Chapter 8

Long Lead Time Production: With Aliza Heching

In the long run we are all dead
– *John Maynard Keynes*

We consider a problem faced by a supplier of custom products that have long production lead times. The problem is inherently multistage with a large number of stages. The majority of these products have no salvage value, so the supplier is exposed to significant risk of excess production. Moreover, customer forecasts will likely err on the upside because the option to purchase has value. This chapter describes a counterbalancing mechanism for the supplier to obtain some compensation for part of the inventory risk.

From a modeling perspective, the most challenging part here is to combine in a meaningful way stages and time periods and then fit in the long lead times. Some of the details will get very involved, but try to follow us on the main modeling questions.

8.1 Supplier-Managed Inventory

A supplier-managed inventory (SMI) program is a way to compensate a manufacturer for inventory risks. The basic steps of the program are as follows:

1. Customers provide the supplier with a demand forecast schedule.
2. The supplier responds with production forecast schedules.
3. The SMI program determines what part of the production forecast falls within the production lead time. This is a *commitment* since it cannot be altered.
4. Periodically—for example, at the end of each quarter—the supplier compares the commitments with the actual customer orders.
5. When orders are lower than commitments, the supplier has the right to ship part of this *underorder quantity* to the customer.

A.J. King and S.W. Wallace, *Modeling with Stochastic Programming*, Springer Series in ORFE, DOI 10.1007/978-0-387-87817-1_8,

The SMI program reduces the incentive for customers to overforecast their demand. Customers have a natural incentive to overforecast because the access to production facilities gives them an option to increase their own production while also denying their competitors access to a scarce production resource.

8.2 Supplier-Managed Inventory: Time Stages

Our goal is to develop a model of the SMI program that indicates the dependence of the supplier's decisions on the information flow. We also illustrate the need for flexibility in modeling decision stages.

8.2.1 Modeling Time Stages

One of the most important details in stochastic modeling is to sort out the stages. Stages define the boundaries of time intervals. The conceit of stochastic programming is that actions and events whose time of occurrence or observation falls *within* a given time interval are actually accounted for at the *end* of that time interval.

Let us think a bit more about why this is so. In the SMI program, the supplier will be collecting information about customer orders and will be making production decisions. At the review point, the supplier closes the books and makes the underorder decision. We imagine the action of closing the books as happening *at the same time* as the history of orders and production since the last review point became known since we make the decision with this information in mind. Technically, this means that time intervals between review points are "open" on the left and "closed" on the right and that events and actions that occur within time intervals are labeled with the right-hand endpoint. Such considerations are only important in models handling information uncertainty. In deterministic models, this issue is irrelevant.

As we have emphasized throughout this book, the first issue of substance when dealing with uncertainty in decision problems is to decide how "clock time" maps to "decision time." Production decisions are made at regular clock-time intervals since every production facility (or, for that matter, any collection of coordinated processes) needs a reference clock to synchronize the component tasks. So from a "clock-time" perspective we say that a certain product takes a certain number of clock-time periods (e.g., minutes, days, months) to be produced, but we need a procedure to map these clock lead times into decision lead times.

The SMI program declares a review-time process that is likely to consist of many clock-time intervals. A natural incentive arises from the *complexity of solving* the problem to reduce the number of time intervals, of course. For this reason alone we will want to focus on review times since these are the least frequent time points that are relevant for the problem. However, easing the burden of solving the stochastic program is a very natural and important consideration.

We need to identify natural features of the modeling that justify the choice of stages. The most natural feature to consider is *when we learn something.*

> In stochastic programming, we identify *stages* with *learning.*

At review times the supplier must study the relationship between customer forecast behavior and actual customer orders because this is the supplier's opportunity in the SMI program to counter the overforecasting incentive. Hence the intervals between review times are the natural candidates for decision stages.

Of course, as the review time approaches, the supplier will be able to form hypotheses about the likely underorder quantity at review time. Perhaps it is sensible to subdivide the review intervals to implement decision stages that react to the increasing reliability of the underorder estimates. In natural systems, like processes inside living cells, such anticipation makes sense.

In supply chain models, the supplier cannot control the customer's order behavior or develop perfect visibility into the processes that influence customer order behavior. Customers could decide to place a huge order just before the review period. Such behavior may even make sense, from the customer's perspective, because the customers of our customer may in turn have their own end-of-period incentives. For example, it is well known that budget cycles drive institutional orders. The "use it or lose it" principle drives institutional members to rush to use up their budget allocation before the end of their fiscal year. (Just think about the importance of Christmas for retail sales in countries with a strong Christian tradition!) And of course, a large end-of-period order may just be irrational. The point is that we cannot control it.

The SMI program is a program that enables a supplier to gain more reliable information about customer demand. Of course, it makes sense for supplier and customer to coordinate the review process timing to account for the natural order cycles in their respective businesses.

For concreteness, let us index clock time by $t = 0, 1, \ldots, T$. Time intervals are open on the left, closed on the right, and labeled with the right endpoint, so that events falling in the interval $(t-1, t]$ are said to occur during period t. Production lead time L_i, for now, will denote the number of clock-time periods needed to manufacture product i.

The review process takes place at time steps that are multiples of the clock time. Let us say that $\eta = 1, 2, \ldots$ indexes review times, and t_η is the time stamp for the review time η. Thus at time t_1 the SMI program will review the production completed before t_1, at time t_2 review production completed since t_1, and so forth.

Some gray areas need to be examined here. For example, how do we handle a product that was committed before the review period ends but arrives in the following period? There will always be time-boundary issues like this, and the problem formulation process needs to address special ways to handle them. Such discussions often lead to the development of a deep insight into the decision process. In the SMI model, we can handle this issue by allowing the customer to order a product that is committed to manufacture but has not yet arrived in inventory—but of course this introduces a more complex inventory update equation, as we will see below.

8.3 Modeling the SMI Problem

The actions that occur at every clock-time stage are as follows:

- Customer provides demand forecasts for multiple future clock-time periods.
- Supplier indicates what product is now committed to production, by period.
- Customer places orders.
- Supplier makes production decisions.

At review periods, in addition, there are the following actions:

- Supplier reviews orders and commitments and determines underorder quantities.
- Supplier ships orders, including a proportion of the underorder.

8.3.1 First- and Last-Stage Model

The first stage is always special, and the flexibility to model first-stage considerations is one of the great strengths of the stochastic programming methodology. The first decision can occur at any clock-time point, and because time starts at 0, we say the first stage is $t = 0$. The information observable at the first decision stage is

- Product in inventory.
- Product commitment and product orders since last review period.
- Customer order and supplier production history since the beginning of time.

The second decision stage corresponds to the interval $(0, t_1]$, where t_1 is the first review time following time 0, or "today." The third decision stage will be $(t_1, t_2]$, where t_2 is the second review time, and so forth.

The final consideration is how to handle the terminal stage. Stochastic programming cannot rely on "long-run" limiting arguments, which discount the horizon into irrelevance—in the long run we are all dead, as the great economist pointed out. So we need to decide what the horizon should be and account for the real choices that exist there. The horizon should be beyond the first review time, but how far beyond? Perhaps it makes sense to select a horizon that is the longest lead time for the current planning cycle? Many arrangements are possible here.

To make the development of the model concrete, we will implement the SMI program as a three-stage problem. Stage 1 is "today," or time 0; stage 2 includes the time until the first review point $(0, t_1]$; and stage 3 is $(t_1, t_2]$, where t_2 is the second review point following today.

8.3.2 Demand Forecasts and Supply Commitments

The managed inventory model begins with an exchange of clock-time demand forecasts and supply commitments between customer and supplier. Let the horizon for the forecasts be τ. Denote the customer's forecast issued at clock time t for product i demand during periods $t+1, \ldots, t+\tau$ by

$$\mathbf{F}_t^i = \left\{ F_t^{i,1}, \ldots, F_t^{i,\tau} \right\}. \tag{8.1}$$

The supplier reviews the forecasts $\mathbf{F}_t^i$ and responds with the volume of product projected to be available for delivery during each future period $t+1, \ldots, t+\tau$. Denote the supplier response by

$$\mathbf{C}_t^i = \left\{ C_t^{i,1}, \ldots, C_t^{i,\tau} \right\}. \tag{8.2}$$

The rules of the managed inventory contract dictate that those forecast periods that fall within the production lead time are *commitments* of the supplier and the customer.

At stage $t = 0$, which in our modeling framework is "today," the commitments are the responses $C_0^{i,1}, \ldots, C_0^{i,L-1}$ because these are already in production. The response $C_0^{i,L}$ is the new commitment.

8.3.3 Production and Inventory

The supplier plans the release of new production and manages the inventory on a clock-time frequency. The release of product into production will create inventory but, due to manufacturing lead times, will not be available until L periods have passed. Most inventory models will incorporate customer deliveries, but in the SMI program, actual customer orders are relevant only at review periods.

Q_t^i = quantity of product i released to manufacturing during period t
X_t^i = on-hand inventory of product i at the end of period t
D_t^i = product i delivered to customer during period t

The supplier makes the production decision at t, but the actual quantities will be released to manufacturing in the interval $(t, t+1]$. They will arrive at inventory during the interval $(t+L, t+L+1]$. Stated another way, the production arriving at inventory during interval $(t-1, t]$ was released to manufacturing L periods before, or $(t-L-1, t-L]$. Thus, the basic inventory update equation is given by

$$X_t^i = X_{t-1}^i + Q_{t-L}^i - D_t^i. \tag{8.3}$$

The supplier commitments require that product available for delivery be greater than the commitment, namely,

$$X_t^i \geq C_0^{i,t}. \tag{8.4}$$

8.4 Capacity Model

The manufacture of new products consumes the services of labor and equipment. The availability of these resources is constrained. Let us say that a unit of product i at manufacturing stage $k = 1, \ldots, L$ consumes a vector of resource capacities α_{ik}. In this section, we are not concerned with the details of modeling the allocation of resources, so we simply assume the existence of an exogenously given resource capacity schedule A_t. The capacity constraint is then

$$\sum_{k=1}^{L} \sum_{i \in I} \alpha_{i,k} Q_{t-k}^i \leq A_t. \tag{8.5}$$

This is a hard constraint, which should concern you; after all, we have made a big point of using soft constraints. Can you work out a version of this problem that has soft constraints for capacity?

8.4.1 Orders and Review Periods

Customer orders are under the control of the customer and observed by the supplier. Accumulated orders since the last review period are denoted Y_η^i. The accumulated deliveries cannot be greater than the accumulated orders, hence

$$\sum_{t \in (t_{\eta-1} - t_\eta]} D_t^i \leq Y_\eta^i. \tag{8.6}$$

At the review point, the supplier compares the total orders with the commitments and calculates the underorder, namely,

$$U_\eta^{i+} - U_\eta^{i-} = Y_\eta^i - \sum_{t \in (t_{\eta-1}, t_\eta]} C_0^{i,t}, \tag{8.7}$$

where the quantities U_η^{i-} and U_η^{i+} are nonnegative. The underorder is the quantity that picks up the negative amount. According to SMI rules, this amount is available in inventory and a proportion γ_i can be delivered immediately. Hence we need to update the inventory equation in the first period following the review period with the underorder quantity:

$$X_{t_\eta}^i = X_{t_\eta - 1}^i + Q_{t_\eta - L}^i - D_{t_\eta}^i + 1_{t=t_\eta}\gamma_i U_\eta^{i-}, \tag{8.8}$$

where the expression $1_{t=t_\eta}$ equals 1 when the subscript condition is satisfied and zero otherwise.

Finally, revenue is generated when product is delivered:

$$R_\eta^i = \rho_i \left[\sum_{t \in (t_{\eta-1}, t_\eta]} D_t^i + \gamma_i U_\eta^{i-} \right], \tag{8.9}$$

and we model this as a quantity to be maximized by the supplier.

At this point, poor reader, you do have our sympathies. There is certainly a great deal to keep in mind in this complicated model. Perhaps the following questions may help to direct your attention:

1. Can you verify that our model is actually "implementable"? Do prior time quantities depend directly on future time quantities without provision for recourse? To tell the truth, the authors worried a great deal about this aspect of the problem formulation. Did we get it right?
2. If we presume that the supplier maximizes revenue, do the delivery variables get set to the right quantities?

8.4.2 The Model

We arbitrarily elected to model a problem with three time stages: today, the first review stage, and the second review stage. The order of the arrival of information, the decisions to be made, and the consequences of these decisions are described in Fig. 8.1. It is useful to consult this figure while reading the rest of this section. Note that customer orders and demand forecasts are revealed before the supplier makes supply commitments and production decisions.

8.4.3 Objectives

Large manufacturers typically have three objectives in mind: high customer satisfaction, high net revenue, and low inventory charges. Assuming that the first objective is satisfied by successfully implementing the terms and conditions of the SMI program, we now develop an optimization formulation of a three-stage inventory management model that emphasizes the attainment of revenue and management of inventory expense targets.

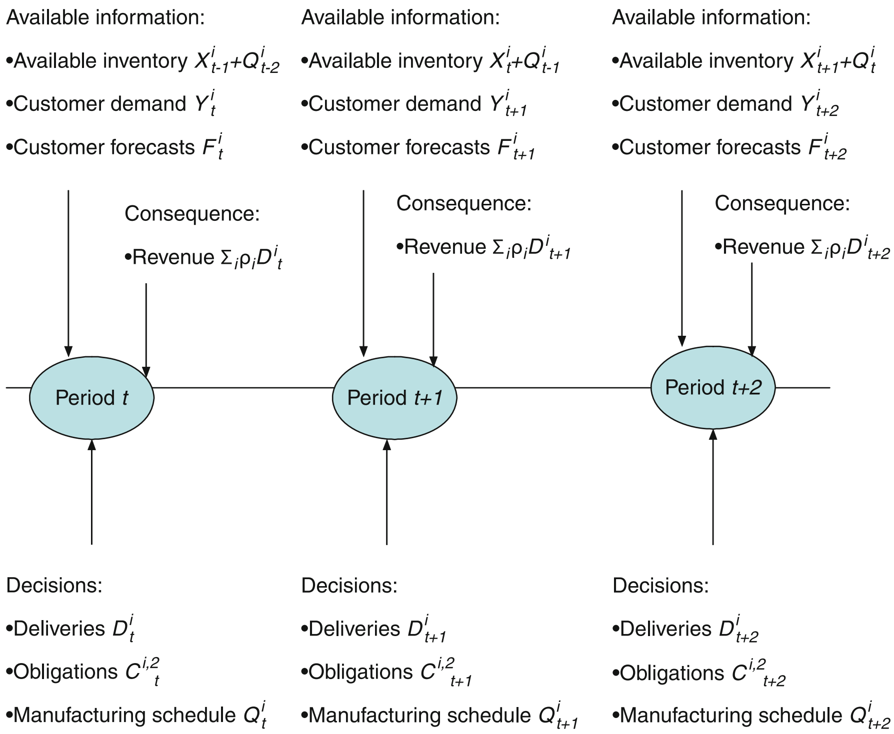

Fig. 8.1: Information revealed/decisions over time

Let
R_η = net revenue target in review periods $\eta = 1, 2$
E_η = inventory expense target in periods $\eta = 1, 2$
ρ_i = revenue per unit of item $i \in I$ ordered
χ_i = inventory cost per unit of item $i \in I$ in on-hand inventory

Expense and revenue targets will be enforced through the objective function as "soft constraints" since it may be impossible to satisfy both targets simultaneously. Also, as *targets* they obviously are not absolutely required, and, hence, the target constraints belong among the soft constraints.

Targets should always be modeled in a soft way by incurring a penalty whenever they are broken.

To that end, we introduce nonnegative variables $Z^+_{R\eta}$ and $Z^-_{R\eta}$ and an equality constraint that defines the variables as the excess and deficiency, respectively, of the revenue with respect to the target:

$$\sum_{i \in I} R^i_\eta - Z^+_{R\eta} + Z^-_{R\eta} = R_\eta.$$

To penalize solutions that do not achieve revenue targets, we incorporate the penalty term $-\lambda Z^-_{Rt}$, with $\lambda > 0$, into the objective function. Similarly, we introduce nonnegative variables Z^+_{Et} and Z^-_{Et} and define them as the excess and deficiency, respectively, of the inventory expense relative to its target:

$$\sum_{i \in I} \chi_i X^i_\eta - Z^+_{E\eta} + Z^-_{E\eta} = E_\eta.$$

A second term added to the objective function, $-\mu Z^+_{E\eta}$, with $\mu > 0$, penalizes solutions with above-target inventory holding costs. Of course, the penalty factors λ and μ can depend on i and η. The supplier may wish additional parametric degrees of freedom to adjust penalties by product or review time—for example, to assess the impact of the inventory charges relative to the production capacities.

Since this style of soft constraints penalizes falling on the wrong side of a target, these are sometimes referred to as "shortfall penalties"; see Sect. 3.4 for a discussion of soft constraints and targets.

8.5 Uncertainty

There are many potential sources of uncertainty. The two major sources are the capacity model (8.5) and the customer order processes, Y^i_η. In this chapter we focus on uncertainty associated with customer orders.

8.5.1 Uncertain Orders

The SMI agreement requires a customer to provide τ period rolling horizon order forecasts. The supplier has different uses for forecasts with different lead times. Longer lead-time demand forecasts are typically used for strategic planning. Shorter lead-time forecasts are used to schedule actual production. Intermediate lead-time forecasts are used for capacity planning; the supplier reviews expected demand and may decide to acquire capacity from outside sources if demand forecasts indicate that demand exceeds production capacity. Thus, the supplier faces multiple types of risk due to the uncertainty associated with the demand forecasts with different lead times. Further, errors in forecasts at different lead times can have very different financial impacts on the supplier.

The model under consideration in this chapter addresses risks of inventory imbalances by modifying production schedules. Therefore, what is of most interest is the short-term forecast uncertainty.

Suppliers depend upon customer forecasts for strategic and tactical planning, but the reasons why customers may report inaccurate order forecasts are quite varied.

8.5.2 Inaccurate Reporting

When customers do not anticipate tight production capacity, they may often treat these forecasts as a courtesy to the supplier. Thus, customers may not invest adequate time in properly estimating expected demand. In particular, where the penalty associated with inaccurate forecasts is not significant, the supplier may see sizeable discrepancies between the forecasted demand that the customer reports and the realized demand.

Anticipation of constrained supply may also lead customers to provide the supplier with inaccurate demand forecasts. When a customer anticipates that production capacity is constrained, the customer may forecast higher-than-expected demand on the assumption that the rationed quantity of product that he receives is closer to his true needs.

Discrepancies between forecasted and realized demand are not always due to intentional actions taken by the customer. For example, if a supplier is manufacturing custom products, forecasted demand for a specific prototype will not materialize into actual demand if changes are made to the prototype. Forecasts for products with mature and successful designs will not suffer from this specific cause for forecast uncertainty.

As another example, customers may be incorporating these products into end products that have a "fashion" component, i.e., where end-consumer interest cannot be established up front. Thus, once the customer begins to sell the end product on the retail market, he may find that end-consumer interest is not as anticipated. For a more direct discussion of the difficulties of "fashion" products, see Chap. 6. Finally, unanticipated changes in global market conditions can impact demand and cause actual demand to diverge from forecasted demand.

8.5.3 A Stochastic Programming Model

A model's stochastic version models uncertainty in demand forecasts. As we indicated above, we wish to simplify the model by incorporating uncertainty only at review periods. There will be different realizations of forecasts and orders depending on what scenario $s \in S$ occurs. The objective function is an integral over the scenarios, weighted by probabilities p_s.

Then the supplier's objective is to solve

$$
\begin{array}{lll}
\max\limits_{(Q,\mathcal{C},D,U)} & \sum\limits_{s\in S} p_s \sum\limits_{\eta=1}^{2} \left[\sum\limits_{i\in I} R^i_\eta(s) - \lambda Z^-_{R\eta}(s) - \mu Z^+_{E\eta}(s) \right], & \\
 & \sum\limits_{i\in I} R^i_\eta(s) - Z^+_{R\eta}(s) + Z^-_{R\eta}(s) & = R_\eta, \\
 & \sum\limits_{i\in I} \chi_i X^i_\eta(s) - Z^+_{E\eta}(s) + Z^-_{E\eta}(s) & = E_\eta, \\
 & \rho_i \sum_{t\in(t_{\eta-1},t_\eta]} D^i_t(s) + \gamma_i U^{i-}_\eta(s) & = R^i_\eta(s), \\
 & U^{i+}_\eta(s) - U^{i-}_\eta(s) + \sum_{t\in(t_{\eta-1},t_\eta]} C^{i,t}_0 & = Y^i_\eta(s), \\
\text{such that} & \sum_{t\in(t_{\eta-1}-t_\eta]} D^i_t(s) & \leq Y^i_t(s), \\
 & X^i_t(s) - X^i_{t-1}(s) - Q^i_{t-L}(s) + D^i_t(s) + 1_{t=t_\eta}\gamma_i U^{i-}_\eta(s) & = 0, \\
 & X^i_t(s) - C^{i,t}_0 & \geq 0, \\
 & C^{i,t}_0 & \leq F^{i,t}_0, \\
 & \sum_{k=1}^{L} \sum_{i\in I} \alpha_{i,k} Q^i_{t-k}(s) & \leq A_t, \\
 & Q, \mathcal{C}, D, X, U, Z & \geq 0,
\end{array}
\tag{8.10}
$$

where all the appropriate initial values are given. (Can you determine what these should be?) This formulation is a three-stage stochastic program with recourse. Decisions in the second stage are made after having observed actual events in the first stage, and so forth.

8.5.4 Real Options Modeling

A stochastic programming formulation requires the supplier to model risk preferences and distributions. The previous formulation used a target penalty approach to deal with the uncertainty in revenues and expenses. But where did the probability measure come from?

We close this chapter with a brief discussion of this issue in the light of "real options modeling" (ROM), as we described it in Chap. 3. The basic idea is to apply *forward-looking information*, such as may be found in financial markets, to evaluate the risk of operational decisions using a *stochastic discount factor* (SDF) as presented in Chap. 7. The key step in the ROM approach is to develop a state space from which we can obtain calibrations for the SDFs, as discussed in Sect. 7.1.4.

The main source of uncertainty we consider is the uncertainty in customer orders Y_η. We require sources of information that can calibrate SDFs on this (or, perhaps, a related) state space. Some possible sources of information include:

- Forecasts for customer orders.
- Internal opinions on forecast errors.
- Supplier-company options, given some model in which supplier-company stock values Or payments are impacted by forecast variations.
- Customer-company options, given some model in which customer-company stock values are affected by forecast variations.

The supplier's internal opinions on forecast errors are probably the most valuable source of calibration. In Chap. 3 we suggested a form of internal market where employees could buy or sell option contracts on future customer revenues. These option contracts can be modeled as virtual "hedges" for the revenue and expense cash flows. Employee opinion generated by the creation of an internal exchange of option contracts would probably be the best source of opinion. Of course, the validity of opinions can be weighted by the volume of the bets entered into the exchange, as discussed in Sect. 7.1.4.1.

One way to obtain option information in the case of large corporate entities is to look at options that are traded on open exchanges. It would be logical to think that robust order growth from a customer should be reflected in the stock prices.

However, there are difficulties with this source of information. First, multiple factors drive stock prices, and the line of business modeled by SMI may only be one of these. Second, the trading public may not have sufficient visibility on the SMI business to form opinions regarding its likely impact on stock prices. The authors themselves have observed examples where customer orders shot up but customer-company stock first declined followed by a sharp increase more than a year later.