Strategic Lines and Substations in an Electric Power Network

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1 Introduction

The development of tools for the analysis of critical infrastructure, and particularly of the power system grid, has been an intense activity during the last several years. A number of different techniques have been proposed for vulnerability analysis of power system and for security in cases of catastrophic events. Such methods include controlled islanding or intentional islanding by using decision analysis techniques and also graph algebraic approaches. Also, more sophisticated techniques such as algebraic network theory, polyhedral dynamics and artificial intelligence-based search methods have been proposed to find critical nodes, critical transmission lines and system vulnerability indices.

A number of these methods allow identification of the critical elements in a bulk power system and the quantification of the consequences of their failures. These techniques are intended to go further in security analysis than the traditional contingency analysis or $n - m$ approach. The objective is more to evaluate the structural robustness of a network or to identify the more vulnerable or the more

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fragile elements in a power system than to determine the response of the power system to a single or several contingencies.

The concepts of reliability, security, survivability and vulnerability of a system to different threats like natural failures, external attacks or operator failures and the means of improving the system response in complex systems like computer and communications networks are included in a broader concept of dependability. Dependability encompasses also concepts of fault-tolerant systems, which accelerated the emergence of the terminology, definitions and a wider view of the problem of system reliability and security.

Dependability in computing and concepts of the fault-tolerant systems has been part of the computer field since its origins but with the formation of the corresponding IEEE-CS TC, this field formally emerged in 1970 [3]. In the field of power systems reliability, security and vulnerability have been systematically studied since the beginning of power grids. However, more recently, a need to consider telecommunications and other infrastructures as interdependent has been recognized and the importance of taking into account cascading and catastrophic events with a broader point of view has been stressed. This led to the application of the concepts of dependability in power systems, which are similar to the ones used in the computer networks [3].

On the other hand, graph theory or network theory and matrix and linear algebra have been applied extensively in power systems since the beginning of the application of computer methods in recognition of the network structure of a power system. In the last decade, many developments in graph theory have been applied to complex networks such as internet, social networks, communication and VLSI circuits. In particular, concepts in spectral graph theory are been used to identify complex network characteristics and to have a deeper insight in the behaviour of such networks.

Spectral Graph theory is a study of the spectra of matrices that identify properties of a graph. Matrices like the node-element matrix, adjacency matrix, Laplacian matrix and in the case of power system, the bus admittance and bus impedance matrices have eigenvalues and eigenvectors that can give important information about the structure of the system and its intrinsic robustness. Also, eigenvalue analysis can lead to important conclusions about possibilities of system partitioning and node criticality.

Spectral graph theory has been applied to chemistry for molecules stability analysis, in quantum mechanics and more recently in communication networks. Spectral analysis have been extensively used to identify the characteristics of the dynamics of linear systems and stochastic processes, in analogous way spectral graph theory could be used to identify the properties of the electrical networks.

Spectral graph theory offers an interesting potential when applied to power systems grids. The study of graph eigenvalues and eigenvectors of a power network provides an insight in the intrinsic structure of the system and allows us to identify structural modes and fragilities. So, graph eigenvalues analysis could be used to design security strategies for the system when affected by massive number of events or cascading as well as catastrophic failures.

The potential of graph theory to identify structural properties of power grids could be advantageously used in the network dependability analysis. In this Chapter, two main aspects will be presented. The identification of principal nodes or a method of ranking the importance of nodes in a power grid and a method of partitioning the grid as a conceptual way to avoid the effect of cascading or catastrophic events by dividing the grid into different self-sustainable subsystems.

This chapter presents the basic concepts of the spectral graph theory as could be applied to power system networks and particularly to the identification of the strategic substations and transmission lines from the point of view of reliability and security of the bulk power system. Some examples are presented using the IEEE test system with different number of nodes to show the simplicity of the applications and the type of results that can be obtained. Further and more complex analysis can be done in reliability and security studies when a power system is affected by catastrophic events. However, the main aim of this chapter is to present a method to identify critical nodes and links within a Bulk Electric Power System (BES), which is an important element of the infrastructure of any country.

Critical or strategic nodes or in this specific case, critical substations are those that, are important to maintain the integrity of the network, or in other words if affected by natural events or terrorist actions, would cause a large disruption by itself or by following cascading events.

On the other hand, strategic transmission lines are those that if open could divide the system in self-sustained islands that could be operated independently in the case of extended events that may cause a system blackout. This separation could be considered as a strategic security action after severe disturbances or cascading events. Intentional-controlled islanding is a strategic action to separate the system into the self-healing islands which generators exhibit slow-coherency behaviour regarding angular stability [16, 20].

Spectral graph theory as applied to power systems also could be used, for example, to identify the number of spanning trees of the graph representing the system and to use the identified trees to define the nodes to minimize the number of phase measurement units (PMUs) in a observability problem.

2 Graphs Linear Algebra

2.1 Definitions of Matrices Associated with a Graph G

In creating the ranking measures for electric power system substations, we will use the following definitions concerning various matrices associated with a graph G [6, 9, 11].

- A graph G: a simple undirected graph whose vertices are positive integers. The order of G is the number of vertices.
- Node degree: the degree of vertex k, $\deg_G k$ is the number of edges incident with k. The graph G is regular of degree r if every vertex has degree r.

- External node degree due to node loads external factors for nodes (EFN): graph node degree could be affected by other ranking criteria that consider aspects of node importance beyond the network connectivity.
- External node degree due to node connecting transmission lines. External factor for lines (EFL): graph node degree could be affected by other criteria applied upon the importance of connecting transmission lines that consider aspects of node importance beyond the network connectivity.
- Adjacency matrix **A**: $\mathbf{A} = [A_{ij}]$, where $A_{ij} = 1$ if $\{i, j\}$ is the edge of G and $A_{ij} = 0$ otherwise.
- Degree matrix **D**: is a diagonal matrix $\mathbf{D} = \text{diag}(\text{deg}_G 1,\ldots,\text{deg}_G n)$
- Node-element (vertex-edge)incidence matrix **N**: is the n (number of vertices) $\times$ m (number of edges) 0, 1-matrix with rows indexed by the vertices of G and columns indexed by the edges of G, such that the i, j entry of **N** is 1 if edge j is incident with vertex i, and 0 if not.
- Orientation of G_p: is the assignment of a direction to each edge, converting edge $\{i, j\}$ to either arc $\{i, j\}$ or arc $\{j, i\}$.
- Oriented incidence matrix: the oriented incidence matrix $\mathbf{N}_p$ of an oriented Graph G_p with n vertices and m arcs is the $n \times m$. 0, 1, -1 matrix with rows indexed by the vertices of G and columns indexed by the arcs of G such that the i, k entry of $\mathbf{N}_p$ is 1 if arc is directed leaving node i and -1 if arc is directed toward node i and all other entries are 0.
- Transformation matrix **T**: $\mathbf{T} = \text{diag}((\text{deg}_G 1)^{1/2},\ldots,(\text{deg}_G n)^{1/2})$
- The Laplacian matrix **L**: $\mathbf{L} = \mathbf{D} - \mathbf{A}$
- Normalized adjacency matrix An: $An = T^{-1}AT^{-1}$
- The normalized Laplacian matrix Ln: $Ln = T^{-1}L\,T^{-1} = I - An$
- The signless Laplacian matrix Ls: $Ls = D + A$
- The normalized signless matrix Lns: $Lns = T^{-1}(D + A)\,T^{-1} = I + An$
- Transition matrix of the random walk M: $M = AD^{-1}$
- Weighted graphs and matrices: all definitions given for a simple graph can be applied to weighted graphs. A weighted undirected graph G has an associated weight function satisfying $w(i, j) = w(j, i)$ with $w(i, j) \geq 0$. Unweighted graphs are a special case where all weights are 0 or 1. In the case of an electrical power grid where we will consider external criteria for node ranking, weights are to be considered not only on the links but also for the nodes. So, we have a special case of weighted graph. A graph with node weight. Node weight will be converted to edge weights adding links between the node and the reference node. In this way, we can have the corresponding matrices A_{ext}, D_{ext}, L_{ext}, L_{next} and Lns_{ext}.

2.2 Eigenvalues of a Graph

One of the main issues of the spectral theory is the analysis of the impact of eigenvalue bounds of a graph matrix and the interpretation of the eigenvalues regarding the structure and the properties of the graph.

Considering that all graph matrices are real and symmetric, we can define the ordered spectrum $\sigma(A)$ of an $n \times n$ matrix $\mathbf{A}$ as the list of eigenvalues, repeated according to multiplicity in a non-decreasing order. Thus, $\sigma(\mathbf{A}) = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ with $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n$.

The eigenvalues of the Laplacian matrix are represented by the spectrum

$$\sigma(\mathbf{L}) = (\lambda_1, \lambda_2, \ldots \lambda_n) \tag{1}$$

and the eigenvalues of the sign-less Laplacian matrix:

$$\sigma(\mathbf{Lns}) = (\mu_1, \mu_2, \ldots \mu_n) \tag{2}$$

The matrices **A**, **An**, **L** and **Ln** are all non-negative, and if G is connected, they are all irreducible. Also, the matrices **A**, **D**, **An**, **L**, **Ln**, **Ls** and **Lns** are all connected through the incidence matrix **N**.

By using the Sylvester's Law of Inertia [14], one can prove that the spectral radius (the supremum among the absolute values of the elements in its spectrum), is 2, it is to say that $\sigma(\mathbf{L}) \subset [0, 2]$, $\sigma(\mathbf{Lns}) \subset [0, 2]$ and $\mu_{\mathrm{ns\ n}} = 2$, and $\lambda_1 = 0$.

3 Level of Importance of Substations in a Bulk Power System

3.1 General Definitions for Node Importance

One step toward security strategies is to identify critical grid elements, it is to say, a node or link that is of a paramount importance due to its role in the interconnection, its role in services to customers in the grid or other intrinsic characteristic like its cost or repair difficulty.

A substation will be called "strategic substation" when it plays a significant role in the connectivity of the grid for the normal operation of the Bulk Power System. It means that the importance of a substation depends of its location within the grid and how it is related to the rest of the power system, so the criteria to determine this importance include parameters that characterize the substations and the system in which the substations of interest are located.

The criteria for importance here is how a substation is necessary or essential for the integrity of the connectivity or interconnection in the BES. This will be considered as a network based criterion. Other criteria could be formulated to define the importance of a node in a BES due to the relative importance of the loads or the relative importance of the connecting transmission lines. So, we could include in the ranking aspects regarding substation loads and connecting transmission lines.

Examples regarding substation loads are (EFN):

- public health and safety,
- national security,

- regional economy,
- location in respect to population centres, transportation corridors and other lifelines,
- and so on.

Examples regarding connecting transmission lines are (EFL):

- underground cables,
- double circuit transmission lines,
- special structure transmission line,
- transmission lines with limited or congested right-of-way.

In general, the performance measures should cover a variety of station-related performance aspects such as reliability, utilization, security and safety to personnel. In this section, we also consider network external factors for node ranking besides purely connectivity properties of the electric power networks [2].

The measures described in this section use the well-known centrality aspects of the spectral graph theory and a concept of weighted centrality due to network external factors and do not require development of any new analytical tools nor do they use complex computer programs. The new measures are based on the analysis of the structure of the electric power network and weights or factors given by experts. Required data can be readily found in the raw data file for the power flow programs used by the electric utilities and weights for externalities could be established independently.

3.2 Network Centrality

Spectral graph theory is a study of the spectra of matrices that identify properties of a graph. Matrices like the node-element matrix, adjacency matrix, Laplacian matrix and in the case of power system, the bus admittance and bus impedance matrices have eigenvalues and eigenvectors that can give important information about the structure of the system and its intrinsic robustness. Also, eigenvalue analysis can lead to important conclusions about node criticality.

Network centrality defined as the relative importance of a vertex within a graph could be measured with several different indices, such as:

- Eigenvector centrality: is a measure of the importance of a node in a network. It assigns relative scores to all nodes in the network based on the principle that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. This means that the importance of a given node depends not only on the number of incident edges but also on the relative importance of the nodes to which it is connected.
- Electrical centrality: recently, Hines and Blumsack [8] proposed an "Electrical Centrality Measure" calculated from the Zbus Matrix that can be considered a betweenness centrality measure. The importance of a substation is inversely

proportional to the Thévenin equivalent circuit seen in the substation. In other words, the electrical equivalent distance (Z Thévenin) between the generation nodes and the substation determines its importance. So, the importance of a substation is represented by its maximum level of the short circuit power.

3.3 Criteria for Node Importance or Node Centrality

For a power system grid, we define the matrix **A** and all other system matrices including the reference node, so that generators represent a link in the network. Also, for a power system network, each substation is represented by a node and each transmission line by a link. We can have multiple links between nodes. Also, further approximations or simplifications could be applied. We could consider only a system with one voltage level, so for example, if the power system to be represented includes voltage levels of 230 and 500 kV, every link at higher voltage than 230 kV should be represented by the 230 kV equivalent links. Transformers will not be represented as links since the total substation is represented by one node. However, we may like to include factors exogenous to the grid structure itself to have a node ranking that not only include the node centrality concept but the node service importance and the relative physical importance of links.

To determine the external factor or weight for a node the following considerations could be applied:

- Substations: for substations with two levels of high voltage (500/230/220 kV), the two nodes could be combined as a "super node" considering all the links for both of them. Substations with special loads could have a weight proportional to the load importance. For example, a substation with an important load due to National Security could have a weight of 2 or 3 instead of the standard weight of 1 for all substations.
- Equivalent links: a 500 kV transmission line is equivalent to two 230 kV lines. So the total number of links for each substation is referred to 230 kV. In this case, a weight of 2 could be associated with the line.
- Generators: all machines connected to a low voltage node are represented as an equivalent generator connected to a node
- Transformers: they are ignored as links for the nodes. However, we could include an external factor in the node to consider special loads or external criteria for node ranking.
- Compensation elements: they are ignored as links for the nodes. However, we could include an external factor in the node to consider special cases or external criteria for node ranking.

Association of a node weight as explained above allow us to identify strategic or important substations regardless of the voltage level or to classify substations that could have two or more voltage levels and substations with important loads and transmission lines. Eigenvector centrality intrinsically recognizes the relative importance of the linked nodes due to the location of a node within the network.

An eigenvector $\mathbf{x} = (x_1, x_2,\ldots,x_n)$ and the corresponding eigenvalue λ, of a matrix $\mathbf{A}$ satisfy the following equation:

$$\lambda \mathbf{x} = \mathbf{A}\mathbf{x} \tag{3}$$

For node i, we can write

$$x_i = \frac{1}{\lambda}\sum_{j=1}^{n} A_{i,j}x_j \tag{4}$$

If we define the centrality index for a given node i as proportional to the average of the centrality indices of neighbouring nodes, then the element x_i of the eigenvector $\mathbf{x}$ of the adjacency matrix is the centrality index for node i [8]. A centrality index should be a non-negative number. So, all elements of the eigenvector $\mathbf{x}$ should be non-negative numbers.

According to the Perron–Frobenius Theorem for square non-negative matrices [14], an $n \times n$ non-negative matrix $\mathbf{A}$ has a real eigenvalue r such that any other eigenvalue satisfies $\lambda \leq r$ and its corresponding eigenvector has non-negative entries. Every other eigenvector for smaller eigenvalues has negative components. Therefore, λ should be the greatest eigenvalue r of the adjacency matrix $\mathbf{A}$ and $\boldsymbol{x}$ is its eigenvector, it is to say, the eigenvector corresponding to the first eigenvalue when they are ordered.

With this criterion the importance of a node is not only determined by the node degree but by the number and quality of its connected nodes as well [18].

3.4 Examples

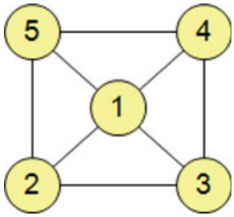
Three examples are presented below to show the application of the concepts presented above. First example uses a five-node system and matrices are presented together with the results. A second example uses the IEEE-39 node test system. For these two examples we show the effect of considering weighted nodes and edges. The third example considers a larger real-life system.

3.4.1 Example 1: Five-Node System

Figure 1 shows a five-node system. It is easy to see that considering network centrality, node one is the most important and that all other nodes are equally important since their role in the interconnection is similar and there is no other criterion for discrimination.

Matrices for system shown in Fig. 1 are shown in Fig. 2. Now, let us assume that link 1–2 is of higher relative importance than any other for some reason (voltage, underground cable, etc.) and due to this fact we assign a factor 3 for this link, as shown in Fig. 3.

Fig. 1 Five-node graph



$$A=\begin{pmatrix}0&1&1&1&1\\1&0&1&0&1\\1&1&0&1&0\\1&0&1&0&1\\1&1&0&1&0\end{pmatrix}\qquad An=\begin{pmatrix}0&0.289&0.289&0.289&0.289\\0.289&0&0.333&0&0.333\\0.289&0.333&0&0.333&0\\0.289&0&0.333&0&0.333\\0.289&0.333&0&0.333&0\end{pmatrix}$$

$$\sigma(A)=\begin{pmatrix}-2&-1.236&0&0&3.236\end{pmatrix}\qquad \sigma(An)=\begin{pmatrix}-0.667&-0.333&0&0&1\end{pmatrix}$$

$$L=\begin{pmatrix}4&-1&-1&-1&-1\\-1&3&-1&0&-1\\-1&-1&3&-1&0\\-1&0&-1&3&-1\\-1&-1&0&-1&3\end{pmatrix}\qquad Ln=\begin{pmatrix}1&-0.289&-0.289&-0.289&-0.289\\-0.289&1&-0.333&0&-0.333\\-0.289&-0.333&1&-0.333&0\\-0.289&0&-0.333&1&-0.333\\-0.289&-0.333&0&-0.333&1\end{pmatrix}$$

$$\sigma(L)=\begin{pmatrix}0&3&3&5&5\end{pmatrix}\qquad \sigma(Ln)=\begin{pmatrix}0&1&1&1.333&1.667\end{pmatrix}$$

$$Ls=\begin{pmatrix}4&1&1&1&1\\1&3&1&0&1\\1&1&3&1&0\\1&0&1&3&1\\1&1&0&1&3\end{pmatrix}\qquad Lns=\begin{pmatrix}1&0.289&0.289&0.289&0.289\\0.289&1&0.333&0&0.333\\0.289&0.333&1&0.333&0\\0.289&0&0.333&1&0.333\\0.289&0.333&0&0.333&1\end{pmatrix}$$

$$\sigma(Ls)=\begin{pmatrix}1&2.438&3&3&6.562\end{pmatrix}\qquad \sigma(Lns)=\begin{pmatrix}0.333&0.667&1&1&2\end{pmatrix}$$

Fig. 2 Graph matrices for system in Fig. 1

Figure 4 shows node ranking for the original graph and with the weight due to external factor considering an external importance of line 1–2. We can see that node 2 is now more important than nodes 3, 4 and 5. External factors signify importance not due to the network structure itself.

3.4.2 Example 2: IEEE 39-Node Test System

Figure 5 shows an example with a larger network, the IEEE 39-node test system. The first six nodes in the ranking are shown for two cases. One with all lines having the same importance and the second when the lines between nodes 3–4, 3–2 and 2–25 have special weights assigned. The analysis is conducted with a weighted graph. Matrices are not shown due to their sizes.

$$EFL=\begin{pmatrix}1&2&3\\1&3&1\\1&4&1\\1&5&1\\2&5&1\\2&3&1\\3&4&1\\4&5&1\end{pmatrix}\quad EFN=\begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix}\quad A=\begin{pmatrix}0&1&1&1&1\\1&0&1&0&1\\1&1&0&1&0\\1&0&1&0&1\\1&1&0&1&0\end{pmatrix}\quad Aext=\begin{pmatrix}0&3&1&1&1\\3&0&1&0&1\\1&1&0&1&0\\1&0&1&0&1\\1&1&0&1&0\end{pmatrix}$$

$$D=\begin{pmatrix}4&0&0&0&0\\0&3&0&0&0\\0&0&3&0&0\\0&0&0&3&0\\0&0&0&0&3\end{pmatrix}\quad Dext=\begin{pmatrix}6&0&0&0&0\\0&5&0&0&0\\0&0&3&0&0\\0&0&0&3&0\\0&0&0&0&3\end{pmatrix}$$

$$L=\begin{pmatrix}4&-1&-1&-1&-1\\-1&3&-1&0&-1\\-1&-1&3&-1&0\\-1&0&-1&3&-1\\-1&-1&0&-1&3\end{pmatrix}\quad Lext=\begin{pmatrix}6&-3&-1&-1&-1\\-3&5&-1&0&-1\\-1&-1&3&-1&0\\-1&0&-1&3&-1\\-1&-1&0&-1&3\end{pmatrix}$$

$$Ln=\begin{pmatrix}1&-0.289&-0.289&-0.289&-0.289\\-0.289&1&-0.333&0&-0.333\\-0.289&-0.333&1&-0.333&0\\-0.289&0&-0.333&1&-0.333\\-0.289&-0.333&0&-0.333&1\end{pmatrix}\quad Lnext=\begin{pmatrix}1&-0.548&-0.236&-0.236&-0.236\\-0.548&1&-0.258&0&-0.258\\-0.236&-0.258&1&-0.333&0\\-0.236&0&-0.333&1&-0.333\\-0.236&-0.258&0&-0.333&1\end{pmatrix}$$

$$xL_{ns}(n)=\begin{pmatrix}0.5&0.433&0.433&0.433&0.433\end{pmatrix}\quad xL_{nsex}(n)=\begin{pmatrix}0.548&0.5&0.387&0.387&0.387\end{pmatrix}$$

Fig. 3 Graph matrices for system in Fig. 1 with external factors for link 1–2

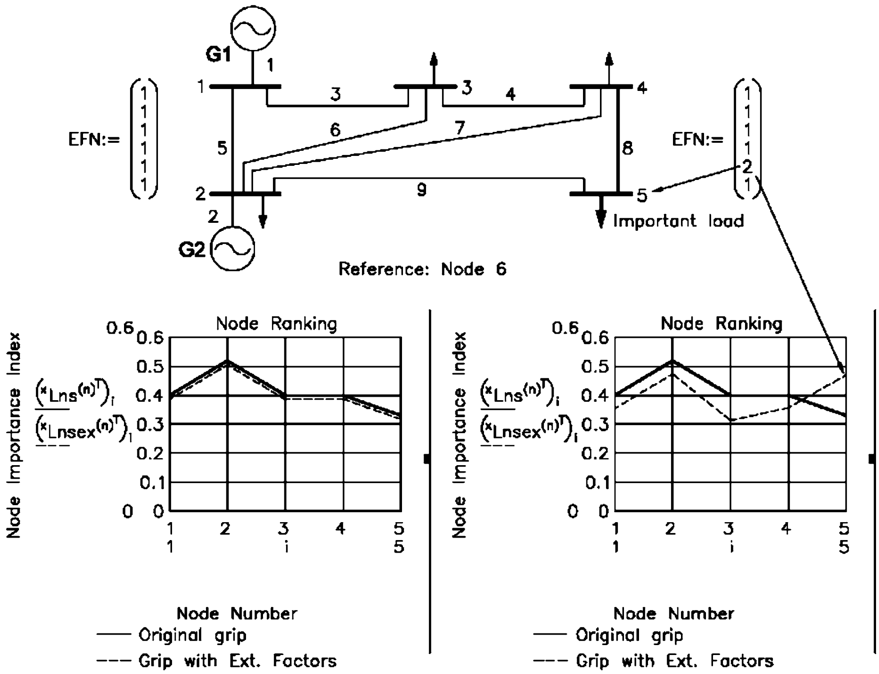


Fig. 4 Node order for the five-node system considering external factors

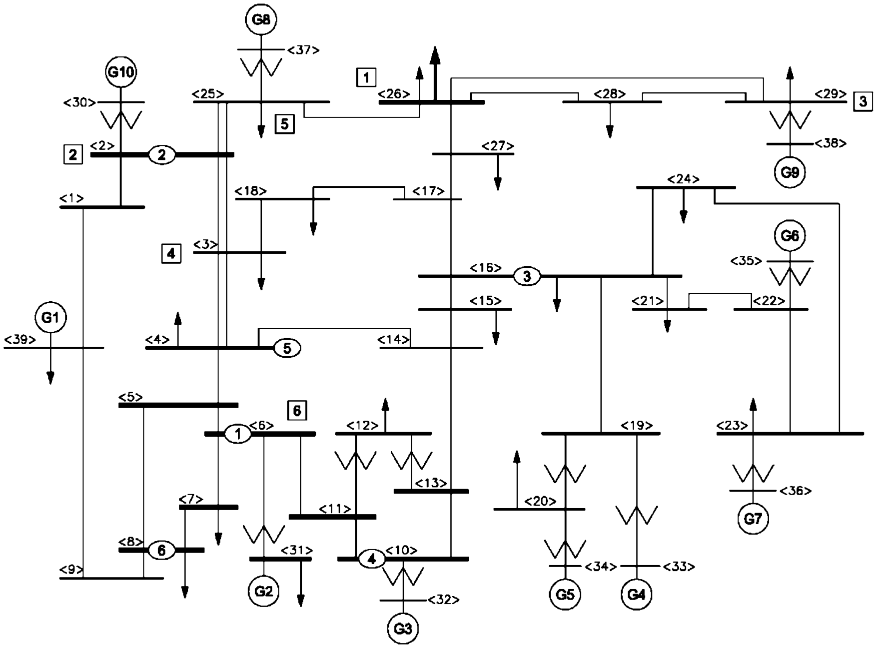


Fig. 5 IEEE 39-node test system. Importance node ranking without external factors (*squares with numbers*) and considering the importance of the transmission lines between nodes 3–4,3–2,2–25 (*ovals with numbers*)

3.4.3 Example 3: A Real Bulk Power System

The criteria presented in this chapter have been used to rank the importance of the substations in a real 87-node 230–500 kV Bulk Power System shown in Fig. 6. The studies were conducted to determine its role in the interconnection or integrity of the interconnected system.

Several ranking methods using the centrality concept will be investigated. Ranking method may be simple or sophisticated; their use depends on the information available and the desired precision of the required results. The proposed methodology looks for a compromise between application, in terms of tools and needed information, and the precision of the results.

Since the eigenvector centrality considers not only the number of edges incident on a node but also the relative importance of the other connected nodes, and it is a method easy to implement, it is a good method to recommend. Also, with this method, a weighted adjacency matrix could be considered by using the imaginary part of the electrical admittances. However, using the un-weighted adjacency matrix we can obtain satisfactory results as well. Figure 7 shows the ranking of the substations for this network and a comparison of the ranking given by different criteria.

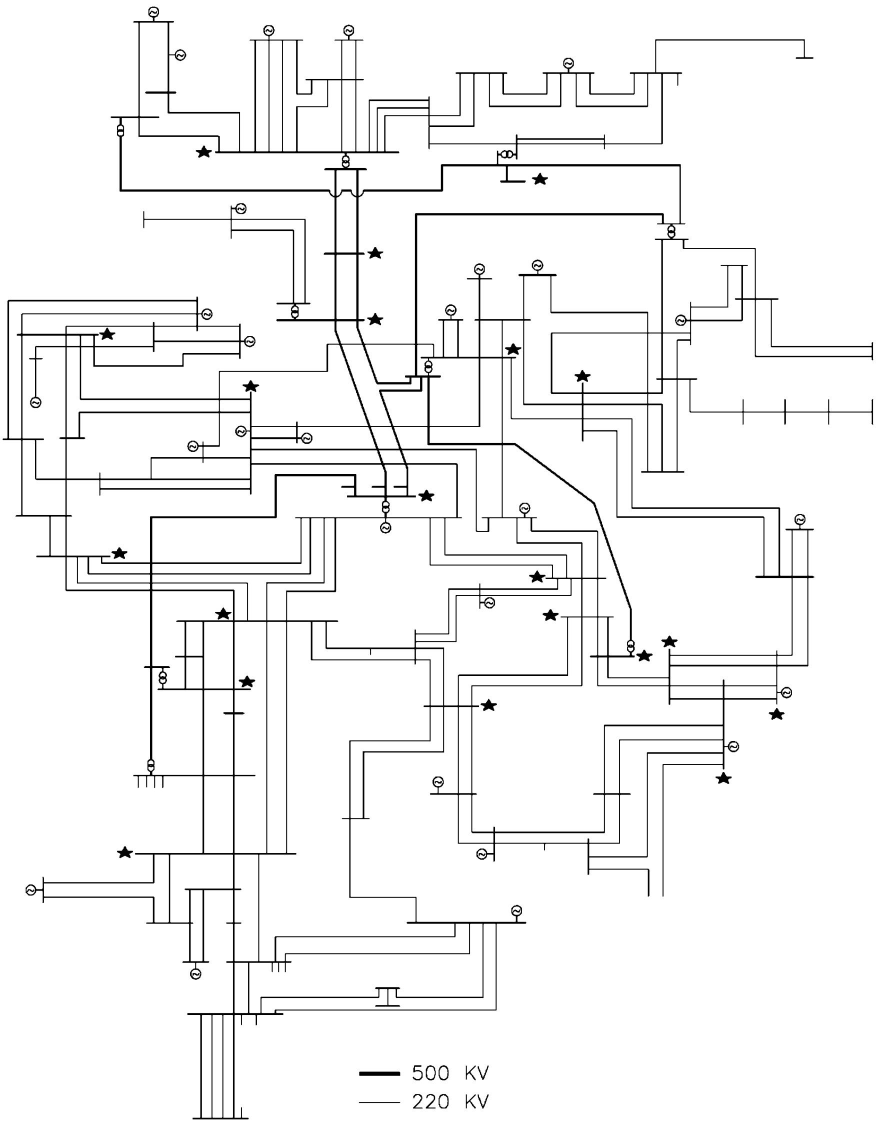


Fig. 6 87-node system

4 Graph Partitioning

Graph partitioning is a problem in graph theory related to a division of the vertices of a graph into two or more sets, while cutting as few edges as possible. There are many restrictions that can be introduced to the problem to consider the number of edges or weights or sizes of the sub-graph to be produced by a partitioning.

NODE IMPORTANCE RANKING BY DIFFERENT CRITERIA					
			SELECTED		
SUB. NAME	Short Circuit (MVA)	Number Equivalent Edges	Ranking By Eigenvector Centrality	Ranking By Degree Centrality	Ranking By Connectivity
SAB	8612	16	1	1	2
PRI	7888	16	2	1	6
SANCA	12748	14	3	3	4
ESME	7211	11	4	4	1
CERRO	7275	10	5	5	5
GUAT	10995	10	6	5	3
VIR	6175	9	7	7	8
GUA	10716	9	8	7	11
PUR	7211	8	9	9	7
MES	8007	8	10	9	13
CHIN	6322	8	11	9	16
COP	4070	7	12	12	9
CHIV	9720	7	13	12	10
GUAT	4103	7	14	12	12
NORO	8645	6	15	15	17
SANMA	6972	6	16	15	43
BAC	8605	6	17	15	48
YUM	7171	6	18	15	19
ANCO	7011	6	19	15	30
TOR	7967	6	20	15	18
BAR	6821	6	21	15	23
SANBE	3227	6	22	15	25
FUND	6097	6	23	15	29
JAM	2669	6	24	15	26
BET	3546	6	25	15	24

Fig. 7 Ranking of the first 25 most important substations for the 87-node system

A graph partitioning could be done by a progressive bi-sectioning of a graph or by a direct *k*-way partitioning. Both procedures could be obtained by matrix spectral analysis, so called spectral partitioning. These techniques have been used intensively in image partitioning when an image is considered as a large connected network.

A high interest in power network partitioning dates to the early days of diakoptics methods introduced by Gabriel Kron to break a matrix problem into small sub-problems, which can be solved independently and then to associate the solutions to obtain the global solution. Also, the problem of partitioning appears in the decomposition methods for state estimation or computational parallel processing for very large systems. The spectral graph theory has been applied to image segmentation and digital image processing [15].

More recently, the problem of power system network partitioning has appeared in security problems. On the one hand, we can see problems where early detection of island formation in power system networks under multiple line outages is important for the study of security and control strategies. On the other hand, intentional islanding and adaptive load shedding schemes have been proposed to avoid cascading outages due to large catastrophic events or simultaneous events that usually precede the occurrence of blackouts [1, 16, 20].

The study of predetermined islanding scenarios with load-shedding schemes and distributed generation strategies may constitute the basis for the predetermined action plans against system collapse when system security is compromised due to a large number of line outages. To detect the formation of islands inside a power system subjected to multiple line outages and for controlled islanding to prevent blackouts, several methods based on minimum cut sets, decision trees and generator coherency have been proposed [4, 7, 10, 16].

Graph spectral analysis allows identification of the natural structure of the system and how it can be partitioned by minimizing a specially defined performance index. The aim of this section is to show the application of the graph spectral theory for power system partitioning to be used in controlled islanding strategies for security enhancement.

4.1 Graphs Cut Problem

In this section, we analyze a graph partitioning problem from the point of view of its spectral characteristics to determine how to divide a graph cutting as few edges as possible. In the case of a power system, the structure of the resulting subsystems includes slow coherent generators under transient system stability conditions given that generator connections are properly defined.

The graph bisection problem is to find a set S of a graph G, $S \subset G$ such that approximately:

$$|S| = \left\lceil \frac{|S|}{2} \right\rceil \tag{5}$$

This problem can be formulated as follows:

$$\min |\{(i,j) \in G : i \in S, j \notin S\}| \tag{6}$$

Set $S \subseteq G$ is referred to as a cut, which represents the partition of G into S and its complement.

Since a perfect bisection may not be possible, it could be good enough to obtain an acceptable value of a performance index or cut quality index like the number of edges cut to the number of vertices or edges removed, the so called ratio of a cut defined, or the conductance of a cut.

The conductance of a set S is defined as:

$$\phi(S) = \frac{|\partial(S)|}{d(S)d(G-S)} \tag{7}$$

where $|\partial(S)|$ is called the boundary of S, which means all edges with one end point in S and the other outside of S and $d(S)$ is the number of edges attached to each side of the graph. It is to say that $d(S)$ is the sum of the degrees of vertices in S. In a weighted graph, the weighted degree of a vertex is the sum of the weights of the edges attached to it.

So, the problem is to find the set S of minimum conductance:

$$\phi(G) = \min_{S \subset G} \phi(G) \tag{8}$$

We have to find a relationship between the graph conductance and the graph eigenvalues and eigenvectors. This is given by the Cheeger inequality, which could be formulated as [5]:

$$\varphi(G) \leq 2\sqrt{2\lambda_2(\mathbf{L}n)} \tag{9}$$

where $\lambda_2(\mathbf{L}n)$ is the second smallest eigenvalue of the normalized Laplacian matrix $\mathbf{L}ns$, which is considered as the most important information in the spectrum of a graph [6]. It is to say that for some j

$$\varphi(S_j) \leq \sqrt{2\lambda_2(\mathbf{L}n)} \tag{10}$$

This means that for a small $\lambda_2(\mathbf{L}n)$ we will have a cut of small conductance, but is difficult to prove that it is the minimum conductance. However, we could consider it as a good solution for a bi-section of a graph.

The procedure is to compute the eigenvectors and to partition the graph into two subgraphs using the second smallest eigenvector. In some cases, the eigenvector takes negative and positive values and the signs will give the exact partition of the graph. However, we can choose the splitting point for better convenience.

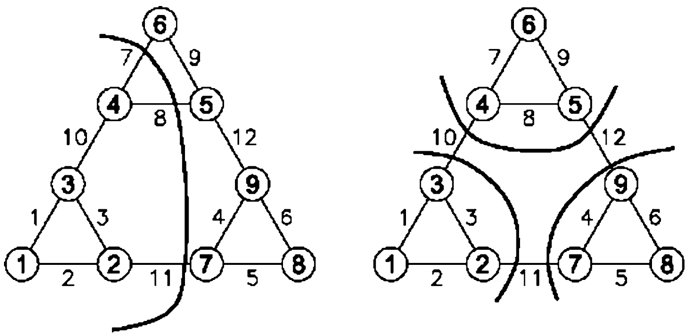
To divide a graph in more than two parts, we can use recursive bi-partitioning with the second smallest eigenvalue and its corresponding eigenvector for each sub-graph successively. One disadvantage of the recursive bi-partitioning is the treatment of oscillatory eigenvectors. However, one can decide the splitting point to reduce their effect.

Although there is no complete knowledge about the meaning of many spectral characteristics of a graph, the other eigenvectors besides the corresponding to the second smallest eigenvalue also contain useful information. We can use the higher eigenvectors for a direct k-way partitioning of a graph. The higher eigenvectors are indicators of how to partition a graph in more than two cuts. However, avoiding the effect of oscillatory eigenvectors is an important task to [15].

4.2 Examples

We will discuss three examples to show the application of the presented concepts. First example uses a nine-node graph as shown in Fig. 8 with 12 links.

Second example uses the IEEE 39-node Reliability Test System. We can consider weighted edges with line and generator admittances by using the $\mathbf{Y}$ admittance matrix with the modification of considering the positive values instead of the normal $\mathbf{Y}_{\mathrm{BUS}}$ admittance matrix. In this example, we go further to test the slow coherency of the subsystems once they are intentionally separated after a severe set of contingencies.



$$\sigma(\mathrm{Lns}) = (0.354 \quad 0.401 \quad 0.5 \quad 0.5 \quad 0.734 \quad 0.905 \quad 1.698 \quad 1.908 \quad 2)$$

$$\sigma(\mathrm{Ln}) = (0 \quad 0.092 \quad 0.302 \quad 1.095 \quad 1.266 \quad 1.5 \quad 1.5 \quad 1.599 \quad 1.646)$$

$$x_{Ln2} = (-0.404 \quad -0.404 \quad -0.403 \quad -0.108 \quad 0.108 \quad 0 \quad 0.404 \quad 0.404 \quad 0.403)$$

$$x_{Ln3} = (-0.296 \quad -0.296 \quad -0.144 \quad 0.425 \quad 0.425 \quad 0.497 \quad -0.296 \quad -0.296 \quad -0.144)$$

Fig. 8 Graph partitioning for a nine-node, 12-link graph

The third example shows the IEEE 69-node system. In this example, we will test the effect of network partitioning to be used for splitting power system when a disturbance occurs. Power system analyses were performed with the Power System Analysis Toolbox PSAT [10].

4.2.1 Example 1: Nine-Node System

Figure 8 shows the nine-node system. Eigenvalues for the Laplacian matrix **L***n* are shown with the eigenvector for the second smallest and third smallest eigenvalues. Eigenvector for the second smallest eigenvalue allow us to define the most suitable bisection of the graph and the eigenvector entries for the third smallest eigenvalue allow us to define a suitable partition in three sub graphs. Graph partition defines link importance.

To find a cut to separate the graph into two parts, we use the eigenvector corresponding to the second smallest eigenvalue. This approach gives the result shown in Fig. 8.

When using the weighted matrices, we may have slightly different results. However, the results obtained with the un-weighted matrices are good enough considering that simpler calculations and less information are required.

4.2.2 Example 2: IEEE 39-Node Reliability Test System

The same approach was applied to a larger system, the IEEE Reliability Test System. Results for a two-way partitioning and important nodes are presented in

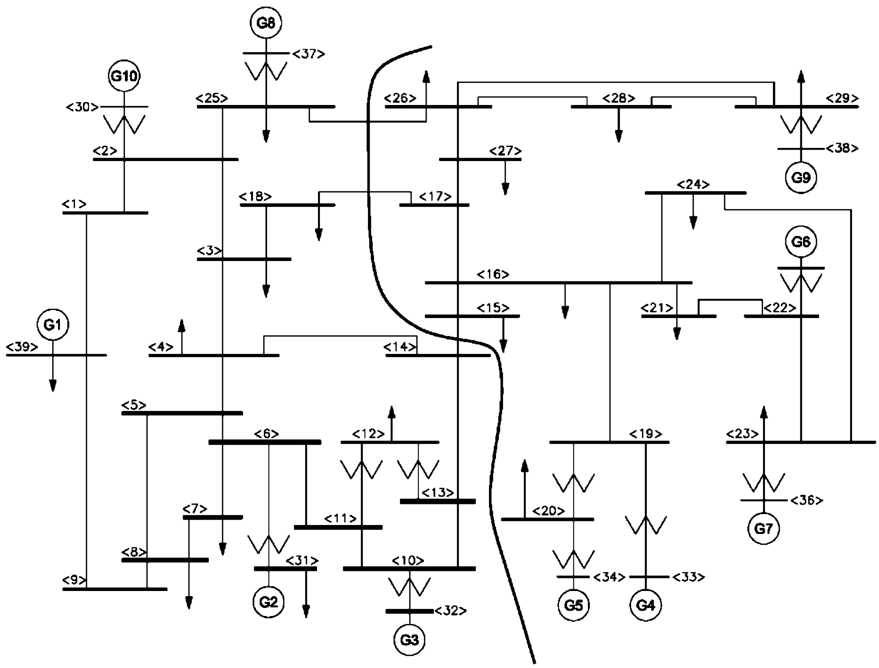


Fig. 9 Two-way partitioning for the IEEE 39-node reliability test system. System diagram taken from [19]

Fig. 9. Figure 10 shows the application of a higher eigenvector for a three-way partition problem.

The three-way partitioning obtained by using the eigenvectors of the third highest eigenvalue is similar to the partition obtained by the slow coherency methods [21]. Figure 10 shows the three-way partition and the entries of the eigenvector for the third highest eigenvalue. The main difficulty may reside in defining the dividing value for the eigenvector entries to obtain the desired partitioning.

The slow coherency in a power system is defined by the connection matrix of the system and the generator inertias. If we consider the linear electromechanical model for an n-machine power system and neglecting damping and the off-diagonal conductance terms, we have:

$$\begin{aligned} \frac{\mathrm{d}^2\Delta\delta}{\mathrm{d}t^2} &= \Delta\ddot{\delta} = \mathbf{M}^{-1}\mathbf{K}\Delta\delta \\ K_{ij} &= E_i E_j Y_{ij}\cos(\delta_i - \delta_j) \end{aligned} \tag{11}$$

where δ is the vector of machine rotor angles, $\mathbf{M}$ is the $n \times n$ diagonal matrix of machine inertias and $\mathbf{K}$ is the $n \times n$ connection matrix whose (i, j) entry is given in

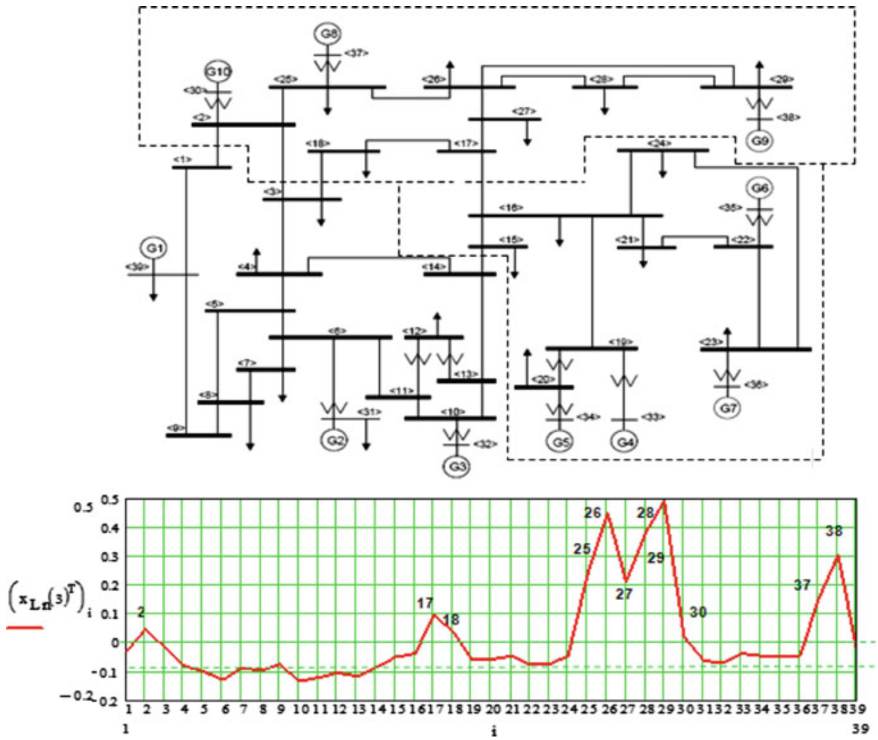


Fig. 10 Three-way partitioning for the IEEE 39-node reliability test system. System diagram taken from [19]

terms of the bus voltages and the admittance matrix. Matrix **K** is dominated by the element-node connection or the node-element incident matrix of the power network.

The slow-coherency method for coherency determination requires the calculation of the slow eigenbasis matrix of the electromechanical model of the power system where the Jacobian matrices allow for the linearization of the nonlinear system equations around an operating point [4].

Once the system state matrix is calculated and the number r of desired coherent areas is chosen, the eigenvector of the r smallest eigenvalue is calculated. It means that the eigenvector corresponding to the r smallest eigenvalue of the system state equation will allow us to obtain r slow coherent areas in the power system [17]. Although to obtain coherent areas the matrix of machine inertias is required, the main component is the connectivity matrix which represents the structural characteristic of the network. It is to say that obtaining coherent groups by eigenbasis analysis of the electromechanical model is similar to the k-way network partition using the spectral graph theory approach, as shown in Fig. 10 in which the obtained partitioning is approximately equal to the one shown in [21].

4.3 Graph Partitioning and Controlled Islanding for System Security

The determination of the strategic transmission lines in a power system as proposed in this Chapter has the main aim of identifying those circuits which will allow us to separate the system into islands in a controlled way to reduce the possibility of system collapse when subjected to severe disturbances.

The strategy of controlled island separation of a power system is becoming of paramount importance considering the need to prevent blackouts. System blackouts could occur as a consequence of severe disturbances or cascading events that could be produced by catastrophic atmospheric conditions or man-produced events such as terrorism acts.

The island-separated system provides temporal conditions that will allow for a controlled operation and response to the extreme conditions. Once the effects of the disturbances no longer present a threat to the system integrity and security, the system will go back to the interconnected operation.

To show this strategy, the 68-node system shown in Fig. 11 has been used. This system is a reduced equivalent of the interconnected New England test system (NETS) and New York power system (NYPS). There are five geographical regions; areas 3, 4 and 5 are approximated by equivalent generators models [13]. Network partitioning is obtained by using the second smallest eigenvalue and the results show the strategic lines for security islanding. The strategic circuits are connecting busses 60–61(2), 27–53 and 53–54(2) [17].

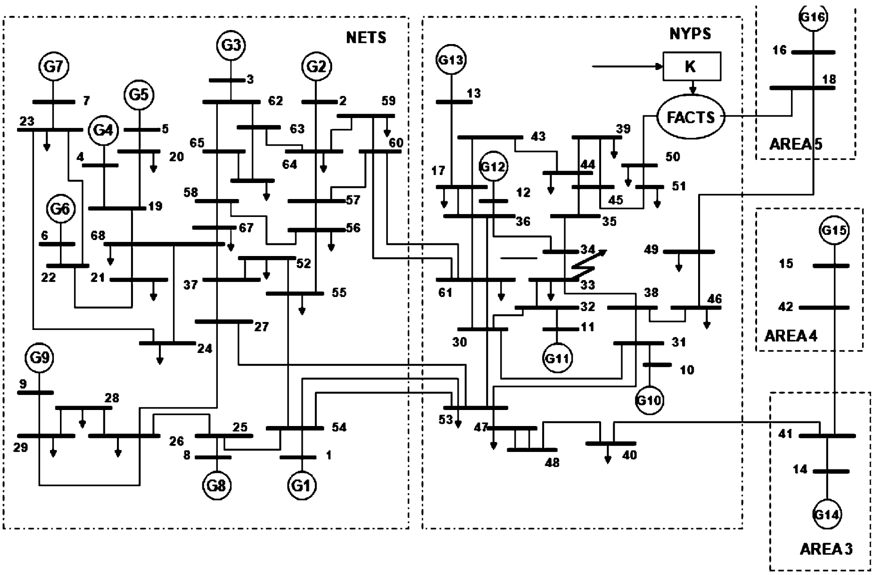


Fig. 11 68-Node system

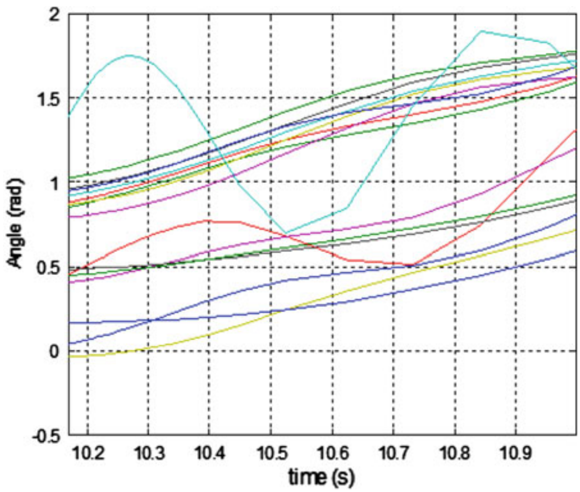


Fig. 12 Angular swings for generators in the 68-node system during a fault. Taken from [17]

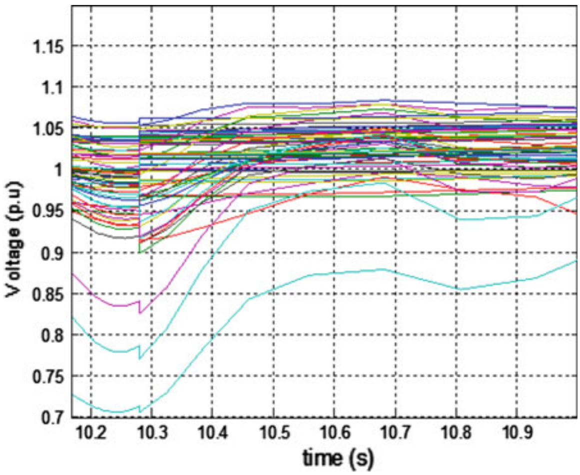


Fig. 13 Bus voltages in the 68-node system during a fault. Taken from [17]

In order to show the robustness of the strategy for separating the system to respond to a fault, the response of the 68 bus system to a critical failure was analyzed. A three-phase fault occurs near bus 33. For simulation purposes, the fault happened at the time $t = 10$ s, time to clear the fault is $t = 10.17$ s, and time for system restoration is $t = 50$ s.

Figures 12 and 13 show the angular rotor swings and voltages of all buses when the fault occurs and the system is maintained interconnected. We can see a loss of synchronism of generators and important deviations on the voltage profiles.

Power system is split into two islands by opening the strategic lines. Time required to perform system islanding by opening all selected strategic lines is $t = 10.28$ s. Machines oscillate coherently and bus voltages behave much better. Figure 14 shows bus voltages after system interconnection is reestablished.

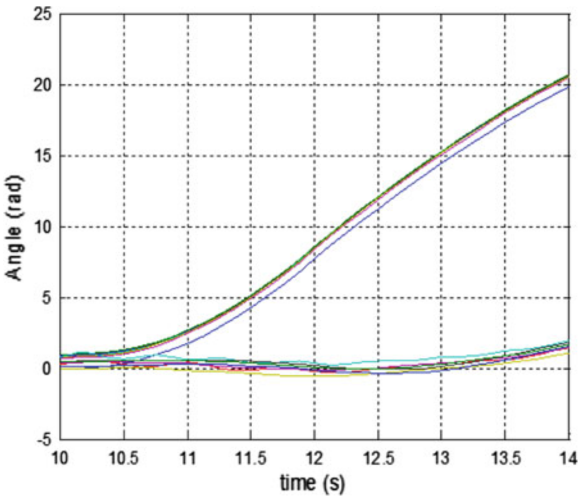


Fig. 14 Bus voltages after interconnection is re-established. Taken from [17]

5 Conclusions

The bulk transmission system is made up of different components such as lines, transformers, and so on, and these components are connected together to perform the function of transmitting electric energy, economically, reliably and safely from generating stations to distribution systems. In a competitive electricity market, owners of the transmission systems are required by regulatory rules to manage effectively their assets and resources and to maintain their systems to specific performance standards. Meeting these standards could be a challenging task for transmission owners or providers particularly if they want to maintain a good investment rate of return for their shareholders. In order for utilities to meet such challenges, new power assessment methods are required.

The identification of strategic substations will allow the transmission system owners and regulators to provide security standards and requirements that will focus on those substations that are more important for the system security and interconnection integrity. The concept of network centrality based on spectral graph theory provides a method to rank substation based on the intrinsic structural characteristics of a network.

The identification of strategic transmission lines from the point of view of the network properties and not from the considerations of transmission capacity or line cost or its properties, provides a method for security enhancement by splitting the power system as a defensive strategic response for severe or catastrophic events.

Graph theory has been used traditionally for matrix analysis in power systems. Spectral graph theory is a promising field of research in reliability, security, observability and state estimation.

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References

1. Amin M, Wollenberg BF (2005) Toward a smart grid. IEEE Power and Energy Magazine, Sept–Oct 2005, pp 34–41
2. Anders GJ, Torres A, Da Silva A (2010) Guide to identify and rank critical facilities. CEATI report T083700-3357
3. Avizienis A, Laprie J, Randell B, Landwehr C (2004) Basic concepts and taxonomy of dependable and secure computing. IEEE Trans Dependable Secur Comput 1(1):11–33
4. Chow JH, Cullum J, Willoughby R (1984) A sparsity-based technique for identifying slow-coherent areas in large power systems. IEEE Trans Power Apparatus Syst PAS-103(3):463–473
5. Chung Fan (2007) Four proofs for the Cheeger inequality, graph partition algorithms. ICCM II:1–4
6. Fiedler M (1973) Algebraic connectivity of graphs. Czech Math J 23(98):298–305
7. Güler T, Gross G (2007) Detection of island formation and identification of causal factors under multiple line outages. IEEE Trans Power Syst 22(2):505–513
8. Hines P, Blumsack S (2008) A centrality measure of electrical networks. In: Proceedings of the 41st Hawaii international conference on system sciences
9. Hogben L (2005) Spectral graph theory and the inverse eigenvalue problem of a graph. Electron J Linear Algebra 14:12–31
10. Liu Y, Liu Y (2006) Aspects on power system islanding for preventing widespread blackout. In: Networking, sensing and control. ICNSC '06. Proceedings of the 2006 IEEE international conference, pp 1090–1095
11. Milano F (2010) Power systems analysis toolbox, PSAT. University of Castilla. http://www.power.uwaterloo.ca/~fmilano/psat.htm
12. Ostrowski AM (1960) A quantitative formulation of Sylvester's law of inertia. Proc Natl Acad Sci 46(6):859–862
13. Pal B, Chaudhuri B (2005) Robust control in power systems. Springer, New York, pp 40–45
14. Perron-Frobenius Theorem (2010) http://www.prenhall.com/divisions/esm/app/ph-linear/leon/html/perron.html
15. Shi J, Malik J (2000) Normalized cuts and image segmentation. IEEE Trans Pattern Anal Mach Intell 22(8):888–905
16. Senroy N, Heydt G, Vittal V (2006) Decision tree assisted controlled islanding. IEEE Trans Power Syst 21(4):1790–1797
17. Torres A, Moreno R (2010) A strategy of security in power system: controlled islanding of PowerSystems. University of Los Andes (in press)
18. Torres A, Anders GJ (2009) Spectral graph theory and critical nodes. Fourth international conference on dependability of computer systems DepCoS-RELCOMEX 2009, Wrocław University of Technology, Institute of Computer Engineering, Control and Robotics, June 2009
19. University of Winsconsin, Electrical and Computer Engineering, IEEE benchmarks (2010) http://psdyn.ece.wisc.edu/IEEE_benchmarks/index.htm
20. You H, Vittal V, Wang X (2004) Slow coherency-based islanding. IEEE Trans Power Syst 19(1):483–491
21. Yusof SB, Rogers GJ, Alden RTH (1993) Slow coherency based network partitioning including load buses. IEEE Trans Power Syst 8(3):1375–1382