5 Parallel Projections

The projections discussed in this book transform a scene from three dimensions to two dimensions. Projections are needed because computer graphics is about designing and constructing three-dimensional scenes, but graphics output devices are two-dimensional. Figure 5.1 illustrates what can happen when a dimension is added to space. The figure shows an impossible object, an object that cannot exist in three dimensions, yet it can be drawn in two dimensions.

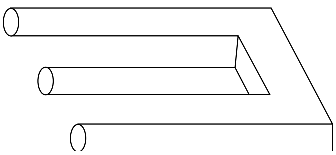

Figure 5.1: An Impossible Fork.

This figure and others like it show how careful we must be when projecting an object.

There are several variants of parallel projections, but they are all based on the following principle: Select a direction $\mathbf{v}$ and construct a ray that starts at a general point $\mathbf{P}$ on the object and goes in the direction $\mathbf{v}$. The point $\mathbf{P}^*$ where this ray intercepts the projection plane becomes the projection of $\mathbf{P}$. The process is repeated for all the points on the object, creating a set of parallel rays, which is why this class of projections is called parallel. Figure 5.2 illustrates the principle of parallel projections. In Figure 5.2a the rays are perpendicular to the projection plane and in Figure 5.2b they strike at a different angle. This is why the latter method is called *oblique projection* (Section 5.3).

Figure 5.2c shows a different interpretation of parallel projections. Because the rays are parallel, we can imagine that they originate at a *center of projection* located at infinity. This interpretation unifies parallel and perspective projections and is in

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accordance with the general rule of projections (Page 200) which distinguishes between parallel and perspective projections by the location of the center of projection.

The three types of parallel projections are orthographic, axonometric, and oblique.

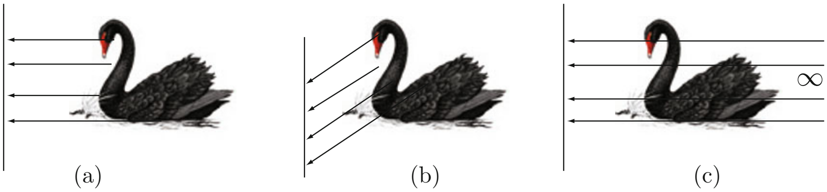

Figure 5.2: Parallel Projections.

> I will sette as I doe often in woorke use, a paire of paralleles, or [twin] lines of one lengthe, thus =, bicause noe 2. thynges, can be moare equalle.
>
> —Robert Recorde, 1557.

5.1 Orthographic Projections

The term orthographic (or orthography) is derived from the Greek $o\rho\theta o$ (correct) and $\gamma\rho\alpha\varphi o\zeta$ (that writes). This term is used in several areas, such as orthographic projection of a sphere (Page 415) and the orthography of a language. The latter is the set of rules that specify correct writing in a language. An example of an orthographic rule in English is that i comes before e (as in "view") except after a c (as in "ceiling").

The family of orthographic projections is the simplest of the three types of parallel projections. The principle is to imagine a box around the object to be projected and to project the object "flat" on each of the six sides of the box (Figure 5.3a). If the object is simple and familiar, three projections, on three orthogonal sides, may be enough (Figure 5.3b). If the object is complex or is unfamiliar, a perspective projection may be needed in addition to the three or six parallel projections. For even more complex objects, sectional views may be necessary. Such a view is obtained by passing an imaginary plane through the object and drawing a projection of the plane.

If one side of the box is the xy plane, then a point $\mathbf{P} = (x, y, z)$ is projected on this side by removing its z coordinate to become $\mathbf{P}^* = (x, y)$. This operation can be carried out formally by multiplying $\mathbf{P}$ by matrix $\mathbf{T}_z$ of Equation (5.1). Similarly, matrices $\mathbf{T}_x$ and $\mathbf{T}_y$ project points orthographically on the yz and the xz planes, respectively.

$$\mathbf{T}_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{T}_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{T}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5.1}$$

The object of Figure 5.3 has two properties that make it especially easy to project. It is similar to a cube, and its edges are aligned with the coordinate axes. In general, if

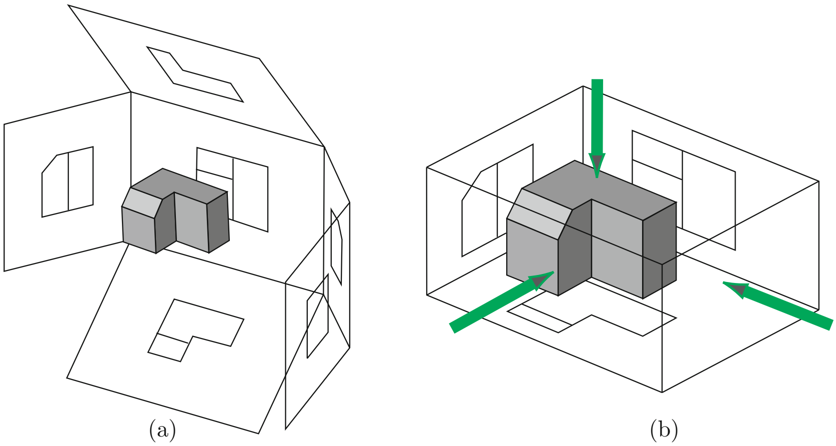

Figure 5.3: Six and Three Orthographic Projections.

the main edges of the object are not aligned with the coordinate axes, its orthographic projections along the axes may look unfamiliar and confusing, and it is preferable to rotate the object, if possible, and align it before it is projected. If the object is not cubical, the best option is to select on the object three axes that are judged the "main" ones and align them with the coordinate axes. The object is then surrounded by a bounding box (Figure 5.4) and the box is projected. Once this is done, the object is transferred into the projected bounding box in a process similar to that described in Section 6.3. If the object is so complex that it is impossible to find three such axes, then the designer should consider projecting several sectional views of the object or using a nonorthographic projection.

Figure 5.4: Orthographic Projection of a Curved Object.

◇ **Exercise 5.1:** Try to interpret the three orthographic projections of Figure 5.5.

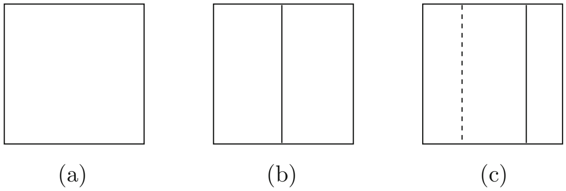

Figure 5.5: Three Orthographic Projections for Exercise 5.1.

The main advantage of orthographic projections is the ease of measuring dimensions. The projection of a segment of length l on the object is a segment of length l (or of a length related to l in a simple way) on the projection plane. This helps in manufacturing an object directly from a drawing and is the main reason orthographic projections are used in technical drawing.

Figure 5.6 shows a side view and the top view of a thin hexagon. It is easy to see that a segment of length l on side a becomes a segment of the same length on the projection, while a segment of length l on side b becomes a segment of length $l\cos\beta$ on the projection (where $\beta = 270° - \alpha$).

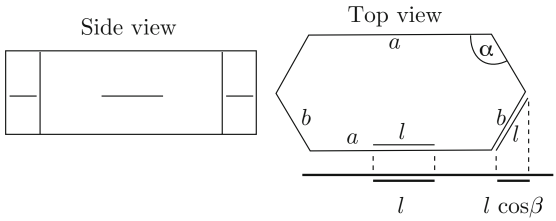

Figure 5.6: Segments on the Sides of a Hexagon.

I feel like I am diagonally parked in a parallel universe. —Unknown.

5.2 Axonometric Projections

The term axonometric is derived from the Greek $\alpha\xi\omega\nu$ or $\alpha\xi o\nu\alpha\zeta$ (axon, axis) and $\mu\epsilon\tau\rho o\nu$ (metron, a measure). We approach this type of parallel projection from two points of view.

Approach 1: Linear perspective, the topic of Chapter 6, was developed in the West during the Renaissance and is based on geometric optics. The observer is considered a point that receives straight rays of light and senses only the color, the intensity, and the direction of a ray but not the distance it has traveled. Oriental art, in contrast, has developed in a different direction and has adopted a different system of perspective, one that is suitable for scroll paintings.

A Chinese scroll painting is normally executed on a horizontal rectangle about 40 cm high and several meters long. The painting is viewed slowly from right to left while unrolling the scroll, and it tells a story in time. As the eye moves to the left, we see later occurrences of the same scene, not new views. We can call this approach to art "narrative," in contrast to Western art, which is situational. Figure 5.7 is an example of this type of art. It is a 33-foot-long scroll titled *A City of Cathay* that was painted by artists of the Qing court (1662–1795).

Figure 5.7: *A City of Cathay.*

Because of the temporal approach to scroll art, Chinese (and other Oriental artists) had to develop a system of perspective with no vanishing points, no explicit light sources, and no shadows. The result was a special type of parallel perspective, known today as "Chinese perspective" or axonometric projection. If we imagine the scroll to be the xy plane and we view it along the z axis, then lines that are parallel to the z axis are drawn parallel on the scroll instead of converging to a vanishing point.

Approach 2: An orthographic projection of an object shows the details of only one of its main faces, which is why three or even six projections are needed. Each projection may be detailed and it may show the true shape of that face with the correct dimensions, but it shows little or nothing of the rest of the object. Thus, interpreting and understanding orthographic projections requires experience. Viewing an object from above, from below, and from four sides tends to confuse an inexperienced person. Engineers, architects, and designers may be familiar with orthographic projections, but they have to draw plans that will be viewed and comprehended by their superiors and customers, and this suggests a projection method that will include some perspective, will show more than one face of the object, and will also make it easy to compute dimensions from the drawing. Linear perspective is easy to visualize and understand, but for engineers and designers it has at least three disadvantages: (1) it is complex to compute and draw, (2) the relation between dimensions on the diagram and real dimensions of the object is complex, and (3) distant objects look small. A common compromise is a drawing in one of the three varieties of axonometric projections.

Axonometric projections show more of the object in each projection but at the price of having wrong dimensions and angles. An axonometric projection typically shows three or more faces of the object, but it shrinks some of the dimensions. When a dimension is measured on the drawing, some computations are needed to convert it to a true dimension on the object. This is an easy, albeit nontrivial, procedure. An axonometric projection shows the true shape of a face of the object (with true dimensions) only if the face happens to be parallel to the projection plane. Otherwise, the shape of the face is distorted and its dimensions are shrunk.

Before we get to the details, here is a summary of the properties of axonometric projections:

- Axonometric projections are parallel, so a group of parallel lines on the object will appear parallel in the projection.
- There are no vanishing points. Thus, a wide image can be scrolled slowly while different parts of it are observed. At every point, the viewer will see the same perspective.
- Distant objects retain their size regardless of their distance from the observer. If the parameters of the projection are known, then the dimensions of any object, far or nearby, can be computed from measurements taken on the projection.
- There are standards for axonometric projections. A standard may specify the orientation of the object relative to the observer, which makes it easy for the observer to compute distances directly from the projection.

To construct an axonometric projection, the object may first have to be rotated to bring the desired faces toward the projection plane. It is then projected on that plane in parallel. We assume that the projection plane is the xy plane, so the projection is done by clearing the z coordinates of all the points or, equivalently, by multiplying each point, after rotating it, by matrix $\mathbf{T}_z$ of Equation (5.1). Assuming that we first rotate the object ϕ degrees about the y axis and then θ degrees about the x axis, the combined rotation/projection matrix is [see Equation (4.30)]

$$\mathbf{T}=\begin{pmatrix}\cos\phi & 0 & -\sin\phi\\ 0 & 1 & 0\\ \sin\phi & 0 & \cos\phi\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi\sin\theta & 0 \\ 0 & \cos\theta & 0 \\ \sin\phi & -\cos\phi\sin\theta & 0 \end{pmatrix}. \tag{5.2}$$

To find how various dimensions are affected by these transformations, we start with the vector $(1, 0, 0)$. This is a unit vector in the direction of the x axis. Multiplying it by $\mathbf{T}$ gives another vector, which we denote by $(x_1, x_2, 0)$. Its magnitude is $s_x = \sqrt{x_1^2 + x_2^2}$ and since the original vector had magnitude 1, the quantity s_x expresses the ratio of magnitudes or the factor by which all dimensions in the x direction have shrunk after the transformation/projection $\mathbf{T}$. Similarly, selecting unit vectors $(0, 1, 0)$ and $(0, 0, 1)$ in the y and z directions and multiplying them by $\mathbf{T}$ produces vectors $(y_1, y_2, 0)$ and $(z_1, z_2, 0)$ and shrinking factors $s_y = \sqrt{y_1^2 + y_2^2}$ and $s_z = \sqrt{z_1^2 + z_2^2}$ in the y and z directions, respectively.

Figure 5.8a shows a unit cube rotated such that its three sides, which used to be parallel to the coordinate axes, seem to have different lengths. Such an axonometric projection is called *trimetric*.

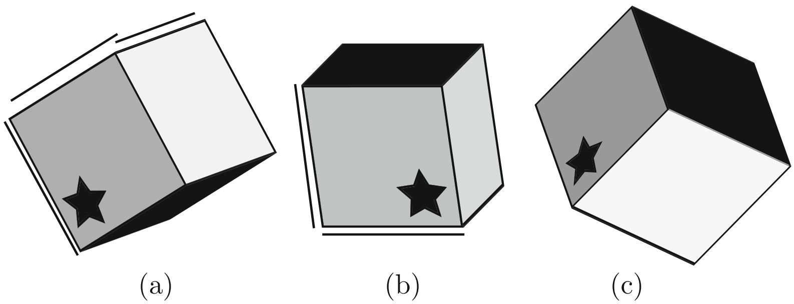

Figure 5.8: The Three Types of Axonometric Projections.

Figure 5.8b shows the same unit cube rotated such that two of its three sides seem to have the same length, while the third side looks shorter. Such an axonometric projection is called *dimetric*. Similarly, Figure 5.8c shows the same unit cube rotated such that all its sides seem to have the same length. This type of axonometric projection is called *isometric*.

Matrix $\mathbf{T}$ of Equation (5.2) can be used to calculate the special rotations that produce a dimetric projection. Consider the product of a unit vector in the x direction and $\mathbf{T}$:

$$(1, 0, 0) \begin{pmatrix} \cos\phi & \sin\phi\sin\theta & 0 \\ 0 & \cos\theta & 0 \\ \sin\phi & -\cos\phi\sin\theta & 0 \end{pmatrix} = (\cos\phi, \sin\phi\sin\theta, 0). \tag{5.3}$$

This product shows that any vector in the x direction shrinks, after being rotated by matrix $\mathbf{T}$, by a factor s_x given by Equation (5.4). The same equation also produces the shrink factors s_y and s_z of any vector in the y and z directions.

$$s_x = \sqrt{\cos^2\phi + \sin^2\phi\sin^2\theta}, \quad s_y = \sqrt{\cos^2\theta}, \quad s_z = \sqrt{\sin^2\phi + \cos^2\phi\sin^2\theta}. \tag{5.4}$$

If we want a dimetric projection where equal-size segments in the x and y directions will have equal sizes after the projection, we set $s_x = s_y$ or, equivalently,

$$\cos^2\phi + \sin^2\phi\sin^2\theta = \cos^2\theta,$$

which produces the relation

$$\sin^2\phi = \frac{\sin^2\theta}{1-\sin^2\theta}. \tag{5.5}$$

Equation (5.5) together with the expression for s_z^2 yields

$$\begin{aligned} s_z^2 &= \sin^2\phi + \cos^2\phi\sin^2\theta = \sin^2\phi + (1-\sin^2\phi)\sin^2\theta \\ &= \sin^2\phi(1-\sin^2\theta) + \sin^2\theta \\ &= \frac{\sin^2\theta}{1-\sin^2\theta}(1-\sin^2\theta) + \sin^2\theta, \end{aligned}$$

or $2\sin^4\theta - (2+s_z^2)\sin^2\theta + s_z^2 = 0$, a quadratic equation in $\sin^2\theta$ whose solutions are $\sin^2\theta = s_z^2/2$ and $\sin^2\theta = 1$. The second solution cannot be used in Equation (5.5) and has to be discarded. The first solution produces

$$\theta = \sin^{-1}\left(\pm\frac{s_z}{\sqrt{2}}\right) \quad \text{and} \quad \phi = \sin^{-1}\left(\pm\frac{s_z}{\sqrt{2-s_z^2}}\right). \tag{5.6}$$

Since the sine function has values in the range $[-1,1]$, the argument of $\sin^{-1}$ must be in this range. The expression $s_z/\sqrt{2}$ is in this range when $-\sqrt{2} \le s_z \le +\sqrt{2}$, and the expression $s_z/\sqrt{2-s_z^2}$ is in this range when $-1 \le s_z \le +1$. Since s_z is a shrinking factor, it is nonnegative, which implies that it must be in the interval $[0,1]$. Also, since Equation (5.6) contains a $\pm$, any value of s_z produces four solutions.

Example: Given $s_z = 1/2$, we calculate θ and ϕ:

$$\begin{aligned} \theta &= \sin^{-1}\left(\pm\frac{0.5}{\sqrt{2}}\right) = \sin^{-1}(\pm 0.35355) = \pm 20.7^\circ, \\ \phi &= \sin^{-1}\left(\pm\frac{0.5}{\sqrt{2-0.5^2}}\right) = \sin^{-1}(\pm 0.378) = \pm 22.2^\circ. \end{aligned}$$

The two rotations are illustrated in Figure 5.9.

◇ **Exercise 5.2:** Repeat the example for $s_z = 0.625$.

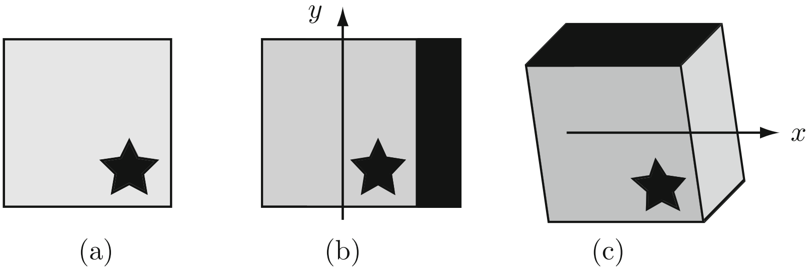

Figure 5.9: Rotations for Dimetric Projection.

◇ **Exercise 5.3:** Calculate θ and ϕ for $s_x = s_z$ (equal shrink factors in the x and z directions).

The condition for an isometric projection (Figure 5.8c) is $s_x = s_y = s_z$. We already know that $s_x = s_y$ results in Equation (5.5). Similarly, it is easy to see that $s_y = s_z$ results in $\cos^2\theta = \sin^2\phi + \cos^2\phi\sin^2\theta$, which can be written

$$\sin^2\phi = \frac{1 - 2\sin^2\theta}{1 - \sin^2\theta}. \tag{5.7}$$

Equations (5.5) and (5.7) result in $\sin^2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = 1/3$, yielding $\theta = \pm 35.26°$. The rotation angle ϕ can now be calculated from Equation (5.5):

$$\sin^2\phi = \frac{1/3}{1 - 1/3} = 1/2, \quad \text{yielding } \phi = \pm 45°.$$

The shrink factors can be calculated from, for example, $s_y = \cos^2\theta = \sqrt{2/3} \approx 0.8165$.

We conclude that the isometric projection is the most useful but also the most restrictive of the three axonometric projections. Given a diagram with the isometric projection of an object, we can measure distances on the diagram and divide them by 0.8165 to obtain actual dimensions on the object. However, the diagram must show the object (whose main edges are assumed to be originally aligned with the coordinate axes) after being rotated by $\pm 45°$ about the y axis and by $\pm 35.26°$ about the x axis. If these rotations result in obscuring important object features, a less restrictive projection, such as dimetric or trimetric, must be used.

Figure 5.10 shows isometric and perspective projections of a simple stair-like object and it is clear that the former looks distorted and unnatural (the side away from the viewer seems too large and bent), while the latter looks real.

Standards for Axonometric Projections

Several common standards for axonometric projections exist and are described here. We start with a simple 30° standard for isometric projections whose principle is illustrated in Figure 5.11. Part (a) of the figure shows a cube projected in this standard after it

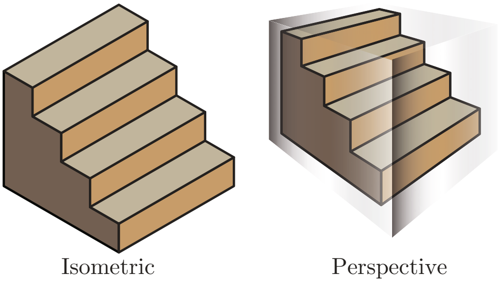

Figure 5.10: Isometric and Perspective Projections.

has been rotated $\phi = 45°$ about the y axis and $\theta = 35°$ about the x axis. Part (b) shows the same cube with dimensions and angles. It is not difficult to see that α satisfies $\tan\alpha = h/w$, which is why $\alpha = \arctan(h/w)$. The standard specifies the ratio $h/w = 1/\sqrt{3}$, which results in $\alpha \approx 30°$. The $30°$ angle is convenient because $\sin 30° = 1/2$. This part of the figure also shows that $\theta = \arcsin(h/w)$, a quantity that happens to be close to $35°$. This projection is attributed by [Krikke 00] to William Farish, who developed it in 1822.

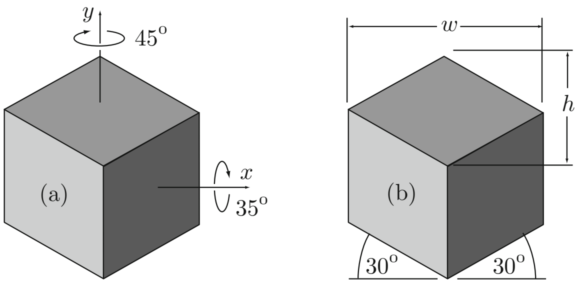

Figure 5.11: The $30°$ Standard for Isometric Projections.

A $30°$ angle is convenient for drafters because $\sin 30° = 1/2$. However, in our age of computers and computer-aided design, virtually all graphics output devices (monitors, plotters, and printers) use a raster scan and are based on pixels. A line is drawn as a set of individual pixels, and even a little experience with such lines shows that a line at $30°$ to the horizontal looks bad. Much better results are obtained when drawing a line at about $27°$ because the tangent of this angle is 0.5, resulting in a line made of identical sets of pixels (Figure 5.12).

As a result, the $27°$ standard for axonometric projections (Figure 5.13) makes more sense. This standard is sometimes also called the 1: 2 isometric projection because it is based on the ratio $h/w = 1/2$.

Figure 5.12: Pixels for 30° and 27° Lines.

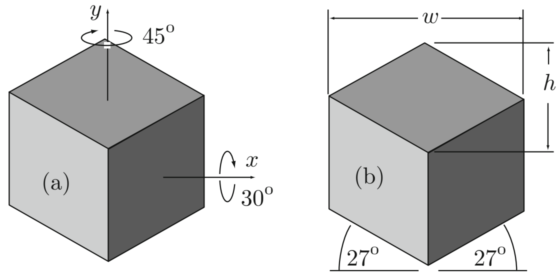

Figure 5.13: The 27° Isometric Projection.

A similar standard is based on the ratio $h/w = 1$, which leads to $\alpha = 45°$. This case is also known as the military isometric projection. This projection is suitable for applications where the horizontal faces of the projected object are important. Figure 5.14 shows that the xz plane becomes a regular rhombus in this projection, which makes it easy to read details and measure distances on this plane.

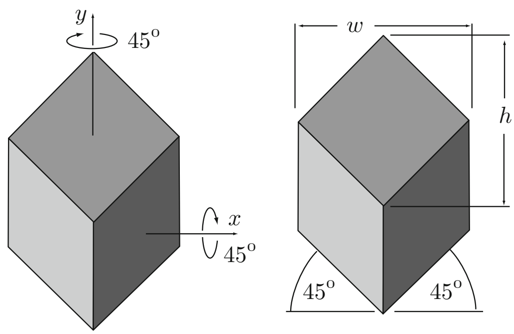

Figure 5.14: The 45° Isometric Projection.

A Dutch standard for dimetric projections is based on the ratio $h/w = 0.33$. It is known as the $42°/7°$ standard because it results in angles α and β of these sizes (Figure 5.15). The z axis (the one that's drawn at $42°$) is scaled by a factor of $1/2$.

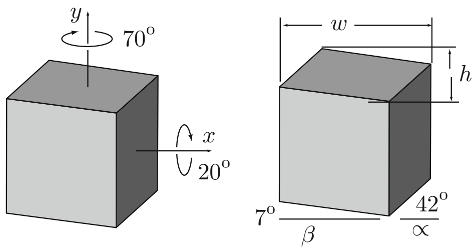

Figure 5.15: The $42°/7°$ Dimetric Projection.

5.3 Oblique Projections

An oblique projection is a special case of a parallel projection (i.e., with a center of projection at infinity) where the projecting rays are not perpendicular to the projection plane. We have already seen that axonometric projections show more object details than orthographic projections but make it more cumbersome to compute object dimensions from the flat projection. Similarly, oblique projections generally show more object details than axonometric projections but distort angles and dimensions even more. In an oblique projection, only those faces of the object that are parallel to the projection plane are projected with their true dimensions. Other faces are distorted such that measuring dimensions on them requires calculations.

> The diagram can be drawn quite quickly because the designer used a style of drawing called oblique projection. So long as basic rules are followed, oblique projection is quite easy to master and it may be a suitable style for you to use in a design project. The basic rules are outlined below.
>
> `http://www.technologystudent.com/designpro/oblique1.htm`

Figure 5.16 illustrates the principle of oblique projections. A three-dimensional point $\mathbf{P} = (x, y, z)$ is projected obliquely onto a point $\mathbf{P}^*$ on the xy plane. We denote the point $(x, y, 0)$ by $\mathbf{Q}$ and examine the angle θ between the two segments $\mathbf{PP}^*$ and $\mathbf{P}^*\mathbf{Q}$. A cavalier projection is obtained when $\theta = 45°$ and a cabinet projection is the result of $\theta = 63.43°$.

Because of the special 45° angle, the three shrink factors of a cavalier projection are equal, as will be shown later. In a cabinet projection, the shrink factors in the x and y directions (assuming that the object is projected on the xy plane) equal 1/2.

Figure 5.17a illustrates the geometry of oblique projections and can be used to derive their transformation matrix. We assume that the projection plane is $z = 0$ (the xy plane) and that all the projecting rays hit this plane at an angle θ. Two projecting rays are shown, one projecting the special point $\mathbf{P} = (0, 0, 1)$ to a point $(a, b, 0)$ and the other projecting $\mathbf{Q} = (0, 0, z)$, a general point on the z axis, to a point $(A, B, 0)$. The origin $(0, 0, 0)$ is projected onto itself, so the projection of the unit segment from the origin to $\mathbf{P}$ is the segment of size s from the origin to $(a, b, 0)$. The value s is therefore the shrink factor of the oblique projection. The three quantities a, b, and s are related

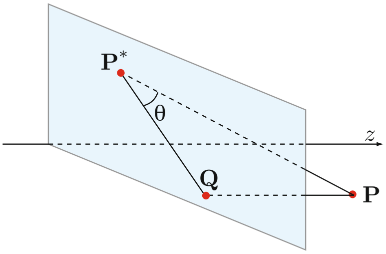

Figure 5.16: Oblique Projections.

by $a = s\cos\phi$ and $b = s\sin\phi$, where ϕ is measured on the projection plane. The shrink factor s is also related to the projection angle θ by $\tan\theta = 1/s$ or $s = \cot\theta$.

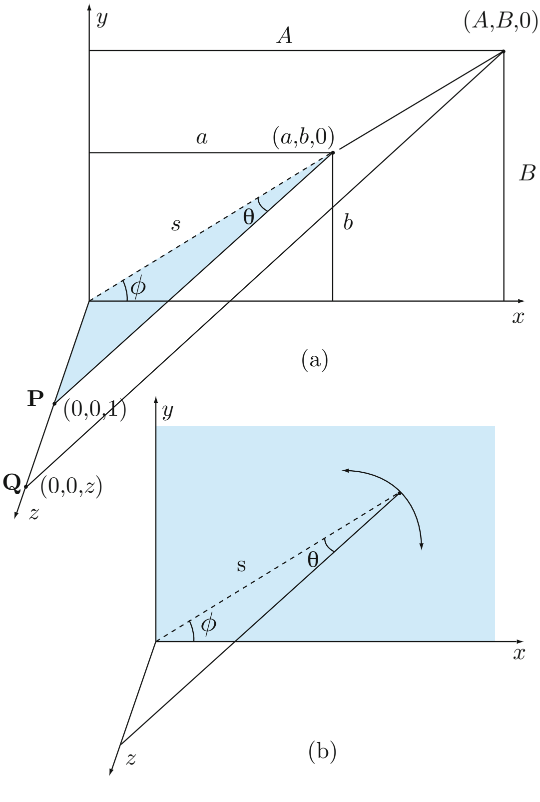

Figure 5.17: Oblique Projections.

Oblique (Adj.) Neither parallel nor at a right angle to a specified or implied line; slanting.

We now consider the projecting ray from $\mathbf{Q}$ to $(A, B, 0)$. Since $\mathbf{Q}$ is at a distance z from the origin, the distance on the projection plane between the origin and point $(A, B, 0)$ is sz. From this we obtain the relations $A = sz\cos\phi$ and $B = sz\sin\phi$. The next step is to consider the projection of a general point (x, y, z). All the projecting rays are parallel, so a little thinking shows that moving a point from $(0, 0, z)$ to $(x, 0, z)$ moves its projection from $(A, B, 0)$ to $(x + A, B, 0)$. Similarly, moving a point from $(0, 0, z)$ to $(0, y, z)$ moves its projection from $(A, B, 0)$ to $(A, y + B, 0)$. A general point located at (x, y, z) is therefore projected to a point at $(x + A, y + B, 0)$. Thus, the rule of oblique projections is

$$(x, y, z) \longrightarrow (x + sz\cos\phi, y + sz\sin\phi, 0), \tag{5.8}$$

which can be written in terms of a transformation matrix

$$\mathbf{P}^* = \mathbf{PT} = (x, y, z)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ s\cos\phi & s\sin\phi & 0 \end{pmatrix}. \tag{5.9}$$

With the help of this matrix we examine the following special cases.

1. A cavalier projection. It is defined as the case where the projection angle is 45°, which implies $s = \cot(45°) = 1$. Thus, all edges and segments have shrink factors of 1.

2. A projection angle of 90°. A value $\theta = 90°$ implies a shrink factor $s = \cot(90°) = 0$. Matrix $\mathbf{T}$ of Equation (5.9) reduces to matrix $\mathbf{T}_z$ of Equation (5.1), showing how the oblique projection reduces in this case to an orthographic projection.

3. A cabinet projection. It is defined as the case where the projection angle is 63.43°, which implies $s = \cot(63.43°) = 1/2$. All edges and segments perpendicular to the projection plane have shrink factors of 1/2.

Figure 5.17b shows how ϕ and θ are independent. For a given projection angle θ, it is possible to assign ϕ any value by rotating the triangle in the figure. In practice, this means that an object can be projected several times, with different values of ϕ but with the same projection angle θ. Such projections may give all the necessary visual information about the object while having the same shrink factors.

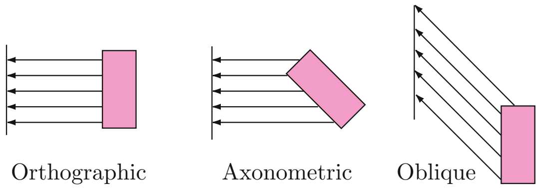

Figure 5.18: Comparing Parallel Projections.

Axonometric and oblique projections are generally considered different, but Figure 5.18 shows that the difference between them is a matter of taste and terminology. If we rotate the object and light rays of the oblique projection 45° counterclockwise, the result on the projection plane is identical to the axonometric projection.

> She could afterward calmly discuss with him such blameless technicalities as hidden line algorithms and buffer refresh times, cabinet versus cavalier projections and Hermite versus Bézier parametric cubic curve forms.
>
> —John Updike, *Roger's Version* (1986)