Chapter 6
व्यतीपातप्रकरणम्
Vyatīpāta

६.१ व्यतीपातसम्भवः
6.1 The possibility of *vyatīpāta*

अर्केन्द्वोर्हीयते चैका यदान्या वर्धते क्रमात् ।
क्रान्तिज्ययोस्तदा साम्ये व्यतीपातो न चान्यथा [1] ॥ १ ॥
वैधृतोऽयनसाम्ये स्यात् लाटः स्यादेकगोलयोः ।

arkendvorhīyate caikā yadānyā vardhate kramāt |
krāntijyayostadā sāmye vyatīpāto na cānyathā || 1 ||
vaidhṛto'yanasāmye syāt lāṭaḥ syādekagolayoḥ |

Of [the two objects] the Sun and the Moon, when [the magnitude of the declination of] one is decreasing and the other is increasing steadily, and when the [magnitudes of] the Rsines of their declinations become equal, then it is *vyatīpāta* and not otherwise; [The same is called] *vaidhṛta* if the *ayanas* are the same and *lāṭa* when the hemispheres are the same.

Condition for the occurrence of *vyatīpāta*

Let δ_s and δ_m be the declinations of the Sun and the Moon at any given time. Then the condition to be satisfied for the occurrence of *vyatīpāta* is given to be

$$|\delta_s| = |\delta_m|, \qquad (6.1)$$

with the constraint that the variation in the two declinations should be having opposite gradients. That is, if $|\delta_s|$ is increasing, $|\delta_m|$ should be decreasing and vice versa. Such a situation is schematically depicted in Fig. 6.1.

[1] The prose order of this verse is: यदा अर्केन्द्वोः (मध्ये) एका क्रान्तिः क्रमात् हीयते अन्या च वर्धते तदा (क्रान्त्योः) साम्ये व्यतीपातः अन्यथा न च।

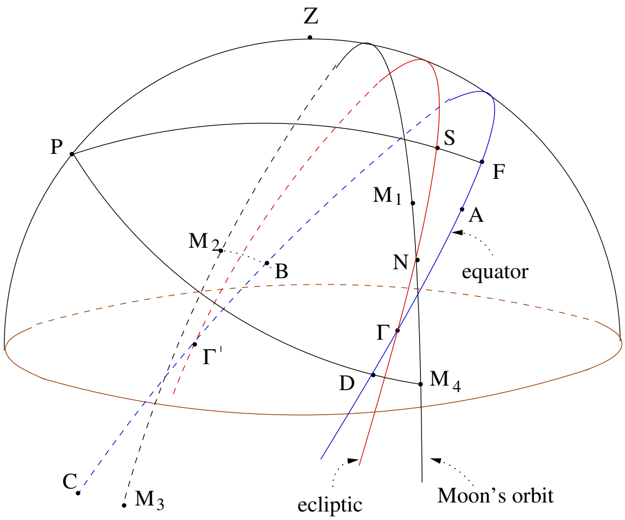

Fig. 6.1 Positions of the Sun and the Moon during *vyatīpāta*.

Occurrence of *lāṭa* and *vaidhṛta*

In the Fig. 6.1, Z represents the zenith, P the north celestial pole, Γ the vernal equinox and N the ascending node of the Moon's orbit. The Sun is at S whose declination $|\delta_s| = SF$. M_1, M_2, M_3 and M_4 correspond to different positions of the Moon, which lie in four different quadrants, at which

$$|\delta_m| = M_1A = M_2B = M_3C = M_4D = |\delta_s|. \tag{6.2}$$

Out of these four positions of the Moon—since $|\delta_s|$ is increasing at S—only M_2 and M_4 correspond to *vyatīpāta*, as $|\delta_M|$ is decreasing only at these two positions. Moreover, it may be noted that when the Moon is at M_2, the *ayanas* of the Sun and the Moon are different but lie in the same hemisphere. Hence it is an instance of *lāṭa-vyatīpāta*. On the other hand, when the Moon is at M_4, the *ayanas* of the Sun and the Moon are the same (both are northerly). Hence this is an example of *vaidhṛta-vyatīpāta*.

The commentary begins with the following *avatārikā*: [2]

एवं रवीन्द्वोः ग्रहणद्वयं दृग्गोलविषयं स्पष्टतरं प्रदर्शितम्। इदानीं भगोलविषयं तयोरेव क्रान्तिसाम्यजनितं व्यतीपातं प्रदर्शयितुमाह।

Thus the two eclipses of the Sun and the Moon, related to the observer-centred celestial sphere (*dṛggola*), were clearly demonstrated. Now in order to explain the concept of *vyatīpāta*—that arises owing to the equality of declinations of them [the Sun and the Moon]—related to the geocentric celestial sphere *bhagola*, [the following] is stated.

[2] The word *avatārikā* refers to succinct introductory remarks.

६.२ अर्केन्द्वोः क्रान्त्यानयनम्

6.2 Finding the declination of the Sun and the Moon

संस्कृतायनसूर्येन्द्वोः क्रान्तिज्ये पूर्ववन्नयेत् ॥ २ ॥

saṃskṛtāyanasūryendvoḥ krāntijye pūrvavannayet || 2 ||

From the *ayana*-corrected longitudes (*sāyana* longitudes) of the Sun and the Moon, let the Rsines of their declinations be determined as earlier.

In Fig. 6.2*a*, Γ is the vernal equinox, S the Sun, M the Moon and N_1 its ascending node. The meridian passing through the Sun meets the equator at G. If λ_s is the longitude of the Sun at S, then its declination is given by

$$R\sin\delta_s = R\sin\varepsilon\sin\lambda_s. \tag{6.3}$$

The secondary to the ecliptic passing through the Moon intersects the ecliptic at I. If λ_m and δ' are the longitude and declination of this point, then considering the triangle ΓIJ and applying the sine formula we obtain

$$R\sin\delta' = R\sin\varepsilon\sin\lambda_m. \tag{6.4}$$

It is the LHS of (6.3) and (6.4) that are referred to as Rsines of the declination (*krānti-jyā*) of the Sun and the Moon in the above verse. Though (6.4) does not give the actual declination of the Moon, which will be derived in the subsequent sections, it can be taken as a reasonable approximation when the latitude of the Moon is small. In fact, as we will see in the next section, the derivation of the actual expression for the Moon involves the declination of the point I given in (6.4) and that's precisely the reason for Nīlakaṇṭha's statement that it may be determined as earlier.

६.३ चन्द्रस्य इष्टक्रान्त्यानयने विशेषः

6.3 Speciality in the determination of the desired declination of the Moon

पातोनेन्दोर्भुजा जीवा परमक्षेपताडिता ।
त्रिज्याभक्ता विधोः क्षेपः तत्कोटिमपि चानयेत् ॥ ३ ॥
परमापक्रमकोट्या विक्षेपज्यां निहत्य तत्कोट्या ।
इष्टक्रान्तिं चोभे त्रिज्याप्ते योगविरहयोग्ये स्तः ॥ ४ ॥
सदिशोः संयुतिरनयोः वियुतिर्विदिशोरपक्रमः स्पष्टः ।
स्पष्टापक्रमकोटिर्द्युज्या विक्षेपमण्डले वसताम् ॥ ५ ॥
इत्युक्तात्र स्फुटा क्रान्तिः गृह्यतां गोलवित्तमैः ।

pātonendorbhujā jīvā paramakṣepatāḍitā |
trijyābhaktā vidhoḥ kṣepaḥ tatkoṭimapi cānayet || 3 ||
paramāpakramakoṭyā vikṣepajyāṃ nihatya tatkoṭyā |
iṣṭakrāntiṃ cobhe trijyāpte yogavirahayogye staḥ || 4 ||

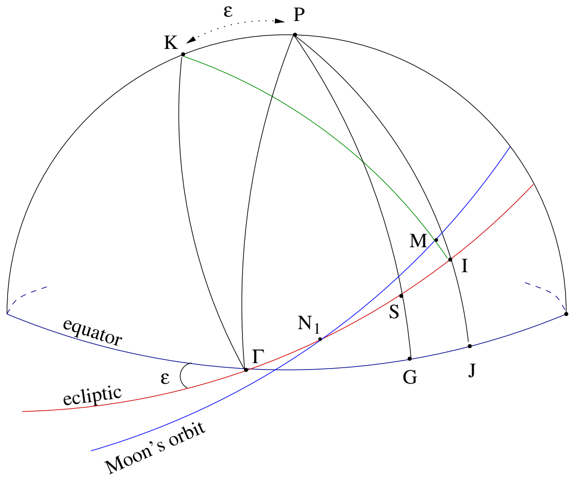

Fig. 6.2*a* Finding the declinations of the Sun and the Moon.

sadiśoḥ saṃyutiranayoḥ viyutirvidiśorapakramaḥ spaṣṭaḥ |
spaṣṭāpakramakoṭirdyujyā vikṣepamaṇḍale vasatām || 5 ||
ityuktātra sphuṭā krāntiḥ gṛhyatāṃ golavittamaiḥ |

The Rsine [of the longitude] of the node subtracted from [the longitude of] the Moon, multiplied by the maximum deflection [of the Moon's orbit] and divided by the *trijyā*, gives the latitude of the Moon (β). Let the Rcosine of it also be obtained.

Having multiplied the Rsine of the latitude of the Moon (the *vikṣepajyā*) by the cosine of the maximum deflection [of the ecliptic from the equator], and having multiplied the Rcosine of that (the latitude of the Moon) by the [Rsine of the] desired declination [of the Moon determined earlier], the two [products] divided by the *trijyā* are readily suited for addition or subtraction.

If these two are in the same direction then they must be added, and if they are in different directions then their difference must be found. [Now] the true declination [of the Moon is obtained]. The Rcosine of the true declination will be the day-radius (*dyujyā*) for those residing in the *vikṣepamaṇḍala*. Let the process of the [determination of] true declination [of the Moon] thus explained be understood by the experts in the spherics.

Considering the triangle PKM in Fig. 6.2*b* and applying the cosine formula, we have

$$\cos PM = \cos PK \cos KM + \sin PK \sin KM \cos PKM. \tag{6.5}$$

Let λ_m, β and δ_m be the longitude, the latitude and the declination of the Moon. Then

$$KM = 90 - \beta \qquad \text{and} \qquad PM = 90 - \delta_m. \tag{6.6}$$

P and K being the poles of the equator and the ecliptic, the arc $PK = \varepsilon$. Γ is the pole of the great circle passing through K and P. Therefore

$$\Gamma\hat{K}P = 90 \qquad \text{and} \qquad P\hat{K}M = 90 - \lambda_m. \tag{6.7}$$

Using (6.6) and (6.7) in (6.5) we obtain

$$\sin\delta_m = \cos\varepsilon\sin\beta + \sin\varepsilon\cos\beta\sin\lambda_m. \qquad (6.8)$$

This is the true declination of the Moon with latitude β. The *krāntijyā* of the Moon denoted by δ', and given by (6.4), is the declination of a point on the ecliptic which has the same longitude as the Moon (point I in Fig. 6.2*a*). It can be easily seen that the RHS of (6.8) reduces to the RHS of (6.4) when $\beta = 0$. Now, using (6.4) in (6.8),

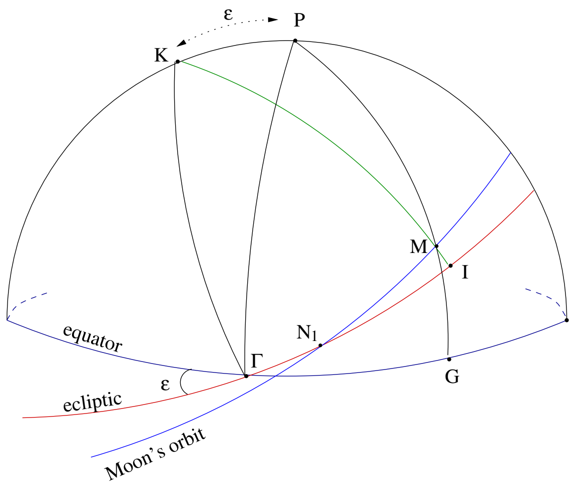

Fig. 6.2*b* Determination of the true declination of the Moon.

we have

$$\sin\delta_m = \cos\varepsilon\sin\beta + \cos\beta\sin\delta'. \qquad (6.9)$$

This is the formula for the true declination (*spaṣṭāpakrama*) of the Moon given in the verses, where it is stated in the form

$$|R\sin\delta_m| = \frac{R\cos\varepsilon\;|R\sin\beta|}{R} \underset{\sim}{+} \frac{R\cos\beta\;|R\sin\delta'|}{R}. \qquad (6.10)$$

The value of the latitude of the Moon (β) used in the above equation is found by using the formula

$$\sin\beta = 270 \times \sin\lambda_m, \qquad (6.11)$$

where the inclination of the Moon's orbit with the ecliptic is taken to be 270 minutes. Further, it may be noted from Fig. 6.2 that, when β and δ' are in the same direction, then the two terms are to be added, and if they are in different directions, that is, if $\sin\beta$ and $\sin\delta'$ have opposite signs, then $|\sin\delta_m|$ would be the difference of two positive terms. This is what is mentioned in the first half of verse 5 in the text, and is indicated by $\underset{\sim}{+}$ in (6.10).

In a passing remark the text also mentions that: 'The Rcosine of the true declination will be the day-radius (*dyujyā*) for those residing in the *vikṣepamaṇḍala*'. Here, the term *vikṣepamaṇḍala* refers to the declination circle of the Moon whose radius is given by

$$dyujy\bar{a} = R\cos\delta_m = \sqrt{R^2 - (R\sin\delta_m)^2}. \qquad (6.12)$$

६.४ इन्दोः प्रकारान्तरेण क्रान्त्यानयनम्

6.4 Determination of the declination of the Moon by another method

अथवा क्रान्तिरानेया परक्रान्त्या[3] विधोरपि ॥ ६ ॥
परमक्षेपकोटिघ्नं जिनभागगुणं हरेत् ।
त्रिज्यया क्षेपवृत्तेऽस्य नाभ्युच्छ्रय इहाप्यते ॥ ७ ॥
पातस्य सायनस्याथ दोःकोटिज्ये उभे हते ।
क्षिप्त्या परमया त्रिज्याभक्ते स्यातां च तत्फले ॥ ८ ॥
अन्त्यद्युज्याहतं तत्र कोटिजं त्रिज्यया हरेत् ।
नाभ्युच्छ्रये च तत् स्वर्णं मृगकर्क्यादि पातजम् ॥ ९ ॥
तद्बाहुफलवर्गैक्यमूलं क्रान्तिः परा विधोः ।
त्रिज्याघ्नं दोःफलं भक्तं तया चलनमायनम् ॥ १० ॥
जूकक्रियादिगे पाते स्वर्णं तत् सायने विधौ ।
तद्बाहुज्या हता क्रान्त्या तदा परमया स्वया ॥ ११ ॥
त्रिज्याप्तापक्रमज्येन्दोः स्फुटा तात्कालिकी भवेत् ।

athavā krāntirāneyā parakrāntyā vidhorapi || 6 ||
paramakṣepakoṭighnaṃ jinabhāgaguṇaṃ haret |
trijyayā kṣepavṛtte'sya nābhyucchraya ihāpyate || 7 ||
pātasya sāyanasyātha doḥkoṭijye ubhe hate |
kṣiptyā paramayā trijyābhakte syātāṃ ca tatphale || 8 ||
antyadyujyāhataṃ tatra koṭijaṃ trijyayā haret |
nābhyucchraye ca tat svarṇaṃ mṛgakarkyādi pātajam || 9 ||
tadbāhuphalavargaikyamūlaṃ krāntiḥ parā vidhoḥ |
trijyāghnaṃ doḥphalaṃ bhaktaṃ tayā calanamāyanam || 10 ||
jūkakriyadige pāte svarṇaṃ tat sāyane vidhau |
tadbāhujyā hatā krāntyā tadā paramayā svayā || 11 ||
trijyāptāpakramajyendoḥ sphuṭā tātkālikī bhavet |

Otherwise the [true] declination of the Moon may be obtained from its maximum declination. The Rsine of 24 (degrees) multiplied by the Rcosine of maximum inclination is divided by the *trijyā*. The quantity obtained is called the *nābhyucchraya* of the *kṣepavṛtta*.

The Rsine and the Rcosine of the *sāyana* longitude of the node, multiplied by the maximum deflection [of the Moon's orbit] and divided by the *trijyā*, will be those *phalas* [i.e. the

[3] In another reading of the text, we find the term स्फुटक्रान्ति instead of परक्रान्ति। That the latter is correct gets confirmed from the procedure and formulae given in the text. The commentator Śaṅkara Vāriyar has also adopted the reading परक्रान्ति।

> *doḥphala* and the *koṭiphala*]. Of them, the *koṭiphala* is multiplied by the Rcosine of the maximum declination of the Sun and divided by the *trijyā*. The result is added to or subtracted from the *nābhyucchraya* depending upon whether the [*sāyana*] longitude of the node lies within six *rāśis* beginning with *Mṛga* or *Karkaṭaka*. The square root of the sum of the squares of that and the *doḥphala* is the maximum declination of the Moon.
>
> The *doḥphala* multiplied by the *trijyā* and divided by that [i.e. the quantity obtained above] is defined as the *ayanacalana* [of the Moon]. This has to be added to or subtracted from the *sāyana* longitude of the Moon depending upon whether the node lies within six *rāśis* beginning with Libra (*Jūka*) or with Aries (*Kriyā*). The Rsine of that is multiplied by the maximum declination and divided by the *trijyā*. The result is the refined (*sphuṭā*) instantaneous [value of the] Rsine of the declination of the Moon.

An expression for the declination (δ_m) of the Moon which is similar to (6.3) is presented in the above verses. We may write such an expression as

$$\sin\delta_m = \sin I \sin\eta, \tag{6.13}$$

where $\eta = (\lambda_m - A)$; λ_m and A refer to the longitude and *ayanacalana* of the Moon. I represents the maximum declination of the Moon which keeps varying and depends upon the position of the Moon's ascending node along the ecliptic. It is also the inclination of the Moon's orbit with the equator. For instance, when the ascending node N_1 coincides with the vernal equinox, then the inclination of the Moon's orbit is

$$I = \delta_{max} = \varepsilon + i, \tag{6.14}$$

which is the same as the maximum declination attained by the Moon. On the other hand, when the ascending node coincides with the autumnal equinox then the inclination of the Moon's orbit is

$$I = \delta_{min} = \varepsilon - i. \tag{6.15}$$

Generally the value of the obliquity of the ecliptic, ε is taken to be 24° and the inclination of the Moon's orbit with the ecliptic, i, to be 4.5°.

From (6.13) it may be noted that the expression for the Moon's declination involves obtaining expressions for two intermediate quantities, namely

1. the maximum declination of the Moon in its orbit, which is called the *parā-krānti*, denoted by I, and
2. the right ascension of the point of intersection of the Moon's orbit and the equator. This is called the *ayanacalana* and is denoted by A.

The desired true declination of the Moon, denoted by δ_m, is expressed in terms of these quantities.

Expression for the *parā-krānti* and *ayanacalana*

The expression for the *parā-krānti*, in turn requires the defining of a few intermediate quantities. A term called the *nābhyucchraya* (x) is defined as

$$x = \frac{R\sin\varepsilon\, R\cos i}{R}, \tag{6.16}$$

Then the *doḥphala* (D) and the *koṭiphala* (K) are defined to be

$$D = \frac{R|\sin\lambda_n|\, R\sin i}{R},$$

$$\text{and} \qquad K = \frac{R|\cos\lambda_n|\, R\sin i}{R}. \tag{6.17}$$

We introduce yet another quantity (y), defined by

$$\begin{aligned} y &= R\cos\varepsilon \times K \\ &= \frac{R\cos\varepsilon\, |\cos\lambda_n|\, R\sin i}{R}. \end{aligned} \tag{6.18}$$

Using x and y, one more term (z) is defined to be

$$\begin{aligned} z &= x - y \qquad \text{when } 90 < \lambda_n \leq 270, \\ &= x + y \qquad \text{otherwise.} \end{aligned}$$

$$\text{Essentially,} \qquad z = x + R\cos\varepsilon\cos\lambda_n \sin i. \tag{6.19}$$

Now the *parā-krānti*, the maximum declination I of the Moon, is given as

$$\begin{aligned} R\sin I &= \sqrt{z^2 + D^2} \\ &= \sqrt{(R\sin\varepsilon\cos i + R\cos\varepsilon\sin i\cos\lambda_n)^2 + (R\sin\lambda_n \sin i)^2}. \end{aligned} \tag{6.20}$$

The *ayanacalana* (A) of the Moon is defined in terms of the maximum declination through the relation

$$R\sin A = \frac{R \times D}{R\sin I}. \tag{6.21}$$

This is also referred to as the *vikṣepacalana*.

Expression for the *iṣṭakrānti*

Having obtained the *ayanacalana*, it is added to the true longitude of the Moon when $180^\circ \leq \lambda_n \leq 360^\circ$, and subtracted from it otherwise. The Rsine of the result is multiplied by the Rsine of the maximum declination and divided by the *trijyā* to get the Rsine of the desired declination. That is,

$$\begin{aligned} R\sin\delta_m &= \frac{R\sin I \times R\sin(\lambda_m \pm A)}{R} \\ &= R\sin I \sin\eta, \end{aligned} \tag{6.22}$$

where η is the angle of separation between the Moon and the point of intersection of its orbit with the equator, along the orbit of the Moon. In the following we provide the rationale behind (6.20), (6.21) and (6.22) with the help of Figs 6.3*a*, 6.3*b* and 6.3*c*.

Derivation of the expression for the *parākrānti*

While the *Yuktibhāṣā* derivation of the expression for the *parākrānti* is given in Appendix E, here we derive the same using modern spherical trigonometry.

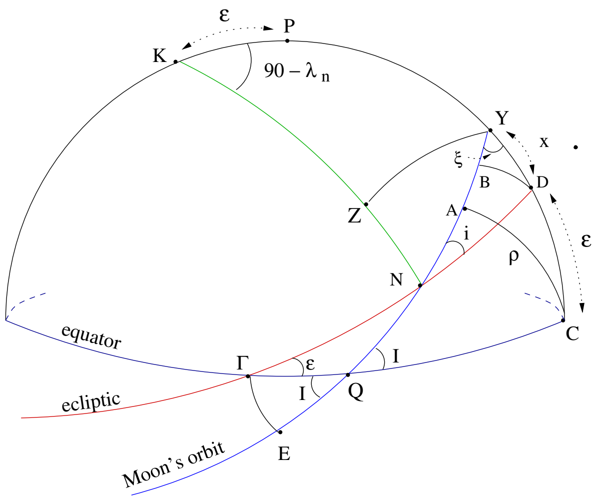

Fig. 6.3*a* Determination of the *parā-krānti*, the greatest declination that can be attained by the Moon at a given point in time.

In Fig. 6.3*a*, P is the celestial pole, K the pole of the ecliptic, Γ the vernal equinox and N the node of the Moon's orbit. Let I be the inclination of the Moon's orbit to the equator. Draw a great circle arc ΓE which is perpendicular to the Moon's orbit at E. Considering the triangle ΓEN and applying the sine formula, we have

$$\sin \Gamma E = \sin i \sin \lambda_n. \tag{6.23}$$

Here $\lambda_n = \Gamma \hat{K} N$ is the *sāyana* longitude of the node. Similarly, applying the sine formula to the triangle ΓEQ, we have

$$\sin \Gamma E = \sin I \sin \Gamma Q, \tag{6.24}$$

Hence

$$\sin I \sin \Gamma Q = \sin i \sin \lambda_n, \tag{6.25}$$

where I is the angle of inclination of the Moon's orbit with respect to the equator.

In the figure, C and D are points that are 90° away from Γ along the equator and ecliptic respectively. Let ρ be the arc from C, perpendicular to the Moon's orbit. Considering the triangle QAC, which is right-angled at A, and using the sine formula,

$$\begin{aligned}\sin\rho &= \sin I \sin QC \\ &= \cos\Gamma Q \sin I \qquad (\Gamma Q + QC = 90).\end{aligned} \tag{6.26}$$

Let $BD = \widetilde{K}$ be the arc from D, perpendicular to the Moon's orbit. Considering the triangle NBD, which is right-angled at B, and using the sine formula,

$$\begin{aligned}\sin\widetilde{K} &= \sin i \sin ND \\ &= \cos\lambda_n \sin i \qquad (\Gamma N + ND = 90).\end{aligned} \tag{6.27}$$

Let the Moon's orbit be inclined at an angle ξ to the prime meridian $KPYDC$. Let $YD = x$. Therefore, $YC = YD + DC = x + \varepsilon$. Now considering the triangles YBD and YAC and using the sine formula, we have

$$\sin\widetilde{K} = \sin x \sin\xi \qquad \text{and} \qquad \sin\rho = \sin(x+\varepsilon)\sin\xi. \tag{6.28}$$

Therefore,

$$\begin{aligned}\frac{\sin\rho}{\sin\widetilde{K}} &= \frac{\sin(x+\varepsilon)}{\sin x} \\ &= \frac{\sin x\cos\varepsilon + \cos x \sin\varepsilon}{\sin x} \\ &= \cos\varepsilon + \frac{\cos x}{\sin x}\sin\varepsilon.\end{aligned} \tag{6.29}$$

In the above equation, we would like to express $\frac{\cos x}{\sin x}$ in terms of other known quantities. From now on, all the intermediate steps till (6.35) are worked out for that purpose. Let $NY = \chi$ in the triangle NDY, which is right-angled at D. Using the sine formula, we have

$$\sin x = \sin\chi \sin i. \tag{6.30}$$

Let YZ be perpendicular to the secondary to the ecliptic passing through N. Considering the triangle NYZ which is right-angled at Z, we have

$$\sin YZ = \sin\chi\cos i. \tag{6.31}$$

Now $N\hat{K}Y = 90 - \lambda_n$. Further,

$$\begin{aligned}KY &= KP + PY \\ &= \varepsilon + (90 - (x+\varepsilon)) \\ &= 90 - x.\end{aligned} \tag{6.32}$$

Considering the triangle KYZ, which is right-angled at Z, we have

$$\begin{aligned}\sin YZ &= \sin KY \sin(90-\lambda_n) \\ &= \sin(90-x)\cos\lambda_n \\ &= \cos x \cos\lambda_n. \end{aligned} \tag{6.33}$$

From (6.31) and (6.33),

$$\sin\chi\cos i = \cos x \cos\lambda_n. \tag{6.34}$$

Replacing $\sin\chi$ in the above equation using (6.30), we have

$$\cos x \cos\lambda_n = \frac{\cos i}{\sin i}\sin x$$

$$\text{or} \qquad \frac{\cos x}{\sin x} = \frac{\cos i}{\sin i \cos\lambda_n}. \tag{6.35}$$

Using the above in (6.29), we obtain

$$\frac{\sin\rho}{\sin\widetilde{K}} = \cos\varepsilon + \frac{\cos i}{\sin i\cos\lambda_n}\sin\varepsilon. \tag{6.36}$$

Further, eliminating $\sin\widetilde{K}$ using (6.27) in the above equation, we have

$$\sin\rho = \sin i\cos\lambda_n\cos\varepsilon + \cos i\sin\varepsilon. \tag{6.37}$$

From (6.26) and (6.37), we get

$$\sin I\cos\Gamma Q = \sin i\cos\lambda_n\cos\varepsilon + \cos i\sin\varepsilon. \tag{6.38}$$

Now squaring and adding (6.25) and (6.38), we obtain

$$\sin^2 I = (\sin\lambda_n\sin i)^2 + (\sin i\cos\lambda_n\cos\varepsilon + \cos i\sin\varepsilon)^2. \tag{6.39}$$

Therefore,

$$\sin I = \sqrt{(\sin\lambda_n\sin i)^2 + (\sin i\cos\lambda_n\cos\varepsilon + \cos i\sin\varepsilon)^2}. \tag{6.40}$$

This is the formula for the inclination of the Moon's orbit to the equator presented in the text as given in (6.20), which is also the maximum declination of the Moon (at any given time). It is known that the nodes of the Moon's orbit complete one revolution in about 18.6 years. During that period, it could happen that the Moon's orbit lies in between the ecliptic and the equator as indicated in Fig. 6.3*b*. In such a situation, the expression for $\sin\rho$ in (6.28) will have $\sin(\varepsilon - x)$ instead of $\sin(x+\varepsilon)$. The effect of this in the final expression for the *parā-krānti* (maximum declination) would be

$$\sin I = \sqrt{(\sin\lambda_n\sin i)^2 + (\cos i\sin\varepsilon - \sin i\cos\lambda_{n'}\cos\varepsilon)^2}, \tag{6.41}$$

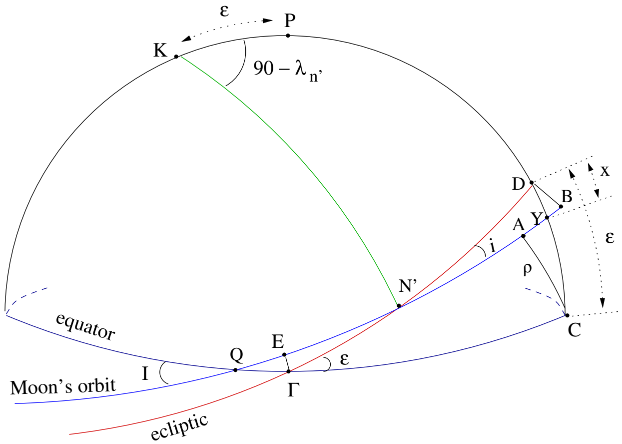

Fig. 6.3*b* Determination of the *parā-krānti*, when the Moon's orbit is situated between the equator and the ecliptic.

where N' is the descending node of the Moon's orbit and $\lambda_{n'} = \lambda_n + 180°$. Then it can easily be seen that the above equation is also the same as (6.20) given in the text.

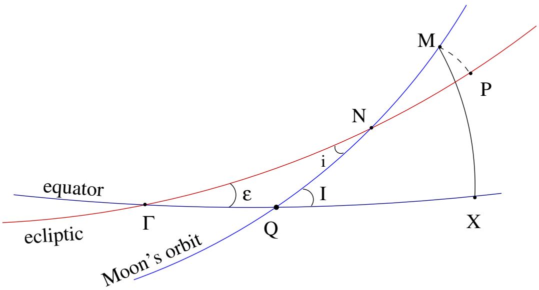

Fig. 6.3*c* Determination of the *iṣṭa-krānti*, the actual declination of the Moon at a given point in time.

In Fig. 6.3*c*, M represents the Moon and MX is its declination at a given instant. P is the point where the secondary to the ecliptic passing through M meets the ecliptic. Considering the triangle MQX, which is right-angled at X, and applying the sine formula,

$$\sin\delta_m = \sin MQ\,\sin I. \qquad (6.42)$$

Now

$$\begin{aligned} MQ &= MN + NQ \\ &= MN + \Gamma N + NQ - \Gamma N \end{aligned}$$

$$\approx NP + \Gamma N - (\Gamma N - NQ)$$
$$= \Gamma P - (\Gamma N - NQ)$$
$$\approx \lambda_m - \Gamma Q, \qquad (6.43)$$

where λ_m is the *sāyana* longitude of the Moon. In arriving at the above equation we have used two approximations:

1. $MN \approx NP$. This is a fairly good approximation since i, the inclination of the Moon's orbit is, very small.[4]
2. The other approximation is that $(\Gamma N - NQ) \approx \Gamma Q$. This again is reasonable as i is small.[5]

Applying the sine formula to the triangle ΓQN, we have

$$\sin \Gamma Q = \frac{\sin \lambda_n \sin i}{\sin I}. \qquad (6.44)$$

It may be noted that the above equation is the same as (6.21) presented by Nīlakaṇṭha, once we identify ΓQ with the *ayanacalana* A. Obviously the term *ayanacalana* in this context refers to the right ascension of the point Q.

Again, because i is small, we may write

$$MQ \approx \lambda_m - \Gamma Q = \lambda_m - A. \qquad (6.45)$$

Substituting for MQ in (6.42) we get

$$\sin \delta = \sin(\lambda_m - A) \sin I, \qquad (6.46)$$

which is the same as the expression for the declination (6.22) given in the text.

६.५ व्यतीपातस्य सदसद्भावः

6.5 The occurrence or non-occurrence of *vyatīpāta*

संस्कृतक्षेपचलनसायनेन्दोः रवेः पदात् ॥ १२ ॥
ओजयुग्मतया भेदे व्यतीपातो न चान्यथा ।

saṃskṛtakṣepacalanasāyanendoḥ raveḥ padāt || 12 ||
ojayugmatayā bhede vyatīpāto na cānyathā |

Only if the longitude of the Moon, corrected for the change in *vikṣepa* and *ayana* [as described earlier], is such that the Sun and the Moon lie in the odd and the even quadrants [or vice versa] does *vyatīpāta* occur and not otherwise.

The condition for the possibility of the occurrence of *vyatīpāta* or otherwise, that was hinted at in—and hence to be inferred from—verses 1 and 2a of this chapter, is

[4] It may be recalled that the inclination is taken to be $270' = 4.5^\circ$ in Indian astronomy.

[5] It needs to be verified numerically how good this approximation is.

being explicitly stated here. It is said that the Sun and the Moon must be in odd and even quadrants for the occurrence of *vyatīpāta*. In other words, the gradients with respect to the change in declination must have opposite signs during *vyatīpāta*.

The following verse in *Laghu-vivṛti* succinctly puts forth the criteria to be satisfied for *vyatīpāta* to occur:

क्रान्तिसाम्ये व्यतीपातो भवेद् भिन्नपदस्थयोः ।
नाभिन्नपदयोरर्कचन्द्रयोर्नाप्यतुल्ययोः ॥

Vyatīpāta occurs only when the declinations [of the Sun and the Moon] are equal and they are in different quadrants. And not when they are in the same quadrant or when their declinations are not equal [in magnitude].

We explain this with the help of Fig. 6.4. Here S refers to the Sun and ST its declination. M_1 and M_4 represent the Moon when it lies in the I and the IV quadrant respectively. We have depicted their positions such that

$$AM_1 = ST = BM_4. \tag{6.47}$$

In other words, the magnitude of the declination of the Moon at M_1 is same as that at M_4, which is also equal to that of the Sun. When the Moon is at M_1 there is no *vyatīpāta*, because the declination gradients of the Sun and the Moon have the same sign. On the other hand, when the Moon is at M_4 there will be a *vyatīpāta* since the gradients have opposite signs, and it is *vaidhṛta* since the Sun and the Moon lie in different hemispheres.

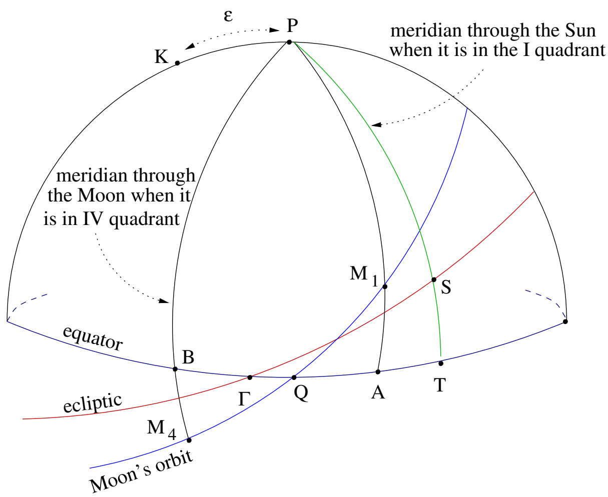

Fig. 6.4 Criterion for the occurrence of *vyatīpāta*.

For the sake of clarity and completeness we present in Table 6.1 all the different possible cases that could give rise to a *vyatīpāta*. The term 'quadrant', occurring as the heading of the first column of the table, has been given a special connotation that suits the present context. From this, the origins of the quadrants for the Sun and the Moon are taken to be the (ascending) points of intersection of their own orbits with the equator. They are referred to as *gola-sandhis*. In the case of the Sun it is the same as the vernal equinox, marked as Γ. The *gola-sandhi* of the Moon is marked by the point Q (see Fig. 6.4). This point moves at a much faster rate than Γ. It completes a cycle in about 18.6 years which amounts to about 20° per year.

Quadrant		Declination		*Ayana*		Nature of
Sun	Moon	Sun	Moon	Sun	Moon	*Vyatīpāta*
I	I	↑	↑	*uttara*	*uttara*	—
I	II	↑	↓	*uttara*	*dakṣiṇa*	*lāṭa*
I	III	↑	↑	*uttara*	*dakṣiṇa*	—
I	IV	↑	↓	*uttara*	*uttara*	*vaidhṛta*
II	I	↓	↑	*dakṣiṇa*	*uttara*	*lāṭa*
II	II	↓	↓	*dakṣiṇa*	*dakṣiṇa*	—
II	III	↓	↑	*dakṣiṇa*	*dakṣiṇa*	*vaidhṛta*
II	IV	↓	↓	*dakṣiṇa*	*uttara*	—
III	I	↑	↑	*dakṣiṇa*	*uttara*	—
III	II	↑	↓	*dakṣiṇa*	*dakṣiṇa*	*vaidhṛta*
III	III	↑	↑	*dakṣiṇa*	*dakṣiṇa*	—
III	IV	↑	↓	*dakṣiṇa*	*uttara*	*lāṭa*
IV	I	↓	↑	*uttara*	*uttara*	*vaidhṛta*
IV	II	↓	↓	*uttara*	*dakṣiṇa*	—
IV	III	↓	↑	*uttara*	*dakṣiṇa*	*lāṭa*
IV	IV	↓	↓	*uttara*	*uttara*	—

Table 6.1 The different possible cases for the occurrence of *vyatīpāta*.

६.६ व्यतीपाताभावनियमः

6.6 The criterion for the non-occurrence of *vyatīpāta*

अर्केन्द्वोः परमक्रान्त्योः अल्पा त्रिज्याहतान्यया ॥ १३ ॥
भक्ता ततोऽधिके बाहौ महाक्रान्तेर्न तुल्यता ।
तच्चापं भत्रयाच्छोध्यं तदाढ्योनायनान्तयोः ॥ १४ ॥
अन्तरालं गते तस्मिन् क्रान्त्योः साम्यं न जायते ।

arkendvoḥ paramakrāntyoḥ alpā trijyāhatānyayā || 13 ||
bhaktā tato'dhike bāhau mahākrānterna tulyatā |
taccāpaṃ bhatrayācchodhyaṃ tadāḍhyonāyanāntayoḥ || 14 ||
antarālaṃ gate tasmin krāntyoḥ sāmyaṃ na jāyate |

The lesser of the the maximum declinations of the Sun and the Moon is multiplied by the *trijyā* and divided by the other. If the Rsine of the greater is larger than the result, then there will be no equality.

The arc of that has to be subtracted from 90°. The result has to be added and subtracted from the *ayanāntas*. If 'that' lies in between, then the equality of the declinations does not take place.

Like many other verses in *Tantrasaṅgraha*, these have been written in a somewhat terse form and require a detailed explanation. The condition given here for the non-occurrence of *vyatīpāta* may be represented in the form

$$R\sin\lambda_{+} > \frac{R\sin\delta_{-} \times R}{R\sin\delta_{+}}, \tag{6.48}$$

where R represents the *trijyā*, δ_{+}/δ_{-} is the larger/smaller of the *paramakrāntis* of the Sun and the Moon, and λ_{+} the longitude of the Sun/Moon corresponding to δ_{+} (measured from the point of intersection of its orbit with the equator). If the above condition is satisfied then there will be no *vyatīpāta*. The maximum declination of the Moon depends upon the situation of the lunar orbit, which in turn is determined by the location of the Moon's nodes. It is worth while discussing the variation of the maximum declination quantitatively before we take up (6.48).

Variation in the maximum declination of the Moon

Let δ_s^* and δ_m^* be the maximum declinations of the Sun and the Moon. While the maximum value of the Sun's declination is fixed—and is equal to the obliquity of the ecliptic, $\varepsilon = 24°$—the maximum declination of the Moon δ_m^* is a variable quantity. Its value depends upon the position of the ascending node (*Rāhu*, denoted by N_1) of the Moon's orbit with respect to the equinox. The range of its variation is given by

$$(\varepsilon - i) < \delta_m^* < (\varepsilon + i), \tag{6.49}$$

where i is the declination of the Moon's orbit, which is taken to be 4.5° in the text. When *Rāhu* coincides with the vernal equinox, then $\delta_m^* = (\varepsilon + i)$. On the other hand, when it coincides with the autumnal equinox, then $\delta_m^* = (\varepsilon - i)$. The two limiting cases are depicted in Figs 6.5*a* and 6.5*b* respectively.

As the Moon's orbit itself has a retrograde motion, around the ecliptic, the node of the Moon's orbit completes one revolution in about 18.6 years. Hence the interval between these two limiting cases depicted in Fig. 6.5 is nearly 9.3 years. We now analyse the two cases from the viewpoint of the occurence of *vyatīpāta* or otherwise.

Case i: $\delta_m^* \geq \delta_s^*$

When the maximum declination of the Moon is greater than the obliquity of the ecliptic, then invariably the magnitude of the declination of the Moon becomes equal to that of the Sun four times during the course of its sidereal period (section 6.1). Of the four instants at which the declinations are equal, only two correspond to *vyatīpāta*. These two *vyatīpātas*, namely *lāṭa* and *vaidhṛta*, necessarily occur during the course of a sidereal revolution of the Moon.

In Fig. 6.5*a*, we have depicted the limiting case in which the Moon's orbit has the maximum inclination ($I = \varepsilon + i$) to the ecliptic. U and D represent the *ayana*-

sandhis[6] in the northern and the southern hemispheres respectively. M_1 and M_2 represent the positions of the Moon, in the *uttarāyaṇa* and *dakṣiṇāyaṇa* (northern and southern courses of the Sun), when its declination is equal to that of the obliquity of the ecliptic. Let t_m be the time taken by the Moon to travel from M_1 to M_2. During this interval, the declinations of the Sun and the Moon will never become equal and hence there can be no *vyatīpāta*. This is because the declination of the Moon during this period will be greater than that of the Sun. As has been stated in the text:

अन्तरालं गते तस्मिन् क्रान्त्योः साम्यं न जायते।

When it (the Moon) is in that interval, the declinations do not become equal.

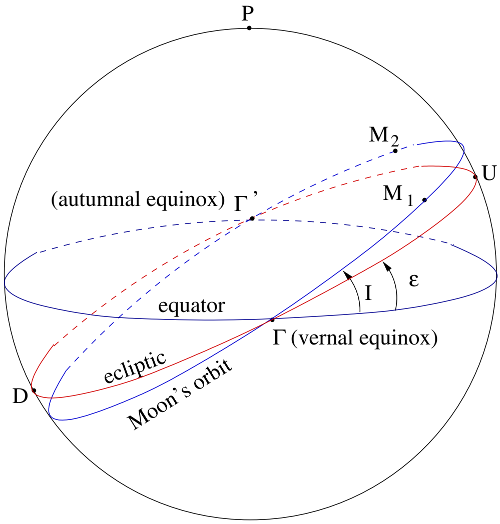

Fig. 6.5*a* Moon's orbit having the maximum inclination, $I = \varepsilon + i$, with the ecliptic.

Case ii: $\delta_m^* < \delta_s^*$

When the Moon's orbit lies completely in between the equator and the ecliptic, then, depending upon the longitude of the Sun, its declination could remain greater than the maximum declination of the Moon—which is the same as the inclination of the Moon's orbit with respect to the equator, denoted by I in Fig. 6.5*b*—for fairly long intervals of time. The said interval may extend even up to two to three months of time when the inclination I has the minimum value. During this period, the declination of the Moon doesn't become equal to that of the Sun and hence *vyatīpāta* does not occur.

In Fig. 6.5*b*, S_1 corresponds the position of the Sun when its declination is just equal to δ_m^*. As the Sun S_1 has northern motion, and is approaching the *ayanasandhi*

[6] The point of intersection of the two *ayanas*, namely the *uttarāyaṇa* and the *dakṣiṇāyana*.

U, its declination will be increasing during the next few days till it reaches the maximum ε. Having crossed the *ayanasandhi*, the Sun starts receding away from it and its declination starts decreasing. When the Sun is at S_2, again its declination will be equal to δ_m^*. Between S_1 and S_2, the declination of the Sun remains greater than δ_m^* and hence there will be no *vyatīpāta*.

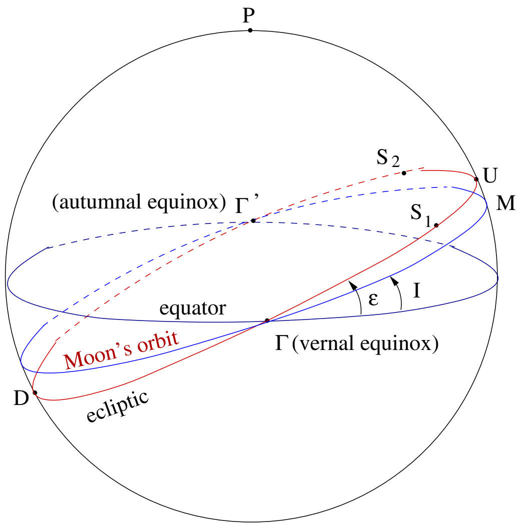

Fig. 6.5*b* Moon's orbit having the minimum inclination, $I = \varepsilon - i$, with the ecliptic.

However, the inclination of the Moon's orbit, which is the same as the maximum declination attained by the Moon, does not change significantly. The change in the maximum declination from $(\varepsilon + i)$ to the minimum value $(\varepsilon - i)$, a difference of $2 \times 4.5^\circ = 9^\circ$, takes place in about 9.3 years. This amounts to hardly one degree per year or around $5'$ per month, whereas the change in the declination of the Sun is around $480'$ per month. Hence, the change in the inclination of the Moon's orbit over a few weeks is negligible compared with that of the Sun. In the following, we make a rough estimate of the duration during which there will be no *vyatīpāta*.

Minimum period for which *vyatīpāta* does not occur

In the latter half of the 14th verse and the first half of the 15th verse, the criterion for the non-occurrence of *vyatīpāta* is given. From this, the minimum period during which *vyatīpāta* does not occur can be estimated. For numerical illustration, we choose the limiting case where the maximum declination of the Moon attains its minimum value as shown in Fig. 6.5*b*. In this case $\delta_m^* = 24.0 - 4.5 = 19.5$. The longitude of the Sun corresponding to this declination is

$$\lambda_s = \sin^{-1}\left(\frac{\sin 19.5}{\sin 24}\right) \approx 55^\circ. \tag{6.50}$$

As the longitude of the Sun increases in the odd quadrants, the magnitude of its declination also increases. Hence, when the longitude[7] of the Sun is approximately in the range

$$55^\circ < \lambda_s < 125^\circ,$$

or when it is in the range

$$235^\circ < \lambda_s < 305^\circ, \tag{6.51}$$

the magnitude of its declination will always be greater than the maximum declination the Moon can attain. Therefore, there will be no *vyatīpāta* during this period.

Since the rate of motion of the Sun is approximately 1° per day, under the limiting cases the minimum period for which a *vyatīpāta* does not occur is about 70 days. As the longitude of the Sun is 0° around March 21, this period approximately extends from the later half of the second week of May to the last week of July, when the Sun is in the northern hemisphere. When the Sun is in the southern hemisphere, this period would be from from the later half of November to the end of January approximately. With this background, we now proceed to explain the criterion given in the text.

Rationale behind Nilakaṇṭha's criterion for the non-occurrence of *vyatīpāta*

The declination of the Sun and its longitude are related through the formula

$$\sin\delta_s = \sin\varepsilon\, \sin\lambda_s, \tag{6.52}$$

where ε is the obliquity of the ecliptic, which is the same as the maximum declination of the Sun. In otherwords $\delta_s^* = \varepsilon$. The longitude of the Sun λ_s is measured from Γ along the ecliptic and is given by ΓS in Fig. 6.6.

The declination of the Moon is given by

$$\sin\delta_m = \sin I\, \sin QM = \sin I \sin\eta. \tag{6.53}$$

Here, I is the inclination of the Moon's orbit with respect to the equator. As in the case of the Sun, I is the maximum declination of the Moon. That is, $\delta_m^* = I$. $QM = \eta$ is measured along the Moon's orbit from the point of intersection of the equator and the Moon's orbit. We have seen that $\eta \approx \lambda_m - A$, where λ_m is Moon's longitude, and A is its '*ayanacalana*'. Dividing (6.52) by (6.53) and rearranging, we have

$$\sin\eta \times \frac{\sin\delta_s}{\sin\delta_m} = \frac{\sin\varepsilon}{\sin I} \times \sin\lambda_s. \tag{6.54}$$

[7] Since declination is involved, the longitudes that we talk about here are all *sāyana* longitudes.

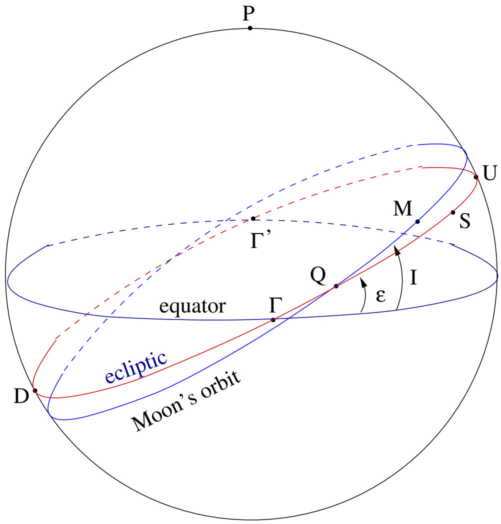

Fig. 6.6 Schematic sketch of the Moon's orbit and the ecliptic, when the maximum inclination of the Moon's orbit, I, is greater than ε.

Depending upon the position of the Moon's ascending node represented by Q in Fig. 6.6, either $\varepsilon > I$ or $\varepsilon < I$. The case $\varepsilon = I$ is true only at one instant, and is a very special case. The other two cases do prevail for an extended period of time. Now let us consider the case $\varepsilon < I$. *Vyatīpāta* occurs under these circumstances when

$$\sin\eta = \frac{\sin\varepsilon}{\sin I} \times \sin\lambda_s. \tag{6.55}$$

As $\sin\lambda_s \leq 1$, this implies that the condition for the occurrence of *vyatīpāta* is

$$\sin\eta \leq \frac{\sin\varepsilon}{\sin I}. \tag{6.56}$$

Hence there is no *vyatīpāta* if

$$\sin\eta > \frac{\sin\varepsilon}{\sin I}. \tag{6.57}$$

The above condition is the same as the one given in (6.48) once we identify that $\varepsilon = \delta_-$, $I = \delta_+$ and $\eta = \lambda_+$, as the maximum declination of the Sun is less than that of the Moon. Similarly, when $I < \varepsilon$, there is no *vyatīpāta* if

$$\sin\lambda_s > \frac{\sin I}{\sin\varepsilon}. \tag{6.58}$$

The equivalence of this condition with (6.48) is also clear once we identify that in this case $I = \delta_-$, $\varepsilon = \delta_+$ and $\lambda_s = \lambda_+$.

६.७ व्यतीपातास्य गतैष्यत्वनिर्णयः

6.7 Determining whether *vyatīpāta* has occurred or is yet to occur

दोर्ज्यां रवेः परक्रान्त्या हत्वा चान्द्र्या तया हरेत् ॥ १५ ॥
लब्धचापसमे चन्द्रबाहौ क्रान्तिगुणौ समौ ।
चन्द्रस्यौजपदस्थस्य दोर्धनुष्यधिके ततः ॥ १६ ॥
व्यतीपातो गतो न्यूने भावी युग्मपदेऽन्यथा ।
तदिष्टचन्द्रधनुषः स्वस्वभुक्तिघ्नमन्तरम् ॥ १७ ॥
गतियोगहृतं स्वर्णं दोषे गम्ये गतेऽपि च ।
सूर्येन्द्वोरन्यथा पाते तावत्कुर्यादिदं मुहुः ॥ १८ ॥
यावदर्कोत्थधनुषा तत्कालेन्दुधनुः समम् ।

dorjyāṃ raveḥ parakrāntyā hatvā cāndryā tayā haret || 15 ||
labdhacāpasame candrabāhau krāntiguṇau samau |
candrasyaujapadasthasya dordhanuṣyadhike tataḥ || 16 ||
vyatīpāto gato nyūne bhāvī yugmapade'nyathā |
tadiṣṭacandradhanuṣaḥ svasvabhuktighnamantaram || 17 ||
gatiyogahṛtaṃ svarṇaṃ doṣe gamye gate'pi ca |
sūryendvoranyathā pāte tāvatkuryādidaṃ muhuḥ || 18 ||
yāvadarkotthadhanuṣā tatkālendudhanuḥ samam |

Having multiplied the sine of the longitude of the Sun by its *parakrānti*, divide [that] by the *parakrānti* of the Moon. If the resulting arc [say x] is equal to the arc corresponding to the Moon (η), then the Rsine of the declination of the Sun and the moon will be equal.

If the arc corresponding to the Moon, in the odd quadrant, is greater than that (x), then *vyatīpāta* has already occurred; if less, then it is yet to occur. It is exactly the reverse [when the Moon is] in the even quadrant.

The difference of the arc corresponding to that ($R\sin x$) and that of the [*ayana*-corrected] Moon must be multiplied separately by their own [i.e. of the Sun and Moon] daily motions and divided by the sum of their daily motions. The results must be added to or subtracted from the Sun and the Moon depending upon whether the *doṣa* (a *vyatīpāta*) is yet to occur or has already occurred. In the case of the node it has to be applied inversely. The process has to be repeated till the arc obtained from the Sun becomes equal to the Moon's arc [found] at that time.

We saw earlier in (6.54) that the ratio of the declinations of the Sun and the Moon satisfy the relation

$$\frac{\sin\delta_s}{\sin\delta_m} = \frac{\sin\varepsilon\ \sin\lambda_s}{\sin I\ \sin\eta}. \tag{6.59}$$

Using the notation

$$\sin x = \frac{\sin\varepsilon}{\sin I} \times \sin\lambda_s \qquad \text{and} \qquad y = \frac{\sin\delta_s}{\sin\delta_m}, \tag{6.60}$$

(6.59) reduces to,

$$\sin\eta \times y = \sin x. \tag{6.61}$$

If $x = \eta$, the above equation implies that $y = 1$, that is, $\delta_s = \delta_m$. This is precisely the condition given here for the declinations of the Sun and Moon to be equal and is stated in the following words:

लब्धचापसमे चन्द्रबाहौ क्रान्तिगुणौ समौ ।

Though the term *cāpa* in general refers to arc, in the present verse it seems to have been used to refer to the sine of the arc. In other words, the term *labdhacāpa* in the above verse refers to $\sin x$. The term *candrabāhu* refers to $\sin\eta$. As mentioned earlier,

$$\text{if } \; x = \eta, \qquad \text{it is the middle of } \textit{vyatīpāta}.$$

The criteria as to whether a *vyatīpāta* has already occurred, or it is yet to occur are given by

$$\begin{aligned} &\text{if } \; x < \eta, \qquad &&\text{already occured,} \\ \text{and} \qquad &\text{if } \; x > \eta, \qquad &&\text{yet to occur,} \end{aligned}$$

in the odd quadrant. It is the other way round in the even quadrant, as $|\delta_m|$ decreases with time. Here η is the angular separation of the Moon from the point of intersection of the Moon's orbit and the equator. In Fig. 6.7 it is given by $QM_i = \alpha_i$ ($i = 0, 1$ and 2).

Rationale behind the given criteria

(a) Criterion for *vyatīpāta* to have occured

Suppose $x < \eta$ at some time $t = t_1$, then we should have $y < 1$ in order that (6.61) is satisfied. Now

$$y < 1 \qquad \Rightarrow \qquad \sin\delta_s < \sin\delta_m. \tag{6.62}$$

This situation is represented by the positions of the Sun and the Moon at S_1 and M_1 in Fig. 6.7. Since the Moon is in the odd quadrant and the Sun is in the even quadrant, the magnitude of the declination of the Moon keeps increasing and that of the Sun keeps decreasing. Since $|\delta_s| < |\delta_m|$ at $t = t_1$, there must be an earlier instant, $t = t_0$, at which $|\delta_s| = |\delta_m|$. This is precisely the condition for the occurrence of *vyatīpāta*. Thus we see that if $x < \eta$ and the Moon is in the odd quadrant, then *vyatīpāta* has already occurred.

(b) Criterion for *vyatīpāta* to occur later

If $x > \eta$, then from (6.61), $y > 1$. Now,

$$y > 1 \qquad \Rightarrow \qquad \sin\delta_s > \sin\delta_m. \tag{6.63}$$

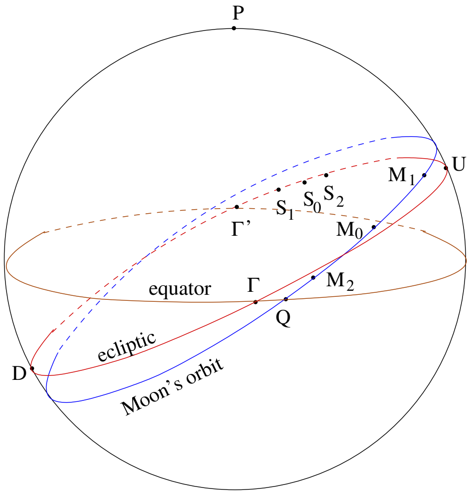

Fig. 6.7 Positions of the Sun and the Moon before *vyatīpāta*, at the instant of *vyatīpāta* and after *vyatīpāta*.

This situation is represented by the positions of the Sun and the Moon at S_2 and M_2 in Fig. 6.7. Again, since the Moon is in the odd quadrant and the Sun is in the even quadrant, the magnitudes of their declinations are increasing and decreasing respectively. Since at $t = t_2$, $|\delta_s| > |\delta_m|$, *vyatīpāta* is yet to occur at $t = t_0 > t_2$.

The situation in the even quadrants can be understood similarly. The above criteria are precisely those given in verses 16b and 17a to find out whether *vyatīpāta* has already occurred or it is yet to occur. In the succeeding verses 17b and 18, a procedure is given for finding the time interval (Δt) between the desired instant and the instant of *vyatīpāta*. Having determined this time interval, an iterative procedure for finding the longitudes of the Sun and the Moon at the instant of *vyatīpāta* is outlined.

The time interval between the desired instant and the middle of *vyatīpāta*

Let λ_s and λ_m be the longitudes of the Sun and the Moon a given instant t, and let the angular velocities (*gati*) of them at that instant be $\dot{\lambda}_s$ and $\dot{\lambda}_m$. It is seen from (6.60) that the quantity x is related to the Sun's longitude and η is related to the Moon's longitude. We denote the difference in arcs between x and η by $\Delta\theta$. That is,

$$x - \eta = \Delta\theta. \qquad (6.64)$$

The significance of $\Delta\theta$ is that it refers to the angle by which the sum of the longitudes of the Sun and the Moon must increase for *vyatīpāta* to occur. It is mentioned that this has to be divided by the sum of the angular velocities of the Sun and the Moon. We denote the result in time units by Δt, which is given by

$$\Delta t = \frac{\Delta\theta}{\dot{\lambda}_m + \dot{\lambda}_s} \qquad \text{(in days)}$$
$$= \frac{\Delta\theta}{\dot{\lambda}_m + \dot{\lambda}_s} \times 60 \qquad \text{(in } ghaṭikas\text{)}. \tag{6.65}$$

The longitudes of the Sun and the Moon at the middle of *vyatīpāta*

The changes in the longitudes of the Sun and the Moon during the above time interval Δt are obtained by multiplying their daily motions with it. That is

$$\Delta\lambda_s = \dot{\lambda}_s \times \Delta t \tag{6.66}$$
$$\text{and} \qquad \Delta\lambda_m = \dot{\lambda}_m \times \Delta t. \tag{6.67}$$

If λ_s and λ_{s_0} are the longitudes of the Sun at the desired instant t and the middle of the *vyatīpāta*, then

$$\lambda_{s_0} = \lambda_s \mp \Delta\lambda_s. \tag{6.68}$$

Similarly, if λ_m and λ_{m_0} are the longitudes of the Moon at the desired instant and the middle of the *vyatīpāta*, then

$$\lambda_{m_0} = \lambda_m \mp \Delta\lambda_m. \tag{6.69}$$

Here we take the sign '$-$' if the *vyatīpāta* has already occurred and the sign '+' if it is yet to occur.

Iterative method

In the procedures described in the previous sections, it has been implicitly assumed that the rates of motion of the Sun and the Moon ($\dot{\lambda}_m$ and $\dot{\lambda}_s$) are constant, which is not true. Hence both Δt and the longitudes λ_{s0} and λ_{m0} obtained are only approximate. As a corrective measure to this, an iterative procedure for determining the longitudes of the Sun and the Moon at *vyatīpāta* is prescribed. The iterative method to be used here is indicated in verses 18b and 19a.

तावत् कुर्यादिदं मुहुः यावदर्कोत्थधनुषा तत्कालेन्दुधनुः समम्।

This [process] has to be repeated till the arc of the Moon at that time will be equal to that of the Sun.

The method indicated above, and further explained in the commentary, may be explained as follows. As Δt given by (6.65) is not exact, we denote it by Δt_1 to indicate that it is the first approximation to the actual value. Having determined Δt_1 we evaluate x and η at time

$$t_1 = t + \Delta t_1, \tag{6.70}$$

and denote their values as x_1 and η_1. The rates of motion of the Sun and the Moon are also evaluated at t_1 and are denoted by $\dot{\lambda}_{s1}$ and $\dot{\lambda}_{m1}$. With them, we find Δt_2

given by

$$\Delta t_2 = \frac{\Delta \theta_1}{\dot{\lambda}_{m1} + \dot{\lambda}_{s1}}\,, \tag{6.71}$$

where $\Delta \theta_1 = x_1 - \eta_1$. The second approximation to the actual instant of *vyatīpāta* is t_2 and is given by

$$t_2 = t_1 + \Delta t_2. \tag{6.72}$$

Again at t_2 the values of x and η denoted by x_2 and η_2 are to be determined. From their difference $\Delta \theta_2$, and the rates of motion of the Sun and the Moon, Δt_3 is found. The process is repeated and, in general,

$$\begin{aligned} \Delta \theta_i &= x_i - \eta_i \\ \Delta t_{i+1} &= \frac{\Delta \theta_i}{\dot{\lambda}_{mi} + \dot{\lambda}_{si}} \\ \text{and} \qquad t_{i+1} &= t_i + \Delta t_{i+1}. \end{aligned} \tag{6.73}$$

The iteration is continued till $\Delta t_r \approx 0$. At this instant (t'), $x = \eta$ to the desired accuracy. Hence, the longitude of the Moon that is determined from η in this process, which in turn is determined by finding x, would be the same as the longitude determined at t' directly. This is what is stated in verse 19a, quoted above. The instant of *vyatīpāta* is then given by

$$t' = t + \Delta t_1 + \Delta t_2 + \cdots + \Delta t_r. \tag{6.74}$$

Here, it should be noted that Δt_r can be positive or negative.

६.८ व्यतीपातमध्यः

6.8 The middle of *vyatīpāta*

क्रान्तिसाम्ये व्यतीपातमध्यकालः सुदारुणः ॥ १९ ॥

krāntisāmye vyatīpātamadhyakālaḥ sudāruṇaḥ || 19 ||

When the declinations [of the Sun and the Moon] are equal, that instant corresponds to the middle of *vyatīpāta*, which is quite dreadful.

६.९ व्यतीपातप्रारम्भः पर्यवसानञ्च

6.9 The beginning and the end of *vyatīpāta*

नवांशपञ्चकं तत्त्वभागौ बिम्बौ स्वभुक्तितः ।
सूर्येन्द्वोर्बिम्बसम्पर्कदलं षष्ट्या निहत्य यत् ॥ २० ॥
गतियोगोद्धृतं तद्धि व्यतीपातदलं विदुः ।
व्यतीपातदले तस्मिन् नाडिकादौ विशोधिते ॥ २१ ॥

मध्यकालाद् भवेत् तस्य प्रारंभसमयः स्फुटः ।
तद्युते मध्यकालेऽस्य मोक्षो वाच्यो हि धीमता ॥ २२ ॥

navāṃśapañcakaṃ tattvabhāgau bimbau svabhuktitaḥ |
sūryendvorbimbasamparkadalaṃ ṣaṣṭyā nihatya yat || 20 ||
gatiyoguddhṛtaṃ taddhi vyatīpātadalaṃ viduḥ |
vyatīpātadale tasmin nāḍikādau viśodhite || 21 ||
madhyakālād bhavet tasya prārambhasamayaḥ sphuṭaḥ |
tadyute madhyakāle'sya mokṣo vācyo hi dhīmatā || 22 ||

The daily motion of the Sun multiplied by 5 and divided by 9, and that of the Moon divided by 25, are the diameters of the discs (*bimbas*) of the Sun and the Moon. Half the sum of the discs multiplied by 60 and divided by the sum of their daily motions is considered to be the half-duration of the *vyatīpāta*.

By subtracting the half-duration of the *vyatīpāta*, in *nāḍikās* etc., from the middle of the *vyatīpāta*, the actual beginning moment is obtained. By adding the same to the the middle of the *vyatīpāta*, the ending moment has to be stated by the wise ones.

If $\dot{\lambda}_s$ and $\dot{\lambda}_m$ are the daily motions of the Sun and the Moon, expressed in minutes, then the angular diameters of their discs α_s and α_m are given as

$$\alpha_s = \frac{\dot{\lambda}_s \times 5}{9}, \qquad \alpha_m = \frac{\dot{\lambda}_m}{25}. \tag{6.75}$$

Now the angular diameter of the Sun

$$\alpha_s = \frac{D_s}{d_s}, \tag{6.76}$$

where D_s and d_s are the Sun's diameter and its distance from the Earth in *yojanas* respectively. The horizontal parallax of the Sun (P), whose value is taken to be one-fifteenth of daily motion of the Sun, is given by

$$P = \frac{R_e}{d_s} = \frac{1}{15}\,\dot{\lambda}_s. \tag{6.77}$$

Using this in (6.76),

$$\alpha_s = \frac{D_s}{R_e}\,\frac{\dot{\lambda}_s}{15} = \frac{2D_s}{D_e}\,\frac{\dot{\lambda}_s}{15}, \tag{6.78}$$

where $D_e = 2R_e$ is the diameter of the Earth. In Chapter 4, the values of D_s and D_e are given to be 4410 and 1050.42 *yojanas* respectively. Therefore

$$\alpha_s = \frac{2 \times 4410}{1050.42 \times 15}\,\dot{\theta}_s = 0.5598\,\dot{\lambda}_s. \tag{6.79}$$

It is this 0.5598 that is approximated by $\frac{5}{9} = 0.5556$ in the text. Similarly, the angular diameter of the Moon is given by

$$\alpha_m = \frac{2D_m}{D_e} \times \frac{\dot{\lambda}_m}{15}, \tag{6.80}$$

where D_m is the Moon's diameter in *yojanas*. As D_m is given to be 315 *yojanas*,

$$\begin{aligned}\alpha_m &= \frac{2 \times 315}{1050.42 \times 15}\,\dot{\lambda}_m \\ &= 0.04\,\dot{\lambda}_m \\ &= \frac{\dot{\lambda}_m}{25}.\end{aligned} \tag{6.81}$$

Using the angular diameters, the half-duration of the *vyatīpāta* is found using the formula

$$\Delta t = \frac{S \times 60}{\dot{\lambda}_m + \dot{\lambda}_s}, \tag{6.82}$$

where S is the sum of the semi-diameters of the Sun and the Moon and is given by

$$S = \frac{d_s + d_m}{2}. \tag{6.83}$$

Let t_b, t_m and t_e be the actual beginning, the middle and the ending moment of the *vyatīpāta*. Here t_m refers to the instant at which (6.1) is satisfied. Then the beginning and the ending moments are given by

$$t_b = t_m - \Delta t \qquad \text{and} \qquad t_e = t_m + \Delta t. \tag{6.84}$$

६.१० विष्कम्भादियोगान्त्यार्धानां त्याज्यत्वम्

6.10 Inauspiciousness of the later half of *viṣkambhayoga* and others

विष्कम्भादिषु योगेषु व्यतीपाताह्वयोऽपि यः ।
तस्य सप्तदशस्यान्त्यमर्धं चाप्यतिदारुणम् ॥ २३ ॥

viṣkambhādiṣu yogeṣu vyatīpātāhvayo'pi yaḥ |
tasya saptadaśasyāntyamardhaṃ cāpyatidāruṇam || 23 ||

The later half of the seventeenth *yoga* commencing with *viṣkambha*, also known as *vyatīpāta*, is extremely inauspicious.

Analogous to the 27 *nakṣatras*, 27 *yogas* (see Table 6.2) are defined in Indian astronomy. They correspond to intervals of time during which the sum of the longitudes of the Sun and the Moon increases by $13°20'$. It may be noted from Table 6.2 that the 17th *yoga* is called *vyatīpāta*. Perhaps, hereby due to the similarity in name, this is also considered inauspicious (particularly its later half).

In this context the following verse is quoted in the commentary *Laghu-vivṛti*:

सूर्येन्दुयोगे मैत्रस्य परार्धं सम्भवेद्यदि ।
सार्पमस्तकसंज्ञः स्यात् तदा दोषोऽतिनिन्दितः ॥

Among the *yogas* of the Sun and the Moon, the later half of *Maitra* is called *Sārpamastaka* and that period is considered to be highly inauspicious.

1. *viṣkambha*	10. *gaṇḍa*	19. *parigha*
2. *prīti*	11. *vṛddhi*	20. *śiva*
3. *āyuṣmān*	12. *dhruva*	21. *siddha*
4. *saubhāgya*	13. *vyāghāta*	22. *sādhya*
5. *śobhana*	14. *harṣaṇa*	23. *śubha*
6. *atigaṇḍa*	15. *vajra*	24. *śukla*
7. *sukarma*	16. *siddhi*	25. *brāhma*
8. *dhṛti*	17. *vyatīpāta*	26. *aindra*
9. *śūla*	18. *varīyān*	27. *vaidhṛti*

Table 6.2 The names of the 27 *yogas*.

Note:

In the above verse, the term *maitrasya* literally means ‘ belonging to *Maitra*’. According to the tradition, each *nakṣatra* is associated with a deity. The deity for the 17th *nakṣatra*, namely *Anūrādha*, is *Maitra*. Hence the 17th *nakṣatra* is called *Maitra*.

While discussing *vyatīpāta*, Bhāskara I states:

सूर्येन्दुयोगे चक्रार्धे व्यतीपातोऽथ वैधृतः ।
चक्रे च मैत्रपर्यन्ते विज्ञेयः सार्पमस्तकः ॥ [8]

When the sum of the [*nirayaṇa*] longitudes of the Sun and the Moon is half a circle (i.e. 180°) it is *vyatīpāta*; when the sum is a full circle (360°) it is *vaidhṛta*. If [the sum] extends to the end of *Maitra* (*Anūrādha nakṣatra*) then it is to be known as *sārpamastaka* [*vyatīpāta*].

६.११ व्यतीपातत्रयाणां त्याज्यत्वम्

6.11 Inauspiciousness of the three *vyatīpātas*

व्यतीपातत्रयं घोरं सर्वकर्मसु गर्हितम् ।
स्नानदानजपश्राद्धव्रतहोमादिकर्मसु ।
प्राप्यते सुमहच्छ्रेयः तत्कालज्ञानतस्ततः ॥ २४ ॥

vyatīpātatrayaṃ ghoraṃ sarvakarmasu garhitam |
snānadānajapaśrāddhavratahomādikarmasu |
prāpyate sumahacchreyaḥ tatkālajñānatastataḥ || 24 ||

The [period of the] three *vyatīpātas* (*lāṭa, vaidhṛta* and *sārpa-mastaka*) is [considered to be] dreadful and is inauspicious for performing all religious rites. But by acquiring the correct knowledge of these periods and performing certain deeds such as having a holy dip, performing charitable deeds or sacrificial deeds, doing penance, oath-taking, performing *homa* etc. one reaps great benefits.

[8] {LB 1974}, (II. 29), p. 39.