

# Chapter 17

## Scheduling Asset Maintenance and Technology Insertions

W. Wang and M. J. Carr

**Abstract** This chapter presents a methodology that has been developed to assist in the planning, scheduling and outsourcing associated with maintenance and service contracting of complex engineering systems. The methodology is based on the delay-time modelling concept and can be used to construct and specify a model of the failure and inspection maintenance processes for a given system or asset. The resulting process model can be used for optimising the maintenance service interval with respect to a number of different criteria including system cost, downtime, availability or reliability. It also caters for a number of analytical scheduling options. Most notably, the model can be used to determine the optimal inspection interval for multiple types of maintenance and service, and can also be used for validator purposes or to improve the actual maintenance processes. This chapter also discusses the application of the methodology in the context of fixed service life contracts and the potential for incorporating technology insertion processes over time in the modelling process. The model developed can be programmed into a software package to facilitate the actual use of the methodology. The chapter concludes with some simple examples to demonstrate the concepts.

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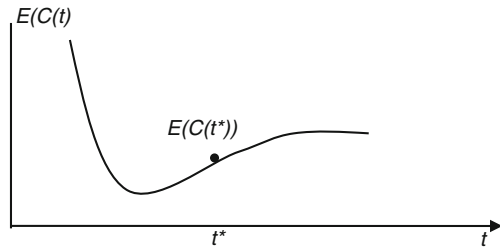
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**Fig. 17.1** The observed and expected cost model v service interval



## 17.1 Introduction

Due to advances in technology and production, longer product life cycles and increased asset availability are increasingly common requirements in many sectors and represent important considerations for the development of service plans and contracts. This is particularly so when both the equipment providers and the maintainers are under the pressure of performance-based contractual arrangements. Improving the provision of contracts to incorporate servicing throughout the proposed life of an asset should theoretically result in a reduction in the acquisition of new engineering systems and equipment and ultimately a behavioural transformation within and between the relevant parties. Longer lifecycles mean that the likelihood of technological obsolescence is substantially increased and the necessity for improved maintenance, repairs and spare parts planning becomes more apparent if the goal of complex system capability enhancement is to be attained.

An important aspect of service contract provision is the development of a cohesive cost-effective maintenance framework with the provision for technological insertion planning and obsolescence management. The transformation from a traditional product supply arrangement to a product supply and service provision contract will involve the realisation of multiple support processes into a single coordinated multi-tier plan. The maintenance plan should support the contract and demonstrate the impact of the transition in terms of cost, downtime, risk, availability or any other criterion.

To clarify the objective of the type of the cost benefit analysis we are concerned here with, considering a plant item with a maintenance practice, or concept, of servicing every period  $t$  hours, says, weeks or months, with repair of failures undertaken as they arise. The service consists of a check list of activities to be undertaken, and a general inspection of the operational state of the plant. Any defect identified leads to immediate repair and the objective of the maintenance concept is to minimise the operational cost. Other objectives could be considered to include downtime, availability, and output, but for now we consider cost reduction.

Conceptually, there is a relationship between the expected cost per unit time  $E(C(t))$ , and the service period  $t$ , see Fig. 17.1. If  $t$  was small, the cost per unit time would be large because the plant would frequently be unavailable due to servicing, and if  $t$  were sufficiently large, the cost per unit time would essentially be under a breakdown maintenance policy. If the chosen service period is  $t^*$ , all that can expect to be

known of  $E(C(t))$  is the observed value  $E(C(t^*))$ , that is the current cost measure. One wishes to reduce  $E(C(t^*))$ , and if a model such as the curve in Fig. 17.1 was available, there would be little difficulty in identifying a good operational period for  $t$ , which could be infinite, that is do not inspect. Unfortunately, in the absence of modelling, all that is generally available is the data of the dot in Fig. 17.1.

To obtain the curve in Fig. 17.1 requires maintenance modelling, and the curve in Fig. 17.1 is a graphical representation of a model with only one type of the service interval.

Plant inspection with only one type of the service interval has been well addressed in the literature, Wang (2008a). However it is noted that multi-level service provision is increasingly common, particularly in the defence sector. The different types of maintenance can be lines with different activities, differing levels of depth, or different sub-systems. In many applications, service provision is grouped into two different levels. The first level consists of on-base processes for maintenance and upgrading activities. The second level consists of on and off-base support providing; labour and management resources, material and repair services, technical information and capability development services. A number of authors have presented models for maintenance service contracts. Lufgheid et al. (2007, 2008) discussed finite horizon problems under a maintenance contract. Lisnianski et al. (2008) and Jackson and Pascual (2008) model increasing failure intensities over the planned service life of an asset. However, the effects of planned inspection maintenance and multiple types of maintenance intervention are neglected in all of these studies.

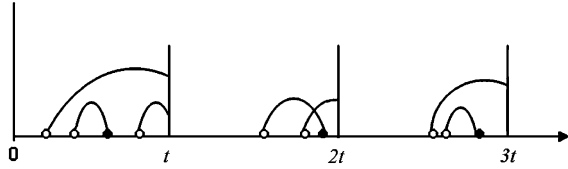
This chapter presents a methodology developed for determining the optimal multiple inspection services and maintenance intervals. The methodology can be utilised for a given application to integrate and schedule maintenance and service activities over different lines or at different levels of depth. The maintenance plan is constructed in an optimal manner and the scheduling is undertaken over the lifetime of an asset in order to provide through-life support while balancing the competing objectives of capability and affordability. The methodology has been developed with consideration given to the limitations associated with operational scenarios and can be tailored to build a model of the failure and planned maintenance processes for any given application. Upgrading and technology insertion activities are regarded as part of the maintenance planning process and capability enhancement trade-off decisions are readily incorporated.

The chapter is organised as follows. Section 17.2 introduces the background of the modelling technique used. Section 17.3 presents the methodology and the model. Section 17.4 develops the model based on a fixed system life, while Sect. 17.5 considers the impact of technology insertions. Section 17.6 illustrates numerical examples and Sect. 17.7 concludes the chapter.

## 17.2 Background

Two maintenance policy extremes are typical in industry. These are (i) ‘break-down’ maintenance policies where the system is only attended to upon the

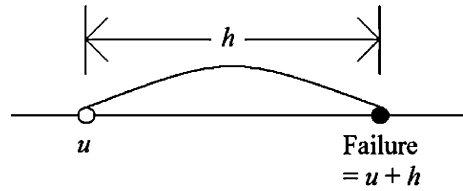
**Fig. 17.2** A defect arrival, failure and inspection process



occurrence of a failure, and (ii) ‘frequent-as-possible’ maintenance scheduling. However, it is rare for either of these policies to produce satisfactory results. Figure 17.2 illustrates a typical defect arrival, failure and planned inspection process for inspections undertaken on a regular interval  $t$ , where multiple defects can be present in the system at the same time and defects that are present are identified and removed or corrected as part of the inspection process. The circles represent the initiation of random defects and the dots represent failures caused by the defects if no maintenance intervention takes place. The arc linking a circle and a dot is the delay time of the defect. Due to the impact of planned inspection interventions, some defects in Fig. 17.2 are identified at inspection and rectified. This reduces the number of system failures from 7 to 3. Clearly, the interval of such an inspection is important since more frequent inspections will identify and remove more defects and consequently reduce the cost associated with failures but will also result in an increased total inspection penalty. We introduce a modelling technique called delay-time modelling which can be used for such a cost benefit analysis to balance the trade-off between these two sources of costs.

Delay-time modelling can be used to optimise the inspection process and avoid the obvious pitfalls associated with ‘breakdown’ or ‘frequent as possible’ maintenance policies. The methodology has proven to be very useful for complex system maintenance scheduling since its introduction in Christer and Waller (1984). Useful summaries of the key delay-time modelling developments are provided in Christer (1999) and Wang (2008a). Typical case studies using the delay-time modelling can be found in, Baker and Wang (1991) for medical equipment, Christer and Waller (1984) for high speed canning lines, Akbarov et al. (2008) for a baking production line, Christer et al. (1995) for an extrusion press, Christer et al. (1997) for a hot steel mill, Jones et al. (2009) for a manufacturing plant and Pillay et al. (2001) for fishing vessel to name a few. Virtually all companies which use some kind of time-based inspection on their assets can benefit from the delay-time model based analysis. Delay-time modelling provides a framework that is applicable to any complex asset maintenance scheduling problem. The delay-time concept defines the failure process of an asset as a two-stage process. The first stage is the normal operating stage from new to the origination of an identifiable defect in the system. The length of this stage depends on the type and nature of the inspection process, the available technology and the associated maintenance team. The second stage is defined as the delay-time of the defect until failure. During the course of the delay-time, preventive maintenance actions can be undertaken to remove the defect and prevent the impending failure.

**Fig. 17.3** The delay-time of a defect from conception to failure



By modelling the process in this manner, an optimal inspection interval can be established based upon downtime, cost, availability, risk or any other criterion. The delay-time concept is similar but has a number of advantages over the much used P–F interval concept in reliability centred maintenance (RCM). However, the P–F interval is only part of the plant life, and the time to P since new is more important than the P–F interval, which was not considered in RCM at all. Moreover, RCM only provided such a P–F interval concept, but the delay-time modelling proposed a comprehensive way to estimate the time to P and the P–F interval. No RCM literature has reported any way for the quantification of such a P–F interval.

Initially, we are concerned with complex assets and inspections taking place in every  $t$  units of time, where the aim of the interventions is to minimise the cost associated with system failures. Figure 17.3 shows that the origin of a particular defect is defined as time  $u$ , while  $h$  denotes the delay-time until failure. A maintenance intervention in the period  $(u, u + h)$  would prevent the failure from occurring.

If the averaged costs of failure, inspection and defect removal are available, we wish to establish the curve depicted in Fig. 17.1 using the delay-time concept. The expected cost per unit time is the sum of the cost of failures, inspection and defect removals occurred within the interval divided by the interval. To present this in a model we have

$$E(C(t)) = \{c_f E(N_f(t)) + c_s + c_d E(N_d(t))\} / t \tag{17.1}$$

where  $C(t)$  denotes the total cost per unit time over  $t$ ,  $c_f$  denotes the average cost per failure,  $E(N_f(t))$  denotes the expected number of failures in  $(0, t)$ ,  $c_s$  denotes the average cost of an inspection service,  $c_d$  denotes the average cost of a defect removal, and  $E(N_d(t))$  denotes the expected number of defects identified at the service point. The cost parameters may be available from the clients, but to get  $E(N_f(t))$  and  $E(N_d(t))$  we need to know the average rate of defect arrival over time and a probability distribution for the delay-time  $f(h)$ . For the derivation of  $E(N_f(t))$  and  $E(N_d(t))$  see Wang (2008a). Clearly Eq. 17.1 is a function of  $t$  through  $E(N_f(t))$  and  $E(N_d(t))$ , and once they and the cost parameters are available, Eq. 17.1 can be readily calculated to assess which  $t$  is the best. Equation 17.1 is the fundamental objective function we shall use throughout this chapter while all others are variants of it.

## 17.3 Multiple Types of Maintenance

### 17.3.1 Methodology

Until recently, the delay-time modelling approach has only been used to optimise the maintenance process for a single line or type of service provision. For complex asset systems with many sub-systems and components, the planned inspection and maintenance will normally be undertaken at different intervals, levels, and depths. Typical practice involves a routine inspection on a more frequent basis to some sub-systems or components which may be subject to more frequent failures and then a longer interval applied to the system as a whole. An example of such maintenance service practice can be seen in aircraft maintenance where up to four different maintenance intervals may be in place (Sriram and Haghani 2003). These different intervals are usually nested, as up-level maintenance with a longer interval will include the content of lower levels of maintenance with shorter intervals (Wang 2000, 2008b). The following summarises the methodology devised for modelling multiple types of maintenance and the associated assessment activities:

1. Collection, storage and pre-processing of events based data.
2. Drawing up a bill of material/product tree to show the structure of the system.
3. For each defined sub-system that will be serviced by a line or type of maintenance service:
  - estimate the defect arrival rate,
  - specify and parameterise the delay-time distribution,
  - estimate the downtime or cost associated with failures and scheduled maintenance servicing.
4. Establish the interactions between the different lines or types of maintenance.
5. Options for analysis include:
  - optimal scheduling algorithms,
  - candidate policy comparisons,
  - reliability and availability evaluation,
  - validation of existing scheduling activities,
  - evaluate efficacy of maintenance activities,
  - technology insertion sensitivity analyses.
6. Assess the performance by comparing the observed and expected output over time.
7. Assess the availability and capability of the system using an expected downtime and reliability evaluation. System reliability functions can be used to assess the system from a risk perspective.

The methodology requires specification and adaptation for a given scenario and is dependent on; the nature of the application, limitations on the availability of data

**Table 17.1** The modelling notation

Notation	Definition
$k$	Defect type, where $k = 1$ or $2$
$\lambda_k$	Rate of type $k$ defect arrival
$h$	Delay-time of a defect
$f_k(h)$	Type $k$ defect delay-time distribution
$c_{kf}$	Average cost measure for a type $k$ failure
$c_{ks}$	Average cost measure for a type $k$ inspection
$c_{kd}$	Average cost measure for a type $k$ defect removal at a type $k$ inspection
$E[N_{kf}(t)]$	Expected number of type $k$ failures over an interval $(0, t]$
$E[N_{kd}(t)]$	Expected number of type $k$ defect removals at a type $k$ inspection after an interval of duration $t$
$E[C(\bullet)]$	Expected cost per unit time with $\bullet$ as the decision variables

and access to design/engineering expertise. However, regardless of the application, the same building blocks are used to model the defect arrival, failure and preventive maintenance processes. It is an extendable and broadly applicable methodology that can be applied to any system. As noted in point 6, for a defined test case we are able to compare the expected cost against the observed operating cost for a specified maintenance service policy when initiated. As such, the comparison can be used to validate the delay-time model of the complex system.

### 17.3.2 Two Types of Maintenance

In this section, we address a simple case when two types of inspection process are in operation. The model can be readily extended to more than two types of inspections. We consider the categorisation of minor and major defects and failures and the scheduling of the corresponding minor and major inspection-based interventions with different cost measures. The different defect arrival rates are estimated independently as  $\lambda_1$  and  $\lambda_2$  for minor and major defects, respectively. Similarly, the individual delay-time distributions are specified as  $f_1(h)$  and  $f_2(h)$  and the average cost associated with failures,  $c_f$ , inspections,  $c_s$  and preventive defect removal,  $c_d$ , are estimated independently for each defect type. Table 17.1 summarises the modelling notation. We specify the following additional assumptions:

- There are two broad types of defects and therefore failures, namely minor and major.
- Minor defects can be identified and rectified by a minor inspection service, while major defects can only be identified and removed by a major inspection service. A major inspection service includes the content of a minor inspection service and occurs at one of the minor inspection service interventions.

### 17.3.3 Common Inspection Model

Firstly, we consider the scenario where minor and major defects and failures are identifiable but the minor and major inspections are undertaken on the same interval. As major inspections incorporate all the activities of minor inspections, we only incorporate the cost associated with major inspections. Extending the concept used in Eq. 17.1, the expected cost per unit time is

$$E[C(t)] = \{E[N_{1f}(t)]c_{1f} + E[N_{1d}(t)]c_{1d} + E[N_{2f}(t)]c_{2f} + E[N_{2d}(t)]c_{2d} + c_{2s}\}/t \quad (17.2)$$

for a common inspection interval  $t$ .

### 17.3.4 Multiple Inspection Model

Now we consider the multiple inspection case where minor and major inspections are scheduled. As discussed in the introduction to this section, major inspections contain all the activities of minor inspections. As such, major inspections are scheduled to occur at integer multiples of the minor inspection interval and the cost associated with minor inspections is assumed to be incorporated in the major inspection penalty. With minor and major inspection intervals of duration  $t_1$  and  $t_2 = mt_1$ , respectively, we have

$$E[C(t_1, m)] = \{m(E[N_{1f}(t_1)]c_{1f} + E[N_{1d}(t_1)]c_{1d}) + (m - 1)c_{1s} + E[N_{2f}(mt_1)]c_{2f} + E[N_{2d}(mt_1)]c_{2d} + c_{2s}\}/mt_1 \quad (17.3)$$

Naturally, when  $m = 1$  the model is equivalent to the common inspection model.

## 17.4 Fixed Service Life

A key assumption used in the construction of the models in the previous section is that of an infinite planning horizon which allows the use of renewal reward theory in the evaluation of the expected cost per unit time. However, many contracting applications involve the scheduling of maintenance and service activities over a proposed service life. As such, further modelling developments are required. Initially some further notation is introduced:

- $T$  is the planned service life, and
- $E[C(\bullet)]$  is the expected cost over  $T$  with  $\bullet$  as the decision variables



Note that for fixed service life applications, we are interested in the total expected cost over the planned service life,  $E[C(\bullet)]$ , rather than the expected cost per unit time,  $E[C(\bullet)]/t$ , used previously. Continuing the multiple inspection interval case with minor and major inspection types, the total expected cost over the service life of the asset is the summation of the cost associated with minor and major failures and minor and major defects identified and removed at inspections. Again, we assume that minor defects can be rectified by minor and major inspections but major defects can only be rectified at a major inspection of the system.

### 17.4.1 Common Inspection Model

If a common inspection on interval  $t$  is scheduled for minor and major inspections, then the total number of inspection intervals during the service life is  $\lfloor T/t \rfloor$  where,  $\lfloor x \rfloor$  is the largest integer less than or equal to  $x$ . As there will be no inspection at the end of the asset service life, it follows that the total expected cost over the planned service life is

$$E[C(t)] \approx \sum_{k=1}^2 ((\lfloor T/t \rfloor - 1) \{E[N_{kf}(t)]c_{kf} + E[N_{kd}(t)]c_{kd}\} + E[N_{kf}(T - (\lfloor T/t \rfloor - 1)t)]c_{kf} + (T/t - 1)c_{2s})$$

However, as we are interested in large values of  $T$  and comparatively small values of  $t$ , the expected cost can be approximated as

$$E[C(t)] \approx \sum_{k=1}^2 ((T/t - 1) \{E[N_{kf}(t)]c_{kf} + E[N_{kd}(t)]c_{kd}\} + E[N_{kf}(t)]c_{kf} + (T/t - 1)c_{2s}) \tag{17.4}$$

### 17.4.2 Multiple Inspection Model

In the multiple inspection case, we define  $t_1$  and  $t_2$  as the minor and major inspection intervals respectively. Assuming that  $t_1 \cdot t_2$  (i.e., the minor and major inspections do not coincide), the expected cost over the contracted service life is

$$E[C(t_1, t_2)] \approx \sum_{k=1}^2 ((T/t_k - 1) \{E[N_{kf}(t_k)]c_{kf} + u_{ks} + E[N_{kd}(t_k)]c_{kd}\} + E[N_{kf}(t_k)]c_{kf}) \tag{17.5}$$

In the nested case, where major inspections are undertaken on an interval  $t_2 = mt_1$  which is an integer multiple of the minor inspection interval  $t_1$ , the expected cost over the service life becomes

$$\begin{aligned}
E[C(t_1, m)] \approx & (T/t_1 - 1)\{E[N_{1f}(t_1)]c_{1f} + u_{1s} + E[N_{1d}(t_1)]c_{1d}\} + E[N_{1f}(t_1)]c_{1f} \\
& + ((T/mt_1) - 1)\{E[N_{2f}(mt_1)]c_{2f} + (c_{2s} - c_{1s}) + E[N_{2d}(mt_1)]c_{2d}\} + E[N_{2f}(mt_1)]c_{2f}
\end{aligned}
\tag{17.6}$$

It is straightforward to show that if technology insertions are incorporated, which reduce the major defect arrival rate over time, the expected cost will be smaller than in the no insertions case. This can be used partially to justify the need for technological insertions if the cost associated with the actual insertion process is not excessive. However, it should be noted that, if the major defect arrival rate is not constant, then a variable inspection interval policy should be employed. This is shown in the next section.

## 17.5 Incorporating Technology Insertions

During the lifecycle of major platforms, the implementation of technology insertions is often carried out to increase the capacity of the platforms. However, technology insertions should only be considered on the assumption that they increase the capability and availability of the system, reduce the cost of planned and unplanned maintenance interventions or prevent technological redundancy over time. Technology insertions can be incorporated into maintenance and service scheduling in a number of different ways and the objective may always not be able to improve the operational capability of equipment. For the example in this chapter, we consider a technological ‘upgrading’ mechanism that is manifested in step-wise decreasing defect arrival rates as more reliable components are assumed to be inserted over time. In many cases, technology insertion may be used to combat natural degradation and obsolescence. This can be regarded as ‘maintaining’ capability for consistent requirements. Alternatively, technology insertion may be used to modify the asset for changing/evolving requirements over time. In these situations, the delay-time modelling scheduling process should be adapted to produce a plan designed to maintain levels of reliability.

Technological insertions are considered in many managerial papers in a qualitative manor; see Kerr et al. (2008a), Strong (2004), and Dowling (2004). These studies mainly address the framework, concepts, components and dimensions of such insertions. However, regardless of the particular application, justification for the technological insertions will be required in the form of a cost benefit analysis that should demonstrate the expected contribution with regard to the minimisation of associated risks and costs. To incorporate an effective technology insertion programme, it is necessary that system architectures are designed on an ‘open’ and ‘modular’ basis, Kerr et al. (2008a). Sensitivity analyses and a comparison of the reduced maintenance costs attributable to failures and inspections and the cost of the insertion activities can then be undertaken. Technological insertions must have an impact upon the failure behaviour of the asset and therefore on the associated inspection policy as well; Dowling (2004).

To incorporate the impact of technological insertions over time for fixed horizon inspection models, we introduce the following additional assumptions;

- New technology that may lead to component upgrading occurs during major inspections at discrete time points and is assumed to impact upon the arrival rate of major defects.
- The arrival rate for major defects is defined as a function of the time since the last major inspection.
- The interval between minor inspections is constant, but the interval between major inspections, which is influenced by aging and upgrading, varies.

Minor inspections are undertaken on an interval of duration  $t_1$  and the time of the  $i$ th major inspection is denoted by  $t_{2,i}$ . To model the impact of technological insertions over time,  $\lambda_2(t_{2,i})$  is defined as the major defect arrival rate after the  $i$ th major inspection. If the major defect arrival rate is influenced by technological insertions over time, we must have a variable inspection scheme for the major inspections. Assuming that there are  $n$  major inspections over the course of the service life  $T$ , where  $n$  is a decision variable, we have the total expected cost associated with maintenance as

$$\begin{aligned} E[C(t_1, n, \{t_{2i}; i1, 2, \dots, n\})] &= (T/t_1 - 1)\{c_{1f}E[N_{1f}(t_1)] + c_{1s} + c_{1d}E[N_{1d}(t_1)]\} \\ &+ c_{1f}E[N_{1f}(t_1)] - nc_{1s} + \sum_{i=1}^{n-1} \{c_{2f}E[N_{2f}(t_{2,i} - t_{2,i-1})] + c_{2s} \\ &+ c_{2d}E[N_{2d}(t_{2,i} - t_{2,i-1})]\} + c_{2f}E[N_{2f}(t_{2,n} - t_{2,n-1})] \end{aligned} \quad (17.7)$$

For the expected number of minor failures over a minor inspection interval,  $E[N_{1f}(t_1)]$  and the expected number of minor defects removed at a planned minor inspection,  $E[N_{2d}(t_1)]$ , see Wang (2008a). Similarly, the expected number of major failures over an interval between major inspections,  $E[N_{2f}(t_{2,i} - t_{2,i-1})]$ , and the expected number of defect removals at the  $i$ th major inspection,  $E[N_{2d}(t_{2,i} - t_{2,i-1})]$ , are also given in Wang (2008a). The expected cost can be compared with proposed budgetary constraints on the maintenance and service activities. This measure can also be used for contract negotiation if the maintenance service is outsourced.

Assuming that major inspections occur at integer multiples of the planned minor inspection interval  $t_1$ , the objective function becomes

$$\begin{aligned} E[C(t_1, n, \{t_{2i}; i = 1, 2, \dots, n\})] &= (T/t_1 - 1)\{c_{1f}E[N_{1f}(t_1)] + c_{1s} + c_{1d}E[N_{1d}(t_1)]\} \\ &+ c_{1f}E[N_{1f}(t_1)] - nc_{1s} + \sum_{i=1}^{n-1} \{c_{2f}E[N_{2f}(m_i t_1)] + c_{2s} + c_{2d}E[N_{2d}(m_i t_1)]\} \\ &+ c_{2f}E[N_{2f}(m_n t_1)] \end{aligned} \quad (17.8)$$

where  $m_i \geq 1$  are the integer values and the major defect arrival rate after the  $i$ th major inspection is

**Table 17.2** The historical failure and inspection data

Type	Total no. of failures	Total failure cost	Total inspection cost	Total defect repairs	Total defect repair cost
1	$TN_{1f} = 300$	$TC_{1f} = 1,200$	–	$TN_{1d} = 900$	$TC_{1d} = 900$
2	$TN_{2f} = 40$	$TC_{2f} = 800$	–	$TN_{2d} = 150$	$TC_{2d} = 750$
Both	$TN_f = 340$	$TC_f = 2,000$	$TC_s = 200$	$TN_d = 1,050$	$TC_d = 1,650$

$$\lambda_2(t_{2,i}) = \lambda_2 \left( \sum_{j=1}^i m_j t_1 \right).$$

An optimisation algorithm is required to jointly optimise the objective function given by Eq. 17.8 with respect to  $t_1$ ,  $n$  and  $m_i$  for  $i = 1, 2, \dots, n$ .

## 17.6 Illustrative Examples of the model developed

### 17.6.1 Example 1: Scheduling multiple inspection service intervals

This example illustrates the potential applications of the proposed methodology for scheduling multiple types of maintenance and the benefits that can be realised for the development of service contracts.

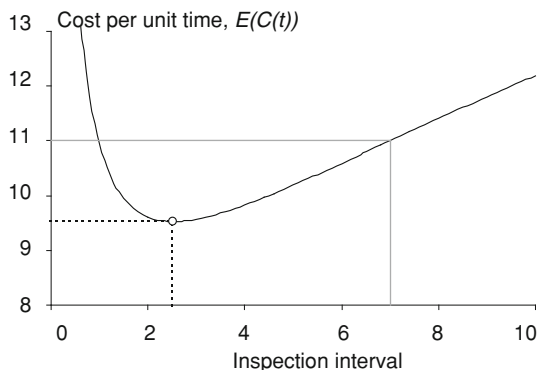
#### 17.6.1.1 Historical Data

We assume that we are presented with  $r = 50$  life cycles of historical failure and inspection maintenance data. A constant cycle length of duration  $t = 7$  days has been used and the cumulative data are illustrated in Table 17.2 for two different failure types. Although the different defect/failure types are identifiable, a single inspection policy has been employed historically.

#### 17.6.1.2 Single Inspection Interval

In the single inspection interval case, the data is analysed collectively, i.e., no distinction is made between different types of defect or failure, as described in Sect. 17.2. As such, the process parameters are estimated from the data corresponding to both failure types as given in the row ‘both’ in Table 17.2. We assume a single defect arrival rate  $\lambda$  and employ an exponential form for the delay-time distribution  $f(h)$  with parameter  $\alpha$ . The cumulative delay-time distribution is therefore  $F(h) = 1 - \exp(-\alpha h)$ . Using the relevant values from Table 17.2 and

**Fig. 17.4** The expected cost per unit time,  $E[C(t)]$ , as a function of the inspection interval,  $t$



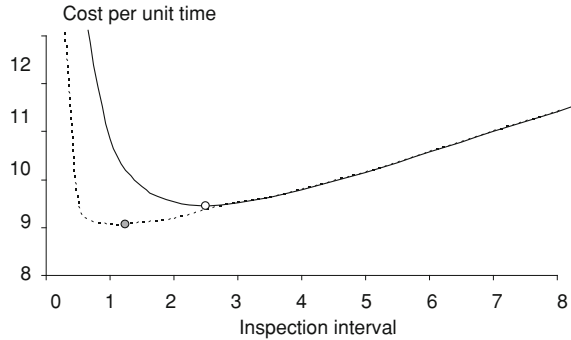
applying a standard statistical procedure we obtain the estimated parameters  $\lambda = 9.267$  defects per day and  $\alpha = 0.197$ . The total number of events and the total associated costs from Table 17.2 are used to estimate the average costs. The average failure cost is estimated as  $c_f = 2,000/340 = 5.882$ , the average inspection cost is  $c_s = 200/50 = 4$  and the average cost for an inspection-based repair is  $c_d = 1,650/1,050 = 1.571$ .

Using the process parameters and the estimated costs, the single inspection type model of Eq. 17.1 can be used to analyse the effects of varying the maintenance interval on the expected future performance of the asset. Using Eq. 17.1, Fig. 17.4 illustrates the expected cost per unit time,  $E[C(t)]$ , against the inspection interval  $t$ . The optimal inspection interval is found to be 2.526 days rather than 7 days employed historically. As shown in Fig. 17.4, changing the single inspection interval from the original 7 days to the optimal interval of around 2.5 days is expected to reduce the expected cost per unit time from 11 to 9.524 which is a 13.4% reduction in the costs associated with maintenance. This illustrates how modelling can be used to develop a tool for decision analysis that is used to inform and assess the benefits of potential maintenance policy changes.

### 17.6.1.3 Two Types of Maintenance

Now we consider the modelling of two different types of maintenance, as described in Sect. 17.3. We consider an infinite planning horizon and seek to minimise the expected cost per unit time. The parameters are estimated for each defect/failure type independently using the same process described in the previous sub-section and the relevant events data from Table 17.2. The estimated parameters are; the rate of type 1 defect arrival  $\lambda_1 = 3.429$ ,  $\alpha_1 = 0.087$  (for exponentially distributed type 1 delay-times) and the rate of type 2 defect arrival  $\lambda_2 = 0.543$ , and  $\alpha_2 = 0.07$  (for exponential type 2 defects). We introduce an assumption that 15% of the inspection activities noted in Table 17.2 are attributable to the investigation of type 1 defects. As such, we have the cost associated

**Fig. 17.5** The expected cost per unit time, for the common and multiple inspection models, as a function of the inspection interval (type 1 interval in the multiple case)



with type 2 inspections as  $c_{s2} = c_s$  (from the previous section) and the cost of a type 1 inspection as  $c_{s1} = 0.15 \times c_{s2}$ . The average costs are;

- Type 1:  $c_{1f} = 1,200/300 = 4$ ,  $c_{1s} = 0.15 \times 200/50 = 0.6$  and  $c_{1d} = 900/900 = 1$
- Type 2:  $c_{2f} = 800/40 = 20$ ,  $c_{2s} = 200/r = 4$  and  $c_{2d} = 750/150 = 1$

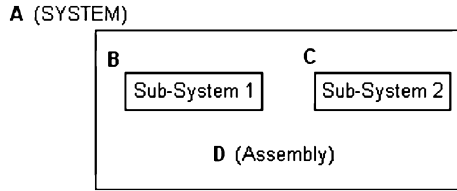
Attention is now turned to comparing the common inspection interval and multiple inspection intervals models, as proposed in Sect. 17.3.3 and Sect. 17.3.4 and given by Eqs. 17.2 and 17.3, respectively. Figure 17.5 illustrates the expected cost per unit time against the inspection interval, where the multiple inspection intervals model is represented by the dashed line. Note that, in the multiple interval case, the representation is for the type 1 inspection interval,  $t_1$ , with the integer value  $m$  maximised for each potential value of  $t_1$ .

It is evident from Fig. 17.5 that a multiple inspection interval policy is superior for this case. In the common interval case, we obtain the optimal interval  $t^* = 2.498$  days, giving the expected cost per unit time  $E[C(t^*)] = 9.456$ . In the multiple interval case, we obtain the optimal type 1 interval  $t_1^* = 1.234$  days and  $m^* = 3$  giving the optimal type 2 interval  $t_2 = m^* \times t_1^* = 3.702$  and the expected cost per unit time  $E[C(t_1^*, m^*)] = 9.053$ .

### 17.6.2 Example 2: Fixed Service Life and Technology Insertions

A hypothetical system with associated maintenance policy is proposed over a specified service life. The potential impact of technology insertion is then assessed by comparing the expected costs over time with and without upgrading activities. The example illustrates how the potential benefits of modelling and incorporating technology insertion can be demonstrated. Alternative candidate maintenance policies and a sensitivity analysis on the impact of upgrading activities can be carried out in a similar manner for a given application. Figure 17.6 illustrates the major aspects of the hypothetical complex system.

**Fig. 17.6** The example system



**Table 17.3** The activities and intervals for each type of maintenance/service

Type	Maintenance activities	Interval duration
1	Assembly (D) inspection and repairs	$\Delta t = 1$ month
2	Sub-System 1 (B) inspection and repairs	$L_2\Delta t$
3	Sub-System 2 (C) inspection and repairs	$L_3\Delta t$
4	System (A) overhaul and technology insertion	$L_4\Delta t$

The system is composed of an assembly supporting two sub-systems and we have  $A = B + C + D$ . The maintenance/service types, the associated activities and the interval between interventions are given in Table 17.3 where,  $L_2$ ,  $L_3$  and  $L_4$  are positive integers. The proposed service life is  $T = 120$  months.

We assume that an overhaul of the system (type 4 intervention) is comprehensive in that it includes all of the activities undertaken in type 2 and 3 interventions and more. As such, when an overhaul is scheduled to occur at the same time as type 2 or 3 interventions, no type 2 or 3 takes place. Technology insertion takes place at the same time as a system overhaul and its impact is manifested in reducing rates of defect arrival for sub-systems 1 and 2 (B and C). Defects are assumed to arise at an average rate of  $\lambda_1 = 2$  a month in the assembly (D). For the purpose of exploring the impact and analysing the cost benefit, we consider the same maintenance schedule with and without technology insertion. Without insertion activities, we adopt average defect arrival rates of 8 and 10 a month in sub-systems 1 and 2, respectively. We now consider the effect on the arrival rates when insertion activities are incorporated. If  $t_{2,i}$  is the time of the  $i$ th overhaul, the average defect arrival rate in sub-system 1 at time  $s$  when we have  $t_{2,i} \leq s < t_{2,i+1}$  is assumed to be  $\lambda_2(s) = 8 - 7t_{2,i}/120$ .

Similarly, in the case of sub-system 2, we assumed the rate to be  $\lambda_3(s) = 10 - 8t_{2,i}/100$ . The mean delay-time from the initiation of an individual defect until the resulting failure is taken to be 1 month for all the different aspects of the system and we assume that the delay-times are exponentially distributed. The various process, system and cost parameters are summarised in Table 17.4.

A total expected cost model of the activities relating to the hypothetical system over a proposed operational life is established using the methodology proposed in Sect. 17.5 but extended to the 4 inspection type case. From the description of the proposed maintenance schedule, we define  $E[C(T)]$  as the expected cost over the proposed asset service life  $T$  and  $t_{2,i} = iL_4\Delta t$  is defined as the time of the  $i$ th overhaul. The parameters  $c_f$  and  $c_s$  represent the costs associated with failures and service interventions respectively and  $\lambda$  and  $F(h)$  are the defect arrival rate and the

**Table 17.4** The activities and intervals for each type of maintenance/service

Parameter	Value	Parameter	Average	Parameter	Estimate
$T$	120	$c_{1f}$	250	$\alpha_1$	1
$\Delta t$	1	$c_{2f}$	300	$\alpha_2$	1
$L_2$	4	$c_{3f}$	250	$\alpha_3$	1
$L_3$	3	$c_{1s}$	60	$\lambda_1$	2
$L_4$	12	$c_{2s}$	75	$\lambda_2(s)$ (with TI)	$8 - 7 s/120$
		$c_{3s}$	50	$\lambda_3(s)$ (with TI)	$10 - 8 s/100$
		$c_{4s}$ (with TI)	10,000	$\lambda_2(s)$ (no TI)	8
		$c_{4s}$ (no TI)	125	$\lambda_3(s)$ (no TI)	10

cumulative delay-time distribution for the relevant aspect of the system. As the delay-time distributions are assumed to be exponential, the parameters are the reciprocals of the mean delay-time. We derive  $E[C(T)]$  as the summation of the expected total cost for each aspect of the system and the associated types of maintenance activity as follows;

*Type 1—Expected Cost*

$$E[C^{(1)}(T)] = (T/\Delta t)c_{1f}E[N_{1f}(\Delta t)] + \{(T/\Delta t) - 1\}c_{1s}$$

*Type 2—Expected Cost*

$$E[C^{(2)}(T)] = \sum_{i=1}^{T/(L_4\Delta t)} \{(L_4/L_2)c_{2f}E[N_{2f,i-1}(L_2\Delta t)] + \{(L_4/L_2) - 1\}c_{2s}\}$$

*Type 3—Expected Cost*

$$E[C^{(3)}(T)] = \sum_{i=1}^{T/(L_4\Delta t)} \{(L_4/L_3)c_{3f}E[N_{3f,i-1}(L_3\Delta t)] + \{(L_4/L_3) - 1\}c_{3s}\}$$

*Type 4—Expected Cost*

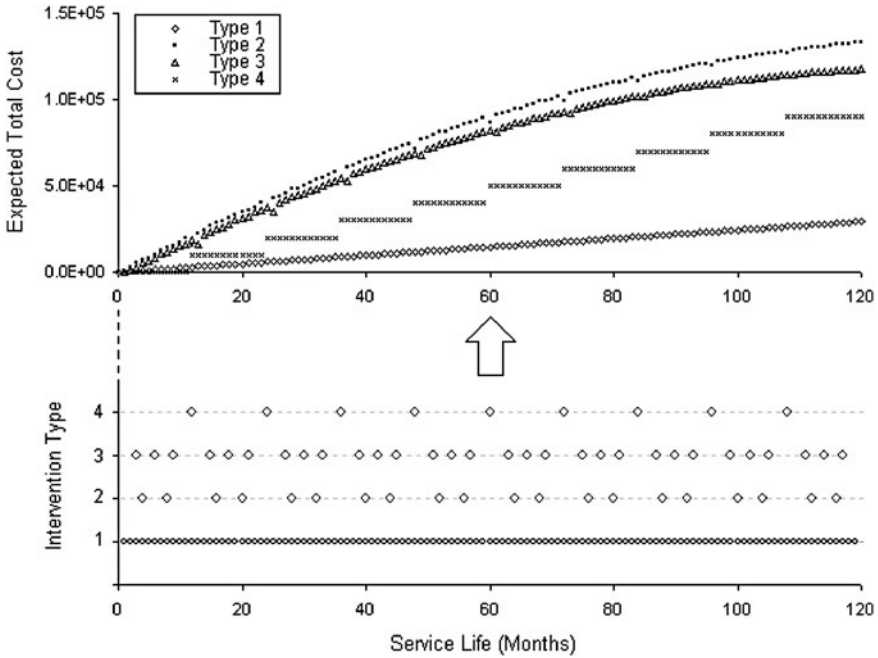
$$E[C^{(4)}(T)] = \{(T/L_4\Delta t) - 1\}c_{4s}$$

The total expected cost over the proposed service life is simply

$$E[C(T)] = \sum_{\text{Type}=1}^4 E[C^{(\text{Type})}(T)]$$

The specific schedule under consideration is a policy as specified in Table 17.4 with  $L_2 = 4, L_3 = 3$  and  $L_4 = 12$  and the question is whether or not to incorporate technology insertion activities over a contracted service life of 10 years. The average costs associated with failures of the assembly, sub-system 1 and sub-system 2 are taken to be 250, 300 and 250, respectively. The average costs associated with interventions of type 1, 2 and 3 are 60, 75 and 50, respectively. A type 4 intervention costs an average of 125 without technology insertion. The cost of a type 4 intervention with upgrading activities (this includes the predicted costs associated with the inserted technologies) is 10,000.

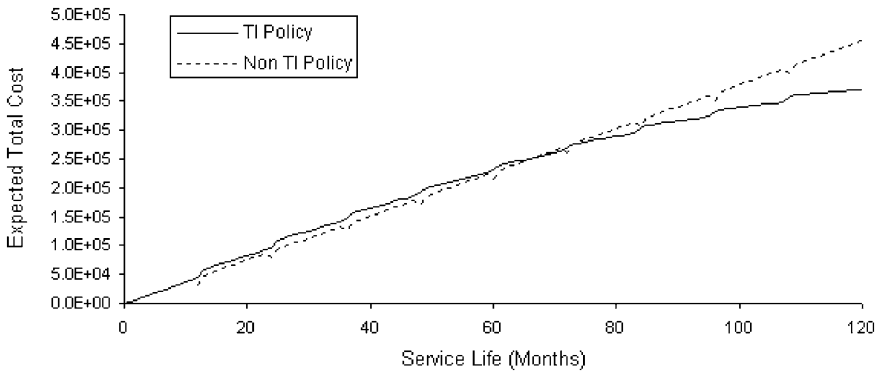




**Fig. 17.7** The events and costs associated with the different lines of maintenance when technology insertion is incorporated

Figure 17.7 illustrates the events and expected costs incurred over time for the four different types of maintenance intervention when technology insertion activities are incorporated. Using the cost model, Fig. 17.8 illustrates the total expected cost incurred over time for the maintenance/service policy with and without technology insertion. From Fig. 17.8, it is evident that half way through the service life, the benefits of the technology insertion programme have not yet been realised. However, by the end of the proposed 10 years service life, the insertion version of the policy is expected to produce a substantial saving; the total expected cost associated with the insertion policy is 370310 compared with 455635 without.

We have illustrated how the methodology can be used to demonstrate the potential benefits gained in the development of an integrated maintenance plan incorporating the various sub-systems and a technology insertion programme for a complex asset. The example demonstrates that any complex system can be modelled if the defect arrival rates and the average costs associated with failures and maintenance interventions can be estimated for each type of inspection or sub-system. The failure process must be characterised and the defect arrival rates estimated using historical events data and/or expert information. The example also illustrates how the potential impact of technology insertion can be assessed and the manner in which a sensitivity analysis could be undertaken for a given case.



**Fig. 17.8** The total expected cost over the asset service life with and without technology insertion

## 17.7 Conclusion and Summary

The methodology presented in this chapter can be used to build a process model for any asset, system or maintenance set-up. In this chapter, the methodology has been used to investigate a two-type maintenance scenario. However, the approach is not limited to the modelling of two types but can be tailored for any number of types of maintenance, with associated interactions, if the process parameters can be estimated using historical data (or subjectively using expert opinion). We have also discussed the potential for scheduling activities over a fixed service life and presented some ideas for the incorporation of technology insertion activities. For the developed models to be useful in practice, software packages based on the models reported in this chapter must be developed. We have developed demonstration prototype software which can be partly used for this purpose. The package can be part of an existing computerised maintenance management system or as a module in an ERP system.

## 17.8 Chapter Summary Questions

The discussion in this chapter raises several questions:

- Whether the delay-time concept used to model the inspection service interval exists or not in practice?
- Why is such a delay-time concept be useful for optimising the service interval?
- Can other measurements such as downtime, availability or capability be optimised instead of the cost measures used in the chapter?

## References

- A. Akbarov, W. Wang, A.H. Christer, Problem identification in the frame of maintenance modeling: A case study. *Int. J. Prod. Res.* **46**(4), 1031–1046 (2008)
- R.D. Baker, W. Wang, Determining the delay-time distribution of faults in repairable machinery from failure data. *IMA J. Math. Appl. Bus. Ind.* **3**, 259–282 (1991)
- A.H. Christer, Developments in delay-time analysis for modelling plant maintenance. *J. Oper. Res. Soc.* **50**, 1120–1137 (1999)
- A.H. Christer, W.M. Waller, Reducing production downtime using delay-time analysis. *J. Oper. Res. Soc.* **35**, 499–512 (1984)
- A.H. Christer, W. Wang, R.D. Baker, J.M. Sharp, Modelling maintenance practice of production plant using the delay-time concept. *IMA J. Manag. Math.* **6**, 67–83 (1995)
- A.H. Christer, W. Wang, J.M. Sharp, R.D. Baker, A stochastic modeling problem of high-tech steel production plant, in *Stochastic modelling in innovative manufacturing. Lecture notes in economics and mathematical systems*, ed. by A.H. Christer, S. Osaki, L.C. Thomas (Springer, Berlin, 1997), pp. 196–214
- T. Dowling, Technology insertion and obsolescence. *J. Def. Sci.* **9**(3), 151–155 (2004)
- C. Jackson, R. Pascual, Optimal maintenance service contract negotiation with aging equipment. *Eur. J. Oper. Res.* **189**, 387–398 (2008)
- B. Jones, I. Jenkinson, J. Wang, Methodology of using delay-time analysis for a manufacturing industry. *Reliab. Eng. Syst. Saf.* **94**, 111–124 (2009)
- C.I.V. Kerr, R. Phaal, D.R. Probert, Technology insertion in the defence industry: A primer. *Proc. Inst. Mech. Eng. Part B J. Eng. Manuf.* **222**, 1009–1023 (2008)
- A. Lisnianski, L. Frenkel, L. Khvatskin, Y. Ding, Maintenance contract assessment for aging systems. *Qual. Reliab. Eng. Int.* **24**, 519–531 (2008)
- D. Lugfigheid, A.K.S. Jardine, X. Jiang, Optimizing the performance of a repairable system under a maintenance and repair contract. *Qual. Reliab. Eng. Int.* **23**, 943–960 (2007)
- D. Lugfigheid, X. Jiang, A.K.S. Jardine, A finite horizon model for repairable system with repair restrictions. *J. Oper. Res. Soc.* **59**, 1321–1331 (2008)
- A. Pillay, J. Wang, A.D. Wall, T. Ruxton, A maintenance study of fishing vessel equipment using delay-time analysis. *J. Qual. Maint. Eng.* **7**(2), 118–128 (2001)
- C. Sriram, A. Haghani, An optimization model for aircraft maintenance scheduling and re-assignment. *Transp. Res. Part A Policy Pract.* **37**(1), 29–48 (2003)
- G. Strong, Technology insertion—a worldwide perspective. *J. Def. Sci.* **9** (3): 114–121 in press doi: [10.1016/j.res.2008.06.010](https://doi.org/10.1016/j.res.2008.06.010) (2004)
- W. Wang, A model of multiple nested inspections at different intervals. *Comput. Oper. Res.* **27**(6), 539–558 (2000)
- W. Wang, Delay-time modelling, in *Complex system maintenance handbook*, ed. by K. Kobbacy, D. Prahakbar Murth (Springer, London, 2008a)
- W. Wang, Condition based maintenance modelling, in *Complex system maintenance handbook*, ed. by K. Kobbacy, D. Prahakbar Murth (Springer, London, 2008b)