Chapter 9
From New Technological Infrastructures to Curricular Activity Systems: Advanced Designs for Teaching and Learning

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What is an "advanced design" of a technology for learning?

For some researchers, the word "advanced" may conjure images of the latest technology. Indeed, it is a common pattern in learning technology research to undertake design studies that investigate the learning potential of the novel technologies (Bell, Hoadley, & Linn, 2004; Barab & Squire, 2004; Dede, 2004). Often the long-term residue of this research lies in its contribution to learning theory (diSessa & Cobb, 2004; Edelson, 2002); contributions to large-scale practice tend to be short lasting and infrequently adopted (Roschelle & Jackiw, 2000).

We argue that the failure of much design research to contribute to large-scale practice emerges from a design flaw: designers fail to notice the infrastructural character of technology and form an unrealistic image of how infrastructure transforms classroom practice. We overestimate the power of technology alone and the proportion of teachers who can realize its potential without extensive guidance. Our minds too often race with thoughts of the power of technology to change classroom practice and underestimate the powerful set of forces in classrooms that conspire to marginalize technological potential (Kaput & Thompson, 1994).

In this chapter, we suggest a different meaning of "advanced design" that is arising in our long-term program of research and development within the SimCalc research program. We suggest that an "advanced design" should offer a plan for bridging the gap between new technological affordances and what most teachers need and can use. We draw attention to three different foci of design in two different SimCalc projects: (a) design of representational and communicative infrastructure (b) design of curricular activity systems, and (c) design of new classroom practices and routines.

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We particularly emphasize curricular activity systems because we are finding that attention to this focus of design has been critically important in our ability to measure learning outcomes at the scale of hundreds of teachers. (Classroom practices and routines are very important too, but research has yet to reduce the vast number of free parameters to a comprehensible design space for replicable classroom practices).

Our chapter begins by briefly reviewing the mission and progress of the SimCalc research program (Roschelle, Kaput, & Stroup, 2000). An important theoretical trend in the project has been the identification of its core technological aims as “infrastructural” (Kaput, Noss, & Hoyles, 2002; Kaput & Hegedus, 2007; Kaput & Schorr, 2008). We review the meaning of this term and the implausibility of jumping from infrastructural technology to scalable, robust effects in classroom practice. We then introduce the concept of a “curricular activity system” as a design emphasis that has emerged in our work on scaling up. To illustrate these concepts, we describe two different curricular activity systems at play within the research program; each supports a different classroom realization of the SimCalc vision (and is funded as a separate project). In closing, we recommend that researchers who aspire to “advanced designs” adopt a view that allows for focused work at the infrastructural, curricular, and classroom routine levels.

About the SimCalc Research Program

The mission of the SimCalc program is to “democratize access to the mathematics of change and variation” (Kaput, 1994). In a chapter in the prior book in this series (Roschelle et al., 2000), we argued that “change” will be a central phenomena of the twenty-first century and therefore that the mathematics of change and variation will become a centrally important strand of mathematics for all students to learn. We argued that the present “layer cake” approach to the mathematics curriculum, in which these important mathematical ideas are restricted to a Calculus layer that is icing on the layer cake of high school algebra, geometry and trigonometry layers is problematic. New approaches are needed and these approaches must introduce the mathematics of change and variation earlier, taking advantage of results in the learning science and the affordances of new technology.

The main software product of the SimCalc program is called SimCalc MathWorlds® (hereon referred to as MathWorlds) and is available at http://kaputcenter.umassd.edu. MathWorlds supports learning about rate and accumulation (Roschelle & Kaput, 1996) by connecting students’ experience of animated motion to mathematical functions, which are portrayed in algebraic, graphical, verbal, and tabular representations (Kaput & Roschelle, 1998). A distinctive feature of MathWorlds is that students can define piecewise linear functions graphically and then “execute” the functions resulting in observed motion in an animated “world.” The characters and background in the world can contextualize students’ experience within familiar experiences and can provide a setting in which mathematical phenomena have more meaning for learners.

As is the case with other "dynamic mathematics" products, such as The Geometer's Sketchpad, TinkerPlots, Fathom, and Cabri Geometre, the software design is strongly rooted in the nature of the mathematics and draws upon "direct manipulation" human–computer interaction paradigms to achieve executable, interactive visualizations of important mathematical concepts (Hegedus, Moreno, & Dalton, 2007).

Our earlier chapter (Roschelle et al., 2000) referred to the representational features of MathWorlds as emerging from a triangulation of perspectives on student learning, technological capabilities, and mathematical epistemology. From detailed development work on student learning, we focused on students' strong abilities to connect graphs to motions, their facility in reasoning intervals of time in graphs and motions, and the power of story telling to inform mathematics learning. The technical capabilities of MathWorlds include most importantly the ability to create "executable representations" (Hegedus, Moreno et al., 2007), representations that control animations and thus have easily perceived links between actions and consequence. Related to this, MathWorlds links representations dynamically ("hot links") so that when a student makes a change in one representation (e.g., increasing the slope of a position graph) they instantly see the corresponding change in another representation (e.g., the rate increases on a velocity graph). From the perspective of mathematical epistemology, the SimCalc team took an approach of "reconstructing subject matter" (Roschelle et al., 2000) – for example, by introducing piecewise functions much earlier in the curriculum, increasing the status and role of graphs (vis-à-vis more traditional algebraic symbols), and returning the phenomenology of motion to its historic place in the development of the mathematics of change and variation. Later, in the section on networked MathWorlds, we will see how these three perspectives were revisited and expanded with the incorporation of network connectivity as an infrastructural element.

The SimCalc research program and its software have been evolving over more than a decade of research, spanning at least eight major funded research projects. In order to contextualize what we have learned about "advanced design for learning technologies," it is worthwhile to recall where we began. In particular, in the early practice of SimCalc design and research, there was a rapid (one might even say feverish) interplay between levels we will soon define as separate. Jim Kaput, the project founder, might describe a new software feature to the developers one morning, write a new curricular lesson plan to exploit the feature that evening, and spontaneously engage in a new pedagogical practice with the lesson plan and feature in class the next day. When watching Jim work, it was easy to see the transformative potential of technology; Jim himself was a whirlwind of transformation that cut across his technological, curricular, and teaching practices. It goes without saying that Jim was fairly unique in this regard; it would be very unrealistic to expect most teachers to follow Jim's model. Further, because the SimCalc Project is deeply committed to scaling up, it has been important to figure out how to design for lasting and democratic access to the mathematical learning opportunities Jim so powerfully envisioned. This involved stabilization of opportunistic development without constraining the ability to generate new activities in the future. Such decision could be said to stabilize the element of the infrastructure.

Representational, Display, and Connectivity Infrastructure

As the SimCalc Project engaged with more and more teachers, Kaput and his colleagues came to articulate the role of the software as infrastructural, stating that their goal was:

> …to provide a framework that helps us to understand the gradual, manifold evolution of the roles of technology in mathematics education. The underlying idea is that changes over the long term amount to a process in which technology is gradually becoming "infrastructural." (Kaput & Hegedus, 2007, p. 173)

Their framework focused on the affordances of "ubiquitous forms of technology in schools" including graphing calculators, sensors and probes, laptop and desktop computers, and digital display technologies such as projectors. Indeed, these forms of technology are becoming fairly common in mathematics classrooms across the world. The SimCalc team also considered networking technologies to be on the cusp of becoming ubiquitous in classrooms such that teachers and students could instantly exchange mathematical objects appearing on their individual devices.

Kaput and colleagues were careful to distinguish between the raw materials and the capabilities that form an infrastructure. By analogy, roads and rails – not concrete and iron – form our transportation infrastructure. They viewed "infrastructure" as the foundational facilities needed for the functioning of a community, in this case, for the function of the classroom mathematical community. Three aspects of the technological infrastructure were highlighted (Kaput & Hegedus, 2007):

1. Representational infrastructure, which provides new ways for students to express, visualize, compute, and interact with mathematical objects.
2. Display infrastructure, which allows for both private (e.g., on a handheld) and public (e.g., projected) views of mathematical representations.
3. Connectivity infrastructure, which allows for rapid communication of mathematical objects among classroom participants and supports operations that distribute, collect, and aggregate student work.

Over time, new capabilities were added to MathWorlds software to integrate these three aspects. These capabilities assume that the students have a personal display and the teacher has a projected display. The new connectivity features of MathWorlds (Hegedus & Kaput, 2003; Kaput & Hegedus, 2002; Hegedus, Dalton, Moniz, & Roschelle, 2007) give teachers flexible capabilities to:

- Set up a classroom roster and cluster students into groups.
- Distribute a configured document to students, giving them a particular "setup" for an activity.
- Control which mathematical functions and representations can be viewed and edited by participants on their handheld or laptop devices.
- Collect (or have students submit) their work to the teacher's machine.

- Hide and show student contributions on the public display, often in meaningful clusters (e.g., by group or by the role in a group).
- Yield control of the main display to a particular students' device.

Each of these infrastructure elements is important because of its deep linkage to the architecture of learning. Display infrastructure is essential for creating shared attention to mathematical objects, and shared attention is a precondition for learning in any social setting, such as a classroom (Barron, 2000). Representational infrastructure is important because how people think (cognition) and come to know (epistemology) are deeply conditioned by the available representations (Kaput, 1992). Papert (2004), for example, has pointed out how difficult it would be to teach students multiplication if we still represented numbers using Roman Numerals; multiplication is much more tractable in the Arabic place value system. Connectivity infrastructure is important because it supports classroom discourse and participation (Hegedus, Moreno et al., 2007), and learning sciences researchers emphasize the importance of discourse and participation in students' development of mathematical meaning (Cazden & Beck, 2003; Cobb, Yackel, & McClain, 2002; Hicks, 1995).

Finally, it is important to note that the MathWorlds infrastructure supports the construction of more specific curricula by way of software documents. A document is a software file that users can "open" or "save." Documents configure all the elements of the infrastructure to enable the enactment of a particular activity while minimizing the amount of time teachers and students spend in preparation. Documents also avoid the need for teachers to master the full set of capabilities of the software, by presenting a narrower set of features, a set tuned to the specific learning goals of an activity. Documents give MathWorlds the advantages of a more open-ended tool (like a graphic calculator) while also appearing ready-at-hand to teachers, like very specific virtual manipulatives or applets.

The Character and Limits of Infrastructural Design Research

Educational researchers who want to study the "advanced design" of learning technologies face the challenge of justifying work that will not have immediate impact. Describing such work as aimed at infrastructure helps to set appropriate expectations. Research on new infrastructure is never undertaken for immediate benefit. Over the long term, however, infrastructural changes can yield sweeping transformations when cleverly exploited through additional layers of design and change in practice.

In infrastructural research with network capabilities, the SimCalc Project's core philosophy was to focus on mathematical content, asking: what types of mathematics can be discovered in new and innovative ways using classroom connectivity? This research attends to the principle that "technologies and tools co-constitute both the material on which they operate and the conditions, particularly social conditions, within which such operations occur" (Kaput & Hegedus, 2007, p. 173). Hence, design research clarifies the most fertile and generative aspects of the technology; less useful capabilities are pruned. Simultaneously, curricular targets are refined.

Infrastructural research can also have the theory-building character often attributed to design research (e.g., Edelson, 2002; diSessa & Cobb, 2004). Hence, SimCalc researchers have theorized about the importance of "identity" in connected classrooms (Hegedus & Kaput, 2004; Hegedus & Penuel, 2008; Kaput & Hegedus, 2002): students can project their mathematical object (and, hence, something that represents themselves) into a public display space. Vahey, Tatar, and Roschelle (2007) examined the importance of transactions between private spaces (only visible to a student) and public spaces (visible to the whole classroom). Stroup advanced the notion that classroom connectivity makes mathematics learning more playful and generative (Stroup, Ares, & Hurford, 2005).

A limit in infrastructural research is that its usefulness depends greatly on the skills and knowledge of teacher users, as well as the particular classroom routines those teachers are comfortable employing (Fishman, 2006). For a new infrastructure to result in transformation of how mathematics is taught in a society, teacher-user communities have to inhabit the infrastructure and fill it with the activities worth doing. They are unlikely to do so if the infrastructure clashes with the comfort zone of their classroom routines and mathematical knowledge.

Although we see the view of technology as infrastructure as empowering, we also worry about two design traps. The first trap is imagining that infrastructure itself will transform educational practice. The trap arises because infrastructure dramatically underspecifies what happens in a classroom among teachers and learners. This can result in false conclusions that the infrastructure "doesn't work" when in fact, a particular realization of classroom activity around the infrastructure did not work. Designers who stop at the infrastructure level, however, have very little control over the classroom realization of their intentions.

The second trap is relying on the availability of "reform-oriented" teachers, who presumably will be ready to tap the potential of new infrastructure. A vast amount of money is directed toward teacher professional development and one might imagine that once teachers have grown through this process, they will be ready to seize new infrastructural affordances and transform their classrooms. Unfortunately, this approach has problems. For instance, it is unclear that there is a single concept of a reform-oriented teacher; rather the goals of teacher professional development tend to loosely overlap around weakly specified beliefs, attitudes, and practices (Ball et al., 2009; Cohen & Ball, 1999). It is thus unlikely that coupling an infrastructure with a particular pool of "reform-oriented" teachers will result in a particular direction of transformation when new technological infrastructure becomes available. In addition, designing effective curricula is hard. Although it is true that a small percentage of teachers can design effective curricula, many more teachers lack either the time or skill to do so.

Thus, we argue that infrastructural design and research, alone, is unlikely to produce desired impacts across a wide variety of classrooms, even if teachers have been prepared through good quality but general-purpose teacher professional development.

The Need for Curricular Activity Systems

Kaput was fond of saying “new technology without new curriculum isn’t worth the silicon it’s written in” (Halverson, Shaffer, Squire, & Steinkuhler, 2006). Similarly, we find that teachers are increasingly attuned to the accountability demands of their environment. Simply put, new technologies must address the core curriculum or face certain marginalization. Infrastructures, however, are not particularly “curricular” in character; they may be designed to a view of the subject matter that transcends the peculiar “school” notions of mathematics of a particular educational regime. Such was the case with MathWorlds; it was designed to address the “mathematics of change and variation” which we argued was important mathematics even if it was not directly obvious in today’s school mathematics standards (Kaput & Roschelle, 1998). Curriculum is thus required to bridge the chasm between infrastructure and what teachers need (Ball & Cohen, 1996).

The word “curriculum” connotes either a framework of teaching objectives or a specific textbook that fulfills such a framework. As the work of the SimCalc Project has evolved, we have begun to design “curriculum” in both senses to complement the representational, display, and connectivity infrastructure, and also to bias teaching and learning with MathWorlds in the right directions.

We call the object of our design efforts a “curricular activity system.” In this phrase, the word “curricular” is meant to convey that we take seriously the need for a learning progression that addresses important mathematics. The progression has to occur over a meaningful number of instructional hours and cover mathematical constructs that lead the learner onward. We chose the word “activity” because the object of our design is not a “lesson” or a “presentation” or a “problem set” – the commonplace objects of curricular design. Instead we design activities that we intend teachers and students to enact and participate in. The responsibility for supporting such activities is distributed across software, paper curriculum, teacher guides, and teacher training workshops. We are appropriately cautious in realizing that we cannot control the exact enactment of an activity. By activity, thus, we do not mean the colloquial sense of “what students and teachers are doing,” but rather we think of an activity in terms of its objective (for the participants), available materials, the intended use of tools, the roles of different participants, and the key things we would like the participants to do and notice. Finally, we use the word “system” because our design aims to engineer an aligned set of related components that coherently support the desired curricular activities. Thus teacher training, curriculum materials, software documents, and so on are all designed together with a singular eye toward enabling classroom realization of our intended activities.

The need to design a curricular activity system has been emergent in our work, particularly as we have attempted to go from small-scale implementation of SimCalc designs (e.g., by Kaput himself or with a few teachers) to implementations involving tens and hundreds of teachers. Building on the work of David Cohen and colleagues (Cohen, Raudenbush, & Ball, 2003), we realized that an ambitious but weakly specified innovation would have little chance of success at scale. While some teachers might understand and implement our intentions, many others might distort the intended use of our infrastructure such that the intended learning gains become unlikely. Indeed, even within a curricular activity system, teachers do not "implement" classroom activities uniformly and unfortunate choices may occur (e.g., we had one teacher who decided that students did not need any hands-on experience with the MathWorlds software). Nonetheless our success in getting quantifiable results with curricular activity systems encourages us to think that this is an important target of design (Roschelle et al., 2007; Tatar et al., 2008).

The "target" of design follows from the learning science principles related to each infrastructural technology (see Table 9.1). Displays afford shared attention, but shared attention to what? We argue that a curricular activity system should afford shared attention to rich mathematical tasks. Hence, one facet of curricular activity system design should be the specification of rich mathematical tasks. We see the representational capabilities of technology as critical to emphasizing mathematical connections. These connections are (a) between students' prior knowledge and mathematical abstractions; (b) among representations of mathematics; and (c) forward and backward along learning progressions within mathematics. The design facet should therefore be knowledge building and learning progressions. Connectivity mediates participation and discourse, relating to a curricular activity system design facet that seeks to foster mathematical argumentation and participation in mathematical practices.

Below, we document examples of two curricular activity systems, each of which builds on the SimCalc technological infrastructure in different ways and is funded as a separate research project. The first draws on the representational and display infrastructure; the latter includes these infrastructural components and adds an emphasis on connectivity infrastructure. In both, we emphasize the rich mathematical tasks, the orientation to learning progressions and knowledge building, and the opportunities for mathematical argumentation and rich mathematical practices.

Example 1: Scaling Up SimCalc

The Scaling Up SimCalc research project investigated, through a randomized experiment, whether a wide variety of teachers could use SimCalc to support their students' learning of conceptually complex mathematics (Roschelle, Tatar,

Table 9.1 Design approach connects technological capability to research on learning

Technological capability	Design approach	Research on learning
Projected displays	Deep mathematical tasks	Enabling shared attention
Linked multiple representations, including animations	Learning progression from more experiential to more abstract mathematics	Emphasizing mathematical connections
Classroom connectivity	Overlapping social and mathematical structures	Engaging student participation in mathematical argumentation

Shechtman, & Knudsen, 2008). This project used MathWorlds in its computer software (not the graphing calculator application) form and did not use the connectivity infrastructure, as this was still under development. Because we were interested in scaling up to a wide variety of teachers, we planned to work simultaneously with over 100 teachers each year. To avoid the "assumption of reform teachers" pitfall, our materials needed to provide supports for teachers who were weak in some areas and they needed to be compatible with pedagogies considered "traditional" as well as "reform oriented." We expected that many of these teachers would be first time users of technology and that pedagogical styles would vary on the spectrum from "traditional" to "reform-oriented." We also wanted to avoid the design trap of under-specifying our intervention by relying too heavily on the representational infrastructure to carry the curriculum. So the complementary resources – student and teacher materials and professional development – had to do much of the specifying, while still providing rich mathematical tasks in which students could experience and be expressive with SimCalc's dynamic representations. These requirements led to the basic components of our curricular activity system: a 2-week replacement unit with a student workbook, brief teacher notes, and software files correlated to the workbook pages. We decided to focus on a replacement unit because replacement units are relatively easy to adopt and offer more breadth and depth than a single lesson. Professional development completed the Scaling Up curricular activity system. Just defining these components helped us in clearly specifying the experiment's "treatment."

Mathematics Content and Learning Progression

The SimCalc representational and display infrastructure has been tested in design experiments with mathematics content ranging in level from middle school through first-year university courses and including topics in algebra, trigonometry, precalculus and calculus courses. A first step in going from an infrastructure to a curricular activity system is to choose a more focused curricular target.

Consideration of the needs of our intended study participants, students and teachers in Texas, led to the selection of a curricular focus. Texas teachers needed materials that addressed their state's accountability requirements, were consistent with locally recommended practices, and used the technology available to them. In Texas' high-stakes testing environment, our curriculum needed to address important state standards – and not just any of the standards, but the ones teachers focused on in preparing students to pass the state test. Because many teachers were following rapidly paced instructional calendars, our unit needed to be short enough to fit in. Texas also has a diversity of students and so we needed to factor in considerations for their needs as well. For example, we needed to lower typical barriers for students who were learning English and for students with low reading levels.

Finding the best intersection of Texas needs and SimCalc offerings was not easy. A signature feature of earlier SimCalc work was exploring the representation of rate in both velocity and position graphs. Prior research gave us strong reasons to believe that we could produce a large learning gain by focusing learning on this SimCalc sweet spot. However, velocity graphs do not fit into Texas middle school standards; it would be hard to convince teachers to spend time teaching velocity graphs and hard to define a fair control condition. After much discussion, we discarded the idea of including velocity graphs and instead focused on a function-based approach to rate and proportionality.

Traditionally, rate and proportionality are taught as separate topics each clearly in the middle school "number" strand. Students are taught to choose appropriate values from a word problem to set up an equation of the form $a/b = c/d$. By filling in three numbers, a fourth can be found using cross multiplication. But implicit in this proportional relationship – and explicit in "rate" problems – is a rate of change which can define a function of the form $y = kx$ where k is the constant of proportionality. With this approach, students are preparing for entry into algebra and later on into calculus, where rates of change are a central topic and are treated algebraically. Moreover, this approach follows naturally from MathWorld's dynamically linked representations of objects in motion and their distances. Rates of change can be identified with slopes of lines that represent the object's speed. This approach leads to a qualitative comparison of different speeds, which can then support analysis of functions tied to their algebraic form.

Fortunately, education leaders in Texas were already advocating that teachers use a function-based approach to teaching proportionality. Texas leaders were also providing professional development, helping teachers consider the standard proportion word problems in a new light. So our curriculum and mathematical approach clearly helped Texas education leaders with one of their goals while remaining true to SimCalc's focus on the mathematics of change and the representation of rate in graphs, tables, equations, and narrative.

With our topic specified, we began to design a learning progression, beginning with simple motion and linear functions, and then developing rich tasks that could develop cognitively complex concepts and skills. The unit had two halves, united by a theme, *Managing the Soccer Team*. Within this theme, lesson-specific activities provided support for understanding the mathematics.

The first half addressed constant speed, comparing simple line graphs and their associated representations. The unit's first activity introduces a single character moving at a constant speed. The character's motion is linked to a graph of time versus position so that as the character moves, the graph builds – with the graph's steepness representing the rate of change or speed of the character. Over the next several activities, complexity develops: Students analyze graphs of characters moving at different speeds and from different starting positions; they recognize faster runs, earlier starting times and races that end in ties through their graphical representations. Students build from the connection between the graph and the situation, to include tables and equations. Culminating this first half, students are asked to translate among graphical, tabular, symbolic and narrative representations of functions of the form, $y = kx$, where time is x, a character's position at time x is y, and the character's speed is k.

The second half moved on to multirate linear functions, where characters in the simulation took on more interesting behavior – e.g., stopping, running backward, and then forward again – all controlled by piecewise linear graphs. The tasks in this half present more challenging mathematics – characters moving at different speeds in a single trip, represented by multisegment line graphs. It was at this point that we moved beyond the state standards for seventh grade. Students learn to interpret horizontal as a stopped motion and a negative slope as a "backward" motion. Through a set of problem-solving activities, students are asked to predict what a simulation or graph will look like, to check their prediction by running the software, and to explain the results in light of the prediction – a routine used across many SimCalc activities.

Overall Support for Teachers and Students

Going from an infrastructure to a curricular activity system requires providing much more support to teachers and students and aligning this support around the key mathematical ideas.

Several features of *Managing the Soccer Team* were aimed at helping a wide variety of teachers to implement the unit. In prior work, we found that teachers often use student materials as their main lesson guide. So we made sure that any crucial information for teachers was not buried in a teaching guide that they might never open. Instead, the student workbook prompts can serve as a kind of "script" for the lesson, though not a prescriptive one. The teacher, then, is not required to develop a sequence of questions and activities to support a learning progression that is likely new to her – but she is free to adapt, edit, and add to the lesson.

Although we did not count on teachers using them, our teaching notes provided simple lesson plans to complement the structure built into the unit – including a page-by-page guide for the "big" mathematical idea for each lesson. Suggested timelines helped teachers figure out how to complete the unit to fit their pacing chart requirements. Lesson planning documents helped teachers make a more detailed "map" of what they intended to do, including specifying what material in their regular curriculum they would "replace" with our unit. In addition, to help the teachers, the Texas standards covered by the unit were listed in the front of the teacher book.

Other features of the unit were designed to address the needs of a wide variety of students. Numbers used in the activities were, for the most part, realistic, so that students could use their knowledge of speed and prices in the real world to gage the correctness of their answers. The text used simple sentence structure and consistent vocabulary, never going beyond a fifth grade reading level, in order to accommodate those with low-level reading skills and to assist English learners in making sense of the context and the mathematics. To help guide and organize students' activities, the workbook used graphical conventions to indicate various kinds of activities and content. For example, definitions appeared inside boxes on the page, as did other critical content information. The amount of white space left after a question indicated the type and length of an expected answer. Simple graphics served as implicit indices for the activities. Even the fact that the workbook contained all the student activities physically bound together provided another organizational aid to students. Lastly, we used as much color as we could afford in reproducing thousands of workbooks, an attempt to appeal to the aesthetic sense of youth who live in a media rich – and colorful – world.

A 3-day teacher workshop was designed to support teachers' effective implementation of the unit. We did not try to change teachers' practices, but instead aimed at providing teachers with a mental image of the unit as a whole and a detailed experience of the unit as learners. The workshop used a standard "teacher-as-learner" approach, providing teachers with an opportunity to experience our intended activities for themselves. We modeled and highlighted use of "predict, check, explain" with students and encouraged teachers to let students use the software. In addition, the workshops provided time for teachers to practice and "play" with the software, to boost their comfort with computer technology. We gave particular focus to the "mathematics knowledge for teaching" that underlies and goes a bit beyond the mathematics students are to learn. Although teachers are familiar with the procedures for calculating using proportional relationships, many middle school teachers are less aware of the critical connections among proportionality and rate, the connections across representations, and how exploring proportionality can become a first step toward algebra.

Design Decisions: From Infrastructure to a Curricular Activity System

A comparison of a "traditional" SimCalc activity with an activity from the Scaling Up unit will illustrate some of our design decisions as we adapted core SimCalc activities for our Scaling Up curricular activity system.

"Sack Race" is a widely used SimCalc activity in which students are asked to create a graph that represents "an exciting sack race" and produce a narrative matching their graph. Using a MathWorlds file that has one character traveling at a constant speed over the course of the race, students create graphs for another character so that it slows down, speeds up, goes backward, and catches up – or any combination of these. By creating different graphs in the software, and trying them out in the simulation, students can explore how different parts of a "multirate linear function" affect the speed and direction of the character. Playfulness is encouraged in students' narration of their race. For example, as their character's race is played out, students often say something like: "Now he has fallen down and can't get up. Finally he struggles to his feet but takes off much more slowly than before."

The resources supporting this activity can be downloaded from the SimCalc website and include one MathWorlds file, an activity sheet and several pages of teacher notes. The original description of Sack Race was follows:

> This is our first 'performance' activity. Its primary focus should be on slope as rate of change and piecewise functions. This activity allows exploration of multiple types of slope; i.e., positive slope, negative slope, or zero slope for students to build their understanding of varying rate. This activity also allows for exploration of intersections of linear functions

> leading to an understanding of solutions to systems of equations. There is no one correct answer to this activity and students should focus on what conditions determine a correct answer…Students' creativity will set the tone for the discussion. You may choose specific students to display their graph and discuss their story one at a time. You want students to pick out the correlation between the action and the function. For example, if someone reads a story where his or her Actor stops, there should be a segment with a zero slope. Students will be excited to share even when their stories are incorrect, be sure to encourage a positive environment for corrections.

The original teacher notes for this SimCalc activity provide guidance in how to structure the lesson for this activity. There are three parts to the lesson: a whole class introduction, individual or group work time, and a whole class discussion of a sampling of students' work. For the introduction, teachers are told to "…decide on as much or as little detail as you wish for an introduction. You should at least introduce adding and manipulating segments to control Actor B's function…" For individual or group work time, the teacher is advised: "This is your opportunity to monitor group progress and determine what students are thinking and/or struggling with. Try not to answer questions directly, give students ways of using the motion to answer their questions."

Teachers create their own lesson plan aligned with this advice. Creating a lesson plan of this sort requires extensive teacher knowledge, some of which would likely be developed over time when using SimCalc materials. Just creating the demonstration MathWorlds file requires design decisions that invoke knowledge of mathematics, theories of learning, and knowledge of students' current levels of understanding. For example, one point of this lesson is that "backward" motion is represented in the graph by segments with negative slope. How should this idea figure into the whole class introduction? Should the teacher leave it out altogether so that students can discover it later? Should it be present in the teachers' demonstration MathWorlds file, without a lot of explicit discussion? Or should the teacher demonstrate and elicit an explanation of backward motion before students do their own work? If so, what is the right introduction? Should the teacher show a segment that is "slanting downward" and ask students what it could mean? Or should she show a backward motion and ask what it is?

In the Scaling Up unit, we include an activity with similar mathematical goals to those of "Sack Race." This activity, called "On the Road" (see Fig. 9.1), differs from the original by being substantially more structured for the teacher and her students. In "On the Road," students are presented with a series of trips between Abilene and Dallas, Texas. Each trip is made by bus and van and each trip is fraught with difficulties: bus breakdowns, forgotten items and bad traffic. The first problem in the activity, shown in Figs. 9.1 and 9.2, asks students to compare the trip of a bus and a van by comparing their graphs on a by-then familiar time versus position graph. This problem introduces a single object moving at two different speeds, but constrains the direction of motion to the familiar moving forward.

The next two problems introduce horizontal lines and then downward slanting lines (all without a formal definition of slope). Once these three ideas have been introduced sequentially, then students work in groups on problems of greater com-

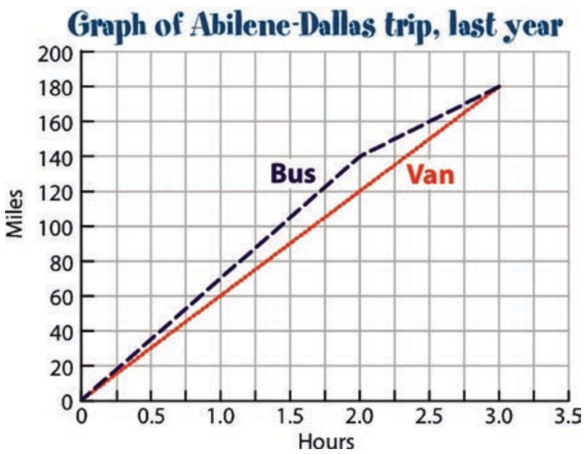


Fig. 9.1 A portion of the student workbook page for the "On the Road" activity

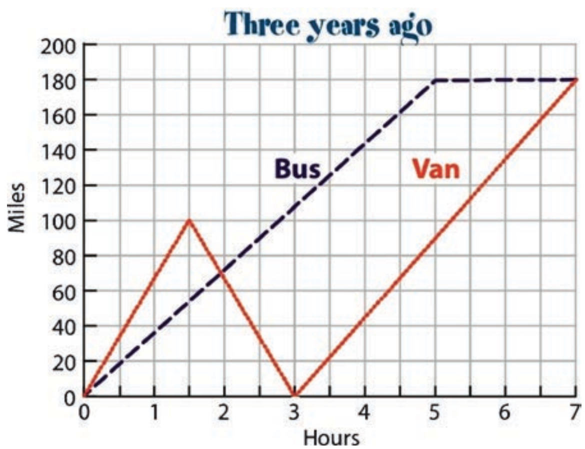


Fig. 9.2 More of the "On the Road" activity, from the student workbook

plexity, combining these different types of motion and their representation graphs. The most complex of these is shown in Fig. 9.2, below.

Relative to the original "Sack Race," we see that designing a curricular activity system involved several additional layers of specification. Both activities fit our notion of a "rich mathematical task," in that comparing the two motions draws forth a set of connected mathematical ideas about slope and rate. But whereas "Sack Race" leaves it to the teacher to find appropriate times and questions to address all the relevant mathematics, "On the Road" sequences the mathematics to start with a simpler situation and build toward the more complex situation. The sequence is structured to direct the teacher's and students' shared attention to relevant aspects of the task in a manageable progression. For example, question 4.a "What did the van do after traveling for one and a half hours?" directs attention to the contrast between positive and negative slopes.

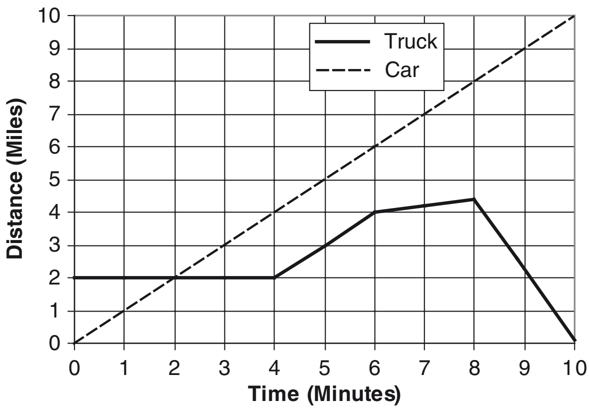
Both activities also engage multiple representations, including narrative, graphs, and motions. The “On the Road” version, however, specifically cues useful knowledge building practices. For example, question 2.a. asks the students to make predictions from the graph before they run it. Note that the question specifically asks students to think about “speed.” One problem with a rich context (e.g., the sack race) is that children tend to dwell on aspects of the story that are irrelevant to the mathematics. In the original, teachers are left to direct students to the mathematically relevant features; the curricular activity system version supports the teacher by including a specific prompt to look at speed. Further, in both questions 2 and 4, the prompts also support the SimCalc routine, “predict, verify, explain.” Student discussion is also specifically scaffolded by prompts in the teacher guide. Mathematical argumentation is encouraged by prompts in the teacher guide, by the questions asked of the students, and by the training that was given to teachers in the summer workshop.

Results

Overall, in our Scaling Up SimCalc project we collected data from 95 seventh grade teachers, half in the control group and half in the SimCalc group. We found statistically significant differences between classrooms that used SimCalc and those that used existing curriculum for the topic of rate and proportionality (Roschelle et al., 2007). Students in classrooms where teachers used SimCalc’s integration of curriculum, software, and professional development had higher learning gains. The gains were higher, specifically on the more advanced aspects of mathematical understand that SimCalc sought to cultivate; students learned about the same amount on the simpler mathematics measured on the Texas state test.

Our findings for both the seventh and eighth grade experiments are presented in detail elsewhere (Roschelle et al., 2007; Roschelle, Tatar, Shechtman, Hegedus et al., 2008). Here, we focus on the specific indicators of the success of the “On the Road” activity in the seventh grade curriculum. Teachers overwhelmingly rated “On the road” their favorite lesson in the unit. In postunit debriefing interviews, we also found that teachers frequently talked about this activity as a highlight. Further, we found high learning gains on test items that are closely related to “On the Road.” Consider the test item in Fig. 9.3

This item targets a common misconception – that the point of intersection on a graph is the time at which the objects are traveling at the same speed. Choosing answer B is an indicator that a student may have this misconception. The correct answer is C, because the objects have the same speed when their graphs have the same slope. On the pretest, only 23% of students got this item correct – and this is about the percentage that would be produced by random guessing. On the posttest, 55% of students who had been in classrooms using SimCalc got the item right, a statistically significant gain. In comparison, only 38.5% of students in non-SimCalc classrooms got the item right at posttest and more students (55%) chose the misconception-based distracter, answer B.



When are the car and truck traveling at the same speed?

A. Between 1 and 4 minutes

B. At 2 minutes

C. Between 4 and 6 minutes

D. Between 6 and 8 minutes

Fig. 9.3 A test item targeting the concept that parallel slopes (not an intersection point) indicate when two objects travel the same speed

On the basis of these results, we argue that the design of the curricular activity system around the "On the Road" activity worked with a wide variety of teachers and students.

Example 2: SimCalc Classroom Connectivity Project

Continuing in the learning progression described in Example 1 (e.g., proportionality and linear function), the next set of mathematical topics occurs in Algebra I (e.g., writing and manipulating linear functions, solving simultaneous equations, etc.). A look at SimCalc's "advanced technology" for Algebra I, however, affords more than a look at additional content because the team working on Algebra I also used new infrastructural capabilities. Consequently, this second example provides a look at a different embodiment of a curricular activity system. It is also different because students used graphing calculator hardware – not computers – connected by the TI-Navigator wireless network.

In addition to two infrastructural aspects of technology leveraged previously, representations and public displays, the SimCalc Classroom Connectivity Project (Hegedus, Dalton et al., 2007) leverages network connectivity among students' devices and the teacher's public display. Within-classroom networks are emphasized rather than connecting to outside-of-class resources over the Internet. Thus, the roots of SimCalc's approach to connectivity are more closely related to prior work on student response systems (a.k.a. "clickers") than to other forms of networked eLearning (Hegedus & Penuel, 2008). Like the work on clickers, the SimCalc Classroom Connectivity Project builds on the opportunity to connect students within a classroom so that they may respond in real-time to a teacher's queries and have their "responses" instantly (and often anonymously) collected and posted to a public display, where they become the focus of classroom discussion. As in certain response system work (Roschelle, Penuel, & Abrahamson, 2004), the major focus is on transforming classroom participation to dramatically increase students' roles in a meaningful classroom discourse. Unlike most of the work on clickers, however, which is content-neutral (the clickers accept only multiple choice responses), the SimCalc connectivity work depends on richer functionality to allow students to construct and contribute mathematical objects, not just select from predetermined multiple-choice responses. These capabilities lead to a *participation infrastructure,* permeated by mathematical considerations, that complements and extends the earlier representational infrastructure.

The rationale for attending to participation infrastructure draws upon the three perspectives (learner, technological, and epistemological) in parallel to the earlier rationale provide for representational infrastructure.

From a learner-centered perspective, the rationale for focusing on participation infrastructure aims to address the sense of alienation that many students experience in typical mathematics courses. Rather than feeling empowered by their inclusion in Algebra classes under the banner of "Algebra for All," many students experience Algebra as affirming their disenfranchisement from mathematics. This outcome, obviously, runs counter to the SimCalc mission of democratizing access to important mathematics.

Ethnographical studies of high school students (Davidson & Phelan, 1999; Phelan, Davidson, & Yu, 1998) reveal a world of alienation with strongly negative responses to standard practices (Meece, 1991) and strong sensitivity to interactions with teachers and their strategies (Davidson, 1999; Johnson, Crosnoe, & Elder, 2001; Skinner & Belmont, 1993; Turner, Thorpe, & Meyer, 1998). Negative responses, particularly as they are intimately connected with self image and sense of personal efficacy, can be deeply debilitating, both in terms of performance variables (Abu-Hilal, 2000) as well as in the ability to use help when it is available (Harter, 1992; Newman & Goldin, 1990; Ryan & Pintrich, 1997). On the other hand, students exhibit consistently positive responses to alternative modes of instruction and content (Ames, 1992; Boaler, 2002; Mitchell, 1993). A recent review by the National Mathematics Advisory Panel (Geary et al., 2008) focused attention on the connection between participation structures and the achievement gap. Evidence suggests that Black and Hispanic students learn particularly well in classrooms that stress a more communitarian outlook and in which they experience the teachers as caring about each child personally (e.g., Fullilove & Treisman, 1990; Ladson-Billings, 1995). More interactive and social classroom participation structures, thus, are seen as a potentially important tool for closing achievement gaps.

A complementary epistemological perspective attends more strongly to mathematics as constructed socially, through argumentation. A watershed event for this perspective with respect to the infrastructure of network connectivity occurred at a 2002 Psychology in Mathematics Education, North America meeting where Stroup led a symposium on the potential interplay of mathematical and social spaces in a connected classroom (Stroup et al., 2002). The symposium emphasized the idea of aligning how students belong to the classroom as a social collective with how each student's mathematical contributions belong to a higher order mathematical object. For example, each student can contribute a single point that fits the equation $y=2x$ and their collective construction, graphed on the shared public display, will be a line with a slope of 2. Likewise, each student can contribute a function in the form $y=2x+b$ with a different value for b and their collective construction, graphed on the shared public display, will be a family of mathematical functions, all with a slope of 2 and parameterized by variation in the y-intercept. An emerging epistemological design principle, then is to overlay mathematical variation onto the social structure of the classroom – mathematically coherent displays arise from socially coherent individual participation. The contrast to the earlier clicker-based approach could not be more stark: clickers emphasize an epistemology of consensus on the right answer; the newer capabilities emphasize the dialectic emergence of coherent mathematical constructions through social argumentation about mathematics that arises from systematically varied individual contributions. Just as mathematical considerations permeate the design of the representations in the earlier version of MathWorlds, mathematical considerations permeate the design of the social infrastructure in the connected version of MathWorlds. A classroom experience that interconnects the social and the mathematical has the potential for increasing students' sense of identity, agency, and belonging because their mathematical contributions remain identifiable in the collective, carry their agency, and belong to the larger group construct.

Technologically, therefore, the right infrastructure needs to do more than collect and display students' mathematical contributions in juxtaposition. In particular, it must make the contributions part of a collective mathematical construction. The newer versions of MathWorlds add a few key features to accomplish this. First, connected MathWorlds provides facilities to collect student contributions into a common motion animation. For example, if each student contributed a function $y=2x+b$, the common animation might look a parade of characters moving at the same speed but with different starting positions. Second, the representational contrasts can be spread out socially in the classroom, for example, so that students contribute a function using Algebraic symbols but see their contribution expressed on the public display as a graph or an animation. Hence, the principle of multiple representations (Goldenberg, 1995) becomes socially distributed in a networked classroom rather than distributed over adjacent windows on the same display. Third, it provides tools for hiding and showing coherent collectives of student work, so that the teacher can focus on comparing and contrasting student work in ways that focus on the relevant mathematics.

Per our central argument, this participation infrastructure remains just that – an infrastructure. And thus it would be unlikely to see transformation in classrooms, even with reform-oriented teachers, without working toward an "advanced design" – that is a design which leverages the new infrastructure in appropriate curricular activity systems.

Algebra I Curricular Activity System Overview

The Classroom Connectivity (CC) curriculum activity system for Algebra I builds upon the lessons learned from the seventh and eighth grade work reported in Example 1. As in Example 1, the mathematical learning progression was designed with close attention to the overlap between state standards and MathWorlds capabilities. In this case, the target state was Massachusetts and about half the topics in the Algebra I standards for Massachusetts were directly amenable to a SimCalc focus, so these conventional topics became the focus of the learning progression. The learning progression built on the succession of topics found in textbooks used in Massachusetts, such as the progression from graphing linear equations to writing linear equations and to solving systems of linear equations. Also, as in the case with the Texas work, the "system" included a student curriculum book, a teacher edition and teacher professional development, with many of the same considerations as discussed earlier.

One new and important element in the CC Algebra I materials is the inclusion of more extensive dialog prompts in the teacher edition. Teachers are deliberately guided to engage their classroom with questions that focus on (a) the activity (b) the connections among multiple representations and (c) the central mathematical ideas of key lessons. This practice aligns with the enhanced emphasis on argumentation in the networked SimCalc classroom. Because the logic of the learning progression and the teacher supports has already been discussed in Example 1 – and not to diminish the role of this logic in the overall curricular activity system – we focus Example 2 on three kinds of activity structures that have emerged to leverage the connectivity infrastructure and support the epistemological alignment of mathematical and social structure in classroom enactment.

The "Where Am I?" Activity

In this activity structure (Hegedus & Kaput, 2002), each student privately constructs a mathematical function in one representation (e.g., an expression for a linear function) and then contributes their function to a collective class representation. Students then participate as a group in trying to find their mathematical objects in a public, collective representation (e.g., an animation where each dot moves according to a student contribution).

Early SimCalc investigations revealed a powerful drive for students to "find themselves" in the collective animation, where students see and talk about contributed functions as extensions of personal identities. Thus, students' attention is drawn to the collective display. Further, the only way to self-identify is to pay attention to the mathematically relevant attributes of the animation, such as the start position and speed of a moving dot. In a classroom discussion, students might, for instance, notice that there is a group of dots that all move at the same speed. The teacher could hide all the other dots and focus on these. Using MathWorlds capability to leave "marks" at positions that are 1 second apart, the class could work to quantify the speed of this subset of dots and discuss which variable, *m* or *b*, is related to the speed. This creates a trajectory from (a) self-identification with a mathematical object to (b) focus of attention on mathematically relevant attributes of the animation and onward to (c) the connection between the perceptual attributes of a motion and the mathematical abstractions of slope and intercept. Hence, instantiating systematic variation socially in the curricular activity system becomes the organizing structure that allows for classroom enactments that personally engage each student, focus their attention on the relevant public mathematics, and move from personal to more abstract mathematics.

Research analyses of SimCalc activities have conceptualized “identity as a form of mediated action” to capture this phenomena (Hegedus & Penuel, 2008). In particular, classroom conversations have many deictic references that connect a person’s name and a public mathematical object. The aspect of identity highlighted here is less one’s attributions to self in relationship to community (a broader, more cultural view of identity) and more one’s projections of self in relationship to the microcommunity constituted in the classroom. One might think of this as a small “i” form of identity that could nonetheless powerfully contribute to a big “I” form of Identity as an adult who feels ownership of the cultural tools of algebra. Indeed, one case study tracked “X” a student who was initially invisible in class, but who became “famous” through the public activity of tracking down his unique identity in the animation, and consequently became a more frequent and vocal participant in the classroom (Kaput & Hegedus, 2002).

Parameterized Variation Activities

A second genre of networked SimCalc activities systematically organizes mathematical variation in a classroom so that the collection of functions submitted by students form a family of functions (Hegedus & Kaput, 2004; Hegedus & Kaput, 2003; Hegedus & Kaput, 2002; Kaput & Hegedus, 2002). These activities rely on a surprisingly simple and robust social idiom, called “counting off” in the United States. To “count off,” each student announces a successive number, thus claiming that number. Working in groups of 4, for example, students can count off numbers from 1 to 4. In some SimCalc activities, groups are also assigned numbers, such that each student has a unique pair of numbers, a group number and a count-off number. Once students have their numbers, a teacher can begin a networked SimCalc activity by asking each student to make a function that uses these uniquely assigned numbers (Fig. 9.4).

One particularly profound use of this capability contrasts functions with the same slope versus functions with the same *y*-intercept. Imagine that each student has made a unique linear function $f(x) = g \times x + c$, where *g* and *c* are a student’s group and count-off numbers, respectively. Using simple networked SimCalc capabilities, a teacher can collect these functions, display them in a graph, and animate moving actors according to them. Further, the teacher can choose settings

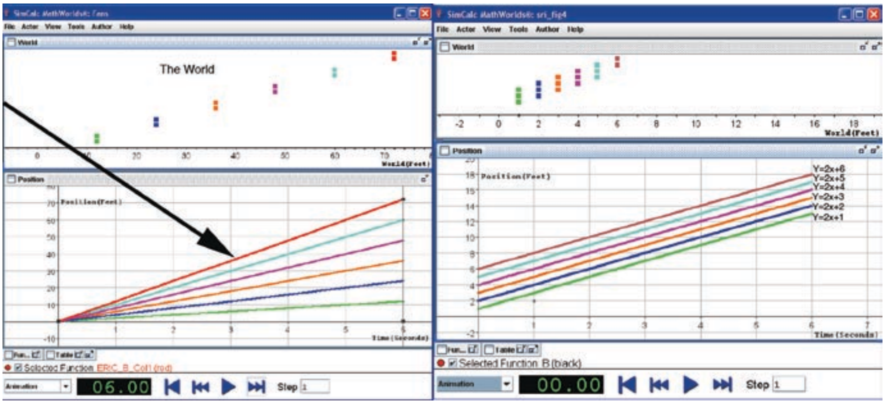


Fig. 9.4 Contrasting lines with the same *y*-intercept versus lines with the same slope

that highlight all students with an equal group number or all students with an equal count-off number.

In classroom practice, these capabilities are deployed with opportunities for cycles of prediction, reflection, and feedback. For example, before showing a graph of all the functions with the group number (and hence, slope) of 2, the teacher should ask students to predict what the graph will look like. Likewise, before animating these graphs as moving characters, the teacher should ask students to predict what the motion will look like. Further, when collecting students' work it is rare that everyone in the classroom made the correct function for their assigned numbers. Hence, cycles of reflections and feedback are appropriate.

The visual and animated results of family-of-function constructions are quite striking and memorable. A collection of functions with the same slope but different *y*-intercepts appears as parallel lines and the motion looks like a series of actors following each other in lock-step (because they move at the same speed). In contrast, a collection of functions with the same *y*-intercept, but different speeds, appears as a "fan" or "spray" of lines emanating from a point. The characters start at the same place but spread apart because they move at different speeds. Indeed, research data show that students remember and recall these patterns. A "fan" can become an iconic representation for "different slopes, same intercept" and parallel lines can become an iconic representation for "same slope, different intercepts." Likewise, "spreading apart" and "marching in lock-step" can become easily remembered correlates of these icons (Piaget, 1970, we note, theorized that "speed" arises as a concept for children in relationship to their perception of "catching up" and "spreading apart" behaviors. Hence, these motion representations are likely deeply connected to the relevant everyday concepts). For example, the SimCalc CC project reports the case study of Erin, who says "we're sandwiching at 12 feet" when asked to predict what would happen if all students make functions with different slopes that intersect at the point (6,12). The same report shows four instances of students' spontaneous gestures in class, using their fingers or other line-like objects to show spread or parallelism.

These mnemonics are important because students have difficulty remembering the different meanings of m and b in $y=mx+b$. Increasing either m or b can be superficially described as "moving up" which misses the difference between "tilting up" and "shifting up." Notice what has happened here, relative to earlier generations of dynamic graphing software. In early generations of software, a mouse drag could be used so that an individual student could see how a changing parameter affected a single line. Now, with networked MathWorlds, a classroom can see how systematic variation in a parameter results in a family of functions. Instead of being contained within an individual eye-hand coordination loop, a group now participates in coordinating the production of a family of functions. Further, social parameterization produces an iconic figure and animation, making it easier (we think) for students to remember the different meanings of "m" and "b." Research within the SimCalc CC Project has begun to quantify the impact of socially distributed parameterization by tracking the increased number of student-to-student conversation sequences, relative to student-to-teacher sequences (Hegedus & Penuel, 2008). Student-to-student conversation sequences would likely increase when variation is distributed between students, rather being contained within the control of the only student with the mouse.

Mathematical Performance Activities

A third kind of activity, a mathematical performance, engages students in using mathematics expressively (Hegedus & Kaput, 2004; Hegedus & Kaput, 2003; Hegedus & Kaput, 2002; Kaput & Hegedus, 2002). Working privately at first, students create a personally meaningful use of mathematics to express a story. The students can then send their mathematics to the teacher via the connectivity infrastructure. The teacher can choose particular students to perform (e.g., tell the story) of their mathematics.

In one such activity, a pair of students position their actors a symmetrical distance away from a meeting point and have to coordinate the creation of a pair of linear functions that will result in a motion animation that shows the actors meeting at the same time at the designated place (Fig. 9.5). The teacher can show successful or unsuccessful meetings and engage students in telling the story of what they had to do to arrange a meeting (since the distance and time to the meeting point are the same, the students need to create functions that model motions of the same speed but opposite direction).

Research on this activity structure suggests that it shares aspects of both prior activities. Organizing students' work as mathematical seems to powerfully engage the identity between a student and the public display of his or her work. Further, different pairs of students naturally vary in *how* they solve the problem (e.g., use different times and speeds), which can result in mathematical debate among students

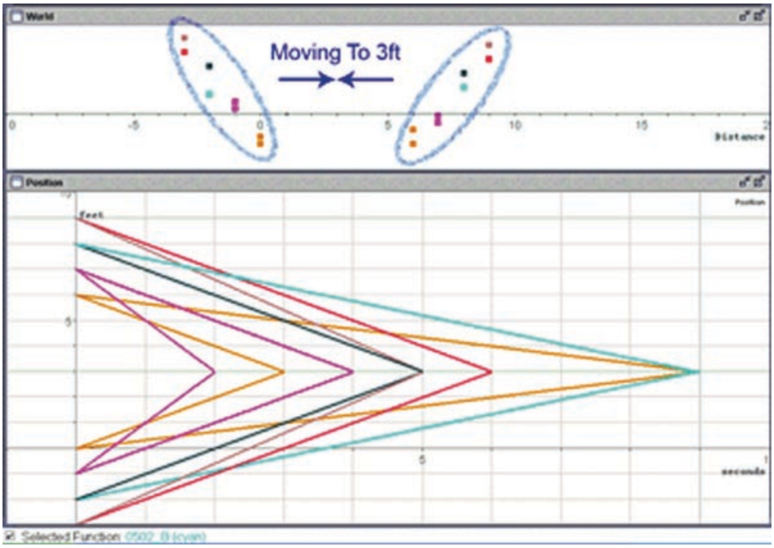


Fig. 9.5 Graphs of functions that converge at 3 feet, but at different times

in the classroom about what was the same and different among their approaches. From such a debate, students can generalize the connections among the equation, graph, and motion form of functions with the same speed by opposite direction.

Results

Research using the connectivity infrastructure is at an earlier stage than the research using only the representational and display infrastructure but is on a similar trajectory toward scale. In a pilot quasi-experiment, a district volunteered seven classrooms of students ($n=133$) to use SimCalc and the remaining eight classrooms of students formed a comparison group ($n=184$). The main effect was statistically significant and showed that students in the SimCalc group had a higher gain on items related to linear functions, slope as rate, proportion, linear variation, seeing across representations, and graphical interpretation, with a medium effect size (Hegedus, Dalton et al., 2007). SimCalc students did especially well, in contrast to the comparison group, on an item that asked students to identify how a position graph represents a change in direction in the corresponding motion. This gain conceivably follows from mathematical performance activities, such as the one described above, which may powerfully address common graph-as-path misconceptions.

The research team is also pursuing classroom changes other than those found via assessments of mathematical content knowledge. For example, as described in the context of the activities above, the team is also searching for methodological refinements that can capture the impact of a networked SimCalc curricular activity system on classroom participation, student identity, and mitigation of students' sense of alienation from mathematics (Hegedus & Penuel, 2008). Promising indicators include the ratio of student-to-student versus student-to-teacher conversational exchanges; an increase in student control of the conversational floor and teacher use of "revoicing" and other facilitation moves; use of deictic markers that connect personal identity and mathematical abstractions. Further, the team is conducting a scale up study looking specifically at longitudinal impacts of participation in networked SimCalc classrooms (using computers and calculators), which might move beyond local engagement to influence on students' proclivity for continuing onward in science, mathematics, technology and engineering studies, and careers.

Discussion

We have presented two design examples from the SimCalc Project, arguing that elaborating a curricular activity system to leverage infrastructure is what makes these designs advanced. In both examples, infrastructure was a target of initial research and emerged from the triangulation of learner-centered, technology-centered, and discipline- or epistemology-centered perspectives, as recapitulated in Table 9.2. An additional common infrastructural element is the use of computer projects to allow a shared, public display. Across the two examples, we have described what elaborating a curricular activity system entails.

Table 9.2 Representational and connective infrastructure from three intersecting perspectives

Perspective	Representation infrastructure	Connectivity infrastructure
Learner-centered	Building on learners' strengths including: • Making sense of motion • Reasoning about intervals • Connecting graphical and linguistic representations	Building on learners' strengths including: • Making sense of motions of groups of actors • Communicating with gestures and informal argument • Identifying with one's contribution in a collective representation
Technology-centered	Presenting new cognitive experiences using: • Executable representations • Dynamically linked representations • Simulated motion	Presenting new social experiences using: • Sharing mathematical objects • Spreading multiple representations across people • Family-of-function-based aggregations of student work
Epistemology-centered	Developing meaning by connecting algebraic symbols and: • Graphs • Piecewise functions • Motions	Developing meaning by connecting algebraic symbols and: • Graphs of families of functions • Relative motions among actors • Parameterized variation

At the heart of our design efforts, the first design focus is always the specification of rich mathematical tasks with an overall learning progression. In our view, these tasks emphasize mathematical connections that are both locally important in terms of the specific curricular objectives but also of longitudinal importance in students' ongoing mathematical development. Rich mathematical tasks often involve multiple representations and involve students in making meaning across representations. Further, within rich mathematical tasks students often have opportunities to practice earlier mathematical skills (for example, identifying point in a Cartesian graph) and procedures (calculating values of mathematical expressions). In the seventh grade curriculum used in the Scaling Up SimCalc project, the "On the Road" activity was a signature mathematical task. Indeed, teachers often commented on this specific activity in the debriefing interviews we conducted after their SimCalc teaching was finished for the year. In the Algebra curriculum used in the SimCalc CC project, the exploration of families of functions with equal slope versus a common point of intersection formed the basis of some of the signature activities. In both cases, these specific tasks occurred as part of a longer-term learning progression that led up to these concepts and built further upon them.

A second design focus is on materials to support teachers and students. It is important to note that these materials are often based on very traditional infrastructure – paper. We continue to find many benefits to a paper workbook to accompany our activities. Some representations (e.g., writing equations, making tables, sketching graphs) continue to be easier for students and teachers to produce in paper form than using a mouse and keyboard. Further, paper extends the real estate available for activities. Instead of overcrowding computer screens, work can extend to extra work surfaces. Indeed, we find that teachers commonly project the MathWorlds display on a screen that is next to their whiteboard, allowing them to work across both the computer display and their whiteboard at the front of the classroom. It is also easier for teachers to markup and comment on student work in paper form. The paper materials provide structure to the activities by introducing the activity and highlighting key concepts, terms, and procedures and by organizing the activity according to some key driving questions. For the teacher, the teacher guide supports enactment of activities by suggesting key questions for classroom discussions. In the SimCalc CC work, for example, the teacher guide offers suggestions for different aspects of the classroom dialog, including a focus on the meaning of the activity, the connections across multiple representations, and essential mathematical ideas of the lesson.

We further consider teacher workshops to be a component of the curricular activity system design, because these workshops are designed to align with and emphasize the enactment of the classroom activities. Consequently, much of the time in teacher workshops is dedicated to working through the student materials, but with commentary and reflection on a teacher level. As these workshops are relatively short in duration and tightly scoped, we tend to think of them as teacher training (to enact the curricular activity system) rather than teacher professional development (with a broader aim in long-term transformation of teaching practice).

The detailed design work involves providing enough structure to enable a broad population to enact the activities without draining the opportunity to struggle with important mathematics out of the activities. As Hiebert and Grouws (2007) argue, two ways that teachers can make a difference in mathematics teaching is to (a) make concepts an explicit focus of classroom discussions and activities, and (b) allow students to struggle with important and meaningful mathematics. In the first example, our design balanced structure and struggle by organizing a progression from simple to more complex mathematics within the activity, but not providing a recipe or procedure for solutions. In the second example, we discussed three specific activity designs, "Where Am I?", Parameterized Variation, and Mathematical Performances, each of which provides substantial structure for classroom enactments but still preserves core conceptual struggles as work for students to do.

We have suggested that recent work in the SimCalc Project has benefited from the growing realization that much of our technology is infrastructural in character and requires the further design of curricular activity systems in order to yield better teaching and learning at scale. The two projects use different combinations of infrastructural features and curricular design principles, making clear that there is not a necessary 1:1 correspondence between a representational infrastructure and a curricular activity system. Further, there is not a 1:1 correspondence between a curricular activity system and how teaching and learning is enacted in particular classrooms. Nonetheless, we have seen that designing a curricular activity system on top of an infrastructure yields enough specificity that a wide variety of teachers can achieve learning gains for students.

We wish to distinguish a curricular activity system from other forms of midlevel design. A curricular activity system is not a web-based repository of teacher-contributed lessons. While such repositories can allow sharing of favorite lessons, they lack the coherent learning progression that we believe is important to strong mathematical growth. A curricular activity system is also not a set of lesson plans or a set of problems, because the design focus is on enacting a classroom *activity*, not on the content of lessons or practicing problem solving. Finally, a classroom activity system is not a technology application. Rather, we have found that the necessary system requires a mix of kinds of materials (e.g., software, paper, teacher guides) and kinds of processes (including teacher workshops but also forms of coaching and peer support) that use the technology infrastructure, but without being exclusively technologically reliant.

Before closing this discussion, we also note that we have focused less on the design of classroom practices and routines, not because we consider these less important but rather because we consider aspects of interventions need much additional research. We do advocate certain classroom moves, for example, asking students to predict what they will see before running the MathWorlds animation or asking students to explain their answers. In general, all aspects of the SimCalc Project value extended mathematical argumentation in the classroom. Overall, however, current research does not provide sufficient guidance to anticipate how to design classroom practices and routines that could scale up to larger numbers of teachers with bounded quantities of professional development.

Conclusion

The central contention of our chapter has been that an advanced technology for learning earns its label not because it uses "advanced technology" but rather because it advances designs for learning. Further, we attend to scaling up as a key goal for advanced technology for learning. Thus, the field needs to become aware of features of its initial designs that may be workable at a small scale but insufficient to structure enactment by a wide variety of teachers across a diversity of school settings. Our work in the SimCalc program suggests that scaling up requires understanding the contributions of different levels of design to successful implementation. At one level, advanced technologies for learning should begin with the identification of new infrastructural capabilities that could profoundly alter students' opportunity to learn. We have argued that these infrastructural capabilities emerge from the joint consideration of students' strengths as learners, specific features of technology, and an epistemological quest for more productive learning progressions that nonetheless honor disciplinary subject matter. At another level, advanced technologies for learning include the design of curricular activity systems. These systems specify rich mathematical tasks within a particular learning progression and include key supports beyond the technology that contribute to successful enactment. A key tension at the curricular activity system level is providing enough structure to make good enactments likely without detracting from a focus on concepts and the opportunity for students to struggle meaningfully with important mathematical ideas. At yet another level, we see long-term teacher professional development as being dialectically coupled to the design of advanced technology for learning. Better enactments are certainly possible when teachers experience carefully designed curricular activity systems that contribute to the vitality of professional development experiences.

Overall, we doubt that mathematics and science education can be improved sufficiently through independently acting, single-factor interventions. Instead, compound interventions are needed and these will include elements designed at different levels of removal from local contexts. Infrastructure, by its very nature, should be designed to provide key capabilities that can be powerfully leveraged in ways that will offer value in many different venues over a long period. Infrastructural design efforts are both important and incomplete with respect to achieving deep educational transformation. We have identified curricular activity systems as another level that is somewhat removed from the specifics of each individual school, teacher, and student but which can provide common structures that make successful enactments likely. Finally, we suspect that improving teaching and learning will always also have a profoundly local aspect, which involves professional development and leadership development at the school level. Thus, we are not recommending an either-or approach to design, but rather that innovators recognize and act more explicitly on their opportunities to create value for teachers and learners at multiple, overlapping levels.

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