Chapter 12

Identifying Critical Measures in DEA

12.1 Introduction

Since each DMU has its own inherent tradeoffs among the multiple measures that significantly influence the performance, it is extremely important for the management to know the critical measures. The current chapter introduces the approach of Chen and Zhu (2003) for identifying the critical measures to DMUs' performance. Note that once the DEA evaluation is done, the management needs to either (i) maintain the best practice for the efficient DMUs or (ii) achieve the best practice for the inefficient DMUs. Thus, when a set of multiple performance measures is determined, measures that are influential to maintaining and achieving the best practice should be regarded as critical to the performance of DMUs. Also, it is believed that a critical measure is signaled by whether changes in its value affect the performance, not by whether inclusion or exclusion of the measure affects the performance. Under the framework of DEA sensitivity analysis, Chen and Zhu (2003) develop an alternative approach, which is independent of identifying DEA weights or DEA multipliers, to identify such critical measures.

12.2 Performance Evaluation and DEA

Regression-based methods can be used in evaluating performance of a set of DMUs. However, they are limited to only one dependent variable. For example,

$$y = \beta_o + \sum_{i=1}^{m} \beta_i x_i + \varepsilon \qquad (12.1)$$

where β_i are estimated coefficients which can be used to determine whether an independent variable has a positive effect on the dependent variable or makes an important contribution. i.e., by estimating the coefficients, we may identify the critical performance measures under the context of average behavior. Also, the estimated regression line can be served as the benchmark in performance evaluation.

In fact, formula (12.1) can be viewed as a performance frontier or tradeoff curve where x_i are inputs and y is the output. However, we are very likely to have multiple outputs y_r (r = 1, …, s). We may rewrite (12.1) as (Wilkens and Zhu, 2001)

$$\sum_{r=1}^{s} u_r y_r = \alpha + \sum_{i=1}^{m} v_i x_i \tag{12.2}$$

where u_r and v_i are unknown weights representing the relative importance or tradeoffs among y_r and x_i .

Suppose we can estimate u_r and v_i , then for each DMU_j , we can define

$$h_j = \frac{\alpha + \sum_{i=1}^{m} v_i x_{ij}}{\sum_{r=1}^{s} u_r y_{rj}} \tag{12.3}$$

as a performance index, where x_{ij}, (i = 1, 2, ..., m) are multiple inputs, y_{rj}, (r = 1, 2, ..., s) are multiple outputs for DMU_j (j = 1, 2, …, n).

In order to estimate u_r and v_i , and further evaluate the performance of j_o th DMU, (denoted as DMU_o) by (12.2), DEA uses the following linear fractional programming problem

$$\min_{\alpha, v_i, u_r} \frac{\alpha + \sum_{i=1}^{m} v_i x_{io}}{\sum_{r=1}^{s} u_r y_{ro}}$$

subject to (12.4)

$$\frac{\alpha + \sum_{i=1}^{m} v_i x_{ij}}{\sum_{r=1}^{s} u_r y_{rj}} \geq 1, j = 1, \ldots, n$$

$$u_r,\ v_i \geq 0 \quad \forall\ r, i$$

where, x_{io} and y_{ro} are respectively the ith input and rth output for DMU_o under evaluation.

When h_o^* = 1, DMU_o is efficient or on the performance frontier. Otherwise, if h_o^* > 1, then DMU_o is inefficient. All the efficient DMUs constitute the performance frontier.

Note that when h_o^* = 1, we have

$$\sum_{r=1}^{s} u_r^* y_{ro} = \alpha^* + \sum_{i=1}^{m} v_i^* x_{io} \tag{12.5}$$

where (*) represents optimal values in model (12.4). That is, DEA estimates the "coefficients" in (12.2). It can be seen that while (12.1) estimates one set of coefficients, DEA model (12.4) estimates one set of coefficients for each DMU, resulting a piecewise linear tradeoff curve represented by several (12.5)-like equations associated with efficient DMUs. Equation (12.5) is theoretically available, but very difficult to obtain empirically.

Obviously, u_r^* and v_i^* represent the tradeoffs among various outputs and inputs. If we can obtain the exact information on u_r^* and v_i^*, the critical performance measures can be easily identified. However, the exact information on u_r^* and v_i^* cannot be obtained because of multiple optimal solutions in the multiplier models.

However, in order to solve model (4), the following transformation is used

$$\mathrm{t} = \frac{1}{\sum_{r=1}^{s} u_r y_{ro}}, \ \omega_i = \mathrm{t} v_i, \ \omega_o = \mathrm{t}\alpha, \ \mu_r = \mathrm{t} u_r \tag{12.6}$$

Based upon (12.6), model (12.4) is solved in the following equivalent linear programming problem (VRS multiplier model, see chapter 1)

$$\begin{aligned} &\min_{\omega_o, \omega_i, \mu_r} \ \omega_o + \sum_{i=1}^{m} \omega_i x_{io} \\ &\text{subject to} \\ &\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \omega_i x_{ij} - \omega_o \le 0 \ \ \forall\ j \\ &\sum_{r=1}^{s} \mu_r y_{ro} = 1 \\ &\mu_r,\ \omega_i \ge 0 \quad \forall\ r, i \end{aligned} \tag{12.7}$$

or the dual to model (12.7) (VRS envelopment model, see chapter 1)

$$
\begin{aligned}
&\varphi_o^* = \max \varphi_o \\
&subject \quad to \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} && i = 1,2,...,m; \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_o y_{ro} && r = 1,2,...,s; \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 && j = 1,...,n.
\end{aligned} \tag{12.8}
$$

Based upon (12.6), we have $\frac{\omega_i}{\omega_k} = \frac{v_i}{v_k}$ and $\frac{\mu_r}{\mu_d} = \frac{u_r}{u_d}$. Thus, μ_r^* and ω_i^* are not the exact weights representing the tradeoffs in model (4). In addition, for efficient DMUs, model (12.7) often yields multiple optimal solutions on multipliers μ_r and ω_i. Also, $\sum_{r=1}^{s} \mu_r^* y_{rj} - \sum_{i=1}^{m} \omega_i^* x_{ij} - \omega_o^* = 0$ may only represent supporting hyperplanes rather than the performance frontier in empirical studies. This further leads to an incomplete tradeoff information. Because of possible multiple optimal solutions in (12.7) and the transformation in (12.6), it is very difficult to back out the tradeoffs represented by u_r^* and v_i^* in model (12.4), i.e., the performance frontier expressed by (12.5) is very difficult to obtain in empirical applications. Chen and Zhu (2003) therefore develop an alternative approach to identifying the critical measures.

Suppose that we obtain the performance frontier. In this case, for example $v_k^* > v_i^*$ indicates that the *k*th input measure is more influential in order for DMU_o to achieve the best-practice. i.e., the *k*th input is more important to DMU_o's performance which is characterized by the efficiency score (h_o^*). Note also that the DEA model (12.4) always tries to assign larger v_i and u_r to smaller x_{io} and larger y_{ro} respectively in order to achieve the optimality. This indicates that when a set of multiple performance measures (inputs and outputs) is determined, the relative importance or tradeoffs is determined by the magnitudes of the inputs and outputs.

It can be seen from model (12.4) that for a specific DMU under evaluation, when a specific input increases, the associated input weight will not increase and when a specific output decreases, the associated output weight will not increase. Consider the frontier represented by ABC in Figure 1 with two inputs and a single output. In Figure 12.1, $v_1 > v_2$ remains true for facet AB if DMU A's x_2 (uncritical one) changes its value, and $v_2 > v_1$

remains true for facet BC if DMU C's x_1 (uncritical one) changes its value. Meanwhile, DMUs A and C remain efficient when the uncritical inputs changes their value, respectively[1]. However, if we increase the x_1 of DMU A or x_2 of DMU C to a certain level, DMU A or DMU C becomes inefficient.

The example in Figure 12.2 indicates that (a) for efficient DMUs, the performance is determined and characterized by the best-practice status, and (b) for inefficient DMUs, the performance is determined and characterized by the distance to the frontier. Thus, a measure that is critical to the performance should be characterized by whether the measure is critical to (i) maintaining the best-practice for efficient DMUs and (ii) achieving the best-practice for inefficient DMUs.

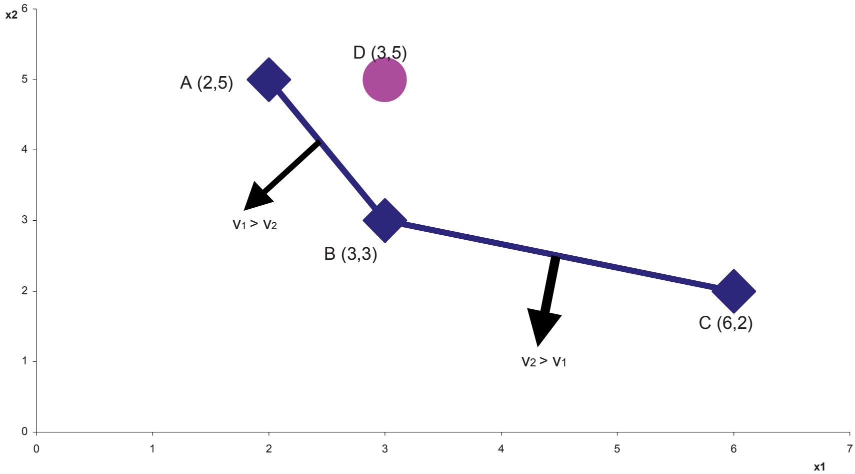

Figure 12.1. Critical Measures and Tradeoffs

The example in Figure 12.1 indicates that (a) for efficient DMUs, the performance is determined and characterized by the best-practice status, and (b) for inefficient DMUs, the performance is determined and characterized by the distance to the frontier. Thus, a measure that is critical to the performance should be characterized by whether the measure is critical to (i) maintaining the best-practice for efficient DMUs and (ii) achieving the best-practice for inefficient DMUs.

[1] Note that for example, if the second input of DMU A decreases its current level to 3, the level used by DMU B, then we no longer have the efficient facet AB. Since DMU B becomes inefficient.

Because a set of multiple performance measures is given prior to the evaluation, a critical measure is signaled by whether changes in its value affect the performance, not by whether inclusion or exclusion of the measure affects the performance.

Definition 12.1 When a set of multiple performance measures is given, a specific measure is said to be critical if changes in its value may alter the efficiency status of a specific DMU.

For efficient DMUs, the performance is determined and characterized by the best practice status. For inefficient DMUs, the performance is determined and characterized by the distance to the frontier. Thus, a measure that is critical to the performance should be characterized by whether the measure is critical to (i) maintaining the best practice for efficient DMUs and (ii) achieving the best practice for inefficient DMUs.

12.3 Identifying Critical Output Measures

Consider the following super-efficiency model where the dth output is given the pre-emptive priority to change

$$
\begin{aligned}
&\max \sigma_d \\
&\text{subject to} \\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{dj} \geq \sigma_d y_{do} \\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r \neq d \\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leq x_{io} \qquad i = 1,\ldots,m \\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j = 1
\end{aligned}
\tag{12.6}
$$

Four possible cases are associated with (12.6): (i) $\sigma_d^* > 1$, (ii) $\sigma_d^* = 1$, (iii) $\sigma_d^* < 1$ and (iv) model (12.6) is infeasible. When $\sigma_d^* > 1$, DMU_o has inefficiency in its dth output, since potential output increase can be achieved by DMU_o. Cases (ii), (iii) and (iv) indicate that no inefficiency exists in dth output.

Now, we consider the efficient DMUs and assume that DMU_o is efficient. Based upon model (12.6) the set of s outputs can be grouped into two subsets: set $O = \{d$: $\sigma_d^* \leq 1\}$ and set $\overline{O} = \{d$: model (12.6) is infeasible for dth output$\}$.

We have when model (12.6) is infeasible, the magnitude of the *d*th output across all DMUs has nothing to do with the efficiency status of DMU_o.

This indicates that the outputs in set $\overline{O}$ are not critical to the efficiency status of DMU_o, since changes in the outputs in set $\overline{O}$ do not change the efficiency classification of DMU_o. The efficiency classification of DMU_o is stable to any changes in the *d*th output across all DMUs when *d* belongs to set $\overline{O}$.

However, decreases in outputs in set *O* to certain magnitudes result in a change of efficiency status (performance) of DMU_o. For example, when the *d*th output of DMU_o is decreased from the current level y_{do} to a level which is less than $\sigma_d^* \; y_{do}$ ($\sigma_d^* < 1$), then DMU_o becomes inefficient. This in turn indicates that the outputs in set *O* are critical to the performance of DMU_o.

Now, let $\mathrm{P}_{d*} = \max\{\sigma_d^*\}$ for the outputs in set *O*. From the above discussion, we conclude that the *d**th output is the most critical output measure to the efficiency of DMU_o. Because, DMU_o's efficiency status is most sensitive to changes in the *d**th output.

Next, we consider inefficient DMUs and assume that DMU_o is inefficient. For inefficient DMUs, the issue is how to improve the inefficiency to achieve the best practice. Since the focus here is how each individual output measure contributes to the performance of DMU_o, we solve model (12.6) for each *d* and obtain $\sigma_d^* > 1$ (d = 1, …, d), where σ_d^* measures how far DMU_o is from the frontier in terms of *d*th output.

As a matter of fact, model (12.6) provides an alternative way to characterize the inefficiency of DMU_o. Each σ_d^* indicates possible inefficiency existing in each associated output when other outputs and inputs are fixed at their current levels. We then can rank the inefficiency by each optimal σ_d^*. Let $G_{d*} = \min\{\sigma_d^*\}$. That is, the *d**th output indicates the least inefficiency. If the DMU_o is to improve its performance through single output improvement, the *d**th output will yield the most effective way. Because G_{d*} represents the shortest path onto the best practice frontier when each output is given the pre-emptive priority to improve. We therefore define that the *d**th output is the most critical output to reach the performance frontier and to DMU_o's performance.

In summary, the critical output is identified as the output associated with $\max\{\sigma_d^*\}$ for efficient DMUs and $\min\{\sigma_d^*\}$ for inefficient DMUs.

12.4 Identifying Critical Input Measures

Consider the following super-efficiency model when the *k*th input measure is of interest.

$$
\begin{aligned}
&\min \tau_k \\
&\text{subject to} \\
&\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{kj} \leq \tau_k x_{ko} \\
&\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij} \leq x_{io} \qquad i \neq k \\
&\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1, \ldots, s \\
&\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j = 1
\end{aligned}
\tag{12.7}
$$

Based upon model (12.7), we have (i) $\tau_k^* < 1$, (ii) $\tau_k^* = 1$, (iii) $\tau_k^* > 1$, and (iv) (12.7) is infeasible. Case (i) indicates that inefficiency exists in DMU_o's *k*th input, since DMU_o needs to decrease its *k*th input to $\tau_k^*\, x_{ko}$ in order to reach the performance frontier. Cases (ii), (iii) and (iv) indicate that no inefficiency exists in DMU_o's *k*th input.

Now, suppose DMU_o is efficient. Based upon model (12.7), the set of *m* inputs can be grouped into two subsets: set I = {*k*: $\tau_k^* \geq 1$} and set $\bar{I}$ = {*k*: model (12.7) is infeasible for *k*th input}.

We have when model (12.7) is infeasible, the magnitude of the *k*th input across all DMUs has nothing to do with the efficiency status of DMU_o.

This indicates that the inputs in set $\bar{I}$ are not critical to the efficiency status of DMU_o, since changes in the inputs in set $\bar{I}$ do not change the efficiency classification of DMU_o. Let $\mathrm{T}_{k^*} = \min\{\tau_k^*\}$ for inputs in set I. We conclude that the k^*th input is the most critical input measure to the efficiency of DMU_o. Because, DMU_o's efficiency status is most sensitive to changes in the k^*th input.

Next, suppose DMU_o is inefficient. We solve model (12.7) for each *k* and obtain $\tau_k^* < 1$ ($k = 1, \ldots, m$), where τ_k^* measures how far DMU_o is from the frontier in terms of *k*th input. Each τ_k^* indicates possible inefficiency existing in each associated input when other inputs and outputs are fixed at their current levels. We then can rank the inefficiency by each optimal τ_k^*. Let $H_{k^*} = \max_k \{\tau_k^*\}$. Similar to the discussion on identifying the critical output measure, we say that the k^* th input is the most critical input to reach the performance frontier and to DMU_o's performance, since the k^*th input indicates the least inefficiency.

In summary, the critical input is identified as the input associated with $\min\{\tau_k^*\}$ for efficient DMUs and $\max\{\tau_k^*\}$ for inefficient DMUs.

12.5 Numerical Example and Extension

To further illustrate the rationale of the approach, consider again the four DMUs shown in Figure 12.1. Table 12.1 reports the optimal value to model (12.7). It can be seen that for DMU D, the first input is the critical measure since DMU D's efficiency can be easily improved if the first input is given the pre-emptive priority to change. For DMU A, the infeasibility associated with the second input indicates that the first input is the critical measure. Note that the efficient facet AB shows that the first input is more important than the second one, since $v_1 > v_2$. Our approach also indicates that the second input is the critical measure to DMU C's performance. This finding is confirmed by the fact that $v_2 > v_1$ in BC. As for DMU B, since it is located at the intersect of AB and BC, it is very difficult to determine which input is the critical factor by looking at the coefficients of efficient facets. Our approach indicates that the second input is the critical one for DMU B, since $\tau_2^* < \tau_1^*$ (17/12 < 14/9).

Table 12.1. Critical Measures for the Numerical Example

DMU	τ_1^*	τ_2^*
A	3/2	infeasible
B	14/9	17/12
C	infeasible	2
D	2/3	3/5

The above discussion assumes that DMUs are able to adjust each input and each output while other inputs and outputs are fixed. Situations when some measures are strongly related with each other may occur. In that case, a set of inputs or outputs has to be adjusted simultaneously and we need to consider the measures in groups. We use the following models.

$$
\begin{aligned}
&\min \mathrm{T}_M \\
s.t.\quad & \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \le \mathrm{T}_M x_{io} \qquad i \in M \\
& \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \le x_{io} \qquad i \notin M \\
& \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \le y_{ro} \qquad r = 1,\ldots,s \\
& \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j = 1
\end{aligned}
\tag{12.8}
$$

and

$$
\begin{aligned}
&\max \Omega_Q \\
s.t.\quad & \sum_{j=1}^{n} \lambda_j y_{rj} \ge \Omega_Q y_{ro} \qquad r \in Q \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \qquad r \notin Q \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} \qquad i = 1,\ldots,m \\
& \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j = 1
\end{aligned}
\tag{12.9}
$$

where inputs represented by set M and outputs represented by set Q are of interest.

Similar to the previous discussions, when DMU_o is inefficient, we use $\max\{T_M^*\}$ and $\min\{\Omega_Q^*\}$ to identify the most critical input and output measures, respectively. When DMU_o is efficient, infeasibility associated with (12.8) and (12.9) indicates the non-critical inputs and outputs.

The above discussion is based upon the assumption that the DEA frontier exhibits VRS. The development can be applied to other DEA models with non-VRS frontiers discussed in Chapter 11.

12.6 Application to Fortune E-Companies

To capture the Internet's effect on the economy, at the end of year 1999, Fortune magazine launched the Fortune e-50 index which consists of 50 corporations who integrate the Internet, computers and enterprise softwares to do the business. As stated in the 1999 December Fortune issue, each of the e-50 is or has the potential to be a major player in the Internet economy. The list of e-corporation is decided by that a company must have been public for at least six months and must have a market capital value that exceeds $100 million. Table 12.2 provides the list of the e-50.

Table 12.2. Fortune's e-corporations

DMU No.	Name	Revenue $ millions	Profits $ millions	Employees	Market Capital $ millions	Year Founded
E-COMPANIES						
1	America Online	4777	762	12100	164308	1985
2	Charles Schwab	4113	498	13300	34194	1986
3	Amazon.com	1015	-291	2100	21202	1994
4	E*Trade Group	621	-54	1735	8341	1982
5	Knight/Trimark Group	618	119	446	4389	1995
6	Yahoo	341	22	803	47946	1995
7	Ameritrade Holding	301	12	985	3740	1992
8	EarthLink Network	254	-88	1343	1409	1994
9	Priceline.com	189	-125	194	7963	1998
10	CMGI	176	476	1024	12567	1986
11	Lycos	136	-52	456	5687	1995
12	Excite@Home	129	-324	570	14647	1995
13	eBay	125	7	138	17106	1995
14	DoubleClick	103	-22	482	5947	1996
15	RealNetworks	89	-4	434	9148	1994
16	CNet	79	40	491	3481	1995
17	Healtheon	68	-68	648	2347	1995
18	eToys	38	-47	306	6276	1996
19	VerticalNet	8	-21	220	2515	1995
NET SOFTWARE AND SERVICE COMPANIES						
20	Microsoft	19747	7785	31396	471573	1975
21	Oracle	9063	1332	44000	85776	1977
22	Intuit	848	377	3675	5942	1983
23	Network Associates	785	-127	2700	2871	1992
24	Cambridge Tech. Partners	628	35	4444	726	1991
25	TMP Worldwide	585	10	5200	2976	1967
26	Ariba	45.4	*	*	*	1996
27	Citrix Systems	323	93	620	7169	1989
28	Macromedia	167	24	553	2690	1992
29	Network Solutions	142	17	385	4801	1979
30	Concentric Network	110	-82	508	1054	1991
31	Exodus Communications	108	-82	472	7080	1992
32	BroadVision	71	10	271	6777	1993
33	Inktomi	71	-24	185	5709	1996
34	Security First Technologies	44	-19	312	1345	1995
35	Razorfish	36	2	414	1896	1995

Table 12.2 Fortune's e-corporations (continued)

NET HARDWARE COMPANIES						
36	IBM	87448	7701	291067	167567	1911
37	Lucent Technologies	38303	4766	153000	211415	1995
38	Intel	28194	7371	64500	285803	1968
39	Dell Computer	21670	1750	24400	110530	1984
40	Cisco Systems	12154	2096	21000	237215	1984
41	Sun Microsystems	11726	1031	29700	85861	1982
42	EMC	4459	967	9700	75371	1979
43	Qualcomm	3937	201	11600	43919	1981
44	Network Appliance	335	42	816	6327	1992
45	Broadcom	335	40	436	15994	1991
46	Juniper Networks	31	-30	190	14455	1992
NET COMMUNICATION COMPANIES						
47	AT&T	56968	6037	107800	154791	1875
48	MCI WorldCom	30720	-883	77000	162492	1983
49	Qwest Communications	3424	-5	8700	27404	1997
50	Global Crossing	691	79	10000	26109	1997

Market capital, profit, revenue and number of employees are provided by the Fortune as the four standard measures to fully characterize the performance of the e-50 corporations. We therefore use them as a set of multiple performance measures. The data on profit, employee and market capital are not available for Ariba (DMU26), and therefore Ariba is excluded from the following analysis.

Because we are interested in the contribution of revenue, profit and employee to the market value, we select the market capital as the DEA output and the other measures as the DEA inputs. Output-oriented DEA model is used, because higher market values are desirable given the current levels of revenue, profit and the number of employees.

The third column of Table 12.3 reports the optimal value to the output-oriented VRS envelopment model. Ten e-corporations are on the performance frontier.

Next, we apply the newly developed method to identify the critical input measures to the market capital under the context of best-practice. Columns 3, 4 and 5 of Table 12.4 report the results from model (12.7).

We use the DEAFrontier software to do the calculation. Once the data are entered into the "data" sheet, we select "Perform Sensitivity Analysis" and then select the input as shown in Figure 12.2. The results are reported in the "Sensitivity Report" sheet. We can select one input at a time.

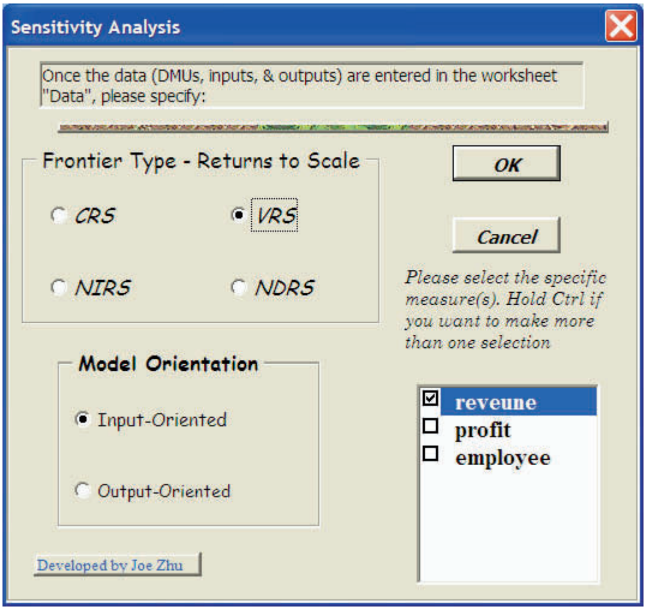

Figure 12.2. Critical Measures and Tradeoffs

For example, consider MCI WorldCom (DMU48), model (12.7) is infeasible when revenue and employee are under consideration (selected) respectively and model (12.7) yields the optimal value of 96.98 when profit is under consideration. This indicates that once the three input measures are determined, the magnitudes of revenue and employee do not affect the efficiency status of MCI WorldCom. However, the value of profit affects MCI WorldCom's efficiency status given the current levels of market value, revenue and employee. Thus, profit is the critical factor to MCI WorldCom's performance.

Consider Charles Schwab (DMU2) which is an inefficient unit. The optimal values to model (12.7) indicate that the profit measure is the critical one for Charles Schwab to achieve the performance frontier.

The sixth column of Table 4 reports the critical measure identified on the basis of model (12.7). However, for efficient DMUs, it is likely that model (12.7) is infeasible for each input measure. Samples can be found in America Online (DMU1), Yahoo (DMU6) and Microsoft (DMU20). This may imply that some measures must be considered in groups. We therefore employ model (12.8) for all possible combinations of the three input measures. The last column of Table 4 reports the results based upon model (12.8). Note that model (12.8) is not applied to the inefficient DMUs.

Table 12.3. Performance evaluation of Fortune's e-corporations

DMU No.	Name	VRS
1	America Online	1.00000
2	Charles Schwab	3.83409
3	Amazon.com	1.05723
4	E*Trade Group	5.31514
5	Knight/Trimark Group	7.15192
6	Yahoo	1.00000
7	Ameritrade Holding	11.63487
8	EarthLink Network	25.09020
9	Priceline.com	1.00000
10	CMGI	2.39677
11	Lycos	4.30319
12	Excite@Home	1.00000
13	eBay	1.00000
14	DoubleClick	3.71566
15	RealNetworks	2.26509
16	CNet	5.64226
17	Healtheon	7.23136
18	eToys	2.32675
19	VerticalNet	1.00000
20	Microsoft	1.00000
21	Oracle	2.31196
22	Intuit	10.30718
23	Network Associates	13.81890
24	Cambridge Tech. Partners	72.12135
25	TMP Worldwide	16.68021
27	Citrix Systems	5.50414
28	Macromedia	10.83562
29	Network Solutions	5.34769
30	Concentric Network	19.73886
31	Exodus Communications	2.90959
32	BroadVision	2.74452
33	Inktomi	2.77061
34	Security First Technologies	11.79142
35	Razorfish	7.90885
36	IBM	2.79636
37	Lucent Technologies	1.72137
38	Intel	1.59834
39	Dell Computer	2.03503
40	Cisco Systems	1.00000
41	Sun Microsystems	2.24478
42	EMC	1.94255
43	Qualcomm	2.22318
44	Network Appliance	1.93360
45	Broadcom	7.47555
46	Juniper Networks	1.00000
47	AT&T	2.64384
48	MCI WorldCom	1.00000
49	Qwest Communications	2.61124
50	Global Crossing	2.18328

Table 12.4. Critical measures for Fortune's e-corporations

DMU No.	Name	revenue	profit	employee	Critical measures	
					(12.7)	(12.8)
1	America Online	infeasible	infeasible	infeasible		{profit, revenue}
2	Charles Schwab	0.0520	0.3619	0.0381	{profit}	
3	Amazon.com	0.5563	0.9817	0.7884	{profit}	
4	E*Trade Group	0.0463	0.6708	0.0945	{profit}	
5	Knight/Trimark	0.0188	0.6297	0.3094	{profit}	
6	Yahoo	infeasible	infeasible	infeasible		{profit,revenue,employee}
7	Ameritrade Holding	0.0344	0.6283	0.1401	{profit}	
8	EarthLink Network	0.1368	0.7066	0.1328	{profit}	
9	Priceline.com	infeasible	1.1200	1.5994	{profit}	{profit, revenue}
10	CMGI	0.1555	0.4180	0.1348	{profit}	
11	Lycos	0.1736	0.7581	0.3700	{profit}	
12	Excite@Home	infeasible	1.4459	infeasible	{profit}	{profit, revenue}
13	eBay	infeasible	infeasible	1.7284	{employee}	infeasible
14	DoubleClick	0.1419	0.7375	0.3361	{profit}	
15	RealNetworks	0.2335	0.7661	0.3639	{profit}	
16	CNet	0.1248	0.7460	0.3329	{profit}	
17	Healtheon	0.3937	0.8759	0.3337	{profit}	
18	eToys	0.6037	0.9469	0.6886	{profit}	
19	VerticalNet	3.8750	infeasible	infeasible	{revenue}	{profit, revenue}
20	Microsoft	infeasible	infeasible	infeasible		{profit,revenue,employee}
21	Oracle	0.1968	0.3002	0.0803	{profit}	
22	Intuit	0.0172	0.4408	0.0376	{profit}	
23	Network Associates	0.0641	0.7296	0.0733	{profit}	
24	Cambridge Tech.	0.0127	0.6063	0.0311	{profit}	
25	TMP Worldwide	0.0152	0.6238	0.0265	{profit}	
27	Citrix Systems	0.0525	0.5797	0.2226	{profit}	
28	Macromedia	0.0499	0.6331	0.2496	{profit}	
29	Network Solutions	0.0874	0.7455	0.3584	{profit}	
30	Concentric Network	0.2942	0.7667	0.3922	{profit}	
31	Exodus Comm.	0.3326	0.7938	0.4261	{profit}	
32	BroadVision	0.2283	0.8749	0.6195	{profit}	
33	Inktomi	0.5639	0.9775	0.9381	{profit}	
34	Security First Tech.	0.1818	0.9023	0.5859	{profit}	
35	Razorfish	0.2222	0.8968	0.4523	{profit}	
36	IBM	0.0564	0.0185	0.0324	{revenue}	
37	Lucent Technologies	0.1846	0.2452	0.0824	{profit}	
38	Intel	0.3794	0.4202	0.2788	{profit}	
39	Dell Computer	0.1258	0.3242	0.2181	{profit}	
40	Cisco Systems	infeasible	1.0754	infeasible	{profit}	{profit, revenue}
41	Sun Microsystems	0.1524	0.2609	0.1192	{profit}	
42	EMC	0.3110	0.4847	0.2870	{profit}	
43	Qualcomm	0.0772	0.5508	0.0617	{profit}	
44	Network Appliance	0.1351	0.7314	0.3165	{profit}	
45	Broadcom	0.0458	0.6096	0.1691	{profit}	
46	Juniper Networks	3.5327	infeasible	infeasible	{revenue}	{profit, revenue}
47	AT&T	0.0775	0.0025	0.0790	{employee}	
48	MCI WorldCom	infeasible	96.9787	infeasible	{profit}	{profit, revenue}
49	Qwest Comm.	0.0441	0.5771	0.0443	{profit}	
50	Global Crossing	0.2010	0.6601	0.0332	{revenue}	

For Yahoo and Microsoft, model (12.8) is feasible (has optimal solutions) when only all three inputs are in set M. For America Online, model (12.7) is feasible (has optimal solutions) when profit and revenue are in set M.

Model (12.8) is also applied to the remaining 7 efficient e-corporations, namely, Excite@Home (DMU12), Vertical Net (DMU19), Cisco System (DMU40), Juniper Networks (DMU46) and MCI WorldCom (DMU48). Model (12.7) is feasible when profit and revenue are in set M.

Except for America Online, Yahoo, eBay, Vertical Net, Microsoft, IBM, Juniper Networks, AT&T and Global Crossing, all the e-corporations indicate profit as their critical measure. This confirms that for the majority of the e-corporations that are rely on the Internet for business, revenue does not necessarily mean profit. In fact, about 40% of the e-corporations had negative profit in year 1999. (The negative values are treated by the translation invariance property in DEA. See Chapter 5's Appendix.)

A closer look at Table 4 indicates that America Online, Yahoo and Microsoft have distinguished themselves from the e-corporations, because the results from model (12.8) imply that their high revenue means profit. Note that among the inefficient units, employee is identified as the critical measure for eBay and AT&T, and revenue is identified as the critical measure for IBM.

The e-corporations actually represent the 21st century new economy where the electronic and information technologies are heavily used. To further illustrate the approach, we next apply models (12.7) and (12.8) to the Fortune 1000 companies in 1995 who represent old economy where the companies design, build and deliver physical, molecular-based products to customer. The purpose is to see whether the new economy e-corporations behave differently compared to the old economy companies in terms of the critical measures.

Since the e-corporations belong to computer and telecommunication industries, we exclude all those Fortune's 1000 companies who are in the computer and telecommunication industries from the analysis. We also exclude those Fortune 1000 companies who do not have complete data on the four performance measures. As a result, we have 51 industries with 760 companies which are different from the e-corporations (see the first column in Table 12.5).

Table 5 summarizes the results from the new approach. The second column reports the number of companies in each industry. The third, fourth and fifth columns report how many companies indicate revenue, profit and employee as their critical measures respectively. For example, the second row in Table 5 indicates that (i) there are 4 companies in the advertising and marketing industry, and (ii) revenue is identified as the critical measure for all companies. In the motor vehicle industry, only two companies (General

Motor and Ford) (9.52%; two out of 21) indicate that profit is the critical measure while other 19 companies indicate that revenue is the critical measure.

Our approach indicates that revenue is the critical factor to 95% of the 760 companies in the Fortune's top 1000 list. In fact, these "old-economy" companies sever relatively mature market or command a lead in markets where they compete. Our finding is consistent with the belief that revenue means a stable proportion of the profit for the old economy companies. Also, our approach does indicate that the e-corporations and the Fortune's 1000 companies behave differently.

Table 12.5. Critical measures for Fortune's 1000 companies

Industry	Companies	Revenue	Profit	Employee
Advertising, marketing	4	100%	0%	0%
Aerospace	11	90.91%	9.09%	0%
Airlines	9	100%	0%	0%
Apparel	5	100%	0%	0%
Beverages	7	100%	0%	0%
Brokerage	7	100%	0%	0%
Building materials, glass	4	100%	0%	0%
Chemicals	39	97.44%	2.56%	0%
Commercial banks	55	98.18%	1.82%	0%
Diversified financials	14	92.86%	7.14%	0%
Electric and gas utilities	73	98.63%	0%	1.37%
Electronics, electrical equipment	41	95.12%	4.88%	0%
Engineering, construction	11	90.91%	0%	9.09%
Entertainment	3	33.33%	33.33%	33.33%
Food	27	92.59%	0%	7.41%
Food and drug stores	20	100%	0%	0%
Food services	5	80.00%	20.00%	0%
Forest and paper products	30	100%	0%	0%
Furniture	5	100%	0%	0%
General merchandisers	16	87.50%	12.50%	0%
Health care	18	100%	0%	0%
Hotels, casinos, resorts	7	100%	0%	0%
Industrial and farm equipment	27	100%	0%	0%
Insurance: life & health	19	94.74%	5.26%	0%
Insurance: prop. & casualty	24	87.50%	12.50%	0%
Mail, package and freight delivery	3	100%	0%	0%
Marine services	2	100%	0%	0%
Metal products	11	100%	0%	0%
Metals	21	100%	0%	0%
Mining, crude-oil production	7	100%	0%	0%
Motor vehicles and parts	21	90.48%	9.52%	0%

Table 12.5 Critical measures for Fortune's 1000 companies (continued)

Industry	Companies	Revenue	Profit	Employee
Petroleum refining	18	50.00%	33.33%	16.67%
Pharmaceuticals	14	85.71%	14.29%	0%
Pipelines	10	80.00%	0%	20.00%
Publishing, printing	17	100%	0%	0%
Railroads	5	100%	0%	0%
Rubber and plastic products	8	100%	0%	0%
Savings institutions	8	100%	0%	0%
Scientific, photo., control equip.	18	94.44%	5.56%	0%
Soaps, cosmetics	8	87.50%	12.50%	0%
Specialist retailers	30	100%	0%	0%
Temporary help	5	100%	0%	0%
Textiles	6	100%	0%	0%
Tobacco	4	75.00%	25.00%	0%
Toys, sporting goods	3	100%	0%	0%
Transportation equipment	5	100%	0%	0%
Truck leasing	2	100%	0%	0%
Trucking	3	100%	0%	0%
Waste management	3	100%	0%	0%
Wholesalers	40	90.00%	0%	10.00%
Miscellaneous	7	100%	0%	0%
Total	**760**	**94.61%**	**3.55%**	**1.84%**

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