierstraß (1882–1898). Reinhard Bölling tells me he is an illegitimate son, probably from Rosa Borchardt. Carl Borchardt, Weierstraß's friend who has already been mentioned, died in 1880. See [Bölling 1993, note 34, p. 359] and also [Biermann & Schubring 1996].

I am very sad not to have Your news. You seem to have forgotten me completely and although it is understandable enough that during the trip You did not have the time to write me, I find it hard to remain so long without hearing from You.

Did You get my letter in Rome that I sent care of the Swedish ambassador?

I have spent a rather sad Christmas. Weierstraß has gone to Weimar for a week to see his sister Clara and "the child". Monsieur Hansemann

[page 2]

my most loyal friend has fallen ill and has been forced to take to his bed for several days so that I am not able to see him. He is somewhat better now but will likely need to stay in his room for several more days.

Runge has left for his mother's in Bremen, which means that I am left completely alone.

As a Christmas present I received from Your sister an article of Strindberg in which he proves as clearly as 2 times 2 = 4 to what extent

This passage is quoted by almost all biographers. See Strindberg's interventions on page 211.

[page 3]

a monstrosity of a female professor of mathematics is pernicious, useless and disagreeable. I find that fundamentally he's right; the only thing I protest against is that in Sweden there is a large number of male mathematicians who are senior to me and that it was not by pure gallantry that I received my appointment.

Consider what a strange irony of fate, the three persons who have befriended me these last years are: — M^me^ Bochardt, Professor Emile Du Bois Reymond and Schwartz [Schwarz]! My three warmest friendships as you know!

This concerns the physiologist Emil Du Bois-Reymond and not the mathematician (and Sofya's former professor) Paul Du Bois-Reymond, who was his younger brother. Runge married Emil's daughter Aimée. Small world!

M^me^ B. has really put herself out in order to entertain me. Du Bois Reymond has gone to the greatest trouble in the world to get me permission [1] to attend courses in the new year and I believe he has succeeded; M. Schwarz wrote me yesterday

[page 4]

an ultra-amiable letter in which he announces that he is coming to Berlin in the new year especially to see me and speak with me.

Schwarz's letter, soliciting Sofya's help in dealing with a partial differential equation that interposed itself in a problem in the calculus of variations, is dated 25 December and is quoted in [Kochina 1985, p. 190].

And that, my dear Monsieur, is everything of interest that I can tell You today. Basically I am in a very sad mood because I have just received very bad news of my sister. Her illness is progressing appallingly. Now it's her eyesight that is suffering and she can no longer read nor write. The cause is always the same; she has a bad heart, the blood congeals in her capillary vessels and a partial paralysis results. I tremble at the thought of the loss that threatens me, perhaps in the very near future.

What a horrible thing life is and how stupid it is to continue living! Today is my very birthday; I am 31 years old today and

1. See pages 46 and 130 regarding this authorization.

This letter was written between Christmas and New Year's. Sofya was born on 15 January, so it is not her birthday, when she will be 35 in two weeks, not 31. She is "obligated" to lie about her age: some weeks before she sent Mittag-Leffler her curriculum vita, which he needed for defending her candidacy of 1884 in Stockholm. She stated there that she was born in 1853 and married in 1869.

I ask myself whether she did not want to thus obliterate the time that was lost during the Russian period.

In the obituary [Mittag-Leffler 1892–93] he published upon Sofya's death, Gösta gave the true birthdate, specifying that he had found a copy of her birth certificate among the papers of the deceased.

In her remembrances of George Eliot [Chapman & Gottlieb 1978, p. 363], Sofya recalls reproaching her for too opportunistically killing off her characters, giving several specific examples in the novels, and that the writer replied that this was actually what happened in life and added that "very often it is the belief in death that has given me the courage to live" [Chapman & Gottlieb 1978, p. 364].

basically it is hideous to think that I have just as long yet to live. But how beautiful it is in plays and novels:

[page 5]

scarcely anyone has discovered that life no longer holds anything pleasant for them without something immediately coming to take them into the hereafter. Reality is much inferior to this viewpoint. People talk so much about the perfections in the organism that living beings are developing in themselves little by little, by means of selection, etc. I find that basically the most desirable perfection would be the faculty to die quickly and easily. From this point of view humans are decidedly retrograde. Insects and lower animals could never decide to die;

[page 6]

it is astonishing how much an articulate can suffer without ceasing to exist; but the higher you go on the ladder of living things, the more the passage becomes quick and easy. For a bird, for a wild animal, a lion, a tiger almost every malady is fatal; either full enjoyment of life, or death. No suffering. But the human being is again close to the insects in this regard and many persons of my acquaintance make me think involuntarily of insects whose wings are ripped off, various joints crushed, their legs broken and nevertheless cannot decide to die.

[page 7]

Excuse me for writing in such a sad way today.

I am in a rather sad mood today. The worst thing is that I cannot feel any inclination for my work.

I haven't yet been able to force myself to seriously come to grips with my course for the next semester. But I muse a lot over the following problem: Take the following system of differential equations

$$\frac{dx}{dt} = ax^2 + by^2 + cz^2 + 2dyz + 2ezx + 2fxy$$
$$\frac{dy}{dt} = a_1x^2 + b_1y^2 + c_2z^2 + 2d_1yz + 2e_1zx + 2f_1xy$$
$$\frac{dz}{dt} = a_2x^2 + b_2y^2 + c_2z^2 + 2d_2yz + 2e_2zx + 2f_2xy$$

by substituting in place of $x\ y\ z$ the linear functions

$$\xi = \alpha x + \beta y + \gamma z$$
$$\eta = \alpha_1 x + \beta_1 y + \gamma_1 z$$
$$\zeta = \alpha_2 x + \beta_2 y + \gamma_2 z$$

we can reduce this system of equations to some simpler *types*.

[page 8]

One of the most interesting is the following

$$\begin{aligned} \frac{dx}{du} &= x(ax + by + cz) \\ \frac{dy}{du} &= y(a_1x + b_1y + c_1z) \\ \frac{dz}{du} &= x(a_2x + b_2y + c_2z) \end{aligned} \tag{1}$$

In the special case where between the constants a, b c etc we have the relations

$$\begin{aligned} a_2 &= a_1 = -a \\ b_2 &= -b_1 = b \\ -c_2 &= c_1 = c \end{aligned}$$

this system of equations can be integrated completely by elliptic functions. The system of general integrals appears in the form of linear functions of the three quotients

$$\frac{\sigma_1(u-u_0)}{\sigma(u-u_0)} \quad \frac{\sigma_2(u-u_0)}{\sigma(u-u_0)} \quad \frac{\sigma_0(u-u_0)}{\sigma(u-u_0)}$$

where the constants g_2 g_3 that enter into the formation of σ are arbitrary. This shows that the system of differential equations (1), for certain values of the constants $a \ldots a_1 \ldots a_2$, can be integrated by uniform functions of u, which have only a unique essential singularity

[page 9]

at $u = \infty$ and for finite values of u only poles of first order.

The first question that arises is whether this property is limited to the special case that we have examined or whether it extends to other values of the constants a b c etc?

To answer this question I reason in the following manner: Let

$$\begin{aligned} x &= x_0 + x_1u + x_2u^2 + \cdots \\ y &= y_0 + y_1u + y_2u^2 + \cdots \\ z &= z_0 + z_1u + z_2u^2 + \cdots \end{aligned}$$

be a system of elements of functions satisfying the equations (1). From a known theorem on the diff. eqns., on the circle of convergence of these series there

The elliptic functions in question here are associated with an elliptic curve that can be defined by two constants, g_2 and g_3 for example, which is to say that this curve can be described by the equation

$y^2 = 4x^3 - g_2x - g_3$.

[page 10]

must be a point u_0 at which at least one of the quantities $x\ y\ z$ becomes infinite. We must thus first of all determine whether a system of functions $x\ y\ z$ satisfying equations (1) can in general admit poles or only essential singular points; in other words, is it possible to satisfy eq. (1) with series of the form

$$(2)\qquad \begin{aligned} x &= x_{-m}(u-u_0)^{-m} + x_{-m+1}(u-u_0)^{-m+1} + \cdots \\ y &= y_{-m}(u-u_0)^{-m} + \cdots \\ z &= z_{-m}(u-u_0)^{-m} + \cdots \end{aligned}$$

where m is a positive integer (or even any positive number). We easily convince ourselves that this is possible in the case

$$m = 1$$

and that then it is always possible.

Each system of functions $x\ y\ z$ satisfying eq. (1) can only admit poles of the first order or essential singular points.

[page 11]

But here is what is most important. If we leave the constants $a\ b\ c\ a_1\ b_1\ c_1$ in the equations (1) arbitrary, the series (2) will be completely determined within a coefficient. This shows that the general integrals of eq. (1) must have yet other singularities other than poles. But in the case where, between the constants of the equations, the relation

$$a_1 b_2 c = a_2 b c_1$$

holds, one coefficient of the series (2) remains undetermined and these series contain, just as in the special case examined above where the integration is realized by means of elliptic functions, three arbitrary constants, which can be determined in such a way that the series take on the values ξ_1, η_1, ζ_1 for an arbitrary value $u = u_1$

[page 12]

This permits us to conclude that in this case the general integrals will also be uniform functions on the entire plane having a single essential point $u = \infty$ and nothing but poles of the first order for finite values of u.

That is where I am at the moment in my research. I don't know if I will be successful in bringing it much further. The question about what these uniform functions are whose existence we have demonstrated seems to me extremely interesting,

all the more so in that the study of the properties of these functions will one day throw light on those of more general functions

$$\frac{dx_\alpha}{dt} = g_\alpha(x_1, \ldots, x_n)$$

where g_α is a quadratic form in n variables.

[page 13]

I very much fear that the difficulties presented in the ensuing research far exceed my powers and that it will need to be undertaken by a geometer who is much cleverer than I.

I am very impatient for the news You will give me of the Paris mathematicians. At one moment I had almost decided to come to join you, but since presently Du Bois Reymond has gone to so much trouble to arrange the matter of my courses, I find it would be impossible to leave right now.

[page 14]

I have spoken with Kronecker regarding the bust of Weierstraß. Basically Knonecker has arranged the matter well enough: he has found a sculptor who has agreed to make a marble bust starting at an amount of 1200 mrk. If we raise more than this amount he will make a larger bust; if we do not raise that much (which would be more than shameful) we would have to settle for a relief.

I still haven't seen Helmholtz: I was at his house but he was not home.

Goodbye for now dear Monsieur. My best regards to Signe.

Best wishes to you S.K.

P.S. The address of my brother-in-law in Paris is Montmartre Rue Lepic 55.

This has to do with the bust that students and friends of Weierstraß were going to have sculpted to present to the master on the occasion of his seventieth birthday, 31 October 1885. For the presents made to Weierstraß on this occasion, see the article by Reinhard Bölling [1989]. See also the summary of the preparations made for this celebration in the book by Pelageya Kochina [1985, p. 150].

Sofya's brother-in-law who lives in Paris is Victor Jaclard, the husband of Aniuta.

Remarks

It is a letter, it is only a letter. I am aware that there is a certain ambiguity in publishing an isolated letter because its mathematical content is interesting. Warning! This really concerns a letter and not a mathematical article that has been revised and published. Sofya says she has mused on a problem ... not that she has solved it. We need to think of this as part of a discussion between colleagues, "look, I've been thinking about this" ... The idea expressed here is more important than the details of its realization.

It is a letter to Mittag-Leffler. Not to Weierstraß. Evidently it is not known and undoubtedly will never be known (unless however ... another box ...) what Sofya wrote to Weierstraß on this subject. But it seems to be an established fact, a fact that can be gleaned by consulting the letters published in [Bölling 1993], that Weierstraß, who was never sparing of commentary or ideas in his correspondence with Sofya and who commented in particular on her solution of differential systems (see [Bölling 1993, letters 148 and 151]), never discussed with her in writing the question of meromorphic solutions. However, one of his students, Paul Hoyer; had in 1879 (during Sofya's Russian period) defended a thesis [Hoyer 1879] in which he studies certain quadratic differential systems; he determined for them solutions expanded in Laurent or Puiseux series. This work never came up in Weierstraß's letters to Sofya [Bölling 1993], nor did it in Sofya's letters [Shtraikh 1951]. The idea of considering systems all of whose solutions are meromorphic as being worthy of interest was not Weierstraß's cup of tea, but seems to have been Sofya's original idea.

Here is what she wrote to Gösta three years before with regard to the solid, 21 November 1881 (there is an English translation of the entire letter in [Cooke 1984, p. 96]):

> The problem involves solving the general case of rotation of a heavy body about a fixed point by means of Abelian functions. M. Weierstrass had once previously suggested that I work on this problem, but all my attempts at the time were fruitless; and M. Weierstrass's own investigations showed that the differential equations of this problem cannot be satisfied by single-valued (*eindeutig*) functions of time. This result compelled me to abandon this problem for a while, but since then the beautiful, still unpublished research of our master on the stability of the solar system and the analogy with other problems of dynamics have renewed my zeal and given me the hope of satisfying the conditions of this problem by Abelian functions whose *arguments are not linear functions of time* ...
>
> This research seems so interesting and so beautiful that I have for the moment forgotten everything else and indulged myself with all the impetuosity of which only I am capable. The route I followed consisted of expressing the variables of the problem by theta functions of two variables which for certain values of the constants reduce to the elliptic theta functions that arise in the particular case of Lagrange, then trying to choose them so as to be able to integrate the differential equations between the theta

> functions and time. The calculations this led me into were so difficult and complicated that I cannot yet say if I will reach the desired end by this route. In any case in two or three weeks at most I hope to know what to think about it and M. Weierstrass is consoling me that even in the worst case I could always reverse the problem and try to find out which forces lead to a rotation whose variables can be expressed by Abelian functions—a poor problem, to be sure, and far from having the same interest as the one I have set myself, but I shall have to settle for it if I have bad luck

At this moment, what Weierstraß proposed was to find which forces yield Abelian solutions, if I understand correctly that it concerns initial conditions under which there exist such solutions. Moreover, he was interested in Abelian functions, not just any meromorphic functions.

Later, in the letter of 5 August 1883 that I mentioned on page 53 and in which he speaks to Mittag-Leffler about employment for Sofya in Stockholm, he describes his work and adds this commentary [Bölling 1993, p. 415]:

> Unfortunately she has lost a good deal of time on the problem of the rotation of a massive solid.

The solid. In this letter Sofya does not mention the solid problem, which does not mean that she was not thinking about it: as we have just seen, on 21 November 1881 she said to Gösta that she had been thinking about it for a long time.

The mathematics of the letter

Sofya considers a differential system of the form

$$\begin{aligned}\frac{dx}{dt} &= Q(x,y,z)\\ \frac{dy}{dt} &= Q_1(x,y,z)\\ \frac{dz}{dt} &= Q_2(x,y,z)\end{aligned}$$

where Q, Q_1 and Q_2 are quadratic forms. We have already encountered, and Sofya knows well, an example of a system of this type, the case where $Q = \lambda yz$, $Q_1 = \lambda_1 zx$, $Q_2 = \lambda_2 xy$, for

which we obtain the system

$$\begin{cases} \dfrac{dp}{dt} &= \dfrac{B-C}{A}\,qr \\ \dfrac{dq}{dt} &= \dfrac{C-A}{B}\,rp \\ \dfrac{dr}{dt} &= \dfrac{A-B}{C}\,pq, \end{cases}$$

which is the Euler case of the solid problem. The systems considered by Hoyer in his thesis are those of the form

$$\begin{cases} \dfrac{dx}{dt} &= ayz + bzx + cxy \\ \dfrac{dy}{dt} &= a'yz + b'zx + c'xy \\ \dfrac{dz}{dt} &= a''yz + b''zx + c''xy. \end{cases}$$

Another family of examples is that of system (1) of the letter,

$$\begin{cases} \dfrac{dx}{dt} &= xL(x,y,z) \\ \dfrac{dy}{dt} &= yL_1(x,y,z) \\ \dfrac{dz}{dt} &= zL_2(x,y,z), \end{cases}$$

where L, L_1 and L_2 are linear forms, which I write as

$$\begin{aligned} L(x,y,z) &= \alpha x - \beta y - \gamma z \\ L_1(x,y,z) &= -\alpha_1 x + \beta_1 y - \gamma_1 z \\ L_2(x,y,z) &= -\alpha_2 x - \beta_2 y + \gamma_2 z \end{aligned}$$

(I have allowed myself a small change in notation).

Here I discuss neither the normal forms of the general system nor Sofya's remark on her page 8 that system (1) is particularly interesting. Likewise I keep the notation t for the variable that Sofya calls u starting on page 8.

This special case is in fact equivalent to Euler's equations.

In the case where (in my notation) $\alpha = \alpha_1 = \alpha_2$, $\beta = \beta_1 = \beta_2$ and $\gamma = \gamma_1 = \gamma_2$, she affirms that the solutions of the system are elliptic functions (with only simple poles). Let us say this in a more geometrical way. If there are elliptic functions, then there has to be an elliptic curve. We easily see that the functions

$$x(\beta y - \gamma z), \quad y(\gamma z - \alpha x), \quad z(\alpha x - \beta y)$$

are first integrals. They are related by the relation

$$\alpha x(\beta y - \gamma z) + \beta y(\gamma z - \alpha x) + \gamma z(\alpha x - \beta y) = 0.$$

For general values of A and C, the curve of the equations

$$\begin{cases} x(\beta y - \gamma z) = A \\ z(\alpha x - \beta y) = C \end{cases}$$

(intersection of two quadrics, here two hyperboloids) is an elliptic curve—which can be viewed as a curve in the plane by eliminating, for example, the variable z

$$\alpha\beta x^2 y - \beta^2 x y^2 - (\alpha A + \gamma C)x + A\beta y = 0$$

(which is the curve shown here). The solutions of the system parametrize this curve. As functions of the variable $t \in \mathbf{C}$, they are doubly periodic (in particular, their poles tend toward infinity—as Sofya remarks at the beginning of her page 9, there is an essential singularity at infinity). Like all elliptic curves, this one is equivalent to a curve with the equation

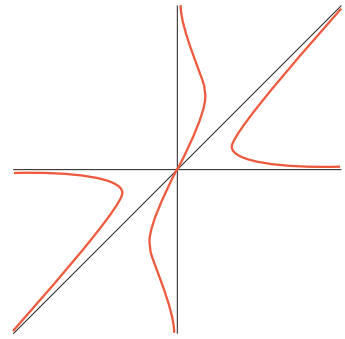
$$y^2 = 4x^3 - g_2 x - g_3,$$

where the constants g_2 and g_3 are mentioned by Sofya on her page 8.

If neither the expression *first integral*, nor any of its synonyms, does not figure in the letter, the notation for elliptic functions, which is not given there explicitly either, necessarily causes some form of the equation of the curve to appear and thus, implicitly, the so-called *conserved quantities*. Let us not commit anachronism here: it is quite clear that for Sofya the first integrals are not the essential object but just auxiliary tools that yield the solution.

She subsequently returns to the general case of system (1) and asks whether there are other cases where the solutions are meromorphic, i.e. have only poles as singularities in the finite plane. The method she proposes, and which she will use for the case of the solid in [Kowalevski 1890–91], is the following: we write the solutions a priori as she does on her page 10, we determine the order of the pole (she finds $m = 1$) and we then try to compute the coefficients in the expansions inductively. For instance, in the sub-example that she considered and which, as I just said, is an integrable example, this computation yields the following result. With

$$\begin{cases} x = t^{-1}(x_0 + x_1 t + \cdots) = t^{-1}X(t) \\ y = t^{-1}(y_0 + y_1 t + \cdots) = t^{-1}Y(t) \\ x = t^{-1}(z_0 + z_1 t + \cdots) = t^{-1}Z(t) \end{cases}$$



Yet another elliptic curve, having a somewhat different aspect from the one on page 95 because it has three points at infinity (corresponding to the three asymptotes) whereas the other had but one.

the differential system becomes

$$\begin{cases} -X + t\dot{X} = X(\alpha X - \beta Y - \gamma Z) \\ -Y + t\dot{Y} = Y(-\alpha X + \beta Y - \gamma Z) \\ -Z + t\dot{Z} = Z(-\alpha X - \beta Y + \gamma Z) \end{cases}$$

(here and in the remainder of the chapter, $\dot{X}$, etc., denote the derivatives of X, etc., with respect to t). We determine x_0, y_0 and z_0 by setting $t = 0$. There is no solution when just one of the coordinates (x_0, y_0, z_0) is zero, but there is when two of them are zero. When all the coordinates have a simple pole at 0, they satisfy

$$\begin{cases} \alpha x_0 - \beta y_0 - \gamma z_0 & = -1 \\ -\alpha x_0 + \beta y_0 - \gamma z_0 & = -1 \\ -\alpha x_0 - \beta y_0 + \gamma z_0 & = -1, \end{cases}$$

a linear system whose determinant is $-4\alpha\beta\gamma$ and which has the unique solution

$$(x_0, y_0, z_0) = \left(\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}\right)$$

assuming that the determinant is nonzero, i.e. that none of the three denominators is zero. It is this that makes for the simplicity of the example in question.

The following coefficients are given subsequently, in succession, by other linear systems. We equate the coefficients of t^m on the two sides,

$$-x_m + m x_m = x_m(\alpha x_0 - \beta y_0 - \gamma z_0) + \cdots + x_0(\alpha x_m - \beta y_m - \gamma z_m)$$

where the terms I have neglected to write only enter into coefficients that have theoretically already been calculated, and we obtain the system

$$\begin{cases} \alpha(m-1)x_m + \beta y_m + \gamma z_m = A_{m-1} \\ \alpha x_m + \beta(m-1)y_m + \gamma z_m = B_{m-1} \\ \alpha x_m + \beta y_m + \gamma(m-1)z_m = C_{m-1}, \end{cases}$$

for which the determinant is $\alpha\beta\gamma(m-2)^2(m+1)$, which is to say that x_m, y_m and z_m are uniquely determined by the x_i, y_i and z_i (for $i \leq m-1$) ... except for $m = 2$, a case where the three left-hand sides are identical. The unique solution for

$m = 1$ is $(x_1, y_1, z_1) = (0, 0, 0)$. The system for $m = 2$ thus reduces to the equation

$$\alpha x_2 + \beta y_2 + \gamma z_2 = 0,$$

which has a space of solutions of dimension 2. So there are two constants of integration in the determination of our formal solutions. With the placement of the pole, we have three parameters for a third order system, thus all the solutions have the desired property. It remains to verify that the solutions obtained are convergent; but Sofya knows this, since she has already written them as elliptic functions (on her page 8).

In fact, the formal solutions thus obtained are always convergent (see [Adler et al. 2004, § 7.2]).

If we return to the general case of the three linear forms L, L_1 and L_2 and if we wish to determine whether the solutions are meromorphic (or to determine the choices of L, L_1 and L_2 for which they are) we are confronted with two problems.

First, the determination of the order of the poles. Then a rather tedious problem of linear algebra: finding the dimension of the space of solutions for a whole family of linear systems. Here is the family in question if we suppose that the three coordinates have a pole of order 1 exactly:

$$\begin{cases} \alpha x_0 - \beta y_0 - \gamma z_0 & = -1 \\ -\alpha_1 x_0 + \beta_1 y_0 - \gamma_1 z_0 & = -1 \\ -\alpha_2 x_0 - \beta_2 y_0 + \gamma_2 z_0 & = -1, \end{cases}$$

then

$$\begin{cases} (m - \alpha x_0) x_m + \beta x_0 y_m + \gamma x_0 z_m & = A_{m-1} \\ \alpha_1 y_0 x_m + (m - \beta_1 y_0) y_m + \gamma_1 y_0 z_m & = B_{m-1} \\ \alpha_2 z_0 x_m + \beta_2 z_0 y_m + (m - \gamma_2 z_0) z_m & = C_{m-1}, \end{cases}$$

The general case of a system $\dot{x}_i = F_i(x)$ where the F_i are quadratic forms, is studied in [Adler et al. 2004]. The matrix in question is called the Kowalevski matrix. The calculation is in fact completely analogous to that of [Kowalevski 1890–91].

and we need to emphasize that, for certain integer values of m, this system does not have a unique solution; in other words, the fact that the matrix whose three rows are $x_0 L$, $y_0 L_1$ and $z_0 L_2$, with (x_0, y_0, z_0) the solution of the first system, has a strictly positive integral eigenvalue (we verify without difficulty that it always has eigenvalue -1).

I said tedious—it is so tedious to do it well that it is quite possible to allow a case to escape! Consider for example the case where

$$L = -y + z, \quad L_1 = x - z, \quad \text{and } L_2 = -x + y,$$

a decent case, studied notably in [Adler et al. 2004, § 6.1]; an integrable case too if the two functions $x + y + z$ and xyz are first integrals, as we can verify immediately, another case whose solutions are elliptic functions, the curves

$$x + y + z = h \text{ and } xyz = k$$

being, in general, elliptic curves. However, in this case the system

$$\begin{cases} & -y_0 & +z_0 & = & -1 \\ x_0 & & -z_0 & = & -1 \\ -x_0 & +y_0 & & = & -1 \end{cases}$$

does not have a solution: the sum of the left-hand sides is 0 but not the sum of the right-hand sides. Clearly it is impossible that x, y and z have a pole simultaneously, since their product is constant. A case that we would have risked missing!

It is a reproach that Марков (Markov) will make regarding Sofya's work (see page 171), and for which Lyapunov will subsequently make another approach to Sofya's property: consider a particular solution of the system and along it the variational equation. He states that, if the nearby solutions are meromorphic, then those of the variational equation must be also. Warning! We are of course dealing only with a necessary condition.

This is what we get following Lyapunov's idea. We suppose for simplicity that one of the diagonal terms α, β_1 or γ_2 is nonzero, for example $\alpha \neq 0$. Then the differential system has the particular solution

$$x = -\frac{1}{\alpha t}, \quad y = 0, \quad z = 0.$$

We note that only x actually has a pole. The linear differential system (variational equation) along this solution is

$$\begin{cases} \dot{X} = XL(x,0,0) + xL(X,Y,Z) \\ \dot{Y} = YL_1(x,0,0) \\ \dot{Z} = ZL_2(x,0,0) \end{cases} \qquad \text{with } x = -\frac{1}{\alpha t}.$$

In his book, Golubev [1960] uses instead a method of small parameters to arrive at the desired conclusion in the case of the solid. I used [Audin 2007] a variant of Lyapunov's method indicated by Luc Haine [1984], to compare Kowalevski's integrability property with Liouville integrability.

We easily solve the last two equations

$$\begin{cases} \dot{Y} = -\dfrac{\alpha_1 Y}{\alpha t} \\ \dot{Z} = -\dfrac{\alpha_2 Z}{\alpha t} \end{cases}$$

yielding

$$Y = Bt^{-\alpha_1/\alpha}, \quad Z = Ct^{-\alpha_2/\alpha},$$

and we then solve the first equation:

$$\dot{X} = -\frac{2X}{t} + \frac{\beta}{\alpha}Bt^{-(\alpha_1/\alpha)-1} + \frac{\gamma}{\alpha}CBt^{-(\alpha_2/\alpha)-1}$$

yielding $X = A(t)t^{-2}$, where A is a primitive of

$$\dot{A} = \frac{\beta}{\alpha}Bt^{-(\alpha_1/\alpha)+1} + \frac{\gamma}{\alpha}Ct^{-(\alpha_2/\alpha)+1}.$$

Liapunov's condition is here that the quotients $-\alpha_1/\alpha$ and $-\alpha_2/\alpha$ be integers. By considering other particular solutions we can find other necessary conditions. I will not discuss the general solution of the problem here—this is not the place for it! The sub-example considered by Sofya corresponds to the case where $-\alpha_1/\alpha = 1$, $-\alpha_2/\alpha = 1$, etc.

As to the remark for the case where $a_1b_2c = a_2bc_1$ (on page 11 of Sofya's letter), here

$$\alpha_1\beta_2\gamma = \alpha_2\beta\gamma_1,$$

I am not able to understand how she found it. It seems erroneous to me: let us write $\alpha_i = m_i\alpha$, $\beta_i = n_i\beta$, $\gamma_i = p_i\gamma$ $(i = 1, 2)$ and suppose that α, β and γ are nonzero. The condition of the letter becomes $m_1n_2 = m_2p_1$—and it suffices to choose these four numbers not all integers and satisfying this equality ($m_1 = m_2$ and $n_2 = p_1$ whatever for example) to Sofya's delight without satisfying Lyapunov. It is likely that Sofya thought of this condition, among others. Once again, we mustn't take what Sofya wrote in this letter for a result in a mathematical article, she is merely "musing" about the problem!

CHAPTER VII

STOCKHOLM

Back before I had read the article on the solid in detail, I mentioned to a colleague that it must be an exceptional piece of work, since the Academy of sciences had increased the amount of the Bordin prize. "Oh, not at all, it was to help her out, because she did not have a position", was the reply he gave me. A bit of condescension once again and a subtle way of putting down work whose exceptional quality was, in fact, recognized by the Academy. And anyway, it was not quite true that Sofya did not have a position. At the time when the Bordin prize was declared in 1886, as well as at the time when it was awarded in 1888, Sofya had a professorial position in Stockholm and was receiving a salary, even if her position was not yet permanent.

Sofya's position in Stockholm

What we are calling "the" position of Sofya in Stockholm is in reality three positions. For which there is a story in three acts.

First act. Privatdozent. This is how Mittag-Leffler tells, in his obituary [1892–93], the history of Sofya's hiring in Stockholm.

> Some years before the death of her husband, Sophie Kovalevsky had expressed the wish to devote herself to teaching as a university professor. Knowing her wishes and having long shared M. Weierstraß' high opinion of his student's exceptional talent, in the autumn of 1880 I began a plan to have Sophie Kovalevsky appointed my *docent* (adjunct professor) at Helsingfors (Helsinki) University, where I held the mathematics chair. My plan failed;

I have used the German spelling *Privatdozent* (but Gösta writes *docent*, which is the Swedish word), mostly because it is said [Leffler 1898] that during her Russian years, Sofya had published a novel entitled *The Privatdozent*, in which she described life in a small German university town (I have never seen a precise reference to the text, which might be described as mythical).

M. Audin, *Remembering Sofia Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_7,

> but when in the spring of 1881 I was called to the newly founded university in Stockholm, (1) I immediately began negotiations with the university authorities to the end of having M^me^ Kovalevsky named my assistant, if she consented.

I interrupt our Gösta's narrative to remind readers who may have forgotten that Russia still administered and controlled Finland at the time when Mittag-Leffler taught at Helsingfors (the tsar of Russia was Grand Duke of Finland) which means that Sofya's sex was not solely responsible for the failure of this first attempt—her expressed political opinions also contributed, the Finns fearing that her nomination could appear as a provocation. And I will take advantage of this break to point out too that Sofya, like Weierstraß, was completely aware of the historical importance of an appointment in Stockholm and the responsibility she would be assuming. Here in fact is what she wrote Gösta on 21 November 1881 (in fact, in reference to the letter that I have already quoted in part on page 116):

> You undoubtedly know as well as I how much respect and friendship bind me to M. Weierstraß and how much interest he consequently always shows in everything concerning me. You may well believe that in such a serious matter I allow myself to be completely guided by him. His opinion in this matter is the following: he thinks that the appearance of a woman in the role of docent to a chaired professor is a serious step, and could have serious consequences for the question of my eagerness to serve, and that I do not have the right to take a decision before showing my capabilities in purely scientific work. M. Weierstraß consequently thinks that it is absolutely essential that I complete the research that occupies me at the moment and to which I have devoted more than a year, and that before its completion I mustn't allow myself to be distracted by anything else nor accept such serious obligations as those you would propose to me. I must admit that I find M. Weierstraß' reasoning so correct that I cannot do other than conform to it entirely. You consequently see, dear Monsieur, that it is unfortunately out of the question that I take a position already this winter. But I repeat

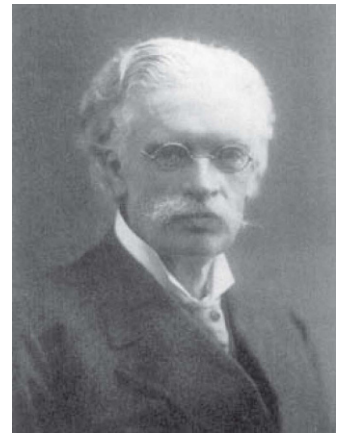
She was working on double refraction, but she was also thinking about the solid, as the portion of her letter already quoted on page 116 shows.

1. I have been calling this institution *Stockholm University* although it did not officially assume this name until 1960. Founded in 1878, this innovative semi-private institution was called *Stockholms Högskola* (Stockholm high school). See [Domar 1978]. A European high school is a post-secondary educational institution.

> that I would be very happy, once my research is complete, if you would once again take this affair in Your hands.

There is also—which she does not say, perhaps because she is not yet aware of it—the delicate status of a woman separated from her husband. Again, it is in 1883 that the situation has changed. We give Gösta the word again, still from [Mittag-Leffler 1892–93]:

> [...] the principal difficulties, which were until then opposing realization of her desires, were to disappear completely upon the death of her husband. In a letter dated 5 August 1883, M. Weierstraß informed me that she was prepared to offer a mathematics course in Stockholm, but at the outset she wanted nothing in the way of publicity given this course.



Gösta Mittag-Leffler

Sofya was thus appointed Privatdozent at Stockholm university in 1883. When today we say that she obtained a position in 1883, we may not know what that means exactly. Well, here it is: this position gave her the right to enter the university (recall that in Berlin she did not have this right and moreover still would not have in 1883) and even to teach a course. These rights were not obtained without some difficulty, and not because the Swedes were not more advanced than the Russians or the Germans, but because this university in Stockholm was new. Gösta Mittag-Leffler, the first professor to be appointed there, explains [1923, p. 191]:

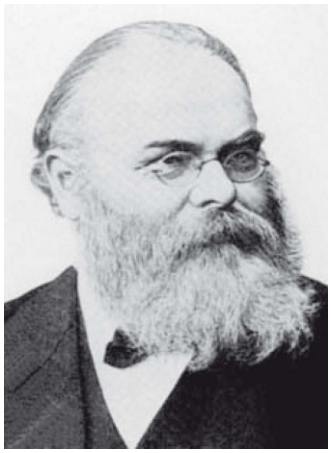
> This will always be an honor for Sweden, the young Stockholm University and the enlightened men and women who were a part of it having a hand in adding such a force as that of Sophie Kowalewsky. Would such a thing have been possible in any other European university? But on the other hand this would be an ill-placed boast to claim that Sonya's engagement was proof of a more advanced social culture from the feminist point of view in Sweden than in other countries. Her appointment above all succeeded by a sort of surprise that did not give the opposition time to organize sufficiently.

The weight of Weierstraß's support was certainly essential. Mittag-Leffler does not say it explicitly, but it is likely mostly due to his own political prowess that the operation succeeded.

Mittag-Leffler. It is more than time for a digression to include a few words on the political skills of Mittag-Leffler. He is without any doubt the first and perhaps to this day the greatest

strategist and tactician for scientific politics that the mathematical community has ever seen. His influence was increased by his marriage; his spouse Signe Lindfors was heiress to a great fortune—it was thanks to this fortune that he was able to get *Acta Mathematica* started—a fortune he used too for establishing his enormous library and for building their Djursholm villa which today houses the Mittag-Leffler institute.

This was an enthusiast for international contact: he knew everyone. He is responsible for the development of mathematics in Sweden, he created and ran the excellent international journal *Acta Mathematica*, to which I will return in more detail later in this chapter, he persuaded the king of Sweden, Oscar II, to establish prizes honoring European mathematicians, he found funds for some of these to come to Stockholm to give prestigious courses, as Painlevé did in 1895 (the *Leçons de Stockholm* [Painlevé 1897] to which allusion was made on page 106 constitute an edited version of his course) and Volterra in 1896, he instituted (that is the right word) the institute that bears his name and he was one of the instigators of the first International Congresses of Mathematicians which take place every four years. Recall (see note 1 on page 46) that the very first version of his article [1923] was the substance of a communication to the 1900 *Congrès* de Paris, the second of these International congresses, the one for which Hilbert drew up his famous list of problems for the 20th century. In homage to Mittag-Leffler, the 1916 Congress would have taken place in Stockholm ... if indeed it had taken place at all.



Hermann Schwarz (1843–1921)

He was not appreciated by everyone. It is known that he was at loggerheads with Schwarz—a rivalry between mathematicians—and also with Alfred Nobel—a rivalry between men of power. Those who knew him certainly describe him as a man of power, but also a man of integrity, and loyal. See [Cooke 1984, pp. 89–91].

It is certain the success in the affair of Sofya's position contributed to his fame, but this was far from being assured in advance and carried risk. Sofya understood this well and feared that his efforts would undermine his position. Gösta was Sofya's special admirer and very sincere friend. We will see, in his letters or in his diary, that he could get irritated with her—without the causes of his irritation ever diminishing the deep affection he found for this colleague and friend (and of which she is very aware, already in 1882, as the letter which is the object of chapter VIII will show).

End of digression. Thus Sofya obtained a position, but one which was not accompanied by a salary, as she ironically remarked:

> Just look at that! They have made me into a princess! I would have preferred that they give me a salary!

while commenting, in a letter she wrote her brother-in-law Alexander Onufrievich, on the following newspaper article (she mentions the same article [2] to Weierstraß in the "burned letter", see [Bölling 1993, p. 427]):

> Today we have to inform you *not* about the arrival of some prince or other equally highly placed but totally ignorant personage. No, it is instead a princess of science, Mme Kovalevskaia, who honors our city with a visit and who will become the first female privat docent in all of Sweden.

Her remuneration would come from her auditors and was thus dependent on their number and the extent of their satisfaction.

Second act. Associate Professor. We again take up the account of [Mittag-Leffler 1892–93].

> In December 1883 Sophie Kavalevsky arrived in Stockholm, and during the spring semester of 1884, and before a limited but attentive audience she expounded, in German, on the theory of partial differential equations. Thanks to the success of the course and the impression made on intelligent circles in Stockholm by the sympathetic personality and genius of the speaker, it was possible for me to come up with the funds for appointing Sophie Kovalevsky professor of higher analysis at Stockholm university for a period of five years. Despite the short time she had lived in Sweden, she already had a good enough command of our language to allow her to teach in Swedish from her debut as professor at the university.

She is thus rapidly appointed (28 June 1884) associate professor (extraordinary professor), but not quite as easily as Gösta seems to imply. There again the words are deceptive. Extraordinary does not mean exceptional and has nothing to do with what today is called *classe exceptionelle* of French professors, but rather to what has here been called, twenty years ago, an assistantship. Moreover, the position was temporary, with a five-year term.

2. The letter and article are quoted in [Koblitz 1993, p. 179].

I don't know precisely what were the standards of the time. The fact remains that when Sofya was appointed to this position, she had but one published article [Kowalevski 1875], that of the Cauchy–Kovalevskaya theorem. The two other memoirs of her thesis were not yet published and the remainder of her output was yet to come. Her work in progress was certainly taken into account. Here is an extract from the report that Hermite sent in support of her candidacy [Dugac 1985, p. 201]:

> Madame Kowalevski's mathematical talent is brilliantly revealed in her inaugural dissertation and in a work of great importance on the theory of light that I have presented to the Paris Academy of sciences. The exposition of Mr Weierstraß's method for establishing the existence of a function satisfying a partial differential equation which was the subject of the inaugural thesis shows a rare gift for extreme clearness of thought as well as of extensive knowledge of analysis at the highest level.
>
> This beautiful work fills in a gap in the science and takes its place in instruction alongside that of MM. Briot and Bouquet on an analogous topic concerning ordinary differential equations.
>
> The mathematical research of Madame Kowalevski in mathematical physics deals with the propagation of light in a crystalline medium. [...] Such a rare talent, such a superior intelligence cause me to wish that, in the interest of mathematical instruction, Stockholm university will enlist Madame Kowalevski's assistance.

This work was the topic of the note [1884a] that Hermite transmitted to the Academy on 11 February 1884.

Here I skip the compliments on a work that turned out to be wrong.

We add, more anecdotally and even if this has already been pointed out, that she and she alone was given the right to go hear Weierstraß lecture in Berlin and even the exorbitant right to enter all Prussian universities. See page 3 of the letter quoted in chapter VI (page 111).

Third act. Full professor. Continuation of Mittag-Leffler's report.

> Before the five years expired, Sophie had won the Bordin prize from the Institut de France [...] This circumstance eased my efforts to gather the funds necessary for permanently establishing the chair in higher analysis at Stockholm university. It was in the spring of 1889 that our university could be assured of the continued services of Sophie Kovalevsky by giving her tenure for life.

According to Jan-Erik Björk [2002, p. 13], the higher analysis chair was opened to competition (as we say today) in March. The two other mathematicians who were capable of occupying it and thus of competing, Lars Edvard Phragmén and Ivar Bendixson, declared that they considered Sophie superior and did not become candidates themselves.

The Bordin prize (to which I will return in chapter IX) helped her obtain a lifetime appointment (but she only had a short time

to live) on 6 June 1889, and although Sofya, to Gösta's great displeasure, did not return to Stockholm at the beginning of the year to celebrate her success (see [Björk 2002] for the details). The letters of recommendation sent by Beltrami, Bjerknes and Hermite, three independent specialists, an Italian, a Norwegian and a Frenchman, were also useful. Although I have trouble assessing the fame of Bjerknes, a Norwegian specialist in hydrodynamics (who is also the author of a biography of his compatriot Niels Abel), I can affirm that all mathematicians today still know the names of Beltrami and Hermite.

That same year, Sofya would receive a prize from the Swedish academy and would be elected corresponding member of the Russian academy of sciences. In France she was even decorated (on 13 July 1889) with the title of officer of public instruction.

Sofya was not the first woman to be elected member of an Academy of sciences, for it was our "first woman", Maria Agnesi, in Bologna.

Life in Stockholm—professional life

Appointed in this city, Sofya thus arrives in Stockholm on 17 November 1883. A few months later, having briefly left her adopted country for a trip to Russia, she writes ([Leffler 1898]):

> I seem to have found a new country in Sweden, a new family, at the moment of my life when I had the greatest need ...

In Stockholm, Sofya learned Swedish to the extent that, although she gave her first lecture series in German, she was able, beginning in September 1884, to do the following in Swedish, she started skating, riding, dancing, she sent for her daughter, she took up her work again. For the first time since her student years in Berlin she found herself doing the work for which she was competent, which she wanted to do, she could set to work in a narrow, but normal, university environment. It was not just that she had a new life, but her most creative period was now beginning.

She attracted attention wherever she went: in her much-admired lectures at the university, in society, in literary circles. The princess of science was accepted enthusiastically ... but not by everyone: for example, the posters announcing her lectures were torn down by her colleagues at Uppsala University.

She participated in the seminars (that were held at Gösta's house). She gave courses at the university—a dozen during the seven years she spent there—with enthusiasm, at least in the

See [Björk 2002, p. 23] for Gösta's description of her courses that he drafted when he wrote the University council nominating her to a full professorship.

According to Jan-Erik Björk [2002, p. 14], Sofya held her first class on 11 February, but Weierstraß writes 1 February in one of his letters (see [Bölling 1993, letter 128]).

beginning. Her first course focuses on the Dirichlet problem, an active subject where she could report on her own research (Cauchy–Kovalevskaya theorem). After her very first lecture on 30 January 1884, she writes in her diary [Kochina 1985, p. 131]:

> Gave the first lecture today. Don't know whether it was good or bad, but I know that it was very sad to go home and feel so lonely in this world. I feel especially lonely at such times. *Encore une étape de la vie derrière moi.* (3)

It is not surprising that Sofya felt alone. Recall that when she arrived in Stockholm two months before she knew no one except Gösta ... whom she had seen previously, if I am counting correctly, but three times: in 1876 and 1880 in Saint Petersburg and then, in Spring 1882, in Paris. In his diary Gösta notes, regarding that same lecture (quoted in [Björk 2002, p. 22]):

> At the outset, Sofya was nervous and had difficulty speaking—but she quickly improved. When the lecture ended, she received applause from her listeners. It was clear from the beginning that she would be an excellent lecturer.

If at the time of the first lecture she looked only at the blackboard and left the hall as soon as she had put down the chalk, she quickly came to feel more at ease. One of her female students relates [Kochina 1985, p. 132]:

> I always felt that Mrs Kovalevskiaya saw through me as if I was made of glass, but at the same time I felt at rest under her tender and sure gaze.

This is what Sophie writes to Gustav Hansemann in 1885. She excuses herself for not writing sooner because of everything she has to do and of which she makes him a list, beginning with [Leffler 1898, p. 223]:

> First I have of course to think about my three lectures on the algebraic introduction to Abel's theory and everywhere in Germany these lectures are considered most difficult. I have many auditors and have kept almost all of them, except for two or three.

Among the auditors of these classes, besides Mittag-Leffler, Bendixso and Phragmén (it was Phragmén who took Sofya's position after her death), we might also note the presence of Ivar Fredholm, whose name is well known to mathematicians today and who completed his thesis only in 1898. Sofya's first

3. Yet another stage of life is behind me.

students presented her, at the end of the course, with a framed photograph of herself [Cooke 1984, p. 103].

The salary issue

We go back several years, to 1881, when she leaves her husband and departs for Berlin and Paris. Weierstraß and Mittag-Leffler began to busy themselves with finding her a position. In June 1881 she writes to Mittag-Leffler from Berlin (quoted in [Leffler 1898, p. 204]): (4)

> I can assure you that if [the position as Privatdozent] were offered to me, I should gratefully accept. [...] Without being rich, I still have the means for living independently. The question of salary is, therefore, of no importance to me in coming to a decision. What I wish, above all, is to serve the cause in which I take so great an interest; and, at the same time, to be able to live for my work, surrounded by those who are occupied with the same questions—a piece of good fortune I have never enjoyed in Russia, but only in Berlin.

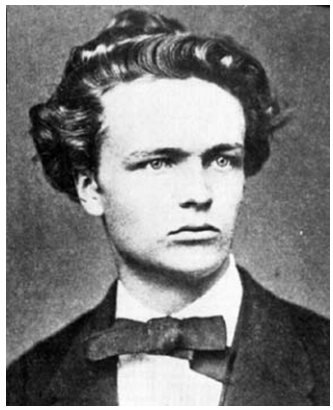
Two years later, as I have said, her husband's suicide finally eased her employment in Stockholm. But remember that the reasons for Vladimir's suicide, which made him bequeath debts to Sofya. In addition, Sofya had to help his younger brother (see [Koblitz 1993, p. 191]). We have seen (page 129) that she sees the question of salary a little differently but is happy to accept the position. In August 1883 she writes, still to Mittag-Leffler, this time from Odessa where she is passing the summer with her daughter at the home of Alexander Onufriévitch, Vladimir's brother ([Leffler 1898]):

Sofya's brother-in-law, Alexander Onufriévitch Kowalevski, was a prominent zoologist.

> I am truly grateful to Stockholm, which is the only European university that will open its doors to me, and I am already prepared to be in love with that city, and to attach myself to Sweden as though it were my native home. I hope that, if I do come there, it will be to find a new fatherland.

After she was appointed to the post for five years, the author August Strindberg wrote (in 1886) to a mathematician (quoted in [Domar 1978, p. 10]):

4. The original was written in French; I assume that Anne Charlotte translated it into Swedish, before it was translated in English.



August Strindberg (1849–1912)

Apparently Strindberg, who perhaps knew the writer Anne Charlotte Leffler better than her mathematician brother, did not know that the latter had attached his mother's name to his birth name.

> Since I have been given the task of writing for a French journal regarding Mrs. Kovalevski but have not the ability to judge her mathematical work, I am taking the liberty of requesting your statement as regards her ability. It would be especially useful for me to know in what relationship her dissertation stands to Professor Weierstrass, and if her mathematical merit is in any proportion to her great scientific reputation.
>
> Further, I would like to learn more about the appointment. Was Leffler's salary divided? And were two professors needed?
>
> With the hope that you will honor me with a frank answer, and with a promise of discretion,
>
> August Strindberg

After having read so many (recent) assertions devoid of the least rigor (some of which I have already cited, but there are more to come) and which have not made me proud of some of my colleagues, I am happy to be able to write here that according to my colleague Yngve Domar [1978], the mathematician in question, Gustav Eneström, who was also secretary of *Acta Mathematica*, did not respond to this letter.

Regarding the question of salary, let us say that the answer is "no", as we would suspect: "Leffler"'s salary was not affected by Sofya's appointment. And, for those who naively ask themselves whether Sofya earned as much as Gösta, why then the answer is, as should be expected: "no". Throughout the excepts from Gösta's diary quoted in [Hörmander 1991] we find the details: at the outset she earned 4,000 Kronor (according to [Björk 2002, p. 22], professors at Uppsala earned 6,000 Kronor per year) and in 1888 she earned 6,000 Kronor (of which 1,000 Kronor came directly from Mittag-Leffler—recall that the financing of the institution was predominantly private; recall too that Gösta was very wealthy (thanks especially to his marriage), independent of the fact that he drew a salary of 7,000 Kronor).

Requests by Sofya for increases eventually irritated her "big brother" Gösta, who comprehended no better (as we shall see) than his sister Anne Charlotte the difficulties faced by Sofya in her daily life. Sofya also experienced the (not yet classic) conflict between career and family life when she requested, in autumn of 1886, leave, which was refused her, in order to go care for her sister Aniuta. That same year her daughter was with her in Stockholm and Sofya experienced the life of a single mother and university professor who was, let us recall, completing an important research project.

Until this time, no professor ever had the idea of making such a request, undoubtedly for the simple reason that none of these great men would ever have gone to care for a sick sister, this is something women are good for.

When Gösta reproached her for complaining too much, she responded [Leffler 1898, p. 225]:

> When a Swedish woman is tired or in a bad mood, she pouts and does not talk. That is why her bad mood enters her organism and becomes a chronic disease. On the contrary, a Russian women moans and wails so intensely that it produces the same mental effect as a limeleaf tea produces physically on influenza. On top of that, I have to tell you that I only moan and start wailing when I am slightly pained. When I am in great distress, I too am silent and no one can detect my anguish.

Life in Stockholm—public opinion

In her sketchy autobiography which can be found in English translation in [Kovalevskaya 1978], Sofya writes:

> As I have said, I have been living in Sweden since 1883 and have adapted so completely to the lifestyle that I feel really at home. Stockholm is a lovely city and its climate is rather good—except for spring, which is unpleasant. I have a large circle of friends and an active social life. I am even received at court.

The Russian ambassador had Sofya received officially by King Oscar II on 28 October 1884, at the request of Sofya herself—she recounts in a letter to Mittag-Leffler that she even gave the King a lesson [Cooke 1984, p. 108].

But Stockholm is a small city, a quite small city in the depths of Europe, at best a provincial capital. And then Sofya, with her bohemian style, her freedom, her political opinions accords poorly with the conventional coldness and reserve of polite Swedish society, always ready to consider and comment upon what she did. She sensed this when she wrote Vollmar from Paris in 1882 the letter which is the object of chapter VIII. And her fears were justified. In the "burned letter", a few days after she arrived in Stockholm, she already wrote, as an accompaniment to the newspaper article where she was appointed princess [Bölling 1993, p. 427]:

> You must know that Stockholm is the funniest little town in the world, where everything is known about everyone and where the smallest incident takes on the proportions of a world event.

It is not clear that she found the situation amusing for long. Here is how she talks about it, *a posteriori*, but as soon as June 1884, in a letter to Mittag-Leffler (or to Anne Charlotte?) [Leffler 1898, p. 214]:

> I am quite willing to submit to the judgment of the Stockholm ladies in all that has to do with the minor details of life, but in serious questions, especially when I do not act in my own interests, but in those of my child, I think that it would be an unpardonable weakness on my part, were I to let myself be influenced by the shadow of a wish to play the part of a good mother in the eyes of Stockholm petticoats.

So there it is her life with her daughter that is in question. She likewise refused to hide her friendship with the socialist leader Karl Hjalmar Branting, which would still be used against her in the debate preceding her appointment as professor in 1889 (see the excerpt from Gösta's diary quoted in [Björk 2002]).

It should be noted that thirty years later Branting, a leader of the social democratic party, became prime minister. He even received the Nobel Peace Prize in 1921 (see again [Björk 2002]).

Life in Stockholm—Anne Charlotte Leffler

To imagine how the right-thinking bourgeoisie of Stockholm might look at Sofya's lifestyle, it suffices to read what her friend Anne Charlotte Leffler says about her interior decoration [Leffler 1898, p. 237]:

> The furniture sent from Russia was very characteristic. It came from her parents' home, and had the old aristocratic look about it. It had occupied a large drawing-room, and consisted in a long sofa, which took up a whole wall; a corner sofa of the old pattern, with floral decorations; and a deep armchair. It was all of rich carved mahogany, upholstered in a bright-red silk damask, now old and tattered. The stuffing was also spoiled, and many of the springs broken. It was always Sonya's intention to have this furniture repaired, newly polished, and newly upholstered; but this was never done, partly because, with Sonya's bringing up, tattered furniture in a drawing-room was nothing astonishing, and partly because she never felt sufficient interest in Stockholm to have things put to rights, feeling sure that her home there was but a half-way house, and she need not therefore trouble to spend money on it.

And we are told that Anne Charlotte was considered a revolutionary in Sweden. What a bourgeoise! And Sofya, who had her furniture sent from Russia, who never had enough money, and her best friend who does not in the least understand and whose respectability is perhaps shocked. In her recollections, Sofya's daughter expresses a similar opinion and comments on the above description thus (from [Kochina 1985, p. 319]):

> Possibly, it seemed so to her for she was accustomed to the respectable Swedish apartments of the well-to-do Swedish families ... However, our apartment seemed gorgeous to me. Our living room with its redwood furniture upholstered in red satin, furniture mother had brought from Russia, seemed magnificent to me, and I hardly noticed the defects that were striking for Anne Charlotte.

Such a small city. Nothing escapes anyone's eye in this hen house. In Heidelberg, in Berlin, in Paris she had her friends, Julia, Maria, Aniuta. In Stockholm she had Anne Charlotte. And it was not the same. From young revolutionaries and scientists—what Anne Charlotte called conspirators—to the stiff bourgeoisie the transition is rather harsh. Sofya certainly found herself isolated, spiritually isolated. It is also the place where she felt the most guilty for not being more militant. What political collusion could there be with Anne Charlotte?

A guilt that she often felt and expressed, as witnessed especially by her correspondence with Georg von Vollmar around 1882 (see e.g. the books [Koblitz 1993, pp. 167–168] and [Tollmien 1995, pp. 109–113]).

And furthermore, what collusion plain and simple? To Anne Charlotte, as she did habitually (see page 209 for what Maria Jankowska said about Sofya's facets), Sofya perhaps showed but one facet of her personality. In any case, Anne Charlotte's biography maintains but one. The brilliant scientist inspired by the beautiful mathematics she has achieved appears only inadvertently in Anne Charlotte's biography [Leffler 1898, pp. 232–233]:

> It now seemed to her [this concerns the meeting with Poincaré and others to whom Sofya spoke about her work on the solid] that nothing was worth living for but science. Everything else—personal happiness, love, and love of nature, day-dreaming—all was vain. The search after scientific truth was now to her the highest and most desirable of things. Interchange of ideas with her intellectual peers, apart from any personal tie, was the loftiest of all intercourse. The joy of creation was upon her [...]

Anne Charlotte, who passed for a champion of women's rights, seems to be persuaded that the quest for love was the ideal of a woman's life. We even see it in the passage where she cannot keep from putting "personal happiness" in opposition to the joy of scientific creation. She tends rather to present Sofya as an unhappy woman (especially in doing mathematics) when she does not wish to love and most of all to be loved. Anne Charlotte's book in fact fulminates with passages such as this [Leffler 1898, p. 231]:

> Sonya could not work, but she maintained with more and more eagerness that work—especially scientific work—was

> not good; it could neither afford pleasure nor cause humanity to progress. It was folly to waste one's youth on work, and especially was it unfortunate for a woman to be scientifically gifted, for she was thus drawn into a sphere which could never afford her happiness.

In the course of the narrations, we nonetheless discover that besides the skating and riding practiced by Sofya with pleasure and good humor, she had enthusiasm for activities such as an excursion into the mountains, for instance [Leffler 1898, p. 233]. It seems to me that if the psychologizing commentaries it contains were removed from the book, it would show a quite different image of Sofya. The book also presents Sofya as a dependent woman, which has certainly not been without influence on her scientific image itself [Leffler 1898, pp. 222-223]:

This supposed absence of practical sense in Sofya—a reputation that is well entreched—seems to me to accord poorly with the jams and embroideries that Sofya made, see page 236.

> She possessed to a high degree that feminine grace so highly appreciated by men. She loved to be protected. To a quite masculine energy and genius, and, in some ways, an inflexible character, she united a very feminine helplessness. [She needed some support—a friend to help her with small problems, and she would almost always find one. Otherwise, she felt miserable and forsaken as a child. She could neither buy a dress by herself nor put her things away.] She never learned her way about Stockholm. She only knew perfectly a few streets—those which led to the University or to the houses of her intimate friends. She could look neither after her money matters, her house, nor her child. The latter she was obliged to leave in the care of others. In fact, she was so impractical that all the minor details of life were a burden to her. When she was obliged to seek work that paid, to apply to an editor or get introductions, she was incapable of looking after her own interests. But she never failed to find some devoted friend who made her interest his own, and on whom she could throw all the burden of her affairs.
>
> At every railway station where she stopped on her many journeys, someone was always waiting to receive her, to procure rooms for her, to show her the way, or to place his services at her disposal. It was such a delight to her to be thus assisted and cared for in trifles that, as I said before, she rather liked to exaggerate her fears and helplessness.

She herself, in a letter to Weierstraß in August 1883, ridicules a young mathematician from Berlin because he had lavished her with advice for her trip to Odessa, "advice that was not the most practical" [Mittag-Leffler 1923, p. 190].

The account of Sofya's last trip and her difficult return across the Danish isles (see here page 61) brings out the same prejudice: because she had no practical sense, she didn't have any Danish money and couldn't pay for a porter, so that she, weak woman, had to carry her bags herself, fell ill and ended up dying.

Note too that Anne Charlotte implies that Sofya left Genova more or less consequent to a dispute with Maxim Kowalevski and that, no doubt desperate, she flung herself into this trip that becomes almost suicidal. This legend is resumed in a condensed, yet more spectacular manner by Marie-Louise Dubreil-Jacotin [1948, p. 265]:

> Not being able to do without him nor to live with him, exhausted, torn by the incessant fights, she waned and died in 1891 at age 41 from a brief attack of influenza.

In reality, as we have seen, Sofya travelled from Genova to Paris, then on to Berlin where she spent several days at the Vollmars in January 1891, and it is from there that she returned to Stockholm. Vollmar describes her as joyous and seemingly happy (see here page 218). The witnesses to her death even relate that her last words were: "too much happiness".

The ravages of this presentation remain amply obvious. The conclusions of Loria [1903, p. 391] concerning Sofya's scientific independence (see also here page 232) are deduced, explicitly, from quotations from Anne Charlotte. The long "psychological" paragraphs that conclude the recent [Kozlov 2000] (including the passage quoted here on page 241) themselves also doubtless come, via various intermediaries, from [Leffler 1898, pp. 222–223].

Another Swedish writer friend of Sofya, Ellen Key, criticized Anne Charlotte Leffler's book by saying that she limited her description to "the woman in the mathematican" while neglecting the "mathematician in the woman" and she did not understand the variable personality of the truly complex woman that was Sofya (see [Björk 2002, p. 41] and our page 226). Let us not be injust. Anne Charlotte's book is one-sided and its effects are pernicious, but it contains some nice things, such as the phrase attributed to Sofya by Anne Charlotte [Leffler 1898, pp. 160–161] and which prefaces our book.

Life in Stockholm—friends

Sofya has friends in Stockholm besides Gösta and Anne Charlotte. There is the writer Ellen Key whom we just mentioned. There is her colleague the explorer Nordenskiöld. There is the astronomer Hugo Gyldén and his family (it is Hugo and Thérèse Gyldén, by the way, that take care of Fufa upon Sofya's death, before she is sent to Russia and adopted by Julia Lermontova).

On 2 December 1889, the centenary of General Schubert's birth, Hugo Gyldén would receive a prize of 1,000 rubles—the biannual prize established in honor of our old friend General Schubert by one of Sofya's aunts, Sofia Schubert.

Hugo Gyldén is an astronomer and mathematician whose fields of interest are not so far removed from those of Sofya—he is moreover mentioned in the very first pages of Poincaré's *Méthodes nouvelles* [1987, p. 3]:

> But the scholar who has rendered the most outstanding service to this branch of astronomy [dealing with the Moon as a three-body interaction with Sun and Earth] is undoubtedly M. Gyldén. His work touches all aspects of celestial mechanics, and he skillfully employs all the resources of modern analysis. M. Gyldén has succeeded in removing entirely from his developments all the secular terms that so plagued his predecessors.

Poincaré was not always so enthusiastic about Gyldén's work; see his letters from 1889 to Mittag-Leffler [Nabonnand 1999].

Here is how Sofya's daughter recalls her mother's entourage in Stockholm [Kochina 1985, p. 320]:

> An old gentleman named Nordenskiöld would sit in this living-room [the living-room that Anne Charlotte and Fufa described to us on page 137] and tell us such interesting stories about his journeys around the shores of Siberia on the ship *Vega*, and we met here the young Nansen, who was just embarking on his career as an Arctic explorer. We were visited here by University Professors, such as Gyldén (an astronomer), Brögger (a geologist [a Norwegian professor in Stockholm]), Leche (a zoologist), Doctor of Medicine Medin (the Heine–Medin illness [polio] was named after him) and Mittag-Leffler, together with his sister, Ellen Key (a writer) and the editor of a newspaper (*Free-Thinker*) Branting, who became very famous later as a representative of the social democratic party in the Rigsdag, but then he was often in prison for his insulting remarks about the King.
>
> From 1888 onwards, Professor Maksim Maximovich Kovalevsky often visited us here; he came to give lectures on sociology. We would entertain Swedish and Norwegian artists, writers, and critics such as Brandes [a Danish critic to whom we will return] and Ibsen and there were many others whose names I've already forgotten.

In 1884 the old gentleman, Adolf Nordenskiöld (1832–1901), was fifty-two years old and he was extremely famous for having opened the *northeast passage* in 1878–79: he made the complete tour of the Eurasian continent: Norway – north of Siberia – Bering Strait – Yokohama – Suez ...

Friends, colleagues, colleagues and friends, but an ambiguous situation. All women mathematicians have, still today, experienced those dinner parties where the men talk about the particular subjects that interest them (mathematics) and the women (their spouses) about pottery, cooking or the garden—and where they have difficulty placing themselves. Alright, suppose you are the first person to whom this has happened. This is Sofya's case.

A remark on Ibsen. Pelageya Kochina begins her book by explaining that Sofya's personality was so very remarkable "that the great writer Henrik Ibsen said that to write her biography required writing a poem". This phrase has been reproduced here and there, but I haven't been able to find an exact reference in Ibsen's works.

There is also the *Nya Idun* society, a social club for women that imitates the masculine *Idun club* (parenthetically, Idun is a goddess, a woman, the one who dispenses the apples of eternal youth, astonishing that a masculine club would take her name). But at *Nya Idun* you get together, give or listen to talks on cultural subjects of general interest, you sing or play the piano. Sofya takes part, likewise Anne Charlotte and Ellen Key, who is the president. Likewise *Heimdall* (from a man's name this time, still a mythological hero but the son of nine women), a mixed club, in which she also participates, and the club of thirteen [Kochina 1985, p. 164] with ... always the same people.

As for love ... Pelageya Kochina mentions [1985, p. 247] the answer that Sophie gave to Maria Jankowska who asked her about her love life:

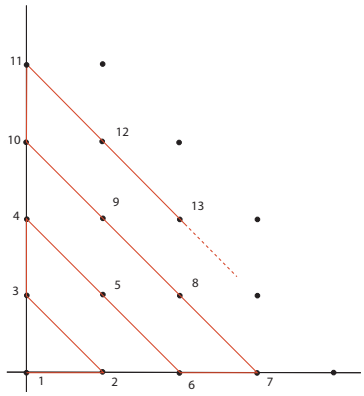
> In Sweden, all young men are born married—my admirers are all venerably old, the three of them totaling more than 200 years in age.

Friends and colleagues, but nonetheless quite a narrow circle. Sofya travels a lot, sees her mathematical friends in Germany, tries to get them to come to Stockholm (see the story of Runge on page 54), meets the French mathematicians, but Stockholm is so small. She begins to feel confined. Especially since her financial means are rather limited. She complains to her friend Gösta that she needs more money and he recounts these demands in his diary (some excerpts have been translated into English and published in [Hörmander 1991]). She even tries to find a position in France (see Hermite's letters quoted on page 165). Who would believe it, but Gösta is sensitive enough to the public opinion, I went to a lot of trouble to get you accepted here and now you threaten to leave, what would I look like. And it is one of the reasons that prompts Sofya to write two plays with Anne Charlotte while she is busy writing her paper on the solid: she hopes that this will bring in some money!

Acta Mathematica

Acta Mathematica is one of Mittag-Leffler's finest successes. He founds the journal in 1882; it is a Swedish journal, a Scandinavian journal, an international journal. The director is Swedish, the editorial committee is Nordic, comprised notably of Bjerknes, Lindelöf, Lie and Sylow, the authors are international. The articles are written in French or German.

It is no big surprise that the French authors would write in French, but it is also French into which Cantor's seven articles are translated in Volume 2 (see also note 5).



The fractions p/q are not more numerous than the pairs of integers (p, q) ... which are exactly as numerous as the integers, since they can be enumerated as shown in the figure: the set of rational numbers is *denumerable* and it is the same with the algebraic numbers. These are results of Cantor that were published in the first issues of *Acta Mathematica*, as was his first proof of the fact that the real numbers are not denumerable.

The fact that Mittag-Leffler is such a personality and above all has connections with the most powerful mathematical communities, the German and the French, is essential for the grandiose beginnings of the journal. Mittag-Leffler looks for articles, his French and German colleagues write them. Sofya will publish some old ones, write some new ones and, what is perhaps more important for the journal, she will attract several. She joins the editorial committee in 1884 and remains until her death. She adds Russian connections to the French and German. Mittag-Leffler also looks for money and the support of renowned institutions. Sofya succeeds in gaining the agreement in principle of the Grand Duke Constantin, president of the Russian Academy, but this institutional support never materializes—because of the Finnish question. The articles themselves arrive anyway, as we shall see.

The brilliant birth of the journal is rather impressive. We find interesting information in the article [Domar 1982]. The idea is supposed to have been suggested to our Gösta by the Norwegian mathematician Sophus Lie. The support of Weierstraß and Hermite played an important role, especially since the new journal seemed to be in competition with Crelle's journal (see the marginal note on page 80), with which Weierstraß was involved along with Kronecker.

Since we are concerned here with Sofya, we recall that the article [Kowalevski 1875] on Cauchy–Kovalevskaya had indeed appeared in Crelle's journal in 1875 ... but that practically all the rest would be published in *Acta Mathematica* (with the notable exception of the article [Kowalevski 1885b] on the rings of Saturn, which she gave upon his request to her friend Hugo Gyldén for the *Astronomische Nachrichten*, whereas he himself also published in *Acta Mathematica*).

Here are some precise data on the first issues of the journal. Volumes 1 through 16 published, from 1882 to 1892–93, articles by a hundred and three mathematicians. There are some regulars, notably Poincaré, who publishes ten articles or memoirs there, often very voluminous (in particular the one that won the prize of King Oscar II [Poincaré 1890], see the correspondence [Nabonnand 1999]) and Cantor, whose seven articles [5] appear in volume 2 and who will publish two others, in volumes 4 and 7.

5. The articles of Cantor are translations into French of articles that appeared in Borchhardt's (namely Crelle's, namely *Journal für die reine und angewandte Mathematik*).

The first volume is already very balanced; there are of course Scandinavian authors (Gyldén, Malmsten, Zeuthen), but Mittag-Leffler obtained German articles (by Fuchs, Netto, Reye, [6] Schering) and French collaborations (with two articles by Appell, two by Poincaré, two by Bourguet, others by Goursat, Hermite, Picard). With the thirteenth issue, an Italian article (by Beltrami).

Yngve Domar [1982] recounts that it was during Mittag-Leffler's honeymoon in 1882 that he made the rounds of his friends for gathering up all these manuscripts. We have already seen, and we will see again, Gösta in Paris in the spring of 1882. He and his wife pass through Berlin at the end of July, they pay a visit to Weierstraß, who recounts in a letter to Sofya, sent from Innsbruck on 5 August [Bölling 1993, letter 111]:

> Mittag-Leffler and Madame were here [in Berlin] last week, from Wednesday to Sunday evening; I have seen them a lot. The young woman pleased much; we admired her simple but remarkably elegant outfit. ML took a very mathematical trip—Straßburg, Heidelberg, Göttingen, Leipzig, Halle, Berlin—not to mention Paris. Very interesting for him—but whether for the young woman too, I really can't say.

In the same letter he also writes:

> Today I am preparing a short paper that I promised ML for the new Swedish journal,

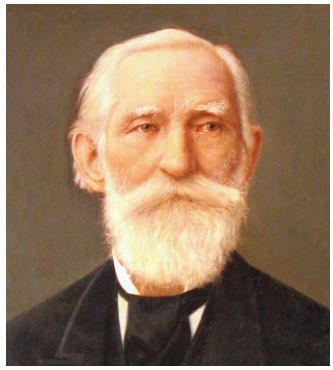
a short paper about which, forty years later, Mittag-Leffler [1923, p. 189] would say:

> This memoir was never written or never left Weierstraß's hands; in any case it must be lost.

Sofya comes onto the editorial committee in 1884 and presents one of the memoirs from her thesis [7] [Kowalevski 1884b] for volume 4; volume 5 witnesses the arrival of two short articles by the Netherlander Stieltjes and three from our old friend Runge, and it also publishes a translation into French of a memoir by Weierstraß on elliptic functions (after articles by Fuchs, Cantor, Du Bois-Reymond, Runge and Sofya,

6. Who writes from *Strassburg i/E* [*in Elsaß*, in Alsace].

7. She will publish a total of five articles in the journal. Besides [Kowalevski 1884b], these will be the article on refraction [Kowalevski 1885a], the two articles [Kowalevski 1889 ; 1890–91] on the solid, finally the short posthumous [Kowalevski 1891].



Chebyshev (1821–1894)

Acta Mathematica has always been one of the best journals that we mathematicians have had at our disposal. As I have already indicated, it is also one of the most beautiful journals in terms of its typography and layout, in particular its large margins.

Mittag-Leffler or Sofya, or both of them, have finally obtained an article by the master himself).

We note in passing the very first article in English, very isolated, due to Hill, arrives from Washington and appears in volume 8 in 1886 (the United States plays a very small role in the mathematical community). Little in English, but especially little by English in these first volumes: after the American Hill, Thomson (being Sir William Thomson, alias Lord Kelvin, who presents an article for volume 11) who is at the time in Glasgow, finally Sylvester, who is at Oxford, publishes an article on Buffon's needle in volume 14.

After the first article in English, the next volume (volume 9) witnesses the arrival in 1886 of the first articles from Russia, with Chebyshev's article already published in Saint Petersburg and translated from Russian into French by Sofya herself together with a letter he sent her, [8] the publication of the translation having renewed Chebyshev's interest in the subject. After Chebyshev, Markov, whom we have already mentioned and to whom we will return. Another Petersburger (but this time a Pole), Ptaszycki, publishes in volume 11. Two more articles by Chebyshev for our period, this time original and translated by I. Lyon, a student of Darboux.

From now on *Acta Mathematica* is one of the most international journals figuring in mathematical publishing.

In addition to the Russian articles, some of those by Poincaré, by Runge (on the Mittag-Leffler theorem, mentioned on page 56), by Minkowski, by Hurwitz (to wit, a function of several complex variables with only poles is rational) and by Beltrami, among others, were solicited and edited by Sofya. See the correspondence [Nabonnand 1999], the article [Koblitz 1984] and the letter to Mittag-Leffler quoted in [Cooke 1984, p. 105]: Sofya's correspondence with Mittag-Leffler displays the enthusiasm with which she dedicated herself to her editorial duties.

The article by Cantor that appears in volume 7, *Über verschiedene Theoreme aus der Theorie der Punctmengen in einem n-fach ausgedehnten stetigen Raume G_n. Zweite Mittheilung*, also passed by her, as evidenced by a letter Cantor wrote her on 7 December 1884 informing her that it would be ready in

8. Many of the short articles published by the journal are letters received by Mittag-Leffler (and, here, by Sofya).

the first months of the coming year "for you to publish in your periodical" (*um Sie in Ihrer Zeitschrift zu publiciren*). (9)

Birefringent media

This concerns a problem that was probably proposed to Sofya by Weierstraß when she was wanting to take up mathematics again following her bad Russian period. It also concerns the propagation of light in a crystalline medium and in particular double refraction. The problem was modeled by Lamé, there is a system of partial differential equations that can be written

$$\frac{\partial^2 u}{\partial t^2} + \operatorname{rot} a \wedge \operatorname{rot} u = 0 \text{ and } \operatorname{div} u = 0,$$

a system not unlike Maxwell's equations. Lamé found solutions for them. Weierstraß, who had an idea for a method for solving partial differential equations of this type, thought that Lamé's solution was not the most general possible and proposed to Sofya that she solve the equations.

The beautiful and precious French word *biréfringeant* comes to us here from the title of the article by Volterra ... and from a time where mathematicians used more than fifty words to write their papers, even if not in their native language.

Which she does. She is being distracted by the problem of the solid, for which she begins to have some ideas, as she writes to Mittag-Leffler on 21 November 1881 and as we saw on page 116. It is in this letter that she explains that she needs to complete the work she has started, that on double refraction, before taking up a *Privatdozent* position (see page 126).

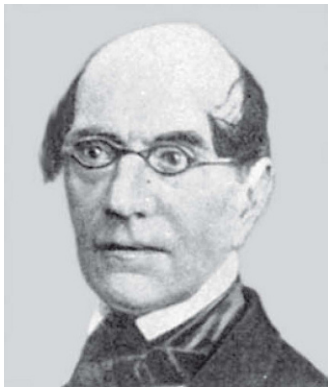
The double refraction problem is Weierstraß's, whereas the solid is truly Sofya's problem.

And then, she writes a note to *Comptes rendus* [1884a], communicated by Hermite, and an article whose first part is dedicated to Weierstraß's method (and is reproduced in the collected works of the master, see below). As I have said, this article was read, revised (see page 54) and then published in 1885. End of the first episode.

The second episode is posthumous. Shortly after Sofya's death, Mittag-Leffler receives a letter from the young Vito Volterra, who remarks that Sofya's formulas do not give, any more than those of Lamé, solutions of the equation in question and also that Sofya's solutions are no more general than Lamé's. He discovered where his two predecessors went wrong, a differentiation under the integral sign, always desired but not always legitimate. Gösta indicates the error in Sofya's obituary [Mittag-Leffler 1892–93]:

Vito Volterra (1860–1940)

9. This letter is reproduced in [Dauben 1990, p.310]. See this book too for the relations between Cantor and *Acta Mathematica*.



Gabriel Lamé (1795–1870)

> Toward the end of June [1883], recovering finally from her illness [following Vladimir's suicide], she went to rejoin her faithful friend and professor, Weierstraß. She ardently resumed her mathematical work and finished the research project that she published under the title: *Über die Brechung des Lichtes in cristallinischen Mitteln*, this Journal, volume 6. In the present volume of this publication, M. Vito Volterra has resumed this same problem: he has shown that the functions given by Sophie Kovalevsky as general integrals of Lamé's differential equations, do not satisfy those equations and he gave reasons for this fact.

And, as we see, he publishes the article by Volterra in his journal.

Returning to the first episode: Hermite transmitted the announcement of the result to the Academy of sciences, Weierstraß did not have the time to read the article in detail, Runge read and revised it, helping Sofya to correct her German, and the journal accepted it, perhaps after the advice of an expert. It seems to me irresponsible to say that this article is wrong because Weierstraß did not have the time to read it carefully, with all this implies regarding Sofya's other articles, as it would be irresponsible to minimize her responsibility by blaming the error on Weierstraß's schedule.

When she wrote this article, Sofya was a mature mathematician, independent and autonomous. She made a mistake, something that happens to the best mathematicians, which happened for example to Cauchy and to Poincaré, in particular in the memoir [Poincaré 1890] which won the prize of King Oscar II, to mention only some of those whose names appear in this book. Let us put the responsibility on those that commit the acts, the errors as well as the triumphs.

And let us leave to the journal the responsibility for the publication of this article, these are things that happen to the best journals and do not hold anything disastrous. In restricting ourselves to articles related to the subject of this book, we mention the article of Roger Liouville [1897], published in volume 20 of *Acta Mathematica* under the title *Sur le movement d'un corps pesant suspend par un de ses points* that comes up here on page 106 and in which a whole collection of integrable cases of the problem are determined, in contradiction to the results explained here in chapter V ... again an incorrect article published in this excellent journal.

The publication of Sofya's article was even a rather good move for the journal. A large part of the article (its pages 254–279) are in fact an edited version of notes of Weierstraß, investigations from the 1860s on partial differential equations with constant coefficients, cited as such (and even in quotation marks) by Sofya. The publication of these notes in *Acta Mathematica* was judged to be so little disastrous that they appear as is in Weierstraß's Complete Works ... some pages after the article [Weierstraß 1861] already mentioned here on page 46, with the indication (p. 296 of the volume) that these notes

> were published, with the permission of the author, by Frau v. Kovalevsky as part of her paper *Über die Brechnung der Lichtes in cristallinischen Mitteln.*

"Do with my notes what you find good", Weierstraß had written to Sofya (letter from 17 October 1884 [Bölling 1993, p. 322]).

It is amusing to see that a recent editor-in-chief of *Acta Mathematica*, Lars Gårding, several years after having expressed his discontent with the publication of this article in this journal—a publication that one of his colleagues, Lars Hörmander, even termed "disastrous" (see our page 238)—wrote a text on Sofya's mathematical work in which he amplified and inflated its role in Sofya's œuvre. This is the chapter dedicated to the papers of Mittag-Leffler and Kovalevskaya in a book on mathematics in Sweden before 1950 [Gårding 1998, Chap. 8], where after eight and a half pages on Gösta, we find two on Sofya. These begin with the text on Cauchy–Kovalevskaya that I have already quoted on page 83 (thirty-two lines) and conclude with a paragraph on the solid (twenty-eight lines). Between the two, forty-two lines are dedicated to "double refraction" (the incorrect article). In defense of the author: it is the part of Sofya's work of which he is incontestably a specialist.

It would not be useful here to reproduce Gårding's text on the solid. I have already dwelled quite long on this problem, and I hope with clarity. But just for fun, because of a delightful typo. The final phrase (which is thus the final phrase dedicated to Sofya in the entire book [Gårding 1998] reads: "In present terminology, this means that most of the motions of a rigid boy [sic] about a fixed point are chaotic". And when will we get a book on typos and their relation to the unconscious?

CHAPTER VIII

A LETTER TO VOLLMAR

The letter which is the subject of this chapter is very different from the one of chapter VI. Sofya sent it to Georg von Vollmar on 12 June 1882.

Why dedicate a chapter to this letter?

We might be astonished to read here a letter dating from 1882. But there is nothing surprising, and not just because this book is not arranged chronologically. Sofya talks at length about her life in Paris and the plan for the position in Stockholm, which I have just discussed.

It seems interesting to me to publish this letter in its entirety. First, because it contains elements about Sofya's reputation, the judgment that society might make concerning her lifestyle for example; second, because she gives a far from neutral opinion about Mittag-Leffler to someone who will never repeat it to him, and also because she puts on stage characters that I have but little discussed elsewhere in this book and who play an important role in Sofya's life, finally because she says things that she perhaps would not say in her correspondence with mathematicians, for example that she's a nihilist or at least not far from being one. She maintains a different tone, she displays her humor, she jests, she expresses herself more freely and perhaps more amicably. We sense a camaraderie that would not have been appropriate with Hermite or even with Gösta.

Two of Sofya's worlds. In Sofya's Parisian life there is the academic world, with Hermite and Bertrand and the *Société Mathématique de France*. There is also (for short) the socialist

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_8,

world. Two very different worlds ... except that she, Sofya, belongs to both.

Practically everything we have seen or are yet to see in this book has to do with Sofya's professional life. Her relations with the Parisian mathematicians will be described in chapter IX. Here is a document that bears witness to her other life—and indeed, via Madame Hermite, to the convergence of these lives. For example, by attentively examining the dates, those of this chapter and those of the following, we learn that Sofya has made the acquaintance of Maria Jankowska, with whom she will later live in Paris, renouncing the furnished rooms about which she will speak to us in this letter, at about the same time as she met Hermite and his colleagues. A luxuriant springtime for Sofya.

It does not seem to me that Sofya described for her beloved master, as she did for Vollmar, her feelings regarding bourgeois Parisian society, Madame Hermite and her like—even if, in a letter from the same period (11 April 1882 [Bölling 1993, letter 104]) Weierstraß described himself as her confessor (*Beichtvater*); he is attentive to the negative effects of her lifestyle, advises her for example to register under the name Frau von Kowalevski at hotels [Bölling 1993, letter 152].

Whence comes the text?

The letter comes from the *International Institute for Social History* in Amsterdam and was kindly communicated to me by Mieke IJzermans. An excerpt from it has already been published in a book on Vollmar [Kampffmeyer 1930]; it was partially translated into Russian in [Shtraikh 1951, letter 42]; an extract also figures in the book by Cordula Tollmien [1995] (it is she who indicated where to find the complete text). Norbert Schappacher deciphered the new portions of this letter for me (it is written in old German script) and helped me translate it. I have already spoken of Sofya's German here and there in this book. Cordula Tollmien brought my attention to the numerous faults that exist in the text and Norbert Schappacher has detailed them for me: prepositions improperly used as in *nach die Schweiz zu gehen* which should be *in die Schweiz zu gehen* or in *Antipathie zum Briefschreiben* which should be *Antipathie gegen Briefschreiben*, bizarre and not very idiomatic turns of phrase such as *nichts wird geschadet* which I have translated as "no harm done", and words are not always in the right order for German. On the other hand, Sofya was not one to let herself be stopped by the poverty of her vocabulary, so she invented the words she needed, as in *Sie verhandeln [...] mit Ihrem armen Fuß* (you negotiate with your poor foot), but you don't negotiate at all with your foot, so I have translated this "you deal with". There are faults in orthography and some lack of agreement as occur also in her French, errors in declining adjectives,

and I will stop there, since my own German is rather poor and I will only try to correct the errors she makes in French.

All of which is no hindrance, and my Germanic expert Norbert Schappacher assures me that the letter is lively and very beautiful. Truly, if Sofya had written it using the Latin alphabet, I undoubtedly would have found it easier to read many texts written by authentic German speakers.

Sofya's friends in Paris

Georg von Vollmar. The friend to whom Sofya addresses herself in this letter is the German social-democrat Georg von Vollmar (1850–1922). He was wounded during the Franco-Prussian war of 1870, from which come both the disability of his foot (brought up at the beginning of the letter) and his socialist opinions. He edited newspapers, the *Dresdner Volksbote*, then *Der Sozialdemokrat* in Zürich, a newspaper that was banned in 1880 for being too revolutionary. He was elected to the Reichstag from 1881 to 1887 and from 1890 to 1918 and then quickly became more than moderate in his socialist opinions. He became a "reformist" and an "opportunist" and as such he was fought by the socialist left, by Engels, Bebel, Rosa Luxemburg especially, and I will not discuss at any greater length his adherence to the *Sacred Union*, i.e. his support of the war in 1914. Trotskyites reproach him still today for having been the true inventor of the reputedly Stalinist concept of "building socialism in a single country".

Still, in 1882, when Sofya wrote him this letter, he is a deputy in the Reichstag but still rather revolutionary; he has contacts with the socialists circles here and there and even requires his parliamentary immunity in order to stay out of prison in Germany. And he's a great friend of Sofya, who seems—just a short time before she wrote this letter—to have considered briefly the idea of falling in love with him (see [Koblitz 1993, p. 155]). It was a Swedish student of Sofya, Julia Kjellberg (whom we have already encountered on page 60) that Vollmar would marry. We note too that, although Sofya is friendly and comradely, she addresses him with the formal *Sie*.

In Paris, they have friends in common in the revolutionary circles, especially Russians and Poles, Lavrov for example and then Maria Jankowska—I suppose they are the ones designated by the initials in Sofya's letter. She was rightly cautious, for

one of the members of her small circle turned out to be a spy, which sowed considerable anxiety [Koblitz 1993, p. 160], and it is undoubtedly not by chance that the only people whose names are spelled out in this letter are Mittag-Leffler and Madame Hermite, who certainly did not risk anything!

Sofya and Vollmar will remain very close their whole lives, until at the end of her life, as we have seen and as we will see again, she paid him a visit just before her death.

Piotr Lavrov. Piotr Lavrov was one of the leaders of the Russian–Polish immigrant community in Paris during the 1880s, a revolutionary with populist tendencies, both a former colleague of general Korvin-Krukovski, a former mathematics teacher and someone who Vladimir saw a lot of in the 1860s. Which amounts to saying that he had known Sofya forever. His revolutionary activity had earned him exile in the Urals in 1868, from whence he escaped and then lived abroad, mostly in Paris.

It was perhaps at Lavrov's house that Sofya met with Maxim Kowalevski, it was at his house or at Aniuta and Victor Jaclard's that she got to know Vollmar, and it was at his house that she met Maria Jankowska ... and this is what she undoubtedly is recounting to Vollmar in our letter.

According to [Björk 2002, p. 45], it was at Aniuta's that Sofya got to know Vollmar, in March of the year 1882.

Maria Jankowska-Mendelson. When Sofya met her, this Polish revolutionary had just gotten out the prison in Poznán. Coming from a wealthy background, she is reputed to have presented herself to the International by proposing to help the workers. Ah yes, like that, in silks and lace, could have been the reply—but she was accepted. From their first meeting she and Sofya were friends. Maria used Sofya's passport to make clandestine trips to Poland, she received Sofya into her home, it was she who took care of Sofya right after Vladimir's suicide, she loaned her apartment to Aniuta at the time of the final phase of her illness from which she would die. She and her second husband Stanislaw Mendelson became members of the Polish Proletariat party before turning, they too, to the right.

The letter

12.6.82

It is not clear that the date is from Sofya's hand. But it is certainly exact, the meetings with "Pani J" and Mittag-Leffler date right from that very moment.

My dear friend!

How hard it is to learn that your health still is not very good! But you treat your foot with so little regard and it is so sensitive that it by itself will remind you when you forget that it is ailing. There is still chance that the session will end early and that you, we hope, will be in Ragaz again, forced to abandon all your activity in order to restore your strength. That my friend is not doing as well as I might hope, I learned even before receiving your letter. And do you know how? From Pani J., the woman traveller whom I met yesterday at L.'s. What a nice woman she is! I liked her a lot, even if we were not able to enjoy her company but for a very little time, since she had to go to a meeting of the new pol. [political? Polish?] society Lud Polski.

I suppose that "L" is Lavrov and that "Pani J." denotes Madame (Pani) Jankowska (the handwritten letter looks a lot like the "J" Sofya wrote in French, but it could just as well be an "I", which could invalidate this assumption).

Lud Polski, the Polish People, was the name of a Polish social-political society that made its program public in 1881.

[page 2]

But she promised me to visit soon, and I am happy to make her closer acquaintance. Unfortunately for me, she will not be staying in Paris much longer and intends to go next to Switzerland.

Mittag-Leffler left Paris yesterday. We spoke a good bit about Stockholm and my future position. He is not just an important and talented scholar, but he's also a very sympathetic person and very well educated in all areas, and if I were to judge all Swedes by him, then I would think that I will be really very happy in Stockholm. Everything he tells me about things in Stockholm has increased my desire to be recruited. For my part, I spoke to him quite frankly and brought his attention to the particulars of my personal situation, which could make a position in a rather bourgeois society disagreeable. For example, I am Russian and as such already suspected of nihilism (which in my case is not far from the truth), secondly, I do not live with my husband, and that, a woman separated from her husband,

[page 3]

for whatever reason, is something dangerous and suspect in the eyes of every right-thinking matron. And educated women are judged more harshly than the others.

([This parenthesis does not close] That I do not exaggerate on these matters, I see perfectly in the behavior of the local mathematicians whose acquaintance I have made recently. They visit me and pay me compliments, but none of them has presented me to his wife, and when I remarked on the fact jestingly to one of the women whom I know from this circle, she laughed and replied "M^me Hermite (the wife of the principal mathematician here) will never receive a young woman in her salon who lives alone in furnished rooms without her husband".

You can imagine that these absurdities bother me little in Paris. In Stockholm this could be completely different. I have also said all that to Mittag-Leffler. He thinks that in Stockholm I will be considered differently. But a fear remains for me: he himself is a great idealist and has so much friendship for me that he thinks everything I do is good and cannot conceive that others could judge me differently.

[page 4]

In any case, after having thought much, I have taken the following decision and ML has also ended up admitting that this is the most practical and most reasonable. Since in Stockholm presently, outside of the university authorities concerned, no one yet knows anything of our plans, I will go at the beginning of November to Stockholm under the pretext of paying a visit to ML. I will give a communication to the mathematical society there and a talk in the mathematical seminar. If this is successful and if I like Stockholm, I can begin teaching at the beginning of the new year. Otherwise I will simply return and everything will be as before, without any damage.

Where I will be until November, I myself do not yet know. For the moment in any event in Paris. I have no need of going to Berlin, since I have to finish my paper in autumn so that it will appear in a Swedish journal. And I confess sincerely that I am very happy not to have to go to Berlin because I find it very sad to arrive there when my friend cannot be there anymore. I must work

[page 5]

a lot now and for the moment I can work here as well as elsewhere.

It is remarkable that my brother still does not come. After he wrote me a very affectionate letter and, all things considered, very reasonable in which he informed me of his imminent arrival. He has not given me any further sign of life. I have already

Later ...

In 1888, on 19 June, Madame Kowalevski, Professor at Stockholm university ... and above all a widow, is invited to dinner at the Hermites with the Picards. M^me (Marie) Picard was Hermite's daughter.

Still in 1888, 25 December, Madame Kowalevski, Professor at Stockholm university, crowned the evening before with the Bordin prize of the Academy of sciences ... and still a widow, is received at the Hermites with the Bertrands [Kochina 1985, p. 169]. M^me (Louise) Hermite was the sister of Joseph Bertrand.

It is a good place to remark that the social role of women in the mathematical collective was not limited to their scientific role.

It is probably because of this passage that the excerpt from the letter published in [Shtraikh 1951] is presented as dating from 1883.

written him two times—in vain. I worry about him a lot and would do so even more if I was not aware of his innate antipathy for letter writing.

I would really very much like to know what your adversaries are writing about you and I regretted very much that the corresponding issue of the *Norddeutsche Zeitung* is so difficult to find here. Share with me a little what will really be written. Or my friend has already become so vain that he does not want to share anything but chants of praise? (I was not able to spare you this tiny bit of malice; I beg you nonetheless to excuse me.) I do not understand either in what way the honor of a personal conflict with the "all powerful" can

I do not see whom this expression can indicate if not Bismarck himself. The *Norddeutsche Zeitung* was a semi-official newspaper.

[page 6]

threaten you? Watch out not to fall into their hands at the instant when your parliamentary immunity ceases to protect you.

And now, goodbye my friend.

With my best and most cordial wishes for your prompt and complete recovery.

Your sincere friend

S. K.

CHAPTER IX

THE BORDIN PRIZE AND SOFYA'S REPUTATION

> Say what you know, do what you must,
> come what may.

At the bottom of the first page of the article [Kowalevski 1889] by Sofya Kowalevski on the solid, we find a note that indicates that this work obtained the Academy of sciences' Bordin prize, raised from 3,000 to 5,000 francs.

It is well established by letters of Hermite to Sofya and to Gösta Mittag-Leffler from June 1886 (quoted in [Cooke 1984] for example), that the problem posed in 1886 for the Bordin prize of 1888, to wit:

> To perfect in an important way the theory of the motion of a solid body

had been custom tailored to publicly recognize the value of a work of which the French mathematical community was not at all ignorant. This was a current practice; another celebrated story is that scarcely three years later the subject of the *Grand Prix des sciences mathématiques* (another prize of the Academy of sciences) was

> Determination of the number of prime numbers less than a given quantity

announced at the end of 1890 at the initiative of Hermite because Stieltjes had hopes of knowing how to prove the Riemann hypothesis (perceiving his error, Stieltjes did not compete and it was Hadamard who received the prize, see [Maz'ya & Shaposhnikova 1998]).

And it is totally well established that it is because the work was exceptional that the prize was augmented. Hermite wrote Gösta on 10 December [Dugac 1985, p. 152]:

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_9,

> In announcing to you that the memoir of Madame Kowalevski would be crowned by the Academy, I asked you to keep my communication for yourself, but today I received from Mr Bertrand the mission, which gives me great pleasure, of *officially* informing Madame Kowalevski that not only has the prize been awarded her, but because of the exceptional merit of her work the Academy, on its proposal, has augmented the value of the prize by means of funds at its disposal, which brings it to 5,000 F.

It is true that Sofya needed money. The amount offered by the prize was not negligible: the annual salary of a teacher at a lycée was of the order of 4,000 francs. A Parisian university professor earned a lot more, especially if he combined his professorial position at the Sorbonne with that of maître de conférences at the École normale supérieure and yet again a professorship at the École polytechnique, as did Hermite, for example (this information was provided me by Catherine Goldstein).

An exceptional work, international glory ... but it is incontestable that today Sofya has a bad reputation—in any case, that she does not have the scientific reputation that she should have. As I have already written elsewhere [2005]: yes, it was the 19th century and Sofya was a mathematician grappling with her century, with the ills of her century, misogyny and pneumonia notably, but also struggling with our centuries where, if there is less death from pneumonia, her brilliant personality is rather suspect, even in a world that should be capable of measuring her contribution to science.

We have already seen, for example in chapter IV, the way in which one of our contemporaries gives account of her work. We will see in chapter X that her private life and its consequences have been an object of insinuations that are perhaps the source of this bad reputation. However, all this is rather recent: in spite of her political opinions and the free life she led, Sofya was a celebrated scientist and appreciated by her colleagues.

The men of her time. Sofya was the victim of the institutional misogyny of her time. This is known. It was not possible for her even to *attend* Weierstraß's courses at Berlin university, much less be appointed professor ... Gösta Mittag-Leffler [1923] explained, and we have reported on page 127 that, if she was able to obtain a position in Stockholm so easily (a relative ease), this is not because the Swedes were more advanced than the Russians, the Germans or the French, but because Stockholm University was a new university (we have also mentioned Gösta's own political abilities). The secretary of the Swedish Academy of sciences, Professor Lindhagen, implicitly confirmed this opinion of Gösta by declaring, (1) in 1886,

> If the Academy starts to elect women among its members, where in God's creation will it stop?

The first woman to be elected into a Swedish academy was to be the writer Selma Lagerlöf in 1914, five years after her Nobel prize in literature.

1. Reported 14 January 1950 by the newspaper *Stockholms Tidnigen*, quoted in [Kochina 1985, p. 148].

Mittag-Leffler adds, perhaps in reference to this type of reaction:

> The real difficulties came later. The manifestations of this hostility are still too recent for me to be able to submit to the whole world the related correspondence that one day will reveal a great number of curious interiors of scholarly republics not only in Stockholm and Uppsala but also in Berlin, St. Petersburg and other centers of international culture.

I don't know precisely to what he was alluding in this passage. (2) Sofya's opponents were numerous enough to express themselves openly, an often-cited case is that of the playwright August Strindberg, whom we have seen and will see again giving his opinion.

However, at this time Sofya had the support of the majority of her colleagues. The Russian historian and the two American historians that I have long and often quoted, Pelageya Kochina [1985], Roger Cooke [1984 ; 1987] and Ann Hibner Koblitz [1987b ; 1987a] affirm that Sofya was a full member of the mathematical community.

Parisian life

We know that Sofya corresponded often with Weierstraß, which was the case too with Hermite, for example. There remain fifteen letters from Hermite to Sofya, but no more letters from Sofya to Hermite than from Sofya to Weierstraß, all (?) having been burned. They met in 1882, thanks again to Mittag-Leffler. We already know, via the letter to Vollmar (chapter VIII) that Sofya saw Gösta in Paris in Spring 1882 and he was surprised at her not having met the French mathematicians, in particular Hermite. Perhaps she was timid. Perhaps she anticipated comments on her lifestyle, a la Madame Hermite (see page 154).

Hermite and perhaps she herself mentioned this meeting in their letters to Weierstraß, who writes to Sofya on 14 June 1882 [Bölling 1993, letter 106]:

2. Even though the article in question, [Mittag-Leffler 1923], dates from 1923, it was likely written before 1900 (we have mentioned the version [Mittag-Leffler 1900] in note 1 on page 29).

Charles Hermite (1822–1901)

In the letter we read in Chapter VIII, Sofya describes Hermite with humor as the principal mathematician ("Hauptmathematiker").
Hélène Gispert [1991] calls him a landmark-mathematician (mathématicien-phare), whereas his brother-in-law Bertrand is "the boss" (le patron).

> That you have gotten to know Hermite, I have learned from him. He wrote me about this very enthusiastically and has related all the problems that you broached with him in this first discussion.

The French mathematicians received her well. They visited her, discussed mathematics with her, even if they did not invite her into their salons (see page 154). In July she was elected member of the *Société Mathématique de France* (SMF).

The *Société mathématique de France*, *cum commento*. At the time of the 7 July 1882 meeting, MM. Stephanos and Picquet present "Madame Sophie de Kovalewski", who is elected member at the following meeting on 21 July [Bsmf 1882–83]. The SMF was founded in 1872. In January 1883 it had about two hundred members, and Sofya appears, the first woman in the brief history of this association, in the list of its members [Bsmf 1883–84] as

> Kovalewsky (M^{me} de), rue des Feuillantines, 9, à Paris.

The second woman elected to the SMF was Nanny Lagerborg, from the meeting of 5 February 1890 [Bsmf 1890]. The first Frenchwomen will not arrive in this society until 1909. As for Nanny Lagerborg, whom Hélène Gispert [1991, p. 150] leaves in anonymity, in an infra-paginal note, as a "Finn without profession", she is not without relevance to our story and is thus the object of a new digression. Ebba Louise Lagerborg, indeed a young Finn, who called herself Nanny Lagerborg, was born in Cannes in 1866. Her mother died bringing her into the world, her father encouraged her to study. She thus spent a year in Geneva where she learned French and where she began to be interested in mathematics and in mountain climbing. At university she took courses in mathematics in which she was the only female auditor and courses in physics where there indeed were other women, mostly Russian nihilists studying medicine, she said later, not a surprise for readers of this book. She is a student at Stockholm University from 1886 to 1889, then goes to Paris where she then becomes a member of the SMF, and obtains, on 30 October 1890, a degree (licence) from the Sorbonne (four years before Marie Curie), a fact duly mentioned by the Finnish press a few days later. The same year, the *Bulletin de la Société mathématique de France* publishes her article [Lagerborg 1890], a remark on a particular case of the solid with a fixed point. On the day of her election, she makes a communication whose content is undoubtedly that of this article but

which she titles *On a new case of integration of the motion of a solid body about a fixed point*. Whose student in Stockholm might this second woman of the SMF have been? Yngve Domar [1978, p. 16] writes with delicate condescendence:

> [...] may be noted a short essay by Nanny Lagerborg on the motion of a body around a fixed point, evoking the image of a young girl who is following very closely in Sonja Kovalevski's footsteps.

And after? Well, Nanny Lagerborg married a baron Cedercreutz, she had a change of name, she also had a change of occupation since she was transformed into a hostess; she however remained a member of the SMF at least until 1920; starting in 1893 she appears in the list of members under the name of

Cedercreutz (baronne Nanny, née de Lagerborg).

She nevertheless wrote short novels, completely forgotten today. She died in 1950. This information was provided me by Agneta Rahikainen, see her article [Rahikainen 1994] for more details. End of digression.

The metamorphosis of brilliant young women into spouses, yesterday and today, is one of the social phenomena that I have the most trouble accepting. See a complementary remark on page 52.

Hermite's letters. But let us return to Sofya's reputation. Catherine Goldstein has confirmed its excellence for me. I quote her message and add the precise references with which she complemented it. The context is that where Mittag-Leffler attempts to have Sofya elected to the Swedish Academy of sciences in 1886, the main opposition coming from Kronecker who, angry at Weierstraß, sent an unsolicited letter to this academy in which he simultaneously attacked Mittag-Leffler, *Acta Mathematica*, Weierstraßanalysis and, of course, Sofya. And she who described to Gösta, in a letter filled with humor, how much she would love to dress up in the pretty robe of the academicians ... Here then is the beginning of what Catherine Goldstein [2006] writes me:

> No doubt about it. Hermite, solicited by Mittag-Leffler, responds positively, but with some hesitation, mainly for lack of a deep knowledge of the articles, but he consults his "band" (Poincaré, Appell) and returns full of enthusiasm. Poincaré in particular is very positive. Hermite then implements a true campaign for support for Sofya (in particular at the moment when she is being refused admission to the Stockholm Academy): he assembles letters from his "band" and also from his semi-enemies (of the Bertrand type), he writes to Genocchi, then president of the Turin Academy of sciences, to enlist his support, etc.

Catherine Goldstein

I could not resist the pleasure of quoting rather lengthily these epistles in a deliciously outdated style ...

On 19 March 1886, Hermite writes to Genocchi in this way [Michelacci 2003, p. 179]:

> Permit me, my dear President, to share with you a circumstance which is of interest to Madame de Kowalevski, the eminent analyst, about which M. Mittag-Leffler has informed me. Several members of the Stockholm Academy of sciences were desirous that she fill a vacant place in this Academy, but a lively opposition has been produced against the proposal of allowing the seating of a woman, whatever her scientific talent, and Madame de Kowalevski was not elected. Not only has she not been elected, but a sort of malevolence has been attached to her person, and one has gone so far as to challenge her mathematical merit. M. Mittag-Leffler asked me to take up her defense and with this intention I have obtained authorization from MM. Camille Jordan, Darboux, Appell, Poincaré, Picard, Tisserand, permission to add their support to mine in her favor. I just ask you, in the most explicit reservations of your agreement, that you join with the French geometers, in case this would not be contrary to your sentiment, to obtain for me the accession of your mathematical friends in Italy. I should not hide from you that, if M. Weierstraß has highly vouched for Madame Kowalevski's scientific worth, M. Kronecker has not acted likewise, and that M. Fuchs has kept Conrart's prudent silence.

Hermite quotes Boileau here: "I imitate Conrart's prudent silence".

And, on the same day, to Gösta (see [Dugac 1985, p. 118]):

> [...] it occurred to me that the best way of defending Madame Kowalevski against the deplorable attacks of which she is the object, is to bring together a stack of opinions from French geometers that are offered to vouch for the superiority of her talent and of the merits of her mathematical writings.
>
> MM. Camille Jordan, Darboux, Appell, Poincaré, Picard, Tisserand, [...], have authorized me to produce their testimonials and join them to my own. M. Maurice Lévy, to whom I also addressed myself, is unfortunately diverted by his position as chief engineer and professor; he told me that he still has not had time to study the work on double refraction, which completely fits into the framework of his research on mathematical physics.

A nice example of solidarity! The mathematical community mobilizes to defend one of its members. Note that, except for Kronecker, the attacks come from outside the community.

Several days later (on 30 March), he thanks Genocchi in these terms (see [Michelacci 2003, p. 182]):

> You have been very good my dear President to respond to my appeal in favor of Madame Kowalevski, and I will be grateful to you to charge you with expressing my thanks to

> your eminent colleague M. d'Ovidio, who permits adding his name to yours. I have already sent to Stockholm the declaration expressly authorized by MM. Camille Jordan, Darboux, Halphen, Appell, Poincaré, Picard, that they judge the memoirs of Madame Kowalevski that recently appeared in the Acta to be *works of first order*, M. J. Bertrand, permanent secretary, has permitted me likewise to add his testimonial to those of the French geometers, and hopefully they will all be taken into account. Yours and that of M. d'Ovidio will be added without delay; may all of them have the necessary authority in Sweden to preclude what menaces the position that has been made for this illustrious woman.

Jealousy. A parenthetical remark on the squabble between Kronecker and Weierstraß. They have antagonistic opinions on what a number is. Kronecker, who contested Cantor's work, is already unhappy that Mittag-Leffler published his articles in his journal. Moreover, Mittag-Leffler has formed a jury for the King Oscar II prize, with Hermite, Weierstraß and himself—but no Kronecker, who becomes furious, writes to Mittag-Leffler that, no, he will not recommend his wife Signe to the famous gynecologist Hegar (!) and threatens to inform the king that he, Kronecker, showed twenty-five years ago that the fourth question posed for the prize is unanswerable. To which Mittag-Leffler responds that Weierstraß is elderly and that was for why he wanted to honor him. Then it is Weierstraß who is offended ... If I report this history (see [Cooke 1984, p. 109]), it is because it shows two respectable German gentlemen in a crisis of jealousy. In this affair, if Mittag-Leffler is in a rage and decides to combat the noxious (!) influence of Kronecker on mathematics, Sofya is more on the side of diplomacy, even if she finds Kronecker's letter a bit strange ... since she has the habit of being a diplomat during bouts of jealousy, this ugly weakness, between these other respectable gentlemen who are Gösta and Schwarz. We read here or there that Sofya was jealous. Gösta himself enters it in his diary (quoted in [Hörmander 1991, p. 203]). Undoubtedly—like most of his colleagues, like everyone! Let us end the remark.

Kronecker was not always hostile to Sofya. In 1881, she and her daughter had celebrated Christmas at the Kroneckers (Leopold and Fanny). Still in 1884, as we saw on page 115, she was on good terms with Kronecker. We also point out the warmhearted homage that he rendered her after her death [Kronecker 1891].

"I cannot understand Madame Kronecker's hostility toward Madame Kowalevski, and I wonder what might be the origin of this malevolence that attacks simultaneously Madame Kowalevski, Mr Weierstrass and you, with me still to come no doubt", wrote Hermite once more to Gösta [Dugac 1985, p. 122] on 8 June 1886.

And let us continue with Catherine Goldstein. There are two moments that she and I recount here:

– 1886—Sofya has, in principle, solved the problem of the solid, she has *nothing more*[3] to do than to write the solutions, she goes to Paris where she encounters a maximum of colleagues and tells them what she has done. It is then that the idea germinates among the Parisian mathematicians to advertise a prize of the academy on the topic of the solid.

– 1888–89—about the award of the prize.

The following happens after the Bordin prize.

> Hermite also recounts a dinner in Sofya's honor organized by Bertrand, with ministers, mathematicians, etc. To the point where Sofya asks herself whether perhaps a position in France, etc. Hermite writes to Mittag-Leffler in order to discourage her (in fact, he speaks directly to Sofya who "is extremely intelligent" and comprehends the situation well: impossible to find a position in Paris except perhaps in Sèvres ["high school" for women], and again, etc.). There remains the province, but that would not be anything satisfying.

For those who are beginning to be astonished over Joseph Bertrand's power ... who, after all, even though he was, as was Pasteur, a permanent secretary of the Academy of sciences, he was not Pasteur, I recommend reading the article [Zerner 1991].

Here is what Hermite writes to Gösta on 12 January 1889, a letter in which he also mentions two dinners, a ball and a lunch, a rich social life [Dugac 1985, p. 156]:

> But I no longer ignore the terrible difficulties, the sorrows that are hidden behind these so brilliant appearances. Madame Kowalevski confided to me that her fortune is mediocre, that she needs her salary as university professor, and that without you and Mr Gylden, who are good and kind to her, her attribute of being Russian and a foreigner, perhaps also a woman scholar, leaves her completely isolated in Stockholm, where she needs to stay in order to live.
>
> [...] There thus remains the province, but the position would not be very desirable and I would never dare advise Madame Kowalevski to risk it and likely be less well off than in Stockholm; what a strange and sad destiny for a woman of genius!

In the letter of 26 January 1889 mentioned by Catherine Goldstein, where Hermite reports what he said to Sofya, he also mentions the difficulties that Stieltjes had, as a foreigner, in Toulouse [Dugac 1985, p.158]:

> Permit me to share with you the concerns I had concerning Madame Kowalevski and her plans to locate in

3. See page 97.

> France which she has formally authorized me to convey to you saying that you have for her the affection of a brother. The day before yesterday, in a long conversation, I believe I succeeded in making her understand the sad and painful difficulties the maître de conférences [senior lecturer] position held for her, or that of university professor in the province. I have in fact put before her what happened at this same moment to Mr. Stieltjes, chargé de cours [lecturer] in Toulouse, for which his being a foreigner is the cause of a thousand miseries, as much on the part of the public and the families of the baccalaureate candidates as with his colleagues. What would this be like then, I ask you, for a woman! But Madame de Kowalevski is extremely intelligent; she realized the scope of everything I said and she has now come to not have more in view than the instruction of young women at the École Normale de Sèvres, so that it would only be in case that she is offered a new position created for her in Sèvres that she would submit her resignation in Stockholm in order to come to France. But this question is not the only one we discussed [...]

Hermite also asks Gösta in this letter what he has already asked of Sofya, that king Oscar II bestow a decoration on his brother-in-law Bertrand on the occasion of the king's sixtieth birthday. Bertrand is already knight ... of the Polar Star (Swedish medal) and would very much like to be promoted!

Catherine Goldstein summarizes:

> So in Paris (and in Italy) she is well known and appreciated, and entirely integrated into the mathematical community. Hermite, who is not generally a progressive, stresses that her talent is "exceptional for a woman". The main problem comes from the fact that in Berlin, it is the moment when Weierstraß and Kronecker are quarrelling violently [...] All of a sudden Kronecker is hostile to Sofya and apparently Fuchs, according to Hermite, keeps quiet as much as possible.

The comments on the political context contained in the letters, from which I here quote some excerpts, leave not the slightest doubt regarding Hermite's political opinions. He writes for example in a letter to Mittag-Leffler on 17 February 1884 [Dugac 1985]:

"But for myself, as good as you are, you cannot prevent that I am a clerical going to mass and a vestry member of the parish."

He is even described as a "fundamentalist catholic" in Hadamard's biography [Maz'ya & Shaposhnikova 1998].

Lissagaray does not tell us whether he saw Hermite in Paris during the Commune or whether he was at Saint Germain with his brother-in-law Bertrand (see the marginal note on page 49), but there is evidence that he will be anti-Dreyfus, even though we are not quite there yet.

On rigor (continuation). Since we are talking about Sofya and about Paris, a small digression to point out the lack of rigor in a text written by a journalist in order to introduce an article [Détraz 2006] on Sofya (it goes without saying that the author of the article has nothing to do with it):

> A brilliant mathematician, Sophie Kovalevskaia had to fight her whole life to be accepted in academic circles. A combat that went from her exile from her native Russia and which led her to Paris [...]

The first phrase is not absolutely true, it's more against institutions that Sofya had to battle; most of her mathematical colleagues, for example, were convinced of her abilities. In the second there is nothing false, but uninformed readers will think

that Sofya came directly from Russia to Paris to do mathematics, which is false (see above the episode of the Commune and also the time she spent in Paris before contacting mathematicians when she returned in 1882) and obscures the fact that her study took place entirely in Germany.

On 24 December 1888 at the Academy

I reveled at reading the summary of this famous session for the award of the Bordin prize [Cras 1888, p. 1031 ff.] ... I revel in recounting it myself (and in my own way). It is the public annual meeting of the Academy of sciences. Solemn. It begins at one o'clock, at one o'clock precisely (we know this from the letter of 18 December signed by Louis Pasteur and Joseph Bertrand, permanent secretaries, inviting Sofya to the ceremony; a photograph of the letter appears in the book by Pelageya Kochina [1985]).

Speech of M. Janssen, the president, an astronomer. Ah! this speech! It begins with

> let us recall the memory of those of our colleagues whom we have had the pain of losing since our last annual session

in particular, of general Perrier (once again, a general) died in February at age fifty-four, who had performed

> this grandiose operation, considered unrealizable until then, namely the geodesic joining of Spain with Algeria across the Mediterranean.
>
> This beautiful success gave Geodesy a continuous arc from north of England to the Sahara, that is to say exceeding in extent the largest arcs measured until then

(again a geodesist, which can't but remind us, who know, of general Schubert, whom Weierstraß mentions [1861], surveyor of the Earth, Sofya's maternal grandfather), and the memory of the great rural engineer Hervé Mangon; I will pass on the others. He then congratulates admiral Jurien de la Gravière on his election to the French Academy (expected, but you never know)—another parenthetical remark is that this admiral, Edmond Jurien de la Gravière was the President of the Academy of sciences in 1886, in which role he met Sofya, as was mentioned, along with other news, by our friend Hermite in a letter to Genocchi from 29 June 1886, end of remark. After which President Janssen announces, finally, what we have all been waiting for:

"Madame Kowalevski likewise attended the session [of the Academy of sciences] and was received courteously and amiably by the President, admiral Jurien de la Gravière, as she was so entitled", wrote Hermite [Michelacci 2003, p. 188]).

Gentlemen, among the crowns that we are about to bestow, there is one of the most beautiful and difficult to obtain which will be placed on a feminine brow.

M^me^ de Kowalewski this year has won the grand prize of the mathematical sciences. Our colleagues from the geometry section, having examined the memoir presented in competition, have recognized in this work not only the proof of extensive and profound knowledge, but in addition the mark of a great inventive mind.

M^me^ de Kowalewski is professor at Stockholm University, where she teaches advanced students. She is descended from the king of Hungary Mathias Corvin, who was not only a great warrier, but who was in addition an enlightened protector of the sciences, arts and letters.

It is clearly in these latter qualities that Mme de Kowalewski takes after her illustrious ancestor, and we congratulate her.

Commentary.

– It is a public session and president Janssen addresses the hall in the masculine. "Gentlemen", he says. There was however at least one woman in the audience, since Sofya herself was present.

– That Sofya was the granddaughter of general Schubert, who was also a geodesist, would not have been known by our president if he had not drawn a parallel with the geodesist general to whom he gave homage, Perrier.

– "Mme de Kowalewksi", he says. A good place to ask where the *de* comes from. Should we take it as a genitive? O.k., I am making a bad joke, because it undoubtedly indicates the nobility that comes from her affiliation with the king Mathias Corvin. Furthermore, after having added the particule, president Janssen stresses her royal ascendance. *Vive la République! Vive la France!* These were the first things I thought when I read the report. More seriously, just like the *von* that Weierstraß liked her to use, she herself signed the paper she sent for the competition with a "de" which would contribute to her respectability.

– President Janssen is so impressed, undoubtedly by the idea of the historic moment he is experiencing that he gets the prize wrong. In that year the grand prize of the mathematical sciences goes to Picard, also with a subject that is custom made:

> To perfect the theory of algebraic equations in two independent variables.

I have reported several poignant remarks by Roger Cooke. Here is one of them, very anecdotal and one that I do not find serious enough to include in Sofya's genealogy, but which I cannot resist the pleasure of quoting here. In his review of the book [Tuschmann & Hawig 1993] for *Mathematical Reviews* he wrote:

"Although many details given in the book are not intrinsically important, they add greatly to the interest of the book. The reviewer [he, Cooke] had not known, for example, that Kovalevskaya's great-great grandfather Johann Ernst Schubert had been friends with a man whose son committed suicide, thereby becoming the model for the central character of Goethe's *Die Leiden des jungen Werthers*."

Who can describe the emotion of the historian who discovers that his heroine is the great-great-granddaughter of the friend of the fellow whose son served as a model for the poet Goethe?

– I have counted, winning or nominated, there are almost sixty this year, but Sofya is the only one mentioned by president Janssen. The minutes of this special session conclude with the program of prizes for the following year. The Bordin prize (reverting to 3,000 Frs) is about examples of surfaces on which

$$ds^2 = (f(u) - \varphi(v))(du^2 + dv^2).$$

Sofya in 1887

Report of the commission. The published version is followed by reports of the commissions on the prizes. As we might expect, mathematicians are the most concise of academicians. If I understand correctly, a prize is given for an anonymous memoir (in the case that interests us and furthermore probably in practically all the cases that could interest us, the anonymity is very relative, the community knows perfectly well who is working on what and how, and Sofya was, as I have said, a full member of the European mathematical community) that is identified by a sort of code, a phrase, a quotation that is found on a sealed envelope that contains the name and address of the author, the said sealed envelope only being opened in case the memoir is chosen as the winner. The losers remain anonymous and can publish their work where they want.

This does not appear in the report that I will reproduce, but Sofya writes to Gösta that there are fifteen candidates (quoted in [Cooke 1984]): well before this, a letter of Hermite in June 1888 reports on two other memoirs (there are actually three memoirs in the file of prizes at the archives of the Academy of sciences). The Commission is composed of Maurice Lévy, Philips, Resal, Sarrau, Darboux (reporter). Darboux begins by recalling the problem, then presents and explains the choice of the Commission.

> With unanimity, the Commission awarded the prize to the memoir inscribed under the n° 2 and bearing the quotation: *Say what you know, do what you must, come what may.* This remarkable work contains the discovery of a new case in which one can integrate the differential equations of motion of a massive body, fixed at one of its points. The author was not content to add a result of the highest interest to those that have been imparted to this subject by Euler and Lagrange: we owe to him a discovery in which almost all the resources of the modern theory of functions are employed. The properties of the ϑ-functions [theta functions] in two independent variables permit giving the complete solution in the most precise and elegant

> form: and one thus has a new and memorable example of a problem in mechanics in which these transcendental functions enter, whose application had until now been limited to pure analysis or geometry.

Appendix: the Bordin prize. C. L. Bordin was a Parisian notary who left the *Institut de France* an allowance of 12,000 francs, the interest to be divided between four of the academies for establishing annual prizes. A decree of 12 November 1835 authorized the Academy of sciences to accept this legacy. Awarding the prize was not proposed until 1856 and this alternatively in two groups, one concerning the mathematical sciences, the other the physical or natural sciences. The first prize proposed in mathematics was in 1862, it was (already) for 3,000 francs. The first subjects related to the mathematics section would be rather, in the actual conception, from physics, as for example those of 1866:

> To determine the indices of refraction of the glasses that are presently used in the construction of optical and photographic instruments, etc.

We have seen in our text the mathematical subjects proposed in 1886, 1888 and 1894. The subject for 1892 was also a true mathematical topic, in the sense we understand it today:

> Applications of the general theory of Abelian functions to geometry.

The last Bordin prize awarded in mathematics was in 1996, the last laureate was a woman, Doina Cioranescu, for her work on materials with cavities and periodic structures with multiple scales, the prize was (as always) 4,000 francs (about 600 euros in 2006, quite far from the annual salary of a lycée teacher). The Bordin prize has since been combined with those with lots of other foundations to create the Grand medal of the Academy of sciences (the other academies continue to award "Bordin prizes").

I owe all this information to Florence Greffe, chief archivist of the Academy of sciences (the commentaries are of course my sole responsibility).

Her reputation, yesterday ... and today?

Sofya participated in this small elite that was the community of mathematicians, she was consulted, for example with regard to the memoir that Poincaré would propose for the prize of the sixty years of King Oscar II. On 31 December 1888, Hermite

writes to Gösta (see [Dugac 1985, p. 154]) while sending him a draft of a report:

> I have conferred with Madame Kowalevski in order to find out your feeling on Poincaré's merit, thinking that I could not address myself better, and it is in complete and absolute concordance with what she said to me, and with what I myself have been thinking for a long time, that I have expressed myself in terms of which I make you the judge. I think I should let you know that, according to Madame Kowalevski, it's the unanimous opinion of the geometers of whom I have made myself the spokesperson [...]

Sofya contributed to ties between the French and German schools of function theory; she was editor of *Acta Mathematica* (as we saw in chapter VII) and in that capacity promoted the publication of articles by her Russian colleagues and thus their dissemination in Western Europe. She also played an important role in the transmission of mathematical ideas.

At the time of the Christiana (Oslo) conference in July 1886, where she did not present a communication, she received an ovation and was elected president of the mathematics section and, at the time of the banquet, Bjerknes gave a long discourse in her honor, followed by enormous applause.

Sofya thus had an excellent scientific reputation. If she was much talked about regarding her bohemian life, her incomprehensible relations with her husband, her political opinions, it seems that no mathematician of her time placed her scientific qualities in doubt. As James says [2002, p. 237]:

> All those who knew her remembered her as a woman of great spirit and originality.

There is no question that her reputation today does not correspond either to what it was during her time or to the quality of what we can read in her articles. I think I have expressed myself clearly on her work on the solid.

Sofya wrote but few articles, her life prior to Stockholm was agitated and difficult (encompassing as we have seen several less productive years mathematically), and she died at the very height of her career.

Election to the Saint Petersburg Academy. In 1889, after the whole series of recognitions that the Bordin prize, the permanent position in Stockholm, the Swedish prize and her election as corresponding member of the Saint Petersburg Academy

constituted, Sofya could dream seriously of a position in a more important European center. The letters of Hermite quoted above give account of her attempts in France. When Буняковский (Bunyakovski) died at the end of the year, thus freeing up a place at the Saint Petersburg Academy, she decided to put forward her candidacy, supported by Mittag-Leffler and by Hermite. Contrary to the title of corresponding member, which is honorary, that of member assures a salary. But there is another serious candidate, Markov, an ambitious young mathematician who would leave his name on an important concept from probability theory, the *Markov processes* (but not yet there). And Markov takes this very seriously, a little like a rugby player takes the game rather seriously by allowing himself to trample on an adversary to gain possession of the ball. He thus causes a rumor to circulate that Sofya's work on the solid, her most beautiful claim to glory, that of the Bordin prize, is false, that Sofya allowed some cases to escape (see the discussion on this subject on page 122). On 18 May 1890, Sofya writes in her diary [Kochina 1985, p. 300]:

> Markov has claimed in public that my memoir is full of errors, but that he would show them only after the respected academicians who nominated me for membership had been kind enough to read my memoir ... But after Markov had been made an extraordinary [associate] Academician, he was so condescending that he said in a private conversation, that my memoir was not as bad as it had at first seemed.

Not only did Markov claim that Sofya's work—with whom he is in competition—is false, he claims that Chebyshev has not read it, thus attacking her defender. Lyapunov silences Markov by proving, using a more secure method than Sofya's, that she was right and had given a complete solution.

Later, on 17 November 1892, the Moscow Mathematical Society, which had had enough of Markov speaking ill of his colleagues, voted the following decision [Kochina 1985, p. 304]:

> The society decided that because unfounded claims, such as those made by Prof. A. A. Markov with respect to the work of S. V. Kovalevskaya, V. G. Imshenetsky, P. G. Bugaev and G. G. Appelrot, serve no purpose to science, and the discussion of such claims has vainly distracted the Society from its work, hereafter the Society will not accept for discussion any unfounded and vicious claims.

The results of this manoeuver of young Markov:

Bunyakovski (1804–1889)

What in Western Europe is called the Cauchy–Schwarz inequality is called in Russia the Cauchy–Bunyakovski inequality ... and for good reasons: Bunyakovski discovered it before Schwarz and, it seems, even before Cauchy.

Liapunov (1857–1918)

Some ten years later the same Lyapunov corrected a false proof of the central limit theorem ... due to the same Markov.

– First, Sofya was not elected. Perhaps she would not have been in any case,

– because her field, complex analysis, was not yet very much in fashion in Russia. Consider for example the illustrious Chebyshev, a powerful and respected mathematician, whom Sofya had met in her youth (Chebyshev kept a salon in Saint Petersburg where you could go to ask him mathematical questions, which Sofya did just after her marriage, in 1868; Roger Cooke [2002a, p. 6] states that if she had been able to study in Russia, Sofya would certainly have become his student). He was famous for his work in differential equations and had almost proved what is known as the prime number theorem ... the one that Hadamard and de la Vallée Poussin proved completely, thanks to complex analysis.

– because the theory of Abelian functions was little known in Russia and considered dry and undeveloped,

– because, according to Pelageya Kochina, there was no way that there would ever have been room for Sofya in tsarist Russia.

– But nothing ever is lost, above all a rumor. Easy to make a rumor spread, but difficult to contain one. Many years later, when Lyapunov and his colleagues had made Markov shut his mouth by proving that this rumor was unjustified, Klein: wrote (in 1926) [4]

> Nor is one completely satisfied with her work on rotation.

We have also seen (page 146) that one of the articles written by Sofya, [Kowalevski 1885a], turned out to be wrong, a not infrequent occurrence in mathematics, which does not take anything away from the quality of the rest of her work.

Clearing hurdles

The scientific careers of women are often compared with a hurdles race. In Sofya's case, I think more about those equestrian contests where the bars are raised at each passage. It seems moreover that the trial is infinitely long in the sense that, even long after her death she has to prove her competencies and

4. In [Klein 1979], the complete text is quoted on pages 234 ff.

the quality of her contribution to science again and again. We find in chapter XI some recent examples of these posthumous obstacles that are constantly accumulating before her.

The article [Koblitz 1987a] studies in a very interesting way the manner in which Sofya's image has (been) modified since her death, especially by Bell [1937] and perhaps also by Klein [1979]. It seems to me, as I have said, that Anne Charlotte Leffler also has some responsibility in the matter (see chapter VII). Here we do not need to reproduce everything that has been said in [Koblitz 1987a]. I will quote Bell's text on Sofya in its entirety in chapter X, and that of Klein along with other assertions by other authors, heirs of Anne Charlotte Leffler, in chapter XI.

Sofya was extremely famous in her lifetime. See for example the way (page 214) that Jules Verne remembered her. The news of her death was announced in newspapers over the entire world (even in Algiers, Istanbul and other remote capitals, says Ann Hibner Koblitz [1987a]) and less than five years afterwards, at least four different biographies had appeared: the fictionalization was *en route*. See, still in [Koblitz 1987a], the list that I can't keep myself from finding hilarious (you'll tell me undoubtedly that it's nothing to laugh about, but I can't help it ...) of articles that appeared shortly after the "novel" by Anne Charlotte Leffler, e.g. a book: *The Russian New Woman. A Cincinnati Writer Finds Proof in Her Sad Life That Women's Sphere is Home*.

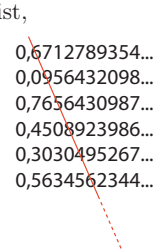
More seriously, the example of Sofya and her career has also been used against the entry of women into higher education and the active life. See especially [Rowold 2001].

She is reproached in astonishing ways. You might find it strange, for example, that mathematicians today, who spend a large part of their time traveling, from a conference in Japan to a week in Germany before a semester in the United States and a stay of two months in Djursholm, can comment on Sofya's life in Stockholm with "She never missed an opportunity of leaving Stockholm". Yes, whatever her feelings were for this city and its society, she felt the need to travel, to meet other colleagues. Like us.

Sofya's scientific independence

I find it rather normal that we ask ourselves, in hiring young mathematicians, how independent they are of their research directors, what original ideas they might have, whether we can expect other new and good ideas.

Suppose, says Cantor, that we can enumerate the real numbers between 0 and 1, writing them one after the other in the order of this list,



Then the number

$$0,706957\ \ldots$$

(created by modifying each of the digits on the red diagonal: we replace 6 by 7, 9 by 0, 5 by 6, etc.) is not in the list: its first digit (7) is different from that (6) of the first number written so it is not that first number. Likewise its n^{th} digit is different from that of the n^{th} number ... This is what is known as Cantor's diagonalization process, a proof from 1890 (different, however, from the one published in *Acta Mathematica* a few years earlier) of the fact that the infinity of real numbers is of a different nature than that of integers.

A remark due to Schwarz and which all mathematics students know, is the fact that, for a $\mathcal{C}^2$-function f of two variables, we have

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \ \ldots$$

a banality henceforth, but so useful!

I also find it normal, when speaking of the numerous students of this monument, of this "father of modern analysis" that is Weierstraß, that we ask ourselves whether they have had original ideas or if it is the "great analyst of the banks of the Spree" who inspired them, who breathed everything into them. We have seen, we will see again, that he was stingy neither with his ideas nor with his advice nor with his unpublished manuscripts. For example, it was Du Bois-Reymond who published his example of a continuous nowhere differentiable function, as we read on the very first page of [Riesz & Sz.-Nagy 1990].

Weierstraß had many students and Sofya is in good and honorable company. Many have left their names with useful and well known mathematical results:

- the Cantor set and diagonization process,
- the Cauchy–Kovalevskaya theorem, the Kowalevski top,
- Fuchs's criterion,
- Frobenius's morphism,
- the Runge–Kutta method,
- the Killing form,
- the Schwarz lemma and reflection principle,
- the Mittag-Leffler theorem and star,

certain among which occupy the margins of these pages.

All have made careers in mathematics, some have lived long, others less. At the time of their death all were recognized and respected mathematicians.

However, most of the texts that mention Sofya raise the question of her independence with respect to her professor, Weierstraß:

> [...] we aren't able to respond to the question of her independence with respect to her professor [Domar 1978, p. 10].
>
> The first striking thing is that these papers are very close to and in the style of those of Weierstraß, to the extent that one cannot see whether they contain independent or personal ideas [Klein 1979].
>
> [...] she succeeded in proving that a linear partial differential equation with analytic coefficients has analytic solutions, an application of ideas of Weierstraß [Klein 1979].
>
> The work of Cauchy on systems was done independently and in an improved manner by Sophie Kowalewsky (1850–91), who was a student of Weierstraß and who applied his ideas [Kline 1972, p. 702].

> On 24 December 1888, the *Institut de France* solemnly awarded her the Bordin prize [...]. It is difficult to determine the part that Weierstrass may have taken in this work [Loria 1903, p. 391].

whereas I haven't seen that anyone has raised the same question for Mittag-Leffler, for example. As Roger Cooke [1984, p. 176] observes:

> It is an implicit premise that one has to determine precisely the originality of work of a woman mathematician before admitting that she is good.

He furthermore says it in regard to the historian and mathematician Gino Loria and about what we have just seen and will again see expressed in this book, and about which he asks:

> How would he like it if this criterion was applied to his own work?

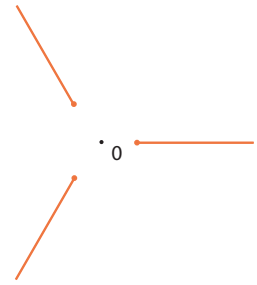
Rumors, rumors ... Look at how rumors have hard sustained lives. In the recent article [Kozlov 2000, p. 1191], we can read, in the spirit of what has preceded:

> [...] her scientific activity was above all a development of ideas of her great professor,

whereas, eight pages above, the author, Valeri Kozlov, asked himself this question ... and has already responded to it:

> [...] it is natural to ask: could this idea have been suggested by Weierstraß? We can respond with confidence in the negative. The proof is in her letter 57 to Mittag-Leffler from 28 December 1884. There she presents her method [...].

This has to do with the letter that was the subject of chapter VI. As is clearly the case for this article, which I have already had occasion to mention (and it's not over ...), the idea of Sofya's (scientific) dependence has certainly been strengthened by her image as a dependent woman that Anne Charlotte constructed (see our page 139).



The Mittag-Leffler star of a power series is the largest starred open set (at 0) on which the sum of the series extends to an analytic function. The figure represents the Mittag-Leffler star of $\sum_{n\geq 0} z^{3n} = \frac{1}{1-z^3}$.

I do not think that there is a total order relation on the set of mathematicians or even on the small subset comprised of Weierstraß' students. My object is not to discuss whether Sofya was more gifted or favored than Schwarz, but to understand why she is no longer there at all.

Sofya or oblivion

It would be interesting to make a more precise study, an authentic work by a historian, on the way that Sofya's memory has been lost in the course of the 20$^{\text{th}}$ century,

– for example, how she who was the most gifted student, the favorite student, of Weierstraß comes not to be mentioned any more at all (for instance [Remmert 1991]),

– or again how she is an active editor, promoting the international renown of the young journal *Acta Mathematica* and how her name absolutely does not appear in the centenary issue [5]—save for a whim of an almost senile Mittag-Leffler in the article [Weil 1982] (see our page 237).

And that is not all. She disappears anew, as if she were a sin of youth about which it is bad taste to speak, once established. Here is a new example of it, in the titles of papers by Adler and van Moerbeke:

– in 1982, they discover the notion of algebraic integrability (and the fact that it is the notion used by Kovalevskaya, as I already pointed out on page 104), the article [Adler & van Moerbeke 1982] is entitled *Kowalewski's asymptotic method, Kac–Moody Lie algebras and regularization*;

– in 1989, they publish [Adler & van Moerbeke 1989], *The complex geometry of the Kowalewski–Painlevé analysis*;

– in 2004, the book [Adler et al. 2004], which makes up the balance, the sum, of the theory of their papers, is entitled *Algebraic integrability, Painlevé geometry and Lie algebras.*

⋆

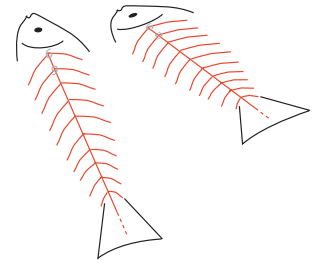
After the Swedish bourgeoisie, *Acta Mathematica*,
Parisian life and Cantor's diagonal proof,
a literary pause.

⋆

5. See the correspondence of Ann Hibner Koblitz with the editors of *Acta Mathematica* in the respectable personages of Gårding and Hörmander in [Koblitz 1984]; also the paragraph on *Acta Mathematica* on page 141 and the "recollections" on page 238 ff.

Pause:
Angels and fishbones

Sonia is a woman who goes everywhere like the wind, in a single bound. Patience is unknown to her, waiting is unbearable.



Nevertheless she waits. Gustave wants to speak to her. He has invited her to lunch, she's at his home and is waiting for him. She is seated in the library she knows well, on a bench covered with a long upholstered cushion, before the window, and looks out at the garden covered with snow at a small brick bridge that spans a frozen stream. The room is beautiful and exotic, furnished in gray, old gold and pink, with a Chinese vase, and the books of the library of old mahogany.

Finally she is summoned. Signe, the lady of the house, is absent. The governess has set lunch by the fire. She does not eat with Gustave and Sonia but enters and leaves several times during the meal, proposing an additional portion that Sonia declines, pouring more wine in the glasses. Gustave and Sonia occupy large chairs on each side of the wood fire. When the governess has left with the trolley, Gustave says: "Sonia, the journal." But Sonia says that she is tired. "Not today." She does not say it to Gustave, but the velvet chairs, the governess's solicitude, the fireplace and also the Chinese vase in the library she feels she will break one day, paralyze her.

"Let's discuss it tomorrow. This time I'm inviting you."

"Alright", says Gustave softly.

⋆

The next day, they are seated in a popular restaurant. It is Sonia who chose the place. The dining room is conventional, with a lustre from the large fire in the fireplace. "Here we can talk peacefully", Sonia says. There are lots of people, three tables where they are speaking loudly, others where they are whispering, but indiscreetly. There are also paintings on the walls, green curtains with white wild rose motifs, and there are red candles on the tables. Sonia orders a bottle of white wine, a bowl of soup and smoked herring with potatoes. Gustave orders the soup, considers the idea of herring, and requests grilled sole. Sonia laughs. She says: "I went skating with your sister this morning. You should come with us. You would be funny." Gustave thinks: how different she is. Lively. Yesterday she

did not even look at me. Should I say it to her? Better not. He watches Sonia turn her soup in all directions, splash the tablecloth, lift her spoon to her mouth, give out a little cry, she has burned herself, and breaks out laughing again.

"The journal," says Sonia, offering a roll to Gustave. "Your journal. How I liked to read there again the mathematics of Cantor's infinite sets." Gustave puts down his spoon and takes a gulp of the wine. "I love the diagonal proof," Sonia says. She smiles. Her smile brightens her face before changing into a new outburst of laughter. She says: "The diagonal proof is a geometrically visible way of relating finite sets to infinite sets. When I was a child there came a day when I understood that there are as many even number as plain numbers, integers. It was paradise." She laughs again. "The fractions." She rummages in the little shapeless bag that she carries everywhere with her, finds a crumpled envelope and a stub of a pencil and makes a sketch of a grid with branches. "I found that all by myself, as many points with integer coordinates as integers, I arranged all the points systematically, in nice order, simply by drawing a set of diagonals, like this." Gustave takes out a morocco notebook. Sonia's slender fingers multiply the lines during all the soup and fish, sliding surely over the paper. "And, says Cantor, we count the fractions in the same way. There aren't any more fractions than there are integers." Gustave watches the fingers come and go with assurance. The wine is acid, the soup rustic, the clientele a little vulgar, they do not talk at all about what he wanted to discuss, but she has such enthusiasm—he can truly imagine her as a little girl in the process of counting fractions and he is simply happy.

"Paradise," says Sonia. "And especially later, when I saw Cantor use another diagonal, but for showing that there are many more numbers, ordinary numbers, real numbers, than there are fractions, the articles I read in your journal ..." Sonia draws more lines and drinks two more glasses of wine, one after the other. She becomes more animated, her face has taken on a shade of velvety ripe peach. Her pencil breaks, Gustave takes out his pen and he watches her strengthen her grip. There appear, between Sonia's fingers, a series of spider webs, of spectres of branches, of forms recalling the graduated dorsal spine of fish skeletons. "The finite numbers are the visible universe. They are there and really there even if our human descriptions of them and their nature are false ... and it is certain that they are incomplete. The infinities are like the angels. That is what

Cantor has constructed for us, a paradise with its angels." She laughs again. "And now, ask for the pear with hot chocolate. Myself, I don't want anything more." But she serves herself another glass of wine.

Gustave swirls the hot liquid and mellow fruit on his tongue. He understands that if he wants to continue on the path of this delicate near-intimacy, he must give up the banal pleasures of conversation, of flirting and of gossip. The sensation is as agreeable as the bitterness of the chocolate, of the white, golden and green sap tint of the wine that is oscillating again in Sonia's glass when she laughs, as the sparkles of light in her gray eyes. Then they speak about the journal, Sonia enthusiastically accepts participating in this adventure. She laughs again: "I will find some Russian articles for you." When they leave the restaurant, Sonia whistles gayly and people turn to look at her, and Gustave cannot even feel uneasy about it.

⋆

The page of *A Whistling Woman* by A. S. Byatt [2003] where Cantor's diagonal process is discussed before sole bones is one of the most beautiful literary texts on mathematics that I have ever read. The two theorems to which Sonia alludes are the denumerability of the set of rational numbers (see here page 142) and the nondenumerability of the set of real numbers (the diagonal process, see here page 174, attentive readers will also have understood that it is also an anachronism, this proof not being the one published in *Acta Mathematica*). I hope that I haven't weakened the text too much in adapting it here, all the more in that the reference to Wittgenstein, essential in the passage by Byatt, became impossible: the anachronism would have been too violent (the paradise that Cantor created for us, an expression due to Hilbert, is also an anachronism, but more discrete).

I also used some phrases from *Babel Tower* [Byatt 1997] and copied the description of Gustave's interior from the one given in *Still Life* [Byatt 1985].

I like to mix things up a bit: Dostoyevsky lovers will have recognized, in the Chinese vase that Sonia surely ended up breaking in this virtual novel, a character from *The Idiot* [Dostoyevsky 1869].

CHAPTER X

THE WOMEN OF *MEN OF MATHEMATICS*

I am interested here again in Sofya's scientific reputation. I have read the commentaries of several specialists who reproach Bell's book [1937] *Men of mathematics* and its responsibility for what I call her "bad reputation". As Roger Cooke [2002a] says

> [...] since she could not attend classes legally, he proposed private lessons. (I am trying hard not to make that proposal sound sinister. There is not the slightest reason to believe their relationship was anything but that of a friendly professor and student, despite some outrageous insinuations to the contrary in the book by E. T. Bell, in which Kovalevskaya, the only woman in a book appropriately named *Men of Mathematics*, is trivialized and depicted as a seductive playmate.)

This book, which seems to have had an important influence on generations of mathematicians, is famous today for the slight (or worse) distortions and the romanticized biographies that it contains. The case of Galois is particularly well known for its invention. I refer to a very interesting article by Tony Rothman [1982].

> As an inventor of fairy tales, one can enjoy Bell; as a biographer it is unclear how far one can forgive him,

he writes with regard to the chapter dedicated to our young genius (but the remark also applies to Sofya's case), whereas Ann Hibner Koblitz [1993, p. 279] herself says that Bell

> was not a man to let the facts spoil a good story.

I have thus decided to go there and have a closer look. I present here a few aspects of the book related to what interests me. I confess that Bell's style is not entirely my

Roger Cooke

I admit that I like this remark by Roger Cooke a lot, typical of the fondness, reasoned but profound, that he evidently feels for Sofya. "It is impossible to doubt" (see the marginal note on page 207) that the historian has fallen in love with the object of his study.

M. Audin, *Remembering Sofya Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_10,

Maria Agnesi (1718–1799)

Maria Gaetana Agnesi, the "first woman", whom I have already mentioned several times, is the author of a work on analytic geometry in two volumes that appeared in 1748. She was elected member of the Bologna Academy of sciences, then appointed to a chair at Bologna university by Pope Benedict XIV, but turned down the position to dedicate her life to religious study. She has left her name with an algebraic curve of degree 3, a cubic, the *cubic of Agnesi*, which she may not have invented herself and which because of a mistranslation from Italian into English was transformed into *witch of Agnesi*. Was there a connection between woman and witch in the head of the translator? This is the curve I used on page 32 to illustrate the approach to an asymptote.

literary ideal, but that will not stop me, as you will see, from appreciating, or even admiring, its processes.

In this chapter I present the seven pages Bell dedicates to Sofya and Weierstraß, but first, to familiarize readers with Bell's style and opinions, I shall page through the book starting with the names of the women mentioned in his index.

Remark. The French translation by Ami Gandillon [Bell 1939] is occasionally a bit attenuated, as its title *Les grands mathématiciens* and the interchange of titles and subtitles of the chapters might lead us to suspect.

The women in the index

It is not entirely true that Sofya is the only woman in the book. Let us look for the others. The index of *Men of mathematics* [Bell 1937] offers us a goodly portion (but not all) of the women who appear in the body of the text. Here is the alphabetical list of these women. The majority are mothers, sisters or spouses of mathematicians (some are simultaneously daughters of remarkable men). A few are writers, friends of or esteemed by our men of mathematics. Some are students of these men. One did innovative work as a mathematician but this is not mentioned. Others are women of power. The witch of Agnesi is not there at all ... nor are Hypathie, Émilie du Châtelet, Mary Fairfax Somerville,[1] nor is Ada Byron Lovelace, Byron's daughter, who might have inspired Bell, nor is Grace Chisholm Young.

It is rather instructive, in order to get an idea of what Bell is capable of writing, to run through this list.

– Antoinette, Marie (*sic*, Bell seems to think that Antoinette was her last name) is mentioned in connection with Lagrange, who would become one of her favorites after being invited by Louis XVI to join the Academy of sciences.

– Austen, Jane, Cayley's favorite writer after (albeit) Walter Scott.

– Boole, Mary Everett, who, after having been the daughter of the Greek professor at Queen's College, married Boole and "became his devoted disciple". A mathematician then?

1. Just as Émilie du Châtelet translated Newton from Latin to French, Mary Somerville translated Laplace from French to English.

– Brochard, Jeanne, mother of Descartes.
– Cantor, Marie Bohm, mother of Cantor.
– Catherine I, who appears in the chapter *Analysis Incarnate* dedicated to Euler, was the widow of Peter the Great. She ruled for two years following the death of her husband, during which she had time to found the Academy of sciences. A first indication (alphabetically) of Bell's intent: Catherine I appears under the name of "the Grand Catherine" (*Catherine the Great* is Catherine II, who ruled Russia a little later).

An inexactitude that permits him some stylistic effects, so why not? Catherine the Great is praised as the equal of the no less great Frederick the Great (Frederick II) for her open-mindedness and her liberality, but (superfluous gossip) the Great Catherine was the mistress of Peter the Great before being his wife (the Greatness of the bed used is not specified). In the French translation, Ami Gandillon has made the correction and translated Catherine the Great by "Catherine Ière".
– Christina, queen of Sweden, plays a quaint second role in *Gentleman, soldier, and mathematician* (under these qualifiers we perhaps can recognize Descartes). The legends, the gossip and the mud-slinging have obscured the historical truth of this exceptional woman ... whose main fault was occupying a position of power (in French history, the case of Catherine de Medici is a bit analogous). We can admire the courageous manner in which Bell unleashes her.

Christina of Sweden
(1626–1689)

> This somewhat masculine young woman was then nineteen, already a capable ruler [...] a wiry athlete with the physical endurance of Satan himself, a ruthless huntress, an expert horsewoman who thought nothing of ten hours in the saddle without once getting off, and finally a tough morsel of femininity who was as hardened to cold as a Swedish lumberjack. With all this she combined a certain thick obtuseness toward the frailties of less thick-skinned courtiers. Her own meals were sparing; so were those of her courtiers. Like a hibernating frog she could sit for hours in an unheated library in the middle of a Swedish winter [...] Her cabinet, she noted without a

> qualm, always agreed with her. She knew everything there was to be known; her ministers and tutor told her so. [...] The very hour that this holy terror saw Descartes's philosophy she decided that she must annex the poor sleepy devil as her private instructor [...]

Let us say right away that this poor devil Descartes accepted (unenthusiastically) and that he died soon thereafter, on 11 February 1650, he too in Stockholm, he too of pneumonia.

But let us return to Christina. Dense and stupid, she understood nothing, even in Greek she had to do battle with grammatical puerilities that Descartes had mastered when he was still a little boy. And I pass on the rest. She reappears in *Greatness and misery of man*, a chapter whose hero, Pascal, dazzled by her transcendent brilliance, offers her his calculating machine. The incorrigible Bell comments: "What Christina did with the machine is not known."

– Dedekind, Julie, Dedekind's sister.
– De Long, Claire, Fermat's mother.
– De Long, Louise, Fermat's wife.
– Edgeworth, Maria, didactic novelist and friend of the Irish mathematician Hamilton.
– Elisabeth, princess, preferred disciple of Descartes. This was a Bohemian princess who was exiled to the Netherlands, with whom Descartes maintained a long correspondence.

> Officially, he declared "of all my disciples, she alone has understood my works completely". There is no doubt that he was genuinely fond of her in a fatherly cat-looking-at-a-king's-female-relative sort of way, but to believe that he meant what he said as a scientific statement of fact is to stretch credulity to the limit [...]

so this is something that we will not do. Descartes taught her analytic geometry. She solved "by Descartes' method" a problem of elementary geometry, to construct a circle tangent to three given circles, a problem to which this method is probably not the best suited. "She was quite proud of her exploit, poor girl", in Bell's words. *Poor girl*, he writes. As

for Descartes, who let her do it, he "would not undertake to carry out her solution and actually construct the required tangent circle in a month" (there are eight solutions in general). "If this does not convey his estimate of her mathematical aptitude it is impossible to put the matter plainer".
– Euler, Catharina Gsell, daughter of the painter Gsell and Euler's wife.
– Euler, Marguerite Brucker, Euler's mother.
– Galois, Adélaïde-Marie Demante, Galois's mother and his teacher until he reached the age of twelve.
– Gauß, Dorothea Benz, Gauß's mother.
– Gauß, Johanne Osthof, Gauß's first wife.
– Gauß, Minna Waldeck, Gauß's second wife.
– Germain, Sophie, mathematician mentioned along with the three preceding women in the chapter on Gauß, the romantic (but veracious) story of Sophie Germain hiding under the name of Monsieur Leblanc is of course reported, but this is not our subject. We rather note the allusion to another Sophie, our Sophie on p. 261:


Sophie Germain (1776–1831)

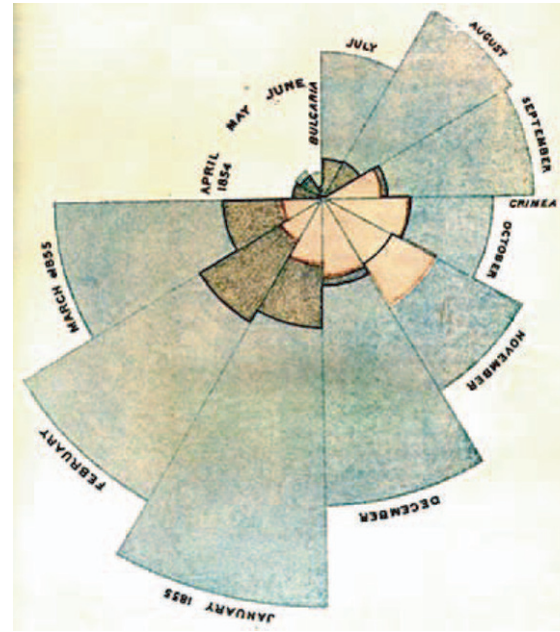
> She and Gauß never met, and she died (in Paris) before the University of Göttingen could confer the honorary doctor's degree which Gauß recommended to the faculty. By a curious coincidence we shall see the most famous woman mathematician of the nineteenth century, another Sophie, getting her degree from the same liberal university many years later after Berlin had refused her on account of her sex. Sophie seems to be a lucky name for women in mathematics provided they affiliate with broadminded teachers.

– Hamilton, Eliza, Hamilton's sister.
– Hamilton, Sarah Hutton, mother of Hamilton, who likely had from her his "extraordinary intellectual briliance", since her family was renowned for their brains.
– Kowalewski, Sonja.
– Lagrange, Marie-Thérèse Gros, daughter of a wealthy physicist and Lagrange's mother.
– Lobatchewsky, Praskovia, Lobatchewsky's mother.
– Newton, Hannah Ayscough, Newton's mother.

– Nightingale, Florence, appears in the chapter *Invariant twins* dedicated to Cayley and Sylvester. The latter, actuary in an insurance company, had retained as his only mathematical activity the private lessons he gave to certain students, one of whom

Florence Nightingale
(1820–1910)

> was to leave a name known and revered today in every country of the world. This was in the early 1850s [...] when young women were not supposed to think very much beyond dabbling in paints and piety. It is rather surprising to find that Sylvester's most distinguished pupil was a young woman, Florence Nightingale, the first human being to get some decency and cleanliness into military hospitals [...] Sylvester at the time was in his late thirties, Miss Nightingale six years younger than her teacher. Sylvester escaped from his makeshift ways of earning a living in the same year (1854) that Miss Nightingale went out to the Crimean War.



Strangely (?) the next phrase begins with "Before this, he had taken another false step [...]". We will encounter Sylvester again in chapter XI.

Bell's readers will not know that in addition to being a courageous nurse whom even Bell is capable of admiring, Florence Nightingale had also been a mathematician, a pioneer in the application of techniques of statistical analysis, developing the *polar-area* diagrams, known also as *coxcombs* in English (and ancestors of the *diagrammes en camembert* in French), for showing the needless deaths caused by the deplorable sanitary conditions and the reforms that were needed. The above figure shows one of these diagrams, in which the exterior parts (in blue-green) represent the number of young men who died in a military hospital of a cause other than their wounds during the Crimean war.

She was one of the first to show that a social phenomenon could be measured and subjected to mathematical analysis.

– Noether, Emmy, appears only in a note at the bottom of a page about Sophie Germain, herself mentioned, as I have said, in the chapter on the *Prince of mathematicians*, and about Göttingen university which, having missed giving an honorary doctorate to Sophie Germain in 1831, having given a genuine doctorate to Sofya in 1874, was the *alma mater* of Emmy Noether before the anti-Semitic politics of nazi Germany expelled

> Fräulein Noether [...], the most creative abstract algebraist in the world [...] Göttingen lost the liberality which Gauß cherished. (2)

But let's not exaggerate, Göttingen never gave Fräulein Noether a permanent position. It was the American women's college Bryn Mawr that did so.

– Pascal, Antoinette Bégone, Pascal's mother.
– Pascal, Gilberte, Pascal's sister.
– Pascal, Jacqueline, Pascal's sister.
– Sophie von Brandenburg, electress, a relative of Princess Elisabeth and student of Leibniz.
– Storey, Miss, fiancée of Newton.

Emmy Noether

If, because of her anonymity, she does not appear in the index to the book, Bell does not miss mentioning in the body of the work the celebrated "infamous coquette" responsible for the death of Galois, whom to no one's surprise he calls a prostitute, another rumor for whose propagation he bears a portion of the responsibility, another rumor without foundation (see [Rothman 1982]).

2. Regarding the dismantling of mathematics at Göttingen by the Nazi regime, see the article by Norbert Schappacher [1993].

Master and pupil

Now that the stage is set, let's go to the chapter on *Weierstrass and Sonja Kowalewski*, the chapter entitled *Master and pupil*. Bizarrely, the women named in this chapter, Weierstrass's mother Theodora Forst and his sisters Klara and Elise, with whom he lived his entire life, do not appear in the index, which thus does not contain a complete list of the women in the book.

Here is the part of the text in which Sofya appears (seven pages). Having attentively read this text and the article by Mittag-Leffler [1923], it seems clear to me that the latter, several excerpts of which we have already encountered, is the principal source for the former. Mittag-Leffler's article is written in French, except for the letters by Weierstraß quoted, which appear in their original German.

I quote Bell's text, with interruptions by commentary:

> The years (1864–97) of Weierstrass's career at Berlin as Professor of Mathematics were full of scientific and human interests for this man who was acknowledged as the leading analyst in the world. One phase of these interests demands more than the passing reference that might suffice in a purely scientific biography of Weierstrass: his friendship with his favorite pupil, Sonja (or Sophie) Kowalewski.

In other words, if we were just speaking about science, we could then omit Sofya?

> Madame Kowalewski's maiden name was Sonja Corvin-Krukowsky; she was born in Moscow, Russia, on 15 January 1850 and died at Stockholm, Sweden, on 10 February 1891, six years before Weierstrass.
>
> At fifteen Sonja began the study of mathematics. By eighteen, she had made such such rapid progress that she was ready for advanced work and was enamoured of the subject. As she came from an aristocratic and prosperous family, she was enabled to gratify her ambition for foreign study and matriculated at the University of Heidelberg.

The political convictions, the white marriage, the way a woman enrolled in Heidelberg?

> This highly gifted girl became not only the leading woman mathematician of modern times, but also made a reputation as a leader in the movement for the emancipation of women, particularly as regarded their age-old disabilities in the field of higher education.

> In addition to all this she was a brilliant writer. As a young girl she hesitated long between mathematics and literature as a career. After the composition of her most important mathematical work (the prize memoir noted later), she turned to literature as a relaxation and wrote the reminiscences of her childhood in Russia in the form of a novel (published first in Swedish and in Danish). Of this work, it is reported that "the literary critics of Russia and Scandinavia were unanimous in declaring that Sonja Kowalewski had equaled the best writers of Russian literature in style and thought". Unfortunately, this promising start was blocked by her premature death, and only fragments of other literary works survive. Her one novel was translated into many languages.

I am sure that Bell would not have used the feminine word *écrivaine* if he were writing today in French. We will see also that he would not have written that Sofya was a brilliant (feminine) *mathématicienne* (not even that she was a brilliant mathematician, as did Mittag-Leffler). I do not know what the career was that little Sofya might have been dreaming about. Perhaps it would be better to say that she vacillated "between mathematics and a literary career" or "between mathematics and literature" as does the French translator Ami Gandillon. Finally, we know that Sofya wrote another novel, but Bell may very well not have known it, he being a mathematician and [Kovalevskaïa 2004] not yet having been translated. We resume:

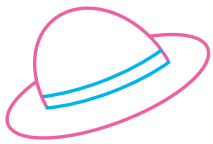
> Although Weierstrass never married he was no panicky bachelor who took to his heels every time he saw a pretty woman coming. Sonja, according to competent judges who knew her, was extremely good looking. We must first tell how she and Weierstrass met.
>
> Weierstrass used to enjoy his summer vacations in a thoroughly human manner. The Franco-Prussian war forced him to forego his usual summer trip in 1870, and he stayed in Berlin, lecturing on elliptic functions. Owing to the war, his class had dwindled to only twenty instead of the fifty who heard the lectures two years before. Since the autumn of 1869, Sonja Kowalewski, then a dazzling young woman of nineteen, had been studying elliptic functions under Leo Königsberger (born 1837) at the University of Heidelberg, where she had also followed the lectures on physics by Kirchhoff and Helmholtz and had met Bunsen, the famous chemist, under rather amusing circumstances—to be related presently. Königsberger, one of Weierstraß's first pupils, was a first-rate publicity agent

I have not succeeded in finding out who Ami Gandillon was. I am rather certain that the rare first name (which means "Friend") is masculine. Payot, Bell's French publisher, assures me of not having any information in its archives on this translator. Internet searches seem to suggest that our Ami might have been from Geneva, Switzerland, that he was perhaps a member of the *International*, perhaps before emigrating to the United States in 1911. Perhaps simply a young man working on this translation, which was published in 1939, and who disappeared in the turmoil.

> for his master. Sonja caught her teacher's enthusiasm and resolved to go direct to the master himself.
>
> The status of unmarried women students in the 1870s was somewhat anomalous. To forestall gossip, Sonja at the age of eighteen contracted what was to have been a nominal marriage, left her husband in Russia, and set out for Germany. Her one indiscretion in her dealings with Weierstrass was neglecting to inform him at the beginning that she was married.

We know, Weierstraß knew, and we know that Weierstraß knew that Sofya was married and moreover that her husband had gone with her to Germany. What she did not tell Weierstraß—and why would she have done so?—is that her marriage was fictitious. Here the real "novel" begins. Bell, while soberly avoiding the image of knowledge flowing from the lips of the master to the pupil seated at his feet (see the quotation from Mittag-Leffler on page 44), will excel. Attention to style!

> Having decided to learn from the master himself, Sonja took her courage in her hands and called on Weierstrass in Berlin. She was twenty, very earnest, very eager, and very determined; he was fifty-five, vividly grateful for the lift Gudermann had given him toward becoming a mathematician by taking him on as a pupil and sympathetically understanding of the ambitions of young people.



Unfortunately for French readers, the pathos of "She was twenty [...]; he was fifty-five [...]" was destroyed in the translation of Ami Gandillon who says "the professor was fifty-five years old".

> To hide her trepidation Sonja wore a large and floppy hat, "so that Weierstrass saw nothing of those marvelous eyes whose eloquence, when she wished it, none could resist."

The quotations are not attributed in the text, it involves here a phrase from Mittag-Leffler [1923]. Translation to translation, it becomes, under the pen of Marie-Louise Dubreil Jacotin [1948, p. 263]: "such that Weierstrass saw nothing of her marvelous eyes, the eloquence of which nothing could resist when she wanted to obtain something". The novel continues.

> Some two or three years later, on a visit to Heidelberg, Weierstrass learned from Bunsen—a crabbed bachelor—that Sonja was "a dangerous woman". Weierstrass enjoyed his friend's terror hugely, as Bunsen at the time was

> unaware that Sonja had been receiving frequent private lessons from Weierstrass for over two years.
>
> Poor Bunsen based his estimate of Sonja on bitter personal experience. He had proclaimed for years that no woman, and especially no Russian woman, would ever be permitted to profane the masculine sanctity of his laboratory. One of Sonja's Russian girl friends, desiring ardently to study chemistry in Bunsen's laboratory and having been thrown out herself, prevailed upon Sonja to try her powers of persuasion on the crusty chemist. Leaving her hat at home, Sonja interviewed Bunsen. He was only too charmed to accept Sonja's friend as a student in his laboratory. After she left he woke up to what she had done to him. "And now, that woman has made me eat my own words", he lamented to Weierstrass.

Genesis of a rumor:

First act, 1869, Sofya convinces Bunsen to accept Julia Lermontova.

Second act. Five years later, Bunsen complains to Weierstraß (the "two or three years" come from [Mittag-Leffler 1923] and is an error, the letter from Weierstraß is dated 21 September 1874, after the theses of Sofya and Julia).

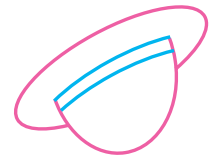
Third act. Weierstraß, who is a good friend of the two young women, finds this amusing and relates to Sofya in a letter what Bunsen said to him, see the story told on page 43.
And Weierstraß's letters remain.

Fourth act. Gösta quotes (innocently, I believe) the phrase about the "dangerous woman" in [Mittag-Leffler 1923] to express his appreciation for the power of Sofya's eyes.

Bell seizes at it.

Marie-Louise Dubreil-Jacotin did nothing to dispel the rumor: the phrase about the irresistible eyes is immediately followed in her article by

"Weierstraß was seduced nonetheless."



Ami Gandillon, who decidedly did not seem to appreciate the original text, did not dare to translate *Poor Bunsen*. But we, we sympathize with this poor Bunsen.

And we are amazed by the intelligent figuration played by the hat. We return to the main novel.

> Sonja's evident earnestness on her first visit impressed Weierstrass favourably and he wrote to Königsberger inquiring about her mathematical aptitudes. He asked also whether "the lady's personality offers the necessary guarantees".

We know, but perhaps Bell did not know it, that he also gave "Sonja" some problems to solve and was impressed by the solutions she brought him.

> On receiving an enthusiastic reply, Weierstrass tried to get the university senate to admit Sonja to his mathematical lectures. Being brusquely refused he took care of her himself in his own time. Every Sunday afternoon was devoted to teaching Sonja at his home, and once a week Weierstrass returned her visit. After the first few lessons Sonja lost her hat.

This is the moment I like, the one where she "loses" her hat. Bell actually wrote "Sonja lost her hat", but Ami Gandillon preferred "Sonia removed the hat". What a pity!

> The lessons began in the autumn of 1870 and continued with brief interruptions due to vacations or illnesses till the autumn 1874. When, for any reason the friends were unable to meet they corresponded. After Sonja's death in 1891 Weierstrass burnt all her letters to him, together

> with much of his other correspondence and probably more than one mathematical paper.

The phrase about the burned letters is taken from [Mittag-Leffler 1923].

> The correspondence between Weierstrass and his charming young friend is warmly human, even when most of a letter is given over to mathematics. Much of the correspondence was undoubtedly of considerable scientific importance, but unfortunately Sonja was a very untidy woman when it came to papers, and most of what she left behind was fragmentary or in hopeless confusion.
>
> Weierstrass himself was no paragon in this respect. Without keeping records he loaned his unpublished manuscripts right and left to students who did not always return what they borrowed. Some even brazenly rehashed parts of their teacher's work, spoiled it, and published the results as their own. Although Weierstrass complains about this outrageous practice in letters to Sonja his chagrin is not over the petty pilfering of his ideas but over the bungling in incompetent hands and the consequent damage to mathematics.

It is not evident that Weierstraß was so very disinterested; it seems on the contrary that he often complained about not being cited enough.

> Sonja of course never descended to anything of this sort, but in another respect she was not entirely blameless. Weierstrass sent her one of his unpublished works by which he set great store, and this was the last time he ever saw it. Apparently she lost it, for she discretely avoids the topic—to judge from his letters—whenever he brings it up.
>
> To compensate for this lapse Sonja tried her best to get Weierstrass to exercise a little reasonable caution in regard to the rest of his unpublished work. It was his custom to carry about with him on his frequent travels a large white wooden box in which he kept all his working notes and various versions of papers which he had not yet perfected. His habit was to rework a theory many times until he found the best, the "natural" way in which it should be developed. Consequently he published slowly and put out a paper under his own name only when he had exhausted the topic from some coherent point of view. Several of his rough-hewn projects are said to have been confided to the mysterious box. In 1880, while Weierstrass was on a vacation trip, the case was lost in the baggage. It has never been heard of since.

The lost manuscripts and the white wooden case come from [Mittag-Leffler 1923], a bit transformed by Bell.

> After taking her degree in absentia from Göttingen in 1874, Sonja returned to Russia for a rest as she was worn out by excitement and overwork.

We do not learn what she did for her doctorate (see chapter IV). Nor do we learn very much about her work.

> Her fame had preceded her and she "rested" by plunging into hectic futilities of a crowded social season in St Petersburg while Weierstrass, back in Berlin, pulled wires all over Europe trying to get his favourite pupil a position worthy of her talents. His fruitless efforts disgusted him with the narrowness of the orthodox academic mind.
>
> In October 1875 Weierstrass received from Sonja the news that her father had died. She apparently never replied to his tender condolences, and for nearly three years she dropped completely out of his life. In August 1878 he writes to ask whether she ever received a letter he had written her so long before that he has forgotten its date. "Didn't you get my letter? Or what can be preventing you confiding freely in me, your best friend as you so often called me, as you used to do? This is a riddle whose solution only you can give me ..."

This is translated, by Bell or by somebody else, from German. We know that Weierstraß used the familiar *Du* when writing Sofya, since we know his letters through [Mittag-Leffler 1923] and through [Bölling 1993], and we know that Sofya did the same in writing Weierstraß *Du*, *Deine Sonia*—thanks to the recovered letter (see pages 63 et seq. and [Bölling 1992 ; 1993]).

> In the same letter Weierstrass rather pathetically begs her to contradict the rumour that she has abandoned mathematics: Tchebycheff [Chebyshev], a Russian mathematician, had called on Weierstrass when he was out, but had told Borchardt that Sonja had "gone social", as indeed she had. "Send your letter to Berlin at the old address", he concludes; "it will certainly be forwarded to me."
>
> Man's ingratitude to man is a familiar enough theme; Sonja now demonstrated what a woman can do in that line when she puts her mind to it. She did not answer her old friend's letter for two years although she knew he had been unhappy and in poor health.
>
> The answer when it did come was rather a let-down. Sonja's sex had got the better of her ambitions and she

In his very first letters, Weierstraß addresses Sofya very formally—*verehrte Frau* and surely with the formal *Sie*—and this lasted two years. On 26 October 1872 ([Bölling 1993, letter 8]) he calls her *Meine theuere Sophie* (my dear Sophie). This is the letter written just after he learned that her marriage was fictitious, the one I quoted on page 71. He begins using *Du* in the letter from 4 November 1872 ([Bölling 1993, letter 9]). This belated use of familiar address is little compatible with the insinuation contained in the loss of the hat.

had been living happily with her husband. Her misfortune at the time was to be the focus for the flattery and unintelligent, sideshow wonder of a superficially brilliant mob of artists, journalists, and dilettante litterateurs who gabbled incessantly about her unsurpassable genius. The shallow praise warmed and excited her. Had she frequented the society of her intellectual peers she might still have lived a normal life and kept her enthusiasm. And she would not have been tempted to treat the man who had formed her mind as shabbily as she did.

We know that this period was the unhappiest of Sofya's life. She and Vladimir had returned to Russia hoping to find positions that corresponded to their qualifications.

In October 1878 Sonja's daughter "Foufie" was born.

The forced quiet after Foufie's arrival roused the mother's dormant mathematical interests once more, and she wrote to Weierstrass for technical advice. He replied that he must look up the relevant literature before venturing an opinion. Although she had neglected him, he was still ready with his ungrudging encouragement. His only regret (in a letter of October 1880) is that her long silence has deprived him of the opportunity of helping her. "But I don't like to dwell so much on the past—so let us keep the future before our eyes."

Material tribulations aroused Sonja to the truth. She was a born mathematician and could no more keep away from mathematics than a duck can from water. So in October 1880 (she was then thirty), she wrote begging Weierstrass to advise her again. Not waiting for his reply she packed up and left Moscow for Berlin. His reply, had she received it, might have caused her to stay where she was. Nevertheless when the distracted Sonja arrived unexpectedly he devoted the whole day to going over her difficulties with her. He must have given her some pretty straight talk, for when she returned to Moscow three months later she went after her mathematics with such a fury that her gay friends and silly parasites no longer recognized her. At Weierstrass's suggestion she attacked the problem of the propagation of light in a crystalline medium.

Henri Poincaré (1854–1912)

In 1882 the correspondence takes two new turns, one of which is of mathematical interest. The other is Weierstrass's outspoken opinion that Sonja and her husband are unsuited to one another, especially as the latter has no true appreciation of her intellectual merits. The mathematical point refers to Poincaré as a coming man and hopes that he will outgrow his propensity to publish too

> rapidly and let his researches ripen without scattering them over too wide a field. "To publish an article of real merit every week—that is impossible", he remarks, referring to Poincaré's deluge of papers.
>
> Sonja's domestic difficulties presently resolved themselves through the sudden death of her husband in March 1883. She was in Paris at the time, he in Moscow. The shock prostrated her. For four days she shut herself up alone, refused food, lost consciousness the fifth day, and on the sixth recovered, asked for paper and pencil, and covered the paper with mathematical formulae. By autumn she was herself again, attending a scientific congress at Odessa.

To which domestic problems does he refer? Bell however tells us that her marriage was happy. Even if Weierstraß thought that it must not have been. And what did Sofya do then in Paris? How and why did Vladimir die? We will not find out. Up until now Sofya has been called "Sonja". The status of widowhood renders her respectable and she now becomes "Madame Kowalewski":

> Thanks to Mittag-Leffler, Madame Kowalewski at last obtained a position where she could do herself justice; in the autumn of 1884 she was lecturing at the University of Stockholm, where she was to be appointed (in 1889) as professor for life.

a small error in dates

> A little later she suffered a rather embarrassing setback when the Italian mathematician Vito Volterra pointed out a serious mistake in her work on the refraction of light in crystalline media.

"Poor" (as Bell would say) Sofya never suffered this setback, since she was already dead when Volterra revealed this error.

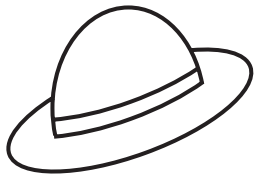
> This oversight had escaped Weierstrass, who at the time was so overwhelmed with official duties that outside of them he had "time only for eating, drinking, and sleeping ... In short", he would say, "I am what the doctors call brain-weary". He was now nearly seventy. But as his bodily ills increased his intellect remained as powerful as ever.

The first work of Sofya that appears in Bell's book is therefore an incorrect paper. And it is incorrect, that paper, because Weierstraß has not had the time to check it.

> The master's seventieth birthday was made the occasion for public honours and a gathering of his disciples and former pupils from all over Europe. Thereafter he lectured publicly less and less often, and for ten years received a few of his students at his own house.

Bell doesn't tell us whether these happy young people lose their hats after a few visits.

> When they saw that he was tired out they avoided mathematics and talked of other things, or listened eagerly while the companionable old man reminisced of his student pranks and the dreary years of his isolation from all scientific friends. His eightieth birthday was celebrated by an even more impressive jubilee than his seventieth and he became in some degree a national hero of the German people.
>
> One of the greatest joys Weierstrass experienced in his declining years was the recognition won at last by his favourite pupil. On Christmas Eve, 1888, Sonja received in person the Bordin prize of the French Academy of Sciences for her memoir *On the rotation of a solid body about a fixed point*.
>
> As is the rule in competition for such prizes, the memoir had been submitted anonymously (the author's name being in a sealed envelope bearing on the outside the same motto as that inscribed on the memoir, the envelope to be opened only if the competing work won the prize), so there was no opportunity for jealous rivals to hint at undue influence. In the opinion of the judges the memoir was of such exceptional merit that they raised the value of the prize from the previously announced 3,000 francs to 5,000. The monetary value, however, was the least part of the prize.



This is certainly true. Recall however (see page 133) that Sofya had her husband's debts and those of her brother to pay, no personal fortune, a daughter to raise and a small salary.

> Weierstrass was overjoyed. "I do not need to tell you", he writes, "how much your success has gladdened the hearts of myself and my sisters, also of your friends here. I particularly experienced a true satisfaction; competent judges have now delivered their verdict that my 'faithful pupil', my 'weakness' is indeed not a 'frivolous humbug"'.
>
> We may leave the friends in their moment of triumph. Two years later (10 February 1891) Sonja died in Stockholm at the age of forty-one after a brief attack of influenza which at the time was epidemic. Weierstrass outlived her

> six years, dying peacefully in his eighty-second year on 19 February 1897 at his home in Berlin after a long illness followed by influenza. His last wish was that the priest say nothing in his praise at the funeral but restrict the services to the customary prayers.
>
> Sonja is buried in Stockholm, Weierstrass with his two sisters in a Catholic cemetery in Berlin. Sonja also was of the Catholic faith, belonging to the Greek Church.

Sofya's belief in science, her political convictions, her faith in the emancipation of people by science, nothing of what motivated her and inspired her is mentioned. But the Catholic faith for Sofya? The Greek Church? Does he mean "Orthodox"? Seen from America, is it synonomous with "Catholic"?

Without pronouncing on this question, for which I have no competence, and for which the only interest is to confirm Bell's lack of seriousness, I mention that:

– Sofya's (Orthodox) baptismal certificate from 17 January (old calendar) 1850 mentions that her father is of Orthodox faith and the mother Lutheran (quoted in [Cooke 1984, p. 4]),

– her grave in Stockholm, of which we find photos in [Kochina 1985] and in [Björk 2002], is adorned by an indisputable Orthodox cross.

And, since I have mentioned Sofya's grave ... it was a crown of white lilies with the inscription

"To Sonja, From Weierstraß"

that the master sent for the funeral of his dearest (*theuerste*) student.

CHAPTER XI

I REMEMBER SOFYA, BY GEORGE, GÖSTA, JULIA AND ALL THE REST

I remember little Sofya in 1858, by Joseph Malevich, 1890

At my first meeting with my gifted pupil in October 1858 I saw an eight-year-old girl rather strongly built, of a pleasant and attractive appearance, whose brown eyes shone with a receptive intelligence and a heartfelt kindness. [...] Three or four years of entirely successful lessons passed without the occurrence of anything notable. But when our study of geometry reached the ratio of the circumference of a circle to its diameter [...] my pupil, when explaining this topic in the next lesson, astonished me by arriving at the same result in a completely different way using her own reasoning. (1)

I have never understood which result is referred to in this narrative, which seems to deal with the definition of π rather than with a "result".

⋆

I remember the little sparrow before our marriage, by Vladimir Onufrievich Kowalevski, 1868

Though she is only eighteen, the little sparrow is well educated, knows languages as well as her native tongue and, she is at present learning mathematics, and poring over spherical trigonometry and integrals, she is as busy as an ant, from morning till night, but she is also lively, sweet and has very good looks. On the whole, this is a happiness that has come like a bolt from the blue. (2)

⋆

1. Quoted in [Cooke 2002a].
2. Letter to his brother Alexander, quoted in [Kochina 1985, p. 44].

M. Audin, *Remembering Sofia Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_11,

What with all my experience, book-reading and push, I cannot grasp and comprehend political and economic problems half as rapidly as she does, and rest assured, she is not just carried away, this is a sober analysis. [...] I think she will make of me a decent person, that I shall drop my publishing business and start research, although I cannot conceal from myself that her nature is a thousand times better, more intelligent and talented than mine, to say nothing of her diligence: they say she works in her country house for twelve hours a day without a stop, and as far as I can see for myself here, she can work like I never would. [...] On the whole, she is a young phenomenon, and why I have got it I can't imagine. (3)

⋆

I remember my cousin in 1868, by Sofya Adelung, 1896

[She] was always ready to go through fire, to die like a martyr for her lofty ideals, for humanity. (4)

⋆

I remember Königsberger's Russian student, by Thomas Hirst, July 27th, 1869

Thomas Hirst (1830–1892) was an English mathematician who wrote a thesis in 1852 in Marburg on conjugate diameters of a triaxial ellipsoid, travelled widely and most importantly kept a diary, a blessing for historians. It is perhaps through him that George Eliot heard about Sofya before meeting her.

27 July 1869,
journal

After another bath in the Neckar I attended Königsberger's lecture on Theory of Determinants. He introduced me to a young Russian lady [Sofya Kovalevskaya] ... who attends his lectures and is at home in Elliptic Functions. She belongs to the mathematically gifted family of Schuberts. She is pretty and exceedingly modest. (5)

⋆

3. Idem, [Kochina 1985, p. 49], I do not know if this is from the same letter.
4. Sofia Adelung was a cousin of Sofya, quoted in [Kochina 1985, p. 35].
5. Quoted in [Gardner & Wilson 1993].

I remember Mme. Kovilevsky, by George Eliot, 1869

5 October 1869,
diary

On Sunday an interesting Russian pair came to see us—M. and Mme. Kovilevsky [*sic*]: she, a pretty creature with charming modest voice, who is studying mathematics (by allowance, through the aid of Kirchoff) at Heidelberg: he, amiable and intelligent, studying the concrete sciences apparently—especially geology; and about to go to Vienna for six months for this purpose, leaving his wife at Heidelberg! (6)

⋆

14 October 1869,
letter to Oscar Browning

[...] A week or two ago we had a Russian lady here, a perfect Hebe in face, who by favour of Kirchoff is studying mathematics in the University at Heidelberg. Her husband was with her—himself clever and scientific—contending for women's equal right of study. (7)

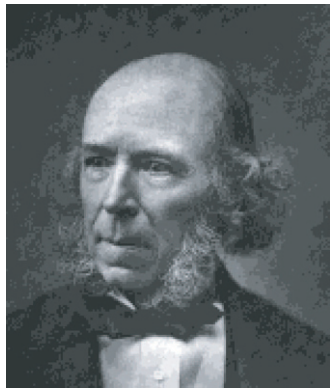
⋆

I remember myself during one of my visits to George Eliot in 1869, by Sofya Kowalevski 1886

In high Victorian England, George Eliot (the pen name of Mary Ann Evans) lived with George Lewes, who was separated from his wife. Ah! The "decent" women of high society! Here at last are women whose opinion counts (see also the allusion to Mme Hermite on page 154).

It was one of the Sundays after I had paid my first visit to George Eliot. About twelve people were already gathered in her drawing room. The company was rather mixed; there was, I seem to remember, a certain young lord who had only just returned from a distant voyage to a little-known country, several musicians and painters, two or three other people who did not follow any definite profession; there was only one lady apart from myself, a very young one, the wife of one of the painters present. As I have said before, few ladies of "decent" English society dared to appear in the drawing room of George Eliot. Mr. Lewes presented every new visitor to me and usually

6. Quoted in [Koblitz 1993, p. 90].
7. Quoted in [Chapman & Gottlieb 1978].



Herbert Spencer (1820–1903)

The "evolutionary philosopher" Herbert Spencer provided a global explanation of the evolution of beings and gave evolution its conceptual framework. The "old man" was forty-nine in 1869 and was very famous, in any case in the circles that Sofya frequented, in particular in London, where she and Vladimir had gone to meet Darwin. Jan-Erik Björk [2002] states that "everyone" in Stockholm knew that Spencer was the spiritual father of the playwright Strindberg, see below page 211.

also told me anything about the new arrival which might be of interest to me.

I had already been in the drawing room for some time when an old man with gray side whiskers and a typical English face entered. This time no one gave me his name, but George Eliot at once turned to him. "I'm so glad that you have come today," she said, "I can introduce you to the living refutation of your theory—a woman mathematician. Allow me to present my friend," she continued, turning to me still without mentioning his name, "only I have to warn you that he denies the very existence of a woman mathematician. In exceptional circumstances there may from time to time appear women who by their mental abilities rise above the average level of men, but he maintains that such women would always direct their intellect and their acumen towards the analysis of their friends and would never fix their attention on a sphere of pure abstraction. Try to make him change his mind!"

The old man seated himself next to me and looked at me with some curiosity. I never suspected who he was, the more as his manner did not betray anything "impressive". The conversation turned to the perpetual, never-ending theme of the rights and abilities of women and whether it would be harmful or beneficial for mankind as a whole if a great number of women were to devote themselves to the study of the sciences. My companion made a few half-ironical remarks which, as I can judge them now, were chiefly calculated to provoke me to raise objections. I should say that at that time I was not yet twenty years old; those few years which separated me from childhood I had spent in a continual struggle at home upholding my right to devote myself to my favorite study; it is, therefore, not surprising that at that time I felt for the so-called "question of women" the whole enthusiastic ardor of a neophyte and that all shyness disappeared when I had to break a lance for the just cause. In addition, as I have already mentioned, I was not aware of what an opponent I had to face; also, George Eliot on her part did her best to egg me on to this argument. This was by no means difficult. Carried away by the argument, I soon forgot my surroundings and at that moment I did not even notice how all the rest of the guests had gradually fallen silent, listening with interest to our conversation, which was becoming more and more lively.

Our dual lasted a good three quarters of an hour, before George Eliot decided to stop it. "You have defended our common cause bravely and well", she said to me at last, smiling, "and if my friend Herbert Spencer has still not let us change his opinion, then, I am afraid, he must be considered incorrigible". Only then did I realize who my opponent was, and it can be imagined how surprised I was at my courage. (8)

⋆

After this visit to George Eliot,
a literary pause.

⋆

George Eliot (1819–1880)

Pause:
Three days in the life of a woman

PALIBINO. Malevich very much liked hearing her chatter and they were on excellent terms. She knew that she could learn geometry because she had looked at books and she had seen what A, B, C mean, they're the names of lines.

"All right, I'm sure you won't be able to learn that", said cousin Mikhail; "why, I'll ask Malevich if you're able to.

"I don't care, responded this proud little rascal, I'm going to ask him myself."

And the same day, when they were in the library, she asked: "Master Malevich, can I learn geometry and have the same lessons as Mikhail?"

"No you can't", said Mikhail indignantly. "Girls can't learn geometry, can they, sir?"

"They can grasp a bit of anything, I believe", Malevich said. 'They have lots of superficial intelligence, but they can't go far in any field. They have quick minds, but no depth."

8. Excerpt from [Chapman & Gottlieb 1978], Sofya's reminiscences which were published in Russia in 1886. A preliminary version had appeared in a Swedish newspaper in 1885 (see also the marginal note, page 211).

Mikhail, delighted with this verdict, expressed his triumph, behind Malevich's chair, by making gestures that functioned like a telegraph. As for Sofya, she's never been so mortified. She was so proud that everyone said she was quick-witted, throughout her short life, and now this quickness seemed suddenly to be a mark of inferiority. Malevich's intent was to make Mikhail feel ashamed because his cousin, a girl two years his junior, was far ahead of him, but Sofya didn't realize this.

"Oh, oh! miss Sofya", said Mikhail when they were alone, "you see, it's nothing special to be quick, You won't get far in any field, you know."

Very happy to brag and to argue with her, he grabbed her by the waist and began jumping with her around the big table of the library. Sofya struggled with such ardor that her hair came loose from behind her ears and whirled around like a mop that had come to life. But the orbits that the children made around the table became more and more irregular and they ended up hitting Malevich's desk, which they sent to the floor with its huge lexicons in one big crash.

"Really, Mikhail", said Sofya in righting the desk, "we should stay calm here, you know. If we break something, mama will make us cry *peccavi*".

"What's that?" asked Mikhail.

"Oh, that's Latin for a good dressing-down", said Sofya, not without a bit of pride in her knowledge.

"You mean she gets angry?" asked Mikhail.

"I think women get angry more than men", said Sofya. "Mama scolds me more than papa."

"Yes, but you'll be a woman one day", said Mikhail, "so you'd better shut up."

"Oh, but I'm going to be an intelligent woman", said Sofya, lifting her head.

"Oh, I'm sure, and a stuck-up wench. Everyone will detest you."

"Oh, but I'll be very nice to everyone. But you're annoying me, it's time to work."

⋆

"I've finished my problem", said Sofya, grasping the physics book that she borrowed from her father and hid under a cushion.

"But look at what I've done. It's difficult and you would never be able to do that", said Mikhail, bringing Sofya closer to show her his problem, while she pushed her hair behind her

ears and got ready to show him who was better in geometry. She began reading with a total confidence in her methods, and she pointed out the solution to each of the problems. He was completely baffled and flushed with irritation. He didn't like being humiliated.

"Oh, miss Know-it-all! That doesn't mean anything! First of all, no one needs to know all that stuff."

"So look, little smarty!" said Sofya, reopening her book and lifting her head, "you see I'm not so stupid as you think. Let me read my book and try to understand this thing about the sine, and you solve your problems by yourself."

⋆

MOSCOW. Everyone who is interested in the story of Sofya and Vladimir learn without surprise that these two people, splendidly equipped with hope and enthusiasm, did not seem happy and were frustrated by lack of mutual tolerance and patience with the world. Vladimir had surprised his friends in several respects. He had acquired a certain distinction as a paleontologist and published several books that brought great praise from the Academy. In Moscow, admiration was more reserved: most of their friends were inclined to believe that the authorship belonged to his wife, because they did not believe Vladimir would ever be able to write.

But when Sofya wrote a short story and had it printed in a Moscow magazine, all those who did not know them were inclined to attribute the honor of this book to Vladimir, declaring that he could have become a professor if he had wanted and especially if his wife allowed him to pursue a normal career.

⋆

STOCKHOLM. We find Sofya in her kitchen, her sleeves rolled up, making meat pies on the fir-wood table, pushing the rolling-pin, looking through the open door at the gestures of the young servant girl while expressing with grammatical fervor the rules about agreement of verbs and pronouns or about the diagonals of a parallelogram that intersect at their middle points, lessons for her daughter seated across from her at the same table, trying to reread the mathematical portion of a letter that is placed in front of her and not quite covered with flour, a pot of jam on the stove, a wash tub and a clothes rack at the other end of the kitchen, indicating that a small wash

The sources for the text of this pause are *The mill on the Floss* [Eliot 1860]—just as the irregular solid that appears here is featured in chapter V, the more and more irregular orbits, described by the two children are really from George Eliot—and *Middlemarch* [Eliot 1872, Chapter 24]. It was by reading this source passage, describing the activities of a woman in her kitchen, and by thinking of the multiple activities of women scientists (Mrs. Garth is not a scientist but just a "simple" housewife) at the same time I was reading texts that marvel at Sofya's supposed lack of practical sense that I had the idea of writing not just the text of this particular pause, but all the others too. I was very happy to discover—afterthought—that Sofya made excellent jam (see page 236)!

is also intermittently underway. Could she do a proper laundry while integrating a differential equation, while preparing a class? Let's look. "Fufa", she would say, closing up the remaining pies, not letting the pastries distract the child's attention to the lesson, "for a parallelogram to be a rectangle it's sufficient that the diagonals be equal, repeat what that means".

⋆

End of the pause,
so let us resume our recollections.

⋆

I remember Sofya in Heidelberg in 1869,
by Julia Lermontova

Her outstanding abilities, passion for mathematics, unusually attractive appearance, and great modesty won the sympathies of everybody she met. There was something really fascinating in her. All the professors she studied with were delighted by her ability; she was very industrious and could sit at a table doing her mathematical calculations for hours on end.

Her moral appearance was complemented by a profound and complicated spiritual life, such as I have never seen in anyone else [...] She won everyone's heart by her ingenuous charm, which distinguished her at this period of her life. Everyone, old and young, men and women, were captivated by her. She was completely natural in her manner, without a trace of coquetry, and did not seem to notice the adoration she induced. (9)

⋆

9. Excerpt from [Kochina 1985, p. 86], I don't know when Julia wrote her memories of Sofya.

Wie schön würden wir hier, von Karl Weierstraß, 1873

During my stay here I have often thought of you and I have imagined how nice it would be if I could spend some time with you, my dear friend (*Herzensfreundin*), several weeks in such splendid nature. How nice it would be here with you—you with your imaginative soul, and me stimulated and refreshed by your enthusiasm ... to dream about and to contemplate all the problems that remain for us to solve, in finite or infinite spaces, on the stability of the solar system, and the other great problems of mathematics and physics of the future. But it has already been some time since I have come to accept that all these nice dreams will never be realized. (10)

Weierstrass is on vacation on Rügen, an island in the Baltic sea, northeast of Rostock, famous in Germany for its beauty. Its chalk cliffs particularly inspired the romantic painter Caspar David Friedrich.

Regarding this letter, from which he also quotes an except, Roger Cooke [2002a, p. 8] writes:

"It is impossible to doubt that Weierstrass was in love with Kovalevskaya, in a way that only a middle-aged bachelor can be, when an emotional bond that he has long ago renounced and believed impossible suddenly becomes a glorious reality."

⋆

Ich erinnere mich an Sophie Kowalevski, von Hermann Schwarz, 1873

Oh, she is a wonderful woman; our common great teacher Professor Weierstraß writes to me so much about her studies. Recently he sent me her compilation of his lectures on Abelian functions. You will not have been able to study them yet, they are the most difficult subject in mathematics, and few men dare tackle them. (11)

Elizaveta Litvinova also reports that in the same year, while visiting Zürich, Sofya accompanied her to one of Schwarz's courses and that her presence lit a fire in his eyes.

⋆

A young woman appeared, by Joseph Malevich, 1874

But then a young woman appeared, firm willed and decisive, determined to pursue her most praiseworthy but extremely difficult goal. She devoted herself to one of the most challenging branches of science, and worked indefatigably in the area of pure mathematics. She married an enlightened man who fully shared her opinions, and did not in the least prevent her from moving forward.

Malevich gave lessens to Aniuta, Sofya and their little brother, up until the time the latter entered a lycée in St. Petersburg. When he retired, he returned to live with Sofya's parents. It is there that he recited this discourse, 29 September 1874, at the celebration of the feast day of Saint Sofya.

10. From the letter of 20 August 1873 [Bölling 1993, letter 30], partially quoted in [Mittag-Leffler 1923]. Our translation.

11. From recollections of Litvinova quoted in [Kochina 1985, p. 74].

She abandoned the pleasures of the world, she sacrificed the best years of a woman's life, she ignored all fatigue, and with rare energy studied her subject in one of the best German universities. But her brilliant successes, achieved in Heidelberg in the course of several semesters, convinced her that her full potential could not be realized there. She moved to the center of German scholarship—to Berlin—and was drawn to Weierstraß, a luminary in science, one of the most famous professors of Europe. She astounded him with her knowledge, and he met with her often to give her valuable advice and instruction. Thus, in the course of five years, she attained the highest academic degree, that degree which in the mathematical sciences is given to very few men. [...]

I salute you, Sofia Vasilevna! You stand on a high pedestal in the ranks of scholars! I salute you also in the name of our small circle, gathered today to celebrate the day of your patron saint! I salute you in the name of our native land, as the first Russian woman to attain the highest academic degree in one of the most difficult areas of science! (12)

⋆

I remember my first encounter with Madam Kowalewsky in 1876, by Gösta Mittag-Leffler, 1923

It was during the period 1875–78 that I made Sonya's acquaintance. At the beginning of February 1876 on the way to Helsingfors I passed through St. Petersburg and, to satisfy my own curiosity as much as to grant a wish I made to Weierstrass, I went to visit the woman who was making such stir in the scholarly world. Without seeking to reconstruct from memory the impressions I experienced, I reproduce a few words from a letter I wrote to Malmsten: "What interested me most in St. Petersburg was making the acquaintance of Mrs. Kowalewsky. Today (10 February 1876) I spent several hours at her home. As a woman, she is delightful. She is beautiful and, when she speaks, her face lights up with an expression of feminine goodness and higher intelligence, so that you cannot help but be dazzled. Her manners are simple and natural, without any trace of pedantry or affected knowledge. For the rest in all respects a lady of high

12. Quoted in [Koblitz 1993, p. 2].

society. As a scholar she is distinguished by an unusual clarity and precision in expression and by a singularly prompt wit.

One easily perceives too the depth to which she has pushed her studies, and I comprehend completely that Weierstrass regards her as the most gifted of his disciples." (13)

⋆

I remember my second encounter with Sonya, in 1880, by Gösta Mittag-Leffler, 1923

At the end of 1879 and the beginning of 1880 a congress of Russian naturalists took place in St. Petersburg, in which I participated. I was then professor at the University of Helsingfors. I had my second encounter with Sonya there and made the acquaintance of her husband and of "Foufie". Although Sonya attended the congress, she took no part in its work. It was clear that, on the one hand, she had lost contact with her mathematical career and that, on the other, she was burning with desire to get back to it.

I knew nothing of her private and material situation. However, the latter had become so difficult by October 1880 that she saw it as hopeless. She then showed that she was a born mathematician, for she found her moral equilibrium in the resumption of her studies. (14)

⋆

I remember the facets of Sofya in Paris in 1882, by Maria Jankowska-Mendelson, 1912

Each of Sofya's numerous friends preserved in his or her memory a different image, because to each one she presented herself in a completely different light. But there was not the least falseness in this; it was just that her rich nature gave to the person who interested her at that moment exactly what seemed to suit that person. Her interest in the given personality was exclusive;

13. From [Mittag-Leffler 1923].
14. From [Mittag-Leffler 1923, p. 173].

it did not permit her to involve herself with others at the same time. (15)

⋆

I remember my first mathematical discussion with a woman by Carl Runge, 1883

On Saturday we had a very interesting party at her flat. The company consisted of Mrs. Kovalevskaya and four young mathematicians, and we talked as we usually do. She is about 30 years old, her face is delicate, thoughtful, a little sad [this was two months after Vladimir's suicide], and quite charming when she smiles. It was strange for me to talk of mathematics with a lady and to be able to discourse with complete freedom. She knows the subject well. I knew this especially when she asked me about my work by the excellent questions she put. Before, I had imagined her to be sharp-nosed, old-looking, and with spectacles, but I was amazed to find that a scientific education can match such a perfect feminity. (16)

Where we see that the stereotype can exist before the species (here women mathematicians).

⋆

I remember Madame Kowalevski, by Charles Hermite, 2 September 1884

You have given me great pleasure, my dear friend, with the issues you sent me of the two illustrated Danish and Swedish journals that give Madame Kowalevski's portrait. But as the original is infinitely superior to these two portraits, where a certain delicate nuance of totally gracious kindness is completely absent, leaving a gap which must have struck you too. (17)

⋆

15. Quoted in [Koblitz 1993, p.164], the original has been translated from Polish to Russian, then to English.
16. Letter to his mother, summer 1883, quoted in [Kochina 1985, p. 196].
17. Letter to Mittag-Leffler [Dugac 1985, p. 92].

I don't wish to remember this monstrosity, by August Strindberg, 1884–1886

[...] the Lady of mathematics may only create unnecessary ambitions in the minds of young girls ... To invite a Russian Lady to Stockholm was only an expression of old-fashioned gallantry—and did not respond to the need of mathematics for the citizens in Stockholm. At this moment the world has far more need of able mothers than professors in mathematics [...] (18)

⋆

A female professor is a pernicious and unpleasant phenomenon, even, one might say, a monstrosity. The Swedes invited her simply because of their famous gallantry toward the weaker sex. (19)

⋆

When the University of Stockholm divided the salary of the male professor to give half of it to a woman, it was a crime—against justice. And the men rejoiced. (20)

⋆

Jan-Erik Björk says that Sofya admired the writer. He adds that her statement, to the same newspaper in which Strindberg published his invective, about her encounter with Herbert Spencer (see page 202 ff.), who was well known to be considered a great philosopher by Strindberg (see note 8 on page 203), was our heroine's subtle and brilliant response to the great misogynist playwright.

I remember that learned mathematician by Henri Poincaré, 1885

I have just been sent a memoir by M^me^ de Kowalevski of which I had no knowledge at the time I wrote the present work [having to do with the form of Saturn's rings]. Although the problem treated by the learned mathematician is not completely the same as the one with which I was occupied, her analysis is very close to mine and I haven't added much to the results that cannot easily be deduced from her memoir. (21)

⋆

In his treatise on hydrodynamics, Horace Lamb [1932, p. 708] studies various forms of fluids under rotation, especially rings, and he mentions in succession the articles by Sofya and Poincaré.

18. A paper published in Swedish in December 1884, translated in English in [Björk 2002, p. 36].

19. Quoted in [Koblitz 1993, p. 230], probably from the same article to which Sofya alludes in the letter published here in Chapter VI.

20. In *Giftas* (*Getting married*), quoted in [Hörmander 1991]. See here page 134.

21. In [Poincaré 1885].

I remember Sonya, by Gösta Mittag-Leffler, 1888

10 August 1888

I went for a walk this evening with Sonya. During the walk from the 7th, she told me that she did not want to marry Kovalevsky, but wanted to live with him secretly, so only their most intimate friends would be aware. He did not want to, he wanted to marry and felt that another sort of relationship would be degrading for her.

On Sofya and the marriage, see the marginal note on page 52. It is interesting that Sofya, who knew which facet of her personality to show to whom, would confide on the question of spouse/mistress in Gösta, who certainly was one of her closest friends, but who was also a seducer and had a reputation as such.

⋆

13 August 1888

This afternoon I paid a visit to Weierstrass's sisters. Sonya, Cantor and I took a guide for climbing the Brocken. En route, various confidences from Sonya. She did not want to marry, she feared that Kovalevsky wanted to marry her out of vanity and would take other mistresses. [...] Anyway, she wanted to be a mistress and not a spouse. [...] She found it so hard to impress on people that [...] no one thinks she could be a mistress and not a spouse.

Unfortunately for her, she's not the sort that a man would want for a mistress, although many would want her as a wife. Men require first of a mistress physical seduction, but in a wife they accept that she doesn't have any when this absence is compensated by other qualities, which she possesses to a high degree.

Anyway, the man who took her as his spouse would be unhappy. Her exceptionally developed egoism and the cold indifference she hides so nicely under her lively and interested air would quickly lead this man to desperation. [...] No, she is certainly right, it would be better that she not marry. (22)

⋆

I recall that I could have loved her, by Fridtjof Nansen

The celebrated Norwegian explorer responds to a Soviet journalist many years later (he lived until 1930).

Kovalevskaya? She was a person of rare intellectual and physical refinement, the most clever and fascinating woman in Europe.

22. Gösta's diary, quoted in [Hörmander 1991].

Yes, no doubt, I was attracted to her and I guessed the feeling was mutual. But I could not break my word, and I returned to the woman I had promised to marry. I am no longer sorry about it. (23)

⋆

I remember the gifted lady Professor in the University of Stockholm, by James Joseph Sylvester, 1886

New College, 15 November (24)

Yesterday afternoon meeting at a friend's house a lady visitor to Oxford who was to sing that evening at one of the hebdomadal concerts at Balliol College, and the conversation happening to turn on the gifted lady professor in the University of Stockholm, my thoughts shaped themselves, as I was walking home, into the following lines, which, if likely to interest any of your readers, I shall be happy to see appear in the world-wide-diffused columns of Nature.

Sonnet

To a Young Lady about to sing at a Sunday Evening Concert in Balliol College

Fair maid! whose voice calls Music from the skies
Weaving amidst pale glimpses of the moon
Tones with fresh hues of glowing fancy strewn
And soft as dew that falls from pitying eyes–
Let from their virgin fount those accents rise
That bid sad Philomel suspend her tune,
Thinking the lark doth chant his lay too soon–
Whose else that trill which with her own note vies!
To her whose star shines bright o'er Maelar lake
And thee who beautifi'st glad Isis' shore
Grant! I one joint harmonious garland bind:
Thou canst with sounds our senses captive take–
She the true Muse, fond poets feigned of yore,
Strike Heaven's own lyre, Nature's o'erruling mind.

⋆

The letter is dedicated to a young singer although inspired by Sofya and where she appears, in the ninth line of the poem as

"her whose star shines bright o'er Maelar lake",

referring to lake Mälaren, west of Stockholm.

Sylvester sent a handwritten version to Sofya (a reproduction of which appears in [Koblitz 1993]).

James Joseph Sylvester (1814–1897)

23. Cited in [Kochina 1985, p. 146].

24. This letter and the accompanying sonnet appeared in *Nature*, under the title *Music and mathématics*, on 9 December 1886.

I remember M^{me} de Kowalewski, by Jules Verne, 1889

"Then Mr Maston, you pretend that a woman has never been able to make mathematical or experimental-science progress?"

"To my extreme regret, I am obliged to, Mrs. Scorbitt", answered J.T. Maston. "That there have been some very remarkable women in mathematics, especially in Russia, I fully and willingly agree with you. But, with her cerebral conformation, she cannot become an Archimedes, much less a Newton."

"Oh, Mr. Maston, allow me to protest in the name of my sex."

"A sex, Mrs. Scorbitt, much too charming to give itself up to the transcendental studies."

"Well then, according to your opinion, no woman seeing an apple fall could have discovered the law of universal gravitation, as did the most illustrious English scientist of the seventeenth century?"

"In seeing an apple fall, Mrs. Scorbitt, a woman would have but the single idea, to eat it, following the example given by our mother Eve."

"Pshaw, I see very well that you deny us all aptitude for high speculations."

"All aptitude? No, Mrs. Scorbitt, and in the meanwhile I would like to prove to you that since there are inhabitants on earth, and consequently women, there has not one feminine brain been found yet to which we owe any discoveries like those of Aristotle, Euclid, Kepler, Laplace, etc."

"Is this a reason? And does the past always prove the future?"

"Well, something that never happened in a thousand years, without a doubt, never will happen."

"I see now that we have to make up our minds, Mr. Maston, and that we are just good enough ..."

"To be good!" answered J. T. Maston."

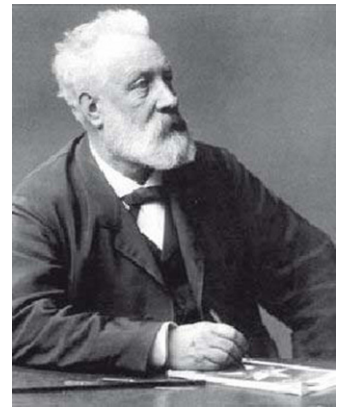
And this, he said with as much politeness as a scientist full of x could command. (25)

⋆

25. The first pages of Jules Verne's novel *Topsy Turvy* [Verne 1890], a novel published in France in 1889, just after the award of the Bordin prize, when Sofya was at the height of her fame in Paris.

Before addressing a more devious misogyny,
a literary pause.
Why Jules Verne would rather not write ...

⋆



Jules Verne (1828–1905)

Pause:
Topsy Turvy

Although he had no taste for mathematics, Alcide Pierdeux appreciated M^me de Kowalewski. He considered her a being of a superior species. Just think! A head where x and y are tossed about like nuts in a sack, a brain that plays with algebraic symbols, hands that juggle triple integrals like an equilibrist with his glasses and bottles, an intelligence that comprehends such things as formulas of the type

$$0 = \frac{ds_1}{\sqrt{R(s_1)}} + \frac{ds_2}{\sqrt{R(s_2)}}$$

$$dt = \frac{s_1\, ds_1}{\sqrt{R(s_1)}} + \frac{s_2\, ds_2}{\sqrt{R(s_2)}}.$$

Yes! To him she seemed worthy of all admiration and made well for a man to feel attracted to her proportionate to the masses and inversely to the square of the distance.

⋆

It was from that brain, where ideas cooked in a cerebral material at a perpetual boiling, that emerged the plan for this great work and the way to bring it to a good conclusion. We cannot repeat too often that M^me de Kowalewski was a remarkable calculator—we would say "emerita", if this word did not have a significance diametrically opposed to that commonly given it. It was child's play for her to solve the most complex problems of the mathematical sciences. She laughed at difficulties both in the science of magnitudes, which is algebra, and in the science of numbers, which is arithmetic. You should also have seen her manipulate symbols, the conventional signs that comprise algebraic notation, whether—letters of the alphabet—they represent quantities of magnitudes, whether—coupled or crossed

lines—they represent relations between quantities and the operations we submit them to. Ah! the coefficients, the exponents, the indices and other arrangements used in this language! Like all these signs fluttered under its plume!

And the letters with which she established her formulas, the capitals A, B, C, which served to represent the known or given quantities, the inertia of the solid, and the lower case p, q, r, which served as the unknown quantities to be determined! And what a turn of hand, her Greek letters, the σ and especially the ϑ of which Weierstraß was so proud!

⋆

Topsy Turvy [Verne 1890] is the most misogynist novel by one of my preferred writers. It begins, as we have seen, with an allusion to Sofya. The mainspring of the plot of this little-known novel is "women and mathematics": the great calculator, the hero, is a man, his grandiose plans fail lamentably and the more naïve readers do not understand until the end of the novel that it is because of an error he committed ... because he was disturbed while making a calculation! Disturbed by a woman!

Neither differential calculus nor integral calculus nor even the calculus of variations were strangers to her. And it is with a sure hand that she traced the famous integral sign, this letter, frightening in its simplicity,

$$\int$$

sum of an infinity of infinitely small elements! It was the same for the $\sum$ sign, which represents the sum of finitely or infinitely many finite elements, and for all the symbols used by this language incomprehensible to common mortals.

Finally, this astonishing woman was capable of rising to the ultimate levels of higher mathematics.

M$^{\text{me}}$ de Kowalewski estimated at five years the time she needed to accomplish her truly delicate and complex task, requiring the solution of diverse equations relating to mechanics, analytic geometry, complex analysis and ϑ-functions of two variables.

⋆

End
of the pause
and return to I remember.

⋆

I remember this learned mathematician, by Henri Poincaré, 1890

Cauchy had already applied the procedure from the calculus of limits to partial differential equations. M^{me} de Kowalevski considerably simplified Cauchy's proof and gave the theorem its definitive form. (26)

⋆

I remember this stupefying example of genius, by Georg Brandes, 1889

The Danish critic Georg Brandes (1842–1927), a radical and passionate personality, here gives an account of Sofya's childhood reminiscences. We will also see him give his opinion on [Kovalevskaïa 2004], on [Leffler 1898] and on a text by Ellen Key.

Until now in Scandinavia we have only known Madam Kovalevsky as the renowned mathematician who teaches in Stockholm and received European consecration in Paris. We have read, without letting ourselves be convinced, Strindberg's angry diatribe against the men who were crazy enough to grant a woman a university chair of this order. All those who have had the pleasure of meeting her take her at once for the same type of cosmopolitan Russian and for a stupefying example of genius in the matter of the exact sciences, so rare in a woman.

In the book that has just appeared, we make the acquaintance of the human qualities of this interesting person. From all evidence, the form of a novel is only a disguise. Tania Rajevski is Sonya Kovalevsky herself and we discover here an autobiography that is masterfully [mistressfully?] written, devoted to her childhood years in Russia.

[...]

The narrator is but thirteen or fourteen years old at the end of the book; we close it in impatient anticipation of the remainder of the story, which does not stop there. (27)

⋆

26. From the crowned memoir [Poincaré 1890] (King Oscar II prize, mentioned on pages 142, 146 and 169). This has to do with what was then not yet called the Cauchy–Kovalevskaya theorem.

27. This review of the Swedish original of [Kovalevskaya 1898] appears in [Brandes 1900, vol. III, pp. 682–683], translated from Danish to French by Mary-May Nielsen.

Ellen Key (1849–1926)

I remember that she was in love, by Ellen Key

I saw her every week in the course of several years, but I met Sonya Kovalevskaya essentially only once. It happened to be at a concert where Beethoven's Ninth Symphony was performed. Contrary to her habit, Sonya was elegantly attired: she wore a black silk dress with lace ... The object of her love, her countryman, was sitting beside her. The divine sounds of Beethoven's music flew around. The commonly nervous features of Sonya Kovalevskaya were lucid and calm. She seemed transformed. She was in love, and the music carried her away into a world of bright dreams ... I only saw this expression on her face twice, the other time Sonya Kovalevskaya was dead. (28)

⋆

I remember Sofya's last visit to Berlin in January 1891, by Georg von Vollmar, 1891

She arrived from the sunny South of Europe, from Italy, where she had taken her leave of absence as a professor of Stockholm University, and she appeared gay and happy. She ... brimmed with creativity in the arts and sciences, full of plans for the years to come, in particular a travel to Caucase she planned to do the next summer with her daughter and Maksim Kovalevsky. She was speaking of the future with so much enthusiasm. Her entire being was so sparkling, and her friendliness was as generous as always. We friends parted merry and smiling and hoping we would meet soon, be it in Berlin or Scandinavia or Paris or wherever. Nobody would imagine that she would die so soon. (29)

⋆

We remember Sofya's final words, by Thérèse and Elsa Gyldén

She would repeat: "Too much happiness". (30)

⋆

28. Quoted in [Kochina 1985, p. 277].

29. These lines of Vollmar were written after Sofya's funeral and are quoted in [Kochina 1985, p. 279] and [Björk 2002, p. 47].

30. In [Kochina 1985, p. 283]. These would be Sofya's last words.

**I remember that she had great thoughts,
and a good heart,
by Ellen Key, 1892**

The quotation of Ellen Key which opens this book is the concluding phrase of an article from which we here have an excerpt. The quotation of Goethe provides the title for this "I remember". On Ellen Key, see too the marginal note page 227.

It was in this high season of seething creativity that death entered the scene. Sonya had been familiar with the thought of death for many years; she was aware of her weak heart, but if there were any period in her life when she would have liked to avoid death it was at this very moment when she was in harmony and full of energy.

She had spent her Christmas holidays in southern France where she had met Russian friends. During her trip back she visited Berlin and caught cold during the travel through Denmark in stormy and cold weather. On Wednesday February 4 she arrived in Stockholm. She did not want to show that she was ill and kept her state of heath secret to those around her. On Friday she gave the first lecture of the term. In the evening she was invited to dinner but retired early and it was not until Saturday morning that she went to bed. Even if her disease—pleurisy—was quite serious nobody realized the full extent of the gravity. All of us who surrounded her were following the struggle between life and death during the forthcoming days and night. Death was caused by suffocation—presumably caused by the violent infection which had produced purulence in the lungs. With a stronger heart the outcome might have been delayed, but the autopsy showed that rescue was not possible.

Sonya was conscious most of the time. She expressed an almost indescribable and patient concern for her surrounding. The near approach of death was not anticipated by the doctors of her close friends during Sunday and Monday but on Tuesday morning February 10 the heart was paralyzed. The very last hours she was unconscious. Death entered as a calm sleep into the vast unknown. Her palish face showed serenity and peace. (31)

⋆

31. This is a short excerpt of an article that appeared in the Swedish newspaper *Dagny*, quoted in [Björk 2002, p. 51].

I remember the sad news of her death, by Charles Hermite, 16 February 1891

Flanville by Noiseville (Lorraine), 16 February 1891

My dear friend,

I was called to Lorraine by the death of one of my aunts, the last remaining and who takes with her the oldest memories of my life, those of my infancy.

In returning from the funeral service to pass a few hours with the family at Flanville, I received the sad news of the death of M^{me} Kovalewski, who was taken from us on the 10th at 4 in the morning by a pleuro-pneumonia. It has been about fifteen days since I saw her in full health when she went through Paris on her way back to Stockholm to give her course at the University, after having spent Christmas vacation at Cannes. I will let you know what they can tell me about her final moments and what will be done to honor her memory. But this deplorable loss for science unfortunately will not be the only one; I am informed that M. Weierstrass, who has long been sick, is now in a deplorable state and that there is no hope remaining for keeping him. I am struck hard by so much death: *sunt lacrymæ rerum et mentem mortalia tangunt.* (32)

Weierstraß's health was very bad, but he survived Sofya by six years.

The world is full of tears, and the burdens of mortality touch the heart (from the *Aeneid*).

⋆

I remember her extreme modesty, by Thomas Stieltjes, 19 February 1891

Toulouse, 19 February 1891

My dear Sir,

I was much saddened by your last letter announcing that you have been struck by new mourning and the so regrettable death of M^{me} de Kovalewski. In a newspaper I was sent from Holland I found some details on her life, she was but 38 years old [Sofya's official age, see the marginal notes on pages 51 and 112] and, what I did not know, it seems that she was also occupied with literary work. I collect piously all description of her, who joined an extreme modesty with such merit. (33)

⋆

32. Excerpt from a letter to Stieltjes, in [Baillaud & Bourguet 1905b, p. 143].

33. Excerpt from Stieltjes's reply to Hermite, in [Baillaud & Bourguet 1905b, p. 144].

I remember her mathematics, by Frits Leffler, 1891

Frits Leffler is the brother of Gösta and Anne Charlotte and a poet by profession. This poem was distributed at Sofya's funeral (the original Swedish version of course).

Has Your thought gone up, designing,
When the stars were saying 'love you',
When great Saturn's ring was shining
In the dark blue sky above you?

Will the numbers' supreme spheres
Analytic functions, dears,
Open to You in condescendence
Immortality's transcendence?

Rays of light, you saw them, gazing
At them with insight amazing.
They refracted were in crystals,
Now what are they in the distance?

[...]
So, farewell and thank you! Covered
By the Swedish soil, you'll lie here
Guarded by your grave. Your young life
Will be long, will stay so far that
Saturn's ring would round the planet
In the midst of bright rotation,
And as far as men would live, your
Image will in mind be stationed. (34)

⋆

I remember they wanted to raise a monument, by Charles Hermite, 3 March 1891

Paris, 3 March 1891

Mon cher Ami,

M. Weierstrass, who reserved for himself the study of Poincaré's memoir and laid claim to the report by having proposed the problem, is unfortunately in no condition to occupy himself with mathematics and I cannot ask anything of him.

34. Excerpt. The original Swedish text and the English translation by M. Burov appear in [Kochina 1985, p. 284].

Sofya in Bordeaux,
bust by Jan-Erik Björk

M^me^ Kovalewski saw him while passing through Berlin on way back to Sweden and let M. Mittag-Leffler know that she found him in a pitiable state, unhappier and sicker than ever. It was she, alas, who was threatened and would die first. You will see from the attached translation from a Swedish newspaper the honors she has just been rendered; M. Mittag-Leffler also informs me that they proposed to publish an edition of her works and to raise a monument to perpetuate her memory. (35)

⋆

I remember her personality, by Gösta Mittag-Leffler, October 1892

Sophie Kovalevsky will maintain an eminent position in the history of mathematics, and her posthumous œuvre, which should soon appear, should preserve her name in the history of literature. But it is perhaps neither as mathematician nor author that it is befitting to judge, first and formost, this woman of such spirit and originality; as a personality she was even more remarkable than we could believe from her works. All those who knew her and were close to her, to whatever circle, to whatever part of the world they belong, remain under the lively and strong impression that her person produced. (36)

⋆

I remember that colossal impression, by Georg Brandes, 1893

The main content of Sonia Kovalevsky's final work, *Vera Vorontzof*, is a short story by the same name.

These hundred and fifty pages leave an impression that we can readily call colossal. What works here is not the artistic bias, for everything is simply stated, in an old-fashioned way, in a language where the principal characters do not speak and which is not the mother tongue of the writer. The facts speak for themselves and it is a gripping subject that imposes itself

35. Excerpt from a letter to Stieltjes, in [Baillaud & Bourguet 1905b, p. 153].

36. These are the concluding lines of [Mittag-Leffler 1892–93].

on the reader in simple and ample lines, thanks to a modest treatment that suits it perfectly. (37)

⋆

I remember the name of M^me Kowalewska, by M. the Dean Darboux, 1893

Miss [Dorothea Klumpke],

You have been occupied by one of the most interesting problems in astronomy. The great names of Galileo, Huygens, Cassini, Laplace, not to mention those of my illustrious colleagues and friends, are attached to the history of serious progress in this equally difficult as attractive theory of the rings of Saturn. Your work has brought us a contribution which is not to be disdained and places you in the ranks of women who have dedicated themselves to the study of mathematics. In the last century, M^lle Marie Agnesi gave us a treatment of the differential and integral calculus. Afterwards, Sophie Germain, as remarkable for her literary and philosophical talent as for her mathematical abilities, attracted the esteem of the great geometers who honored our country at the beginning of our century. It is scarcely several years since the Academy of sciences, by the report of a commission of which I had the honor to be a part, bestowed one of its best prizes which placed the name of M^me Kowalewska beside those of Euler and Lagrange in the history of discoveries related to the theory of the motion of a rigid body about a fixed point. In your turn, miss, you have begun your career; we know that for several years you have been occupied, with the greatest zeal and the greatest success, in the direction of work on measure relative to the astronomical chart. Your thesis, which you have prepared while following, with an assiduousness which cannot be ignored, our courses in higher mathematics, is the first that a woman has presented and successfully defended before our Faculty for obtaining the degree of doctor of mathematical sciences. You worthily open the way and the Faculty hastens to declare you worthy of obtaining the degree of doctor with all the white balls [with unanimity].

⋆

I present here in its entirety—how could I cut it—the allocution of Darboux, who presided over the thesis jury of Dorothea Klumpke, published with its laudatory report on the thesis defense in [Tissandier 1893]. Note that, in this allocution, Dorothea Klumke is called "the young doctor" (the French feminine word *doctoresse* is quite unusual today).
The text in italics at the beginning of the pause on page 85 is inspired by a paragraph from this report.

37. The beginning of Brandes's [1900, Vol. III, p. 684] critical review of [Kovalevskaïa 2004], without the word "nihilist" in the title and published in Swedish. Translated from Danish to French by Mary-May Nielsen.

I remember Sonya's eyes, by Anne Charlotte Leffler, 1895

As is natural, now that Sonya is dead, my first meeting with her is vividly recalled to my mind, even in its most minute details. She arrived from Finland in the evening by boat, and came as a guest to my brother Leffler's house. I went there the day after her arrival.

We were prepared to be friends, for we had heard much of each other, and were eager to become acquainted. Perhaps she had expected more from the meeting than I, for she felt a great interest in that which was my special aim and object. I, on the other hand, rather fancied that a woman-mathematician would prove to be too abstract for me.

She was standing in the window when I arrived, paging through a book. Before she could turn, I had time to see a serious and marked profile, rich chestnut hair arranged in a negligent plait, and a spare figure with a certain graceful elegance in its pose, but not well proportioned, for the bust and upper part of the body were too small in comparison with the large head. Her mouth was large, her lips fresh and humid and most expressive. Her hands were small, almost like a child's; exquisitely modeled, but rather spoiled by prominent blue veins. Her eyes were the most remarkable feature of her face, and gave her countenance the look of lofty intellect which so greatly impressed all who observed her. Their color was uncertain; they varied from gray to green and brown. Unusually large, prominent, and luminous, they had an intensity of expression which seemed to pierce the farthest corner of your soul when she fixed her eyes upon you. But though so piercing, they were also soft and loving and full of responsive sympathy, which seemed to woo those on whom their magnetizing power rested to tell their inmost secrets. So great was their charm that one scarcely noticed their defect—Sonya was so short-sighted that when she was very tired she often squinted.

She turned to me with a quick movement, and came across the room to greet me with outstretched hands. There was a certain shyness about her which made one feel a bit formal at first.

Our first conversation turned on the bad toothache from which she had unfortunately suffered during the voyage. I offered to take her to the dentist. A pleasant object, indeed, for her first walk in a new town! She was, however, the last

person to bestow too much attention or time on so trivial an incident. (38)

⋆

I remember that Weierstrass found Sonya's works original, by Gösta Mittag-Leffler, 1923

I will not occupy myself here with the mathematical part of Weierstrass' communication. I will only remark that the envious have attempted to have us believe that Sonya, in revising her doctoral thesis, was not as independent as she should have been and that she owed more to Weierstrass than she herself has admitted. Weierstrass's own words are for us today a proof to the contrary.

The proof that the differential equation

$$\frac{\partial \varphi(x,t)}{\partial t} = \frac{\partial^2 \varphi(x,t)}{\partial x^2}$$

is satisfied formally by a power series which does not converge for any system of values of the independent variable was one of the most original parts of the thesis and was in this era a discovery of high importance [...] Weierstrass's simple and cordial words show us better than any commentary the relations which existed between the master and his devoted pupil. (39)

This is the example (actually a counterexample) related to the Cauchy–Kovalevskaya theorem, explained here on page 77.

⋆

I remember that short lady of youthful appearance, by Georg Brandes

It is the destiny of a great personality who is presented in this book [of Anne Charlotte Leffler]. It brings us the breath of an existence and of a spiritual life that is richer than those we are accustomed to lead in the Nordic countries and the literary public would do well to surmount the prejudices that still rage in Denmark against reading in Swedish, since writings of this quality must be read in the original language.

38. From [Leffler 1898, Chap. VII].

39. This excerpt from [Mittag-Leffler 1923] regards a letter from Weierstraß to Sofya dated 6 May 1874 ([Bölling 1993, letter 49]).

Not that in the life of Sonia Kovalevsky, as described by the Duchess of Cajanello (Mme Leffler), the artistic accent has focused on detail or petty matters. Nor is this the case with M^me^ Kovalevsky's own novel *Vera Vorontzov*, which too recounts the astonishing story of a remarkable woman who is no less real than the author. But in these two works it is really art that is involved, enriching and essential; in Mme Leffler we have a polyvalent talent of attention and observation and in M^me^ Kovalevsky we have the gift of ample reproduction painted with a large brush. But in both cases art is present in every essential way and the material is astonishingly interesting.

[...]

Those who know this extraordinary Russian only superficially, that a caprice of destiny—let us rather say a bold idea, on which Professor Mittag-Leffler has insisted energetically—led her to settle in Scandinavia, recall a small lady of youthful appearance, with a fine silhouette, a clear and penetrating gaze, and a face with withered complexion but almost beautiful, with incredibly lively features, marked by a deep furrow between the eyes which gives her in the eyes of men an aspect if not less feminine at best neutral. Her mind seemed always in motion and her conversation, imprinted with a restless vivacity, almost tense, could move over the course of half an hour among three or four foreign languages in search of the most adequate expression. (40)

⋆

I remember the mathematician in the woman, by Ellen Key

Sonya's interest for science lent nobility and firmness to her personality; it was in a way her spiritual spinal column. Sofya's relation with the science was neglected in the biography [that of Anne Charlotte, with which this text of Ellen Key is concerned] by Leffler's decision to limit her evocation of Sonya to a subjective description. With the result that Sonya's personality appears in the biography to be less powerful and coherent than it was in reality. Sonya's manner of thinking, scientifically

40. Excerpt from the critical review of [Leffler 1898] in [Brandes 1900, Vol. III, pp. 691–692], translated from Danish into French by Mary-May Nielsen.

oriented—which so profoundly affected her poetry, her conception of life and her emotions—is thus almost absent from the biography. The greatest part of her genius has disappeared. Leffler wanted to show the Woman in the mathematician. But she did not show the Mathematician in the woman. (41)

In homage to the memory of Ellen Key, of whom Jan-Erik Björk [2002] says that she was one of the greatest stylists in Swedish literature.

⋆

I remember that she poked fun at me, by Ilya Metchnikov

She worked under the direction of the famous Berlin mathematician Weierstrass, who was already middle-aged at the time. He became infatuated with her and under the influence of his infatuation he gave her the idea of her work, which she merely carried out. (42)

Metchnikov was a celebrated physiologist and friend of Alexander and Vladimir Kowalevski (according to [Koblitz 1993, p. 169], it is thanks to him that Alexander was not ruined by the bankruptcy of his brother). He had reasons for holding a grudge: according to [Koblitz 1993, pp. 224–225] Sofya had rejected him in 1869 and then, much later, she mocked his discovery of the role of white blood cells, pretending to write a piece on "the combat of red cells and white cells".

⋆

Ich erinnere mich an Sonja, von Karl Weierstraß, 1895, reported by Gösta Mittag-Leffler

Weierstrass had already burned Sonya's letters when Anne Charlotte Leffler's biography appeared. He did not hide from me that he would have preferred that this book not be published. What good is it to present his Sonya's rich personality to the larger public? "*Die Menschen sterben, die Gedanken bleiben*" [people die, ideas remain]: it would have sufficed for Sonya's imposing figure to be passed on to posterity on the sole basis of her mathematics and literary work. (43)

⋆

I remember how we would have found it difficult to follow her, by Fabian Franklin, 1896

The title of this "I remember" was taken from [Koblitz 1993, p. 271].

Her work was of a far higher grade than any that has yet been achieved by any American mathematician. (44)

It should be noted that Fabian Franklin was a professor at Johns Hopkins University, and about whom it can be said that he produced one of the truly important results of American mathematics in the 19th century.

⋆

41. From a review of [Leffler 1898] quoted in [Björk 2002, p. 42].
42. Quoted in [Koblitz 1993, p. 250].
43. From [Mittag-Leffler 1923].
44. Quoted in [Koblitz 1993, p. 272].

I remember the eternal unpredictability of her genius, by Georg Brandes, 1893

Because of the friendship that linked Anne Charlotte Leffler and Sonya Kovalevsky and on the occasion of the obituary that the first wrote about the second, the last part of the book [a book by Ellen Key on Anne Charlotte], which takes the form of a comparison made between these two remarkable beings, their natural talents, their individuality and their unequal faculties, is a piece of feminine psychology so well considered, so conscientiously and spiritually constructed, that it throws a new light, and probably the right one, on these two eminent women who haven't up until now been studied by anyone who, like Miss Key, possesses all the elements for understanding them. Thanks to her description, these two personalities—who subsequent to their death, without familiarity but for more conciseness we call by their first names, Anne Charlotte and Sonya—seem alive in the eyes of the reader.

Anne Charlotte was health itself, a being with balanced feelings and solid and clear intelligence. When she discoursed, she always gave priority to the essential. Sonya, a being of imagination and emotion, presented the eternal unpredictability of the genius. In the course of a conversation she gave, of her own accord, a random course to any subject, but which made it nice and perfect. In the absence of any real subject, she "shaped nutshells", warmed up amusing paradoxes and took pleasure in following them to their conclusion. The serious Anne Charlotte sometimes took this frothy mix of humor and poetry too seriously: it follows that we find in her book imprecision in her judgment of her Russian friend. Anne Charlotte thought in questions and responses, Sonya in images; in nature the former loved the outdoors, the latter poetry.

Anne Charlotte was close to reality, whence her qualities and faults as a writer. She did not have, said Miss Key, "any music in her being". Sonya was a dreamer in steady vibration [...]

"When Sonya said hello, she extended her hand with rapid and surprising movement and the nervous little fingers then

withdrew, rapid as the beating of a bird's wings—that handshake epitomized the impulsive and emotional being that she was [...]" (45)

⋆

I remember her brilliant discoveries, by Elizaveta Litvinova, 1899

Poincaré and the other first-class mathematicians followed the results of her work with great interest. Nevertheless, it is generally believed that Kovalevskaya was not one of the geniuses of mathematics—she did not institute any revolution, but she was without question equal to the most talented male mathematicians of our time. She probed deeply into existing methods, used them in the cleverest ways, shared them with others, and developed them. She made completely new, brilliant discoveries and easily handled the greatest difficulties. Had she lived longer [...]. (46)

⋆

I remember that she was degenerate, by Paul Julius Möbius, 1900

We can thus say that a woman mathematician is counter to nature, a hermaphrodite in a certain sense. Here it is not any different than with other talents. Women scholars and artists are results of degeneration. [...] There is a known saying that brilliant women look like men in disguise. [...] Among women mathematicians, Sophie Germain looks especially masculine. Kowalewsky shows that health and outstanding talent find difficult coexistence in women. She was highly nervous, passionate emotions made her prematurely old and sick. Germain was a benign eccentric. Châtelet in her shamelessness represents the bad type of a degenerate woman. The most good can perhaps be said about Caroline Herschel; she was female in nature, and so healthy and able, reached a very old age. We admittedly

45. Excerpt from a review of a text of Ellen Key on Anne Charlotte Leffler, in [Brandes 1900, Vol. III, p. 702]; translated from Danish to French by Mary-May Nielsen.

46. Quoted in [Koblitz 1993, p. 250].

know too little about most women mathematicians to be able to pass judgment on their pathologies.

It is an exaggeration to speak about mathematical genius in women. No one will doubt that mathematics would have developed just as well if the totality of women mathematicians had not lived. None of them achieved anything essential or devised new methods. They were good pupils and not more. The biography of Kowalewsky says very aptly that her entire work consisted of implementations of Weierstraß's ideas. Caroline Herschel was her brother's faithful assistant; after his death she withdrew into silence. The most original was Germain. (47)

We let the psychologist Józefa Joteyko [1904, p. 13] (for more on Józefa Joteyko, see the marginal note further on) remind us about who Caroline Herschel was:

"[...] a good part of her brother's glory belongs to her. For forty years Caroline was a consoling angel and guardian. Among other things, she discovered of her own accord eight planets [comets] and compiled two precious catalogues encompassing more than a thousand stars and nebulae. The London astronomical society awarded her the grand gold medal in 1845."

⋆

I remember her profoundly agitated life, by Gino Loria, 1903

Even during the heroic period of her mathematical life, while she was working on the memoir that was destined to receive the Bordin prize, Sophie confessed to working "without joy and without enthusiasm". It was with completely youthful ardor that she had committed herself to a way that leads toward science, but her thirst for knowledge was promptly satisfied; after having had all the impulses of her most audacious youth, arriving at a mature age, she declares, mortified and contrite that "a woman teaching mathematics is a useless and repugnant monstrosity", she has reached the point of castigating against scientific work!

We perhaps will recognize the quotation, which occurs for at least the third time in this book (see pages 111 and 211): the anathema in question is due to Strindberg.

The sad example of this woman, on whom nature had heaped its favors and whom possibly clumsy but certainly excessive work had rendered irritable and unsightly, of this woman who at thirty found life too long and died exhausted at thirty-seven, can and should serve as a salutary warning for inexperienced girls who, following suggestions for a real or apparent vocation, propose adopting mathematics as a professional and scientific occupation; they should, before taking an important decision, carefully weigh whether they possess sufficient vigor to support throughout their whole life the heavy burden awaiting those

The thirty-seven years come from Sofya's deception about her age already mentioned several times (see the marginal notes on pages 51 and 112). When Loria wrote this text, the article [Mittag-Leffler 1892–93] had already revealed publicly her real age.

47. From the book *Über die Anlage für Mathematik*, published in 1900, quoted in French translation in [Loria 1904, p. 392].

who aspire to follow in Euclid's glorious tracks. The example of Sophie Kowalewski's profoundly agitated life raises the question whether ruler, compass and table of logarithms are not too weighty instruments for feminine arms. (48)

⋆

I remember the illustrious Sophie Kowalewski, by Józefa Joteyko, 1904

Józefa Joteyko (1866–1928), who signs as "M^{lle} J. Joteyko", was a physiologist of Polish origin. After studying physics at Geneva and medicine at Brussels and Paris, she founded the subdiscipline pedology. She was the first woman (yes!) elected to the Collège de France (1916). She incontestably believed Anne Charlotte, but that did not stop her from responding to the article [1903].

Sophie is not unhappy because her scientific work is not a joy for her (but then we need to explain this absence of joy?), but her nervous temperament, her indisputably confirmed severe neurasthenia, due in large part, if not entirely, to reasons of an emotional nature, taking from her all joy of life and showing the sterility of all effort. At thirty she finds life too long and she dies exhausted at thirty-seven. How interesting are all these details! But can we say with M. Loria that Sophie Kovalewski's sad example should also serve as a "salutary" warning "for inexperienced girls who propose adopting mathematics as a professional and scientific occupation"? It is really doing too much of an honor "to inexperienced girls" to establish a parallel between these girls and the illustrious Sophie Kovalewski! (49)

This response terminates with a sympathetic paragraph:
"We may ask whether a woman will remain the 'eternal devotee'? It is likely that those who, more and more numerous, enter with firm stride into the domain of official science, will no longer follow the example of a Caroline Herschel or of a Sophie Kovalewski, but will know how to live for themselves and for their ideas."

⋆

I remember that she obtained a collaboration with a mathematician of first order, by Gino Loria, 1904

As for Sophie Germain and Sonya Kowalevsky, the collaboration they obtained with first rank mathematicians impedes fixing their role in mathematics precisely. However, what we know about it allows us to give a final brushstroke to the moral portrait of every woman mathematician: *It is always a prodigious child that is admired because of the strangeness of her aptitudes, encouraged and mightily aided by her friends and teachers. In her childhood she succeeds in surpassing her male classmates; in her youth she only succeeds in equaling them; by the end of her studies, while her comrades of the other sex are striding fresh*

In passing from the particular to the general, one announces a law—which is undoubtedly the explanation for the italics.

48. Excerpt from a long article [Loria 1903] on women mathematicians.
49. From [Joteyko 1904, p. 14].

Gino Loria is, among other things, the author of a history of mathematics *from the dawn of civilization to the twilight of the 19th century* [Loria 1950]. The only entry in the index that recalls Sofya's name is "Kowalevsky, V.". Thus not Sofya. A V. Kowalevsky does appear on p. 925 of the book, a fellow who marries fictitiously, so I thought of a misprint ... until I perceived that there was also a "Kruxowski" in the index, referring to the same page 925. How better to hide Sofya!

A strange mix, whose effect is to throw discredit on the work of Sofya, who in any case was not there anymore to defend herself ...

Felix Klein (1849–1925)

and courageous, she's still trying to impress the teacher, friend or parent; and after a small number of years, fatigued by efforts greater than her strength, she ends by abandoning a work that does not bring her any joy, just like the casual walkers who never feel any taste for long marches. Now if this picture is accurate, and I am certain it is, how could we have the courage to advise our sisters or our daughters to take such a model as an ideal? (50)

⋆

I don't remember exactly, but what does that matter? by August Strindberg, 1907

It is a fact that the Moon only shows one side against Earth ... but to say that this depends upon its rotation is a lie! The motion of the Moon has so far defied attacks by mathematicians, because its nineteen years cycle is connected to the unsolvable three-body problem. In 1890 it was all claimed to have been solved by Mrs. Kovalevsky, but that was a mere lie. (51)

⋆

Ich erinnere mich nicht so gut an Sonja Kowalevsky, von Felix Klein, 1919

Finally, [to conclude his chapter on Riemann and Weierstraß]] I will devote a few words to Weierstraß's renowned pupil Sonya Kowalevsky.

She was born in Moscow in 1850 and studied—we can only follow her mathematical fortunes—as a private pupil first with Königsberger in Heidelberg then with Weierstraß in Berlin, becoming very close to the latter. But she could not attend the public lectures, because women auditors were still not allowed. On Weierstraß's recommendation she was graduated in absentia from Göttingen in 1874 on the basis of her work on linear partial differential equations (Crelle, v. 80). In it she arrived at the result that a linear partial differential equation with analytic coefficients has analytic solutions; this was a working out of ideas that Weierstraß had set down in his own youthful work, which

50. From [Loria 1904, p. 339], response to the response [Joteyko 1904].
51. Quoted in [Björk 2002, p. 37].

is now published in Volume 1 of his Werke. (52) Upon the instigation of Mittag-Leffler she became a Privatdozent in 1883, and in 1884 a professor at the private university directed by Mittag-Leffler in Stockholm. She gained international renown: in 1889, also through Mittag-Leffler's application, she was awarded the Great Prize of the Paris academy for her investigation of the rotation of a heavy unsymmetrical top. She died in Stockholm in 1891.

Her nature is in no way exhaustively characterized by her mathematical works. She wrote, among other things, novels, and she experienced them; finally, she became the center of interest in the movement for the emancipation of women. (53) It is therefore very difficult to reach a clear judgment of her scientific character.

On the one side stand the enthusiasts, praising their heroine; on the other side the doubters, who are sooner inclined to condemn her life and her works. Neither party offers us certainties; for we all know how much fame, and too great praise and too harsh blame, distort the true picture of a person. Perhaps the most valuable judgment is in the memorial that Mittag-Leffler devoted to her in *Acta Math.*, Volume 16.

Naturally we can deal with only a small fragment of the fortunes of her life, and this but briefly. Our concern is the significance of her mathematical works. The first thing that strikes us is that her works are closely based on Weierstraß's and are quite in his style, so that one does not see to what extent they contain her own independent thoughts. (54) Doubts have been expressed on the reliability of her later results; see Volterra's critique in Volume 16 of *Acta Math.* of her work on doubly refracting crystals, inaugural dissertation (*Habilitationsschrift*), *Acta Math.* v. 6, 1883, where she is shown to have made a

Grace Chisholm Young was a student of Felix Klein. An impressive list of her activities is drawn up in the article by Mary Lucy Cartwright [1944]: mathematician, housewife, tennis player, author of children's books etc.

Upon her arrival at Göttingen, Grace Chisholm took Klein's course with two other women, Americans (and many male students). Her thesis was on algebraic groups in spherical geometry. She became a specialist in real analysis, we find one of her theorems (relations between right and left derivatives of a measurable function) in [Riesz & Sz.-Nagy 1990].

Here is how Grace Chisholm described (in a letter quoted in [Cartwright 1944]) Klein's position regarding women students:

"[...] he will not countenance the admission of any woman who has not already done good work, and can bring him proof of the same in the form of degrees or their equivalent [...] and further, he will not take any steps till he has assured himself by a personal interview of the solidity of her claims. Professor Klein's view is moderate. There are members of the Faculty here who are more eagerly in favour of the admission of women, and others who disapprove altogether. But the chief difficulty is in Berlin. Were not Hanover reduced to the condition of a province of Prussia, a condition very much disliked by a strong party here, I should have very little doubt of the success of the cause in a few years."

52. This was not the first time a woman obtained the title of Doctor in Göttingen. A hundred years before, Dorothea Schlözer was promoted at age seventeen for a work on Russian financial economy called *De re metallica*. On her diploma, one can find the beautiful expression *virgo erudita* which became later *domina doctissima* and means noting (note of F. Klein).

53. On her life, see the biography, published in the *Reclams Universalbibliothek* and due to A. Ch. Leffler, Mittag-Leffler's sister (note of F. Klein).

54. The letters written by Weierstraß to present the thesis to the Faculty at Göttingen and in which he comments on his closeness with her works appear in Vol. 18 of the *Jahresberichte der* D.M.V. (1909, p. 89, p. 93) by Wentscher and Schlesinger (note of F. Klein). See [Wentscher 1909; Schlesinger 1909] (note of M.A.).

Grace Chisholm Young
(1868–1944)

fundamental mistake. (55) Nor is one completely satisfied with her work on rotation. (56)

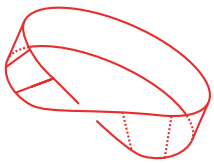
No matter, one thing is sure: Sonya Kowalevsky joined an ardent interest in mathematics with a great power of comprehension and great energy. It is a source of wonder that, in spite of her many interests in quite different fields and her life full of changes, she accomplished so much in mathematics. And in any case, we can be thankful to her for luring Weierstraß out of the reserve he otherwise showed toward everyone in human matters, and for bringing her teacher closer to us through his correspondence with his trusted pupil.

After this singular case, the study of mathematics by women in Germany has proceeded in much clearer paths. Since 1893 the Prussian government has admitted women auditors, first at Göttingen. A doctorate in mathematics on the basis of a regular examination was first given to a woman—Grace Chisholm, now Mrs. Young—in 1895. (57)

⋆

I remember Sonya's mathematical eyes, by Gösta Mittag-Leffler, 1923

Franz Joseph Gall, the "bump man", invented phrenology at the beginning of the 19th century.



Paul Julius Möbius is a grandson of "our" Möbius (for us mathematicians, the one of the "Möbius band" pictured here). Paul Julius Möbius is a neurobiologist that we have already encountered, the one who found women mathematicians degenerate (page 229), and who left his name on quite a band of syndromes. I suppose that Gösta did not know the passage from his book that I have mentioned above.

But she was a born mathematician, she had very definitely that conformation of the left eye that Gall and Moebius recognized as characteristic of mathematical instinct; this feature has, by the way, been removed by the retouching of her portraits. (58)

⋆

I remember that Emmy was not Sonya, by Hermann Weyl, 1935

A comparison with the other woman mathematician of world renown, Sonya Kovalevskaya, is necessary. Sonya certainly had

55. *Acta Math.*, Vol. 16 (1892/93), p. 153 (note of F. Klein). This paper appeared in 1885, see our page 146 (note of M.A.).

56. See our page 172 (note of M.A.).

57. Excerpt from [Klein 1979], lectures dating from the beginning of the 20th century and published in 1926 after the death of the author. For the entry of women into German universities, see also especially [Rowold 2001].

58. From [Mittag-Leffler 1923].

the more complete personality, but she also had a much less happy nature. In order to be able to follow her studies, Sonya needed to overcome the opposition of her parents and to contract a nominal marriage, even if it didn't last. Emmy Noether had, as I have already said, neither a rebellious nature nor bohemian tendencies. Sonya had feminine charm, instinct and vanity; for her social success was not at all immaterial. This was a creature of tension and caprice [...] one sees the tension between her creative spirit and her passionate life, and the self derision which she ironically considered her desperate inner conflicts. So remote from Emmy's possibilities! (59)

This touching text, in which Weyl realizes that two human beings can have different personalities, even when they have the same profession, is a portion of an obituary written for Emmy Noether.

⋆

I believe I remember that she was tormented, by Marie-Louise Dubreil-Jacotin, 1948

The photographer shows Emmy Noether, Marie-Louise Dubreil-Jacotin and her hushand Paul Dubreil in Göttingen in 1931. Marie-Louise Dubreil-Jacotin, a mathematician who specialized both in algebra and fluid mechanics (and in sailing and bridge) and who lacked for neither charm nor perfidy, wrote [1948, p. 266] of Emmy Noether:

"Lacking feminine charms, she had neither feminine cunning nor perfidy. This was a good and simple soul, without ambition, full of courage and life, a friend for all seasons."

Not resisting another digression, I quote here our own André Weil [1991, p. 52]:

"Emmy Noether played the role of mother hen, protective, full of good-will and cackling incessantly [...] Her courses would have been more useful had they been better organized."

She always had, it seems, a certain predilection for tense situations; this was a tortured person; she had to be encouraged in every new idea that was born in her: a feminine weakness,

59. From [Weyl 1935].

Mittag-Leffler said rightly—but how complex and endearing a personality. (60)

⋆

I remember her jam, by a daughter of the Gyldéns, 1950

I remember lots of little things from the daily life of the great Sonya Kovalevskaya. She made very nice jam and offered her friends lovely embroideries. (61)

⋆

I remember her theorem on partial differential equations, by Olga Oleinik, 1975

Her work marked the beginning of the development of a general theory of partial differential equations. (62)

⋆

I remember that she felt a great desire for happiness, by Pelageya Kochina, 1981

[Her] biographers have written about her private life at length. But there still are people who consider Kovalevskaya a "bluestocking". No, she was not! She was in fact a woman with a great desire for happiness; she experienced all the joys and woes that may befall a woman, while her life was a noble struggle to blaze open trails for women. And she won

60. Marie-Louise Dubreil-Jacotin [1948, p. 263] is clearly remembering not just [Leffler 1898] but also what was said by Hermann Weyl.

61. This appeared in the Swedish daily paper *Svenska Dagbladet* on 8 January 1950, quoted in [Kochina 1985, p. 146].

62. This opinion of Olga Oleinik, a specialist in partial differential equations, is reported in [Koblitz 1993, p. 242].

through with flying colors, making her name in mathematics and winning hearts by her literary work. (63)

⋆

I remember what Mittag-Leffler remembered about Sonya Kowalewska, by André Weil, 1982

Mittag-Leffler's photograph in volume 50 of the *Acta* gives a good idea of his appearance at the time. He looked like a bird—a bird of prey of course, such as one could see in the Skansen in Stockholm: frail but still tough, wiry, showing little sign (to my inexperienced eye at least) of his impending death, which was to occur in July. On the day after my arrival, I was called to him to discuss the monograph project [the reason for Weil's stay in Sweden]. That conversation and all subsequent ones on the same subject (perhaps once or twice a week) followed one and the same pattern. He began in French, reminiscing about his earlier work on polynomial expansions, which he remembered in very general terms. Soon his mind turned to his earlier intimate contacts with great mathematicians, chiefly Weierstrass; at this point he dropped into German. Invariably the next topic was Sonia Kowalewska. Then, understandably, he grew tired and lapsed into Swedish; this was puzzling to me at first, but not so after a week or two; nevertheless he stopped himself sharply after a while with the remark: "But I was forgotting that you don't know Swedish; we will continue next time". (64)

An official Sofya: although post-soviet, a respectable matron

In this article published in the centenary number of *Acta Mathematica*, Weil tells of a month's stay in Djursholm in 1927, at what was not yet called the Mittag-Leffler Institute. He was then twenty-one and Mittag-Leffler was eighty-one.

⋆

I prefer not to remember ... by Lars Gårding, 1983

The editors of *Acta Mathematica* are sorry not to agree with you that 30 January 1884 was an event of some importance in the history of mathematics. In the history of the feminine movement yes, but not in the history of mathematics which is a subject independent of sex considerations. Further, we still

This date refers to Sofya's first class in Stockholm. See page 132.

63. From the preface of [Kochina 1985].
64. From [Weil 1982].

feel unhappy about having accepted, in 1884, Kovalevskaya's unfortunate article about wave propagation in crystals. (65)

⋆

... me neither, by Lars Hörmander, 1983

[...] she became an editor from volume 5 on but what active participation that involved nobody knows now. What is mentioned in Gårding's letter is the notoriously incorrect paper published in *Acta Mathematica* by Sonja Kovalevsky in 1884. No doubt you are aware that the error in question is mentioned in many places in the literature, such as Mittag-Leffler's obituary, the paper by Volterra in the same volume of *Acta Mathematica*, Klein's *Entwicklung des Elementarmathematik*? [sic] and even Bell's *Men of Mathematics*. I doubt that the editors of any journal like to commemorate such a disaster no matter who the author was. And I hope that this can close the discussion. (66)

I am not so sure that this was very difficult to ascertain in 1983. It is even simpler presently with so much material accessible electronically, the tables of contents of the volumes, for instance. The correspondence between Mittag-Leffler and Poincaré (in [Nabonnand 1999]) is also enlightening.

After publication of their letters in [Koblitz 1984], our two great men had opportunity to express themselves anew about Sofya, Gårding [1998] in the book I have already mentioned several times, Hörmander [1991] in the article where he has selected and translated into English excepts from Mittag-Leffler's diary, some of which clarify Sofya's salary and her desire to leave Stockholm. See our page 212.

⋆

I remember now that there was also a mathematician, by Yuri Manin, 1983

I used to think of Kovalevskaia as a heroine of the women's movement rather than as a mathematician. Now, with her name being mentioned in the context of research articles in mathematical physics, I have changed my mind. Renewed interest in the work of a mathematician so long dead is unusual. There has to be some special substance to her work. (67)

⋆

A century has passed since the death of Sofya Kovalevskaya, yet everything her life represented is of undiminished relevance today. Her scientific achievements and her social activities, her

65. Letter to Ann Hibner Koblitz quoted in [Koblitz 1984]. See pages 145 ff.

66. Quoted in [Koblitz 1984]. It seems Gösta failed to bequeath his instinct for diplomacy to his successors. The excerpts from the texts [Bell 1937] and [Klein 1979] have been given above.

67. Reported by Ann Hibner Koblitz [1984].

strength of character and her talent, all strike a resonant chord in the latter part of our century. (68)

⋆

I remember her ideas, by André Weil, 1983

For this "I remember" as well as the one for Weil that follows, I have permitted myself to reconstruct what Manin and Weil said to Ann Hibner Koblitz: she does not quote them word-for-word, but reports on conversations.

She had the distinction of being a "two-idea" mathematician. Many mathematicians, including even some well-known ones, have one good idea (which is of course better than having none), after which the rest of their work consists in elaborating upon it or trivially extending it. Look at Mittag-Leffler. In contrast, Kovalevskaya had the idea which led her (independently from Cauchy) to the so-called Cauchy–Kovalevskaya Theorem, and then, ten years later, the idea leading to her solution of the problem of the Kovalevskaya top. (69)

In a certain great French tradition: a complement for one, a bit of nastiness for the other ...

⋆

Ich erinnere mich nicht an Sonja Kowalewski, von z. B. Reinhold Remmert, 1991

(70)

A book, with a lot of information on Karl W. (Sonja's prof.), quotations of big pals, such as Schwarz, Cantor, Hurwitz, Killing, Gösta, but not Sonja, not at all—a void.

⋆

Finally, I'd like to remember a bit ... by Lars Hörmander, 1991

In Sweden it is proclaimed that Sonja Kovalevsky is the first woman in the world to become a university professor. This is perhaps an exaggeration, but it is true that she was the first in Sweden, the second was appointed in 1938. (71)

⋆

68. Quoted in [Kochina 1985, p. 290].
69. Reported by Ann Hibner Koblitz [1984].
70. In [Remmert 1991], for instance, that I chose because I like that book.
71. From the beginning of [Hörmander 1991].

... and me too,
by Lars Gårding, 1994

Sonja Kovalevski defied a masculine society, which has colored her posthumous fame both negatively and positively. She constituted a triumph for Mittag-Leffler, who proved that he was master at Stockholm University, the author of an act that has passed into history. (72)

⋆

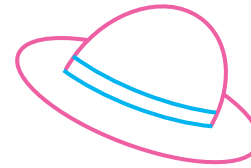
I remember that she didn't like painting
—but I don't remember where I heard that—
by Valeri Kozlov, 2000

It is interesting to note that Kovalevskaya, who had so many talents, did not love and made no attempt to understand painting, sculpture, or architecture. Despite living in Paris for such a long time, she never visited the Louvre, and was always absolutely indifferent to the interior décor of rooms. (73)

The last part of the phrase undoubtedly comes from reading Anne Charlotte. I have never read anywhere that Sofya never went to the Louvre, an affirmation which deserves a precise reference (her diary?). In any case it is a proven fact that she visited museums and art galleries in England during the summer of 1888.

⋆

I remember Halloween parties in Stockholm,
by Steven Krantz, 2002



One of Gösta Mittag-Leffler's most famous associates was Sonja Kowalewska (1850–1891). She lived in Mittag-Leffler's house for some time, and it was rumored that they were intimate (she was *not* his wife). Among Mittag-Leffler's photographs are snapshots taken at a Halloween party. In one of these, one can see the lovely Sonja Kowalewska dressed up as a kitty kat. (74)

"Kowalewska", the italics and "Halloween" are really in the text ... which does not divulge whether the dinner for this memorable evening was ordered from MacDonald's. The photo also appears in the book.

⋆

72. Excerpted from, translated into English by Gårding himself from a Swedish text from 1994.
73. From [Kozlov 2000, p. 1191].
74. From [Krantz 2002, p. 124].

I remember hearing that she had no practical sense, by Valeri Kozlov, 2000

In everyday life Kovalevskaya was rather absent-minded and impractical, which often caused her all kinds of domestic problems. And most of the time there were some men somewhere around who were honored to help and support her in any way. (75)

I really like the idea of Gösta doing Sofya's shopping and Maxim the housekeeping to help Sofya with domestic problems while Nordenskiöld takes care of little Fufa. Concerning Sofya's dependency, see also pages 139 and 175.

⋆

I remember gossip about Sofia, by Ann Hibner Koblitz, 2004

Twenty-five years ago when I began research on my doctoral dissertation on the Russian mathematician, writer, and social activist Sofia Kovalevskaia (1850–1891), I was surprised to learn that most mathematicians I encountered had some smattering of information (often false) about her. The more diffident and cautious among them would recount their "knowledge" in the form of questions, such as "Is it true that she slept with her advisor and that he did all of her best work?" or "Did she really abandon math for literature?" But I also met a few mathematicians whose thoughtlessness or arrogance led them to state confidently that all of Kovalevskaya's papers were erroneous or that she was awarded the Prix Bordin of the French Academy of Sciences out of gallantry. One Swedish mathematician (who will remain nameless) condescendingly chided me for being interested in Kovalevskaia at all, saying that although she might have been something of an amateur mathematician, the Cauchy–Kovalesvskaia Theorem (one of her best-known results) was her husband's, not hers. And several people took great glee in recounting to me the aphorism often attributed to Hermann Weyl that there have been only two women in the history of mathematics, and one of them was not a mathematician (Kovalevskaia), while the other was not a woman (Emmy Noether). (76)

Decidedly, certain Swedish mathematicians have trouble, even decades later, stomaching Sofya's appointment in Stockholm.

⋆

75. From [Kozlov 2000, p. 1191].
76. From the review [Koblitz 2004, p. 39] of the book [Spicci 2002].

Emmy Noether

I very much like Emmy Noether's fine smile in this photo and I have difficulty believing that Weyl, who, as we have seen, knew her well, actually could have said that the serene mathematician portrayed here is not a woman ...

Unfortunately, it seems as if some mathematicians are more comfortable with the old rumors and tales about their predecessors than they are with the reasoned results of careful scholarship. Eric Temple Bell's *Men of Mathematics*, for example, still appears to be popular despite the fact that it is riddled with errors and his section on Kovalevskaia is inaccurate and coyly sexualized. And Felix Klein's idiosyncratic two-volume history of nineteenth-century mathematics (which casts doubt on Kovalevskaia's originality) is still commonly regarded by mathematicians as authoritative, even though, as Jeremy Gray noted, historians have learned "to handle it with care". [...] And just last year in *Mathematical Apocrypha* Steven Krantz perpetuated the canard that Kovalevskaia and her colleague Mittag-Leffler had been sexually intimate. Krantz also saw fit to illustrate his little anecdote with a photograph of the "the lovely Sonja Kowalewskaia dressed up as a kitty kat". (77)

⋆

77. From [Koblitz 2004, p. 41], also see page 240.

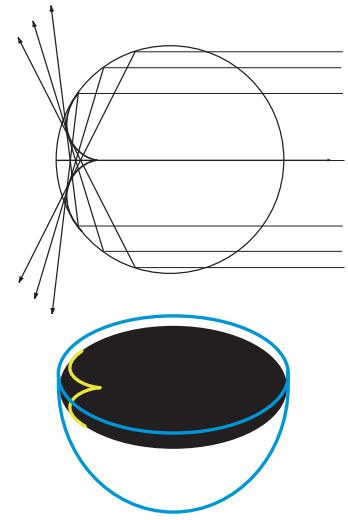
CHAPTER XII

I TOO REMEMBER SOFYA

> Roland had never been much interested in Randolph Henry Ash's vanished body; he did not spend time visiting his house in Russell Street, or sitting where he had sat, on stone garden seats; that was Cropper's style. What Roland liked was his knowledge of the movements of Ash's mind, stalked through the twists and turns of his syntax, suddenly sharp and clear in an unexpected epithet.
>
> A. S. Byatt [1990]

It was through mathematics that I encountered Sofya so that there will be, in this chapter too, some mathematical words—but without technical details. It can be read without any knowledge of the concepts I allude to, at least I hope so ...

In mathematics I specialize in integrable systems and more generally in what is called "symplectic geometry". This subdiscipline of mathematics comes from classical mechanics and, more precisely, from the way that Hamilton presented it in the 19$^{\text{th}}$ century. This is true, but I have to say that my interest in the subject, at the outset, had nothing to do with mechanics. It comes rather, if we absolutely have to give a reference to physics, from the caustics of geometrical optics; these are the lovely curves you see on the surface of a bowl of coffee when there is sunlight. It is by the contact between the algebraic and symplectic geometries that I began to be interested in integrable systems (of which "the case of Sophie Kowalevski" is an example), a frontier of symplectic geometry for which the objects are in fact close to mechanics.



The rays of light reflect off the wall of the bowl and trace out the luminous curve, the caustic, on the surface of the coffee.

Around 1990 we had a seminar on integrable systems in Strasbourg in which my colleague Jean-Yves Merindol (who came

M. Audin, *Remembering Sofia Kovalevskaya*, DOI 10.1007/978-0-85729-929-1_12,

more from the algebraic geometry side) was going to talk about a very nice article of Griffiths [1985].

A few technical terms. Everything I had previously read on these systems and their treatment by algebraic geometry conformed to the following scheme: we exhibit a Lax pair, we put the differential system in the form of a Lax equation that gives an algebraic curve—the spectral curve—and thus also its Jacobian and often (this is what Griffiths studied precisely in his article) we can linearize the solutions on this Jacobian (the Jacobian is a torus and has a canonical linear structure, which gives a sense to the word "linearization"). Whereas in the symplectic geometry the phase space is foliated by tori, called Liouville tori, and the differential system is written in linear fashion on these tori. The relation between these algebraic tori and the topological Liouville tori is not perfectly clear (a euphemism!). Thanks to Griffiths (and to Jean-Yves) I thought that everything was becoming clear: the eigenvalues of the Lax matrix give the curve, the eigenvectors relate the Jacobian of this curve to the Liouville tori.

So I had the idea (which seems to me so natural that I am astonished that no one had thought of it before me) that we might be able to use these eigenvectors and make some topology for comparing the real part of the Jacobian (a disjoint union of tori) at the level of the first integrals considered (another union of disjoint tori). I thus set out to find a pertinent and nontrivial example on which to try out this strategy.

Robert Silhol

Sofya, Robert and I

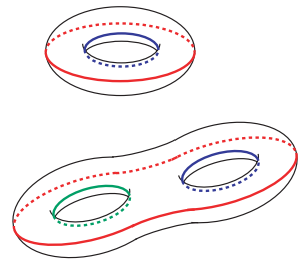
In all the articles, a list of "classic" cases is drawn up, so that the way the methods that I was going to evoke were applicable in these cases, with, at the time, the ritual comment: up to the present no one knows how to put the equations of the solid in the Kowaleski case [1] into the Lax form (see for example [Verdier 1981]). Fortunately, this situation would not last long.

It was just at this moment that I chanced to come across an article in which the authors claimed to understand the Liouville tori for the Kowalevski top by describing the real part of the Jacobian of the curve to genus 2 used by Kovalevskaya for solving the equations of the top. I did not understand completely what

1. The case I described in chapter V.

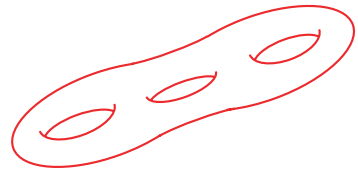
they did precisely and furthermore their result seemed dubious to me. I had just encountered the "Kowalevski case" for the first time.

It happens that at the same moment, in April 1991, I had the opportunity to go to Toulouse to make one or two presentations and that, by chance, the Toulouse algebraic geometers had simultaneously invited Robert Silhol, who is one of the authors of the article I was just discussing. That is when Robert and I met. I am not very diplomatic generally but in addition, on that day I had a violent migraine so I began by attacking him a bit, oh!, you're the one who wrote this paper, you can't be serious, can you? But fortunately he took it well, we discussed it, I suggested using eigenvectors, and we decided to work together on the problem.



The genus of a complex curve (or of a real surface) is the number of holes, 1 and 2 respectively for the complex curves in the figures on pages 95 and 98.

And the Lax pair of the remarkable article [Bobenko et al. 1989] had just appeared (I explained briefly in § V.2.5 in what way this article is remarkable). The spectral curve (after a first reduction) is of genus 3 instead of 2 as we would have expected (since, as Sofya showed, the solutions are expressed in terms of ϑ-functions of two variables), but there is a symmetry and the eigenvectors nonetheless relate the topology of the Liouville tori to the real geometry of an Abelian variety of dimension 2, a *Prym variety*. Robert knew a lot about modular spaces of real Abelian varieties, we worked a lot, we met again in July in Strasbourg, in September in Paris, this was not a very easy example, but we finished the calculations and the writing of the paper [Audin & Silhol 1993] in 1992.



A (complex) curve of genus 3

Next I, and Robert too, made a good many presentations, here and there, on the eigenvector method applied to the Kowalevski top and on the nice Prym varieties that are one of the tools.

There are a good many mysteries in Sofya's famous article [Kowalevski 1889]. For example, there is a curve of genus 2 (that of chapter V) the Jacobian of which is where took place the arguments that I found so suspicious in the article Robert had co-authored. The Lax pair of [Bobenko et al. 1989] provides, as I have said, a curve of genus 3, and already at that moment we found that the relations between all these objects were not absolutely clear.

And then I read several articles in which the "mysterious Kowalevski change of variables" arises, which intrigued me and I would (finally) read the original article. It was on this occasion that I discovered the pleasure of sifting through the coffers of the

Strasbourg library where are stored, among other things, the old volumes of our periodicals, in particular the 1889 volume of *Acta Mathematica*. I confess to admiring above all the beautiful typography of the formulas (see the equations reproduced in chapter V).

Sofya, Anne Charlotte and Jacqueline. During these years, there was also the conference of the *European Women in Mathematics* association. It was at CIRM, a meeting center for mathematics in the Calanques, near Marseille, in December 1991. I also used my stay in the south for working a bit with Robert, but above all it was the occasion where I made the acquaintance of Jacqueline Détraz, who was professor at University of Marseille and who was finishing the book [Détraz 1993] in which she presents the French version of Sofya's childhood recollections, Anne Charlotte's biography and Sofya's scientific papers.

Jacqueline Détraz

It was my first contact with Sofya's life, her biography. Of course I read Jacqueline Détraz's book when it appeared. You will undoubtedly have guessed it, but I immediately detested Anne Charlotte Leffler's text, with its melodramatic tone; I am not able to believe, to imagine, that Sofya or anyone who had been able to do such beautiful mathematics would have been so unhappy, even less that the search for love was her main occupation. May readers who think I am biased read her text and form their own opinion!

It was also at the CIRM meeting that I began to be afraid of the female ghetto: I am a woman, so I worked on the Kowalevski top because Kovalevskaya is a woman; that is what everyone thinks (except maybe the integrable systems specialists, but there are so few of them). I even acquired a reputation as a feminist (which in my case is not far from the truth, as Sofya said about her nihilist reputation, see page 154), so much a feminist that I have even worked on the papers of a woman ...

An interlude: the women's ghetto. It indeed exists, at least in our heads, in all our heads, men and women. I have just illustrated it. It is also what ensures that it is a woman who is asked to write a book about another woman—and that a woman is not asked to write anything else. There are numerous examples, one of which I have already mentioned several times in this book, having to do with the anthology [Le Lionnais 1948] with its article written by a woman, the one by Marie-Louise

Dubreil Jacotin [1948], from which we have encountered several excerpts ...

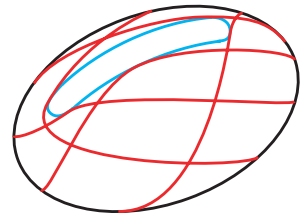
Participation in such an enterprise does not imply any feminist position, as this example shows.

Sofya was not Poincaré. End of the interlude, return to Sofya. We don't speak much about her mathematics, we never say what she really did, no one says how interesting this can be. We have rather the air of excusing ourselves from speaking about it, a backlash perhaps, not wanting to speak approvingly for fear others will think it's because it's a woman that we are speaking of, she didn't discover anything extraordinary, she made brilliant discoveries, it's true, she had these two ideas, especially the one about the solid, sure, this was not a great genius, but nonetheless, she overcame great difficulties, she didn't revolutionize mathematics, but her results are of very high quality, it's very ingenious, perhaps if she had lived longer, a top-drawer mathematician, she didn't found a school, but she was among the better mathematicians of her time, in any case no worse than this one or that one, anyway she had mathematical taste, it was she who urged Mittag-Leffler to read Poincaré, she was so enthusiastic, whereas he understood nothing, she could argue as an equal with the best, anyway Weierstraß said she was the most gifted of his students, but you've caught on, we don't have to specify the meaning of the words, the most gifted, that's perhaps not the same as saying the best ...

Because, of course, to declare her "good" we would need to rank her, better than this one but not as good as that one, as if we determined our interest in a scientific contribution by ranking it, as we rank the candidates for entry to this or that school, yes, she was good, but of course she was not Poincaré ...

Oh, no, she was not Poincaré, how many times have I heard it? She was not Poincaré, she was Sofya Kovalevskaya. Anyway, who was Poincaré, except Poincaré himself, of course?

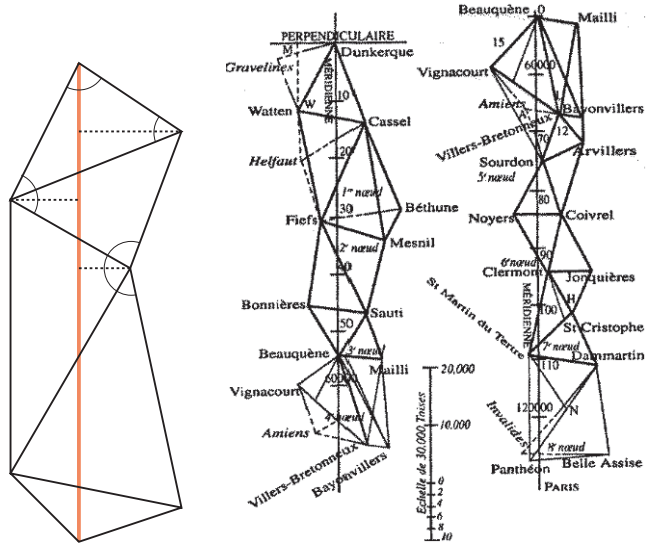
As for me, alright, I am a mathematician (and I am not Poincaré either), so I continue to work, I still apply the idea of eigenvectors, and yes, as André Weil explained to us and as I reported on page 239, when you have a great idea, exploit it! I use another example, the geodesics on a quadric surface—in more comprehensible terms it has to do with the "shortest paths from one point to another" on spheres that are a bit flattened (ellipsoids) or other surfaces of the same nature—about which I write an article [1994].



A geodesic of an ellipsoid

In this figure, the geodesic is the red curve. The blue curve is a *line of curvature*, bounding a portion of the ellipsoid that the geodesic cannot enter.

I love stories about those who actually have measured the earth, after all we are in the bicentenary of the measurement of the meridian by the astronomers Delambre and Méchain, so I began that article by mentioning the Russian general Schubert, whom I know as the person mentioned by Weierstraß [1861] because he measured the axes of the terrestrial ellipsoid (the measurements that I reported in chapter III). The thought does not even occur to me for an instant that our general has a connection with Sofya, about whom I am not thinking anyway.



Measuring a meridian arc by triangulation

We measures angles while aiming at a tower, a church and, when we know the angles and one side of a triangle, we can deduce by a trigonometric calculation the lengths of the remaining sides. Thus we determine the length of the meridian arc.

I think a little more about another integrable system, that of Toda, I write other articles, a book [1996], in which I include a chapter on the Kowalevski top and the figures reproduced in the article of Аппельрот (Appelrot) [1940]—a photocopy of which I just got from somebody who returned from Moscow—and I mention the book by Golubev [1960]. I think of something else, I now have a certain recognition, in this area in particular. I have long forgotten Sofya.

Sofya at the library. She is, however, present, Sofya. For example, because there is a photo of her on one of the walls of the Strasbourg library, one of the large posters from the mathematical publisher Springer Verlag. She is there in the form of the portrait that we saw here on page 168. She's not alone, of course, several of her colleagues keep her company. Apart

from her, I remember Hilbert with his hat. She has acquired a decorative function.

Sofya in Russia. She is also present thanks to Russian colleagues, Alexei Reyman especially, with whom I was conferring a fair amount in those years, but also thanks to many others, who clearly have a much more positive appreciation of the article [Kowalevski 1889], which of course has been translated into Russian and is more popular there (as is shown in the article [Appelrot 1940]—Appelrot must have been a very old man, he wrote an article on the solid in 1892—and the book by Golubev [1960])—as though nationalism is sometimes good. The Russian establishment waited until Sofya was famous and accepted elsewhere before recognizing and rehabilitating her, but in the end it finally did so.

Sofya ambiance

I continue to work. The years 2000 ff. come. There is the theorem of Morales and Ramis, to whose popularity I will contribute (through the book [Audin 2008] and the expository paper [Audin 2002]), which relates the integrability of a system to a certain form of regularity of the solutions, that I mentioned toward the end of chapter V. So I am inspired to go back and read Sofya's article.

Andrei Bolibrukh
(1950–2003)
left on the tribune, at the time of the Congress celebrating the 150th anniversary of Sofya's birth, in Moscow in 2000

Andrei and Sofya. There is also Andrei Bolibrukh, who comes to Strasbourg each year. We see each other a little, for

both professional and family reasons. Andrei loves mathematics. He loves to discuss it. With everyone. He is passionate. He speaks with me, he talks to me in particular about Sofya, whose work and brilliant personality he admires. It is perhaps because of the women's ghetto that he talks to me about her, perhaps too because he knows I wrote the article with Robert, who can say? We'll never know.

Then there is an article that I read rather extensively, since I am the referee used by the journal, in which Andrzej Maciejewski and Maria Przybylska [2005] reprove, using the Galois criterion of Morales and Ramis, a theorem of Ziglin [1983] (that I mentioned in chapter V) which states that the solid body is not integrable except in the cases mentioned above (Euler, Lagrange, Kowalevski).

A Sofya ambiance is in the air, in mine in any case, but not only mine, as we will see. At the end of 2004, things speed up and she begins to take up some space, at least my space.

In October 2004 I go to Bordeaux, I give a series of lectures, I speak with colleagues, Alain Yger has me visit the library, adorned with a blue-green bust realized by Jan-Erik Björk and representing Sofya (a colored version of the one that was made for the garden of the Mittag-Leffler Institute in Djursholm), the one presented here on page 222.

In November, I am once again at CIRM in Marseille when I decide to have a look for myself at the solid. Because of Andrei. I give a talk about Sofya's article to the "Bolibrukh conference", an international congress organized in his memory at Strasbourg. I will speak about the article again in January 2005 at the Franco-Nordic congress in Reykjavík, this article written in French by a Russian woman and published in a Scandinavian journal! What a bonanza! I am candid and I absolutely do not know at that moment the violently negative opinions about Sofya expressed scarcely twenty years earlier by the publishers of this journal—see [Koblitz 1984] and in this book in the "I remember" quotations of Gårding and Hörmander in chapter XI. Anyway, the congress was opened by the Icelandic minister of education, a woman, who explained to us that Icelandic youth lack models of mathematicians. I throw myself at the theme, so we have a model.

Had I known the opinions of the Swedish editors of *Acta Mathematica* in 1982, I would have made the same presentation, but perhaps while feeling more militant ...

I write an article [2007] for the Bolibrukh conference volume, less than I would have liked; thanks to Andrzej Maciejewski for pointing out some errors in a draft.

Sophie at the BHV. Several months earlier the French translation [Kovalevskaïa 2004] of the novel *Nigilistka* (*Nihilist Girl* [Kovalevskaya 2001]) appeared. I spot it on a table of the Kleber bookstore in Strasbourg (while Jean-François Peyret discovered it at the BHV, but that I did not know) and, of course, I bought it and read it. Without really much enthusiasm, I admit.

I found many more interesting things in this novel when I came back to it several months later after having begun to address Sofya's personality.

The BHV (*Bazar de l'Hôtel de Ville*) is a Paris department store known for home improvement and furnishings.

Le cas de Sophie K, by Jean-François Peyret

One evening in the spring of 2005 I get a phone call from Jean-François Peyret, whom I do not yet know, calling to speak with me about the theatre and Sofya. More precisely, he tells me he is staging a play (that is what I understand) on Sofya. Ignoramus that I had become with regard to the theatre over several years, I admit I feared the worst, imagining a play based on Anne Charlotte's vision, a tearful heroine dying of love before the curtain falls—in Puccini's *la Bohème* style. Jean-François tells me nonetheless that he has read [Détraz 1993] and has contacted Jacqueline Détraz and that she told him to call me too. I don't know it yet but in fact Jean-François has read "everything". It is thanks to him that I will subsequently read most of the texts mentioned in this book ... His mention of Jacqueline, a mathematician, reassures me a little, but I remain reticent, because of the women's ghetto. In addition, it's a bit difficult for me to leave Strasbourg this spring.

The laboratory. I end up going anyway to spend an afternoon at the Chaillot theatre with Peyret's troop and I admit too that I was completely seduced. Let me tell you how this happened.

So it was in June and it was raining—I am setting the stage—I found Jean-François in a bistro in *place du Trocadéro*, we got to know each other and then crossed the square—in the rain—and we entered Chaillot—by the artists' entrance, I was very impressed—then in the rehearsal hall, a space that I did not very well know, but I quickly recognize an essential subspace, the table. We introduced ourselves and they—mainly the women anyway—started asking me questions, around the table where we spent the best part of the day. If I had believed that I came to relax by spending an afternoon in the theatre,

well ... I answered questions, mathematical and technical questions, on various levels, questions about the way I work, because anyway it had to do with a play about Sofya, so they wanted to know how that works, the brain of a mathematician, in Sofya's case, in ours. I am not the first one they had met.

Le cas de Sophie K
A play by Jean-François Peyret
and Luc Steels
Scenography Nicky Rieti
Music Alexandros Markeas
Video Pierre Nouvel
and Valère Terrier
Dramaturgy Marion Stoufflet
Web Agnès de Cayeux
Producer Claire Béjanin
with
Olga Kokorina,
Elina Löwensohn,
Alexandros Markeas
Nathalie Richard,
Graham F. Valentine
premièred at the
Avignon Festival in 2005

Elina Löwensohn, with her gaze, asks me x, y, that's abstract, it says nothing to us, what do you see when you hear x and y? Curves, I draw curves. Olga Kokorina discovers that she was not aware that she didn't know about a square root for negative numbers, I would remember it in July at the Chartreuse when hearing her "imagine the square root of -1" ... This goes on for several hours, there are tops, curves of genus 2, complex numbers. I circumvent the difficulties, explain and go on drawing. Then the three actresses begin to improvise a bit. Nathalie Richard chases a fly that none of us sees but that bothers her, pure silent Chekhov. It is in a preliminary state; the text—I should say the score—is not yet fixed. As Clio Lacroix [2006] expresses it so nicely:

> At the beginning of the work, there were texts of all sorts that served as material for improvisations by the actors according to what "resonates with them". Only the written words will be pronounced and only by "erosion" over the course of the rehearsals will the play take form.
>
> For example, for Sophie K "there are several hundreds of pages of material about the mathematics of Sophie Kowaleski, the scientific revolution of her time, the theory of chaos, the crisis of determinism, texts of Poincaré, biographical material about her literary manuscripts, etc. to end up with 45 pages". The play adopts the discontinuous form of vagabond memories following associations of ideas.

However there was already a division into several levels, a cubist play, the critics will say, always one of the actresses filmed in video in the corridor and projected close-up on the bottom of the platform, yes, there are three actresses, Elina, Olga, Nathalie, there is also an actor, Graham Valentine, but he is not there on that day, the amazing musician, Alexandros Markeas, is not there either, the piano is there, we are not in a garret but in a bourgeois salon, as for the romantic consumptive heroine, you should know that there are not any camellias either, and in this play Sofya does not die, in any case we will not be told how she dies. Jean-François Peyret is not of the biography kind. A pause for the actresses, return to the table, and the questions resume. I speak of algebraic equations, no

longer knowing why, and I find myself saying $-b \pm \sqrt{b^2 - 4ac}$ ... over $2a$ adds Claire Béjanin ...

We then go to eat, still at Chaillot, magnificent panoramic view, the Eiffel tower, that of Sofya, still talking mathematics. But I have already been there rather long.

Then there is Avignon, where I spend a few days at the Chartreuse with the same troop and where I adore the finished (?) play. There are things to tell about that too, but enough digressions! As far as this book is concerned, we just need to know it was then and there that I encounter and adore the text of A. S. Byatt that we grind to dust on pages 177 ff.

I write a review with the title *Le cas de Sophie K. (by Jean-François Peyret)* for the *Gazette* (the journal that all members of the *Société Mathématique de France* receive) [2005]. I know this is the sort of thing that Jean-François is fond of ... the *Gazette* is not *Nature*, but anyway ...

Sofya's springtime

For me the spring of 2006 is Sofya's springtime.

Sofya at the National Library (BNF). The biggest piece is the episode of the BNF, the series *A text, a mathematician*, in which Martin Andler asked me to make a presentation, suggesting that I do it starting with a text by Sofya, a proposition that interests me (and honors me). This will be on 5 April, in a large auditorium of the BNF a hall with 350 places almost all filled, in a somewhat difficult political situation, where the majority of the youth were protesting against the CPE (a project called "contract for a first job"), a circumstance that costs Sofya almost all potential listeners from the Lycées.

But before that, there would be the reprinting of my article [2005] from the *Gazette* in the newsletter of Chaillot—it goes without saying that Jean-François Peyret is happy to be talked about in the scientific journals, I am very proud to have my article republished here—I would also write the summary of the conference, a rather widely disseminated article [2006] which appeared in a collection of articles dedicated to Sofya by the magazine *Tangente*.

During this time I begin to write a text entitled *Mon choix de Sophie* (My choice of Sofya) which was the embryo of this book and for which I set out to read pretty much everything that

I could find on Sofya. I read first Mittag-Leffler's article [1923] on Sofya and Weierstraß, the one I have abundantly quoted here. If I start with that, it was because the phrase about the shape of the left eye of mathematicians, heard in *le Cas de Sophie K* and found here on page 234 intrigues me ... The rest plays out quite naturally.

And then there is the theatre. For in parallel with all this activity, there is the revival of *le Cas de Sophie K* at Chaillot.

Sofya, Jean-François and Émilie at the BNF. On 3 May another meeting, with the theme theatre and mathematics, at the BNF, with Jean-François Peyret. I am surprised that there are so few people, many fewer people than for the mathematics talk. According to Jean-François, "theatre and blah", here blah = mathematics, never works and, anyway, no one ever goes to the theatre in May.

It must be said that "our" debate was an unfair competition ... I had the impression of trying to set some of our listeners onto the steep path of free creation by using this athlete of thought (here I am trying my hand at Gösta's metaphoric style) who is Jean-François (whereas the cultural superhighways drain off the masses into the wake of personalities as mediated as the writer Élisabeth Badinter and the critic Tzvetan Todorov) toward Émilie du Châtelet who has become, in three hundred years, a capital of Enlightenment (and, irresistibly, a light of the capital).

The approach which leads us in these places, Jean-François and I and with us Sofya, is certainly more singular ... Émilie du Châtelet versus Sofya Kowalevski, bizarre programming. In any case, our hall is far from full. Several weeks later, I console myself with this setback by telling myself that a big soccer match France-against-somebody would have perhaps emptied the cultural superhighway without affecting the climb.

Sofya at Chaillot. Then the next day, *le Cas de Sophie K* again, at Chaillot this time, still more beautiful, more precise, more perfect than at Avignon, the three sisters, perfect, a new actor, Étienne Oumedjkane, Markeas again, the brilliant musician improvising at the piano, his music causing a top to turn on a screen, (2) and let me evoke here again the perfection of the video image, closeups on the eyes, the mouths of two of the

Émilie du Châtelet (1706–1749) perhaps was not a mathematician as we understand the term today, but was certainly a scientist and an enlightened woman: we owe to her especially the translation into French (with commentary) of Newton's *Principia*, which appeared in 1759, so she is quite appropriately featured in this book.
In an article that I have already mentioned, Loria [1903] introduced her as the "prototype of learned women who were swarming at the time in Parisian salons". Elegant, isn't it? In a rare violent paragraph he gives the "belle Émilie" an extremely sexualized image and absolutely denigrates everything she did, with her "repulsive physiognomy". Józefa Joteyko responds, regarding our marquise:

"Émilie du Châtelet, proclaimed by Arago as 'a genius in geometry', author of numerous volumes, is unworthy of any mention according to M. Loria because of her wanton life. How more just and humain seems Rebière's opinion, affirming that the works of the marquise du Châtelet protect her memory. And then, the scientific criticism has no business occupying itself with people's private lives: the marquise du Châtelet sinned only by loose morals, while her famous partners are not criticized."

In this her tricentenary year, we render homage to this exceptional woman and to her works.

2. Alexandros Markeas was recognized by a critics prize in 2006 as best composer for the theatre, for *le Cas de Sophie K*.

actresses who take turns with a rather difficult text by Poincaré on scientific creation, they're filmed in the dressing room, during their makeup, whose is the eye?, whose is the mouth?, yes, there is a mirror, no, it was not an easy play, and right here I think, mathematics, thy name is woman ...

A magnificent and difficult play, I am satiated, but an exhausting debate follows, organized by the *Bars des Sciences*, we remain standing for a whole hour, facing the hall where some spectators remain, and was not very interesting, the extraordinary persistence of the idea of Sofya's misfortune continues to wreak havoc, the journalist who leads the discussion either didn't like or didn't understand the play. The scene, I mean to say this discussion, takes place in a theatre, and, as we know, the theatre does not always click.

On rigor (continuation). Another example of the lack of rigor, rather funny. It concerns poor Strindberg (as Eric Temple Bell perhaps would write), who, as we know, said or wrote this or that about Sofya, I have already quoted him extensively. But let's not add any more: in the course of the debates in which I participated in the spring of 2006, I heard someone quote the beginning of the phrase

> [...] of Strindberg revolted by the idea that Sophie, a woman, was appointed [nommée, named, in French] to the position of professor [...]

(which figures in the presentation of *le Cas de Sophie K*) in the form "it seems that Strindberg thought that a woman should not be named Sophie". A lovely misinterpretation! But once again a remark that is inexact and especially not very rigorous. At least let's read the phrases to their end!

Sofya at *Mauvais Genre*. And the following week, at the request of Rebecca Rogers, a presentation for the *Mauvais Genre*[3] group in Strasbourg, women and men, historians, sociologists, Germanists, Anglicists, Slavicists, a diverse and difficult audience for me, who likes to talk about mathematics. It is however in this seminar that I get to know Nadezhda Suslova, about whom I spoke on page 38, thanks to Natalia Tikhonov, a historian specializing in the admittance of women to the universities.

3. Literally, wrong gender, but to have *mauvais genre* also means to look disreputable.

Sofya on the radio. I also participate, on 31 March, in a radio broadcast, on *France Culture*, announcing the talk at the BNF, of course, but nonetheless almost an hour devoted to mathematics, to Sofya and to the theatre. I say "almost" because the same decidedly contrarian political newscast had scheduled a speech of the President right in the middle of the broadcast and no, it is not interrupted but it is shortened. The *le Cas de Sophie K* team is represented by the dramaturgist Marion Stoufflet, I say a few words about things I like, I speak a little about mathematics—and Martin Andler too, even if he will say, he too, there again, that, anyway, Sofya was not Poincaré ... And then, cherry on the cake, on 25 May, again the radio, an hour on *France Inter* where the broadcast, more superficial, is called "When I grow up", with a journalist who every day greets a soccer player, a nurse, an opera singer and thus also, sandwiched between a famous reporter (the day before) and a former gangster (the day after), a mathematician. That was part of the promotion for *le Cas de Sophie K*, so I also spoke about Sofya.

At the end of all that, I finish by reading almost everything that had been written or translated (in French, English and, with more difficulty, in German) about Sofya, all the passages or chapters dedicated to her in books about mathematical people, important mathematicians, remarkable mathematicians, the development of mathematics in the 19$^{\text{th}}$ century, among others the books [Klein 1979 ; Bell 1937 ; Kline 1972 ; Wußing & Arnold 1975 ; James 2002].

No one is a mathematician who is not a bit of a poet. As you see, I too am (finally!) seduced by Sofya's personality. Thanks to the play, I discovered that Sofya said that, to be a mathematician, was not necessary to be a poet, an opinion that Weierstraß also expressed (as we saw on page 44 ... and also on page 69), an opinion I have defended, in my small way, for a long time. Mathematics is a language lots of people speak, read or even write (as in a natural language we write, for example, to our tax inspector), in which only these or those who have ideas, imagination and a certain esthetic sense can be creative. As for Sofya? It does not take me long to think, like Roger Cooke [2002a]:

> [...] the more I reflect on her life and consider the magnitude of her achievements set against the weight of the obstacles she had to overcome, the more I admire her. For me she has taken on a heroic stature achieved by very

few other people in history. To venture, as she did, into academia, a world almost no woman had yet explored, and to be consequently the object of curious scrutiny, while a doubting society looked on, half-expecting her to fail, took tremendous courage and determination. To achieve, as she did, two results of lasting value to scholarship, is evidence of considerable talent, developed through iron discipline.

Some stories I haven't told

I had almost finished writing this book when I realized that I have talked rather little about Sofya's emotional life. This was not deliberate, in any case not consciously, but in the end I am rather satisfied.

What I know about it, what's important to me: her difficult life with her husband Vladimir, which clearly sometimes played a determining role in her choices, in her career; her (final) love for Maxim, because finally she saw herself happy professionally, intellectually, personally up to the moment of her death.

What I know, what I have recounted, are confirmed stories, ones she spoke about to her friends, in her correspondence. The rest? It is certain that a unique and remarkable woman who evolves in a masculine environment will likely fall in love here or there, and we can easily find allusions to this or that name in other books on Sofya, in authentic biographies. Here we have encountered Vollmar and Nansen whom she could have loved. But the majority of these stories bring up insinuation and rumor. It is more certain that she herself met with much success (see the Metchnikov affair on page 227). I could well see a "feminist" and ultimately plausible way of reversing Bunsen's insinuations: would it be so extraordinary to imagine that he made them because she had repulsed his advances? But after all, does this really concern us? I do not raise this question out of prudery, I ask myself whether this really interests us. I feel myself rather in sympathy with the opinion expressed by Józefa Joteyko regarding the marquise du Châtelet on private life and refer here to page 254.

Most probably the answer is no, this did not interest me ...

Our contemporaries surpass their predecessors: Bell contented himself with the insinuation about the relations between Sofya and Weierstraß, whose purpose—to imply Sofya's scientific dependence—was clear ... The insinuation of Kozlov on her relations with Runge (here page 81), as well as that of Krantz about Mittag-Leffler (here on page 240) seem to be free from the same motivation.

An honorable woman

I have also—in the course of my readings, of the meetings, debates or discussions that I have brought up, or while hearing here or there this or that reflection—been struck by the persistence of what I have called Sofya's bad reputation. Why and how can such a rich personality, such a clearly competent scientist, be always again the object of suspicion, of rumors, of stupid legends, not to mention sarcasm and dubious aphorisms? The self-contradictory excerpts from the article [Kozlov 2000], a recent article by a competent mathematician from which I quoted, give a troubling example of their persistance.

How can Sofya become once again the "honorable mathematician" that she would never cease to be? I am referring to the book which French title means *An honorable woman* dedicated some twenty-five years ago to Marie Curie by a media personality [Giroud 1986]. Marie Curie displays in fact, without any doubt, the image of an "honorable woman"; her image has been polished by the recognized quality of her scientific work, by the edifying biography written by one of her daughters [Curie 1937], by the sympathetic but real power of great scientists, her descendants and her allies, making up what we could call the "Curie clan". I read Eve Curie's book *Madame Curie* when I was a child. In it there are good, nice stories to tell, the one for example about the young woman who only nourished herself on science and cherries ... Even if, as in the case of Sofya, the details of Marie Curie's private life do not interest me enormously, I admit to having been astonished and even touched, in reading [Giroud 1986], to discover a fault, a love story, a human side, in this marble statue.

Marie Curie (1867–1934)

The physicist Françoise Balibar asks

> Throughout the whole world, children are taught the gilded legend of Marie Curie, this Holy Virgin of science, presented as an (inaccessible) model to girls whom the school is supposed to emancipate. But can we really believe that girls are capable of identifying with such an image?

in the context of the last chapter, *the Myth*, of her well-named recent short book *Marie Curie, femme savante ou Sainte Vierge de la science* [2006]. It is in this chapter that she answers the question which appears in the frontispiece of this book:

> [...] science is a serious matter, whose celebrants should be serious individuals, responsible and composed; but the

> image of the young woman, in our societies, retains something of its proximity to the girl: she is fickle, irresponsible, frivolous. Marie Curie, who has become a legend because she was a woman of science, the exception that makes us forget the rule, or the tree that hides the absence of a forest, cannot be anything but serious and responsible. That she was a widow at a rather young age seems to offer solid guarantees in this regard; that her features in pictures show a constant sadness does not hurt either.

The first woman (again!) I began this book by saying that Sofya was not the first woman to ... In ending it, I have found respectable scientific and medical institutions around Europe that bear the name of Marie Curie and announce on their website that Marie Curie was the first woman in Europe to obtain a doctorate in science ... A point in common between her and Sofya!

But no, Marie Curie was not the first woman to have obtained a doctorate in science, not even the first in chemistry.

Marie Curie defended her doctoral thesis in 1903. Among just the protagonists of the present book and limiting ourselves to the non-medical sciences, we have seen occur the theses of

Sofia Kovalevsakaya (mathematics, *in absentia*, Göttingen, 1874),
Julia Lermontova (chemistry, Göttingen, 1874),
Elizaveta Litvinova (mathematics, Bern, 1877),
Dorothea Klumpke (mathematics, Paris, 1893),
Grace Chisholm (mathematics, Göttingen, 1895) ...

The rumor. Sofya's biography contains more than one common point with that of Marie Curie—more or less seriously, the gray eyes, the way in which they learned to read, their letter writing, their love of liberty, in particular for Poland, their belief in the importance of public education, jams, the title of "first woman to ...", the widowhood that allowed them access to a professorial position ... Sofya's professional life was shorter, less constant, less dense, less productive, she did not found a dynasty, having just a single daughter who became a simple soviet physicist, herself without progeny. The differences between our two scientists are flagrant, they too. We add that at the age where the one obtained her doctorate after having confronted, as we know, administrative difficulties over the course of years, the other managed to overcome financial problems and arrive finally in Paris where she began her studies; but she could enroll at the Sorbonne where she passed brilliantly the examinations in which Sofya never had the right to participate—Sofya Kowalevski is nonetheless an honorable woman too, with whom many an alert and determined young woman in the future could identify in following her way in science.

However, we have seen (page 242) Ann Hibner Koblitz say (and I repeat):

> Unfortunately, it seems as if some mathematicians are more comfortable with the old rumors and tales about

> their predecessors than they are with the reasoned results of careful scholarship.

How to stop an old rumor? Regarding the fictionalization of Galois's biography, one concludes that a serious historic review does not count for much against a pretty novel. I who as a child read and reread the pretty novel in question [Infeld 1978] (which alas contains the same errors as the corresponding chapter of [Bell 1937]) and thrilled for the misunderstood young mathematician and freedom-loving victim of a reactionary conspiracy, I would not say otherwise. Sofya's case seems nonetheless easier to deal with: against Bell's bad novel and those of his followers, a nice serious presentation would undoubtedly suffice. And if we do not try ...

I try.

BIBLIOGRAPHY

Adler (Mark) & van Moerbeke (Pierre)

[1982] Kowalewski's asymptotic method, Kac–Moody Lie algebras and regularization, *Comm. Math. Phys.*, 83(1) (1982), pp. 83–106.

[1989] The complex geometry of the Kowalewski–Painlevé analysis, *Invent. Math.*, 97(1) (1989), pp. 3–51.

Adler (Mark), van Moerbeke (Pierre) & Vanhaecke (Pol)

[2004] *Algebraic integrability, Painlevé geometry and Lie algebras*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics, vol. 47, Berlin: Springer-Verlag, 2004.

Apollinaire (Guillaume)

[1965] *Alcools,* in *Œuvres poétiques*, Bibliothèque de la Pléiade, Paris: Gallimard, 1965.

Appelrot (German Germanovitch)

[1940] Incompletely symmetric heavy gyroscopes, in *Motion of a Solid About a Fixed Point*, Academy of Sciences USSR, 1940, Russian.

Audin (Michèle)

[1994] Courbes algébriques et systèmes intégrables: géodésiques des quadriques, *Expositiones Math.*, 12 (1994), pp. 193–226.

[1996] *Spinning tops, a course on integrable systems*, Cambridge University Press, 1996, Russian translation, Regular and chaotic dynamics, Moscow, 1999, Japanese translation, Kyoritsu, 2000.

[2002] Intégrabilité et non-intégrabilité de systèmes hamiltoniens, [d'après S. Ziglin, J. Morales-Ruiz, J.-P. Ramis...], in *Séminaire Bourbaki 2000/2001*, Astérisque, vol. 282,

M. Audin, *Remembering Sofia Kovalevskaya*, DOI 10.1007/978-0-85729-929-1,

Paris: Société Mathématique de France, 2002, pp. 113–135.

[2005] Le cas de Sophie K. (de Jean-François Peyret), *Gazette des Mathématiciens*, 106 (2005), reprinted in le journal de Chaillot, n° 8, April–June 2006, see also http://www-irma.u-strasbg.fr/~maudin/cas-sophie-k.pdf.

[2006] Le cas de Sophie Kowalevski, *Tangente*, 109 (2006), http://www-irma.u-strasbg.fr/~maudin/sophietangente.pdf.

[2007] Two notions of integrability, in *Differential equations and quantum groups* IRMA lectures in mathematics and theoretical physics, European Mathematical Society, 2007.

[2008] *Hamiltonian systems and their integrability*, SMF/AMS Texts and Monographs, vol. 15, Providence, RI: American Mathematical Society, 2008, Translated from the 2001 French original.

AUDIN (Michèle) & SILHOL (Robert)

[1993] Variétés abéliennes réelles et toupie de Kowalevski, *Compositio Math.*, 87 (1993), pp. 153–229.

BAILLAUD (Benjamin) & BOURGUET (Henry)

[1905a] *Correspondance d'Hermite et de Stieltjes, tome I*, Paris: Gauthier-Villars, 1905.

[1905b] *Correspondance d'Hermite et de Stieltjes, tome II*, Paris: Gauthier-Villars, 1905.

BALIBAR (Françoise)

[2006] *Marie Curie, Femme savante ou Sainte Vierge de la science?*, Paris: Découvertes Gallimard, 2006.

BELL (Eric Temple)

[1937] *Men of mathematics*, New York: Simon and Schuster, 1937.

[1939] *Les grands mathématiciens*, Paris: Payot, 1939, Traduction et préface d'Ami Gandillon.

BIERMANN (Kurt-R.)

[1966] Die Berufung von Weierstraß nach Berlin, in *Festschrift zur Gedächtnisfeier für Karl Weierstraß, 1815–1965*, Köln und Opladen: Westdeutscher Verlag, 1966.

BIERMANN (Kurt-R.) & SCHUBRING (Gert)

[1996] Einige Nachträge zur Biographie von Karl Weierstraß, in *History of mathematics*, San Diego, CA: Academic Press, 1996, pp. 65–91.

BJÖRK (Jan-Erik)
[2002] Sonja Kovalevsky: her life and professorship at Stockholm, in *Operator methods in ordinary and partial differential equations (Stockholm, 2000)*, Oper. Theory Adv. Appl., vol. 132, Basel: Birkhäuser, 2002, pp. 11–53.

BOBENKO (Alexandre Ivanovitch), REIMAN (Alexei Georgevitch) & SEMENOV-TIAN-SHANSKY (Mikhail Arsenievitch)
[1989] The Kowalevski top 99 years later, *Commun. Math. Phys.*, 122 (1989), pp. 321–354.

BÖLLING (Reinhard)
[1989] A Birthday present, *Math. Intelligencer*, 11(4) (1989), pp. 20–35.

[1991] Sonja Kovalevskaja (1850–1891) zum Gedenken, *Journal für die reine und angewandte Mathematik*, 421 (1991).

[1992] ... Deine Sonia: a reading from a burned letter, *Math. Intelligencer*, 14(3) (1992), pp. 24–30, Translated from the German by David E. Rowe.

[1993] *Briefwechsel, Karl Weierstraß, Sofja Kowalewskaja*, Berlin: Akademie Verlag, 1993.

[1994] *A photo album for Weierstrass*, ed. by Bölling (Reinhard), Braunschweig: Friedr. Vieweg & Sohn, 1994, Dual German–English text.

[1998] An unknown photograph of Kovalevskaya, *Math. Intelligencer*, 20(3) (1998), pp. 27–28.

BRANDES (Georg)
[1900] *Samlede Skrifter*, Kjøbenhavn: Gyldendalske Boghandels Forlag, 1900.

BSMF
[1882–83] Compte-rendu des séances, *Bull. Soc. Math. France*, 10 (1882–83), pp. 253–254.

[1883–84] Liste des membres, *Bull. Soc. Math. France*, 11 (1883–84), pp. 6–10.

[1890] Compte-rendu des séances, *Bull. Soc. Math. France*, 18 (1890), p. 163.

BYATT (Antonia Suzanna)
[1985] *Still Life*, London: Chatto & Windus, 1985.

[1990] *Possession*, London/New York: Vintage, 1990.

[1997] *Babel Tower*, London/New York: Vintage, 1997.

[2003] *A Whistling Woman*, London/New York: Vintage, 2003.

CALVINO (Italo)
[1976] *Cosmicomics*, San Diego: Harcourt, Brace, Jovanovich, 1976.

CARTWRIGHT (Mary Lucy)
[1944] Obituary: Grace Chisholm Young, *J. London Math. Soc.*, 19 (1944), pp. 185–192.

CHAPMAN (Raymond) & GOTTLIEB (Eleanora)
[1978] A Russian view of George Eliot, *Nineteenth-Century Fiction*, 33 (1978), pp. 348–365, English edition of *My recollections of George Eliot*, by Sofia Kovalevskaya.

COOKE (Roger)
[1984] *The mathematics of Sonya Kovalevskaya*, Springer, 1984.

[1987] Sofia Kovaleskaia's place in nineteenth century mathematics, in [Keen 1987], Proceedings of a Symposium sponsored by the Association for women in mathematics and the Mary Ingraham Bunting Institute.

[2002a] The life of S. V. Kovalevskaya, in *The Kowalevski property (Leeds, 2000)*, CRM Proc. Lecture Notes, vol. 32, Providence, RI: Amer. Math. Soc., 2002, pp. 1–19.

[2002b] The Cauchy-Kovaleskaya theorem, *preprint*, (2002), http://www.cems.uvm.edu/~cooke/ckthm.pdf.

CRAS
[1888] Séance publique annuelle, *Comptes rendus hebdomadaires de l'Académie des sciences*, 107 (1888), p. 1031.

CURIE (Ève)
[1937] *Madame Curie*, New York: Doubleday, 1937.

DAUBEN (Joseph Warren)
[1990] *Georg Cantor, His mathematics and philosophy of the infinite*, Princeton, NJ: Princeton University Press, 1990.

DÉTRAZ (Jacqueline)
[1993] *Kovalevskaïa: l'aventure d'une mathématicienne*, Paris: Belin, 1993.

[2006] Le rêve parisien de Sophie Kovalevskaïa, *La Recherche*, 397 (2006), pp. 54–56.

DOMAR (Yngve)
[1978] Mathematical research during the first decades of the University of Stockhom, 1978, A lecture given in conjunction with the university of Stockholm's centennial celebration, Stockholm University, Department of Mathematics.

[1982] On the foundation of Acta Mathematica, *Acta Mathematica*, 148 (1982), pp. 3–8.

Dostoïevski (Fedor Mikhailovitch)
[2002] *Le Joueur*, Folio classique, Gallimard, 2002, traduction de Sylvie Luneau de 1956.

Dostoyevsky (Fyodor Mikhailovitch)
[1869] *The Idiot*, Penguin Classics, 1869.

[1875] *A Raw Youth*, Penguin Classics, 1875.

Dubreil-Jacotin (Marie-Louise)
[1948] Figures de mathématiciennes, in *Les Grands Courants de la Pensée Mathématique, édité par F. Le Lionnais*, Cahiers du Sud, 1948, pp. 258–269.

Dubrovin (Boris Anatolievitch), Matveev (Vladimir Borisovitch) & Novikov (Sergei Petrovitch)
[1976] Nonlinear equations of Korteweg–de Vries type, finite-band linear operators and Abelian varieties, *Uspehi Mat. Nauk*, 31(1(187)) (1976), pp. 55–136.

Dugac (Pierre)
[1985] Lettres de Charles Hermite à Gösta Mittag-Leffler (1884–1891), *Cahiers du Séminaire d'Histoire des Mathématiques*, 6 (1985), pp. 79–216, Transcription et annotations par Pierre Dugac.

Eliot (George)
[1860] *The Mill on the Floss*, New York: Harper, 1860.

[1872] *Middlemarch*, London: Blackwood, 1872.

Engels (Friedrich)
[1884] *The Origin of the Family, Private Property and the State*, Penguin Classics, 1884.

Fruttero (Carlo) & Lucentini (Franco)
[1988] *L'amant sans domicile fixe (L'amante senza fissa dimora)*, Paris: Seuil, 1988, Traduit de l'Italien par F. Rosso.

Gårding (Lars)
[1998] *Mathematics and Mathematicians, Mathematics in Sweden before 1950*, American Mathematical Society, London Mathematical Society, 1998.

Gardner (Helen J.) & Wilson (Robin J.)
[1993] Thomas Archer Hirst — Mathematician Xtravagant VI, *Amer. Math. Monthly*, 100 (1993), pp. 907–915.

Giroud (Françoise)
[1986] *Marie Curie: A Life*, London/New York: Holmes and Meier, 1986.

GISPERT (Hélène)
[1991] *La France mathématique*, Cahiers d'histoire & de philosophie des sciences, vol. 34, Paris: Société française d'histoire des sciences et des techniques, 1991.

GOLDSTEIN (Catherine)
[2006] Private communication, 2006.

GOLUBEV (Vladimir Vassilievitch)
[1960] *Lectures on integration of the equations of motion of a rigid body about a fixed point*, Translated from the Russian (1953) by J. Shorr-Kon, Published for the National Science Foundation by the Israel Program for Scientific Translations, 1960.

GRIFFITHS (Philip A.)
[1985] Linearizing flows and a cohomological interpretation of Lax equations, *Amer. J. of Math.*, 107 (1985), pp. 1445–1483.

HADAMARD (Jacques)
[1923] *Lectures on Cauchy's problem in linear partial differential equations*, New Heaven: Yale University Press, 1923.

[1945] *Psychology of Invention in the Mathematical Field*, Princeton: Princeton University Press, 1945.

[1968] *Œuvres*, Paris: CNRS, 1968.

HAINE (Luc)
[1984] The algebraic complete integrability of geodesic flow on SO(n), *Comm. Math. Phys.*, 94 (1984), pp. 271–287.

HAYWARD (Robert Baldwin)
[1854] On a direct method of estimating velocities with respect to axes movable in any manner in space, *Trans. of the Cambridge Phil. Soc.*, 10 (1854), pp. 1–22.

HÖRMANDER (Lars)
[1991] The first woman professor and her male colleague, in *Miscellanea mathematica*, Berlin: Springer, 1991, pp. 195–211.

HOYER (Paul)
[1879] *Ueber die Integration eines Differentialgleichungssystems durch elliptische Funktionen*, Dissertation, Friedrich-Wilhelms-Universität zu Berlin, 1879.

HØYRUP (Else)
[2004] Sofja Kovalevskaja (1850–1891): Verdens første kvindelige, professionnelle matematiker, *Matilde*, 20 (2004), pp. 21–26.

HUSSON (Édouard)
[1906] Recherche des intégrales algébriques dans le mouvement d'un solide pesant autour d'un point fixe, *Ann. Fac. Sci. Toulouse*, 8 (1906), pp. 73–152.

INFELD (Leopold)
[1978] *Whom the Gods Love: The Story of Evariste Galois*, National Council of Teachers of Mathematics, 1978.

JAMES (Ioan)
[2002] *Remarkable mathematicians*, Cambridge University Press, 2002.

JOTEYKO (Józefa)
[1904] À propos des femmes mathématiciennes, *Revue Scientifique (Revue rose)*, 21 (1904), pp. 12–15.

KAMPFFMEYER (Paul)
[1930] *Georg von Vollmar*, München: G. Birk & Co, 1930.

KEEN (Linda), ed.
[1987] *The legacy of Sonya Kovalevskaya*, ed. by Keen (Linda), Contemporary Math., vol. 64, Amer. Math. Soc., 1987, Proceedings of a Symposium sponsored by the Association for women in mathematics and the Mary Ingraham Bunting Institute.

KENNEDY (Don H.)
[1983] *Little sparrow: a portrait of Sophia Kovalevsky*, Athens, OH: Ohio University Press, 1983.

KIRILLOV (A. A.)
[1976] *Elements of the theory of representations*, Berlin: Springer-Verlag, 1976, Translated from the Russian by Edwin Hewitt, Grundlehren der Mathematischen Wissenschaften, Band 220.

KLEIN (Felix)
[1979] *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, Berlin: Springer-Verlag, 1979, Two volumes reprinted as one, with a foreword by R. Courant and O. Neugebauer.

KLEIN (Felix) & SOMMERFELD (Arnold)
[1897] *Theorie des Kreisels*, Teubner, Leipzig, 1897.

KLINE (Morris)
[1972] *Mathematical thought from ancient to modern times*, New York: Oxford University Press, 1972.

KLUMPKE (Dorothea)
[1895] Contribution à l'étude des anneaux de Saturne, *Annales de l'Observatoire de Paris*, 21 (1895), pp. C1–C60.

KOBLITZ (Ann Hibner)

[1984] Sofia Kovalevskaia and the mathematical community, *The Mathematical Intelligencer*, 6 (1984), pp. 20–29.

[1987a] Changing views of Sofia Kovalevskaia, in [Keen 1987], Proceedings of a Symposium sponsored by the Association for women in mathematics and the Mary Ingraham Bunting Institute.

[1987b] Sofia Kovaleskaia, A biographical sketch, in [Keen 1987], Proceedings of a Symposium sponsored by the Association for women in mathematics and the Mary Ingraham Bunting Institute.

[1993] *A convergence of lives*, Lives of Women in Science, New Brunswick, NJ: Rutgers University Press, 2nd edition, 1993, Sofia Kovalevskaia [S. V. Kovalevskaya]: scientist, writer, revolutionary.

[2004] Review of *Beyond the limit: The dream of Sofya Kovalevskaya*, *Notices of the AMS*, 51 (2004), pp. 39–42.

KOCHINA (Pelageya Yakovlevna)

[1985] *Love and mathematics, Sofia Kovalevskaya 1850–1891*, Moscow: MIR, 1985.

KOSTANT (Bertram)

[1970] Quantization and representation theory I: prequantization, in *Lectures in modern analysis and applications III*, Lecture Notes in Math., vol. 170, Springer, 1970.

KOVALEVSKAÏA (Sofia)

[2004] *Une nihiliste*, Paris: Phébus, 2004.

KOVALEVSKAYA (Sofya)

[1898] *Her Recollections of Childhood*, New York: The Century, 1898, Published in a single volume with [Leffler 1898].

[1978] *A Russian childhood*, Springer, 1978, Translated and introduced by Beatrice Stillman.

[2001] *Nihilist Girl*, Modern Language Association of America, 2001.

KOWALEVSKI (Maxime)

[1890] *Tableau des origines et de l'évolution de la famille et de la propriété*, Stockholm, 1890.

KOWALEVSKI (Sophie)

[1875] Zur Theorie der partiellen Differentialgleichungen, *Journal für die reine und angewandte Mathematik*, 80 (1875), pp. 1–32.

[1884a] Sur la propagation de la lumière dans un milieu cristallisé, *C. R. Acad. Sci. Paris*, 98 (1884), pp. 356–357.

[1884b] Über die Reduction einer bestimmten Klasse Abel'scher Integrale 3$^{\text{tem}}$ Ranges auf elliptische Integrale, *Acta Math.*, 4 (1884), pp. 392–414.

[1885a] Über die Brechnung der Lichtes in cristallinischen Mitteln, *Acta Math.*, 6 (1885), pp. 249–304.

[1885b] Zusätze und Bemerkungen zu Laplace's Untersuchung über die Gestalt der Saturnringe, *Astronomische Nachrichten*, 111 (1885), pp. 37–48.

[1889] Sur le problème de la rotation d'un corps solide autour d'un point fixe, *Acta Math.*, 12 (1889), pp. 177–232.

[1890–91] Sur une propriété du système d'équations différentielles qui définit la rotation d'un corps solide autour d'un point fixe, *Acta Math.*, 14 (1890–91), pp. 81–93.

[1891] Sur un théorème de M. Bruns, *Acta Math.*, 15 (1891), pp. 45–52.

Kozlov (Valeri Vassilievitch)

[2000] Sofya Kovalevskaya: a mathematician and a person, *Russian Math. Surveys*, 55 (2000), pp. 1175–1192.

Krantz (Steven G.)

[2002] *Mathematical apocrypha*, MAA Spectrum, Washington, DC: Mathematical Association of America, 2002, Stories and anecdotes of mathematicians and the mathematical.

Kronecker (Leopold)

[1891] Sophie von Kowalevsky, *Journal de Crelle*, 108 (1891), p. 88.

Lacroix (Clio)

[2006] L'irréductible Jean-François Peyret, 2006, non publié.

Lagerborg (Nanny)

[1890] Sur le problème du mouvement d'un corps solide autour d'un point fixe, *Bull. Soc. Math. France*, 18 (1890), pp. 118–122.

Lamb (Horace)

[1932] *Hydrodynamics*, Cambridge University Press, 1932.

Lautréamont (Isidore Ducasse)

[1869] *Les chants de Maldoror*, Poésie, Gallimard, 1869.

Le Lionnais (François), ed.

[1948] *Les grands courants de la pensée mathématique*, ed. by Le Lionnais (François), Cahiers du Sud, 1948.

LEFFLER (Anne Charlotte)
[1898] *Biography of Sonya Kovalevsky*, New York: The Century, 1898, Published in a single volume with [Kovalevskaya 1898].

LIOUVILLE (Roger)
[1897] Sur le mouvement d'un corps pesant suspendu par un de ses points, *Acta Mathematica*, 20 (1897), pp. 239–284.

LISSAGARAY (Prosper-Olivier)
[1976] *History of the Paris Commune of 1871*, New York Publications, 1976.

LORIA (Gino)
[1903] Les femmes mathématiciennes, *Revue Scientifique (Revue rose)*, 20 (1903), pp. 385–392.

[1904] Encore les femmes mathématiciennes, *Revue Scientifique (Revue rose)*, 21 (1904), pp. 338–340.

[1950] *Storia delle matematiche, dall'alba della civiltà al tramonto del secolo XIX*, Milano: Ulrico Hoepli, 1950.

MACIEJEWSKI (Andrzej) & PRZYBYLSKA (Maria)
[2005] Differential Galois approach to the Non-Integrability of the Heavy Top Problem, *Ann. Fac. Sci. Toulouse*, 14 (2005), pp. 123–160.

MARKUSHEVICH (Dimitri Genrihovitch)
[2001] Kowalevski top and genus-2 curves, *J. Phys. A*, 34(11) (2001), pp. 2125–2135, Kowalevski Workshop on Mathematical Methods of Regular Dynamics (Leeds, 2000).

MAZ'YA (Vladimir) & SHAPOSHNIKOVA (Tatyana)
[1998] *Jacques Hadamard, A Universal Mathematician*, American Mathematical Society and London Mathematical Society, 1998.

MICHELACCI (Giacomo)
[2003] Le lettere di Charles Hermite ad Angelo Genocchi (1868–1887), *Quaderni Matematici, Università degli Studi di Trieste*, 546 (2003), Trascrizione, introduzione, commenti, note ed indici a cura di Giacomo Michelacci.

MITTAG-LEFFLER (Gösta)
[1892–93] Sophie Kovalevsky, *Acta Math.*, 16 (1892–93), pp. 385–392.

[1900] Une page de la vie de Weierstrass, *Congrès International des Mathématiciens, Gauthier-Villars, Paris*, (1900), pp. 131–153.

[1923] Weierstrass et Sonja Kowalewsky, *Acta Math.*, 39 (1923), pp. 133–198.

Morales-Ruiz (Juan)

[1999] *Differential Galois theory and non-integrability of Hamiltonian systems*, Progress in Math., Birkhäuser, 1999.

[2000] Kovalevskaya, Liapounov, Painlevé, Ziglin and the differential Galois theory, *Regular and chaotic dynamics*, 5 (2000), pp. 251–272.

Nabonnand (Philippe)

[1999] *Correspondence of Henri Poincaré with Gösta Mittag-Leffler*, Publications des Archives Henri Poincaré, Wiley, 1999.

Painlevé (Paul)

[1897] *Leçons de Stockholm*, Hermann, 1897, Notes manuscrites lithographiées.

[1902] Sur les équations différentielles du second ordre et d'ordre supérieur dont l'intégrale générale est uniforme, *Acta Math.*, 25 (1902), pp. 1–85.

Pascal (Pierre)

[1970] *Dostoïevski, l'homme et l'œuvre*, Slavica, Lausanne: L'âge d'homme, 1970.

Perec (Georges)

[1978] *Je me souviens*, P.O.L., Hachette, 1978.

[1987] *Life a user's manual*, Godine, 1987, Translated from the French by David Bellos.

[2005] *A Void*, Verba Mundi, 2005.

[2008] *Species of spaces*, Penguin Classics, 2008.

Poe (Edgar)

[1844] *The Purloined letter,* in *Complete Tales and Poems*, Penguin Classics, 1844.

Poincaré (Henri)

[1885] Sur l'équilibre d'une masse fluide animée d'un mouvement de rotation, *Bulletin astronomique*, 2 (1885), pp. 109–118.

[1886] Sur les fonctions abéliennes, *American Journal of Mathematics*, 8 (1886), pp. 289–342.

[1890] Sur une classe nouvelle de transcendantes uniformes, *J. Math. Pures Appl. (4)*, 6 (1890), pp. 313–365.

[1921] Analyse de mes travaux sur les fonctions abéliennes, *Acta Mathematica*, 38 (1921), pp. 79–85.

[1987] *Les méthodes nouvelles de la mécanique céleste. Tome I*, Paris: Librairie Scientifique et Technique Albert Blanchard, 1987, réimpression de l'édition originale de 1899.

POLUBARINOVA-KOCHINA (Pelageya Yakovlevna)

[1966] Karl Theodor Wilhelm Weierstrass (On the 150th anniversary of his birth), *Russian Math. Surveys*, 21(3) (1966), pp. 195–206.

[1978] On the scientific work of Sofya Kovalevskaya, in [Kovalevskaya 1978], 1978, Translated and introduced by Beatrice Stillman.

RAHIKAINEN (Agneta)

[1994] Nanny Cedercreutz — författare, matematiker och bergsbestigare, *Källan*, 1 (1994).

REMMERT (Reinhold)

[1991] *Theory of complex functions*, Graduate Texts in Mathematics, vol. 122, New York: Springer-Verlag, 1991, Translated from the second German edition by Robert B. Burckel, Readings in Mathematics.

REYMAN (Alexei Georgevitch) & SEMENOV-TIAN-SHANSKI (Mikhail Arsenievitch)

[1994] Group theoretical methods in the theory of finite dimensional Integrable systems, in *Dynamical systems VII*, Encyclopaedia of Math. Sci., vol. 16, Springer, 1994, pp. 116–225.

RICHTER (Peter), DULLIN (Holger) & WITTEK (Andreas)

[1997] *Kowalewskaja Kreisel*, Göttingen: IWF, 1997, DVD, http://www.iwf.de.

RIESZ (Frigyes) & SZ.-NAGY (Béla)

[1990] *Functional analysis*, Dover Books on Advanced Mathematics, New York: Dover Publications, 1990.

RIMBAUD (Arthur)

[1972] *Poésies,* in *Œuvres complètes*, Bibliothèque de la Pléiade, Paris: Gallimard, 1972.

ROTHMAN (Tony)

[1982] Genius and biographers: the fictionalization of Évariste Galois, *Amer. Math. Monthly*, 89 (1982), pp. 84–106.

ROWOLD (Katharina)

[2001] The many lives and deaths of Sofia Kovalevskaia: approaches to women's role in scholarship and culture in Germany at the turn of the twentieth century, *Women's History Review*, 10 (2001), pp. 603–627.

Schappacher (Norbert)
[1993] Questions politiques dans la vie des mathématiciens en Allemagne (1918–1935), in *La Science sous le Troisième Reich,* sous la direction de J. Olff-Nathan, Paris: Seuil, 1993, pp. 51–89.

Schlesinger (Ludwig)
[1909] Die Briefe von K. Weierstraß an L. Fuchs, *Jahresbericht der deutschen Mathematiker-Vereinigung*, 18 (1909), pp. 93–99.

Senon (Alfredo), Evero (Bernardo), Eben (Carlo) & Trovato (Daniele)
[1982] For whom Bell tolls, *Per. Appl.*, 17 (1982), pp. 259–272.

Shtraikh (Solomon Yakovlevitch)
[1951] *S.V. Kovalevskaya, Vospominaniya i Pis'ma*, Presses de l'Académie des Sciences de l'urss, 1951.

Spicci (Joan)
[2002] *Beyond the limit: The dream of Sofya Kovalevskaya*, Forge Books, 2002.

Tissandier (Gaston)
[1893] M^{lle} Klumpke docteur ès sciences mathématiques, *La Nature*, 1073 (1893), p. 62.

Tisserand (François-Félix)
[1891] *Traité de mécanique céleste, tome II, théorie de la figure des corps célestes et de leur mouvement de rotation*, Paris: Gauthier-Villars, 1891.

Tollmien (Cordula)
[1995] *Fürstin der Wissenschaft*, Weinheim: Beltz & Gelberg, 1995.

Tuschmann (Wilderich) & Hawig (Peter)
[1993] *Sonja Kowalewskaja, Ein Leben für Mathematik und Emanzipation*, Basel: Birkhäuser, 1993.

Vallès (Jules)
[2006] *L'enfant, le Bachelier* et *l'Insurgé*, Omnibus, 2006.

Verdier (Jean-Louis)
[1981] Algèbres de Lie, systèmes hamiltoniens, courbes algébriques [d'après M. Adler et P. van Moerbeke], in *Séminaire Bourbaki, 1980-81*, Berlin: Springer, 1981, pp. 85–94.

Verne (Jules)
[1890] *Topsy-Turvy*, New York: Ogilvie, 1890.

WEIERSTRASS (Karl)
[1861] Über die geodätischen Linien auf dem dreiachsigen Ellipsoid, *Math. Werke I*, (1861), pp. 257–266.

WEIL (André)
[1982] Mittag-Leffler as I remember him, *Acta Math.*, 148 (1982), pp. 9–13.

[1991] *Souvenirs d'apprentissage*, Vita Mathematica, vol. 6, Basel: Birkhäuser, 1991.

WENTSCHER (M.)
[1909] Weierstraß und Sonja v. Kowalewsky, *Jahresbericht der deutschen Mathematiker-Vereinigung*, 18 (1909), pp. 89–93.

WEYL (Hermann)
[1935] Emmy Noether, *Scripta Mathematica*, 3 (1935), p. 219.

WUSSING (Hans) & ARNOLD (Wolfgang)
[1975] *Biographien bedeutender Mathematiker*, Aulis Verlag Deubner & Co KG, 1975.

YUSHKEVICH (Adolf Pavlovitch), ed.
[1984] *Perepiska S. V. Kovalevskoi i G. Mittag-Lefflera*, ed. by Yushkevich (Adolf Pavlovitch), Научное Наследство [Scientific Heritage], vol. 7, Moscow: "Nauka", 1984, Compiled and with commentaries by P. Ya. Kochina and E. P. Ozhigova.

ZAKHAROV (Vladimir Evgenievitch), ed.
[1991] *What is integrability?*, ed. by Zakharov (Vladimir Evgenievitch), Berlin: Springer, 1991.

ZERNER (Martin)
[1991] Joseph Bertrand, in [Gispert 1991], pp. 298–322.

ZIGLIN (Sergei Lvovitch)
[1983] Branching of solutions and non existence of first integrals in Hamiltonian mechanics II, *Funct. Anal. Appl.*, 17 (1983), pp. 6–17.

INDEX