

Chapter 4

1920 – 1929

1920-01-25

From H.A. Lorentz — 25.I.1920

Haarlem
Scientific Committee
for Advice and Research

Amice,

It seems to me too that, replying to the enclosed communication, you could draw attention to our Subcommittee for Photogrammetry. On my part I would be happy, also in view of the plan to establish this new committee, to have a talk in the next two weeks with the Minister of Education and maybe others about the Scientific Committee for Advice and Research. But can I say then, that I have learned about the plan from the Minister of War? The letter that you sent me was marked ‘secret’. ⁽¹⁾

Perhaps you can ask the Commander of the Aviation Department whether you are allowed to inform me, as chairman of the Scientific Committee for Advice and Research, about the plan without mentioning the names of the persons considered for the Committee. The ‘secret’ probably will refer to those names and I don’t have to know these in order to bring up the matter.

You will understand that I have heard with great interest about your nominations ⁽²⁾ in Berlin and Göttingen. I am delighted about the great ap-

⁽¹⁾The letter is not extant. It is not unreasonable to assume that it is another copy of the letter of 19.I.1920. ⁽²⁾Lorentz erroneously writes ‘appointments’ (*benoemingen*).

preciation of your merits thus shown from the side of you German colleagues. I understand very well that especially the Berlin proposal has, apart from its many drawbacks, its attraction, and that you must seriously consider it. But I very much hope that you will come to the decision to stay in the Netherlands.

With amicable greetings

t.t.

H.A. Lorentz

[Signed autograph – in Brouwer]

1920-02-04

To Mayor Amsterdam — 4.II.1920

Berlin S.W.
Hospiz St. Michael
Wilhelmstrasse 24

Dear Mayor, [Hooggeachte Burgemeester]

Should my wishes find a favorably reception with the curators,⁽³⁾ would you then perhaps have the kindness inform me about it by a few words to the above address? In view of the way I have been received here in Berlin, and the courtesy shown to me, I would appreciate very much to convey my decision orally to my Berlin colleagues (especially if it is unfavorable for them).

Assuming that I can stay in Amsterdam, I would like to make yet another proposal to you, namely that Mayor and Aldermen try to find a way to authorize me already now to put the credit of f. 10,000 for the reference library⁽⁴⁾ at my disposal. For I believe that I can *now* and *personally* make purchases in Leipzig that will be two to three times cheaper, than they would have to be *later* and *from Amsterdam*. It would be simplest if the city of Amsterdam or one of its institutions had an account with a German bank, and that it would be prepared to transfer money in German currency to the account of German booksellers, following my instructions. In that case I

⁽³⁾The mayor was, ex officio, president of the board of curators of the University of Amsterdam. ⁽⁴⁾*handbibliotheek*.

could conclude the transactions irrevocably by payment in cash, and check the shipping to Holland in person.

With my apologies for the trouble I cause,

Sincerely yours ⁽⁵⁾

L.E.J. Brouwer

[Signed autograph – in GAA]

1920-02-12

From Mayor of Amsterdam — 12.II.1920

Amsterdam

Dear Professor, [Hooggeleerde Professor]

I received your letter of February 4 last only yesterday evening. I am very much pleased that I can inform you that the Curators have declared to be prepared to transmit your wishes to the City Council, and as the City Council has yesterday said that it does not object to consent to your wishes, trusting that you will be retained for Amsterdam and the fatherland. To avoid misunderstandings and to confirm our conversation on January 26 last I mention your three wishes below. ⁽⁶⁾

I. Your annual salary will be raised to the maximum of f. 10,000, effective January 1, 1920.

II. An amount of f. 10,000.- is made available for buying back volumes of mathematics journals,

III. The number of teaching staff is increased by two lecturers, for teaching the undergraduate students. ⁽⁷⁾

I discussed with Prof. Hendrik de Vries the possibility that, to save expenses, the lecturer's positions could be combined with teaching a not too large number of hours at the Gymnasium ⁽⁸⁾ or one of the high schools ⁽⁹⁾ I informed the Council that a solution in this direction would be looked for, without committing myself.

⁽⁵⁾ *gaarne Uw dienstwillige.* ⁽⁶⁾ These three desiderata are the basis of the promise of a 'Göttingen in Amsterdam'. ⁽⁷⁾ In Dutch 'candidaten' i.e., students who have passed their first university examination after about two years of study. ⁽⁸⁾ A secondary school with Latin and Greek. ⁽⁹⁾ *Hoogere Burgerscholen*, secondary schools without Latin or Greek, but with a strong science program.

The council members have been bound to secrecy in this matter under reference to article 43 of the Municipal Law.

Concerning your proposal to open a credit for the acquisition of journals – I cannot possibly consider this at this moment. Before taking the required steps in this matter, I would in the first place need a statement from you that you will remain at the University of Amsterdam. Moreover, purchases in Germany require extreme caution, *especially with respect to the required export permit*. The biology department of your faculty has experienced a few months ago a great disappointment in this domain. So if it is necessary to go to Leipzig, then this always can be done later, after the necessary arrangements have been made with the financial experts of the city.

Sincerely yours ⁽¹⁰⁾

Your
T. ⁽¹¹⁾

[Initialled autograph draft – in GAA]

1920-02-21

To Mayor of Amsterdam — 21.II.1920

Laren

Dear Mr. Mayor, [Hooggeachte Burgemeester]

Having received your letter of the 12th of this month at this address, I have the pleasure to confirm once more in writing that I fully agree with the contents of your letter, and once more to thank you for having made it possible by your efforts to make me remain in the fatherland. Also with respect to the board of curators, I beg you as Chairman to accept my gratitude for their cooperation.

Sincerely yours ⁽¹²⁾

L.E.J. Brouwer

[Signed autograph – in GAA]

⁽¹⁰⁾ *Met de meeste hoogachting.* ⁽¹¹⁾ J.W.C. Tellegen. ⁽¹²⁾ *Gaarne Uw dienstwillige.*

1920-03-00**From Brouwer et al. to KNAW⁽¹³⁾ — III.1920** **Amsterdam**

The undersigned propose for the Foreign Membership of the Royal Academy of Sciences, Mr. Jacques Hadamard in Paris, without any doubt the most versatile, astute and fertile of the living French mathematicians. Among the very diverse domains of research in which Hadamard had a key role in the last 30 years, the undersigned mention the theories of analytic continuation, entire functions, orbits in mechanics, wave propagation, vibration modes of plates, distribution of prime numbers, functional calculus, integral and integrodifferential equations, and calculus of variations.

The undersigned are of the opinion that the place left vacant by Poincaré among the Foreign Members of our Academy cannot be filled better than by the man who also was his successor in the Section de Géométrie de l'Académie des Sciences in Paris.⁽¹⁴⁾

D. Korteweg
 H.A. Lorentz
 W. Kapteyn
 J.C. Kluyver
 Jan de Vries
 J. Cardinaal
 Hk. de Vries
 L.E.J. Brouwer

[Signed autograph – in KNAW]

1920-03-25b

From J.A. Schouten — 25.III.1920^b (15) **Delft**
 Rotterdamsche Weg 2⁵

Dear Sir, [Weledele Heer]

In polite reply to your letter of the 20th, I inform you that the promise contained in my letter of November 20, 1919, copy enclosed here, does admit

⁽¹³⁾In Brouwer's handwriting. ⁽¹⁴⁾Hadamard was appointed in Paris in the year of Poincaré's death, in 1912. He was duly appointed in Amsterdam. ⁽¹⁵⁾Erroneously dated 1919.

no other interpretation than the relinquishing of one of the *manuscripts* in the state it has been submitted during the summer of 1917 to you. Concerning the state of the *binding* and the *manner of binding* no promise whatsoever has been given by me. For your further information, the manuscript intended for you was divided for my own convenience into two parts of a more convenient thickness. My plan was, as already announced to you, to send you consecutively both parts, each of course neatly bound. In my humble opinion I thus would have completely fulfilled the promise I made, because I promised the *manuscript*, not the *binding*, and a manuscript doesn't change of course by an artifice as mentioned.

Meanwhile I have taken proper notice of your statement that there is no possibility of restitution of the manuscript in parts. So you refuse acceptance of the manuscript now offered to you in a completely respectable form and completely as agreed upon, just because of the fact that this was bound in two volumes instead of in one. As you have no grounds at all for demanding that I reunite the manuscript in a single binding, I consider myself relieved of the obligation to satisfy the promise made by me at the time.

I record that even after a not particularly polite request from you in November 1919, I have immediately *kindly promised* the manuscript to which you didn't have any *legal claim*. Furthermore, I have out of *kindness* informed you telegraphically about the contents of a letter which you hadn't read yet, in order to save you the costs of having a manuscript re-typed, of which the possession was already promised to you weeks ago. So on my side there was no lack of consideration and patience. Where you have reacted since the second half of 1917 to this, for reasons as yet unknown to me, with unkindness and with misrepresentations, it cannot be expected from me that I am forthcoming with respect to a legally unsupported *demand* now formulated by you, where a *request* would have been more appropriate.

Sincerely yours ⁽¹⁶⁾

J.A. Schouten

[Signed typescript – in Brouwer]

⁽¹⁶⁾ *Hoogachtend.*

1920-04-01

From O. Blumenthal — 1.IV.1920

Kreis Heinsberg
Waldhotel Wasserberg

Dear Brouwer! [Lieber Brouwer]

First of all I want to tell you, while awaiting your promised letter, that I am away from Aachen for 14 days. I am staying quite close to the Dutch border, namely close to Dalheim, which is the German border station on the line between Roermond and Mönchen Gladbach.⁽¹⁷⁾ *It would mean a great deal to me to meet you soon.* The forest here is very beautiful, maybe that attracts you.

I come back to the dating problem. You misunderstood the agreement of the editorial board. It has after all been laid down that the date of *acceptance* will be shown. It is the intention of the proposer, that the date should not be a ground for priority claims of the author, but it would give the public a possibility to check how much time elapses between acceptance and publication. The date of acceptance, not of submission, was chosen because we are afraid of disputes with the authors in case of returning manuscripts for revision, about what the date of submission is: the author thinks the date of reception of the manuscript that was returned later, and the editor thinks the date of reception of the manuscript that is ready for printing. When ‘date of reception’ is chosen, the author has more rights, and in case of ‘date of acceptance’ the editor. I admit that one can disagree about the efficiency, and I am quite ready to enter an argument with you.

Your misunderstanding originates from the following: as implementation for the agreement that the date of acceptance should be shown, I have proposed that *in general the acceptance day should be the day that the print-ready manuscript is in the incoming mail.* In that way it should, in the interest of the author, be prevented that an editor will have a paper unnoticed with him for months, which could also happen.

You admit that apart from the date of acceptance there is justification for an author-date. Nonetheless, I would, also now, give the editor the right to reject an author-date that seems unjustified. This, as opposed to my earlier view.

On the other hand there are also cases where there is justification for an author-date next to a date of *reception*. I have just now seen such a case: an

⁽¹⁷⁾ nowadays ‘Mönchengladbach’.

article was for three months at the *Mathematische Zeitschrift* and then was sent to me by Lichtenstein with the request that we take it: not because the article was bad, but because the journal already had an article by the same author. In this case the author-date undoubtedly is justified.

Maybe we will find in face to face discussion a solution, which is correct for all cases. In writing one gets involved in complications. As Clemenceau once said: *Je suis dans l'incohérence, j'y suis, j'y reste.*⁽¹⁸⁾

Best greetings and 'auf Wiedersehen'!

Your
O. Blumenthal

[Signed autograph – in Brouwer]

1920-05-06a

From H. Weyl — 6.V.1920^a

Zürich

Dear Brouwer! [Lieber Brouwer]

Finally I have sent the long promised [object] off to you.⁽¹⁹⁾ It should not be viewed as a scientific publication, but as a propaganda pamphlet, thence the size. I hope that you will find it suitable to rouse the sleepers; that is why I want to publish it. I would be grateful for your opinion and comments. Did I enclose everything that you let me have only as a loan? If not, please reclaim it; the lecture on Formalism and Intuitionism⁽²⁰⁾ was already in my possession in the old days; at that time I did not pay attention to it or understand it ...

At the moment the matter of the appointment is finally approaching a decision. The reason for the delay was Berlin; and after Herglotz apparently turned it down, I have been offered Berlin in addition to Göttingen. The day after tomorrow I depart. I feel rather loosely tied to Zürich. Neither for mathematics, nor for myself I can realize here something. I'll write to you

⁽¹⁸⁾I am in [in a state of] incoherence, here I am, here I stay. The second part is in fact a famous quotation by itself, namely of general MacMahon in the Crimean War (1855). ⁽¹⁹⁾Manuscript of the 'New Crisis' paper, [Weyl 1921]. ⁽²⁰⁾[Brouwer 1912a], [Brouwer 1914]

about the result. Today a couple of cordial greetings from your

Hermann Weyl

[Signed autograph, postcard – in Brouwer]

Editorial supplement

D. Hilbert to H. Weyl — 16.V.1920

Göttingen⁽²¹⁾

Sontag

Lieber Weyl,

[.....]

I heartily wish that you can improve your financial situation in Zurich as far as your wishes go. Should you, however, decide for Germany, then it is not clear to me why you should prefer Berlin. What I can quite understand with Brouwer and Landau—Brouwer wanted just temporarily to stay in Berlin, and to get familiar with Berlin and the nimbus to be appointed in the capital, were his motives, and Landau has his roots in Berlin and also the financial basis, which cannot be replaced by any salary, necessary for Berlin— does not apply to you: moreover, you can in a few years time obtain a transfer to Berlin, when later the extremely unpleasant and not to be envied circumstances in Berlin have been improved.

With best greetings to also to your wife,
your
Hilbert

[Signed autograph – in Weyl]

1920-05-06b

To H. Weyl — after 6.V.1920^a ⁽²²⁾

Your unreserved scientific assistance has given me an infinite pleasure. The reading of your manuscript was a continual delight and your exposition,

⁽²¹⁾Only the for Brouwer relevant part of the letter is reproduced. ⁽²²⁾This draft is poorly readable. Some sentences have been left unpolished or unfinished. C.f. [Van Dalen 1995].

it seems to me, will also be clear and convincing for the public . . . That the two of us have different opinions some side issues, will only be will only stimulate the reader. However, you are completely right in your formulation of these differences of opinion; in the restriction of the objects of mathematics you are in fact more radical than I am; however, one cannot argue about this, these matters can only be decided by individual concentration.

Referring to your expositions on the concept of a continuous function I would like to draw your attention to my concept of a completely defined function of the continuum. I mean by that a law which assigns to each point of a point species that locally coincides⁽²³⁾ with the continuum a further point of the continuum. Such a function can very well be discontinuous without being in any manner generated by putting together continuous functions on separated continua; one can, by the way, operate with them in many ways (one can, for example, integrate them in certain cases without having information about their continuity or discontinuity).

Apart from our points of difference, I have the following remarks:

To the non-existence proofs (to which belong for example the cardinality theorems on p. 13 and 43 of my first treatise⁽²⁴⁾ and also the Hilbert finiteness theorem for complete systems of invariants in his first proof) you don't devote any space in your enumeration of mathematical judgments. On p. 3, l. 8 (and likewise on the analogous place on page 13 of 'The Continuum' the meaning of the word 'Sachkenntnissen'⁽²⁵⁾ is obscure to me.

It seems to me that the whole point of your paper is endangered by the end of the second paragraph of page 34⁽²⁶⁾. After you have roused the sleeper, he will say here to himself: "So the author admits that the real mathematical theorems are not affected by his expositions? Then he should no longer disturb me!" and turns away and sleeps on. Thereby you do our cause an injustice, for together with the existence theorem of the accumulation point of an infinite point set, many a classical existence theorem of a minimal function, and also the existential theorem of the geodetic line without the second differentiability condition, loses its justification!

The statement you formulate on p. 37, l. 3–6, which by the way, as you know, contradicts my opinion, should be explained a bit more in detail. It seems to me that also the reader who has followed you closely so far, will have problems with this passage. Your discrete function and mixed function to me seem, just as well as the continuous function, to be contained in my

⁽²³⁾ A notion from [Brouwer 1919a]: A locally coincides with B if $\neg\exists a \in A \forall b \in B (a \# b) \wedge \neg\exists b \in B \forall a \in A (a \# b)$. ⁽²⁴⁾ [Brouwer 1918a]. ⁽²⁵⁾ Factual knowledge. ⁽²⁶⁾ [Weyl 1921]

spread concept. My spread law can very well give in advance for every choice sequence the certainty that after it once has generated a sign, it will henceforth generate again and again nothing. ⁽²⁷⁾

I am tremendously curious about your decision between Göttingen, Berlin and Zürich. May you see clearly and make the right choice. That won't be easy for you!

I can keep the copy of your manuscript you sent me, right? You don't have to send me back anything. Because some of my reprints have been printed anew, I want to ask you to inform me which of the following publications of mine you have at present:

1. Intuitionisme en formalisme (Dutch)
2. Intuitionism and formalism (English)
3. De onbetrouwbaarheid der logische principes (Dutch)
4. Het wezen der meetkunde (Dutch)

I can now supplement the possibly missing items. Once more, many sincere thanks for the joy and satisfaction that your text has given me, cordial greetings also to your wife and 'auf Wiedersehen'!

Your Egbertus Brouwer.

[Draft handwritten – in Brouwer]

1920-07-26

From H. Dingler — 26.VII.1920

Munich

Clemensstr. 47-III, München

Dear Professor! [Sehr geehrter Herr Professor]

Please accept my warm thanks for your kind, rich package; I have immediately occupied myself with the extremely interesting reading of the material. I was very much interested to find in you a strong inclination also towards epistemological problems, that have been occupying me already for many years (until now I was only familiar with your more mathematical articles). I have yet to get acquainted with your set theory. For the time

⁽²⁷⁾This is part of Brouwer's definition of 'spread', see e.g. [Brouwer 1981] p. 14, [Van Dalen 1999] p. 314.

being I don't quite understand how you want to get around the fundamental theorem of the excluded middle. However, something like that is after all certainly possible, just as it is possible to construct non-Euclidean geometries. I enjoyed very much your demand for a constructive (I would rather say 'synthetic') set theory.¹ Fortunately the holidays will start in a few days, and then I'll find more time to go into your valuable writings.

Thanking you again,
Sincerely yours⁽²⁸⁾

H. Dingler.

[Signed autograph – in Brouwer]

1920-08-07

To F. Klein — 7.VIII.1920

Bad Harzburg

Krodothal 4

Dear Mr. Geheimrat, [Hochgeehrter Herr Geheimrat]

Refereeing an article by Schouten amounts in my opinion to first translating the cumbersome and worthless symbolism into common language,⁽²⁹⁾ then sifting from among the great mass of trivialities thus obtained, the few theorems that matter, and finally figuring out on which places, unquoted by the author, these theorems, insofar they are correct, have appeared earlier in the literature. Then the result, certain from the beginning, is the rejection.

¹Also the definition of a set by a law, I find highly sympathetic in the case of the higher sets, as well as your set theoretical theorems (until now only in the formulation; I have yet to learn to understand more closely the meaning and proof).

⁽²⁸⁾*Mit verbindlichen Empfehlungen und nochmaligem besten Dank, Ihr ergebenster.*

⁽²⁹⁾Although Brouwer was no admirer of formalisms — see e.g. Brouwer, *Intuitionism and Formalisme* (inaugural address, University of Amsterdam, 1912), [Brouwer 1913b] p. 84 — he would not object to efficient notations. Schouten's formalism, however, was more than he could take. Brouwer was not alone in this view, cf. *Brouwer to Klein 19.IX.1919* and [Van Dalen 1999] section 8.3. Klein did not share Brouwer's negative opinion (see below); he asked Weyl for a second opinion, (15.I.1920), but Weyl was not forthcoming, cf. [Van Dalen 1999] p. 298.

But to carry out the justification of the rejection in a logical and matter of fact manner, demands not only a large and unrewarding investment of time, but also a library that completely contains the newest literature, hence for this reason already I am unable to undertake this assignment here in Harzburg. Also, on the other hand I by now feel justified, after having protected the *Annalen* already a few times from the embarrassment of accepting a Schouten article (indeed, the article about the classification of associative number systems ⁽³⁰⁾ has been accepted by Hölder), to waste no more time and effort on this author, and to restrict myself to declining any responsibility for the publication of his productions.

I apologize for expressing myself somewhat bluntly, but I see no other possibility to express my point of view clearly in any other way.

As regards Haalmeyer, in the past months he has submitted his article two more times to me; both times it seemed to me capable of improvement and I have handed it back to him.

In my further publications about topological groups I will probably have more often the opportunity to refer to the ‘*Theorie der automorphen Functionen*’ by Fricke and you, especially where I prove the topological equivalence of the topological and linear infinite discontinuous groups.

With many greetings

Yours truly and cordially ⁽³¹⁾

L.E.J. Brouwer

[Signed autograph – in Klein]

1920-08-20

From G. Mittag-Leffler — 20.VIII.1920

Tällberg ⁽³²⁾

Dear Colleague, [Tres honoré Collègue]

Are you still in Amsterdam? I have been told that you have accepted to become the successor of Carathéodory in Berlin, but I don't know any details. If that is the case, could you not think of Frédéric Riesz as your

⁽³⁰⁾[Schouten 1918]. ⁽³¹⁾*Ihr wie immer hochachtungsvoll und herzlich ergebener.*

⁽³²⁾Letter forwarded to Villa Friedwalt, Krodotal 4, Bad Harzburg (envelope).

successor? It is improbable that you could find a worthier one. He is now in a very unhappy position, being fired from Koloszwär (Klausenburg), because he couldn't give courses in Romanian.

I allow myself to send you three brochures of mine, and I would be happy if you would always send me reprints of what you publish yourself.

Please accept the expression of my great respect and my admiration for your beautiful works,

Yours truly, ⁽³³⁾

Mittag-Leffler

[Signed typescript – in Brouwer]

1920-08-28

From J. Wolff — 28.VIII.1920

Groningen

Amice,

Enclosed I submit to you an article, ⁽³⁴⁾ for which I give you full authority: if you think it good enough for the Academy, ⁽³⁵⁾ then you would do me a very great pleasure to present it. In case of the least doubt I ask you urgently not to present it, and then I'll hear from you about it some time.

As an extension of the notion of limit, I assign to each set V_δ depending on $\delta > 0$, with $V_{\delta'} < V_\delta$ if $\delta' < \delta$, a limit set (L), which is the intersection of the closed hulls ⁽³⁶⁾ of V_δ . In that way one can for example speak about the limit set ⁽³⁷⁾ of a function 'in a point'. Usually one only considers the extremal elements, those are the two limit functions. Wouldn't it be nice to classify functions according to the nature of their limit sets? As appears from my article, functions for which the limit sets are all points or continua must form an important class, see for example p. 4 § 10.

I have meticulously checked in the Revue ⁽³⁸⁾ whether functions have been studied at all according to this program and I come to the conclusion that this is not the case. I think it is interesting to examine the kinds of

⁽³³⁾ *Agréez, je vous en prie, très honoré Collègue, l'expression de ma haute considération et de mon admiration de vos beaux travaux.* ⁽³⁴⁾ Possibly [Wolff 1920]. ⁽³⁵⁾ KNAW

⁽³⁶⁾ The author uses here a German term. ⁽³⁷⁾ in Dutch: *limesverzameling*. ⁽³⁸⁾ *Revue semestrielle*.

continua that occur in the complex plane near the differential quotients of a function: if they are all points, then the function is holomorphic.

With friendly greetings

t.t.

Wolff.

[signed autograph – in Brouwer]

1920-09-07

To H. Weyl — 7.IX.1920

Bad Harzburg ⁽³⁹⁾

Krodotal 4

Dear Weyl [Lieber Weyl]

As a supplement to my postcard from Switzerland, first the following: in building up mathematics in Amsterdam I don't pursue at all the plan of establishing there an intensive lecture and seminar business, but only to bring together a circle of people whose mathematical work is mainly a stimulating and controlling side phenomenon of their general spiritual development, in other words people who feel themselves to be more or less the thinking organ of the community and who unabashedly relegate the directly tangible academic teaching activities to the second place, after this calling. (Indeed, I see the drive for mathematical knowledge — which is fundamentally different from the joy of solving mathematical problems — as a characteristic of a mental attitude that safeguards a free and wide view on the most diverse moral and practical domains, which is considerably superior to the prevailing view.) To this I add that we mathematicians in Amsterdam have secured in the last years a very large degree of academic freedom and that we use this in the above sense. Moreover, we are respected in our faculty (of natural sciences) and our subject is held there in an esteem that is free of skepticism. However, in the other faculties (maybe with exception of the medical faculty) we have more or less the reputation of Bolsheviks.

⁽³⁹⁾The unmentioned topic of this letter is Brouwer's attempt to get Weyl to accept a chair in Amsterdam.

Concerning assistants: I have one who manages the reading room and who has worked out a few of my lectures. My colleagues don't have and don't want one. You certainly can get one, as soon as you want. Besides the salary (which by the way is expected to be increased again in the near future) there are *no* tuition fees.⁽⁴⁰⁾

When you prefer, you can live of course in a suburb — like I do, for example at the North Sea in Zandvoort, about 30 minutes by train from Amsterdam. Rent and taxes together will amount to between 1000 and 2000 guilders; both are substantially lower in suburbs than in town.

My colleagues De Vries (Vossiusstraat 39) or better even Mannoury (Koninginneweg 192), who has four children of school going age, will be able to inform you precisely about schools; just ask them specific questions. The schools in town are excellent, of those in the suburbs I have heard less praise, but also there they are certainly bearable. In Amsterdam there is even a German school, but I don't know anything about its quality.

As far as your official language is concerned, you have automatically permission to teach for two years in a language other than Dutch; this permission will then be extended when needed, I believe one year at a time. Ehrenfest lectured already in Dutch the second year he was in Holland. For Denjoy it will be already the fourth year that he lectures in French in Utrecht.

At the end of the week I'll be in Holland again, and from there I'll come to Nauheim.⁽⁴¹⁾ Please write the rest to me in Laren. Our lectures start again on the first of October.

Please recommend me to your wife and accept with my wife's greetings a warm handshake from

Your
Egbertus Brouwer

[Signed autograph – in Weyl]

⁽⁴⁰⁾I.e. money paid directly by the students to teachers. ⁽⁴¹⁾Where the 1920 Naturforscherversammlung was to be held.

1920-10-04

A. Denjoy to O. Blumenthal — 4.X.1920

Utrecht ⁽⁴²⁾
Stationsstraat 12^{bis}

Sir, [Monsieur]

Back from a holiday and from the Congrès International des Mathématiciens held in Strasbourg from 22 to 30 September, I find at my return in Utrecht a postcard written in pencil, which you were so kind to address to me from Mr. Brouwer's house.

Despite the warm memories I have kept ever since our meeting in Rome, I do not believe this is the time to renew our personal relations.

As long as the governments that belong to the League of Nations have not arrived at an unanimous decision about the admission of Germany, mathematicians from my country will keep, I believe, their reservations with respect to colleagues from yours.

The visible reasons for a renewal of the conflict between our two countries are from gone. One must have seen the devastation of certain regions in the North and North-East of France and measured the amount of work and expenses necessary for rebuilding to realize that the people of France, heavily reduced as they are in their means of production, will not consent in assuming that task alone, and in exonerating yesterday's enemies, less tested than they are.

In case Germany would rise to escape her obligations and France would have to resort to force in order to submit her, the initiative, taken already by a French scholar, to ignore prematurely all reservations with regard to a German colleague, that position, taken in an offhand manner, would be regarded as thoughtless and irresponsible.

I am in no doubt that such is the opinion of all of us French mathematicians. If it is seen with disapproval across the border, that is of no concern to us, if only because the war has strengthened us in our firm belief in our better judgment, despite the low esteem in which it used to be held in the old days.

More often than not the past six years have proved us right. It is not the French way of thinking that did not stand the test of the facts. Accordingly, we will continue to give it credit, even if it results in contempt again: costly as the effects of it are for us, they cost others even more.

⁽⁴²⁾This letter has been reproduced in *Brouwer's to the Minister of Education 27.IX.1922*.

I conclude. The day the French government will believe to have received proofs of good will on behalf of your government and will judge them to be sufficient as a justification for the restituting to Germany the rank of an ordinary nation; that day, too, I will no longer see any fundamental obstacles to engage in, or renew, relations with those of my German colleagues at any rate who will not have provoked, by their resounding manifestations, the personal feelings of resentment of scholars belonging to the Entente.

Meanwhile, I obey the orders dictated by the attitude of the government of my country.

Yours sincerely, ⁽⁴³⁾

A. Denjoy

[Typewritten signed original – in Brouwer]

1920-10-17

To A. Denjoy — 17.X.1920 ⁽⁴⁴⁾

Laren

Sir, dear colleague, [Monsieur et cher collègue]

Our colleague Mr Blumenthal has shared with me your letter of 4 October, by which you thought fit to answer a postcard he had sent you whilst staying with me.

I have no doubt that you realize the consequences of that incident for our personal relations: the laws of hospitality oblige me to see the attitudes taken towards one of my guests as engaging me personally. Allow me to tell you that my opinions on the political responsibility of scholars (especially of us, members of the academy of a neutral country) are diametrically opposed to yours.

Rest assured, Sir, dear colleague, of my due respect. ⁽⁴⁵⁾

(signed) L.E.J. Brouwer.

[typescript copy – in Brouwer]

⁽⁴³⁾ *Je vous prie d'agréer, Monsieur, l'assurance de ma considération distinguée.*

⁽⁴⁴⁾ Reproduced in the letter from Brouwer to the minister of education 27.IX.1922.

⁽⁴⁵⁾ *Soyez assuré, Monsieur et cher collègue, que je vous portie la grande estime qui vous est due.*

1920-10-20

From A. Denjoy — 20.X.1920

Utrecht ⁽⁴⁶⁾
Stationsstraat 12^{bis}

Sir and dear colleague, [Monsieur et cher collègue]

I take pleasure in acknowledging the receipt of your letter of 17 October. I deeply deplore its conclusions, if not, the very natural course of action as regards Mr Blumenthal, you claim to be the reason.

First of all I have my reservations on whether it is right for a host to take remarks addressed to someone staying under his roof, as being directed against himself.

Undoubtedly you would not allow me to take offence if it is your pleasure to welcome at your home people I cannot possibly meet. So why would you be offended if I decline the offer of a conversation on behalf of one of your guests?

Recently you went to Nauheim to attend the Conference organized by the Germans, at the same time as the Strasburg Conference, the scientific interest of which presumably was not less great than that of the first. If you had written me from Nauheim I would have replied with my customary cordiality but the Germans would have been gravely mistaken if they had believed that my sympathies towards their guest was also extended towards his hosts.

If, in due time, when the suspicions towards Germany will be lifted officially, you wish to do your best to bring together scholars that were former enemies, your actions would be seen with great interest. But take my cordial advice and believe me when I predict that it will be more effectual if you take more and better notice of what is going on in the scientific circles of the Entente and if you also heed more carefully the indispensable precept not to impose to friends of one camp your sympathies for those of the other.

The French do not like orders — neither to give nor to receive them. For four years we have received orders of a different force but we preferred not to listen. Eventually, on the contrary, when our orders were finally heard, nobody failed to obey.

Your letter contains a phrase which I cannot help being somewhat disturbed by, namely, on the ‘political duty of us, members of academies of the neutral countries.’

⁽⁴⁶⁾Reproduced in Brouwer’s letter to the minister of education 27.IX.1922

If last April there had been among the mathematicians staying in Holland, anyone deserving better than I do your votes and those of your colleagues, nothing should have prevented you from electing him. If on the other hand I seemed to be the least unworthy candidate for being one of yours, I do not believe your appeal asked anything from me beyond accepting it.

Accordingly, I do not believe to be ungrateful for the honor that has been done to me, if I acknowledge no other duty towards the Academy than to contribute to her works with all my efforts.

I am convinced that if the Paris Académie des sciences had you as a correspondent or foreign associate, she would not ask more than that from you.

If the post of member of a Dutch Academy would involve obligations of a different kind, especially of a political nature, I would be weak enough to have no hesitation between a foreign title, however honorific it is, and my being a simple French citizen, which does not lend itself to being compromised.

In the same domain there is one rule, though, to which I would believe to be bound. If a section of the Academy would address a letter of rebuke to a German or Austrian learned society, elementary tact would prevent me from lending my French name to a manifestation that could damage the reputation of Dutch science beyond your borders.

You have my approval if you believe that the *neutral* members of the neutral academies can be useful in bringing together scholars from different European countries, provided that for the time being you limit yourself to establish and maintain contact between their *works*. Some weeks ago you asked me, on behalf of some Germans whose name I do not know and about whom I do not care, some of my latest articles. I have never ceased to be willing to send you copies of those as long as I have enough reprints. I have no objection whatsoever to contributing in that way to lessen the lack of publications the Germans complain about.

But it would be counterproductive to strive, prematurely, for the reunion of the authors themselves.

Public opinion attributes to scholars, more than to the majority of other individuals, like businessmen for example, a kind of national character, which must make scholars very cautious when it comes to lending their personality for informal contacts, which could be criticized by sensible patriots.

Towards Germany, grooved by ambivalent tendencies, France feels, among other things, like a self-conscious neutral party confronted with a conflict in which others are engaged but in which she could be dragged along.

For more than four years France has resisted under difficult conditions. For a few months more we can have the patience

to tolerate a government that I wish will make the transition from a state of war to a state of peace.

There should be no doubt that the feelings expressed in this letter do not diminish the great esteem in which other mathematicians hold you and in which I join without any reservation. Believe me to be, Sir, dear colleague, your devoted,

Sincerely yours, ⁽⁴⁷⁾

A. Denjoy

[Signed typescript – in Brouwer]

1920-10-27

To A. Denjoy — 27.X.1920

Amsterdam ⁽⁴⁸⁾

Sir, dear colleague [Monsieur et cher collègue]

Thank you for your letter of 20 October. You will no doubt agree with me, if I don't see the usefulness to continue a discussion on hospitality (neither on the consequences of my own extended to Blumenthal, nor on the acceptability of the hospitality extended to me by Strasbourg for reasons of the accident of the place of my birth), if I of course do not dream of interfering with your political views as French citizen; and finally if, that in case you are interested in the way I think of the tribute we scholars ought to pay to opinion (whether it be in the country of our birth or in that in which we are active or indeed in the world at large), I limit myself to sending you the official report of the session of our Academy of 31 October 1914 (p. 828).

As regards the way you see the role of an ordinary member (the question is not about correspondents or foreign members) of the Amsterdam Royal Academy (especially with respect to art. 2, sub c.,² of the Rules of the

²It is because of this article, that M. Blumenthal, citizen of a country that has friendly relations with the government of The Netherlands, quite naturally turned to you, an ordinary member of the Academy of Sciences of The Netherlands, when there was reason to talk to you about certain scientific matters.

(signed) L.E.J. Brouwer

⁽⁴⁷⁾ *Croyez-moi, Monsieur et cher collègue, votre tout dévoué.* ⁽⁴⁸⁾ Reproduced in the letter from Brouwer to the minister of education 27.IX.1922.

Academy), I must admit my surprise, but, all things being considered, in the present situation this is a matter of concern only to you and the Dutch government.

Needless to say, my dear sir and colleague, on one hand that I infinitely regret the circumstances that remove me from a man of your worth; on the other hand that those circumstances do not diminish the feelings of respect I have for you.

(signed) L.E.J. Brouwer

[Typescript, copy – in Brouwer]

1920-10-29

From A. Denjoy — 29.X.1920

Utrecht ⁽⁴⁹⁾
Stationsstraat 12^{bis}

Mr. A. Denjoy ⁽⁵⁰⁾

Utrecht

Dear Sir [Den Heer A. Denjoy]

I do not wish to keep the letter below, which I am really not able to take cognizance of.

I urgently request you to direct no further letters to me.

In the mean time, please be assured of my sincere respect.

L.E.J. Brouwer.

Sir, dear Colleague, [Monsieur et cher Collègue]

I am not going consult article 2 sub c of the rules of the Academy. ⁽⁵¹⁾ I would be surprised if the statutes of this society allowed her ordinary members to break off even their epistolary relations and obliged them to open the doors of their apartments to any visitor who has nothing to do with that Academy.

‘In the present situation’, you say, ‘my way of seeing the role of an ordinary member (I had only hypothetically assimilated you to a correspondent or foreign member of the Paris Académie des Sciences, given the fact that

⁽⁴⁹⁾The original letter was returned to Denjoy; it is likely that the notes on this typed duplicate were made known to Denjoy. ⁽⁵⁰⁾Note on top of page in Brouwer’s handwriting.

⁽⁵¹⁾KNAW

only French citizens can be ordinary members), ‘is a matter of concern only to the Dutch government’ and myself.³

I will not scrutinize the mysterious meaning of those sybilline words. I am perfectly tranquil. The Dutch government will not expel from her academy a Frenchman to punish him for having declined for the time being the invitation of a German to meet him.

Your government will have no wish to please, by an incident of this nature, the enemies of the good relations between our two countries.

As far as I am concerned, I will scrupulously avoid widening our differences as long as no qualified person interferes.

French public opinion — which is all that matters to me, my letter to Mr Blumenthal has clarified that point, French public opinion is already not too well disposed towards Holland. It is felt too clearly that certain people here would have seen the disappearance of France and her civilisation as a minor accident. They would not deplore it if the world had become German. The aggression of 1914, four years of German crimes, on land as well as on sea — all that would be no more than peccadilloes. It is in nobody’s interest to confirm the belief of my compatriots — which for that matter is not exact — that all Dutch people think this way.

Your letter shows me that you acknowledge only a vague attachment to Holland, created by the accident of place of birth. Are you not exaggerating your indifference? If the Belgians or the English just had been invading your country, and had pillaged and destroyed the wealthiest region, from Rotterdam to Amsterdam, killed 300,000 young men, maybe you would have felt enough aversion towards the aggressor to make you feel Dutch.

Your obligations towards what happens to be the place of your birth do not allow you to visit a conference at Strasburg but they do allow you to visit one at Nauheim. I would have understood if you declined both the German and the French invitation. Your duties towards Holland entail rigors and accommodations that strike me as strange.

Except for the kinds of countries you mentioned, one recognizes also the country of affinity, a category the existence of which you will find difficult to challenge.

³[Brouwer’s note in margin; in the pamphlet it is a footnote:] ‘This quotation is incorrect and must *perhaps* explain the unreadable sequel. If you really think that the Dutch Government has nothing to do with the manner in which Rules established by the government are interpreted by an official involved, then I don’t want to quarrel with you about that either. What I meant to write was not more than that *except* the official involved and the Government, *certainly no third party* needs to bother with it, and apparently you agree with that.

But above all you forget the country of nationality. To belong to a nation implies charges but also advantages. Any man should see it as an honor — and for any man it is also wise — to be attached to a people under all circumstances.

I dare to congratulate myself for being able to reunite in one country those of nationality, of affinity and of birth.

It is no coincidence that my origins are in Gascogne. Given the fact that for many generations all my ancestors have been living in that corner of France, it would have been against all the odds had I been born elsewhere.

I have no hesitations in feeling myself a member, and a very humble member, of one family, together with all those who have made my language, incomparably superior to any other because of its rigor, its precision, its immaterial and energetic vigor. That language is perfectly apt to give expression to certain spiritual meanings I see in myself [het Frans begrijp ik strikt genomen niet — er moet een transcriptiefout gemaakt zijn]. And it does not easily lend itself to translate confused mental dispositions that my nature dislikes but in which many a foreign soul finds pleasure.

I can recognize myself in the aversion of French intellect from vainglory from charlatanry and from appeals to superficial curiosity.

Among the dominating traits that are most characteristic for the French people is that I quite enjoy to rebel with all that is in me against characters opposed to mine.

I know of no people with a greater inclination to criticize themselves and greater aversion from admiring themselves.

There are no others on whom arguments of noblesse have more effect and contemptible reasons less impact. They are not like those to go to war hoping to come back rich.

All these affinities determine my impression that I am not a Frenchman by accident.

Your respect touches me, but I have never asked for it. Less respect for me, and less antipathy for my country would be more to my satisfaction

Sincerely yours ⁽⁵²⁾

(signed) A. Denjoy

[Typescript copy with notes signed by Brouwer – in Brouwer]

⁽⁵²⁾ *Veuillez agréer, Monsieur et cher collègue, l'assurance de mes sentiment dévoués.*

1921-01-01

To H. Weyl — 1.I.1921

Laren ⁽⁵³⁾

Dear Weyl, [Lieber Weyl]

Many thanks for your letter, which just arrived.

When a while ago your telegram came, my disappointment was, to my own surprise, very great; and from that it became clear to me how much I would have liked to have you here. I was already prepared for this negative result, and when no message was forthcoming for such a long time, I had, as I thought, completely reconciled myself with it; the effect of the definitive message shows me that I succeeded only quite imperfectly in this reconciliation. Well, let us hope that you have made the right choice for you and your family, and that the matter will bring all that is beneficial and desirable to your work environment in Zürich.

Perhaps we will succeed in accomplishing that the position intended for you will be offered to another young person with a very outstanding reputation, and as such Bieberbach (whom I would have liked the best) cannot very well be considered, because he is too little known beyond the narrow circle of mathematical professionals, and he also doesn't have a completely unchallenged name as initiator. His prospects for Berlin seem to be fairly good, if however he cannot hold his own here too, he should at least get Leipzig or Hamburg (assuming that Blaschke goes to Berlin; I already wrote to you about that earlier).

For your position, in case the vacancy will be maintained, Birkhoff, who is also known to people in the fields of astronomy and mechanics and who is especially considered a star of the first magnitude, is now the first who comes to mind, and he moreover belongs to a Dutch family (both his parents were born here) and who can perhaps be won over, because in America he isn't yet a full professor.

If it doesn't come to that, and if one returns to the original arrangement of two extraordinary professors or lecturers then I will ask you maybe a few specific questions about Polya. Anyway, I will keep you informed.

What Klein means by a 'reconciliation' between Schouten and me, is not clear to me. For Klein there can be only one relation between Schouten and me, namely that I have rejected papers by Schouten, that were given to me to referee; but Klein knows that this was because of the plagiaristic

⁽⁵³⁾ 'Laren, New Year's Day 1921'.

character of these articles, and not at all on personal grounds. Anyway, recently I haven't seen any publications by Schouten. As nothing is more pleasing than to revise an unfavorable judgment about an author as soon as there is an occasion, I would like to ask you to indicate to me the place in the journal of the 'positive achievement' of Schouten that you mentioned (or rather to let me borrow the journal issue or a reprint for some time); if I then find about the relevant mapping problem a new theorem that hasn't been copied from somewhere (e.g. from Cartan Bull. Soc. Math. 45, p. 57–81), then I would be glad to recognize and appreciate it.

Now dear chap, here's to your health and that of your family for 1921. May the mountains bring you health and vigor. I long to stay there: I don't feel well at all the last few weeks and every day that I don't lie down for a few hours I have a considerable temperature every evening. By itself that is no reason to worry, because I have more often such periods, but if I stay like that for a longer time, then I will request a vacation and then I must go Switzerland for a few weeks. Then we will be able to meet very soon again. Otherwise hopefully in next summer or at the next congress.

Cordial greetings to you and your wife, also from mine, and believe in my faithful friendship

Your
Egbertus Brouwer

[Signed autograph – in Weyl]

1921-01-17b

To A. Schoenflies — 17.I.1921^b

Laren

Confidential

Dear Mr. Schoenflies [Lieber Herr Schoenflies]

No doubt, you will have received these days several recommendations regarding the Bieberbach-vacancy that has arisen with you: may I also for my part direct your attention to a colleague, with whom I am certain you would make an excellent choice? I am thinking of Blumenthal, with whom I have been in contact for roughly a decennium, and whom I have

learnt to appreciate more and whom I have learned to appreciate more and more during this period, and also in more and more aspects. In particular I am convinced that he hardly has an equal among our confreres in all-round mathematical knowledge, in energy for work, in helpfulness and moreover in honesty and decency. The activity in which I have been able to observe him first hand (apart from personal contacts), so that the above mentioned opinion has become firmly rooted in me, is the publication of the *Annalen* (in which I was involved as a silent assistant of the editorial board in 1911 – 1914, and from 1914 on officially as an editor), which before the war was for three quarters in his hands, and similarly from the beginning of 1919 on. By far the larger part of the refereeing was done by him, either alone or together with a specialist engaged by him for this purpose; and if the *Annalen* of Klein and Hilbert have stood their ground in the first ranks of the mathematical journals, then it owes it in first place to the untiring, unselfish and expert work of Blumenthal, and this work must be valued all the more because it requires on the one hand considerable talents, and on the other hand it brings no honor at all, because for the wider public it takes place completely in the shadows. That nonetheless Klein and Hilbert never got Blumenthal a university chair,⁽⁵⁴⁾ I can only explain by the Machiavellian principle ‘le premier devoir des rois, c’est l’ingratitude’,⁽⁵⁵⁾ and in addition Blumenthal’s excessive modesty (he never tried to get a professorship himself) has played a role. How much Blumenthal formed the core of the editorial board of the *Annalen* is shown by the years 1914 – 1918, during which the journal was most dangerously ailing because of Blumenthal’s military service, and it would have certainly succumbed, if, immediately after his return, Blumenthal would not have given all his energy to it, so that the old ‘Standing’ was recovered in a few months.— Furthermore I have been able to observe in the last months that Blumenthal is realizing the injustice done to him, and is starting to become embittered; I believe that his wife feels the injustice even more keenly than he does and she longs to get away from Aachen.

Now, my conclusion is as follows: if you should get Blumenthal to Frankfurt, then an old debt of the mathematical community to Blumenthal will be settled, and on the other hand you would get the headquarters of the *Mathematische Annalen* in Frankfurt and moreover an enthusiastic and very energetic colleague, and whose modesty will moreover preserve in undamaged form the leading position taken by yourself.

⁽⁵⁴⁾ Blumenthal was a professor at the Technische Hochschule (Institute for technology) at Aachen, which did not count as a university. ⁽⁵⁵⁾ The first duty of kings is ingratitude.

In any case, I hope you don't blame me for writing the above to you: I saw it as my duty.

With most cordial greetings from house to house

Your

L.E.J. Brouwer.

[Signed autograph – in Brouwer]

1921-01-31

From R. Weitzenböck — 31.I.1921

Graz

Glacis-Strasse 59

Dear Brouwer, [Lieber Brouwer]

Thank you very much for your letter of the 24th of this month! First of all I want to thank you most cordially for having thought of me and for doing so much work in advance.

In this matter I might then first say the following. In the seventh year of my marital state, I have, in November 1920, become settled halfway, meaning that as full professor ⁽⁵⁶⁾ I have a corresponding position, and I can in the local circumstances live my life with my family and with my work. I don't want to give up this situation without sound reasons, and more in particular I don't want to start more or less at the bottom.

What is a lector at your university? Does it correspond to our Extraordinarius (extraordinary professor)? Are they civil servants with normal retirement rights? For example, when I would become lector and then die after half a year, would my wife then get anything like a pension? The legal position with respect to the board of the university will not cause me any special discomfort. I believe that once I am over there, it will be straightened out in due time.

The teaching duties you indicated (number-forms and theory of invariants) would *suit me very well*, and would delight me. How many hours per week would be considered there? Of course I would commit myself to lecture in Dutch after at most two years. Also I would, in case the matter is

⁽⁵⁶⁾ *Ordinarius*.

arranged, apply for the Dutch nationality. My brother wants, as far as I know, now to do likewise.

Now about the question of money. Can one live in your country with 6000 fl⁽⁵⁷⁾ in roughly the following manner: (with family) three or four rooms and the rest – in particular a bathroom, in a modest neighborhood or outside of town, a housemaid, every day enough to eat, but otherwise no special demands for clothes and amusement. Maybe once a year a trip to here? For me two things matter most: in the first place to be with my family free of all the petty material worries that one has here every day and every hour – roughly to the degree of the way we lived before the war. And in the second place to be able to dedicate myself in peace and quiet to science and finally get around to intensive work.

If you believe that the sofar mentioned considerations in their two main points can be arranged, then I am happy to agree with your propositions, and will come to you. Let me repeat the two main points: the first one is the status of a lecturer (I am yielding secure ground here!) and the second concerns the material aspects.

In this letter I include a curriculum vitae and a publication list for the case that you think you can pursue the matter further for me, and that you need data about me.

Of course there are many other questions to be settled, as you mention with justification. More specifically I raise two matters, namely the question whether housing situation is also so bad in your country and the matter of moving. The latter must of course be payed for me, I could not afford this myself.

Please inform me if you know more about these things. Especially when you are in the position to tell me something about the point of view of the faculty and the board of curators. Could it not be to the advantage of your university to point out that Delft has created so many new positions for mathematicians, and thus make a fourth professor possible? I want on no account to view your country as a so-called ‘milk cow’. But with us it is, as in Germany, now a custom that the rich foreign countries buy us up. And then I am thinking in the first place of that what is most precious, namely human material. I believe I have told you already in Nauheim⁽⁵⁸⁾ that my brother has during his vacation in Europe recruited engineers and building technicians for your colonies, or that he has recommended them to The Hague. As I have heard, some 30 of these have been hired until now.

⁽⁵⁷⁾ *florijnen* – i.e. Dutch guilders. ⁽⁵⁸⁾ The Nauheim conference of 1920, cf. [Van Dalen 1999].

I have witnessed the thing here, at least in part, and I can assure you that your government really got good people.

My wife and I would be *very glad*, if you would pay a visit over Easter to our beautiful Styria and we could have you with us for some time.

Best greetings and recommendations from house to house

Your ⁽⁵⁹⁾

R. Weitzenböck

[Signed autograph – in GAA]

1921-02-01a

From H. Kneser — 1.II.1921^a

Göttingen

Annistr. 2 II

Dear Professor [Hochgeehrter Herr Professor]

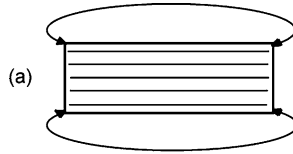
Departing from a problem of differential geometry ‘in the large’, I had conceived some time ago the idea of examining the topologically different types of families of curves ⁽⁶⁰⁾ on closed surfaces. In this context I consider the family of curves given by an everywhere regular differential equation satisfying the Lipschitz condition (but other assumptions that are invariant under topological mappings would also suffice). The results are the following:

1) The surface is a one or two-sided annular surface.

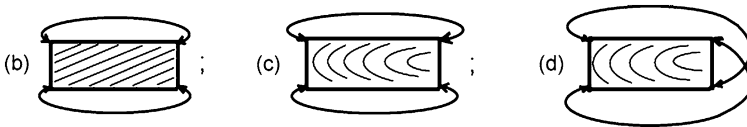
2) If the family of curves doesn’t contain a closed curve, the annular surface is two-sided and it can be mapped one to one and continuously onto the square $0 \leq x \leq 1, 0 \leq y \leq 1$ with identification of opposite points $(0, y$ and $1, y; x, 0$ and $x, 1)$, in such a way that each line $y = \gamma x + c$ with fixed irrational γ and arbitrary c corresponds to a curve from the family, up to at most countably many lines, that each correspond to a complete band of curves from the family.

3) One obtains all other types by joining bands that are bounded by closed curves of the family, whose types I might describe by figures:

⁽⁵⁹⁾ *Ihr ergebenster.* ⁽⁶⁰⁾ *Kurvenscharen*, see [Kneser 1921].



(only closed curves);



If one joins finitely or countably many bands (a) and (b) together, finitely many bands (c) and one or two bands (d) to form a closed surface, then one obtains all possible types.

Theorem 1 is hardly new; for the others I have found in the literature only Bohl, *Acta Mathematica* 40. ⁽⁶¹⁾ In particular your articles about vector fields seemed to pursue different aims.

However, now I would like to investigate whether an extension to more dimensions is feasible. Here the question about the behavior of a topological mapping of a surface onto itself in case of unbounded iteration, becomes important, especially the question about the properties of such maps that don't have a fixed point for any iteration. Hence I must in any case study your articles about mappings of surfaces in the *Amsterdam Verslagen* ⁽⁶²⁾ and I would be very grateful to you if you would send them to me, and actually I would like to use the Dutch edition rather than the English one.

Sincerely yours ⁽⁶³⁾

Hellmuth Kneser

From 1.III to 1.V my address is: Breslau 16, Hohenlostrasse 11.

[Signed autograph – in Brouwer]

⁽⁶¹⁾[Bohl 1916]. ⁽⁶²⁾Proceedings KNAW. ⁽⁶³⁾*Mit den besten Empfehlungen, Ihr ganz ergebener*

1921-02-14

From A. Schoenflies — 14.II.1921

Frankfurt am Main

Grillparzerstr. 59

Confidential

Dear Mr. Brouwer [Lieber Herr Brouwer]

I have shown your letter on Blumenthal to both Bieberbach and Hellinger. Our staffing is very, very difficult. A full replacement for a versatile character as Bieberbach⁽⁶⁴⁾ is not at all available. But we want in the first place a man who exerts a great scientific stimulus on students and PhD students, which was the main strength of Bieberbach, and which hardly anybody can emulate. This does not merely include command, even comprehensive command of the subjects, but also an agility to take up and formulate problems, which is a characteristic of Bieberbach.

In the first place we think of Lichtenstein and Polya. I am afraid that the government won't even approach Lichtenstein, who is just now going to Münster. In the case of Polya there is perhaps a personal obstacle. If neither Lichtenstein nor Polya are nominated, then we are so to speak desperate. We have thought of Radon and Rosenthal, and also we seriously consider the name Blumenthal, however as a last resort. But I myself will also leave soon: according to the law I will go into retirement on October 1. In my opinion Blumenthal would then be a very good replacement. But we cannot completely ignore Hellinger and Szász either— you see, the situation is complicated in every respect.

On this occasion I would like to allow myself a wish for Amsterdam. Hanna, who was here last Sunday — I gave a Rector's ball on Saturday and I had invited her —, told me that after Weyl declined your offer, you no longer have a position for a full professor, but that you think of establishing two extraordinary professorships. If that is so, and if you haven't made your choice yet, then I would like to recommend Szász most warmly. For, he is in the first place an arithmetician and a number theorist, and that's what is still lacking you in Amsterdam. He would also constitute an excellent and at the same time necessary completion of your mathematical circle. He is capable, has many interests and as far as I can judge, he is also a good teacher. We find him here very pleasant, and he is also someone whom we would miss if he weren't there. So you may wonder why I recommend him so warmly.

⁽⁶⁴⁾Bieberbach was appointed in Berlin.

It is only because of the uncertainty of the situation here. I don't know if it would be possible to give him a promotion here. That would in any case only be possible by calling attention to him for the arithmetical completion of the completeness program. ⁽⁶⁵⁾ And exactly for that reason I mention him out to you too. He is a completely honorable decent personality. As you know, he has published much, though his papers don't always go so deep.

That is what I would like to give to you in consideration.—

With cordial greetings from house to house

Your

A. Schoenflies

[In the margin:]— Hanna has told us the most wonderful stories about the stay at your home; please accept our thanks for all the lovely hospitality!

[Signed autograph – in Brouwer]

1921-04-10

From A. Fraenkel — 10.IV.1921

Amsterdam

Dear Professor, [Hochgeehrter Herr Professor]

Returning from a trip to London to visit the siblings of my wife, I found here, just now, your kind lines from Italy. I hope that the article from Crelle ⁽⁶⁶⁾ that I sent to you in the beginning of March has got or will, in spite of your absence, get into your hands. Because towards April 20 we want to return to Marburg, we will postpone our visit to you to another time.

Yet, I should like to use the opportunity to make a few remarks about the treatise by Schoenflies 'Zur Axiomatik der Mengenlehre' ⁽⁶⁷⁾ that was communicated by you to the Amsterdam Academy. I have addressed these remarks already about a quarter of a year ago to Schoenflies himself. The heart of Schoenflies's article is his treatment of the problems of comparability (p. 794 and p. 808). In more detail: first of all it should be remarked that the proof for the possibility $(dd) = (a)$ on p. 808 has gaps, because the

⁽⁶⁵⁾Probably connected with Szasz' research in the area 'Completeness and Closure'; he published on completeness of function systems ⁽⁶⁶⁾[Fraenkel 1921]. ⁽⁶⁷⁾On the axiomatics of set theory. [Schoenflies 1920].

system treated there is not only in contradiction with axiom I on p. 803, but also with the very fundamental axiom II on p. 787. Incidentally, this second contradiction can be easily remedied by changing the system treated by Schoenflies into another one. But apart from these single gaps: the alternative ‘either $(dd) = d$ or $(dd) = a''$ ’, from which the comparability of sets follows, i.e. according to Hartogs, eventually the axiom of choice, seems to me a totally arbitrary requirement that goes beyond the principle of choice. If one wants to go beyond Zermelo, then one should allow both possibilities without excluding one axiomatically, and then one will doubtlessly discover that one cannot do without the axiom of choice.

Allow me to venture my opinion beyond this main point of the Schoenflies article, to the effect that an axiomatic foundation with such an extensive axiom system (and moreover with a relatively large number of undefined fundamental concepts) has only little value, when the independence proof is not provided. With Zermelo this is completely different, because with his very few axioms it is *evident* that they are *more or less* independent. But the axiom system of Schoenflies will show itself without any doubt to a large extent reducible when the independence is tested, much to his advantage; the comparison with the axiom systems of geometry fails, because set theory is much less complicated than geometry (and must therefore restrict itself also to very few fundamental notions).

My interest in this matter was stimulated by an investigation completed in the beginning of this year,⁽⁶⁸⁾ in which I axiomatically developed the theory of cardinal numbers and cardinalities on the basis of ten axioms, whose independence I completely proved. Meanwhile I have also looked more closely into the independence of Zermelo’s axiom system. Unfortunately the work on Gauss’ algebra⁽⁶⁹⁾ doesn’t leave me as much time for my own research as I would like.

In case this letter is forwarded notwithstanding my counter-indication, then I wish you and your honored spouse a really pleasant further journey. With the best greetings of my wife and me I sign

yours truly⁽⁷⁰⁾

A. Fraenkel — Marburg

[Signed autograph – in Brouwer]

⁽⁶⁸⁾[Fraenkel 1922]. ⁽⁶⁹⁾[Fraenkel 1920]. ⁽⁷⁰⁾*Ihr ganz ergebener.*

1921-04-11**To R.C. Mauve — 11.IV.1921****Firenze**

Greetings from he Piazza Michelangiolo, where twenty years ago I have so often eaten with you both. Since that time nothing has changed here; maybe not even a single new house has been built; there are only new nameplates under many of the most important paintings; there must be a new Director of Fine Arts; for Da Vinci and Michelangelo there is almost nothing but ‘già attribuito’;⁽⁷¹⁾ in your house lives a Dottore Medico Chirurgo; on the little omnibus of the Porta Romana you still pay the old Lumps[?]; that meanwhile the prudent Italians have won a war is a scream; it’s swarming here with Dutchmen; besides many colleagues, I met here in the streets within one week Eisenloeffel, De Winter and Spigt.⁽⁷²⁾

Bye!

Brouwer and Lize Brouwer

[Signed autograph, postcard (picture) – in Collection v.d. Noort]]

1921-11-27**To Algemeen Handelsblad**⁽⁷³⁾ — **27.XI.1921****Laren**⁽⁷⁴⁾

As a supplement and for clarification of the fragmentary report in your Evening Edition of 26 this month⁽⁷⁵⁾ of an incident occurring during a session in the meeting of the Academy, the undersigned would like to remark the following:

The minutes⁽⁷⁶⁾ of the Ordinary Meeting of October 29, 1921 will inform the international readership of the Works of the Academy, that on that date the Ordinary Members of the Division of Mathematics and Physics, convening publicly, on the instruction of the Dutch Government and at the expense of the state, have deviated from using the official language of the

⁽⁷¹⁾Formerly attributed. ⁽⁷²⁾J. Eisenloeffel (1876 – 1957) silversmith; A.J.J. (Janus) de Winter (1882 – 1951), painter. ⁽⁷³⁾A newspaper based in Amsterdam. ⁽⁷⁴⁾Published as a ‘letter to the editor’ in *Algemeen Handelsblad* 27.XI.1921 ⁽⁷⁵⁾See below. ⁽⁷⁶⁾in Dutch ‘*zittingsverslag*’.

Kingdom of the Netherlands, ⁽⁷⁷⁾ The undersigned, wishing to be relieved, before the mentioned readership, of being partly responsible for this act, sent the Meeting the motivated notice of absence, which was printed in your Evening Edition of October 29, and mentioned again in the Evening Edition of the 26th of this month; and pointed out to the Secretary in an enclosed letter, that this notice was meant as a public protest, and intended for the Minutes of the Session.

When it appeared yesterday at the reading of the minutes that the notice of absence had not been presented by the Board to the Meeting, and that the inclusion in the Session Proceedings of October 29 as intended by the undersigned would not be realized, the undersigned requested to be allowed to read the motives of his absence during the last meeting to the meeting, in order to effectuate the release of the co-responsibility, as requested by him at least by means minutes of the Session of November 26.

The refusal of this request, which implies a violation of elementary minority rights of the undersigned, will have as a consequence that the readers of the Works of the Academy will get the incorrect impression that the national character of the Academy can be disregarded without serious objection from its midst.

L.E.J. Brouwer.

[Typescript, copy – in Brouwer; handwritten draft also in Brouwer]

Editorial supplement 1

[*Handwritten private note of Brouwer, 26.XI.1921, written on the backside of the envelope of a letter to Hk. de Vries*]

Jaeger, Winkler, Eykman, Magnus, v.d. Hok, v. Everdingen, Lorentz, Julius, Jaeger, Haga

voted against incorporating my protest in the session proceedings ⁽⁷⁸⁾

In favor only H. de Vries and myself

⁽⁷⁷⁾ Denjoy lectured in French. The lecture was announced in the convocation of the KNAW, *Bolk to Members KNAW, 24.X.1921* as ‘De Heer Denjoy zal een mededeeling doen, getiteld ‘*Recherches récentes sur les séries trigonométriques*.’ [Mr. Denjoy will present a communication, entitled: Recent investigations on trigonometric series] ⁽⁷⁸⁾ of a KNAW meeting of 24.IX.1921.

Board Members Went and Bolk did not express themselves in voting (however, the former had declared earlier to be in favor, the latter against)

Editorial supplement 2

[*Report in the Algemeen Handelsblad of 26.XI.1921 in the section Science (Wetenschap)*]

Royal Academy of Sciences

In the meeting today of the section of the Academy of mathematical and physical sciences, prof. Brouwer asked, after the minutes were read, that a letter he sent to the Section would be read.

The chairman prof. Went said the this letter was dealt with in the extraordinary meeting.

Prof. Brouwer remarked that the letter was directed to the ordinary meeting, and he maintained his request to read, appealing to the meeting.

Only one member supported prof. Brouwer, and after this he asked to be allowed to make a statement.

Secretary prof. Bolk pointed out that prof. Brouwer's desire already had been satisfied, because he had published the letter in the 'Handelsblad'.

Prof. Brouwer then asked that in the minutes and in the proceedings of the ordinary meeting it should be noted that he was refused to read a note to the meeting.

1922-00-00

L.E.J. Brouwer, Note on Weitzenböck

1922 ⁽⁷⁹⁾

R. Weitzenböck

Weitzenböck wrote in 1908 at the age of 23 years as an officer the book *Complex symbolism, an introduction to the analytical geometry of multidimensional spaces*,⁽⁸⁰⁾ which appeared in the Schubert Collection; a book

⁽⁷⁹⁾1922, or later. Probably part of the appointment procedure. ⁽⁸⁰⁾*Komplexsymbolik, eine Einführung in die analytische Geometrie mehrdimensionaler Räume.*

which is partly a compilation, but which also contains many new things and which has remained both in set-up and subject unique in its kind, even though unfortunately some minor errors occur in it. With this book Weitzenböck's transition from a military career to a mathematical career became visible, even though he remained active as an officer in the Austrian army for several years afterwards. Between 1908 and 1912 about ten articles follow in the tracks of this book.

Since 1912 he occupied himself mainly with the problems originating from Klein and Study, of finding complete systems of invariants for figures of several classical transformation groups (more in particular the projective and affine groups and the group of motions). All these problems, which had withstood the efforts of Klein and Study (except for the simplest cases) were completely conquered by Weitzenböck in a series of about 25 articles (which mostly appeared in the *Wiener Berichte*), and not only for the projective and affine groups and the group of motions, but also for the Galilei-Newton-group (the group of classical mechanics). The basis of this series of investigations is formed by the so-called fundamental theorems of the symbolic method, which have been given their definitive form by Weitzenböck, and which reduce the invariants of an arbitrary system of algebraic figures to the ones of a certain *linear* system of figures, and which moreover enumerate in the first place all possible types and in the second place all possible rational relations between these types of the above mentioned invariants. Weitzenböck has done this work between 1913 and 1919, where one must keep in mind that he was in active service,⁽⁸¹⁾ almost until the end of the war.

In recent years Weitzenböck has extended his invariant theoretic methods to differential invariants, and he has been able to give among other things quite a number of applications to the theory of general relativity. More in particular he has enumerated all possible mutually independent simultaneous second order differential invariants of a tensor of the first and of a tensor of the second rank (in four-dimensional space); there are only six of these; thereby he has made an important contribution to the questions about the Hamiltonian in general relativity.

At the moment Weitzenböck is indeed the foremost authority on invariants in the world. His article in the Enzyklopädie *Neuere Arbeiten der algebraischen Invariantentheorie. Differentialinvarianten*⁽⁸²⁾ and his book *Invariantentheorie* (Groningen, Noordhoff, 1922) must also be mentioned.

⁽⁸¹⁾in German 'im Felde'. ⁽⁸²⁾New publications on the Algebraic Theory of Invariants. Differential Invariants.

Furthermore Weitzenböck has shown to be an excellent academic teacher, and in daily contact he is a man of rare simplicity, sociable, honest and co-operative.

LEJB

[Initialled autograph – in Brouwer]

1922-04-21

From T. Ehrenfest-Afanassjewa — 21.IV.1922

Leiden

Dear Mr. Brouwer! [Sehr geehrter Herr Brouwer]

Many months ago I have written you a begging-letter, and I have not received any answer at all. It concerned your articles for professor *B. Kagan in Odessa*. *However difficult it is for me*, I must repeat my request, because Kagan is a person who really deserves that one does something for him. Now he is in the greatest misery — hunger and lack of even the most primitive things in clothing — and yet the first things he begs for are — books, necessary for the continuation of his scientific work. He manages not only to work very hard himself, but also to interest people around him in scientific work.

In the latest letters he explicitly asks for your articles about the foundations of mathematics.

Would you please for once be so kind as to either send me the things, so that I can pass them on (nowadays it is very easy by mail), or tell me that you cannot or will not do it. Then I will try to buy them for Kagan. Until now I didn't do that because I must save as much as possible for shipments of food for our Russian friends and relatives.

With best greetings, also for your wife

T. Ehrenfest-Afanassjewa

[Signed autograph – in Brouwer]

1922-04-26

To T. Ehrenfest-Afanassjewa — 26.IV.1922

Bad Harzburg

Krodotal 4

Dear Mrs. Ehrenfest [Verehrte Frau Ehrenfest]

Please excuse me for not having answered your previous letter; since the Dutch Academy of Sciences is acting as n -th pair of oxes to draw the loot wagons of the Parisian Shylock gang (and has its members that don't agree with this humiliation without any protest, scolded by the Shylock lackeys) I have lapsed into such a state of disillusion and apathy that most of the incoming letters remain unanswered. This may explain and excuse that I have let it come to a reminder from you. But that you imagine the possibility that I possibly don't *want* to concede to your plea, and that you might request a confirmation of this, which would compel you to *buy* my articles (are reprints then for sale at all?), adds to the numerous incomprehensibilities that nowadays pour down on me.

Perhaps, however, you didn't mean it literally this way, and did you expect just my wholehearted promise, which I now make, that I will deal with the subject of your letter immediately after my return home (in so far as no overly paralyzing events or situations await me at home).

With best greetings from house to house

Your L.E.J. Brouwer

[Signed autograph, draft – in Brouwer]

1922-10-10a

From A. Dresden — 10.X.1922^a

Madison (Wisconsin)

University of Wisconsin, Madison

Department of Mathematics

2114 Vilas Street

Dear Professor Brouwer, [Waarde Professor Brouwer]

I am planning to write an article for the *Bulletin of the American Mathematical Society*⁽⁸³⁾ in connection with Weyl, *Mathematische Zeitschrift*; Hilbert, *Hamburger Abhandlungen* etc.⁽⁸⁴⁾

⁽⁸³⁾[Dresden 1924]. ⁽⁸⁴⁾[Weyl 1921], [Hilbert 1922].

The main point will be to explain in more detail your criticism of the logical foundations. Among my colleagues there is quite some interest in this, but it isn't easy for them to get a clear idea about it. If I want to clarify it, I have in the first place to be certain that I understand it. So I take the liberty to ask you for clarification concerning a few points in your 'Foundation of set theory', ⁽⁸⁵⁾ part 1.

On p. 3 you define the concept 'element of a spread' ⁽⁸⁶⁾ as a 'sequence of signs'. Why is then in the case of finite groups of signs or of sequences of type ζ , ⁽⁸⁷⁾ only the single sign an element?

Why do you speak about 'digit complexes' ⁽⁸⁸⁾? Does that mean only 'group of digits, among which the 0 may occur'?

On p. 4 it says that the 'spreads' are a special kind of 'species of the first order' ⁽⁸⁹⁾ — but then they are properties, not laws. Is the idea that the 'spreads' are a kind of 'elements of species of the first order' ⁽⁹⁰⁾?

Could you give a few examples of the 'species'.

What is your view on the Kronecker program of arithmetization; and do I understand your view correctly, when I say that an indirect proof is only permitted when one first has *proved* that of the two cases between which the proof has to decide, at least one occurs?

And what is the meaning of the n on line 7, p. 80 of the JDMV, V. 23? ⁽⁹¹⁾

What would you think of a translation into English of 'The unreliability of the logical principles' in the Tijdschrift voor Wijsbegeerte, 1908?

I enjoyed meeting your brother this summer. Won't you come to America for a trip? For the time being I don't see a possibility to come to Holland.

Sincerely yours ⁽⁹²⁾

Arnold Dresden.

[Signed autograph – in Brouwer]

⁽⁸⁵⁾ *Begründung der Mengenlehre*. ⁽⁸⁶⁾ *Element der Menge*. ⁽⁸⁷⁾ ζ is the sequence of natural numbers. ⁽⁸⁸⁾ *Ziffernkomplexe*. ⁽⁸⁹⁾ *Species erster Ordnung*. ⁽⁹⁰⁾ *Elementen der Species erster Ordnung*. ⁽⁹¹⁾ [Brouwer 1914]. ⁽⁹²⁾ *Steeds gaarne de Uwe*.

1922-11-24

To G. Mannoury — 24.XI.1922

Laren

Dear Gerrit [Beste Gerrit]

In the document of Van Ginneken,⁽⁹³⁾ I read for the first time a formally pronounced ruthless negation of the only thing that attracts me to significs: the hope of the creation of linguistic social means of reform, independent of all existing (in my view mostly obsolete) formation of groups, and by people that in a neutral and humanitarian community would rise above their respective groups. Indeed, this view has in our circle been relegated more and more to the background, but I have patiently allowed that to happen, firstly by acknowledging my learning capacities in this matter, and secondly in the expectation that the community I hoped for would finally be established and would function, notwithstanding all difficulties.

I must now definitively give up this expectation after the experience that one of my fellow members now derives inspiration even from the rejection of my (unaltered) principle, and the consequence of this can be none other than my resignation from our circle. I am even of the opinion that it would tend to unfairness and lack of character if I would under these circumstances keep publishing our joint manifest, knowing it is followed by Van Ginneken's postscript.

Notwithstanding the above, I have the feeling that there is something that ties us four more to each other than to others, but it seems that this *je ne sais quoi* cannot be admitted into the realm of conscious reality.

With a handshake⁽⁹⁴⁾
your

[Carbon copy – in Brouwer]

⁽⁹³⁾ Van Ginneken's statement as a member of the Signific Circle contained a plea for understanding and communication within small coherent groups, in his case the Roman Catholic Community. See [Brouwer 1937] and [Schmitz 1990] p. 425. ⁽⁹⁴⁾ *Met handdruk.*

1922-11-25**To F.A.F.C. Went** ⁽⁹⁵⁾ — **25.XI.1922****Laren**

Dear Colleague, [Hooggeachte Collega]

After ample considerations I have come to the opinion that it is rather difficult for me to ask *on my initiative* a fellow member of the Academy to put himself at disposal for nomination into a Committee of advice concerning my address to the minister. ⁽⁹⁶⁾ This fellow member would involuntarily start to feel himself to be my advocate and this would be improper, because I can, nor may be a party in the treatment of this matter in the Academy. So I withdraw my remark about this point, made in the October meeting.

However, I can say to you that most particulars that could shed light on my conflict with Denjoy are known to our colleague Hendrik de Vries; also, that if our colleague Winkler would like information from my side, I would be completely at his service.

With friendly greetings

Your

(w.g.) L.E.J. Brouwer

[Signed autograph, draft – in Brouwer]

1922-12-16**To G. Mannoury** — **16.XII.1922****Laren**

Dear Gerrit [Beste Gerrit]

I suddenly notice that you have convened me tomorrow morning (in my mind it was the 24th), and because tomorrow Corrie is going abroad with her sister, for which still a lot has to be organized, there is a big chance that

⁽⁹⁵⁾Chairman section Physics KNAW. ⁽⁹⁶⁾See *Brouwer to Minister of Education 27.IX.1922* and [Van Dalen 1999] section 9.3.

I can't combine that with my presence at our meeting. So in case I have to be absent again, I want to answer *now* your letter of a few weeks ago, also for the others.

What attracts me to significs has always been and remained: the hope for the creation of linguistic social means of reform, independent of all existing group-forming, and by people that in a neutral and humanitarian community would rise *above their respective groups*. Indeed, this view has been relegated more and more to the background in our Circle, but I have allowed that to happen, firstly allowing for the (until now not realized) possibility that I see the light in this matter, and secondly hoping that the community that was before my mind's eye would still in the end be established and would function, notwithstanding all difficulties.

In your encyclopedia program ⁽⁹⁷⁾ I hardly find anything at all of my ideal expressed, and an even stronger indication of the solitude of my path is given by the document of Van Ginneken, which not only rejects my principle, but that even derives inspiration from this rejection. ⁽⁹⁸⁾

Much more important than the professional and recreational activity of a philological or psychological character, which makes up your encyclopedia, is for me the fulfillment of primary humanitarian duties with signific basis, such as the struggle for the morality of international science, into which I have been driven ⁽⁹⁹⁾ in the last few years, and which I have seen absorb a great deal of my mental powers. The world needs the spiritual struggle of practical significs more than the accompanying linguistic and psychological theories.

Greetings to Van Eeden and van Ginneken, and a handshake from

your
Bertus.

[Signed autograph – in Mannoury]

⁽⁹⁷⁾See [Schmitz 1990] p. 430. This program was proposed by Mannoury when the original plans failed. ⁽⁹⁸⁾See [Schmitz 1990] p. 425. ⁽⁹⁹⁾This is an oblique reference to Brouwer's efforts for the re-establishing of international scientific cooperation and organization, cf. [Van Dalen 1999] Ch. 9.

1923-04-00**To the Dutch Mathematical Society** ⁽¹⁰⁰⁾ — **IV.1923** **Amsterdam**Bijlage F. ⁽¹⁰¹⁾

M.M.H.H.,

Hereby we propose to you to notify the ‘Union, etc.’ ⁽¹⁰²⁾ in Paris:

That the Wiskundig Genootschap has joined the Union at the time in the expectation that this association would develop into an international association, that however until now no events have occurred that justify the hope that this will be the case in a foreseeable future, and that the Wiskundig Genootschap therefore sees itself forced, in view of its very limited financial capacity, to resign its membership.

G. Mannoury
Hk. de Vries
L.E.J. Brouwer

[Typescript draft/copy – in Brouwer]

1923-04-18**From A. Fraenkel — 18.IV.1923****Amsterdam**

Dear Professor, [Hochverehrter Herr Professor]

as I didn’t find you today in the University — I only there heard that the lectures hadn’t started yet — I allow myself to say goodbye in this way. I have received your very friendly card with the proofs that were recently taken to Laren; ⁽¹⁰³⁾ I believe that I may infer, if I don’t hear anything to the contrary, that you will essentially agree with the proof sheets sent through the mail (insofar as they interest you).

⁽¹⁰⁰⁾Wiskundig Genootschap. ⁽¹⁰¹⁾Handwritten remark. ⁽¹⁰²⁾Union internationale de Mathématique. ⁽¹⁰³⁾Proofs of [Fraenkel 1923].

It only remains to thank you most cordially for your inspection of the proofs, which was very valuable for me, for your kind support of the use of the library, for the interesting and original lecture course, and finally — and at the same time to thank your revered spouse, the booklet of hers she lent my wife was also very interesting for me — for the nice and stimulating hours in Laren. Among other things it was very interesting for me to observe the fresh life in intuitionism, which had already been pronounced dead from many sides; within myself these questions are still fermenting.

Please convey at a suitable opportunity my regards to Mr. Weitzenböck, whose inaugural lectures I can't attend anymore, much to my regret. We travel within the next few days.

With best wishes and greetings from house to house,

Your ⁽¹⁰⁴⁾

A. Fraenkel

[Signed autograph – in Brouwer]

1923-08-25

From T. Ehrenfest-Afanassjewa — 25.VIII.1923

Jena

Dear Mr. Brouwer, [Sehr geehrter Herr Brouwer]

Many thanks for your card. Your consent has made me very happy. Now the big question is to organize the matter really as soon and as well as possible. In any case, please let me know, before my departure, the literature that should be considered in your planned rewriting of the book ⁽¹⁰⁵⁾ — maybe I still could acquire it quickly. If your own papers are among these, then I would be extraordinarily grateful if you would send the relevant reprints to me in Leyden.

I think I'll be in Leyden from August 29, until the end of the first week of September ⁽¹⁰⁶⁾ and then I'll go to Russia via Berlin. So I cannot attend

⁽¹⁰⁴⁾ *Ihr ergebenster*. ⁽¹⁰⁵⁾ Apparently a translation into Russian of Brouwer's dissertation (and later papers) was considered. ⁽¹⁰⁶⁾ I.e. Friday, September 7.

the German Naturforschercongress. I could visit you before September 7 or after November 1. I must confess that I am tremendously looking forward to the opportunity to talk with you about those fine things — whether you will enjoy it as much is another question, the more so as I never studied the topics thoroughly.

One request I still have for you: if you don't want to redo the whole set-up of the book, but only isolated places, then please tell me which chapters I can translate immediately: then I can do that already in Moscow and would be perhaps in the position to do the proof correction of one part over there. Your question about Germany I cannot answer in a few words: it goes without saying that I think the situation is terrible and that I wish that it would change soon and that I would *really feel relieved* when that would happen. But I cannot feel so *uniformly*⁽¹⁰⁷⁾ outraged as you seem to be. By the way, I believe that outrage always contains an element of surprise, and being surprised means that one doesn't completely understand the matter. During the war I have too much put myself in the position of the other party — don't forget I'm Russian, and I know too well the contempt and thirst for power of another nation that is unable to immerse itself into the psychology of your own people. I vividly imagine how *we* would feel if the end result was just the opposite of the present one. That is why I can understand now a bit the certainly all too blind rage of the French, without empathizing of course! But after everything I see here, I am convinced that the cultural consciousness and the inner national coherence and also the many cultural practices are on such a high level here, that a destruction of Germany is impossible.

There, dear Mr. Brouwer! Let me hope that you will not completely wash your hands of me after this short extract from my credo.

Please greet your wife most cordially from me.

Your

T. Ehrenfest- Afanassjewa

[Signed autograph – in Brouwer]

⁽¹⁰⁷⁾ 'einheitlich' in the letter.

1923-09-01**To A. Schoenflies — 1.IX.1923****Laren**

Dear Mr. Schoenflies [Lieber Herr Schoenflies]

Because, according to the newspapers, there is again a severe food shortage in your area, I am sending you right now a charity package with a few daily needed items. Unfortunately I heard at the post office that packages in Germany are no longer delivered for free, but that the recipient has to pay 300,000 Mark custom duty⁽¹⁰⁸⁾ (which the sender cannot prepay in any form), for which I apologize. I was last year with my wife in Seefeld in Tirol. The view from Mösern into the valley of the Inn there is magnificent! I have received the Jordan curve theorem and looked through it and found it really amusing: for the time being I lack the time for precise checking; my own work is resting completely for three years, because my strength is almost completely occupied with the struggle against our annexation by France, which is so industriously promoted by the Lorentz clique.⁽¹⁰⁹⁾ Nonetheless I certainly hope to come to Marburg⁽¹¹⁰⁾ and see you there.

With cordial greetings from house to house

your
L.E.J. Brouwer

[Signed postcard – in Brouwer]

1923-10-24**From P.S. Urysohn⁽¹¹¹⁾ — 24.X.1923****Moscow**

Twerskaja Street, Pimenowski pereulok 8, kb.3, Moskau

Dear Professor! [Hochgeehrter Herr Professor!]

You have requested me in Marburg⁽¹¹²⁾ to communicate to you in writing the objections against your proof in *Crelle's Journal* (Band 142),⁽¹¹³⁾ that

⁽¹⁰⁸⁾The year 1923 was the time of hyper-inflation in Germany, with prices rising about 5 percent *per day*. ⁽¹⁰⁹⁾Lorentz was advocating a compromise policy, see [Van Dalen 2005], p. 510 ff., [Schroeder-Gudehus 1966]. ⁽¹¹⁰⁾meeting of the DMV. ⁽¹¹¹⁾Pavel (Paul) Samuilovich Urysohn. ⁽¹¹²⁾Annual meeting DMV; Urysohn's talk was on 21.IX.1923. ⁽¹¹³⁾[Brouwer 1913d].

I raised in my talk there. Please accept my apologies that I have tarried so long with this letter; I am only since two weeks in Moscow, and after an absence of almost five months I had so much to do that I couldn't get around to writing.

The article concerned is titled 'On the natural dimension concept' ⁽¹¹⁴⁾ and it contains apart from the definition of the dimension concept, the proof of the 'Dimension theorem': *An n -dimensional manifold possesses the homogeneous dimension degree n .* This proof consists of two parts: the reduction to a lemma (p. 149–150) and the proof of the lemma (p. 151–152). The latter is perfectly flawless; but concerning the reduction I have to remark the following.

In the first place — something relatively unimportant — the concept used there, 'the domain set g_1 , bordering on the edge E_1E_2 determined by π_2 in τ_1 ' (p. 150, l. 1) is insufficiently defined. The definition you give in a footnote (p. 150, *), says nothing about the connectivity situation, which is clearly indispensable for the characterization of g_1 . Likewise the concept of the boundary of this domain set is not defined. I will show in a minute that your proof remains inadequate with *any* definition. ⁽¹¹⁵⁾ Hence a more detailed discussion of the possible definitions is superfluous; besides I might remark that in any case a sensible definition of the 'domain set g_1 ' is not easy to give: for one may not be guided by the analogy with ordinary domains and describe g_1 as *the largest connected*⁴ respectively *continuously connected*⁵ *subset of $\tau_1 - \pi_2$* ⁶ bordering E_1E_2 , — because the sets⁷ defined in this way generally don't have to be domain sets.

Now your proof contains two unfounded statements (p. 150, l. 13–16)

I) $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ converge with ε to zero,

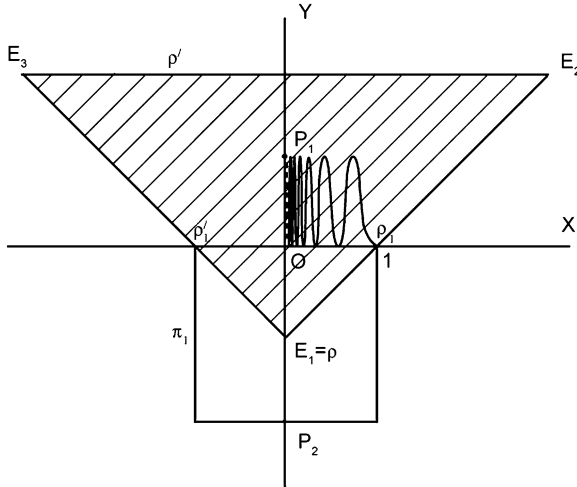
and

II) $\tau_1, \tau_2, \dots, \tau_n$ are contained as subsets in respectively $\pi_1, \pi_2, \dots, \pi_n$.

I will now show by a simple example that — depending on the definition of the domain set g_1 , and its boundary τ_2 — *at least* one of these statements is wrong. It suffices to choose the Euclidean plane as manifold π ; let then $E_1E_2E_3$ be the line triangle with

⁴In the sense of Hausdorff: i.e. g_1 cannot be split into two subsets neither of which contains a boundary point of the other. ⁵Terminology of Mr. Kerékjártó; you call such a set a 'continuum' (loc.cit., p. 147, l. 3). ⁶ $\tau_1 - \pi_2$ is the complementary set determined by π_2 in τ_1 . ⁷For both definitions lead to different sets.

⁽¹¹⁴⁾ *Über den natürlichen Dimensionsbegriff.* ⁽¹¹⁵⁾Here Urysohn was overly pessimistic. See *Brouwer to Urysohn 14.VI.1924* [Brouwer 1924a, Brouwer 1924d] and [Van Dalen 2005] p. 461.



vertices $E_1 = (0, -1), E_2 = (3, 2), E_3 = (-3, 2)$, and let π_1 consist of the following six curves

- | | | | |
|--------------------------------|--------------------|--------------|--------------------|
| 1) $y = \sin^2 \frac{\pi}{x},$ | $0 < x < 1$ | 4) $x = -1,$ | $-2 \leq y \leq 0$ |
| 2) $x = 0,$ | $0 \leq y \leq 1$ | 5) $y = -2,$ | $-1 \leq x \leq 1$ |
| 3) $y = 0,$ | $-1 \leq x \leq 0$ | 6) $x = 1,$ | $-2 \leq y \leq 0$ |

and let finally π_2 be the set consisting of both points $P_1 = (0, 1)$ and $P_2 = (0, -2)$.

If one takes the (only natural) definition of g_1 , according to which g_1 coincides with part 1) of π_1 , then τ_2 is identical with part 2), hence statement II) is incorrect. But when we can define g_1 and its boundary in such a way that statement II) is satisfied, i.e. that τ_2 is a subset of π_2 , then τ_2 consists necessarily of the single point P_1 . One sees immediately that then ε_2 (p. 150, l. 12) does not go to zero with ε , so that statement I) does not apply.

Hence the proof of the ‘dimension theorem’ is not correct. Unfortunately I have not succeeded in deciding whether the theorem itself is correct. In any case not only the theorem but also its proof can be made correct by an appropriate *change of definition* of the dimension degree, or more precisely, — of the basic definition of *separation*. Your definition of this concept (p. 147, l. 15–19) must namely be replaced by the following: ‘ ρ and ρ' are called separated in π by π_1 , when $\pi - \pi_1$ can be split into two subsets λ and λ' , that contain ρ respectively ρ' and such that neither of them contains a boundary point of the other one.’ That this notion of separation differs from yours, one can for instance see from the examples given above. However, the

definition of dimension degree thus obtained is, as I have shown,⁸ at least for F_σ sets⁹ equivalent with the much simpler one that I published last year in the *Comptes Rendus* of the Paris Academy.¹⁰

The latter definition runs as follows. Let C be any set lying in a compact metric space,¹¹ and x a point of it. I say that the subset B of C ε -separates the point x in C , when the complement set $C - B$ can in such a way be split into two subsets A and D , that

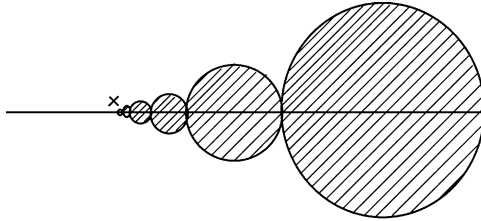
- 1) none of these two sets contains a boundary point of the other,
- 2) x belongs to A ,
- 3) the diameter (the upper boundary of the distance of two points) of A is $< \varepsilon$.¹²

Then I define the dimension inductively as follows:

1. The *empty* set has dimension -1 .
2. When the point x in C does not have dimension $< n$, but when for every ε it can be ε -separated in C by a set B of a dimension $< n$, then we say that x has the dimension n in C .
3. If the set C contains only points of dimension $\leq n$, and among these also points whose dimension $= n$ is, then we say that C has dimension n .
4. Of sets (and points) that do not have a dimension according to 1—3 we say that their dimension is finite.

⁸In Ch. VI of my treatise about the dimensions of sets. The accompanying manuscript is already for several months with the editors of *Fundamenta Mathematicae*; the first part (Introduction and Ch. I – II, maybe also III) will appear in Volume VI of this journal; Ch. VI will appear only in Volume VII or even VIII (the whole treatise is several hundred pages). ⁹I call a set F_σ when it can be considered as lying in a compact metrical space (i.e. (\mathcal{D}) -), which means that it is homeomorphic to such a set, and that can be represented as the union of countably many closed sets; every manifold is clearly an F_σ . ¹⁰Volume 175, p. 440 & 481. The proofs of the theorems that I have stated in these notes without proof, are contained in the above mentioned treatise. [ed. [Urysohn 1922]] ¹¹That is, compared to your assumptions, no restriction at all. Indeed, Mr. Chittenden has shown some years ago in the Transactions of the American Mathematical Society that every (\mathcal{V}) -set is homeomorphic to a (\mathcal{D}) -set, and recently I have proved (I have submitted the proof in July of this year to Mr. Prof. Hilbert for the *Mathematische Annalen*), that every separable (\mathcal{D}) -set (hence *a fortiori* every normal \mathcal{V}) is homeomorphic to a subset of a specific compact metric space H_0 (H_0 is the parallelepiped in the infinite dimensional Hilbert space determined by the equations $0 \leq x_n \leq \frac{1}{n}$). ¹²One can also demand — which, as I have proved, doesn't change the final definition of dimension, — that I) B is closed in C and II) also the diameter of the union set $A + B$ is smaller than ε . When one then uses the (modified) concept of separation and if one denotes K the set of points of C whose distance from the point x is $\geq \varepsilon$, then one can also say that K and x are separated in C by B .

The advantage of this definition is not only that it is much better organized than the previous one and that it yields also for non-normal sets a completely useful result, but it also contains a much sharper definition of the dimension *in a point* as one can see from the following example: let C be a plane closed point set that consists of the point x and a countable number of disks that consecutively touch each other and that converge to x . Then x has dimension 1 in C , although it is a limit point of points of higher dimension.



My definition at last permits one to penetrate very far into the properties of dimension: I refer for example to the theorems stated in my *Comptes Rendus* notes (by the way, since then I have found yet other results).

Dearest professor, if you would be interested in the theory thus established, then I would be glad to communicate more details about it. Finally I permit myself to direct a humble request to you. As I told you already in Marburg, I have thoroughly studied several of your remarkable topological articles. Unfortunately only those were accessible for me that were printed in the German journals. You have published many important articles in the English language (unfortunately the Dutch language is unknown to me) in the Amsterdam Academy, publications of which are not available at all in Moscow. Therefore I venture to bother you, dearest professor, with the request to send me reprints of your Amsterdam articles. ⁽¹¹⁶⁾ My address is as follows:

Moscow (Russia), Twerskaja Street, Pimenowski pereulok 8, kv. 3

I apologize for the laboriousness of this letter

sincerely yours

Dr. Paul Urysohn.

[Signed autograph – in Alexandrov]

⁽¹¹⁶⁾ *KNAW, Proceedings.*

1923-11-29a

To F. Klein — 29.XI.1923^a

Laren

Vertraulich

Dear Mr. Geheimrat, [Hochgeehrter Herr Geheimrat]

I have the honor to send you the enclosed accounting circular of the *Mathematische Annalen* from Blumenthal, which was sent to me with the request to pass it on.

Last summer I was entrusted by the editorial board of the *Annalen* with the refereeing of an article by Mohrmann ‘On curves of maximal class index’⁽¹¹⁷⁾ which had been received by Blumenthal. This refereeing cost me a lot of trouble, both regarding the content matter of the submitted work and the personal priority relations: exactly because of that I was very unpleasantly surprised when Blumenthal a few weeks later indicated that Mohrmann wished to retract his submitted article.

The *Annalen* circular of the last summer mentions both the reception and the withdrawal of the Mohrmann article, and also my name as refereeing editor and the general nature of my objections.

Recently now I got the message from Blumenthal that Mohrmann has again submitted the article in question, and indeed to you. About this I would like to remark that two years ago, on the occasion of an analogous incident, the entire board of editors has jointly and expressly decreed that an editor, once he has been entrusted with the refereeing of an article will remain for his co-editors the one who decides about acceptance, as long as he does not voluntarily part with this duty. Indeed, without this certainty any cooperation between editors is impossible. I am, by the way, the only one who knows the previous history of Mohrmann’s submission and also about the mutual priority rights of Mohrmann and Nagy concerned here, and these are based upon the order in which the letters of both these authors have been received by the editorial board (even the authors themselves cannot exactly know this order).⁽¹¹⁸⁾

⁽¹¹⁷⁾ *Ueber Kurven vom Maximalklassenindex.* ⁽¹¹⁸⁾ The Mohrmann manuscript was probably the cause of Klein’s exit from the editorial board of the *Mathematische Annalen*. The letters *Blumenthal to Ed. Board Math. Ann 16.XI.1928* and *Brouwer to Ed. Board Math. Ann 30.IV.1929* shed more light on Klein’s decision to step down. See also [Van Dalen 2005] p. 613, 631.

In the hope that everything is well with you in so far as the prevailing circumstances permit, I am in sympathy with you, greeting you and the other Göttingen colleagues cordially

Yours truly ⁽¹¹⁹⁾
L.E.J. Brouwer

[Signed autograph – in Klein]

1923-11-29b

To P.S. Urysohn, Summary — 29.XI.1923^b **Zandvoort** ⁽¹²⁰⁾

Saturday 29.IX.23 from Zandvoort, boarding house John Bückmann, written to Dr. P. Urysohn, Mathematical Seminar of the University, Moscow, concerning the pencilled note (at the separation definition) in the margin of my personal copy of *Über den natürlichen Dimensionsbegriff*. ‘This pencilled note, which clears up everything, must date from many years ago; it is very well possible that it has been made after a remark of a colleague (in that case probably Weyl, Gross or Rosenthal). I will try to determine this, and also investigate whether this note was not added to a later publication as Erratum.’

L.E.J. Brouwer

[Signed autograph – in Alexandrov]

Editorial supplement

[We quote from Freudenthal’s comments on Brouwer’s pencil remarks, from *CWII 1967*, p. 549:]

“There is a hardly visible pencil correction, which in the history of Brouwer’s style of writing must be dated before 1923: the word ‘*abgeschlossen*’ ⁽¹²¹⁾ in line 18 is deleted and a line with an arrow is drawn

⁽¹¹⁹⁾ *Ihr ganz ergebener.* ⁽¹²⁰⁾ The following is a private note containing a summary of a letter to Urysohn. ⁽¹²¹⁾ closed.

from this word to the margin, where one reads ‘zu streichen in Übereinstimmung mit S.150 Fussnote *’.⁽¹²²⁾ In a letter to H. Hahn of 4 August 1929 Brouwer asserted that he corrected the text of his own copy as early as March 1913, which certainly means the pencil rather than the ink correction. This is confirmed in the most unexpected way by a note in the proof sheets of A. Schoenflies 1913 (Brouwer read carefully the proofs of Schoenflies’ book and advised the author in the most efficient way (see 1910C and A. Schoenflies 1913, VII Vorwort)). On p. 382 of these proof sheets he elaborated footnote ²⁾ by adding ‘...; ebenso die Untersuchungen Brouwers in Math. Ann. 70, S. 161–165 (an letzter Stelle ist übrigens nach einer Mitteilung Brouwers auf S. 147, Z. 18 das Wort ‘abgeschlossen’ zu streichen).’⁽¹²³⁾

For unknown reasons Schoenflies did not adopt Brouwer’s note.

1924-01-16

From W. Dubislav –16.I.1924

Berlin-Friedenau

Gosslerstr. 6

Dear Professor, [Sehr geehrter Herr Professor]

It might perhaps interest you that it is extraordinary simple to give examples in which the so-called ‘principle of the excluded third’, the validity of which you doubt, is demonstrably *not* true. In its usual fashion the principle — the numerous statements in the literature, that partly also differ in content, that are designated as ‘principle of the excluded third’ had better be ignored — says: ‘statement A or statement non- A is correct’, and as an aside it may be remarked that the formulation one often meets ‘ C is D or not’ can be easily reduced to this. To show now that the principle is not always true, we consider for example the axiom system drawn up by Hilbert in the ‘*Grundlagen der Geometrie*’ Chapter I (4th edition Leipzig 1913), without axiom group {IV} (parallel axiom) and axiom group {V} (continuity axioms). Now, from the axiom system contracted in this way, let’s call it V for short, the following statement, where a is an arbitrary line and A a point outside a : ‘There exists in the plane through a and

⁽¹²²⁾To cross out in agreement with p. 150 footnote *. ⁽¹²³⁾... likewise the investigations of Brouwer in Math. Ann. 70, p. 161–165 (incidentally, in this last place the word ‘closed’ on p. 147, l. 18 must be crossed out, according to a communication from Brouwer.

A at most one line through *A* that does not intersect *a*' is not provable. But also the negation of this statement, namely the statement 'There are in the plane through *a* and *A at least* two lines that pass through *A* and do not intersect *a*' is not provable from *V*. So we have with respect to *V* a meaningful declarative statement, let us call it *S* for which holds that neither *S* nor non-*S* is provable with respect to *V*. So for *S* and its negation — always with respect to *V* — the 'principle of the excluded third' does not hold. The statement *S* is as one says logically independent from *V*, and as one can immediately generalize, if one has a statement that is independent from a totality of statements that together determine a domain of thought, then with respect to that totality the 'principle of the excluded middle' is not valid. So the 'principle of the excluded middle is in its general formulation a logically inadmissible fundamental concept. Q.e.d. ⁽¹²⁴⁾

Nonetheless I believe that it is almost always used in mathematics (set theory included) in a manner that seems admissible to me. Namely one uses it mostly in indirect proofs, when one has obtained a contradiction from assuming the negation of the statement to be proved, and then concludes that the statement to be proved is correct. Because according to the 'principle of contradiction', the principle that both a statement and its negation are true, is wrong; according to the 'principle of the excluded third' one of both theorems, the statement or its negation, must be correct. If one however also would want to doubt this application of 'the principle of the excluded third', then one should consider every indirect proof as an inadmissible justification, which would go too far in my opinion. In other words, insofar as one uses the 'principle of the excluded third' *only* for statements of which one knows or can prove (as in the indirect proof by means of the 'principle of contradiction') that they are *not* logically independent theorems with respect to the valid assumptions, I consider its application is fully legal.

Sincerely yours ⁽¹²⁵⁾

Walter Dubislaw

[Signed typescript – in Brouwer]

⁽¹²⁴⁾Dubislaw erroneously identifies the principle of the excluded middle with completeness of theories. ⁽¹²⁵⁾*In vorzüglicher Hochachtung bin ich.*

1924-01-22

To P.S. Urysohn — 22.I.1924

Laren

Dear Mr. Urysohn [Sehr geehrter Herr Urysohn]

I received your card of 27.XII.23 all right. With the same mail I dispatch a couple of envelopes with reprints to you and some more will follow.

After my return from Marburg the objection you made there, became immediately clear to me on checking my private copy of the article ‘On the natural notion of dimension’, ⁽¹²⁶⁾ where I have on p. 147 an old marginal note at l. 17–20, which says at this place ‘*make it agree with p. 150 at **’. This was the marginal note to which my card from Zandvoort referred.

Coming back now a bit more in detail to the topic, I remark first of all that in my topological articles that appeared in 1908—1914 the expression: ‘the domain set g is determined by the closed set α ’ says exactly the same as ‘the domain set g is bounded by α ’ (c.f. e.g. *Mathematische Annalen* 69, p. 170, where this is explicitly stated). Consequently the quote on p. 150, “*by π_2 in π_1 determined domain set g_1 bordering on the edge E_1E_2 ” on p. 150 at *)”, can in connection with the text have no other meaning than that of the intersection of a domain set γ_1 that is already available in π_1 determined by π_2 , bordering E_1E_2 , however not bordering $E_1E_3 \dots E_{n+1}$, with τ_1 , so that the existence of the latter domain set γ_1 is postulated by the concept of ‘separation of ρ_1 and ρ'_1 in π_1 by π_2 ’. Hence the considerations of the article are actually based upon a separation definition, according to which ρ and ρ' in π are separated by π_1 , only if π_1 determines in π a domain set that contains ρ but not ρ' . The definition that you indicate in your letter of 24.X.1923 says the same thing in another form. As far as the origin of the oversight on p. 147 is concerned, my notes of that time make it probable that the manuscript of the article originally didn’t contain an explicit separation definition, just as in my article that appeared in *Annalen* 71: ‘Proof of the invariance of domain’, and that such a definition only much later has been inserted rather thoughtlessly, after a reader of the page proofs pointed out the absence. When not long after the article appeared, the oversight became clear, a quick correction was not forthcoming, because I expected that the article mentioned on p. 151 of the above mentioned paper on the same subject, promised by Lebesgue, would appear soon, and I was convinced that this article would force me to make a rejoinder, in*

⁽¹²⁶⁾ *Ueber den natürlichen Dimensionsbegriff.*

which I could naturally include the necessary rectification as an addendum. When subsequently, the promised article of Lebesgue failed year after year to appear, the whole matter vanished gradually from my mind, and without your interpellation I would maybe never have thought of it again.

Now I have on the occasion of your remarks also studied the published explanations of Lebesgue, that came out with a delay of ten years (and not as agreed in the Bulletin de la Société Mathématique but in Vol. II. of *Fundamenta Mathematicae*, ⁽¹²⁷⁾) and I have seen that these, just as I expected ten years ago, make a contra-publication necessary, for indeed, the proof of Lebesgue of the lemma formulated on p. 150 of ‘On the natural notion of dimension’, is merely a abbreviated form of my proof of the same theorem. I hope this rejoinder will appear soon. ⁽¹²⁸⁾ It will at the same time (while mentioning your priority) provide the correction of my old oversight.

I would be very grateful for the promised copies of your *Comptes Rendus* Notes, and also for more information about your yet unpublished investigations. To be sure, my own researches are since some years of a different orientation, but my interest in topology has remained, and I consider you as one of the few that really can open new perspectives here.

With best greetings

Yours truly

L.E.J. Brouwer ⁽¹²⁹⁾

[Signed autograph – in Alexandrov]

1924-03-12

From K. Menger — 12.III.1924

Vienna

Dear Professor, [Hochverehrter Herr Professor]

Thank you very much for the kind dispatch of your article about the natural dimension concept. ⁽¹³⁰⁾ When I tried in 1921 to define curves and the dimension concept, I was in the first year of my university study and didn't know your article in the *Journal für die reine und angewandte Mathematik* 142 ⁽¹³¹⁾ at all, in which the definition is essentially anticipated. But also

⁽¹²⁷⁾[Lebesgue 1921]. ⁽¹²⁸⁾Published as [Brouwer 1924a]. ⁽¹²⁹⁾*Ihr sehr ergebener.*

⁽¹³⁰⁾[Brouwer 1923a]. ⁽¹³¹⁾[Brouwer 1913A]

later, after I found the publication when studying the relevant literature, I hoped that I could offer through my results at least a small supplement. For I have investigated the structure of n -dimensional sets, and I have proved as a supplement of the theorem: Every open set of R_n is n -dimensional — the following theorem: Every n -dimensional set of R_n contains an open part. I hope to be able to send you in the course of this year in printed form the second part of my article, which I had prepared already long ago.

Please accept, my dearest professor, my expression of my particular admiration and affection.

Sincerely yours

Karl Menger

[Signed autograph – in Brouwer]

1924-03-25a

To G. Mannoury — 25.III.1924^a

Laren ⁽¹³²⁾

For the undersigned signification does not consist so much of practicing language criticism, but rather of:

- 1°. tracking down affect elements, into which the cause and effect of words can be analyzed. By this analysis the affects that relate to human relations are brought closer to control by conscience.
- 2°. the creation of a new vocabulary which also for the spiritual life tendencies of people opens access to their thoughtful exchanges of ideas and hence to their social organization.

For the realization of the part of the program mentioned under 1°, cooperation is necessary: for countless affect complexes can not be analysed unless by the catalytic action of philosophical discussion between unlike-minded.

Also with regard to the creative work meant under 2°, I have believed for a long time in the great importance of cooperation, here between like-minded. But I have come more and more to the opinion that this

⁽¹³²⁾This letter contains Brouwer's personal statement, which was published as part of the *Beginselverklaring* (declaration of principles) of the Signific Circle. Published much later in the '*Signifische Dialogen*', [Brouwer 1937], see also [Schmitz 1990] p. 423.

higher task of significs can only be accomplished by the utmost concentration of the mind of the single individual.

L.E.J. Brouwer

Dear Gerrit,

Above a new version. In the old form it was really too silly. I am glad that your warning has stopped this in time.

Cordial greetings

your
Bertus

[Signed autograph – in Mannoury]

1924-04-06

To K. Menger — 6.IV.1924

Laren

Dear Mr. Menger [Sehr geehrter Herr Menger]

Many thanks for your letter of March 12. I am glad that you too have noticed that the definitions we both give for an *n-dimensional continuum* are equivalent, and indeed essentially because for a bounding [set] B of a neighborhood $U(A)$ of a closed set A , finitely many points P_1, \dots, P_r of A can be given with neighborhoods $U(P_1), \dots, U(P_r)$, such that B is contained in the union of the bounding sets of the $U(P_\nu)$.

On the other hand we both assign different meanings to the statement: ‘the continuum K is *n-dimensional in the point P*’, as you certainly will have seen.

Why do you embed the sets M considered by you in metric spaces, instead of considering these sets exclusively as Fréchet normal sets *by themselves*? In the latter case a neighborhood of a point P in M becomes simply a ‘domain set’ that contains the point P ; cf. the definition of domain set in footnote ¹⁶⁾ of the reprint I sent you recently.

As far as the rectification of Crelle volume 142 in footnote ⁽¹¹⁾ of this reprint is concerned, you probably will have noticed already that the Crelle text can also be put right by deleting the word ‘closed’ on p. 142 l. 18. But I have preferred a formally thorough change of definition, in the interest of better readability of the new text. ⁽¹³³⁾

That the rectification of this oversight, which was discovered already in 1913, has been postponed so long, is because in order to come back on the matter, I wanted to wait for the article of Lebesgue mentioned in footnote ⁽¹⁹⁾, and this article was postponed for 10 years.

I am curious to see the proof of your theorem that every open set of R_n is n -dimensional, and even more the results concerning the set theoretic characterization of the topological images of intervals of the R_n that you envisage. In case you find a fast publication of your proof in the interest of your priority, I am quite happy to submit it to the Amsterdam Academy. And for an extensive exposition I would be pleased to put the *Mathematische Annalen* at your disposal.

Please greet Prof. Hahn from me.

With the best wishes for further success of your investigations, I remain

Yours truly ⁽¹³⁴⁾

L.E.J. Brouwer

Am I correctly informed that Prof. W. Gross ⁽¹³⁵⁾ is no longer alive? And do you know perhaps when he died?

L.E.J.B.

[Signed autograph – in Menger]

1924-04-09

To P.S. Urysohn — 9.IV.1924

Laren

Dear Mr. Urysohn, [Sehr geehrter Herr Urysohn]

I received your letter of March 20 all right, and also the batch of reprints of yours and Mr. Alexandroff. Unfortunately I must conclude from your

⁽¹³³⁾[Brouwer 1913d], [Brouwer 1923a]. ⁽¹³⁴⁾*Ihr ergebenster.* ⁽¹³⁵⁾Wilhelm Gross, 1886–1918.

letter that of the seven envelopes with reprints, two have been lost. Also a card for which I asked you in the beginning of March to confirm the receipt of my letter of January 22, seems not to have arrived.

The word ‘usual’ in footnote ⁽¹¹⁾ of the new version of ‘On the natural notion of dimension’ was indeed inappropriate, because in mathematical treatises every statement with a subjective or unprovable character must be inadmissible. Consequently I have in turn omitted this qualification in the enclosed communication that appears in Crelle’s journal ⁽¹³⁶⁾. For the same reason I have also left unmentioned any earlier disclosure of the oversight shortly after the article appeared, by myself and by others, because at the moment I do not have any documents about them in my possession.

Meanwhile I have found in my copy of the book by Hausdorff ⁽¹³⁷⁾ in the margin of p. 458, § 7 yet another note, according to which it is absolutely necessary to strike the word ‘*abgeschlossen*’ ⁽¹³⁸⁾ at the place concerned in Crelle 142 p. 147, l. 18, precisely because of the examples Hausdorff gives there. ⁽¹³⁹⁾

This deletion produces exactly the separation definition given by Hausdorff on p. 334 of his book. In the new version of ‘On the natural notion of dimension’, ⁽¹⁴⁰⁾ the now published change of definition, which formally goes deeper, decidedly is to be preferred, in view of readability and coherence.

I have read with great interest the theories that you communicate in your last letter. I hope that you will obtain along these lines the axiomatic characterization of the Cartesian n -dimensional spaces among the Cantorian n -dimensional manifolds; I think you are the right man for that.

I will be happy to submit the results of you and those of Mr. Alexandroff that are connected with my article ‘On linear inner limiting sets’ (which however does not deal with ordinals, but with ‘uniform’ topological homeomorphy and homogeneity, that preserve their meaning for $n > 1$) to the Amsterdam Academy. I would ask you to write the text either in German or English, and if possible adhere in the formulation to the terminology that I introduced in my article ‘Some remarks on the coherence type η ’ (Amsterdam Proceedings 1913 ⁽¹⁴¹⁾).

I hope that the reprint of this article that I recently sent you, reaches you and that it does not happen to be in one of the lost envelopes.

⁽¹³⁶⁾ [Brouwer 1924b]. ⁽¹³⁷⁾ Unfortunately this book, together with Brouwer’s complete library, was sold not long after his death. The whereabouts of the collection has not been discovered. ⁽¹³⁸⁾ Closed. ⁽¹³⁹⁾ [Brouwer’s note on top of page:] *Note not belonging to the letter.* Namely, as connection between $P(0, 0)$ and $Q(\pi^{-1}, 0)$ B winds itself inside of B' through boundaries that lie inside B' , through which no continuum connecting P and Q can wind. ⁽¹⁴⁰⁾ *Über den natürlichen Dimensionsbegriff.* ⁽¹⁴¹⁾ [Brouwer 1913c].

Unfortunately, because of many kinds of obstacles, the manuscript of my third communication about ‘The theory of the finite continuous groups’ is still waiting in a drawer for the ‘finishing touch’,⁽¹⁴²⁾ which however I hope to be able to give it in a not too far future.

I have forwarded your information with respect to the Revue Semestrielle and Matem. Sbornik to the chief editor of the former. I assume that he will shortly write to you in person.

I hope to meet you and Mr. Alexandroff in September at the congress in Innsbruck.⁽¹⁴³⁾ We might also meet somewhat earlier, in case you would be in Western Europe during the coming summer. Recently a colleague here (Prof. Van der Hoeve from Leiden) talked about you both: I believe he had been together with you last summer in Norway.

With warmest greetings for you and Mr. Alexandroff

Your

L.E.J. Brouwer

[Signed autograph, draft – in Alexandrov]

1924-06-14b

To P.S. Urysohn⁽¹⁴⁴⁾ — 14.VI.1924^b

Amsterdam

Dear Mr. Urysohn, [Sehr geehrter Herr Urysohn]

Maybe the enclosed variant on the passage in Crelle’s Journal 142⁽¹⁴⁵⁾ between p. 149 l. 2 from below and p. 150 l. 10 from below, by means of which the proof is adapted to the separation definition on p. 147 as now printed (hence without the erasure of the word ‘*abgeschlossen*’⁽¹⁴⁶⁾ which was needed for the old version of the proof).⁽¹⁴⁷⁾

(In the accompanying text an ‘ η -chain’ means a finite point sequence in which each two consecutive points have a distance $\leq \eta$.)

⁽¹⁴²⁾In the original German text the English expression is used. ⁽¹⁴³⁾Annual meeting of the DMV. ⁽¹⁴⁴⁾Pavel (Paul) Samuilovich Urysohn. ⁽¹⁴⁵⁾[Brouwer 1913d]. ⁽¹⁴⁶⁾closed. ⁽¹⁴⁷⁾This is the notorious *slip of the pen*; by unintentionally adding the adjective ‘closed’, the definition of ‘separation’ became too weak. Cf. [Brouwer 1976] p. 541, 547 ff., [Van Dalen 2005] p. 452 ff. Brouwer observes here that even with the unintended separation of the paper a coherent dimension notion arises.

This variant, which I recently found back among my papers from the years 1912 and 1914, has most probably been communicated at that time in correspondence about dimension with Schoenflies, Gross, and others. I will investigate whether maybe the other parties have preserved their correspondence more carefully than I have. My own interests have been diverted for nine years from these subjects, and unfortunately I have always failed as archivist.

Meanwhile, I consider, as before, the separation definition without the word ‘closed’ more appropriate and productive from the viewpoint of dimension theory.

I am curious to see your article for the Amsterdam communications; ⁽¹⁴⁸⁾ likewise the promised communications of Mr. Alexandroff (whose address still is unknown to me).

With warmest greetings

your

L.E.J. Brouwer

[Signed autograph, draft/copy – in Alexandrov; 1 enclosure, not extant.]

1924-06-21

From P.S. Urysohn — 21.VI.1924

Göttingen

Hospitalstrasse 1b (bei Assmann)

Dear Professor, [Hochgeehrter Herr Professor]

Last year I have sent at the end of July (i.e. two months *before* the Marburg meeting) a note for the *Mathematische Annalen* to Mr. Hilbert in which I criticized your dimension notion. I had long since forgotten about this note, when I suddenly received the proofs the day before yesterday. It is not at all clear to me what I should do with it. Maybe you are satisfied with the ‘Added in proof’ ⁽¹⁴⁹⁾ which I have written. Hence I allow myself to send you these proofs and ask you respectfully to inform me whether you agree with the present version, or what changes you deem necessary, or what else?

⁽¹⁴⁸⁾ *KNAW, Proceedings.* ⁽¹⁴⁹⁾ See below.

If it is not too much trouble, I would like to ask you to answer me as soon as possible, because the enclosed proof⁽¹⁵⁰⁾ is the one that should go to Mr. Blumenthal; moreover, I stay only for 19 more days in Göttingen.

Finally I must thank you for a dispatch of reprints. With warmest greetings

Sincerely ⁽¹⁵¹⁾

Paul Urysohn

Added in proof

In the statements above I naturally have based myself on the assumption that one remains within the definition of the notion of dimension in Vol. 142 of Crelle's journal. But since then Mr. Brouwer has published a *rectification*,¹³ where he in fact changes the definition of *separation* that is at the basis of the dimension concept. *Thereby* the proof is completely correct and I might emphasize that, as I have learned, the necessity of such a change was already known for a long time to Mr. Brouwer and it remained unintentionally unpublished so far. All the same I believe that the above lines may have some use, because Mr. Brouwer did not indicate in his rectification *why* the old definition should be rejected.

Göttingen June 21, 1924.

Register. Deliver by express.

[Signed autograph – in Alexandrov]

Editorial supplement

[*On backside of the envelope in the handwriting of Cor Jongejan:*]

Hello Dad,

This just arrived by mail. The matter is urgent, so I send it on. This evening I sleep here again to check your mail. Tomorrow I sleep on

¹³Crelle **153**; the improved text has also appeared in the Proceedings Akad. Amsterdam **26**, p. 795.

⁽¹⁵⁰⁾In the Brouwer Archive. ⁽¹⁵¹⁾*hochachtungsvoll*.

the Overtoom, ⁽¹⁵²⁾ because little Moek ⁽¹⁵³⁾ has to leave so early. Bye, bye. Good luck with your exam period.

Corus ⁽¹⁵⁴⁾

1924-06-24

To P.S. Urysohn — 24.VI.1924

Bergen aan Zee

Dear Mr. Urysohn, [Sehr geehrter Herr Urysohn]

Many thanks for sending me the proofs of your forgotten small Annalen note and for asking my advice about it. I am of the opinion that in both our interests the publication of this note should absolutely be omitted. For, publication by scholar *A* of an oversight that escaped author *B* is only then compatible with the dignity of scholars, when either the oversight can be understood only by an elaborate exposition of new discoveries by *A*, or when consultations between the parties concerned has become materially impossible (e.g. for political reasons or because of the death of *B*). In any other case such a publication raises the suspicion that either *A* has been carried away by impetuous ambition, and, maybe on purpose, wants to insult *B*, or that *B* refused to acknowledge his oversight to *A*, alternatively refuses public acknowledgment, at least to full extent. Fortunately neither of the mentioned circumstances applies in this case, but rather, in all respects, the opposite.

For the rest I agree with you that it could be useful when the counterexample you put forward would be brought to the attention of the public. As a matter of fact, I must after all come back to the matter myself, in order to show along the lines of my old correspondence with Schoenflies and Gross, how the proof of the dimension theorem can be put in order also on the basis of the erroneous separation condition from the year 1913 (more about that was communicated to you in my letter that I sent to Moscow ten to twelve days ago). With the publication of this proof I will have a good opportunity to insert the counterexample concerned here (naturally mentioning its paternity). ⁽¹⁵⁵⁾

⁽¹⁵²⁾over the pharmacy. ⁽¹⁵³⁾a private pet name for Lize Brouwer-de Holl; one might translate it as 'little Mom'. ⁽¹⁵⁴⁾Nickname for Cor. ⁽¹⁵⁵⁾See [Brouwer 1924a].

I believe this is the only dignified way of dealing with the matter, and I hope you agree with it. The subsequent suppression of your small *Annalen* note will cause no problem: as one of the editors of the *Annalen* I will arrange that with the editorial board and the publisher. Without your message to the contrary the affair will be settled in this manner.

I am looking forward to a, as I hope, reunion before long (you have received my card sent to Göttingen?) and with warmest greetings also to Mr. Alexandroff

Your
L.E.J. Brouwer

On the 27th or 28th of this month I will be in Laren again.

[Signed autograph, copy – in Alexandrov]

1924-06-27b

P.S. Urysohn to W. Sierpiński — 27.VI.1924^b Göttingen⁽¹⁵⁶⁾

Dear Waclaw Constantinovicz, [Hochverehrter Waclaw Constantino-
vicz]

Analyzing my sketch, I have found out that in the introduction of my *Mémoire sur les multiplicités Cantoriennes*⁽¹⁵⁷⁾ I have inserted a remark about the ‘*Natürliche Dimensionsbegriff*’ of Brouwer, in which I have written roughly the following: ‘Now the proof of this theorem contains an error that seems incorrigible to me.’ Although this remark is justified with respect to the earlier formulation of Brouwer (in his rectification he changes the *definition* of the notion of dimension), it seems to me that after the publication of this rectification it is not appropriate that my remark appears in print. So allow me to beg you urgently to modify the criticism; If possible, to replace it by the by the one given below; if this is perhaps impossible for technical reasons, then at least delete it.

⁽¹⁵⁶⁾In the Menger archive there is a number of translations of letters of Urysohn, Brouwer, and Alexandrov in an unknown handwriting. They were based on documents in the possession of Sierpiński. The originals are presumably not extant. ⁽¹⁵⁷⁾Memoir on the Cantorian manifolds.

Here is the text of the desired remark: the memoir was already finished when I learned about the article ‘*Über den natürlichen Dimensionsbegriff*’ published by Mr. Brouwer in 1913 in the *Journal für die [reine und angewandte] Mathematik* (volume 142, p. 146). I hope to come back at another occasion to the definition of Mr. Brouwer and mine.’⁽¹⁵⁸⁾

Paul Sergiewicz ⁽¹⁵⁹⁾ sends his greetings to you and simultaneously sends reprints of his latest article to you, to the editorial board of the *Fundamenta* and to Mr. Rajchmann.

Sincerely ⁽¹⁶⁰⁾

Paul Urysohn

P.S. Just now the three reprints sent by you have arrived, for which the both of us thank you very much. We stay in Göttingen until July 9, (address Hospitalstrasse 1b with Assmann), and then we go for a few days to Bonn (address poste restante) to Hausdorff, and afterwards probably to Paris.

P.U.

[Handwritten translation – in Menger]

1924-07-09b

To P. Zeeman — 9.VII.1924^b

Laren

Dear Colleague [Waarde Collega]

May I ask your assistance for just a moment regarding the enclosed letter? The permission to live outside Amsterdam was at the time one of my conditions to reject the call to Leyden. About this matter there has been correspondence in the summer of 1915 between you as chairman of the faculty and the Mayor of Amsterdam⁽¹⁶¹⁾, and then you promised me that you would keep the letter of Mayor Tellegen to you in which the pertinent permission was granted. Would you maybe willing to lend it to

⁽¹⁵⁸⁾This text was adopted in the published version, followed by a reference to Menger’s work. ⁽¹⁵⁹⁾Alexandrov. ⁽¹⁶⁰⁾*In aufrichtiger Hochachtung Ihr Ergebener.* ⁽¹⁶¹⁾Tellegen.

me for a short time, so I can use it to plead for my rights with Mayor and Aldermen?

Kindly thanking you in advance and with many greetings

Your
L.E.J. Brouwer

[Signed autograph – in Zeeman]

1924-07-29

From P.S. Urysohn, P.S. Alexandrov — 29.VII.1924 Le Batz

Dear Professor, [Hochgeehrter und lieber Herr Professor]

Only today we finally got around to writing.⁽¹⁶²⁾ In Paris we have been walking around every day from 9 o'clock in the morning until 10 o'clock at night¹⁴ — because apart from the city and the museums there was also a police headquarter which made problems for us, and the German consulate, where we asked for a transit visa for the return journey, and so on. After four days we had become so tired,¹⁵ that we decided to postpone the continuation¹⁶ of Paris to the return trip (Urysohn), respectively eternity (Alexandrov). The day before yesterday we arrived here, and it took us a whole day until we could find a quiet place on the coast.

In the same cover you will find our curricula vitae, as well as a letter to you, which should count as the official statement of our wish to come to Amsterdam.⁽¹⁶³⁾

As far as mathematics is concerned we have, naturally, as yet little news. By the way, Urysohn has found a space which not only in the topological sense (like Hilbert space) but also in the metric sense, may be considered the largest metric space with a countable everywhere dense subset. More precisely formulated: there exists a metric space with a countable dense subset which contains for every other metric space with a countable dense subset

¹⁴with the greatest torment: Paris is more horrible than I ever thought. ¹⁵and Alexandrov has cursed so much and has become so unbearable. ¹⁶four days

⁽¹⁶²⁾Alexandrov and Urysohn had visited Brouwer in Laren in the middle of July. As Brouwer had to go to Göttingen, the two moved on to France. ⁽¹⁶³⁾Alexandrov and Urysohn – Rockefeller grant.

a subset that is congruent (= admitting a distance preserving mapping) to the latter.— There are actually several ‘universal’ spaces of this kind, but only one that satisfies certain homogeneity conditions.

We would like to express once more our warmest thanks for the extraordinarily friendly reception, that we found at your place, and we thank also both ladies, whom we caused so much trouble.

Moreover we apologize once more because of the alarm clock. Please write to us whether you didn’t forget to take anything with you to Göttingen because of that. In any case, do write something to us about the trip to Göttingen and your stay there; every detail ¹⁷ (164) will interest us.

With best greetings to you and both the ladies.

Most cordially yours, (165)

Paul Urysohn

Paul Alexandroff

Our address is (until 25 VIII):

Le Batz (Loire Inférieure), Pension de famille ‘Le Val Renaud’

[Signed autograph – in Alexandrov]

1924-08-21

To H. Kneser — 21.VIII.1924

Bad Harzburg

Krodotal 4

Dear Mr. Kneser [Lieber Herr Kneser]

You probably will also have received the crushing message that Urysohn has drowned in France while bathing. ⁽¹⁶⁶⁾ It is an incredible blow of fate. Alexandroff will probably arrive the day after tomorrow in Göttingen. Should you or Miss Noether learn about the hour of his arrival, would you please

¹⁷I totally reject the responsibility for the use of this word in German, and also for its gender. I call Urysohn nowadays ‘Baberuschka’, which always makes him mad. Please explain to him that the Russian sense of the word fits him perfectly!

Furthermore Alexandroff develops language- and other theories that differ only from the ones of Denjoy by sign.

⁽¹⁶⁴⁾Urysohn used in the letter the word ‘Detail’ with neutral gender. ⁽¹⁶⁵⁾*Ihre herzlichst ergebene.* ⁽¹⁶⁶⁾August 17.

inform me immediately, if necessary telegraphically. I, on my part, part will do the same for you.

Your
L.E.J. Brouwer.

[Signed autograph – in Kneser]

1924-08-31

To P.S. Alexandrov — 31.VIII.1924

Laren ⁽¹⁶⁷⁾

Dear Alexandroff,

I have received both your letters, and I am all the time in my thoughts with you. Yet I would not pray, in accordance with your statement, that you will not have a long life. In the first place, because we may not pray on account of objective events, but only for the sake of clarification of our consciousness of duty and for the sake of bearing the trials that are imposed on us.

In the second place, because our existence on earth has been granted to us exclusively for purification of our soul of the original sins of fear and desire, and it is only according to the fulfillment of this goal that the life span of the righteous man is measured.

Just for that reason the death of a righteous man has for himself always the character of a satisfaction, a liberation and a redemption, and we must continue to bring him after his death just our love, and not our pity (compassion), in particular not when his death passage has been light.

And for those who are left behind in mourning the following holds: every grief for the heart that suffers it has its purifying meaning, and in the days of grief it is often easier than in the days of joy, to become aware of the proximity of God, because the grief – to be endured in tranquility – forces to dematerialization.

May this also happen to you.

⁽¹⁶⁷⁾ Addressed: Twerskajastr. Pimenowski pereulok 8kb5, Moskau.

For our coming meeting here in the autumn I will also get the necessary things arranged following your indications.

In faithful friendship

your

L.E.J. Brouwer

[Signed autograph – in Alexandrov]

24-09-07

From Mrs. C. Alexandrov — 7.IX.1924

Smolensk

Dear Professor, [Sehr verehrter Herr Professor]

In that happy time, when both of my boys⁽¹⁶⁸⁾ were still together, I blessed you for your hospitality and for your touching relation to them. Now in my suffering you have understood with your sensitive heart, that only the certainty that you are with my poor lonely son can sooth me.

The other one, a rarely gifted, happy, innocent child, who had never experienced suffering, was taken by his heavenly Father, to relieve him from all earthly worries, that he would have had sooner or later in order to pay for his cloudless happiness. But the remaining one has gone through many sorrows notwithstanding his youth, and now he is completely broken by this last heavy blow.

Words do not suffice, dearest professor,⁽¹⁶⁹⁾ to express my appreciation for your great compassion and your warm sympathy.

In profound gratitude I shake your hand and wish you the best in life. I send the warmest greetings for your dear family members.

Sincerely yours⁽¹⁷⁰⁾

Your devoted
C. Alexandroff

⁽¹⁶⁸⁾ P.S. Alexandrov and P.S. Urysohn. ⁽¹⁶⁹⁾ *verehrter Herr Professor*. ⁽¹⁷⁰⁾ *Mit Hochachtung verbleibe ich — Ihre ergebene*.

With true fear in my heart I await the reunion with my son. I know I will not be able to console him; he is straying for a long time to come, maybe forever, from the path in life that they went together!

[Signed autograph – in Alexandrov]

1924-10-13

To P.S. Alexandrov — 13.X.1924

Laren

My dear friend, [Mein Lieber Freund]

I just received your letter of October 5, with your so beneficent, faithful sympathy. My well-being in bed leaves nothing more to be desired and my recovery is making steady progress, only not particularly quickly, and moreover the doctor made it clear that I have to be careful for a long time to come and that I must take care of my health. I have in fact not been able to get a diagnosis with a scientific of a disease name out of him; he only spoke about ‘influenza with complications’. With the return of well-being also came giving up copious amounts of sputum, which still persists, but every day in smaller amounts.

From America they further ask me how much your trip from Moscow to Amsterdam will cost. Please inform me about this by returning mail, and make your calculation for a *comfortable* trip.

Sierpiński answers me that the introduction and Chapters I and II of Paul’s⁽¹⁷¹⁾ Mémoire will appear in volume VII of the Fundamenta, that he is willing to include the whole remainder (i.e. Chap. III–VI) in volume VIII, and that this volume VIII probably will appear in the fall of 1925. Maybe he expresses himself a little too optimistically, but anyway I am of the opinion that we should take no steps for the time being with Sierpiński, and at least leave the matter for volume VII as now planned. We can discuss the rest, if necessary, here.

Meanwhile it seems that we unfortunately have to take into account the possibility that Kuratowski already has on his own authority declared the introduction of Paul’s Mémoire ready for printing (although on the other hand, such an act without the authorization of Paul’s heirs should appear incomprehensible to me) and that hence the footnote 3) which was criticized

⁽¹⁷¹⁾P.S. Urysohn.

by me ⁽¹⁷²⁾ on the ninth proof page cannot be changed anymore. For the rest the corrected proofs don't look ready for printing at all: they still contain many annoying printing errors.

What you write about the depersonalization of your life, are words to my heart. The new path that you will now follow, will actually connect you ever more with eternity, and the awareness of the 're-connection' (= religio) with eternity will bring you ultimately joy and happiness (even though of a quite different, very quiet and pure sacred 'ultraviolet' kind). I hope with all my heart that you have made the right decision in relation to your wife, in fact I assume you did, because you have reached the decision only after consulting with your mother. ⁽¹⁷³⁾ Unfortunately I have lost the address of your mother, would you please give it me once more?

Preserve your inner peace; my thoughts are with you and I greet you most warmly.

Your Brouwer.

[Signed autograph – in Alexandrov]

1924-10-20b

To P.S. Alexandrov ⁽¹⁷⁴⁾ — 20.X.1924^b

Laren

My dear friend [Mein Lieber Freund]

I am happy that I can tell you that I am going out again, and that I hope to take up my lectures again next Friday. It seems to me as if I have come back into the light from a dark abyss. Whether I will recover without being permanently affected, the doctor cannot say for certain yet, but he is accordingly ⁽¹⁷⁵⁾ on that point rather confident.

I hope to be able to write soon to your mother, I am just waiting for her address. Give the family Urysohn many greetings, and think of me as I do of you.

⁽¹⁷²⁾This is the footnote in which Urysohn refers to Brouwer's dimension paper of 1913. It was revised more than once. See [Van Dalen 2005] section 15.5. ⁽¹⁷³⁾Refers to Alexandrov's divorce. ⁽¹⁷⁴⁾Adressed: Prof.Dr. Paul Alexandroff, Twerskaja str., Staropimenowski pereulok 8kb5, Moskau. ⁽¹⁷⁵⁾The text is rather enigmatic here; 'nonetheless' would fit better. A slip of the pen?

I look at you and shake your hand.

Your
Brouwer

[Signed autograph, postcard – in Alexandrov]

1924-10-21

To H. Kneser — 21.X.1924

Laren

Dear Mr. Kneser [Lieber Herr Kneser]

You probably have received the proofs of your article *Ein topologischer Zerlegungssatz*.⁽¹⁷⁶⁾ Unfortunately I didn't succeed in arranging that you will be sent free of charge more than the statutory number of 25 offprints. If you want more, then I advise you to write beforehand a line on the proofs, in which you inquire about the price, and then let me know the answer given to you, together with your view on the appropriateness of that price, so that I can, if necessary, complain about it.

In a few days a student of mine (or rather of Weitzenböck) will come to Göttingen for the winter semester. He is called Van der Waerden; he is very bright and has published something already (in particular about the theory of invariants). I don't know whether at the moment the required formalities for a foreigner who wants to register as a student are difficult; in any case it would be most valuable for Van der Waerden if he would find there some help and guidance. May he perhaps call on you one day to talk things over? Many thanks in advance.

With best greetings

Your
Brouwer

[Signed autograph – in Kneser]

⁽¹⁷⁶⁾a topological decomposition law [Kneser 1924].

1924-11-02

To K. Menger — 2.XI.1924

Laren

Dear Mr. Menger, [Sehr geehrter Herr Menger]

Enclosed the revision of your note for the Amsterdam Proceedings,⁽¹⁷⁷⁾ which I ask you to return *to me*, if possible ready for printing. I call your attention to both changes (on p. 2 and p. 5) underlined in pencil on the old proofs, because they originate with me and are of a factual nature: I hope you will agree with them. Furthermore I also enclose for your information the proofs of a note by me on the same subject, with the request to return them.

I welcome your plan to come to here after the end of the winter semester. The Easter holidays in Holland are brief, about three to four weeks in April, whereas the summer vacation starts here earlier than in Germany and Austria.

I would be most grateful if in relation with the slip of the pen of my minor oversight in Crelle 142,⁽¹⁷⁸⁾ you would replace in the manuscript of your article submitted to the Monatshefte: ‘On the dimensions of point sets. Part two’⁽¹⁷⁹⁾ on p. 6 in the footnote the words:

‘given in a but little known brief article (Crelle Journal 142, p. 146–152) a definition of n -dimensional continua, which after a correction of a clerical error in the (Amsterdam Academy Proceedings XXVI, 1923⁽¹⁸⁰⁾) is equivalent with our definition of the n -dimensional continuum.’

by the words

‘in a but little known brief article (Journal für [die reine und angewandte] Mathematik 142, p. 146–152; cf. also the correction of a clerical error in there in the Amsterdam Proceedings 26, p. 796), which is equivalent with our definition of an n -dimensional continuum.’

I believe that in this way the reader gets an idea that does more justice to the facts. It is also better to mention the ‘Amsterdam Proceedings’, rather than of the ‘Amsterdam Reports’,⁽¹⁸¹⁾ because the latter usually does not refer to the ‘Proceedings’, but to the ‘Verslagen’.

⁽¹⁷⁷⁾[Menger 1924a]. ⁽¹⁷⁸⁾[Brouwer 1913d]. ⁽¹⁷⁹⁾Über die Dimension von Punktmen-
gen. Zweiter Teil, [Menger 1924b]. ⁽¹⁸⁰⁾[Brouwer 1923a]. ⁽¹⁸¹⁾Amsterdamer Berichte.

With best greetings, and in the hope soon to get to know you personally,

Your ⁽¹⁸²⁾

L.E.J. Brouwer

[Signed autograph – in Menger]

1924-11-06

From C. Carathéodory — 6.XI.1924 ⁽¹⁸³⁾

Blumenthal asks me to answer your last letter in which the passage about Painlevé is contained.

Basically, I completely share your view, but I wonder whether one should forever recall the nonsense that in all countries has been put together during the war; because then one would not have had to stop the shooting at all.

Especially where the Riemann volume is concerned, ⁽¹⁸⁴⁾ in my opinion it isn't really necessary to have French mathematicians there too. But if one so wishes, there is no other way to do this in a decent way, then by turning to Painlevé in the very first place. For Painlevé is among the French mathematicians the only one who holds a sufficiently secure position to take part in the Riemann volume, without running the risk that the whole pack of the narrow minded ⁽¹⁸⁵⁾ starts barking at him. Moreover, through Nernst I know that while he was rector, Painlevé had offered to give a few talks at the University of Berlin, and that – in spite of the fact that the Foreign Ministry was interested in the case – this came to nothing because of the opposition of a few Berlin professors. So you see, that at least according to this report, Painlevé seems to have forgotten the words that you hold against him.

[Typescript – in Einstein ⁽¹⁸⁶⁾]

⁽¹⁸²⁾ *Ihr sehr ergebener.* ⁽¹⁸³⁾ This letter was, according to *Carathéodory to Blumenthal 20.I.1925*, not received by Brouwer. ⁽¹⁸⁴⁾ *Mathematische Annalen* 97, 1927. For the conflict see [Van Dalen 2005] section 13.3. ⁽¹⁸⁵⁾ *Banausen.* ⁽¹⁸⁶⁾ Fragment in collection of letters re Riemann volume.

1924-11-13

From K. Menger — 13.XI.1924

Vienna

Fuchsthallergasse 2, Wien IX

Dear Professor, [Hochverehrter Herr Professor]

After the settlement of a protracted railway strike that endangered the mail service abroad, sending this letter is my first priority. Dear Professor, I cannot thank you enough for the attention you have paid to my small note, also for your kind letter and for sending me your article which I already had read with great interest in the ‘Verslagen’ of April 28, 1924.⁽¹⁸⁷⁾ Although one would at first expect that the N - and the MU -dimension⁽¹⁸⁸⁾ would coincide for non-condensed species, this is generally *not* the case.— Basing dimension theory on separation definition b) would certainly be very interesting — both theories⁽¹⁸⁹⁾ would perhaps in a certain respect relate to each other like the theory of simple curves to that of irreducible continua.

In the duplicated⁽¹⁹⁰⁾ text of my article about dimension (II)⁽¹⁹¹⁾ I have made some small changes before the printing. I have inserted a definition of dimension of sets that are considered in themselves. In particular I have improved in wording the awkward formulation of the footnote on page 6 along the lines that you, dear Professor, suggested, already before receiving your letter.

In the last few days I had to give a talk about research in the foundations of mathematics in a privatissimum of the epistemologist Prof. Schlick. It may have been the first time that an extensive exposition of intuitionism has been given in Vienna. The lecture was followed by a long discussion.

It would be a great joy for me to receive in a few months time instruction from your lectures, dear Professor, about these fundamental questions that

⁽¹⁸⁷⁾[Brouwer 1924c]. ⁽¹⁸⁸⁾ N : Natürliche, i.e. Brouwer (separation) Dimension; MU : Menger-Urysohn Dimension. ⁽¹⁸⁹⁾Brouwer and Urysohn based their definition of dimension on separation; Menger’s dimension definition made use of boundaries. All definitions were inductive. The Menger-Urysohn dimension was a *local* one, in the sense that dimension was considered in points; Brouwer’s dimension was *global* in the sense that it concerned the dimension of the *whole* space. See [Menger 1928b], [Urysohn 1925, Urysohn 1926]. A new and surprising fact on the relation between Brouwer’s dimension and the Menger-Urysohn dimension can be found in [Fedorchuk and Van Mill 2000]. ⁽¹⁹⁰⁾in text ‘opalographiert’. ⁽¹⁹¹⁾Published as [Menger 1924b].

are very close to my heart. Meanwhile, please receive the expression of my deepest reverence and gratitude.

Your ⁽¹⁹²⁾

Karl Menger

[Signed autograph – in Brouwer]

1924-12-21

To P.S. Alexandrov — 21.XII.1924

Laren

Dear chap [Lieber Kerl]

Today it furthermore occurred to me that for the fortification of our point of view (indeed, it is a real war, our point of view plays the role of a fortified encampment, in the wall of which a breach has been shot as a consequence of your ill-fated letter to Sierpiński, which we must close with might and men), that it would also be most important for the fortification of our point of view to insert at the end of the first paragraph of page 2 of the outline to Kuratowski (i.e. after the words ‘share mine with Mr. Brouwer’) more or less the following:

‘Moreover, while writing my unfortunate letter to Mr. Sierpiński, I knew already that Mr. Brouwer had asked to read the proofs as well; I was convinced that thereby the correction of the proofs would adequately take into account the exchange of ideas, that took place before the death of my friend, between him and Mr. Brouwer. And because, moreover, I have agreed with Mr. Brouwer that he would not take any important decision without consulting me, one could in fact hardly dispute the necessity that I receive the proofs too.’ ⁽¹⁹³⁾

Please forgive my insistence in this matter: perhaps I am making a nervous impression on you, but innerly I have the firm and calm conviction of the necessary actions, as well of my own helplessness without your strong support. For, the fact that the people in Warsaw don’t bother even to the slightest degree about me, already follows from the fact that after I asked Sierpiński in September, while sending him the mandate of Paul’s ⁽¹⁹⁴⁾

⁽¹⁹²⁾ *Ihr ganz ergebener.* ⁽¹⁹³⁾ This passage is in French. ⁽¹⁹⁴⁾ Urysohn.

father, to send the proofs *to you and to me*, Sierpiński, in reaction to your fateful letter, withheld the proofs, not only from you, but also from me, and this without any notification.

One more point is unclear to me: now both Sierpiński and Kuratowski write to you, as if there was from the outset never any plan to send out proofs of Paul's *Mémoire*, and as if Paul himself had agreed to that. How then was it possible that you received in Batz the proof of the first sheet of the *Mémoire*!

Should there come a definite refusal from Warsaw, then as last *medicamentum heroicum* ⁽¹⁹⁵⁾ we have still this, that I withdraw the whole printed *Mémoire* in the name of Paul's heirs, who possess the literary property rights. Then the *Fundamenta* *may* not publish it, and the editorial board will, with a probability of 95 percent, back off from the ensuing complications, and at last conform to our wishes. Should the editors even then not give in, then we get the manuscript back, which will be printed again within a few weeks in Amsterdam, and in much better form than in the *Fundamenta* with its bad paper and the many printing errors. In that case the management of the *Fundamenta* will have a damage claim because of the wasted typesetting work, but I'll gladly bear that.

In connection with your pass and your residence permit Pannekoek has now written to Rutgers (a Dutch mountain engineer who has a high position in the service of the Russian government) and I myself to Varjas (professor of the red professorate, Ostoschenka 53, Moscow). Moreover, within a few days a letter will be sent from my faculty to yours.

Now, my dear boy, very soon more. If only you were just here! But for now, we meanwhile stand, distanced as we are, calmly and firmly side by side, in unflinching passive resistance!

With the warmest greetings, also from my family

Most cordially yours ⁽¹⁹⁶⁾

L.E.J. Brouwer

[Signed autograph – in Alexandrov]

⁽¹⁹⁵⁾ Kill or cure medicine. ⁽¹⁹⁶⁾ *Herzlichst Ihr.*

1925-02-11

From K. Menger — 11.II.1925

Vienna

Fuchsthallergasse 2, Wien IX

Dear Professor, [Hochverehrter Herr Professor]

Please accept my sincere thanks for your kind card, which made me very happy. Above all because I see from it that your health has improved and that I may hope to attend in the spring your lectures, dear Professor. And then, because of your great kindness to submit my article about curves to the Amsterdam Academy and to consider the extensive article that I enclose with this letter for the *Mathematische Annalen*.⁽¹⁹⁷⁾ I cannot thank you enough, dear Professor, for the extraordinary support that you give to my work.

Recently I have studied your articles on the foundations of mathematics again, first of all ‘Mathematics, Truth, Reality’.⁽¹⁹⁸⁾ Although I still need ample instruction concerning your positive construction of mathematics,— I feel the urge to tell you, dearest Professor, that your criticism of pure existence statements in arithmetic has now convinced me. Theorems of that kind are empty forms, which only can acquire a meaningful content by constructive realization. That such a constructive realization would always be possible,— for that no reason has been given until now, and when one bases oneself on constructive foundations, may perhaps not be given at all. One can at most *believe* in the possibility of such a completion, but then the rigor of constructive argument has come to an end. Since all of this has become clear to me, I look with deep admiration at your work, by which you take hold of age-old prejudices by the root, and pursue them to their far-reaching consequences.

Dear Professor, accept the expression of my greatest admiration and sincere thanks.

Sincerely yours⁽¹⁹⁹⁾

Karl Menger

[Signed autograph – in Brouwer]

⁽¹⁹⁷⁾[Menger 1925a, Menger 1925b]. ⁽¹⁹⁸⁾*Wiskunde, Waarheid, Werkelijkheid*, [Brouwer 1919e]. ⁽¹⁹⁹⁾*Ihr ganz ergebener*.

1925-06-22b

From B. Kagan — 22.VI.1925^b

Odessa, Ukraina

Tschernomorskaja, 20

Dear colleague, [Sehr geehrter Herr Kollege]

With great joy I have obtained a couple of days ago a series of your articles and the longer treatise on the foundations of science, which you sent to me through my friends P. and T. Ehrenfest. Unfortunately they have been written in a language that I have no command of. But I hope to conquer this obstacle, many a page I have mastered already. Soon the holidays will start, then I will have time enough to master the Dutch language through your works. Probably only the first pages will offer serious problems. In any case, I will not give up this enterprise because the questions to which your works are dedicated interest me highly; they were cultivated in our school in Odessa for quite some time, as I believe not quite without success. The border areas between mathematics and logic pose very great difficulties to a strict scientific treatment, which are mainly rooted in logic.

From a number of references in the literature I have found that you have published in 1920 a treatise about the law of the excluded third. This question was posed already several years ago by Professor S.O. Schatunowsky here and, insofar as I can judge from the scant indications in your article about the ‘Set Theory’ of Schönflies and Hahn, Schatunowsky’s ideas in essence hardly differ from yours. Prof. Schatunowsky has published a substantial treatise in Russian, ‘Algebra as theory of congruences on functional modules,’⁽²⁰⁰⁾ which has mainly (though not exclusively) the aim to develop algebra while completely avoiding the law of the excluded middle, so also while avoiding the theory of irrational numbers based on it. We had planned to have this work published also in German, but the war and further events have prevented that. So we were looking forward to the above mentioned article with special interest, and we regret very much that we didn’t get it. Don’t you have a copy of it? We would be most happy to have this article in our library.

Both in my name and also in that of my colleagues I thank you most warmly for the articles sent to us, and we politely request to make also your

⁽²⁰⁰⁾ [Shatunovsky 1920].

further articles in this way accessible to us.

Sincerely ⁽²⁰¹⁾

Ben. ⁽²⁰²⁾ Kagan

Professor in Odessa.

[Signed autograph – in Brouwer]

1925-07-03

From K. Menger — 3.VII.1925 Semmering (Niederösterreich)

Kurhaus Semmering

Dear Professor, [Hochverehrter Herr Professor]

I thank you most warmly for your kind words of condolence for the terrible stroke of fate that has befallen me.

I cannot express in words what I have lost in my dear mother, her good-heartedness was boundless. And to the sorrow that she has been taken away from me is added the indescribably tormenting thought that she, who has since I was born, done and sacrificed so much for me, died right now, when finally a more peaceful evening of life had begun for her, to which she was looking forward to with great pleasure, still being able to enjoy it.

Deeply interested, she followed from a distance everything that concerned me, with gratitude in particular towards you, dearest Professor, ⁽²⁰³⁾ for all the favor and support you showed me. These tidings were her last joy.

An emptiness that cannot be filled has been struck in my life.

My mother was never ill, except for colds and in the last years occasionally a lumbar pain, which she thought was rheumatic. Now we know that this must have been the unobtrusive symptoms of an advancing nephritis. Because after a seemingly slight indisposition of two days she succumbed to a sudden kidney attack. The slight mental confusion that commonly in the last hours goes with this treacherous disease let her pass away without any inkling of her condition and without pain.

⁽²⁰¹⁾ *Hochachtungsvoll ergebensts.* ⁽²⁰²⁾ Benjamin Fedorovich ⁽²⁰³⁾ *verehrter und gütiger Herr Professor.*

Only after this terrible event the telegram was sent to me; I obtained it the same night in Heidelberg, thanks to the prudent forwarding from Laren, so that I could arrive the next day in Vienna. If the only relatives that I have at all, a sister of my dear mother and her husband, had not taken care of me, taken me in and sacrificed themselves to nurse me with the help of friends,— then I wouldn't know how I could have survived these days without going mad. Even so I laid down for a week, ill and half out of my mind. As of today I am in the Semmering,⁽²⁰⁴⁾ where I must regain my strength through a rest-cure of several weeks in the open.

During this time I often thought of the poor Urysohn and I wished that I had perished in his place. Only the thought that I should not destroy what my beloved mother had built up with so much effort in her life, now gives me the will to regain my health, if possible, and then to achieve something.

I stop for now, dearest Professor, to write again to you as soon as I have gathered more strength. Meanwhile, rest assured of my sincere veneration and gratitude.

Yours devoted⁽²⁰⁵⁾

Karl Menger

P.S. I had written the enclosed letter⁽²⁰⁶⁾ that evening, unaware yet of the events. In the confusion it got into my luggage, where I found it only today.

[Signed autograph – in Brouwer]

1925-07-08

To K. Menger — 8.VII.1925

Laren

My dear Mr. Menger, [Mein lieber Herr Menger]

I thank you for your letter of the third of this month, which gives me in any case the relief that you have withstood the stroke of fate which has struck you so suddenly. For that reason I have worried very much, because during your stay here I have felt strongly to what great extent the aura of your mother irradiated your life. So I surmised how great your loss was, and I expected the crisis that would be unleashed in you by the sudden emptiness

⁽²⁰⁴⁾ a sanatorium. ⁽²⁰⁵⁾ *Ihr ganz ergebener.* ⁽²⁰⁶⁾ Not extant.

and the sudden necessity to assume a different spiritual way of inhaling. But after you have weathered the first crisis, I am certain that you will find the necessary concentration and religious dedication to work your way through, and that the certainty about the wish in that direction of the dear departed, and also the during, serene memory of her will help you with that.

Because I don't know whether your situation has possibly now also worsened in pecuniary respect, I have preferred to propose you already now for the assistant's position, mainly because one also can't be sure whether and when the Rockefeller stipend will be awarded. (I just received a letter from Paris in which in the first place recommendation letters are required from your teachers in Vienna, not including prof. Hahn, and secondly reprints of your publications until now. I would like to ask you to send the reprints directly to Dr. Trowbridge, Agent for Europe of the Rockefeller Foundation, 22 rue de l'Elisée, Paris 8^c; for the recommendation letters Weitzenböck will turn to Wirtinger). I have managed to get a salary of 3000 guilders for the assistant's position, to which can be added a personal extra allowance of 500 guilders, if necessary. Of course you should not come earlier than your health allows; but if you can be here on the first of October, your salary will start on September 16.

Within a few days I will travel with my wife to Switzerland; but until further notice my postal address will remain in Laren. I would appreciate to be kept informed about your well-being; please rest assured that my best wishes accompany you.

With cordial greeting, also from my family ⁽²⁰⁷⁾

Your
L.E.J. Brouwer

[Signed autograph – in Menger]

1925-12-15

To W. von Dyck — 15.XII.1925

Laren

Dear Colleague, [Hochgeehrter Herr Kollege]

At the same time I send you 50 copies of the enclosed document about the Conseil Internationale de Recherches, that I put together earlier. In

⁽²⁰⁷⁾ *Mit herzlichem Gruss, auch von den meinigen.*

fact this is part of the Karo brochure,⁽²⁰⁸⁾ but it may create a stronger impression if read by itself.

I would like to ask you to make arrangements that every member of the science section of the Bavarian Academy of Sciences gets a copy. Because soon the union of the German Academies of Sciences will be invited to join the Conseil internationale de recherches, which was founded only to malign and boycott Germany. Maybe some will say then: ‘Who accepts the League of Nations, can also accept the C.I.R.’⁽²⁰⁹⁾ But that would be wrong, firstly because the material necessity that pushes one to the former, does not exist in case of the latter, and secondly because the League of Nations is in the end a humanitarian American idea, while the C.I.R. is only a product of the French wish for destruction, as the enclosed composition may show unambiguously.

Sincerely yours

your⁽²¹⁰⁾

L.E.J. Brouwer

[Signed typescript, copy – in Brouwer]

1925-12-21a

To H. Hopf — 21.XII.1925^a

Laren

Dear Mr. Hopf, [Sehr geehrter Herr Hopf]

I have read with great interest the proof sheets of your article about vector fields on n -dimensional manifolds⁽²¹¹⁾ (just as, by the way, the ones of your preceding Annalen article). As far as the quotation in § 42 of the Hadamard note⁽²¹²⁾ mentioned by you, is concerned, I agree with you that this is not correct. The explanation is that the contents of both my article *Ueber Abbildungen von Mannigfaltigkeiten*⁽²¹³⁾ and the note by Hadamard in the book by Tannery have been discussed by Hadamard and me around Christmas 1909 in Paris. On that occasion I have—referring to a couple of articles of mine that were in part in print, in part waiting for the final

⁽²⁰⁸⁾[Karo 1926]. ⁽²⁰⁹⁾Conseil Internationale de Recherches. ⁽²¹⁰⁾Mit hochachtungsvollem Gruss – Ihr sehr ergebener. ⁽²¹¹⁾[Hopf 1926]. ⁽²¹²⁾[Hadamard 1910]. ⁽²¹³⁾On mappings of manifolds.

editing—stated, among other things, also the theorem, proved now by you for the first time, and I was holding out the perspective of a publication of its proof in my article ‘On mappings of manifolds’, that was at the time available in preliminary version, and originally intended for publication in the Amsterdam proceedings ⁽²¹⁴⁾

To this circumstance on the one hand the quotation in § 42 of the Hadamard note, and on the other hand the description of the theorem in § 40 as ‘théorème de Brouwer’, are to be attributed. It was an omission on my part that later I didn’t tell Hadamard in time that I would submit the article not to the Amsterdam Proceedings but to the *Mathematische Annalen*, and also that the implementation of the proof of this theorem finally became so complicated that I had to abandon its publication for the time being.

So because of the above I would like to ask you to make the following changes (the present formulation would among others imply that my article ‘On mappings of manifolds’ was based upon the already present note of Hadamard and that the latter note was written independently from me): ⁽²¹⁵⁾

- l. 11 ‘*bereits kurz vor*’ to be replaced by ‘*ungefähr gleichzeitig mit*’
- l. 12 ‘*von Hadamard*’ to be replaced by ‘*von Hadamard ohne Beweis*’
- l. 18–22 ‘*genügt; Hadamard will Beweis befindet.*’ to be replaced by ‘*genügt*³⁾. *Wie mir Herr Brouwer mitteilt, sind übrigens die Brouwersche und die Hadamardsche Arbeit unter Gedankenaustausch zwischen den beiden Verfassern entstanden.* ⁽²¹⁶⁾

I was sorry that I could not get you as assistant in Amsterdam. For a single semester it would serve no purpose to come, I absolutely need someone who can stay long enough to immerse himself thoroughly in the local activity.

In the hope that I can get to know you soon personally, I remain with best greetings

Your ⁽²¹⁷⁾

L.E.J. Brouwer

[Signed typescript – in Hopf]

⁽²¹⁴⁾ *Proceedings KNAW*. ⁽²¹⁵⁾ Hopf adopted Brouwer’s suggestions in his manuscript.

⁽²¹⁶⁾ As Mr. Brouwer informs me, the articles of Brouwer and Hadamard have grown out of an exchange of thoughts between the two authors. ⁽²¹⁷⁾ *Ihr sehr ergebener.*

1926-04-10a

From H. Hahn — 10.IV.1926^a

Vienna

Hahn to Brouwer 10.4.1926⁽²¹⁸⁾

Dear Colleague, [Lieber Herr Kollege]

I will tell you with great pleasure what I know about the genesis of the first articles of Menger. I am in the position to do that as well in my quality as university professor because Mr. Menger sought my advice repeatedly when he was writing his Ph.D. thesis, as in my quality as publisher of the *Monatshefte für Mathematik und Physik*, because I attached importance to the publication of the first results of Menger in this journal.

I conducted a seminar in the summer semester of 1921 on some problems in the theory of point sets. I opened this seminar in the first days of May with a talk in which I pointed out that a fully satisfactory definition of the curve concept didn't exist yet. Quite soon afterwards Menger, whom I had not known until then, came to see me, to find out what I thought about a definition of this concept which he had thought out, stimulated by my talk. I saw immediately that Mr. Menger was on the right track, which had been before my mind's eye since 1914 as the one that should lead to a natural definition of dimension, without however pursuing these quite vague thoughts at that time. I was especially glad that now a young man all by himself followed precisely that direction. As is unavoidable with a young student who is in the stage of familiarizing himself with some field, Menger's definition of a curve at first had an essential defect (but the fundamental idea was already the final one); I pointed out this defect and challenged him to deal with it by thinking a bit more. A first written sketch from that time is still available. Mr. Menger succeeded very quickly in redressing the defect. Also, in February 1922 he had already recognized with complete clarity that the path he took would give a recursive definition of the concept '*n*-dimensional'. This definition is described in extenso in a letter to me of February 15, 1922, which is in my possession. In fact Mr. Menger must have possessed the essential parts of this definition even earlier, because in the letter it says: 'I had ended the small article which you, Professor, have been so very kind to read, with a definition of the *n*-dimensional set, which should have been,

⁽²¹⁸⁾ In pencil in Brouwer's handwriting.

as I now believe, as follows etc.’ But I can’t recall this earlier version anymore.

I received from Mr. Menger a completely revision in November 1922. Now everything was completely correct, only the importance of the covering theorem that now bears Menger’s name wasn’t recognized, which is not essential for the question now at hand.

Summarising, I observe: Mr. Menger was stimulated by my seminar talk in May 1921 to search for a satisfactory definition of the curve concept. In next to no time he had found the right way. Pursuing this route he had found in February 1922 a recursive definition of the concept ‘ n -dimensional’. A final written exposition was in my hands in November 1922. That also elsewhere work had been done on these concepts nobody here in Vienna knew.

I hope that with this I have clarified everything that needs to be known.
With best greetings

Your ⁽²¹⁹⁾

H. Hahn.

[Signed autograph – in Brouwer]

1926-04-10b

From K. Menger — 10.IV.1926^b

Vienna

Fuchsthallergasse 2, Wien

Dear Professor, [Hochverehrter Herr Professor]

I write to you only today, because the priority matter is now completely settled: not only all evidence mentioned by me, but also other material has been found that I myself had already forgotten. I have put all the material in a safe and I will hand over personally all the originals to you. Today I only mention shortly the *officially* certified documents:

1) A manuscript submitted in *June* 1921 to the Monatshefte, containing a definition of the constructs that I later named ‘regular curve’, furthermore a definition of end- & branching points.

⁽²¹⁹⁾ *Ihr ergebener.*

2) The letter at the Academy containing the full curve definition, the definition of surface, the definition of the n -dimensional continuum, of end points and branching points, and a few important theorems about curves.

3) A letter delivered on February 15, 1922 to the Monatshefte, which has been placed in safekeeping by the editors, containing *literally* my general definition of dimension (including the empty set as -1 -dimensional) and other matters.

4) A manuscript entered at the editorial board of the Monatshefte in November 1922, containing numerous theorems *together with full proofs* (among others the theorem that the union of finitely many closed n -dimensional sets is n -dimensional, with a proof, & implicitly the proof of this theorem for the union of countably many sets).

So much for your preliminary orientation. About the tension that the collecting of these documents caused me, I'd rather remain silent: if I had not kept in mind that I had to put the documents into your hands for all that you have written about the theory, and that you have done for me,— then I would not have been able to bear all I had to go through!

Now my nerves have gone completely to pieces. Yesterday I visited an excellent doctor who says that my nerves and my general condition are in a terrible state and who told me that I should spend every day I can possibly make free in absolute rest somewhere around Vienna. At the same time he advised me to be as careful as possible for some time, if I don't want to risk that my ability to work will soon be permanently lost.—

If you, dear Professor, could bring yourself to drop me a line to say that you have received my letter all right, and when you will be in Amsterdam, then I would be very grateful. I would be very happy to hear that you are having a nice vacation and recuperate well.

Sincerely yours

Your grateful ⁽²²⁰⁾

Karl Menger

[Signed autograph – in Brouwer]

⁽²²⁰⁾ *Empfangen Sie inzwischen den Ausdruck meiner verehrungsvollen Ergebenheit – Ihr dankbarer*

1926-05-11d**To A. Heyting — 11.V.1926^d****Laren**

Dear Mr. Heyting [Waarde Heer Heyting]

I have glanced through your manuscript, to my great satisfaction, although I have by no means checked the details (for which I hope to find time later), but so much is clear to me that your work is ready for international publication. So I would like to suggest to you to write a German (or, if you prefer, a French) treatise, which contains both your dissertation and these last results, and which from the outset aims at deducing the non-Pascalian number geometry from the non-Pascalian axioms, while the ‘Pascalian, non-Archimedean’ and the ‘Archimedean’ geometries are dealt with as specializations at the end in an appendix. We can discuss in more detail the manner of publication of this German treatise when it is finished or almost finished; indeed, this will depend on the size and disposition of the work. Maybe the article is suitable for one single *Treatise*⁽²²¹⁾ of the Academy of Sciences; maybe a series in a professional journal will be preferable.⁽²²²⁾

With friendly greetings,

Your
L.E.J. Brouwer

[Signed autograph, postcard – in Heyting]

1926-07-23**From M. Planck — 23.VII.1926****Berlin**

Berlin-Grunewald

Dear Colleague, [Hochverehrter Hr. College]

In the matter about which you were so kind as to inform us recently in the Academy, we meanwhile have received new information, which I think

⁽²²¹⁾Verhandeling. ⁽²²²⁾The papers were published in the *Mathematische Annalen*, [Heyting 1927a, Heyting 1927b].

I should communicate to you, before I wait until I receive the letter you promised me.

Mr. Schuster-Manchester has formally communicated to the secretariat of our Academy ⁽²²³⁾ that, in the statutes of the Conseil des Recherches the passage that referred to the London declarations has been *struck out*. If this is really true — and from the whole nature of the letter we actually have no cause to doubt — then there are two statements that are diametrically opposed, and we would be sincerely indebted to you, if you would be in the position to clarify the matter. Because any further step that we can make depends essentially on what are the facts at hand.

With collegial greeting

Yours sincerely ⁽²²⁴⁾

M. Planck

[Signed autograph – in Brouwer]

1926-08-08

From M. Planck — 8.VIII.1926

Berlin

Preussische Akademie der Wissenschaften

Unter den Linden 38

Dear Colleague, [Hochverehrter Herr College]

I have received your kind note of the 31st of last month, and as agreed I have informed the secretariat of our Academy ⁽²²⁵⁾ insofar as it is represented here during the vacation. It is a mystery to us how the Royal Society acquired the ‘certainty’ that Germany would unconditionally comply with an invitation to join the Conseil, ⁽²²⁶⁾ and I will take the trouble to find out what is the source of this myth.

We don’t think it useful to direct a formal request to the government, because we cannot at the same time produce tangible evidence, and as a consequence we can be certain that we will not get an adequate answer. However, the main thing is that this astonishing statement of the representative of the Royal Society doesn’t have the least significance for us, and that

⁽²²³⁾ *Preußische Akademie der Wissenschaften.* ⁽²²⁴⁾ *Ihr aufrichtig ergebener.*

⁽²²⁵⁾ *Preußische Akademie der Wissenschaften.* ⁽²²⁶⁾ *Conseil Int. des Recherches.*

we have an entirely free hand with respect to our position on the question of Germany's entering the Conseil.

With collegial greetings

Sincerely yours ⁽²²⁷⁾

M. Planck

[Signed autograph – in Brouwer]

1926-08-19

From K. Menger — 19.VIII.1926

Vienna

Fuchsthallengasse 2, Wien

Dear Professor, [Hochverehrter Herr Professor]

After you saw me off at my last visit with the request that I call on you again in a couple of days (in particular for the formulation of my dedication of my *Bericht*), I tried to do so four times, during the next two days, but each time I found that nobody was at home, and the housepainters confirmed this. The following day I woke up with a violent influenza, which tied me for four days to my room. My first steps as soon as I could leave the house, were to you, where I heard that you had left that same morning for an indefinite (and in any case, a longer) time. Miss Jongejan added that you had neither written to me, nor left a message for me, because you thought I had left the country without informing you. — I must tell you, professor, that it is the first time in my life I had to hear such an unjustified attribution of lack of character, education & manners.—

In view of my request concerning the dedication of the *Bericht*, I permit myself to submit it in writing, as I have now gone to my country. I was going to write:

Herrn L.E.J. Brouwer,
 entweder: dem grossen Förderer der Topologie ⁽²²⁸⁾
 oder: dem bahnbrechenden Bearbeiter der Topologie
 zugeeignet.

⁽²²⁷⁾ *Mit der Versicherung ausgezeichnete Hochachtung und collegialen Grüßen – Ihr aufrichtig ergebener.* ⁽²²⁸⁾ Dedicated to L.E.J. Brouwer, either: the great promoter of topology / or: the pioneering developer of topology.

I pray to you, revered professor, to let me know as soon as possible about this matter.

It remains for me to thank you sincerely that I had the honor to be your assistant for a year, and that you have made it kindly and magnanimously possible for me to prepare and publish a series of articles. When I join to my gratitude a plea that you do not effectuate an extension of my assistantship, then this is a decision that was hard to take, but carefully considered, of which I am certain that it also conforms to your own wishes.

I hope, dear professor, that you will soon come to Vienna and visit me, and that I may guide you through Vienna. I hope also that you will like it here. I just want to ask you, in order that I will indeed be in Vienna, that you send me a telegram two days before you arrive, and that you tell me in time what hour of your arrival, so that I can meet you at the station.

I assure you, dearest professor, of my permanent gratitude and unshakeable veneration

Karl Menger

[Signed autograph – in Brouwer]

1926-08-20

From H. Scholz — 20.VIII.1926

Baarn
Huize Ekely
(p.A. Herrn Dr. W.H. Patyn)

Dear Professor, [Hochverehrter Herr Professor]

You may as well hold me for one of the most ungrateful people that you have ever met; but I just had the misfortune, to be so badly pursued by misfortune, that it had really been impossible for me to discuss with you the theorem of the equivalence of the absurdity of the absurdity of the absurdity with the simple absurdity, which has manifested itself in the hardest conceivable form as truth to me, now also in theoretical form.

But that would have been necessary and some other things as well, as is indicated on the enclosed sheet.

I soon understood that now a renewed personal discussion could be a bit more useful for me, and that it can be organized in such a manner that no excessive claims on your time are made.

So now I choose the shortest way, namely that I put the enclosed sheets as reference documents for such a consultation into your hands ⁽²²⁹⁾ and that at the same time I ask you whether you could be so kind as to inform by telephone Mr. Patyn (149 Baarn) if and when I may visit you once more next week Tuesday, Wednesday or Friday about this matter.

For you may not conclude from my silence, that I didn't struggle the whole year with these questions that I was allowed discuss with you in the last summer.

Otherwise I couldn't have given a lecture this summer on the axiom system of classical logic and its correction by Brouwer.

At this occasion I have not only found out how much I still lack, but I believe also that I have brought the problem of consistency into a new light, for which I am indebted to your constitution of concepts as first stimulus.

The problem is this: Can we, in an ultimate reduction, prove at all the consistency of some mathematical concept in any other way than by constructing at least one object that falls under this concept?

Hilbert's consistency proof by means of the inference from n to $n + 1$ stands or falls by such an existence proof is essential, because its consistency can only be shown when there is at least one class of entities to which it can be applied.

I would like to elaborate this a bit more extensively with you.

Because what was shattered last winter, should be accomplished this winter.

In January or February, I will speak about the crisis in the foundations of logic for the Berlin Kant society.

But in any case you must finally know now, that I have a better memory than you thought, and that the severe personal inhibitions that also prevented my access to you don't prove at all that I didn't remember you in the most sincere gratitude.

Sincerely yours ⁽²³⁰⁾

Your
Heinrich Scholz

⁽²²⁹⁾ Enclosures: 1. Classification of consistency [We have translated *Widerspruchsfreiheit* (in Scholz's text systematically abbreviated as 'WF') by 'consistency', where 'freedom of contradiction' would be somewhat artificially archaic.] consistency - propositions, 2. On the place of Hilbert's concept of consistency, 3. Consistency etc. 4. Problematic and unspecified concepts ⁽²³⁰⁾ *In grösster Hochschätzung – Der Ihrige.*

From August 28 on my address will be again:

c/o Mr. Justus Meyer
30 Zandvoortsche Laan
Zandvoort.

Postscriptum: (1) Mr. Patyn will drive me in his car to you, so that we don't have to reckon with the trains.

(2) Only after many doubts I decided to send you the enclosed material. Please consider it merely as a preliminary study, and allow me to ask it back so I can elaborate it further.

[Signed autograph – in Brouwer]

1926-12-13

To H. Hopf — 13.XII.1926

Laren

Dear Mr. Hopf [Lieber Herr Hopf]

I believe that it is best for Miss Gawehn (also in the interest of her possible later scientific career) that she first takes the state examination, and then coming fall applies for a Rockefeller grant to study for a while in Amsterdam. If she qualifies herself well during these studies with me, I would be happy to consider her *subsequently* for an assistantship. At the moment she would not be of use for me notwithstanding her evident talent; she has not enough command of the subject matters and also too easily makes errors. (I base this all on her manuscript that she submitted months ago to the *Annalen* ⁽²³¹⁾ which I see gradually getting ready for printing, and about which Menger as my assistant is corresponding with her.) In case I, as I hope, soon come to Berlin for a few weeks, I will also find an opportunity to speak with Miss Gawehn about her plans for the future. ⁽²³²⁾

Would you be so kind to read through the continuation of the investigation of Wilson on the mapping degree, of which you have refereed the first part during the last summer? I permit myself to send the manuscript concerned with the same mail, together with the page proofs of the first part, which you know already. ⁽²³³⁾ Many thanks in advance for your efforts.

⁽²³¹⁾[Gawehn 1928]. ⁽²³²⁾For Gawehn see [Van Dalen 2005] p. 567. ⁽²³³⁾[Wilson 1928].

I am very eager to see your own further publications and I am very much looking forward to our meeting again, hopefully before long, in Berlin.

Cordial greetings!

Your
Brouwer

[Signed autograph – in Hopf]

1926-12-21

To A. Fraenkel ⁽²³⁴⁾ — 21.XII.1926

Laren

Dear Mr. Fraenkel, [Lieber Herr Fraenkel]

I cannot tell you how dumbfounded I was, when hardly three weeks after I received the first proofs, ⁽²³⁵⁾ you gave me to understand that the time for taking into account possible suggestions for changes had already expired. What kind of wizard you must have taken me for that you required me, in the middle of the semester and with all my time as good as completely occupied with other things, to study a book of more than 100 pages, and do it so thoroughly that I could bear the responsibility for suggestions to change something. Even today I haven't yet finished my judgments concerning details, I will indeed still need also the Christmas week for that. With the inexplicable hurry, which in my opinion is damaging for all parties (author, publisher, public) the only way out for me is, that I incorporate all my marginal remarks into a review of your book, in which I will however have to put right quite a lot (especially as far as intuitionism is concerned), but it is maybe just as well that I have a reason to deal with the erroneous information about intuitionism which is given to the public from so many sides. In order that meanwhile my review can remain as free of personal matters as possible, I would like to suggest to you three small changes, which certainly can still be corrected on the proof sheets: 1) delete the (indeed completely unfounded) insinuation in footnote ¹²⁾ (sheet 18); 2) in the text of sheet 20, lines 21 from below, 15 from below and 7 from below,

⁽²³⁴⁾ Addressed: Breiter Weg 7, Marburg (Lahn). ⁽²³⁵⁾ proofs of Fraenkel's *Zehn Vorlesungen über die Grundlegung der Mengenlehre*. (Ten lectures on the foundations of set theory).

speak of Brouwer rather than of intuitionists in general; 3) include in the literature references to all my intuitionist articles (among which actually the only publications about intuitionism in existence that ‘don’t just talk but do something’ — apart from Heyting’s dissertation — are to be found.).⁽²³⁶⁾

All the best greetings and holiday wishes from house to house

Your Brouwer

P.S. A package of reprints is sent today to you. It is obvious that under the present circumstances I *cannot* allow that anywhere in your book or preface the fact can be mentioned that I have seen your proof sheets.

Your B.

[Signed autograph, postcard – in Fraenkel]

1927-01-12

To A. Fraenkel — 12.I.1927

Laren⁽²³⁷⁾

Dear Mr. Fraenkel, [Lieber Herr Fraenkel]

That the main theorem of Cantor evidently holds for completely deconstructible point sets, but that it is ‘false’ for general point sets, has nothing to do with a ‘gradual refining’ of the fundamental concepts, but only with the fact that the intuitive basic construction of mathematics (which nowhere exceeds the countable, where it occurs with my predecessors) was explained by me first (1907) as *completely deconstructible finite spread*,⁽²³⁸⁾ next as *completely deconstructible (but not necessarily finite) set*, and finally as *spread without further qualifications*, but which was always in the phase of its introduction called “spread”, for short.⁽²³⁹⁾ One cannot keep intro-

⁽²³⁶⁾For a discussion of Fraenkel’s views on intuitionism see [Van Dalen 2000], [Van Dalen 1999] section 10.5. ⁽²³⁷⁾Addressed: Breiter Weg 7, Marburg (Lahn).

⁽²³⁸⁾What Brouwer called *Menge* and *finite Menge* is now known as ‘spread’ and ‘fan’. The notion of ‘deconstructible’ is essentially taken from the transfinite proof of the Cantor-Bendixson theorem. See also [Van Dalen 1999], section 10.5. ⁽²³⁹⁾‘1907’ seems surprising; the notion of *afbreken* occurs in [Brouwer 1917a, Brouwer 1917b], and *abbrechen* and *abbrechbar* occurs in [Brouwer 1918a]. In the dissertation one can however, reading between the lines, recognize the notion of “breaking off” (p. 64 ff.). From the present letter one may conclude that Brouwer had recognized that his implicit notion of ‘fan’ required extra conditions.

ducing new terminology all the time; therefore I have denoted my intuitive basic construction by ‘spread’ again, each time when it needed an extension; even a few months ago such an extension became necessary as one can read in my article ‘Intuitionistic introduction of the dimension concept’⁽²⁴⁰⁾ After this extension too, certain so far ‘self evident’ theorems will turn out to be ‘false’. Nonetheless, admonishments from your side, as in the mentioned footnote, do not have the least justification. Should you want to stick to this humiliating and hollow insinuation, even after my urgent request and my urgent advice to delete it, then the competent reader (I too, claim to qualify as such) can only view that as a declaration of war to me; I am asking myself in vain what grounds I could have given you. Excuse me that I write sharply and clearly; but I will have to do that subsequently in public too, and then it should not be said that I didn’t call your attention to the implications of the statement, and didn’t warn you.

With friendly greetings

Your Brouwer

[Signed autograph, postcard – in Fraenkel]

1927-01-28

To A. Fraenkel — 28.I.1927

Berlin-Halensee
Joachim Friedrichstr. 25^{II} ⁽²⁴¹⁾

1 Enclosure.

Dear Mr. Fraenkel, [Lieber Herr Fraenkel] ⁽²⁴²⁾

You are really mistaken, and you really hurt me again, if you attribute my latest card to an ‘irritability independent of you’; please keep in mind that in your letter of December 31, 1926, you quote my position in 1913 and 1919 on Cantor’s main theorem as an example of the phenomenon that in connection with the gradual sharpening of fundamental concepts the term ‘self evident’ easily gives rise to errors, a claim that after my exposition

⁽²⁴⁰⁾ *Intuitionistische Einführung des Dimensionsbegriffes* [Brouwer 1926]. ⁽²⁴¹⁾ ‘Adresse bis Mitte März’ ⁽²⁴²⁾ For more information on the topics of this letter, see [Van Dalen 2000].

given on my last card must appear to you too as both unfounded and insulting.

How little one can speak of a “declaration of war” on my side, and how strongly, on the contrary, I strive with all my strength to avoid a public fight between the two of us, you can see from the fact that I have succeeded in getting a statement from Teubner that he is willing even now to incorporate substantial changes into your book before the printing.⁽²⁴³⁾ And so I would like to implore you not to continue the expropriation that the German mathematical literature has practiced on me, by making me share what is exclusively my personal intellectual property with Poincaré, Kronecker and Weyl. (By the way, to a certain degree I am to be blamed for that myself, because I have now and then, in a for the superficial reader easily misleading manner, brought myself and my predecessors, with whom I merely share the struggle against formalism, under the common denominator of “Intuitionist”.)

For your information I enclose (with the request to please send it back some time, because it is my last copy) the German translation of a section of an article which I will publish in the *Revue de Métaphysique et de Morale*⁽²⁴⁴⁾ and in line with that I propose the following changes for your book which are minimally required by justice:

- α To edit the second paragraph of §6 of the 3/4-th lecture:⁽²⁴⁵⁾ “[in] this intuitionism two phases can be distinguished, of which the first one is only a phenomenon of reaction [...old text ...] of the last quarter [...old text ...] by Cantor; at the beginning of this century [...old text ...] adopted a far milder position.

The second, much more radical phase, which does not just concern the *founding* of mathematics, but which reshapes *the complete doctrine of mathematics*, was inaugurated by Brouwer, who was joined by Weyl as an adherent. According to a formulation of Brouwer this neo-intuitionism⁽²⁴⁶⁾ is based on the two following principles:

1. *The independence of mathematics* [...old text ...] will be capable.
2. *The constructive definition of set* [spread] [...old text ...] without using the Bolzano-Weierstrass theorem”.

⁽²⁴³⁾‘*Zehn Vorlesungen*’, [Fraenkel 1927]. ⁽²⁴⁴⁾Paper not published. No manuscript extant. ⁽²⁴⁵⁾cf. [Fraenkel 1927] p. 34, 35. ⁽²⁴⁶⁾neo-intuitionism

(These two principles are on the one hand exclusively mine,¹⁸ on the other hand they implicitly embody in a completely rigorous way the whole future rebuilding of mathematics.)

β Lines 15-21 of section §9 of the 3/4-th lectures⁽²⁴⁷⁾ are to be revised for example as:

“[...old text ...] of a real function which is continuous in a closed interval; the deficiency of this proof is matched in intuitionism (cf. Brouwer 5) by the curious (in fact in no way obvious, but rather deep) fact, that each function which is defined everywhere on a continuum,⁽²⁴⁸⁾ is uniformly continuous”.

(In the summer of 1919 I have once in personal conversations with Weyl in the Engadin, as a result of which he was converted to my views, in connection with the definition of the continuous function in §1 of my *Begründung der Mengenlehre unabhängig vom logischen Satz vom ausgeschlossenen Dritten* stated and motivated the conjecture that these functions are the only ones existing on the full continuum (cf. in this connection p. 62 of my paper *Über Definitionsbereiche von Funktionen*, which has just appeared in the Riemann volume of the *Annalen*). The legend which has since then been circulated about Weyl, “that it is obvious in Brouwerian analysis that there cannot exist on the continuum any but uniformly continuous functions”, can only be based on this ([as many?] other ones, half understood by Weyl) conjecture, stated by me).

γ Extend line 16 of the first paragraph of §10 of 3/4-th lecture⁽²⁴⁹⁾ as follows:

“in an inductive (or recurrent) way. Over and above this, Brouwer has subsequently made the step (already mentioned in §6), that he unfolds the ur-intuition further to the general spread construction, and in this manner extends the intuitionistic founding of (discrete and denumerable) arithmetic to (continuous and non-denumerable) analysis. From this ur-intuition, stressed with special emphasis”

δ To complete the part of the References which concerns me, at least as follows:

¹⁸so that it is a crude injustice towards me to claim that “these considerations of the new adherents to intuitionism have emerged, at totally different places, independent of each other, in a remarkable agreement”.

⁽²⁴⁷⁾cf. [Fraenkel 1927] p. 48. ⁽²⁴⁸⁾i.e. a connected compact set. ⁽²⁴⁹⁾cf. [Fraenkel 1927] p. 50.

- “ 1. Begründung der Mengenlehre unabhängig vom logischen Satz vom ausgeschlossenen Dritten I–II. Begründung der Funktionenlehre unabhängig vom logischen Satz vom ausgeschlossenen Dritten. I. *Amsterdamer Verhandelingen*, **12** no. 5, 7, 13, no. 2 (1918–1923).
2. Intuitionistische Mengenlehre. *Jahresbericht der Deutschen Mathematiker Vereinigung*, **28** (1919), p. 203–208.
3. Über die Bedeutung des Satzes vom ausgeschlossenen Dritten in der Mathematik, insbesondere in der Funktionentheorie. *Journal f.d. reine u. angewandte Mathematik*, **154** (1925), p. 1–7.
4. Zur Begründung der intuitionistischen Mathematik I–III. *Mathematische Annalen*, **93** (1925), S. 244–257; **95** (1926) S.453–472; **96** (1926), p. 451–488.
5. Über Definitionsbereiche von Funktionen, *Mathematische Annalen*, **97** (1926), p. 60–75.”

(The citing of the three Amsterdam essays would in any case be more necessary than that of the three *Annalen* papers, which altogether only bring a technical elaboration – without any philosophical addition whatsoever – of the first (least important one) of the three mentioned *Verhandelingen*. And the citation of my paper which appeared in the Riemann volume, which is of central importance for the continuity question for full functions (cf. above under β) and in general for the continuum problem, seems to me of the utmost urgency, where for the rest you mention indeed every philosophy [...] textbooks on set theory).

In the last paragraph of §8 of the 9/10-th lectures, line 17 from the bottom, mention instead of “the opinion of the radical intuitionist”, “the opinion of Brouwer” (this opinion is, even if it has since then been repeated after me by others, nonetheless to no lesser degree my intellectual property).

According to a statement of Schopenhauer, there will be practiced against each innovator, by the automatically appearing opposition, at first the strategy of (factual) ignoring⁽²⁵⁰⁾, and after the failure of this strategy, that of priority theft. Should this also bear on my case, then I am convinced that you do not belong to my enemies, that on the contrary you harbor the wish—and after learning the above—will cooperate to make the above-mentioned strategy against me as little successful as possible.

⁽²⁵⁰⁾ *totschweigen*.

Finally I beg you to believe that the purely objective content of this letter is accompanied only by benevolent and friendly feelings towards you.

With best greetings ⁽²⁵¹⁾

Your Brouwer

[Signed autograph – in Fraenkel]

1927-02-03

To P.S. Alexandrov — 3.II.1927

Vienna

Wien

Greetings and a handshake. (The card will, by the way, not be sent today, because the postal drivers are on strike, so the mailboxes will not be emptied). I am here for a few days for discussions and for a visit to Dutch friends. Tomorrow I will dine with Wirtinger, Ehrenhaft, Hahn, Vitoris and Loewy. In Berlin the colleagues are very nice to me and my lectures are attended very well. ⁽²⁵²⁾ That Blumenthal sent the Kuratowski paper to you, while bypassing me and also without informing me in advance, was against the rules of the editorial board, and it was unfriendly, offensive and inappropriate (maybe offensive on purpose because of the many conflicts between him and me; he still is regularly changing my articles after they have been declared ready to print; in the Riemann volume ⁽²⁵³⁾ he has introduced again a gross error). When he does something like that again, please answer him that you can accept these refereeing requests only from me, because for the outside world I am the editor in charge of topology.

Greetings to your family members. ⁽²⁵⁴⁾

Your Brouwer.

⁽²⁵¹⁾ *Mit den besten Grüßen.* ⁽²⁵²⁾ The Berlin lectures on Intuitionism. ⁽²⁵³⁾ The commemorative volume for Riemann's birth 100 years ago; see also [Van Dalen 2005] section 13.3. ⁽²⁵⁴⁾ *Grüsse an Ihre Hausgenossen.*

Brouwer is celebrated a lot. He will have to buy a dinner jacket! He drags me everywhere with him.

Warmest Greetings Your Corrie Jongejan.

[Signed autograph, picture postcard – in Alexandrov]

1927-03-08

To H. Hopf — 8.III.1927

Berlin-Halensee
Joachim Friedrichstr. 25^{II}

Dear Mr. Hopf, [Lieber Herr Hopf]

It just occurred to me that I owe you and Feigl an addendum to what I said at the end of my talk about fixed point theorems; and before I leave, I want to settle that debt. When I remarked that the classical fixed point theorems *cannot* be saved intuitionistically *as fixed point theorems*, I didn't mean at all that intuitionistically these theorems don't admit an interpretation that is still valid there.⁽²⁵⁵⁾ On the contrary: the classical theorem that the transformation τ of a compact space R (which we will suppose to be a metric space) has a fixed point, has a meaning which remains intuitionistically correct, namely that for every $\varepsilon \triangleright 0$ ⁽²⁵⁶⁾ a point P of R can be determined, that is less than ε removed from its image point. And the classical theorem that the transformation τ possesses n mutually distinct fixed points, has the intuitionistically correct meaning that there exists an $a \triangleright 0$ with the property that for every $\varepsilon \triangleright 0$, there can be determined n points P_1, P_2, \dots, P_n of R , which all are less than ε removed from their image points and of which every two have a mutual distance $\geq a$. But these theorems are not fixed point theorems anymore, because one doesn't have means to indicate a fixed point, i.e. to approximate it.

Please show this card to Feigl too. It is intended for you both.

Cordial greetings!

Your Brouwer

[Signed autograph, postcard – in Hopf]

⁽²⁵⁵⁾ Cf. [Brouwer 1992] p. 56. ⁽²⁵⁶⁾ \triangleright is Brouwer's notation for the natural order relation: $a \triangleright b$ if the difference of a and b is greater than a suitable 2^{-k} .

1927-04-09**To H. Hopf — 9.IV.1927****Laren** ⁽²⁵⁷⁾

Dear Mr. Hopf, [Lieber Herr Hopf]

Many thanks for your letter of March 20. I have written immediately to Dr. Trowbridge ⁽²⁵⁸⁾ in Paris (but I had spoken already about your case with Dr. Tisdale in the fall), and I have received from him an answer that appears to be very favorable.

I was only today in the position to send three copies of my article about domains of functions ⁽²⁵⁹⁾ to Miss Gawehn, ⁽²⁶⁰⁾ one for herself, one for you and one for Feigl. You would do me great favor when you would keep an eye on Miss Gawehn, and try do something so that her philosophical article would be ready to print and printed as soon as possible.

Please give many greetings from me to Mrs. and Prof. Courant, and recover completely.

Cordial greetings from your

Brouwer

If you have time, then go to Arosa to Miss Alice Beyreiss (teacher, lives in Chalet Valbelle, somewhat above Sporthotel Merkur), and bring her my greetings. You would do me a pleasure. Your B.

[Signed autograph – in Hopf]

1927-09-07**From H. Scholz — 7.IX.1927****Baarn**
Huize Ekely

Dear Professor [Hochverehrter Herr Professor]

It is an old experience that one only knows what one lacks, when one has learnt something new.

⁽²⁵⁷⁾ Addressed: Hotel Pratschli. Arosa; forwarded to ‘Prof. Dr. Courant, Universität Göttingen.’ ⁽²⁵⁸⁾ of the Rockefeller Foundation. ⁽²⁵⁹⁾ [Brouwer 1927]. ⁽²⁶⁰⁾ See [Van Dalen 2005] p. 567.

In this sense I would like to ask you kindly to make a few remarks about the enclosed page that would redeem me.

Because by what you said to me today, my interpretation until now

I can neither show: $r = 0$

nor show: $r < 0$

nor show: $r > 0$

is completely thrown into confusion.

I thank you once more most cordially for the two *wonderful* hours of this afternoon and I remain

in the greatest veneration

Yours ⁽²⁶¹⁾

Heinrich Scholz

10.–15. September: 30 Zandvoortschelaan, Zandvoort.

[Signed autograph – in Brouwer]

1927-11-08

From L. Herzberg — 8.XI.1927

Berlin-Tempelhof

Dear Professor, [Sehr geehrter Herr Professor]

The *Berliner Tageblatt* has the plan to acquaint its readers with the main thoughts in the modern dispute about the foundations of mathematics, and for this purpose it wants to make one page of the newspaper available. The theme might be perhaps: ‘What about the validity of the theorem of the excluded middle?’

The *Berliner Tageblatt* would be grateful to you, dear professor, if you could write something on this theme in an article from the *intuitionistic point of view* of about three or four typewritten pages. If you would decline to produce *yourself* a popular article for a mostly lay public, then it would be very kind if you could send me a few statements about this theme in a letter, especially also about the consequences for people’s world view, which

⁽²⁶¹⁾ *in grösster Verehrung – der Ihrige.*

might follow from intuitionistic mathematics. Then I would convert these into a newspaper article and submit this to you to sign.

In case you agree we will ask professor Hilbert in Göttingen to treat the same theme from the formalistic point of view. If you would agree with our request, we would be much obliged.

Sincerely yours ⁽²⁶²⁾
 (signed) Dr. Lily Herzberg.

[Typescript copy –in Brouwer ⁽²⁶³⁾]

1927-11-16

From H. Scholz — 16.XI.1927 ⁽²⁶⁴⁾

Kiel

A) ATTEMPT OF A CONSTRUCTION OF THE BROUWER THOUGHT CONCEPTS.

- (1) Thinking is
 - a) constructing of relations,
for short: constructing
 - b) deducing new relations from relations already constructed,
for short: deducing.
- (2) What constructing and deducing is cannot be defined, but can only be learned by demonstration and imitation.
- (3) Thinking is basically a ‘soundless’ process, i.e. a process which is fundamentally independent of all (symbolic) means of representation by means of speech or without speech, by which we preserve the results for ourselves and for others
- (4) So, thinking is
 - a) a soundless constructing of relations
- (5) More precisely of relations between ‘objects’ and ‘concepts’.

⁽²⁶²⁾ *Mit vorzüglicher Hochachtung — Sehr ergebenst.* ⁽²⁶³⁾ The copy was most likely made at Brouwer’s request; there is no original letter in the archive. ⁽²⁶⁴⁾ Date - postmark. This is more a (drafted) manuscript than a letter. There are some lines of correspondence inserted in the text.

- (6) More precisely of relations between objects and concepts, about which one can think.
- (7) About an object or concept one can only think when both can be clearly grasped.
- (8) This is only the case when they can be ‘constructed’.
- (9) A concept can only be constructed when the objects it encompasses can be constructed.
- (10) Objects cannot be created from nothing, hence also not by ‘purely’ thinking; because then this must create them from nothing. Thinking is itself only a construction tool, but it is not able to produce the construction material by itself.
- (11) This material (and certain intuitions, that are unconditionally necessary for the evaluation of the material) is produced by the sense of time, and only the consciousness of time. (More under ‘mathematics’.)
- (12) The objects thus generated are called, with reference to their number nature, mathematical objects, correspondingly the concepts built from them are called mathematical concepts.
- (13) Hence thinking is
- a) A constructing of relations between mathematical objects and concepts.
- (14) Thinking is
- b) Deducing new relations from already constructed relations; but certainly not according to abstract reasoning schemes given in advance, but so that the ‘deduced’ or ‘deducible’ relations must follow instinctively and evidently from the intuition of the already constructed relations, and only from this intuition.
- Example: I have proved:
- a) Every x from K is an x from K' ,
- b) Every x from K' is an x from K'' .
- Then it is evident that I have proved:
- c) Every x from K is an x from K'' .
- (15) Summary:
- Hence thinking is
- a) in its constructive function:
operating on mathematical objects and concepts.

b) in its deductive function:

interpreting results of constructions based on intuitionistic consideration of one's own constructions.

So all deductive thinking is thinking that is based on the intuitionistic consideration of mathematical constructions, not based on pre-existing abstract reasoning schemes.

- (16) Operating with non-mathematical objects and concepts is only thinking in so far as these non-mathematical objects and concepts can be reduced to mathematical objects and concepts, i.e. can be replaced by these. Whether and into what extent this is possible can only be decided on a case by case basis.

B) THE BROUWERIAN CONCEPT OF LOGIC

(1) Logic, as theory of the forms of valid thinking, is

a) not a system of aprioristic deduction schemes, when 'aprioristic' roughly means 'independent of intuitionistic consideration of mathematical constructions'.

Follows from the nature of reasoning, characterized under A15.

b) and certainly not a system of universally valid aprioristic deduction schemes when 'universally valid' amounts to 'directly applicable to all classes of objects and concepts'.

Already not because then one would burden oneself with the absurd set of all things.

Especially because of A16.

c) and certainly not a system of arbitrary aprioristic, universally valid deduction schemes, if 'arbitrary' = 'only satisfying the postulate of freedom of contradictions'.

Because such a thinking has with the thinking characterized under A simply nothing but the name in common.

(2) Logic, as theory of forms of valid thinking, is the system of those and only those schemes that I obtain, when I

a) have somehow symbolically represented both the 'constructions of constructive thinking'⁽²⁶⁵⁾ that are soundless by themselves and the likewise soundless deductions of deductive thinking from these constructions,

⁽²⁶⁵⁾The end of quote mark seems to be missing here in the original.

b) study the invariants of this symbolically represented thinking.

Hence the theory of the valid invariants of symbolically represented thinking.

Or the system of fundamental reasoning schemes that are abstracted from the symbolically represented accomplishment of thinking and hence, strictly speaking thus only for the fundamental deduction schemes that are basic for these symbolized accomplishments but not for the proper (soundless) thinking.

But now all ‘thinking’ is a mathematical operating.

Consequently the symbolized thinking is a symbolic mathematical operating.

Or shorter: mathematics in a verbal (symbolic) representation.

Consequently logic is the theory of valid forms of a verbal representation of mathematics. (And not the theory of valid forms of mathematical construction as such!)

B’) THE BROUWERIAN CONCEPT OF LOGIC

- | | | |
|--|---|--|
| (1) The assumptions of logic: | } | Executed in
more detail
under B) |
| a) Thinking,
b) the verbal (symbolically represented)
expression of what is thought,
from which thinking as such is
fundamentally independent. | | |
| (2) The object of logic:
the forms of symbolized thinking. | } | |

(3) The task of logic:

the analysis and synthesis of forms of symbolically represented thinking.

Analysis = formation the system of the original forms of thinking,

Synthesis = formation the system of the ‘deducible’ forms of symbolized thinking, where deducibility is determined by well determined formal constellations and substitution rules.

Summarizing: the theory of forms of symbolically represented thinking.

But now thinking is defined as a mathematical operating.

Consequently the verbally expressed (symbolically represented) thinking as a verbally expressed (symbolically represented) mathematical

operating.

Therefore logic is the theory of forms of verbally expressed mathematical operating, or the theory of forms of verbal expression that accompany mathematical operations (but in such a manner that they don't fundamentally depend on these forms) or shorter: the theory of mathematical language.

(4) Consequences

- a) Logic is not a necessary condition for the construction of mathematics; for it is only the theory of the mathematical language (which is as such basically irrelevant for mathematics).
- b) Mathematics is a necessary condition for the construction of logic; because it produces the material for logic, the verbal (symbolical) formulation of which is the object of logic.
- c) From a rational logic must be demanded:
 - 1. that it restricts itself strictly to formulas that admit an mathematical interpretation at all;
 - 2. that it applies these formulas basically only to the extent that they, after a mathematical interpretation has been achieved, can be confirmed by the thinking mathematician.

Already the classical logic has most severely violated 2. Symbolic mathematical logic [logistics] has relieved itself from 1, and consequently compromised itself even more severely than classical logic.

C) THE THREE MAIN FAILURES OF FORMAL LOGIC. ⁽²⁶⁶⁾

- (1) the misuse of the Tertium non Datur:
 - consisting of
 - a) the illegitimate application to arbitrary properties of a given individual,
 - b) in the use of it in the form: either all x from K are also x from K' , or there is at least one x from K , which is not x from K'' for transfinite classes.
- (2) the abuse of the notion of class, resp. property.
 - consisting of the use of the above for the creation of non-constructible sets, and in particular totally unrestricted. or, as this unrestricted use has led to logical 'catastrophes, under the determined conditions of the sharpened axiom of separation of Zermelo.

⁽²⁶⁶⁾Section ends here.

(3) the misuse of the notion of consistency

consisting in the identification of the mathematically totally inconsequential ‘logical’ phenomenon of consistency with constructibility (crucial difference with *Poincaré*)

based on the arbitrary introduction of non-mathematical objects, the existence of which is identified with the consistency of the properties that define them, and which the domain of mathematical objects is allegedly made part of.

D) ON THE BROUWERIAN INTERPRETATION OF LOGIC

For the precise understanding of Brouwer’s notion of logic it is of the greatest importance, that one grasps clearly, what it means that the logic in the sense of Brouwer is the theory of valid forms of symbolized thinking.

This means that the laws of logic are the laws of symbolized thinking, and not the laws of thought in general. This in particular, because they can only be formulated at all for symbolized thinking. I cannot even formulate the excluded contradiction, if I do not have p and $non - p$, resp. p and $abs p$. And I have p and $abs p$ only in the domain of symbolized thinking.

Thinking as such is, strictly considered, just as little contradictory or consistent, yes, even just as little true or false (absurd), as the building of a house, or the experimenting of an experimental scientist.

Contradiction, consistency, truth and falsity (absurdity) are therefore not properties of thinking in general, but properties of symbolized thinking.

Thinking as such can rather, like all constructing, either be carried out (is crowned with some success), or cannot be carried out (ends in failure).

Thinking ends then and only then in failure if the objects, with which it is operating, disintegrate in the course the operation, but then and only then if a distinction ($0 \neq 0$) is intrinsically forced upon the operations that have been tried.

The supreme basic law of Brouwerian thinking could thus be formulated as: each object of thought ‘disintegrates’ when processed, under the influence of thinking, if by means of this processing a distinction from itself is forced upon it. That is to say: if at least one property can be constructed, that is both given to it through the processing by virtue of thinking, and withdrawn.

Thus we are back to the Aristotelian formulation: ‘It is excluded that a (not disintegrating) object has the property E , and also not has the prop-

erty'; but we interpret it now with Brouwer ontologically, and basically with so little concern for any 'logical' interpretation, as has, at least in the domain of mathematics, not happened since Aristotle.

E) THE BROUWERIAN NOTION OF MATHEMATICS

- (1) I operate – I construct relations between “objects”.
- (2) I operate with operations – I derive from already constructed relations new relations.
- (3) I cannot define what it means to construct, but only demonstrate, and learn through imitation.
- (4) Constructing, and operating as well, is a fundamentally speechless act. That is to say: it is basically independent of all symbolizing, communication, linguistic means of expression, by means of which we preserve the results of construction for ourselves and for others.
- (5) Mathematics is not a game of formulas, of the results of which only consistency is required, but an operating with objects.

I add: and with operations and objects; for if I cannot deduce the successful embedding of each x from K in K'' from the successful embedding of each x from K in K' and each x from K' in K'' , then I can not build up a mathematics.

I can thus also say: mathematics is the totality of all results, that I obtain by constructing relations between objects and from the constructed relations derive new relations.

NB. This deriving does not mean a concluding in the logical sense of the word, where it means:

If I have the formula $F = (p \ q)$,
then I can write the formula F' :

$$(p \ q) \ (q \ p);$$

for this is already a statement on sign-complexes, through which we symbolize mathematical constructions, has thus nothing to do directly with mathematics as such. Instead, deriving means here an immediately intelligible drawing of conclusions based on the nature of the executed construction.

(Cf. the syllogistic interpretation of the “Cogito, ergo sum” by Descartes, and Descartes' position on logic at all!)

- (6) Mathematics can only operate with sharply graspable, i.e. with constructible objects.

- (7) Where does mathematics get the material from which it can generate its objects?

Not from logic:

for logic operates either with signs for objects; then it already presumes the objects.

Or it operates with ‘meaningless’ signs: then, in any case, it does not yield material for generating objects.

Finally one can ask, in how far it can do without numbers, which should be created first.

Not from observation either (see below).

There only remains as material-providing principle a field of sources of unfailing intuitive certainties.

Spatial consciousness can not provide this field, for

1. it is so intrinsically vague, that it becomes comprehensible, when it is understood as the expression of a Riemannian manifold.
2. the delicate question, not yet existing for Kant, indeed incorrectly declared to be impossible, arises, which spatial consciousness we should accept as fundamental; for, to each Riemannian manifold (with its own measure of curvature) corresponds then a specific spatial consciousness.

Thus only the consciousness of time remains.

This provides us with

1. distinct ‘now’-points, i.e. points that are separated by means of time; that is, discrete objects, or rather at once natural sequences of such objects.
2. It provides us with these points in arbitrary number, i.e. more precise, with the consciousness that the sequences of these points will never stop.

Comment: Observation can never achieve this; therefore it cannot be the foundation of mathematics either.

3. the equally unfailing certainty that between any two ‘now’-points there can always be interpolated a third.

From 1. and the ability to collect discrete objects, and to create, beginning with one, through repeated addition of a new thing, ever new units, we obtain the natural numbers.

NB. It is not clearly seen, whether first ordinal- or cardinal numbers!

From 2. we obtain the constitutive consciousness of the unbounded continuation of this sequence.

From 3. we obtain the basis for a constructive composition of the continuum.

F) ON THE THEORY OF INDIRECT PROOFS

Euclid 1.1.6: Every triangle with equal base angles is isosceles. =

If x is a triangle satisfying the condition: $\beta = \gamma$, then x is a triangle that satisfies the condition: $b = c$.

Proof:

x is a triangle that satisfies the condition B : $\beta = \gamma, b \geq c$ (1)

$\rightarrow_x x$ is a triangle satisfying the condition B' : I can construct for x a x' with

$b' = c$ and construct the sides b', c' such that x'

is fully contained in x (resp. such that x is fully contained in x').

$\rightarrow_x x \geq x'$

Now, however, I can show: $x = x'$.

Therefore (1) is false (absurd).

Therefore there is no triangle, satisfying the condition: $b = c$.

Therefore every triangle with equal base angles is isosceles.

In this form Euclid's proof seems me to be also intuitionistically completely correct.

But it is only so, if one acknowledges the implication

$$abs(p \text{ abs } q) \rightarrow (p \rightarrow q) \text{ }^{(267)}$$

For without the acceptance of this basic implication, an indirect proof of an implication is not possible at all.

Then we would get: if I can show: the assumption: 'there is at least a triangle x (= I can at least construct an x), for which $\beta = \gamma$ is true, and $b = c$ absurd, is itself absurd; then I have shown: if x is a triangle with equal base angles, then x is a isosceles triangle.

I repeat: If this conclusion is *not* admissible, then I can not see any possibility at all to show the implication indirectly; and in particular: the proof given by himself ⁽²⁶⁸⁾ runs, when precisely analyzed, exactly according to this schema.

I would like to go one step further and claim that the converse:

$$(p \rightarrow q) \rightarrow abs(p \text{ abs } q)$$

⁽²⁶⁷⁾ I.e. $\neg(p \wedge \neg q) \rightarrow (p \rightarrow q)$. ⁽²⁶⁸⁾ possibly 'yourself'?

is also completely intuitionistically correct too.

Then we have the equivalence:

$$(p \rightarrow q) = abs(p \ abs \ q)$$

In words: $(p \rightarrow q)$ = it is absurd that there is an x (= that I can construct ax), for which p is true, and q absurd.

This equivalence is all the more legitimate, as certainly also in intuitionism a ‘there is *no* x , which... (= there is *no* x constructible, which ...) is just as little an existential statement, like any implication.

[handwritten] Thus there is both on the left hand side and on the right hand side of the equation a *non*-existential statement.

If, however, one admits the above equivalence, then the following deep apories:

$abs(p \ q)$	$= abs(p \ q)$
$= p \rightarrow abs \ q$	$= q \rightarrow abs \ p$

I should then have to proceed accordingly:

$abs(p \ abs \ q)$	$= abs(abs \ q \ p)$
$= p \rightarrow abs^2 \ q$	$= abs \ q \rightarrow abs \ p$

I would thus only be able to get $p \rightarrow q$ along this line because in this case, by way of exception, I start with classical logic: $abs^2 q = abs \ q$!

If, in order to avoid this, one does not admit the equivalences, then

1. I do not see how $p \rightarrow q$ can be shown indirectly at all,
2. it remains unclear what the relation between $p \rightarrow$ and $abs(p \ abs \ q)$ is.

In order to make the consequence of these apories quite clear, I add the following confrontation with the table of Wavre:

Scholz	Wavre
$p \rightarrow q = p \rightarrow abs^2 q = abs \ q \rightarrow abs \ p$	$p \rightarrow q \neq p \rightarrow abs^2 q \ p \rightarrow q = abs \ q \rightarrow abs \ p$
$p \rightarrow abs \ q = q \rightarrow abs \ p$	$p \rightarrow abs \ q = abs^2 q \rightarrow abs^2 p$
$abs \ p \rightarrow abs \ q = q \rightarrow abs^2 p = q \rightarrow p$	$abs \ p \rightarrow abs \ q = abs^2 q \rightarrow abs^2 p$

Finally I remark that the for the intuitionistic proof of

$$abs^3 = abs \ p$$

required equivalence

$$p \rightarrow q = absq \rightarrow abs p$$

from my point of view, can only be justified by the evident

$$abs(p abs q) = abs(abs q p)$$

which, however, requires (see above) that one decides at this point to accept $abs p = abs^2 p$.

Otherwise I should have to beg for a precise intuitionistic justification of this equivalence.

[Typescript – in Brouwer]

1928-01-17

From K. Menger — 17.I.1928 ⁽²⁶⁹⁾

Dear professor, [Hooggeachte professor]

Enclosed a typescript of ‘*Allgemeine Räume & Cartesische Räume III*’. ⁽²⁷⁰⁾ Maybe it will give you some pleasure. A detailed exposition of the entire proof will of course greatly exceed the space for a Note.

Coming back to the Encyclopedia article, ⁽²⁷¹⁾ I must confess that perhaps I wouldn’t really like to read the page proofs, and that even if they would be sent to me, I must *reject* any thanks for advice I haven’t given. For, I meanwhile met Vietoris in the Vienna seminar with the proofs, and he refused explicitly to even *show* them to me just for a moment, and he declared that it was the wish of Tietze and himself that among the German scholars only Rosenthal and Kneser receive the proofs, and he added to this verbatim (it is incredible!) that both authors ⁽²⁷²⁾ hadn’t shown the proofs *to me* already last autumn!! Well, in case the authors of the Encyclopedia article expect more help from the two gentlemen than from me, they are welcome to believe that (it doesn’t reflect, I think, on *my* intelligence). It is clear that under these circumstances, and also in view of the fact that Vietoris in the conversation appeared to be totally ignorant of fundamental

⁽²⁶⁹⁾Original in Dutch. ⁽²⁷⁰⁾General spaces and Cartesian spaces III, [Menger 1929].

⁽²⁷¹⁾Cf. *Brouwer to Menger 3.I.1928*; the topic is the contribution of Vietoris and Tietze to the Encyclopedia. ⁽²⁷²⁾‘*uitgevers*’ in original.

dimension theoretical theorems (published in 1926), I am afraid that I will have serious objections to this article. I naturally suppose that you have arranged for the acknowledgments to you in the preface formulated in such a manner, that it will still be possible for me, notwithstanding my highest esteem for you, to express my objections to the fruit of the Tietze-Vietoris labor!—

In my tax affair, I'm sorry to have to bother you again. The letter which you were so kind to send me, I cannot post ⁽²⁷³⁾ because I have not received the tax assessment for 1927/28, and the form of 1926/27 was sent by the Laren/Blaricum tax office, to which I reported my moving, respectively departure, in the summer of 1926. Maybe you can inform me through a word from Hurewicz where I should direct my letter of my checking out.

From what I heard, you will receive one of these days an extensive letter from Ehrenhaft-Hahn-etc.

With respectful greetings, good bye

Yours sincerely ⁽²⁷⁴⁾

Karl Menger

[Signed autograph – in Brouwer]

1928-01-20

From L. Bieberbach — 20.I.1928

Berlin-Schmargendorf

Marienbader Strasse 9

Dear Brouwer, [Lieber Brouwer]

First I would like to thank you cordially for your kind report on Mr. Süss; consequently I have proposed to approve his research grant.

Concerning the proposal of your article ⁽²⁷⁵⁾ for the academy: there is in some cases a difficulty because of § 17 of the academy regulations, which read as follows in paragraph 1:

⁽²⁷³⁾Menger refers here to a tax form. ⁽²⁷⁴⁾*Met waardeeringsvolle groeten en tot ziens – Uw dienstwillige.* ⁽²⁷⁵⁾*Intuitionistische Betrachtungen über den Formalismus,* [Brouwer 1928a, Brouwer 1928b, Brouwer 1928c].

‘A scientific communication intended for the publications of the academy may in no case before it is published there, be published elsewhere in the German language, whether as abstract or in more extended form. If the editing secretary becomes acquainted with a publication that violates this rule, before it is published by the academy, then he must cancel the communication.’

Under these circumstances I ask you to inform me that your article in the ‘*Amsterdamer Berichten*’⁽²⁷⁶⁾ will *not* appear in the *German* language; then I think I may assume that the academy will consent to inclusion in the *Sitzungsberichte*. Unfortunately there is no possibility to deviate from this session regulation.

Finally, in the matter of your statements about the Conseil de Recherches, I see no possibility to include them in the Jahresbericht,⁽²⁷⁷⁾ because it would create a novum when we would accept political statements in the Jahresbericht; thus it has been avoided until now, because of the political aspects, to mention the planned congress in Bologna. It seems to me that the proper place for your statements would be perhaps the *Hochschulnachrichten*. Personally I agree with you and I will not go to Bologna.⁽²⁷⁸⁾

The works of Weierstrass do not belong to the ones that members of the DMV can get for a reduced price. But if you tell me which volumes you want to obtain, and whether you want them bound or unbound, then because of our personal relations I will try to get a cheaper copy in some other way.

I have received now the proof sheets of his first communication, corrected by Mr. Menger. I assume that you would prefer to look at them when the handwritten corrections of Mr. Menger are in print.

With cordial greetings

Bieberbach

For the academy a short abstract is required. Do you think the enclosed one is all right for you?⁽²⁷⁹⁾

[Signed typescript – in Brouwer]

⁽²⁷⁶⁾ KNAW Proceedings. ⁽²⁷⁷⁾ JDMV. ⁽²⁷⁸⁾ Sentence added in handwriting.
⁽²⁷⁹⁾ Added in handwriting.

1928-01-23

To L. Bieberbach — 23.I.1928

Laren

Dear Bieberbach, [Lieber Bieberbach]

The translation of my article in another language than German is so difficult that I must for once, without exception, abandon writing a Dutch text for the Amsterdam communications and restrict myself to the publication of the German text in the Proceedings.⁽²⁸⁰⁾ However I can promise to take care that the publication in Amsterdam in the Proceedings of the already submitted article will happen at least a month after the appearance in the communications of Berlin. It seems that by this promise the rules of the Berlin academy statutes are satisfied. Please tell me whether this solution is also satisfactory to you. In the opposite case I would also agree to a publication in the Jahresbericht,⁽²⁸¹⁾ but only when this publication by way of exception could be effected *immediately*.

As far as my statements about the Conseil des Recherches⁽²⁸²⁾ are concerned, they are only in form, but not in actual content, more political than the invitation to the Bologna Congress (precisely this is explained to each reader by my arguments which expose the hidden meaning of the invitation). So when you cannot print my arguments in the Jahresbericht, then I will have it printed as a pamphlet, and I will ask you to send it together with the Jahresbericht as a separate supplement, just as it was done with the invitation for the Congress.⁽²⁸³⁾ Please let me know whether you or Teubner agree with this proposal. It would be especially pleased if a few German mathematicians would cosign the pamphlet.

In the matter of the Menger proof sheets, the copy with the handwritten corrections would be most welcome, because I know the original text which was written in agreement with me, and I would like to get a quick survey of the subsequent changes.⁽²⁸⁴⁾

In case there would be a publication of an article in the Berlin communications, the abstract you wrote, which I return hereby, is completely adequate.

⁽²⁸⁰⁾[Brouwer 1928a, Brouwer 1928b, Brouwer 1928c]. ⁽²⁸¹⁾JDMV. ⁽²⁸²⁾Conseil Internationale d. Recherches. ⁽²⁸³⁾Bologna congress. ⁽²⁸⁴⁾Refers probably to [Menger 1928a], which deals with spreads from a classical point of view.

With cordial greetings

Your
Prof.Dr. L.E.J. Brouwer

[Carbon copy of typescript – in Brouwer]

1928-02-16b

To H. Weyl — 16.II.1928^b

Laren

Dear Weyl, [Lieber Weyl]

I was really pleased with the card you sent together with Révész and Geiger, from Arosa. Today I repay you with a more businesslike sign of life. For, in Utrecht there is an important mathematical list of candidates of the faculty: Barrau, Beth, Schaake (all three insignificant).⁽²⁸⁵⁾

My (alphabetic) list: Heyting, Hurewicz, Van der Waerden (in that order an intuitionist, a topologist and an algebraist). Heyting and Van der Waerden are Dutch, Hurewicz (my assistant) is in fact of Polish nationality and educated in Moscow and Vienna, but has settled already for a long time in Holland. To document my list for the minister, I need foreign testimonials. For Heyting (until now my only truly gifted intuitionistic student), only you qualify as a suitable author of a testimonial. Such a testimonial should on the one hand in general terms stress the importance of intuitionistic investigations at the present stage of development of mathematics (this is namely not at all believed outside of Amsterdam in Holland), and on the other hand it should qualify Heyting's articles (which I send you simultaneously) as pioneering.

The matter is extra difficult for me, because all three of my candidates are still young (well under 30) and the candidates of the faculty respectively 55, around 45, and around 40 years.

Many thanks in advance, and please rest assured that also without a sign of life your existence is something that is essential for me.

The first half of March I give talks in Vienna. What is that man Scherrer doing who sent me some time ago letters and articles on topol-

⁽²⁸⁵⁾H.J.E. Beth, the father of E.W. Beth.

ogy? Greetings to your wife and also to A[?]la and Mrs. Geiger when you see them.

In true friendship ⁽²⁸⁶⁾

Your Brouwer

[Signed autograph – in Weyl]

1928-03-24

From A. Sommerfeld — 24.III.1928

Munich

Leopoldstrasse 87, München

Dear Mr. Brouwer, [Lieber Herr Brouwer.]

Faber didn't write to you because he thought you would have heard from Bieberbach everything what he had to say to you about his negotiations with Bologna: he didn't know more than what had been discussed in the committee of the Mathematics Society. ⁽²⁸⁷⁾

Today a new invitation to Bologna arrived. It didn't contain a word about the Conseil de Recherches or similar matters. Also the ominous enumeration of congresses was omitted. So I believe that you will have no difficulty with your efforts, for which we are very grateful.

Whether this letter will reach you in Bologna. It was a bit delayed.

Next week I will look more closely into the Michels ⁽²⁸⁸⁾ case and write to you to Amsterdam.

Hopefully you will soon recover. Schönflies would gladly put a few pounds of bacon at your disposal!

With cordial greetings

Your

A. Sommerfeld.

[Signed autograph – in Brouwer]

⁽²⁸⁶⁾ *In treuer Freundschaft.* ⁽²⁸⁷⁾ DMV. ⁽²⁸⁸⁾ Michels, Amsterdam physicist.

1928-04-12b

To R. von Mises — 12.IV.1928^b

Rapallo

Dear Mises [Lieber Mises]

I first have talked to Pincherle, and subsequently corresponded with him. The result is as follows. The gentlemen in Bologna will send a new circular, in which neither the Union ⁽²⁸⁹⁾ nor the *real* congress will be mentioned, but on the contrary it announces a closing session of all congress attendants *and discussion about time, place and modality of the next congress* on the last congress day. So our hosts will organize the congress independently of the Union, and will clearly show this independence, and maintain it towards everybody.

However, they cannot make the facts go away, that the initiative for this congress was taken by the Union, and that the Union will have a meeting simultaneously with the Bologna congress. Just as little can they take the responsibility upon themselves that the Union will not try in Bologna to gain influence on the congress and on this closing session. Under these circumstances it seems to me that adherents and opponents of the Union can equally well take part in the congress, the latter with the intention that they will fight the Union if it should interfere with the congress, and if possible to destroy it. Moreover the congress participants that oppose the Union can continue their struggle against the Union during the months before the congress without being disloyal to the congress.

I spoke also with Levi-Civita in Rome and with Cipota in Palermo, and I have the impression that in Italy hardly anyone takes the Union seriously.

I hope to come to Berlin in the beginning of May, to discuss the matter once more with Schmidt, Bieberbach and you (and if possible also with Planck) on the basis of my correspondence with Pincherle.

Please inform Hahn and Ehrenhaft too about the above situation. Cordial greetings from your

Brouwer

[Signed autograph, postcard – in Mises]

⁽²⁸⁹⁾Mathematical Union.

1928-07-03

From H. Bohr — 3.VII.1928

p.t. Fynshav Als

Dänemark

Dear Mr. Brouwer, [Lieber Herr Brouwer]

Many thanks for your letter, which I got just forwarded from Göttingen. As you have perhaps learnt, I too have had an exchange of letters with Prof. Pincherle, and I even have written on May 26 a long letter (in the name of my good friend prof. Hardy and myself) to Pincherle, and received a detailed answer of Pincherle — unfortunately accidentally much delayed. Hardy and I expressed as strongly as possible our opinion that it would be absolutely necessary that ‘the congress will be in every respect on a completely international footing and that the German participants have no different position from the others.’ As you will probably know, Hardy and I have waged the same fight against the Conseil Internationale⁽²⁹⁰⁾ as you in Holland, and we wrote also in our letter to Pincherle, how sad we were that such a Conseil was established, which carried unjustifiably the name ‘international’. Also we have fought with all means against joining the Union (in Denmark I would certainly have succeeded to obstruct this joining, if Nörlund hadn’t formed a committee in favor of joining, independently from our academy and Math. Society).

Actually, the point of view of Hardy and me was in principle the natural one, namely that we didn’t want to have anything to do with a congress that like the congress in Bologna was so tied in its early history to the Union.⁽²⁹¹⁾ But when we thought (just as I see from your letter you thought) that we should try to help to make all mathematicians of the world come together in Bologna, it was important for us that we heard from all sides that the leading Italian mathematicians, Pincherle, Levi-Civita and so on, were internationally minded in the true sense, but foremost that we heard that in several circles in Germany people were prepared out of deep interest for the internationality of science, to ignore the foolish and sad previous history, *when only the congress itself would be fully international and would meet completely independent of the Union*. Pincherle’s answer, through his letter to Picard (I speak now only about the actual contents and not about the form), of which I have received a copy from Pincherle, and most of all because of the new circular which explicitly gives completely equal rights to all real

⁽²⁹⁰⁾Conseil Int. d. Recherches. ⁽²⁹¹⁾Mathematical Union.

participants (voting rights etc.), the congress has in my view been certainly and factually put on an international footing. From your exchange of letters with Pincherle I see with deep regret that you think that Pincherle did not achieve everything he promised you. Quite apart from this more personal question I find that since it has even been successfully arranged that the congress determines the place of the next congress, the Union is now — even with respect to questions that are not directly connected with this congress — so completely cut out, and ‘we’ internationally minded (i.e. people like you, Hardy etc.) have in fact won so completely, that I from my point of view would think it would neither naturally nor for the future look good, if the congress now would be sabotaged from the side of the Union opponents.

It would all have been much easier if we, who are of completely the same mind for these questions, would have contacted each other sooner, but because we were so outraged about the establishment of the Conseil and didn’t want have anything to do with it, we have somewhat pushed away all questions connected to it. But I would think it just too sad when in the end the instigators of this corporation that is science unworthy, would attain that we, the opponents of the Conseil, having reached the point to score a complete victory, cannot come to agreement about relatively small questions and formalities, and that thereby a division would come between us like-minded.

With the most cordial greetings

Sincerely yours ⁽²⁹²⁾

Harald Bohr

[Signed autograph – in Brouwer]

1928-07-17

To A. Heyting — 17.VII.1928

Laren

Dear Mr. Heyting [Waarde Heer Heyting]

Your manuscript ⁽²⁹³⁾ has interested me very much, and I am sorry that you have to rush me to send it back. In the future I would appreciate it, if you made a copy of your manuscripts before you send them to me, at least if

⁽²⁹²⁾ *Ihr ergebener.* ⁽²⁹³⁾ On the formalization of intuitionistic logic, the sequel to Heyting’s prize winning essay, cf. *Mannoury to Brouwer 26.I.1927.*

you appreciate a more than superficial reading by me. Meanwhile I have already now formed such a high opinion of your work, that I ask you to write it in German for the *Mathematische Annalen* (and rather somewhat more extensive rather than abbreviated). Maybe you can make then an even sharper distinction between the *original* signs and those that are introduced by definitions (as *abbreviations* for other symbols). And perhaps the notion of ‘Law’ can be formalized (in view of § 13). But these are only inessential remarks.

As to your remark concerning the [paper in] *Mathematische Annalen* 93, ⁽²⁹⁴⁾ p. 245, at the occasion of my Berlin lectures several improvements of ‘On the founding of intuitionistic mathematics’ ⁽²⁹⁵⁾ have turned out to be necessary. Among others I assigned then to each property as ‘species’ the ‘identity with an arbitrary thing that possesses the relevant property’. I started then from the species of order zero, by which I mean either a given element of a spread or the identity with an arbitrary element of a given spread. A better way of treatment may however be, to introduce next to the things themselves, the ‘species of identical things’, and to consider the latter in the first place, similar to the manner in which in topological set theory not the points themselves are studied, but the point cores.

The Berlin lectures will soon appear in print. ⁽²⁹⁶⁾ If the publication is delayed then I will send you reprints of ‘On the founding of intuitionistic mathematics’ with the main improvements.

With friendly greetings

Your
L.E.J. Brouwer

[Signed autograph –in Heyting]

1928-09-27

From H. Härlen — 27.IX.1928

Eislingen/Fils ⁽²⁹⁷⁾

Dear Professor, [Sehr geehrter Herr Professor]

Below I allow me to give you a *Report about Bologna*. I must state in advance that I can only render my subjective impressions, and that I don’t

⁽²⁹⁴⁾ [Brouwer 1925]. ⁽²⁹⁵⁾ *Zur Begründung der intuitionistischen Mathematik*.

⁽²⁹⁶⁾ They appeared posthumously in 1991. ⁽²⁹⁷⁾ The last page(s) of the letter are missing.

claim in the least to be complete, only that I report on more or less accidental observations of mine. Moreover, I will not treat the mathematically interesting things.

On arrival in Bologna (Sunday, September 2) I first was struck by a German poster, pointing to the information stand for congress participants. There seemed to be more German than French and English posters in the railway station. In the information stand a German speaking lady. In these external appearances the German was quite satisfactorily taken into account.

Monday, September 3: Opening session, very splendidly done by city and state. Speech of the *podesta*:⁽²⁹⁸⁾ Welcome in the name of the town, the fascist town which is happy to show its foreign guests the achievements of fascism. Praise of fascism. Then welcoming speech by the *rector* to the guests that had responded to the *invitation of the university*. Then opening speech by *Pincherle*. Short report about the previous history of the congress which was the reason for the university to take it in its own hands. Clear effort to offend nobody. All the same he mentioned ‘discordant voices, coming from diametrically opposing sides’, and also that the exclusion of some nations in Strassbourg and Toronto were explained, if not *justified* by the ‘morning after the war’. And later, that this state of mind nowadays wasn’t *justified* anymore.

After *Pincherle Birkhoff* spoke in French and in English, and he thanked the Italian mathematicians for their work to create a *truly international* congress. Quite a few remarked that these thanks were not repeated in German. Afterwards speech by the *minister of education* about the significance of mathematics.

Afternoon: 1st session. Choice of the chairman. Proposal of the meeting: *Pincherle*, adopted with great applause. Then *Pincherle* makes proposals for vice-chairmen, adopted by acclamation: for Belgium: de la Vallée Poussin, France: Hadamard, Germany: Hilbert (very strong applause, very striking), Switzerland: Fehr (as representative of the education committee), England: Young (*as board member of the Union*), United States: *Birkhoff* (representative of the government), Scandinavia: Bohr, Spain & South America: Rey Pastor, Poland: Sierpiński, Russia: Lusin. An error by me in this list, especially in the order, is possible.

After this the first talk by *Hilbert*, who is greeted with a storm of applause. Frequent repetitions; his ability to concentrate clearly much influ-

⁽²⁹⁸⁾An old city governing position, going back to the Middle Ages, comparable to mayor; revived by Mussolini in 1926.

enced by physical suffering. Contents essentially known from recent publications. Great applause. — *Hadamard* is also greeted with great applause, and his talk is also very good in presentation— much more effective than the one of Hilbert. With Hadamard the applause afterwards was much stronger than beforehand. With Hilbert the applause was almost only for the person, with Hadamard also for the talk.

The longer talks that were given:

by Germans 3, French 3, English 1, Americans 2, Russians 1, Italians 6. Lusin and Birkhoff spoke in French, the other speakers in their mother tongues. Of the more than 400 section talks the most part were French, then came Italian, and at distance German and English. Among the participants Germany was strongly represented, also Poland, Hungary, Switzerland, Scandinavia and younger Frenchmen. Of the older ones many, among whom Borel and Painlevé, seemed to be absent because of external circumstances. Noticeably weak was the participation of England. Also the United States were weakly represented.

The participants received insignia on ribbons in the Italian colors. It would have been more tactful when they would have chosen the colors of Bologna. Not only for us Germans is it an ordeal to have to wear the colors of Italy, but also for a few other countries, e.g. Yugoslavs, Swiss and maybe the French. — In the concert given on the occasion of the congress the Italian national anthem and the fascist hymn were played, with Italian manifestations. Such manifestations occurred also at the breakfast organized by the city. At breakfast every menu was decorated with a small Italian flag. It was clearly expected that we would wear these flags, as was done at least by the Italians.

During this breakfast I entered a discussion with Mr. *Stoilow* (Romania) about the position of the Germans at the congress and their attitude to the Union. Mr. Stoilow told that Picard as chairman of the *Conseils des Recherches* could not take part — in his own words — in the congress because the invitation two years ago to Germany to join had remained unanswered. He moreover mentioned that the French were afraid that we would establish a German Union. I rejected this curious fear and represented the point of view: precondition for international cooperation is that the past should be thoroughly stowed away. Violations of one or the other side during or after the war are to be explained by war psychosis and should be considered as dealt with. The mentalities of peoples are too different, so every nation should show the greatest restraint and consideration. A Ro-

manian whom I didn't know and who entered the conversation, recalled the manifest of the 84⁽²⁹⁹⁾ German scholars of 1914, which apparently also today gives offense. Because I don't know the manifest I didn't take a position on it, but I pointed out the situation of Germany then. I added that if in this manifest there are places that can only be explained by the situation of Germany at that time, but that are not justified today, then undoubtedly those scholars would not subscribe to that manifest today. All the time I stressed very much that in Germany the wish for a rapprochement *in all circles* is dominant, also in 'nationalistic' circles, provided this rapprochement does not include a humiliation.— Just as little as the manifest can a corresponding manifestation of the opposite side (Painlevé's introduction speech for the Conseil) constitute a basis for international cooperation. Hence before there is any question of Germany's joining, the Conseil has to base itself on a new foundation, or better yet, a whole new organization should be established.— Essentially Mr. Stoilow had to recognize my point of view, when he also observed that he as Romanian wasn't so sensitive in these matters and that he was amazed about our sensitivity. I have the impression that a rapprochement with the French is possible, even if there are maybe great difficulties to be overcome.

9. The breakfast was Saturday afternoon. In the afternoon an invitation from the Union to its members was distributed for a meeting on Sunday, which should take place during one of the general talks. In the evening I heard from Mr. *von Kerékjártó*, that Prague was considered for the next congress site. The invitation also came from the German university in Prague, because it expected a strengthening of German culture in Bohemia. For Hungary participation in a Prague congress is impossible, because of the situation of the Hungarian minority in Cechoslovakia. Mr. *von Kerékjártó* pleaded for Switzerland. Even though the idea of the Germans from Prague appealed to me, I have to admit that with the present situation in the world only a congress in a truly neutral country like Switzerland is possible.— Until now I haven't used your file for reasons I already informed you about, except that I told to some gentlemen the matters related to the final session. I left it further in the hands of Mr. *von Kerékjártó*.

The final session in Florence on Monday, September 10, started with a welcome by a Florentine magistrate and a talk by *Birkhoff*. Then the choice of the next meeting place followed. *Pincherle* announced that an invitation from Switzerland had come. So he proposed Switzerland. The proposal

⁽²⁹⁹⁾Most likely 'of the 93'. This was manifest signed by 93 prominent German scholars, who reports about misconduct of the German military. C.f. [Van Dalen 1999] p. 337.

was adopted with great applause. The *representative of Switzerland* (name unknown to me) presented his invitation first in German, then in French, and then thanked in the German language the Italians and especially Pincherle for the magnificent course of the congress.

After the final session Mr. Stoilow told me in short about the Union meeting. Pincherle had resigned as chairman of the board, but remained a board member. A request to appoint a committee to clear up the relations with Germany, is superseded because the board has been charged with that problem. A suggestion of Holland as next congress location was mentioned. But because of uncertainty about your position this idea was abandoned, and also no further proposals have been made.— Because of the method used in Florence, the Union is for the time being without any influence on the organization of future congresses.

About the mood during the congress it must be said that overall it was good. The relations between the subjects of different nations were friendly. Where there were dangerous moments, one really managed very well to take away all conflict matter.

Finally, let me say a few words about us Germans. For us the trip to Bologna was very taxing because of the German-Italian relations. The Italians celebrate the date of their declaration of war ⁽³⁰⁰⁾ as a national holiday; they know that for us that war declaration has a special meaning. But what is much worse, is the situation in South Tirol. I know the situation from own experiences, and I have to say that they are much worse than one can imagine from even the most detailed press reports. Such a horrible brutality against a minority has no precedent in the entire civilized world. In view of this fact it is actually impossible that a German accepts the hospitality of the Italian government. That this was the case in Bologna, was because we had no influence on the choice of the congress location and because of the role of mediator of the Italians, a rejection would be misunderstood.— Whether the trip to Lake Ledro has gone through, I don't know. Most congress participants joined the trip to Ravenna. Incidentally, Lake Ledro lies in territory that is undoubtedly Italian ⁽³⁰¹⁾

[Typescript – in Brouwer]

⁽³⁰⁰⁾ May 23, 1915 Italy declared war, in the hope to gain pieces of territory such as South Tirol, i.e. the region around Bolzano. ⁽³⁰¹⁾ Document breaks off here.

Editorial supplement

H. Härten to Ms. I. Gawehn⁽³⁰²⁾ — 27.IX.1928

Dear Miss Dr. Gawehn!⁽³⁰³⁾

Would you please tell Professor Brouwer:

To my report I must still add that a committee of representatives of the whole world has deliberated about the site of the next congress. So the meeting in Florence received an already established proposal. How the choice of the representatives for this committee was made I don't know. I only know that *Landau* belonged to it (he is said to have proposed Jerusalem) and probably also *Hahn*.

With friendly greeting

Yours sincerely⁽³⁰⁴⁾

H. Härten

[Signed typescript, postcard – in Brouwer]

1928-10-25a

From D. Hilbert — 25.X.1928^a

Göttingen⁽³⁰⁵⁾

Dear Colleague, [Sehr geehrter Herr Kollege]

Because it is not possible for me, given the incompatibility of our views on fundamental questions, to cooperate with you, I have asked the members of the editorial board of the *Mathematische Annalen* for the authorization, and received that authorization from Messrs. Blumenthal and Carathéodory, to inform you that henceforth we will forego your cooperation in the editing of the *Annalen*, and that consequently we will delete your name from the cover page.

⁽³⁰²⁾Brouwer's assistant. ⁽³⁰³⁾'Sehr geehrtes Fräulein Dr. Gawehn'. In handwriting preceding the typescript. ⁽³⁰⁴⁾*Ihr sehr ergebener*. ⁽³⁰⁵⁾The letter was not opened by Brouwer, see [Van Dalen 2005] section 15.3. The text is taken from *Hilbert to Einstein 25.X.1928*.

At the same time I thank you in the name of the editorial board of the *Annalen* for your past activities in the interest of our journal.

Sincerely yours ⁽³⁰⁶⁾

D. Hilbert

[Typescript copy – in Einstein]

1928-11-02a

To O. Blumenthal — 2.XI.1928^a

Laren

Dear Colleague, [Werter Kollege]

On October 27 I received simultaneously a ‘*Kennisgeving*’ ⁽³⁰⁷⁾ concerning two registered letters from Göttingen and a telegram from Erhard Schmidt, ⁽³⁰⁸⁾ which made me postpone the collection of the letters for the time being, but to wait with that until the visit of Carathéodory that was announced in the telegram.

During this visit, which took place on October 30, both letters were present, unopened, and from the statements of Carathéodory I gathered:

about one of the letters (which had no sender’s address on it).

1. That the communication in this letter should have, according to the rules, either several signatures or yours.
2. That in the letter the name Carathéodory is mentioned not in accordance with the facts (but that Carathéodory will not disavow the letter, should I have learned the contents).
3. That the sender of the letter would within a few weeks probably seriously regret sending it.

Thereupon I have decided not to open or read the letter.

about the second letter.

1. That your name as sender on the envelope was incorrect and that the letter was written by Carathéodory.
2. That Carathéodory regretted the contents of the letter.

⁽³⁰⁶⁾ *und ergebnst.* ⁽³⁰⁷⁾ Notification (from the postal office). ⁽³⁰⁸⁾ *Schmidt to Brouwer 27.X.1928.*

Thereupon I gave the letter back to Carathéodory unopened.

Furthermore Carathéodory informed me that the board of chief editors of the *Mathematische Annalen* planned to remove me from the board of editors of the *Annalen* (and that it felt legally entitled to do so). This because Hilbert wished that removal, and because his state of health demanded indulgence. Carathéodory asked me, out of compassion with Hilbert, who was in such a state that one could not hold him accountable for his misdemeanor, that I would accept this infuriating insult with equanimity and without resistance.

With respect to this plea of Carathéodory I have made a reservation to decide after calm deliberation. Today I have decided. You find enclosed the copy of a letter to Carathéodory.⁽³⁰⁹⁾

Your
(signed) Brouwer.

[Carbon copy – in Brouwer]

1928-11-02b

To C. Carathéodory — 2.XI.1928^b

Laren⁽³¹⁰⁾

Dear Colleague, [Werter Kollege]

After careful consideration and extensive consultations I have to take the point of view, that the plea you directed to me, namely to treat Hilbert as of unsound mind, could only be complied with, if it would have reached me in writing, and in fact jointly from Hilbert's wife and his family doctor.

Your
(signed) Brouwer.

To Prof. C. Carathéodory.

[Typescript copy – in Brouwer]

⁽³⁰⁹⁾ *Brouwer to Carathéodory 2.XI.1928.* ⁽³¹⁰⁾ A copy was enclosed in *Brouwer to Blumenthal 2.XI.1928*

1928-11-05a

To Eds. *Mathematische Annalen* — 5.XI.1928^a

Laren

To the publisher and the editors of the Mathematische Annalen.

[An Verleger und Redakteure der Mathematischen Annalen]

From information communicated to me by one of the chief editors of the *Mathematische Annalen* on the occasion of a visit on 30-10-1928 I gather the following:

1. That during the last years, as a consequence of differences of opinion between me and Hilbert, which had nothing to do with the editing of the *Mathematische Annalen* (my turning down of the offer of a chair in Göttingen, conflict between formalism and intuitionism, difference in opinion concerning the moral position of the Bologna congress), Hilbert had developed a continuously increasing anger against me.

2. That lately Hilbert had repeatedly announced his intention to remove me from the board of editors of the *Mathematische Annalen*, and this with the argument that he could no longer ‘cooperate’⁽³¹¹⁾ with me.

3. That this argument was only a pretext, because in the editorial board of the *Mathematische Annalen* there has never been a cooperation between Hilbert and me (just as there has been no cooperation between me and various other editors). I have not even exchanged any letters with Hilbert since many years and that I have only superficially talked to him (the last time in July 1926).⁽³¹²⁾

4. That the real grounds lie in the wish, dictated by Hilbert’s anger, to harm and damage me in some way.

5. That the equal rights among the editors (repeatedly stressed by the editorial board within and outside the board¹⁹ allow a fulfillment of Hilbert’s

¹⁹From the editorial obituary of Felix Klein, written by Carathéodory ‘He (Klein) has taken care that the various schools of mathematics were represented in the editorial board and that the editors operated with equal rights alongside of himself—He has (...) never heeded his own person, always had kept in view the goal to be achieved.’ (From a letter from Blumenthal to me, 13-9-1927). ‘I believe that you overestimate the meaning of the distinction between editors in large and small print. It seems to me that we all have equal rights. In particular we can speak for the *Annalenredaktion* if and only if we have made sure

⁽³¹¹⁾ *zusammenarbeiten*. ⁽³¹²⁾ Brouwer lectured on July 22 in Göttingen on *Überall und scheinbar überall definierte Funktionen* (Functions that are defined everywhere and functions that are defined apparently everywhere). At that occasion there was a reconciliation between Brouwer and Hilbert, see [Alexandrov 1969] and [Van Dalen 2005], p. 571.

will only in so far that from the total board a majority should vote for my expulsion. That such a majority is scarcely to be thought of, since I belong to the most active members of the editorial board of the *Mathematische Annalen*, since no editor ever had the slightest objection against the manner in which I fulfill my editorial activities, and since my departure from the board, both for the future contents and for the future status of the *Annalen*, would mean a definite loss.

6. That, however, the often proclaimed equal rights, from the point of view of the chief editors, was only a mask, now to be thrown off. That as a matter of fact the chief editors wanted (and considered themselves legally competent) to take it upon themselves to remove me from the editorial board.

7. That Carathéodory and Blumenthal explain their cooperation in this undertaking by the fact that they estimate the advantages of it for Hilbert's state of health higher than my rights and honor and professional prospects, ⁽³¹³⁾ and than the moral prestige and scientific status of the *Mathematische Annalen*, that are to be sacrificed.

I now appeal to your sense of chivalry and most of all to your respect for Felix Klein's memory, and I beg you to act in such a way, that either the chief editors abandon this undertaking, or that the remaining editors split off and carry on the tradition of Klein in the management of the journal by themselves.

L.E.J. Brouwer

[typescript copy – in Brouwer]

of the approval of the editors interested in the matter under consideration. — Although I too take the distinction between the two kinds of editors to be more typographical than factual (I make an exception for myself as managing editor), I understand your wish for a better typographical make up very well. You know that I personally warmly support it. However, we can for the time being, as long as Hilbert's health is in such a shaky state as it is now, change nothing in the editorial board. I thus cordially beg you, to leave your wish for later. At the right moment I will certainly and gladly bring it out.

⁽³¹³⁾ *Wirkungsmöglichkeiten.*

1928-11-06b

O. Blumenthal to Editors Math. Annalen — 6.XI.1928^b Aachen

To the editors of the *Mathematische Annalen*. [An die Redakteure der
Mathematischen Annalen]

Dear Colleague,

I accidentally learned that in the affair, you are familiar with, B r o u w e r has written a letter to the joint editors and the publisher. I beg you not to answer this letter, before you have received from me an extensive exposition of some new events, which appear to me essential for judging the situation. You will receive this exposition within a few days.

Best greetings,

Your

O. Blumenthal

[Copy of signed typescript – in MA collection]

1928-11-16a

O. Blumenthal to Eds. Math. Annalen — 16.XI.1928^a Aachen

To the publisher and the editors of the *Mathematische Annalen*
[An Verleger und Redakteure der *Mathematische Annalen*]⁽³¹⁴⁾

As manager of the editorial board of the *Annalen* I feel obliged to make a rejoinder to Brouwer's circular to the publisher and editors of the *Mathematische Annalen*.⁽³¹⁵⁾ I base my explanations partly on letters from Hilbert, Carathéodory and Brouwer, and partly on a long and detailed conversation that I had with Hilbert in Bologna.

⁽³¹⁴⁾ According to *Blumenthal to Courant 12.XI.1928* no copy was intended for Brouwer. Eventually Blumenthal sent a copy of the final version to Brouwer (see *Blumenthal to Bohr & Courant 4.XII.1928*). In the Brouwer archive there are typescript copies of the circular. It is plausible that one of the editors – most likely Bieberbach, but possibly Carathéodory – sent a copy to Brouwer. ⁽³¹⁵⁾ *Brouwer to Publisher and editors Math. Annalen, 5.XI.1928*.

I want to remark in advance that the formulation of Brouwer's letter is misleading: one might get from it the impression that the editor that visited Brouwer on October 30 (Carathéodory) has drafted statements 1–7. This is of course not the case with any of them, they are rather the viewpoints that Brouwer has formed for himself.

In the following I give a brief representation of the events and I will go into Brouwer's letter in the appropriate places.

1. Hilbert's letter and his reasons.

The letter that Hilbert sent to Brouwer on October 25, is as follows.

Dear Colleague!

Because it is not possible for me, given the incompatibility of our views on fundamental questions, to cooperate with you, I have asked the members of the editorial board of the *Mathematische Annalen* for the authorization, and received that authorization from Messrs. Blumenthal and Carathéodory, to inform you that henceforth we will forego your cooperation in the editing of the *Annalen*, and that consequently we will delete your name from the cover page.

At the same time I thank you in the name of the editorial board of the *Annalen* for your past activities in the interest of our journal.

Sincerely yours
D. Hilbert.'

This letter has not been opened by Brouwer, as I must remark already here, and motivate later. He was, however, informed by Cara⁽³¹⁶⁾ about its contents, more specifically about the reasons of Hilbert's actions mentioned in the first sentence. Brouwer's points 2 and 3 refer to this. About this I want to say the following:

On point 2 and 3. Brouwer interprets the idea of cooperation in an extrinsic sense (point 3). This is a complete misjudgment of the true interpretation. It is rather so, that Hilbert has acquired the firm conviction that Brouwer's actions are damaging for the *Annalen*, and that he therefore cannot take the responsibility to act as chief editor in a board to which Brouwer belongs. So it is in no way a pretext.

⁽³¹⁶⁾Nickname for Carathéodory.

On point 1 and 4. The grounds given by Brouwer for Hilbert's acts don't apply. The reason given in point 4 is spiteful and hence needs not be answered. Also the scientific difference of opinion concerning the foundations of mathematics plays no role. More specifically, it is not correct, as Brouwer seems to suggest in point 5, that the mathematical direction he represents will in the future be heard less in the *Annalen*. Also Brouwer's circular letter before the Bologna congress, by the wording of which Hilbert felt insulted, has only together with other, maybe more important factors, brought about the decision. The motives are lying much deeper. I give them in my formulation, but I am certain that they completely represent Hilbert's meaning.

Felix Klein has been until his resignation from the editorial board ⁽³¹⁷⁾ a kind of highest authority among us, who was called upon in difficult cases or who acted on his own initiative to support important decisions (e.g. the transfer of the *Annalen* to the Springer Verlag), or to resolve differences within the editorial board. It is good and necessary that in a numerous board such as ours there is such a higher authority, who is not concerned with the details but keeps an eye on the general context and feels responsible for it. After the death of Klein ⁽³¹⁸⁾ Hilbert has thought himself obliged to fulfill this function, and he already has acted in this sense, and I at least have personally always recognized him as such. Hilbert has seen in Brouwer an obstinate, unpredictable and dominant character. He was afraid that once he would resign from the editorial board, Brouwer would bend it to his will and he has considered this such a serious danger for the *Annalen*, that he wanted to counteract him when he still could do so. Probably under the influence of his recent illness he felt obliged, in the interest of the *Annalen*, to effectuate Brouwer's exit from the board, and to implement this measure right away and with all his energy.

Cara and I who have been friends with Brouwer for many years, had to recognize the objective correctness of Hilbert's objections to Brouwer's editorial activities. Although Brouwer was a very conscientious and active editor, he was really difficult in his contacts with the management and meted out difficulties to authors that were hard to bear. For example, manuscripts that had been sent to him for refereeing were stored for months, because he, on principle, first had copies made of all articles refereed by him. (I just recently had an example of that.) There is no doubt whatsoever that Klein's resignation from the editorial board goes back to Brouwer's rude behavior (although in an affair where Brouwer formally was right ⁽³¹⁹⁾). The further

⁽³¹⁷⁾ 1924. ⁽³¹⁸⁾ 1925. ⁽³¹⁹⁾ The Mohrmann affair.

developments (see below) have shown that Hilbert was even more right than we thought at that time.

Because we could not ignore the factual justification of Hilbert's point of view, and because we saw ourselves confronted with his irrevocable determination, we consented to Brouwer's removal from the board. Only we wished — unjustified, as I see now — a milder form, in that Brouwer should be persuaded to resign his editorship himself. But Hilbert could not be persuaded, and finally we have, although reluctantly, decided to give him a free rein. Einstein has not consented, with the motivation that one should not take Brouwer's peculiarities seriously.

Point 5 and 6. I will not examine here in how far it is justified that the other editors were not informed in advance of Hilbert's plan. Formally the justification seems to be given by the distinction made on the Annalen cover between 'advisors' and 'editors'.⁽³²⁰⁾

II. The events after the letter was sent

On October 26 and 27 Cara and I were in Göttingen to discuss the situation. Then Cara went on to Berlin to discuss the matter. Although he objectively held the removal of Brouwer from the board for unavoidable, he decided in Berlin to make a last effort to come to an amicable agreement by softening the categorical form of the dismissal. So he came on the 30th⁽³²¹⁾ to Laren, after Brouwer had been telegraphically requested not to take any steps. Because Brouwer hadn't opened Hilbert's letter, Cara told him the contents (but not the formulation) and proposed to him to step down voluntarily from the board of editors of the Annalen, and leave the letter unopened. He thus wanted to prevent Brouwer to feel insulted by the form, and felt justified because it seemed to him that its rudeness was partly caused by Hilbert's ailing condition. He left Brouwer in the dark about the fact that in our opinion he should step down from the board, and asked him, out of compassion with Hilbert and his disease to resign by himself. Brouwer reserved a decision until after calm deliberation. He has left Hilbert's letter unopened and on November 2 wrote the following letter to Cara:

Dear Colleague,

After careful consideration and extensive consultations I have to take the point of view that the plea you directed to me, namely

⁽³²⁰⁾ 'Mitwirkenden' und 'Herausgebern'; the present day formulation would be 'associate editors and (chief) editors. ⁽³²¹⁾October

to treat Hilbert as of unsound mind, could only be complied with, if it would have reached me in writing, and in fact jointly by Hilbert's wife and his family doctor.

Your Brouwer.'

For this horrible and repulsive letter, which Brouwer has communicated also to me by means of a copy, I have only this one explanation, that Brouwer (intentionally or involuntarily) has put together for himself the ugliest view, from Cara's utterances and pleas. I have to admit — and Cara has written the same to me —, that I have thoroughly misjudged Brouwer's character, and that Hilbert understood him and judged him more accurately than we did. I too am unable to cooperate further in the editorial board with the writer of this letter, and I now also actively take Hilbert's side. I can't understand that Brouwer after this letter can appeal in the final paragraph of his circular to the chivalry of the editors and the memory of Felix Klein.

I ask you gentlemen either to speak out soon, or for your tacit approval that from the next issue on Brouwer's name is omitted from the cover page of the *Annalen* and that he receives no further *Annalen*-information.

Yours sincerely
O. Blumenthal

[Copy of signed typescript – in Einstein, typescript copy in Brouwer]

1928-12-22

**D. Hilbert, F. Springer to Eds. *Math. Annalen* — after 22.XII.1928
Göttingen, Berlin ⁽³²²⁾**

[note on Brouwer's carbon copy in his handwriting:] received 27.12.28

Dear Sir, [Hochgeehrter Herr]

The editors until now of the *Mathematische Annalen* have together with the publisher agreed that with the publication of the 100th volume the old contract will be terminated and replaced by a new one at the publication of volume 101.

⁽³²²⁾This letter is dated XII.1928; the same letter has been sent to Courant with the date 22.XII.1928 [copy in Brouwer archive]; this suggests a date between 22.XII.1928, and the date of delivery: 27.XII.1928.

At the same time a change will take place insofar that Carathéodory and Einstein have withdrawn themselves, and Hecke has joined.

The revision of the publisher's contract is combined with a fundamental change in the manner of management. It has been shown desirable, that for the acceptance or rejection of articles only the real editors take the full responsibility, and that they will be satisfied with soliciting referee reports from colleagues outside, without burdening them with a final responsibility. Accordingly only the names of the responsible editors will be shown on the title page, starting from volume 101.

The publisher and the editors use this occasion, to express our warm thanks to all those who have until now regularly taken part in the publication of the *Annalen* as associate editor, ⁽³²³⁾ for the rendered exceptionally valuable work, and to combine this with an appeal, that the cooperation in the form of referee reports also in the future will not be refused. Independent of this the publisher wishes to show his gratitude for the shown help, by making available to all of the gentlemen concerned a free copy of the *Annalen*, as before.

For the Editorial board of the *Mathematische Annalen*
(signed) D. Hilbert

For the Publisher of the *Mathematische Annalen*
(signed) F. Springer

[Signed typescript – in Einstein; signed carbon copy of typescript – in Brouwer]

1928-12-23b

R. Courant, H. Bohr to C. Carathéodory — 23.XII.1928^b
Göttingen ⁽³²⁴⁾

Dear Carathéodory, [Lieber Carathéodory]

Many thanks for your letter of December 19, and most of all for the announcement of your visit.— The *Annalen* matter is now formally wrapped up: the new contract has been signed and the circular of Springer and Hilbert has been sent. Bohr and I are like you very happy about the conclusion of

⁽³²³⁾ *Mitwirkende*. ⁽³²⁴⁾ There is a draft and a (presumably) final version with a letter of Bohr appended. The corrections in the text are clearly Bohr's.

this affair that has worried us so much during the last month and that made great demands on our thoughts and time for work.

Our satisfaction about the solution of the crisis would be even greater if not a couple of phrasings of your letter worried us because they suggest a possibility for new misunderstandings. It is about the question what the real motives of Hilbert were.

When I first heard of Hilbert's intention, the immediate reaction was a shock.⁽³²⁵⁾ Because at first Hilbert also did not⁽³²⁶⁾ explain his motives to me, I have only gradually understood these clearly. But now, where it is again possible to speak calmly and in detail with Hilbert, all doubts have vanished that Hilbert's motives were absolutely objective, based on his sense of responsibility for the *Annalen*, and moreover on his understanding that Brouwer's personality could be dangerous for the *Annalen* when Hilbert wouldn't be able anymore to act as a counterbalance. Hilbert has stressed again and again to us that he has no personal feelings of hate, anger or offense against Brouwer, and that he rather deemed a factual separation necessary and that he wanted to carry that through with all his strength. The more radical solution to abolish the whole advisory board was immediately and eagerly adopted by Hilbert, not only because he thought it objectively useful, but also he was very happy with it because thereby the personal edge against Brouwer was taken from the whole action.

So it is nothing less than a construction after the fact, if one now, at the winding up of the matter, stresses these objective motives, even though the first step taken by Hilbert under such singular circumstances could create a different impression.

To point emphatically to this state of affairs, seems — in the very first place because of Hilbert — to be our duty. In the whole affair we have acted in his name, and we cannot admit that a version about his intentions becomes public that does him no justice. When already you accept such a view, what should we expect from those who are farther removed? Our responsibility to Hilbert on this point is all the greater, because until now Hilbert hasn't been informed about all details of the development of the conflict; more specifically he is totally unaware of your visit to Laren and the outrageous representation of that by Brouwer.⁽³²⁷⁾ So he doesn't know that the reproach of subjectivity and personal wish for revenge has been

⁽³²⁵⁾In draft: 'a mild shock' ⁽³²⁶⁾The word corresponding to 'not' is missing, but comparison with the draft learns that this is a copying error, caused by a slight rephrasing.
⁽³²⁷⁾In draft this part runs slightly different: 'in particular nothing about your visit to Laren and the distorted representation as reported by Brouwer, and he does not suspect, that the reproach ...'

raised against him; he cannot defend himself against that and we must take that task upon us as long as we haven't informed Hilbert about all details, which we would so much like to avoid in the interest of all concerned.

It remains to take the future relations between German mathematicians among each other into consideration. When some of the colleagues do not learn to understand, what is really at the bottom of Hilbert's mind, then the bad feelings won't go away and can erupt here and there. When such a latent tension — which won't come from the circle of Hilbert — in the future is to be avoided, then we must use this moment now to remove any unjustified ugly appearance from the matter, and enter into a basis of mutual understanding and trust. It would be very gratifying and reassuring if you could help with it, that all concerned, especially also the Berlin colleagues, take this attitude.

Many cordial greetings and also Christmas wishes from house to house

Your
Courant

Dear Carathéodory, ⁽³²⁸⁾

I add two words to Courant's letter. First to say how much I and my wife are looking forward to seeing you and your wife in January in Göttingen. But secondly also, because I want to tell you of my own accord personally how much you have, in my opinion, misunderstood Hilbert, when you think that he wanted Brouwer removed from the board, just because he felt personally insulted. I had never doubted that you, like and me in the discussion with you, were quite clear about it that Hilbert (correctly or not) thought *that Brouwer's stay in the board would constitute a danger for the future*. When you are not completely convinced yourself then the only right thing to do is really that you ask Hilbert quite openly about his reasons, because Hilbert — without him knowing it, so he can't defend himself — is first considered of unsound mind and then as not-objective, ⁽³²⁹⁾ this is a situation which I, as representative of Hilbert, in the long run cannot bear standing by idly.

With best greeting, sincerely yours ⁽³³⁰⁾
H. Bohr

[Copy of carbon copy – in MA collection]

⁽³²⁸⁾Lieber Carathéodory. ⁽³²⁹⁾'unzurechnungsfähig', 'unsachlich'. ⁽³³⁰⁾Mit den besten Grüßen, Ihr ergebener.

[*Editorial supplement: Carathéodory re Hilbert's motives. From Carathéodory to Courant — 19.XII.1928.*]

I am tremendously happy about the final settlement of the Annalen Affair and also about the fact that Hilbert has acknowledged that I have done the best possible for him. I have admired from the beginning the strength with which he attacked Brouwer. He has, however, indicated as the sole grounds for his decision at the time, that Brouwer had insulted him; I would find it unworthy if one would construe after the fact, that he was motivated by impersonal grounds.

1929-01-23

To Editors *Mathematische Annalen* ⁽³³¹⁾ — 23.I.1929 Laren

To Messrs. Bieberbach, Bohr, Carathéodory, Courant, von Dyck, Einstein, Hoelder, von Karman, Sommerfeld. [An die Herren Bieberbach, Bohr, Carathéodory, Courant, von Dyck, Einstein, Hoelder, von Karman, Sommerfeld.]

Because I persist for the time being in the interest of the decorum of the mathematical community in the point of view that I expressed in my circular of December 23, 1928, namely to await the result of Carathéodory's efforts, and only correct the erroneous impressions contained in Blumenthal's circular, if the possibility of a rectification by the other side cannot be counted on anymore, I restrict myself right now to take position on the Hilbert-Springer circular of December 1928, which I only received after I had sent my circular of December 23, 1928.

1. The *Mathematische Annalen* constitute a spiritual heritage, a common spiritual property of the whole editorial board, which has got together to serve the collective progress of mathematics without regard for personal scientific activity. The so-called chief editorial board was established by free choice of the joint editors²⁰ and occupied a merely representative position

²⁰This character of appointment doesn't change by the fact that usually a formal choice by a majority vote is replaced by informal discussions within the total board.

⁽³³¹⁾Blumenthal and Hilbert excluded.

with respect to the public.²¹ The formal right with respect to contracts with the publisher constitutes therefore for the chief editors not an inherent possession, but something that has been entrusted to them. And if Messrs. Hilbert and Blumenthal purloin these entrusted goods from their principals, then they commit a misappropriation, also when this accidentally cannot be challenged legally.⁽³³²⁾

2. The role of Blumenthal as revealed in the Hilbert-Springer circular can be described as a breach of trust and faith on the following grounds:

Firstly, Blumenthal has in his quality of managing editor repeatedly and in the most unambiguous way acknowledged the structure of our circle as explained above. An even clearer example than the one mentioned in my circular of November 5, 1928, is the following statement in a letter of October 12, 1924: ‘The editorial board of the *Annalen* was from the outset a democratically organized institution where all editors have equal rights. We would like to uphold this principle or rather to revive it.’

Secondly, in the summer of 1925, when in my opinion the amount of irregularities committed by Blumenthal as manager had become excessive because of a very serious infringement, and I demanded a full session of the whole board to discuss this and to prevent repeats, I only relinquished this request on the explicit announcement of Blumenthal’s plan to stay on as manager at most until volume 100.²² Volume 100 has just now, on December 28, 1928, been wound up.

According to the above the editors of the *Annalen* have to recognize as the contents proper of the Hilbert-Springer circular, that Hilbert and Blumenthal as editors and Springer as publisher have thus advocated their dismissal. The remaining editors therefore have the task to further administer the inheritance of Felix Klein together with a new publisher and continue

²¹If this interpretation of the structure of our circle hadn’t since 1914 been repeatedly emphasized to me by several editors, especially by our leader Felix Klein (who also took this most conscientiously into account during the handling of several incidents in which he and I were involved), then the responsibility experienced by me as editor and also the activity I took upon me would never have reached the magnitude which in fact existed and actually is known among my co-editors only to Blumenthal. ²²When I reminded him orally of this in August 1927, I received from Blumenthal the evasive answer that it was very difficult, as long as Hilbert was alive, to change anything in the board. Blumenthal himself has given a striking refutation of this pretext.

⁽³³²⁾Observe the similarity to Brouwer’s comment on consistency proofs, [Brouwer 1923b] p. 3.

the Klein tradition of running a mathematical journal.

(signed) L.E.J. Brouwer

[Carbon copy of typescript – in Brouwer]

1929-04-30

To Editors *Mathematische Annalen* — 30.IV.1929

Laren

To the publisher and the editors of the *Mathematische Annalen*. [An Verleger und Redakteure der *Mathematischen Annalen*]

1.

To my amazement and disappointment, notwithstanding my demand, so far no rectification from the other side, of the false expositions contained in the Blumenthal circular of November 16, 1928 has appeared. My amazement and disappointment concern most of all the circumstance that Carathéodory did not feel it his duty of honor to gainsay Blumenthal's statements concerning his visit to me on October 30, 1928, and to confirm the statements in my circular of November, 5, 1928.

Therefore I take the floor myself.

The points 1–7 formulated in my above mentioned circular are not, as the Blumenthal circular falsely pretends, 'viewpoints that Brouwer has formed for himself', but *viewpoints that during the mentioned visit came up between me and Carathéodory in mutual agreement, i.e. that each time was enunciated by one of us and accepted by the other.*

To substantiate this I provide details about the visit of Carathéodory, pointing out that I defend myself against Blumenthal's slander, and how I was driven to the general statements in my circular of November 5, 1928 concerning the earlier mentioned visit, by the necessity to defend myself against Hilbert's attack that was announced in the course of visit.

As was already stated in the annex of my circular of December 23, 1928, I received on October 27, 1928 simultaneously a 'Notice' of two registered letters from Göttingen and the following telegram from Berlin that made me to collect the letters at a later time: 'Professor Brouwer. Laren N.H.— Please do not undertake anything until you have talked to Carathéodory,

who must inform you about a matter unknown to you with the greatest consequences. The matter is completely different from what you must believe from the letters received. Carathéodory comes to Amsterdam on Monday. Erhard Schmidt.’

During his visit on October 30, 1928, Carathéodory informed me first, while the two letters that just had been collected were lying unopened before us, that the ‘matter unknown to you with the greatest consequences’ consisted of the following: recently the taking of a wrong medicine had produced in Hilbert such a state that he on the one hand ‘could not be taken seriously anymore at all’ (words of Carathéodory),²³ and that on the other hand the slightest resistance to his will could be fatal to him.

In this situation the idea had come up to remove me from the editorial board of the *Annalen* and he wanted to carry out this idea with all means.— It was evident that the realization of Hilbert’s plans would constitute a grievous injustice. In order not to endanger Hilbert’s life, he (Carathéodory) begged me not to undertake anything against this for the time being. Hopefully Hilbert would soon use the right medicine again, and as a consequence of the improvement of his situation, come to better views, before anything definitive had happened.

One of the closed letters present was from Hilbert. The statement in it, that Hilbert fired me as editor, ‘authorized by Blumenthal and Carathéodory’ were unjustified; because when he (Carathéodory) after his return from America had been requested in writing by Hilbert for this authorization, he answered: he would in principle not put any obstacle in Hilbert’s way, but he would come to Göttingen to discuss the matter. When he arrived in Göttingen, he heard from Blumenthal that Hilbert had already dispatched his letter of dismissal, under reference to the mentioned authorization. In the subsequent discussion of half an hour with Hilbert the matter was not touched on, as little then as today.²⁴ — With reference to the second letter (which carried on the envelope Blumenthal’s name as sender), this was written by him (Carathéodory), and in this he asked me to resign voluntarily

²³One could think for a moment that communicating such utterances is somewhat incorrect, because naturally one assumes a certain degree of confidentiality with reference to these. But the assumption of confidentiality and the ensuing solidarity can certainly not, insofar as they have not become null and void because of the further course of the conversation as sketched below, be brought into agreement with Carathéodory’s later silence upon Blumenthal’s false impressions. Moreover, also justified scruples must in the case at hand, where it concerns the clarification of a scandalous calumny and robbing someone’s position, yield — in analogy to the case of hearing witnesses in a criminal process.

²⁴Einstein, too, was asked by Hilbert for authorization, but he refused.

from the board of editors out of consideration with Hilbert's state of health. But now he regretted the formulation of this letter.

Thereupon I have returned this latter letter closed to Carathéodory, and I have told him that I considered my possible removal from the editorial board not only a grievous injustice, but also a serious damage to my scope of action, and as an insult of my honor in the public opinion; and that, if this unheard of event really would come to happen, my honor and my scope of action could only be restored by a most extensive appeal to the public, and hence that an atrocity committed against me would result in a public scandal.— Carathéodory answered that he had been prepared for such a standpoint on my part, that in his opinion the *Annalen* would be ruined through the realization of the plans hatched against me, and that he himself already had taken the decision to resign from the board, a decision which actually — again out of consideration with Hilbert's state of health — could for the time being not be carried out.

The further course of the discussion then brought the seven points mentioned in my circular of November 5, 1928.

With respect to the desired consideration for Hilbert's state of health of me by Carathéodory, I expressed my opinion that in case there was a direct risk of Hilbert's life, it would be a crime to be an accessory to see him ending his life with a crime; but on the other hand unreasonable tolerance could increase his petulance and lust for power in way that could put the happiness of his life in danger. I promised however that I would discuss this last psychological question with appropriate acquaintances. In case my point of view would not change after closer consideration, then yielding to Carathéodory's plea to undertake nothing for the time being against the realization of Hilbert's plans, would be equivalent for me to the probability that these plans would be cancelled without active interference by me.— The discussion closed with Carathéodory repeated pointing at Hilbert's terrible situation, and the words that he (Carathéodory) under these circumstances 'appealed to my mercy'.

During this discussion of two hours in the morning of October 30, Carathéodory's attitude was all the time that of a confidant, friend and ally, who advised me on the possibilities and means to prevent a calamity. The discussion seemed to be concluded in full agreement, notwithstanding the tentative differences in our opinion on details of the affair. Accordingly, Carathéodory stayed still several hours together with me and a few guests, who were invited because of him, who all had the impression of an untrammelled atmosphere. Only at the farewell, when I was alone again with Carathéodory, I expressed the thought that occurred to me only at that mo-

ment, that since Hilbert had been able to face Einstein's objections to his plan, he also could bear without any danger a repudiation of the unjustified authorization in his letter to me. Only when I didn't get a logical answer from Carathéodory to this remark, but only (maybe to be attributed to the agitation of the farewell) received as answer cries like 'What should one do' and 'I don't want to kill people', I started to feel surprise, uncertainty, and irritation with respect to Carathéodory's attitude, which, in a complete change of mood, on my side found their expression in remarks like 'I don't understand you anymore', 'I consider this visit as a final parting' and 'I am sorry for you'.

The impressions that Carathéodory's visit left with me were basically confirmed 14 days later at the occasion of a discussion with Erhard Schmidt in Berlin, but completed in the following manner. I heard in the course of that discussion:²⁵

1. That Carathéodory had visited me at the instigation of Schmidt.

2. That the aim of this visit, in Schmidt's opinion, mainly had been this: to offer me in advance some satisfaction for the planned injustice to me, and in fact in the form of a open admission of the circumstance why I had to forego the protection of my co-editors against this injustice (Hilbert's state of health).

3. That according to remarks of Carathéodory to Schmidt, Hilbert's wrath against me was caused, even more than the three points mentioned in the first point of my circular of November 5, 1928, by my obstruction of the invitation of French mathematicians to contribute to the Riemann volume of the *Mathematische Annalen*.

Concerning the matter of satisfaction, the thought then came up between Schmidt and me that for a public insult a private satisfaction of course is insufficient, and that Carathéodory at least had the duty to make this private satisfaction a public one from the moment that this could be done without damage for the situation of Hilbert's health.

2.

From the arguments in Blumenthal's circular of November 16, 1928, under the caption 'Hilbert's letter and his reasons', I have gathered *that for the treacherous attack on me, apart from Hilbert's wrath, there had been a second reason: a strong desire of Blumenthal to remove me from the board.*

²⁵ Although I am aware of the confidential atmosphere of the talk with Schmidt, I must with respect to the communicability of its contents consider the argument valid that I gave at the end of footnote 1) above.

Because the purported ‘grounds’ that lie in my activity as editor, which Hilbert — suddenly proclaimed as the supreme authority by Blumenthal, ignoring all claimed equal rights — should have had for his action against me, could only have been suggested to him by Blumenthal himself.

1. Because the complaint brought against me, when traced back to Hilbert, would degenerate into an anecdote. Indeed, for years already he counts so little as editor⁽³³³⁾ that it even has proved dangerous for the orderly handling of the business to submit manuscripts to him. Consequently Hilbert himself doesn’t dare to mention this ‘ground’ in his dismissal letter, the content of which has become known through Blumenthal’s circular of October 25, 1928.

2. Because Blumenthal is the only one who, except me, can judge my total activity as an editor.

If therefore Blumenthal, before as well as after the start of the campaign against me, is responsible for the complaints raised in his circular, then I claim furthermore that those are to be considered as mere pretexts, behind which is Blumenthal’s above mentioned desire. In connection with the nullity of Blumenthal’s accusations, to be explained below, the fact comes to the fore that Blumenthal might by my removal be liberated from the following inconveniences:

1. The obligation to fulfill his promise mentioned in my circular of January 23, 1929, to resign from the management after the winding up of volume 100.⁽³³⁴⁾

2. My frequent admonishments concerning the arbitrariness in the management and the fact that this is damaging for the *Annalen*.

I now proceed to the discussion of Blumenthal’s accusations. I am blamed for the following:

1. That I have been rude in my behavior as an editor.
2. That I should have caused Klein’s resignation.
3. That manuscripts sometimes remained for months in storage with me.
4. That I made on principle a copy of each manuscript that was submitted to me.

⁽³³³⁾ Cf. *Blumenthal to Courant 9.II.1929*. ⁽³³⁴⁾ In defense of Blumenthal it should be pointed out that he tried to withdraw in 1925 from his editorial position (*Blumenthal to Hilbert 15.XI.1925*). His attempt was vigorously suppressed by Hilbert (*Hilbert to Blumenthal 18.XI.1925*). See [Van Dalen 2005] p. 626

Ad 1. One can very well speak about a reality corresponding to the word ‘rude’, ⁽³³⁵⁾ if the meaning is determined as follows: the will to integrity (duty to people), extended by the will to clarity (fate of the mathematician).— It came with me to an expression of this will, whenever the honor and the prestige of the *Annalen* were at stake. (Incidentally, among these were cases where Blumenthal himself had called me in.) In those cases neither the vanity of the authors, nor the tendency of Blumenthal to please everybody, could be taken into consideration.— When I occasionally made my will prevail against that of the manager, then the latter must have found no support from his colleagues in the board, or he had reasons not to elicit such support.

Ad 2. The event to which Blumenthal refers in his statement on Klein’s resignation, hardly can be any other than the following: I had a discussion with Klein about an article that I had already dealt with, whose author ⁽³³⁶⁾ had appealed to Klein as chief editor in the matter of changes demanded by me, and in an oral discussion he had made his views sound reasonable. When I talked it over with Klein, he understood that the author was wrong (not formally as Blumenthal would have it, but in matter of content), and that he therefore could not honor his given promise. During the further course of this talk, Klein expressed his view that the manner in which the chief editors were mentioned on the cover apparently gave the wrong impression to the public and he personally could, insofar he himself was involved, hardly bear the responsibility for this impression.— Some time later he resigned as chief editor.— Such a behavior speaks as much in favor of Klein, as it speaks against Hilbert, who with a much smaller share in the editorial activity than Klein’s at the time of his resignation, used the possibility to deploy the inner weakness of his position for its outer confirmation.

Ad 3. Because I spent on average about one thousand hours per year on my editorial activities, it is almost obvious that submitted manuscripts usually remained for months in my possession. Only the word ‘stored’ is misleading, because never were articles temporarily forgotten by me or even lost without a trace (as has happened with Hilbert), but they constituted each time the object of the most intensive editorial activity, by which their content usually was substantially influenced. As I have kept manuscripts longer than the normal deadline for printing only in the extremely rare

⁽³³⁵⁾The German word ‘*schroff*’ can mean all kinds of things like abrupt, blunt, brusque, curt, gruff, harsh, inaccessible, and is translated here by ‘rude’. ⁽³³⁶⁾Brouwer refers to Mohrmann. See *Blumenthal to Brouwer 23.VII.1924*, *Brouwer to Klein 29.XI.1923*. Mohrmann had gone over Brouwer’s head to Klein in the matter of a paper of his. See [Van Dalen 2005] p. 631.

cases where very large defects came to light, the articles were taken care of by me much better than if they had been ‘stored’ with Blumenthal during the same time.— Blumenthal held, by the way, until recently the opinion that my method was normal and conscientious, otherwise he would not have asked me for refereeing, even in the case of articles where I could not all be counted as expert, considering their subject.

Ad 4. Although Blumenthal can give an ‘example’ of my ‘basic method’ of making a copy of every submitted manuscript, and although I consider such an act as an elementary right of a refereeing editor, since many years it has come to that only in cases where an article seemed quite acceptable, but only after revision or after considerable extension. Then I considered this measure a duty with respect to the historiography of mathematics, indeed because the possibility should be taken into account of an incorrect reference to the submission date.

I challenge Blumenthal to produce the Annalen archive, especially with the complete correspondence between him and me. I claim that precisely these documents will refute his accusations in the most complete manner.

(signed) L.E.J. Brouwer.

[Carbon copy – in Brouwer]

Editorial comment

The following letter is part of a long series of exchanges concerning the priority of the theory of dimension. Its history is complicated and drawn out. The major players are Brouwer, Menger and Urysohn. The last one acknowledged Brouwer’s claims, but since he died in 1924, his view played no role in the discussions. There is no doubt that Menger, already during his stay in Amsterdam, developed the conviction that his role in dimension-theory was not given its rightful place by Brouwer and the Russian topologists. The reader should consult [Van Dalen 2005], ch.12, and section 15.5 for the historical background. The present letter and its sequel are concerned with the discussion in which Menger’s book *Dimension Theory* (1928) was the first volley. Brouwer reacted in the paper *On the historiography of dimension theory* (1928). Hahn and Brouwer at one point decided that

the conflict should be closed with a reply from Menger in the Proceedings of the Dutch academy (where Brouwer's paper as published). In spite of the efforts of Hahn, all attempts at a reconciliation failed; resulting in the end in plain hostility and irreconcilable differences.

1929-07-11

To H. Hahn — 11.VII.1929

Laren

Dear colleague Hahn, [Lieber Kollege Hahn]

Many thanks for sending me the manuscript of Menger and for your accompanying letter. I think that the manuscript really turned out well; both the general structure and the treatment of most details seem fit to me to bring the matter to a conclusion in that way, so neither from me nor from Moscow objections will be necessary. I hope that I can restrict myself to expressing in a short postscript (which I will show you beforehand) to the Menger note, the hope that the discussion that took place may be a useful contribution to clarification of the historical development of dimension theory, and to observe that the attentive reader can see from the reading of *both* notes that there are hardly any essential points of difference between Menger and me left. Naturally I will furthermore see to it that in the review of Menger's book ⁽³³⁷⁾ in the Jahresbericht of the D.M.V., the meanwhile obtained clarification of the situation and agreement between Menger and me will timely be taken into account.

I would like to discuss with you in person a few details in Menger's manuscripts that seem amenable to improvement as soon as possible, and I will arrange my travel plans (leading southwards anyway) accordingly. So please tell me until what day you will still be in Vienna, and which address you will have after your leave from there. It would be best to meet you in Vienna, where I have a chance to take a look at the relevant documents that I don't know yet (just as I on my part have, by the way, to show you some more documents).

Concerning the Menger documents in my safekeeping, this safekeeping is explicitly mentioned by me in the Amsterdam Proceedings, ⁽³³⁸⁾ so that I think it is more appropriate towards the public that in the future I myself

⁽³³⁷⁾[Menger 1928b]. ⁽³³⁸⁾[Brouwer 1928d].

also will function as trustee for these items. But maybe we can find a fitting modus to meet your wishes in this. We can discuss this point too in person.

With warm greetings, hoping to meet you again ⁽³³⁹⁾ soon

Your
L.E.J. Brouwer

[Typescript draft with handwritten corrections – in Brouwer ⁽³⁴⁰⁾]

1929-08-09a

To H. Hahn — 9.VIII.1929^a

Brussels
Brüssel

Dear Colleague Hahn [Lieber Kollege Hahn]

As a consequence of an disruption in carrying through my travel plan, your letter from Belgio has reached me after considerable delay. The interruption was caused by a great calamity: four days ago my briefcase ⁽³⁴¹⁾ which also contained my scientific diary was stolen from me on the front platform of a Brussels' tram, by a pickpocket, and both the police and the detectives consider the case as hopeless. Since in this diary my collected scientific thoughts and ideas of the last three years, which have largely disappeared from my memory, and of which only a few have already found a registration elsewhere, had been recorded, this event means for my scientific personality a serious personal mutilation ⁽³⁴²⁾, roughly the same as what 'decapitation' (elimination of the central process) means for a pine tree. To my amazement, I remain so far, fairly calm under this blow of fate; I believe, however, from certain phenomena, that I have nonetheless suffered a nervous collapse, the consequences of which will perhaps only later become visible, together with a disorganization of my scientific thoughts.

In my present condition, my power of judgement is, as you will understand, at the moment somewhat uncertain; and it is with this reservation, that I believe to have to consider the counterproposals of Menger that are

⁽³³⁹⁾ *Mit herzlichem Gruss auf hoffentlich baldiges Wiedersehen.* ⁽³⁴⁰⁾ Carbon copy of the letter itself also in Brouwer. ⁽³⁴¹⁾ Brouwer uses *Brieftasche* (wallet); it is more likely that he was carrying a small type of briefcase that was very common at the time, than a wallet. ⁽³⁴²⁾ *Verstümmelung*

contained in your letter, as unacceptable (in particular in as far as according to these the slip of the pen of my Crelle paper has not been freed from the implicit doubt contained in Menger's book²⁶).

With respect to these counterproposals I also have, for the time being, to take back my liberty concerning the postscript planned by me. As soon as I have regained somewhat my balance, I will write to you in extenso on this matter; as a follow up, we will be able, as I hope, to have a definitive fruitful discussion in the Tessin; the problem of the mutually satisfactory version has indeed its objective solution.

Anyway, even in the most unfavorable case that we should not discover the solution, and that therefore the postscript had to be given up, I would not consider the situation as desperate. The main thing is that Menger rehabilitates himself, by representing his disputed views to the public in a chivalrous way in person, and in the same journal where he was attacked, and to explain these, even when in the conflict with me, these should retain their one-sided character, in an acceptable way.

With warm greetings I remain
always your⁽³⁴³⁾

(signed) L.E.J. Brouwer

[Carbon copy of typescript – in Brouwer]

Re enclosure

[Of the enclosure two carbon copies have been preserved in the Brouwer archive. The first one only contains the texts of two letters from Blumenthal (*Blumenthal to Brouwer 3.II.1912*, *Blumenthal to Brouwer 12.II.1912*); at the top of the first one finds in Brouwer's handwriting '*Copies, enclosures to the letter of Brouwer to Hahn of 9.VIII.1929*'; the second one contains the same text and is preceded by the following lines:]

Enclosure to the letter of Brouwer to Hahn of 9.VIII.1929, containing a copy of documents (known to Menger since the year 1925), from

²⁶To facilitate clearing up of this point of difference, I send at the same time to Menger the document that is enclosed here in copy.

⁽³⁴³⁾ *Mit herzlichen Grüßen verbleibe ich – stets Ihr.*

which it appears that I have extensively refereed a paper of Lennes, which was intended as an extension of the paper published in 1911 in the *American Journal of Mathematics* 33 by the same author: Curves in non-metrical analysis situs with an application in the calculus of variations (these documents offer a rebuttal of the insinuation, contained both in Menger's book 'Dimensionstheorie', as in the note that was submitted to the Amsterdam Proceedings⁽³⁴⁴⁾ on July 1, 1929, that I could not have known in 1913, at the time of writing my paper on dimension theory in *Crelle's journal*, the above mentioned paper of Lennes of 1911).⁽³⁴⁵⁾

1929-10-07

From A. Heyting — 7.X.1929

Enschede

Dear Professor, [Hooggeleerde Heer]

I am most grateful for the sending of the documents about the coup in the editorial board of the *Mathematische Annalen*.⁽³⁴⁶⁾ I summarize in what follows my opinion about some important points.

Anybody who has in recent years taken a look at an issue of the *Mathematische Annalen*, could recognise in it the important results of your activity as editor. If he moreover knows from experience, that you always took an interest in helping to make each article appear in the best possible form, then he must share your indignation about the attempt to remove you from the board of editors, and admit that the term 'grievous injustice' is a correct qualification.

The conditions that you put to Mr. Carathéodory in your letter of November 2,⁽³⁴⁷⁾ are logical and correct; only about the question whether the form of this letter was fortunate, a difference of opinion is possible.

I share your view about the effectuated change of the editorial board, as expressed in your circular of January 23. For the many who kept primarily in touch with contemporary mathematical research through this journal, the fact that it has lost now a great deal of its representative character,

⁽³⁴⁴⁾ *KNAW, Proceedings*. ⁽³⁴⁵⁾ The upshot of Menger's claim was that Brouwer was not aware of the modern definition of connectedness ⁽³⁴⁶⁾ Brouwer had put together a file of documents relevant to the *Mathematische Annalen* conflict. The Brouwer archive contains presumably most of the material he collected. ⁽³⁴⁷⁾ *Brouwer to Carathéodory 2.XI.1928*.

constitutes a heavy blow. I want to support your attempt to fill the void thus created by establishing a new journal to my best ability, even though I am of the opinion that it is in general undesirable to increase the number of mathematical journals, and that the *Annalen* with their important historical tradition will not be replaced easily.

I appreciate that you don't want to expose my manuscripts that have been deposited with you, to an indeterminate delay. I will consider it an honor if they can be published in the *Bericht* of the Berlin Academy. I hope that I can soon send you the third article⁽³⁴⁸⁾ which has to be revised because of your changes in '*Zur Begründung der intuitionistischen Mathematik*'.

Many thanks for the improved copy of the above mentioned article. My own copy shows so many traces of frequent use, that I cannot send it back to you. I have copied all changes and return the copy that has been amended by you.

[Carbon copy – in Heyting]

1929-10-26

From T. de Donder — 26.X.1929

Brussels

5 Rue de l'Aurore, Bruxelles
 Université Libre de Bruxelles,⁽³⁴⁹⁾
 Faculté des Sciences, 50,
 Avenue des Nations

Dear colleague, [Très honoré Collègue]

I have had the honor to present in 1927 and 1928 several notes, written by Messrs. Barzin, A. Errera, Glivenko, Paul Lévy, etc., to the Royal Academy of Belgium (Science Division). These notes refer to your new logistic system. By presenting these notes, I nourished the hope to stimulate discussions that would throw more light on your ideas.

A recent article by Messrs. Barzin and A. Errera '*Sur le principe du tiers exclu*' (Bruxelles; Archives de la Société Belge de Philosophie, 1929)⁽³⁵⁰⁾ gives me the impression that your ideas have been erroneously interpreted. You certainly would render a great service to Science by letting me know

⁽³⁴⁸⁾[Heyting 1930a] ⁽³⁴⁹⁾[letterhead] ⁽³⁵⁰⁾On the principle of the excluded third, [Barzin, M 1929].

what you think about the articles mentioned, *more in particular those of Messrs. Barzin and A. Errera*. I would please me very much to present your note to the Royal Academy of Belgium (Science Section); if you prefer, you can write *in Dutch*.

Sincerely yours ⁽³⁵¹⁾

T. De Donder

[Signed autograph – in Brouwer]

⁽³⁵¹⁾ *Veuillez agréer, Monsieur et très honoré Collègue, l'expression de mes sentiments les meilleurs.*