Chapter 13

Returns-to-Scale

13.1 Introduction

As demonstrated in Figure 1.3, the VRS envelopment model identifies the VRS frontier with DMUs exhibiting IRS (increasing returns to scale), CRS (constant returns to scale), and DRS (decreasing returns to scale). In fact, the economic concept of RTS (returns to scale) has been widely studied within the framework of DEA. RTS have typically been defined only for single output situations. DEA generalizes the notion of RTS to the multiple-output case. This, in turn, further extended the applicability of DEA.

Seiford and Zhu (1999a) demonstrate that there are at least three equivalent *basic* methods of testing a DMU's RTS nature which have appeared in the DEA literature. Based upon the VRS multiplier models, the sign of the optimal free variable (μ^* or ν^*) indicates the RTS (Banker, Charnes and Cooper, 1984). Based upon the CRS envelopment models, the magnitude of optimal $\sum_j^n \lambda_j^*$ indicates the RTS (Banker, 1984). These two methods may fail when DEA models have alternate optimal solutions. The third method is based upon the scale efficiency index (Färe, Grosskopf and Lovell, 1994). The scale efficiency index method does not require information on μ^* or ν^* or $\sum_j^n \lambda_j^*$, and is robust even when there exist multiple optima. However, the scale efficiency index method requires the calculation of three DEA models.

Seiford and Zhu (1999b) and Seiford and Zhu (2005) study the sensitivity of RTS classification. Seiford and Zhu (1999c) provide a use of RTS sensitivity analysis in improving performance of a two-stage process.

13.2 RTS Regions

It is meaningful to discuss RTS for DMUs located on the VRS frontier. We discuss the RTS for non-frontier DMUs by their VRS efficient targets as indicated in Table 1.1. Because a VRS envelopment model can be either input-oriented or output-oriented, we may obtain different efficient targets and RTS classifications for a specific non-frontier DMU.

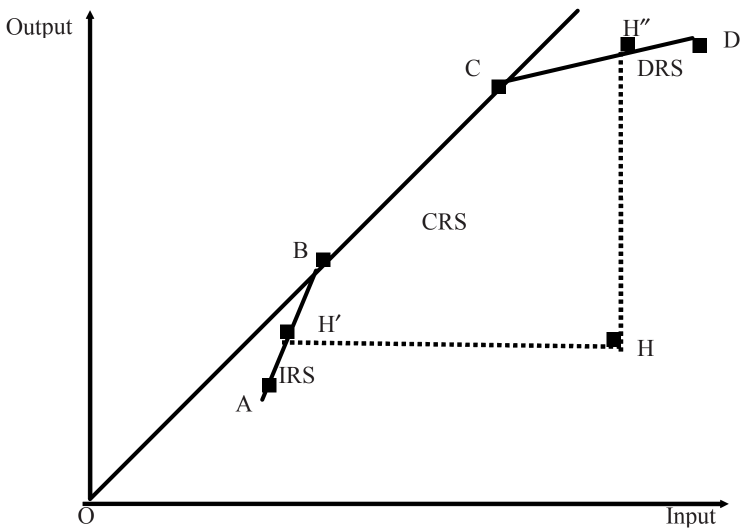


Figure 13.1. RTS and VRS Efficient Target

Suppose we have five DMUs, A, B, C, D, and H as shown in Figure 13.1. Ray OBC is the CRS frontier. AB, BC and CD constitute the VRS frontier, and exhibit IRS, CRS and DRS, respectively. B and C exhibit CRS. On the line segment AB, IRS prevail to the left of B. On the line segment CD, DRS prevail to the right of C.

Consider non-frontier DMU H. If the input-oriented VRS envelopment model is used, then H′ is the efficient target, and the RTS classification for H is IRS. If the output-oriented VRS envelopment model is used, then H″is the efficient target, and the RTS classification for H is DRS.

However some IRS, CRS and DRS regions are uniquely determined no matter which VRS model is employed. They are region ‘I’ – IRS, region ‘II’ – CRS, and region ‘III’ – DRS. In fact, we have six RTS regions as shown in

Figure 13.2. Two RTS classifications will be assigned into the remaining regions IV, V and VI. Region 'IV' is of IRS (input-oriented) and of CRS (output-oriented). Region 'V' is of CRS (input-oriented) and of DRS (output-oriented). Region 'VI' is of IRS (input-oriented) and of DRS (output-oriented).

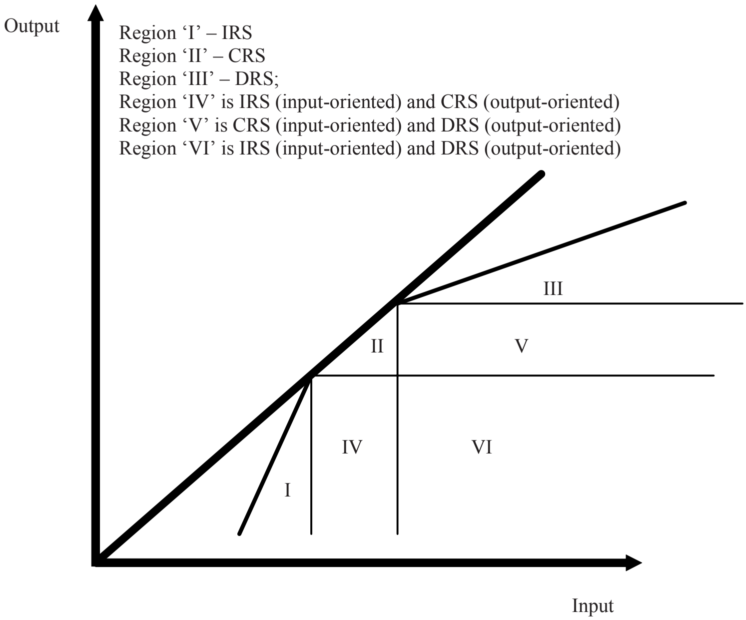


Figure 13.2. RTS Region

The RTS regions can provide a DMU classification. See also Gregoriou and Zhu (2005).

13.3 RTS Estimation

3.3.1 VRS and CRS RTS Methods

Let μ^* represent the optimal value of μ in the input-oriented VRS multiplier model, and ν^* the optimal value of ν in the output-oriented VRS multiplier model, then we have the VRS RTS method.

Theorem 13.1
(i) If μ^* = 0 (or ν^* = 0) in *any* alternate optima, then CRS prevail on DMU_o.
(ii) If μ^* > 0 (or ν^* < 0) in *all* alternate optima, then IRS prevail on DMU_o.
(iii) If μ^* < 0 (or ν^* > 0) in *all* alternate optima, then DRS prevail on DMU_o.

Note that the VRS frontier can be expressed as $\sum_{r=1}^{s}\mu_r y_{rj} = \sum_{i=1}^{m}\nu_i x_{ij} - \mu$ (or $\sum_{r=1}^{s}\mu_r y_{rj} = \sum_{i=1}^{m}\nu_i x_{ij} + \nu$). Thus, geometrically, in the case of single output, $-\mu^*$ (or ν^*) represents the y-intercept on the output axis. Consider Figure 13.1. The intercept is positive for line segment CD so μ^* < 0 (or ν^* > 0) and RTS is decreasing for any DMU on CD (excluding C), whereas the intercept is negative for line segment AB so μ^* > 0 (or ν^* < 0) and RTS is increasing for any DMU on AB (excluding B). The intercept for line OBC is zero so μ^* = 0 (or ν^* = 0) and RTS is constant. However, in computation, we may not obtain the unique optimal solution (the frontier), and we may obtain supporting hyperplanes at VRS frontier DMUs. Consequently, we have to check all optimal solutions as indicated in Theorem 13.1.

Table 13.1 presents five VRS frontier DMUs with two inputs and one output. The last column indicates the RTS classification.

Table 13.1. DMUs for RTS Estimation

DMU	input 1 (x1)	input 2 (x2)	output (y)	RTS
1	2	5	2	CRS
2	2	2	1	CRS
3	4	1	1	CRS
4	2	1	1/2	IRS
5	6	5	5/2	DRS

Table 13.2. Optimal Values for RTS Estimation

DMU	$\mu^* \in [\mu^-, \mu^+]$	λ_j^*
1	[-7, 1]	λ_1^* = 1; $\sum_{j=1}^{6}\lambda_j^*$ = 1
2	[0, 1]	solution 1: λ_2^* = 1; $\sum_{j=1}^{6}\lambda_j^*$ = 1
		solution 2: λ_1^* = 1/3, λ_3^* = 1/3; $\sum_{j=1}^{6}\lambda_j^*$ = 2/3
3	[-5/3, 1]	λ_3^* = 1; $\sum_{j=1}^{6}\lambda_j^*$ = 1
4	[1/2, 1]	$0 \le \lambda_1^* \le 1/12$, $\lambda_2^* = 1/4 - 3\lambda_1^*$, $\lambda_3^* = 1/4 + \lambda_1^*$
		$5/12 \le \sum_{j=1}^{6}\lambda_j^* \le 1/2$
5	(-∞, -3/37]	$\lambda_1^* = 35/48 - \lambda_2^*/3$, $0 \le \lambda_2^* \le 35/16$, $\lambda_3^* = 25/24 - \lambda_2^*/3$
		$85/48 \le \sum_{j=1}^{6}\lambda_j^* \le 15/6$

The second column of Table 13.2 reports the optimal μ^*. μ^* can take all the optimal values in the interval [μ^-, μ^+]. μ^* = 0 is found in DMUs 1, 2, and 3, therefore the three DMUs exhibit CRS. All μ^* are positive and negative in DMU5 and DMU6, respectively, therefore IRS and DRS prevail on DMU5 and DMU6, respectively.

The above RTS method uses the VRS multiplier models. In fact, we can use CRS envelopment models to estimate the RTS classification (Zhu, 2000a). Let λ_j^* be the optimal values in CRS envelopment models. We have

Theorem 13.2

(i) If $\sum_j^n \lambda_j^*$ = 1 in *any* alternate optima, then CRS prevail on DMU_o.
(ii) If $\sum_j^n \lambda_j^*$ < 1 for *all* alternate optima, then IRS prevail on DMU_o.
(iii) If $\sum_j^n \lambda_j^*$ > 1 for *all* alternate optima, then DRS prevail on DMU_o.

From Table 13.2, we see that DMU2 has alternate optimal λ_j^*. Nevertheless, there exists an optimal solution such that $\sum_j^n \lambda_j^*$ = 1 indicating CRS. DMU4 exhibits IRS because $\sum_j^n \lambda_j^*$ < 1 in all optima, and DMU5 exhibits DRS because $\sum_j^n \lambda_j^*$ > 1 in all optima.

3.3.2 Improved RTS Method

In real world applications, the examination of alternative optima is a laborious task, and one may attempt to use a single set of resulting optimal solutions in the application of the RTS methods. However, this may yield erroneous results. For instance, if we obtain λ_1^* = λ_3^* = 1/3, or μ^* = 1 for DMU2, then DMU2 may erroneously be classified as having IRS because $\sum \lambda_j^*$ < 1 or μ^* > 0 in one particular alternate solution.

A number of methods have been developed to deal with multiple optimal solutions in the VRS multiplier models and the CRS envelopment models. Seiford and Zhu (1999a) show the following results with respect to the relationship amongst envelopment and multiplier models, respectively.

Theorem 13.3

(i) The CRS efficiency score is equal to the VRS efficiency score *if and only if* there exists an optimal solution such that $\sum_j^n \lambda_j^*$ = 1. If The CRS efficiency score is not equal to the VRS efficiency score, then
(ii) The VRS efficiency score is greater than the NIRS efficiency score *if and only if* $\sum_j^n \lambda_j^*$ < 1 in all optimal solutions of the CRS envelopment model.
(iii) The VRS efficiency score is equal to the NIRS efficiency score *if and only if* $\sum_j^n \lambda_j^*$ > 1 in all optimal solutions of the CRS envelopment model.

Theorem 13.4
(i) The CRS efficiency score is equal to the VRS efficiency score *if and only if* there exists an optimal solution $\mu^* = 0$ (or $\nu^* = 0$). If The CRS efficiency score is not equal to the VRS efficiency score, then
(ii) The VRS efficiency score is greater than the NIRS efficiency score *if and only if* $\mu^* > 0$ (or $\nu^* < 0$) in all optimal solutions.
(iii) The VRS efficiency score is equal to the NIRS efficiency score *if and only if* $\mu^* < 0$ (or $\nu^* > 0$) in all optimal solutions.

Based upon Theorems 13.3 and 13.4, we have

Theorem 13.5
(i) If DMU_o exhibits IRS, then $\sum_j^n \lambda_j^* < 1$ for *all* alternate optima.
(ii) If DMU_o exhibits DRS, then $\sum_j^n \lambda_j^* > 1$ for *all* alternate optima.

The significance of Theorem 13.5 lies in the fact that the possible alternate optimal λ_j^* obtained from the CRS envelopment models only affect the estimation of RTS for those DMUs that truly exhibit CRS, and have nothing to do with the RTS estimation on those DMUs that truly exhibit IRS or DRS. That is, if a DMU exhibits IRS (or DRS), then $\sum_j^n \lambda_j^*$ must be less (or greater) than one, no matter whether there exist alternate optima of λ_j.

Further, we can have a very simple approach to eliminate the need for examining all alternate optima.

Theorem 13.6
(i) The CRS efficiency score is equal to the VRS efficiency score *if and only if* CRS prevail on DMU_o. Otherwise,
(ii) $\sum_j^n \lambda_j^* < 1$ *if and only if* IRS prevail on DMU_o.
(iii) $\sum_j^n \lambda_j^* > 1$ *if and only if* DRS prevail on DMU_o.

Thus, in empirical applications, we can explore RTS in two steps. First, select all the DMUs that have the same CRS and VRS efficiency scores regardless of the value of $\sum_j^n \lambda_j^*$. These DMUs are in the CRS region. Next, use the value of $\sum_j^n \lambda_j^*$ (in any CRS envelopment model outcome) to determine the RTS for the remaining DMUs. We observe that in this process we can safely ignore possible multiple optimal solutions of λ_j.

Similarly, based upon VRS multiplier models, we have

Theorem 13.7
(i) The CRS efficiency score is equal to the VRS efficiency score *if and only if* CRS prevail on DMU_o. Otherwise,
(ii) $\mu^* > 0$ (or $\nu^* < 0$) if and only if IRS prevail on DMU_o.

(iii) $\mu^* < 0$ (or $\nu^* > 0$) if and only if DRS prevail on DMU_o.

3.3.3 Spreadsheets for RTS Estimation

We here develop spreadsheet models for RTS estimation based upon Theorem 13.6. The RTS spreadsheet model uses VRS and CRS envelopment spreadsheets. Figure 13.3 shows a spreadsheet for the input-oriented CRS envelopment model where CRS efficiency scores and the optimal $\sum_j^n \lambda_j^*$ are recorded in columns J and K, respectively. The button "Input-oriented CRS (RTS)" is linked to a VBA procedure "RTS".

```
Sub RTS()
    Dim i As Integer
    For i = 1 To 15
'set the value of cell E18 equal to i (1, 2,..., 15)
    Range("E18") = i
'Run the Solver model. The UserFinish is set to True so that
'the Solver Results dialog box will not be shown
    SolverSolve UserFinish:=True
'Place the efficiency into column J
    Range("J" & i + 1) = Range("F19")
'Place the sum of lambdas into column K
     Range("K" & i + 1) = Range("B25")
    Next i
End Sub
```

	A	B	C	D	E	F	G	H	I	J	K
1	Company	Assets	Equity	Employees		Revenue	Profit		λ	CRS Efficiency	$\Sigma\lambda$
2	Mitsubishi	91920.6	10950	36000		184365.2	346.2		0	0.662831738	1.101942
3	Mitsui	68770.9	5553.9	80000		181518.7	314.8		0	1	1
4	Itochu	65708.9	4271.1	7182		169164.6	121.2		0	1	1
5	General Motors	217123.4	23345.5	709000		168828.6	6880.7		0	1	1
6	Sumitomo	50268.9	6681	6193		167530.7	210.5		0.47	1	1
7	Marubeni	71439.3	5239.1	6702		161057.4	156.6		0	0.971966637	0.956252
8	Ford Motor	243283	24547	346990		137137	4139		0	0.737166307	0.99751
9	Toyota Motor	106004.2	49691.6	146855		111052	2662.4		0	0.524557613	0.819264
10	Exxon	91296	40436	82000		110009	6470		0	1	1
11	Royal Dutch/Shell Group	118011.6	58986.4	104000		109833.7	6904.6		0	0.84142373 1	1.067172
12	Wal-Mart	37871	14762	675000		93627	2740		0.01	1	1
13	Hitachi	91620.9	29907.2	331852		84167.1	1468.8		0	0.386057261	0.62692
14	Nippon Life Insurance	364762.5	2241.9	89690		83206.7	2426.6		0	1	1
15	Nippon Telegraph & Telephone	127077.3	42240.1	231400		81937.2	2209.1		0	0.348577853	0.619252
16	AT&T	88884	17274	299300		79609	139		0	0.270381772	0.481746
17											
18		Reference		DMU under	**15**	Efficiency					
19	**Constraints**	set		Evaluation		0.270382					
20	Assets	24032.61	$\le$	24032.613		Input-oriented CRS (RTS)					
21	Equity	3338.645	$\le$	4670.5747							
22	Employees	12922.98	$\le$	80925.264							
23	Revenue	79609	$\ge$	79609							
24	Profit	139	$\ge$	139							
25	$\Sigma\lambda$	0.481746									

Figure 13.3. Input-oriented RTS Classification Spreadsheet Model

In order to obtain the RTS classification, we need also to calculate the input-oriented VRS envelopment model. This can be achieved by using the spreadsheet model shown in Figure 1.8 (Chapter 1). We then copy the VRS efficiency scores into column L, as shown in Figure 13.4. Cells M2:M16 contain formulas based upon Theorem 13.6. The formula for cell M2 which is copied into cells M3:M16 is

```
=IF(J2=L2,"CRS",IF(AND(J2<>L2,K2<1),"IRS",IF(AND(J2<>L2,K2>1),"
DRS")))
```

To obtain the output-oriented RTS classification, we use the spreadsheet for output-oriented CRS envelopment model. Figure 13.5 shows the spreadsheet, and Figure 13.6 shows the Solver parameters. Note that range names are used in the spreadsheet shown in Figure 13.5 as in the spreadsheet for output-oriented VRS envelopment model shown in Figure 1.27. For example, cell E18 is named as “DMU”, cell F19 is named as “Efficiency”, and cell B25 is named as “SumLambda”. The button “Output-oriented CRS” is linked to a VBA procedure “GeneralRTS” which automates the calculation, and records the efficiency score and $\sum_j^n \lambda_j^*$ into columns J and K, respectively.

```
Sub GeneralRTS()
Dim NDMUs As Integer, NInputs As Integer, NOutputs As Integer
```

```
    NDMUs = 15
    NInputs = 3
    NOutputs = 2
    Dim i As Integer
    For i = 1 To NDMUs
    Range("DMU") = i
    SolverSolve UserFinish:=True
Range("A1").Offset(i,NInputs+NOutputs+4) = Range("Efficiency")
Range("A1").Offset(i, NInputs+NOutputs+5) = Range("SumLambda")
    Next
End Sub
```

M2 =IF(J2=L2,"CRS",IF(AND(J2<>L2,K2<1),"IRS",IF(AND(J2<>L2,K2>1),"DRS")))

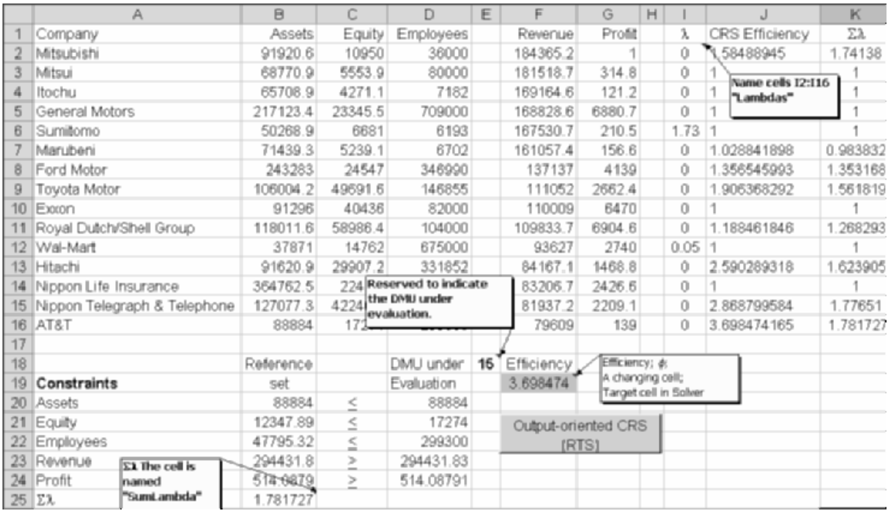
	J	K	L	M	N	O	P
1	CRS Efficiency	Σλ	VRS Efficiency	RTS	Company		
2	0.662831738	1.101942	1	DRS	Mitsubishi		
3	1	1	1	CRS	Mitsui		
4	1	1	1	CRS	Itochu		
5	1	1	1	CRS	General Motors		
6	1	1	1	CRS	Sumitomo		
7	0.971966637	0.956252	1	IRS	Marubeni		
8	0.737166307	0.99751	0.737555958	IRS	Ford Motor		
9	0.524557613	0.819264	0.603245345	IRS	Toyota Motor		
10	1	1	1	CRS	Exxon		
11	0.841423731	1.067172	1	DRS	Royal Dutch/Shell Group		
12	1	1	1	CRS	Wal-Mart		
13	0.386057261	0.62692	0.557595838	IRS	Hitachi		
14	1	1	1	CRS	Nippon Life Insurance		
15	0.348577853	0.619252	0.470610997	IRS	Nippon Telegraph & Telephone		
16	0.270381772	0.481746	0.533543522	IRS	AT&T		

Figure 13.4. Input-oriented RTS Classification

Note that we can assign “RTS” to the button “Output-oriented CRS (RTS)”. In fact, when the range names are used, Range(“DMU”), Range(“Efficiency”), and Range(“SumLambda”) are equivalent to Range(“E18”), Range(“F19”), and Range(“B25”), respectively. The procedure “GeneralRTS” can be applied to other data sets with the range names.

With the output-oriented VRS efficiency scores and Theorem 13.6, we can obtain the output-oriented RTS classification shown in Figure 13.7.

Based upon Figures 13.4 and 13.7, we obtain the RTS regions (see column O in Figure 13.7).



	A	B	C	D	E	F	G	H	I	J	K
1	Company	Assets	Equity	Employees		Revenue	Profit		λ	CRS Efficiency	Σλ
2	Mitsubishi	91920.6	10950	36000		184365.2	1		0	1.58488945	1.74138
3	Mitsui	68770.9	5553.9	80000		181518.7	314.8		0	1	1
4	Itochu	65708.9	4271.1	7182		169164.6	121.2		0	1	1
5	General Motors	217123.4	23345.5	709000		168828.6	6880.7		0	1	1
6	Sumitomo	50268.9	6681	6193		167530.7	210.5		1.73	1	1
7	Marubeni	71439.3	5239.1	6702		161057.4	156.6		0	1.028841898	0.983832
8	Ford Motor	243283	24547	346990		137137	4139		0	1.356545993	1.353168
9	Toyota Motor	106004.2	49691.6	146855		111052	2662.4		0	1.906368292	1.561819
10	Exxon	91296	40436	82000		110009	6470		0	1	1
11	Royal Dutch/Shell Group	118011.6	58986.4	104000		109833.7	6904.6		0	1.188461846	1.268293
12	Wal-Mart	37871	14762	675000		93627	2740		0.05	1	1
13	Hitachi	91620.9	29907.2	331852		84167.1	1468.8		0	2.590289318	1.623905
14	Nippon Life Insurance	364762.5	224[illegible]	[illegible]		83206.7	2426.6		0	1	1
15	Nippon Telegraph & Telephone	127077.3	4224[illegible]	[illegible]		81937.2	2209.1		0	2.868799584	1.77651
16	AT&T	88884	172[illegible]	[illegible]		79609	139		0	3.698474165	1.781727
17											
18		Reference		DMU under	15	Efficiency					
19	**Constraints**	set		Evaluation		3.698474					
20	Assets	88884	≤	88884							
21	Equity	12347.89	≤	17274		Output-oriented CRS [RTS]					
22	Employees	47795.32	≤	299300							
23	Revenue	294431.8	≥	294431.83							
24	Profit	514.0879	≥	514.08791							
25	Σλ	1.781727									

Figure 13.5. Output-oriented RTS Classification Spreadsheet Model

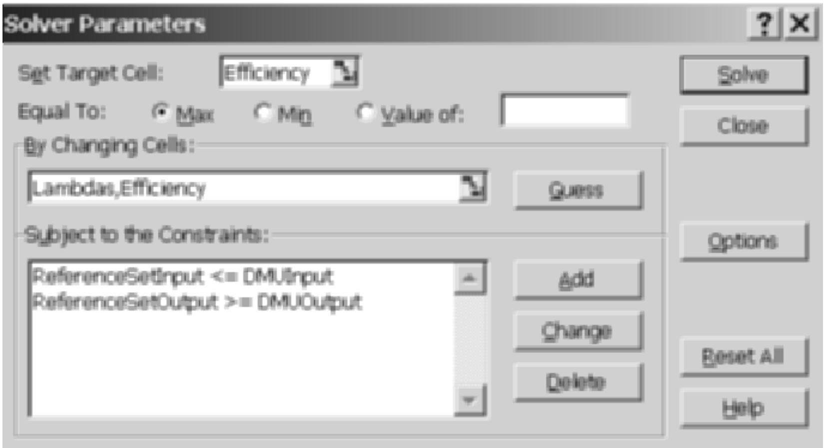


Figure 13.6. Solver Parameters for Output-oriented CRS Envelopment Model

M2 =IF(J2=L2,"CRS",IF(AND(J2<>L2,K2<1),"IRS",IF(AND(J2<>L2,K2>1),"DRS")))

	J	K	L	M	N	O	P
1	CRS Efficiency	Σλ	VRS Efficiency	RTS	Company	RTS Region	
2	1.58488945	1.74138	1	DRS	Mitsubishi	Region III	
3	1	1	1	CRS	Mitsui	Region II	
4	1	1	1	CRS	Itochu	Region II	
5	1	1	1	CRS	General Motors	Region II	
6	1	1	1	CRS	Sumitomo	Region II	
7	1.028841898	0.983832	1	IRS	Marubeni	Region I	
8	1.356545993	1.353168	1.158414974	DRS	Ford Motor	Region VI	
9	1.906368292	1.561819	1.371588284	DRS	Toyota Motor	Region VI	
10	1	1	1	CRS	Exxon	Region II	
11	1.188461846	1.268293	1	DRS	Royal Dutch/Shell Group	Region III	
12	1	1	1	CRS	Wal-Mart	Region II	
13	2.590289318	1.623905	1.898938938	DRS	Hitachi	Region VI	
14	1	1	1	CRS	Nippon Life Insurance	Region II	
15	2.868799584	1.77651	1.892916538	DRS	Nippon Telegraph & Telephone	Region VI	
16	3.698474165	1.781727	2.311193684	DRS	AT&T	Region VI	

Figure 13.7. Output-oriented RTS Classification

13.4 Scale Efficient Targets

By using the most productive scale size (MPSS) concept (Banker, 1984), we can develop linear programming problems to set unique scale efficient target. Consider the following linear program when the input-oriented CRS envelopment model is solved (Zhu, 2000b).

$$\begin{aligned}
&\min \sum_{j=1}^{n} \lambda_j \\
&\text{subject to} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta^* x_{io} \qquad i = 1,2,\dots,m; \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \qquad r = 1,2,\dots,s; \\
&\lambda_j \ge 0. \qquad j = 1,2,\dots,n.
\end{aligned} \tag{13.1}$$

where θ^* is the input-oriented CRS efficiency score.

Based upon the optimal values from (13.1) (i.e., $\sum \lambda_j^*$), the MPSS concept yields the following scale-efficient target for DMU_o corresponding to the largest MPSS

$$\text{MPSS}_{\max}: \begin{cases} \tilde{x}_{io} = \theta^* x_{io} / \sum \lambda_j^* \\ \tilde{y}_{ro} = y_{ro} / \sum \lambda_j^* \end{cases} \tag{13.2}$$

where (~) represents the target value.

If we change the objective of (13.1) to maximization,

$$
\begin{aligned}
& \max \sum_{j=1}^{n} \hat{\lambda}_j \\
& \text{subject to} \\
& \sum_{j=1}^{n} \hat{\lambda}_j x_{ij} \le \theta^* x_{io} \qquad i = 1,2,\ldots,m; \\
& \sum_{j=1}^{n} \hat{\lambda}_j y_{rj} \ge y_{ro} \qquad r = 1,2,\ldots,s; \\
& \hat{\lambda}_j \ge 0. \qquad j = 1,2,\ldots,n.
\end{aligned} \tag{13.3}
$$

then we have the scale efficient target corresponding to the smallest MPSS.

$$
\text{MPSS}_{\min}: \begin{cases} \tilde{x}_{io} = \theta^* x_{io} / \sum \hat{\lambda}_j^* \\ \tilde{y}_{ro} = y_{ro} / \sum \hat{\lambda}_j^* \end{cases} \tag{13.4}
$$

Note that models (13.1) and (13.3) are based upon the input-oriented CRS envelopment model. However, by using the relationship between the input-oriented and output-oriented CRS envelopment models (see Lemma 13.2), it is trivial to show that $\text{MPSS}_{\max}$ ($\text{MPSS}_{\min}$) remains the same under both orientations. Consequently, $\text{MPSS}_{\max}$ and $\text{MPSS}_{\min}$ are uniquely determined by θ^* and $\sum \lambda_j^*$ ($\sum \hat{\lambda}_j^*$).

We can select the largest or the smallest MPSS target for a particular DMU under consideration based upon the RTS preference over performance improvement. For example, one may select the smallest MPSS for an IRS DMU and the largest MPSS for a DRS DMU. Further, if the CRS envelopment models yield the unique optimal solutions, then the $\text{MPSS}_{\max}$ and $\text{MPSS}_{\min}$ are the same.

The spreadsheet model for calculating the scale efficient target involves (i) calculating CRS envelopment model, and (ii) calculating model (13.1). We demonstrate (ii) using the input-oriented CRS envelopment model shown in Figure 13.3.

In Figure 13.8, the target cell is B25, and contains the formula "=SUM(I2:I16)", representing the $\sum \lambda_j^*$. Cell F19 is no longer a changing cell, and contains the formula "=INDEX(J2:J16,E18,1)". This formula returns the CRS efficiency score of a DMU under evaluation from column J.

The changing cells are I2:I16. The constraints in the Solver parameters for the input-oriented CRS envelopment model shown in Figure 1.24 remain the same. Figure 13.8 also shows the Solver parameters for calculating the model (13.1). Select "Max" if model (13.3) is used.

To automate the computation, we remove the statement Range("J"& i+1)=Range("F19") from the procedure "RTS", and name the new procedure "MPSS".

```
Sub MPSS()
  Dim i As Integer
    For i = 1 To 15
'set the value of cell E18 equal to i (1, 2,..., 15)
    Range("E18") = i
'Run the Solver model. The UserFinish is set to True so that
'the Solver Results dialog box will not be shown
    SolverSolve UserFinish:=True
'Place the sum of lambdas into column K
     Range("K" & i + 1) = Range("B25")
    Next i
End Sub
```

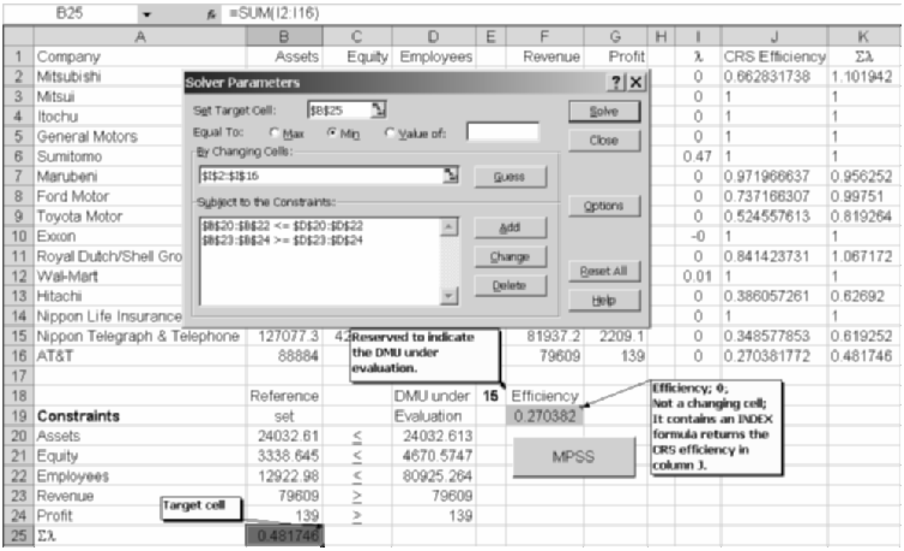


Figure 13.8. Largest MPSS Spreadsheet Model

It can be seen that the maximum $\sum \lambda_j^*$ is the same as that obtained from the input-oriented CRS envelopment model shown in Figure 13.3. This is due to the fact that we have unique optimal solutions on λ_j^*. As a result, minimum $\sum \hat{\lambda}_j^*$ = maximum $\sum \lambda_j^*$. We can apply (13.2) or (13.4) to obtain the scale efficient targets for the 15 DMUs.

13.5 Solving DEA Using DEAFrontier Software

RTS Estimation can be found at the Returns-to-Scale menu item, as shown in Figure 13.9.

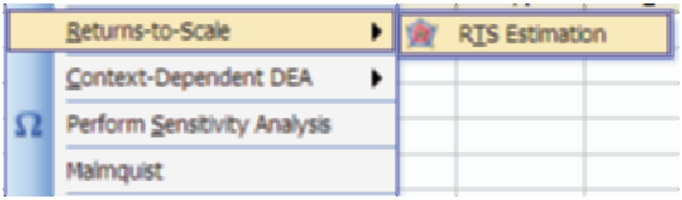


Figure 13.9. Returns-to-Scale Menu

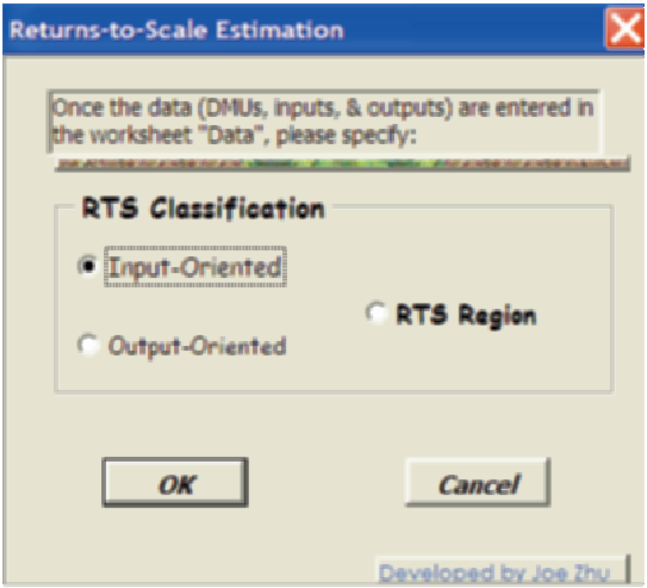


Figure 13.10. RTS Estimation

The RTS Estimation menu will provide (i) the RTS classifications, and (ii) RTS regions as shown in Figure 13.2. (see Figure 13.10).

If RTS Region is selected, the software will run both the input-oriented and output-oriented envelopment models. The results are reported in the "RTS Region" sheet.

If Input-Oriented is selected, then the software will generate the RTS classification based upon the input-oriented envelopment models and report the results in the sheet "RTS Report". If Output-Oriented is selected, then the software will generate the RTS classification based upon the output-oriented envelopment models.

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