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Summary

This volume presented the fundamentals of probability, parts of information theory, differential geometry, and stochastic processes at a level that is connected with physical modeling. The emphasis has been on reporting results that can be readily implemented as simple computer programs, though detailed numerical analysis has not been addressed. In this way it is hoped that a potentially useful language for describing physical problems from various engineering and scientific fields has been made accessible to a wider audience. Not only the terminology and concepts, but also the results of the theorems presented serve the goal of efficient physical description. In this context, efficiency means that the essence of any stochastic phenomenon drawn from a broad set of such phenomena can be captured with relatively simple equations in few variables. And these equations can be solved either analytically or numerically in a way that requires minimal calculations (either by human or computer). This goal is somewhat different than that of most books on stochastic processes. A common goal in other books is to train students of mathematics to learn how to prove theorems. While the ability to prove a theorem is at the center of a pure mathematician's skill set, the results that are spun off during that process sometimes need reinterpretation and restatement in less precise (but more accessible) language in order to be used by practitioners. In other words, rather than stating results in the classical definition–theorem–proof style aimed at pure mathematicians, this book is intended for mathematical modelers including engineers, computational biologists, physical scientists, numerical analysts, and applied and computational mathematicians.

A primary goal of mathematical modeling is to obtain the equations that govern a physical phenomenon. After that point, the rest becomes an issue of numerical implementation. In this volume many equations have been provided that can serve as potent descriptive tools. The combination of geometry, information theory, and stochastic calculus that is provided here can be applied directly to model engineering and biological problems. The numerous explicit examples and exercises make the presentation of what would otherwise be an abstract subject much more concrete. Additional examples can be found in the author's technical articles.

The emphasis here has been on continuous-time processes. This emphasis will continue in Volume 2, in which the topic of stochastic processes on Lie groups is addressed. The first three chapters in Volume 2 define, in a concrete way, the properties of Lie groups. These are special mathematical objects that have the benefits of both group theory and differential geometry behind them. Since a Lie group is both a group and a manifold, more detailed results about the theoretical performance of stochastic processes

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on Lie groups can be made than for abstract manifolds. In addition, numerical methods based on harmonic analysis (Fourier expansions) on Lie groups become possible.

Topics that received considerable attention in the current volume, but which were not directly applied to stochastic processes here, are used to a large degree in Volume 2. These include differential forms, Weyl's tube theorem, Steiner's formula, and curvature integrals over bodies and their bounding surfaces. It will be shown that such things play important roles in the area of mathematics known as *integral geometry* or *geometric probability*.

Many other topics are covered in Volume 2 including: variational calculus, Shannon's information theory, ergodic theory, multivariate statistical analysis, statistical mechanics, and Fourier methods on Lie groups.

The topics covered in Volume 2 are not the only ones that follow naturally from the background that has been established in this volume. For those readers with more theoretical interests, but who are not inclined to go on to Volume 2, plenty of pointers to the literature have been provided throughout this volume. The following references cover material not addressed in Volume 2 that will also be of interest: [1, 2, 3, 4, 5].

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