Human Behavioral Modeling Using Fuzzy and Dempster–Shafer Theory

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Abstract: Human behavioral modeling requires an ability to represent and manipulate imprecise cognitive concepts. It also needs to include the uncertainty and unpredictability of human action. We discuss the appropriateness of fuzzy sets for representing human centered cognitive concepts. We describe the technology of fuzzy systems modeling and indicate its the role in human behavioral modeling. We next introduce some ideas from the Dempster-Shafer theory of evidence. We use the Dempster-Shafer theory to provide a machinery for including randomness in the fuzzy systems modeling process. This combined methodology provides a framework with which we can construct models that can include both the complex cognitive concepts and unpredictability needed to model human behavior.

1 Human Behavioral Modeling

Two important classes of human behavioral modeling can be readily identified. The first is the modeling of some physical phenomenon or system involving human participants. This is very much what is done in social sciences and is clearly inspired by the classical successful use of modeling in physics and engineering. The modeling here is from the perspective of an external observer. We can refer to this as E-O modeling. The second type of modeling, of much more recent vintage, can be denoted as I-P modeling as an acronym for Internal Participant modeling. This type of modeling has arisen to importance with the wide spread use digital technology. It is central in the construction of synthetic agents, computational based training systems and machine learning. It is implicit in our attempts to construct intelligent systems. Here we are trying to digitally model a "human" or "human like" agent that interacts with some more complex environment which itself can be digital or real or some combination.

In either case, and perhaps more so in the I-P situation, human behavioral modeling requires an ability to formally represent sophisticated cognitive concepts that are often at best described in imprecise linguistic terms. Set based methods and more particularly fuzzy sets provide a powerful tool for enabling the semantical modeling of these imprecise concepts within computer based systems [1-2]. With the aid of a fuzzy set we can formally represent sophisticated imprecise linguistic concepts in a manner that

allows for the types of computational manipulation needed for reasoning in behavioral models based on human cognition and conceptualization. Central to the use of fuzzy sets is the ability to capture the "grayness" of human conceptualization. Most concepts used in human behavioral modeling, both from the E-O and I-P perspective, are not binary but gradually go from clearly yes to clearly no. Furthermore in discussing the qualities of important social relationships such as political ties, kinship obligations and friendship we use attributes such as intensity, durability and reciprocity [3]. These attributes most naturally evaluated in imprecise terms. In modeling the rules determining the behavior of some simulated agent we must have the ability to model the kinds fluidity central to the human capacity to adapt and deal with new situations.

Fuzzy systems modeling (FSM) [4] is a rule based technique that allows for formal reasoning and manipulation with the types of imprecise concepts central to human cognition. It can use a semantic understanding of an age related concept such as *old* in order to be able how well a particular individual satisfies the concept.. Clearly FSMs can be used to model the types of complex relationships needed in human behavioral modeling. It is the basic technique used in the development of many successful applications [5]. FSM helps simply the task of modeling complex relationship and processes by partitioning the input (antecedent) space into regions in which one can more easily comprehend and express the appropriate consequents. In FSM the rules are expressed in linguistic terms with a representation using fuzzy subsets. An important feature of the FSM is that it can create and formulate new solutions. That is the output of an FSM does not have to be one of the consequents of a rule but can be constructed out of a combination of outputs from different rules.

In addition to the imprecision of human conceptualization reflected in language many situations that arise in human behavioral modeling entail aspects of probabilistic uncertainty. This is true in both E-O and I-P applications. Consider an observation such as "Generally women of child bearing age do not get to close to foreigners" Here we see imprecise terms such as "child bearing age" and "close" as well as the term "generally" conveying a probabilistic aspect. In this this work we describe a methodology for including probabilistic uncertainty in the fuzzy systems model. The technique we suggest for the inclusion of this uncertainty is based upon the Dempster-Shafer theory of evidence [6, 7]. The Dempster-Shafer approach fits nicely into the FSM technique since both techniques use sets as their primary data structure and are important components of the emerging field of granular computing [8, 9].

We first discuss the fundamentals of FSM based on the Mamdani reasoning paradigm. We next introduce some of the basic ideas from the Dempster-Shafer theory which are required for our procedure. We then show how probabilistic uncertainty in the output of a rule based model can be included in the FSM using the Dempster-Shafer (D-S) paradigm. We described how various types of uncertainty can be modeled using this combined FSM / D-S paradigm.

2 Fuzzy Systems Modeling

Fuzzy systems modeling (FSM) provides a technology for the development of semantically rich rule based representations that can model complex, nonlinear multiple input output relationships or functions or systems.

The technique of FSM allows one to represent the model of a system by partitioning the input space. Thus if $U_1, \ldots U_r$ are the input (antecedent) variables and V is the output (consequent) variable we can represent their relationship by a collection *n* of "rules" of the form,

When U_1 *is* A_{i1} and U_2 *is* $A_{i2}, \ldots$ and U_r *is* A_{ir} then V *is* D_i.

Here each A_{ij} typically indicates a linguistic term corresponding to a value of its associated variable, for example if U_j is the variable correspond to age then A_{ij} could be "young." or "child bearing age." Furthermore each A_{ij} is formally represented as a fuzzy subset over the domain X_j of the associated variable U_j Similarly D_i is a value associated with the consequent variable V that is formally defined as a fuzzy subset of the domain Y of V.

In the preceding rules the antecedent specifies a condition that if met allows us to infer that the possible value for the variable V lies in the consequent subset D_i. For each rule the antecedent defines a fuzzy region of the input space, $X_1 \times X_2 \times ... \times X_m$, such that if the input lies in this region the consequent holds. Taken as a collection the antecedents of all the rules form a fuzzy partition of the input space. A key advantage of this approach is that by partitioning the input space we can allow simple functions to represent the consequent.

The process of finding the output of a fuzzy systems model for given values of the input variables is called the "reasoning" process. One method for reasoning with fuzzy systems models is the Mamdani-Zadeh paradigm. [10].

Assume the input to a FSM consists of the values $U_j = x_j$. In the following we shall use the notation $A_{ij}(x_j)$ to indicate the membership of the element x_j in the fuzzy subset A_{ij}. This can be seen as the degree of truth of the proposition U_j *is* A_{ij} given that $U_j = x_j$. The procedure for reasoning used in the Mamdani-Zadeh method consists of the following steps:

1. For each rule calculate its firing level $\tau_i = \text{Min}_j[A_{ij}(x_j)]$
2. Calculate the output of each rule as a fuzzy subset F_i of Y where

$$F_i(y) = \text{Min}[\tau_i, D_i(y)]$$

3. Aggregate the individual rule outputs to get a fuzzy subset F and Y where
$$F(y) = \text{Max}_i[F_i(y)].$$

F is a fuzzy subset of Y indicating the output of the system. It is important to emphasize that F can be something new, it has been constructed from distinct components of the rule base.

At this point we can describe three options with respect to presenting this output to the final user. The simplest is to give them the fuzzy set F. This of course is the least appealing especially if the user is not technically oriented. The second, and perhaps the most sophisticated, is to perform what is called retranslation. Here we try to express the fuzzy set F in some kind appropriate linguistic form. While we shall not pursue this approach here we note that in [11] we have investigated the process of retranslation. The third alternative is to compress the fuzzy set F into some precise value from the space Y. This process is called defuzzification. A number techniques are available to implement the defuzzification. Often the choice is dependent upon the structure of the space Y associated with variable V. One approach is to take as the output the element in Y that has the largest membership in F. While available in most domains it loses a lot of the information. A preferred approach, if the under lying structure of Y allows, is to take a kind of weighted average using the membership grades in F to provide the weights. The most commonly used procedure for defuzzification process is the center of gravity. Using this method we calculate the

defuzzification value as $\overline{y} = \dfrac{\Sigma_i y_i F(y_i)}{\Sigma_i F(y_i)}$.

3 Dempster-Shafer Theory of Evidence

In this section we introduce some ideas from the Dempster-Shafer uncertainty theory [6, 7]. Assume X is a set of elements. Formally a Dempster-Shafer belief structure *m* is a collection of q non-null subsets A_i of X called focal elements and a set of associated weights $m(A_i)$ such that: (1) $m(A_i) \in [0, 1]$ and (2) $\sum_i m(A_i) = 1$.

One interpretation that can be associated with this structure is the following. Assume we perform a random experiment which can have one of q outcomes. We shall denote the space of the experiment as Z. Let P_i be the probability of the ith outcome z_i. Let V be another variable taking its value in the set X. It is the value of the variable V that is of interest to us. The value of the variable V is associated with the performance of the experiment in the space Z in the following manner. If the outcome of the experiment on the space Z is the ith element, z_i, we shall say that the value of V lies in the subset A_i of X. Using this semantics we shall denote the value of the variable as V *is* m, where m is a Dempster-Shafer granule with focal elements A_i and weights $m(A_i) = P_i$.

A situation which illustrates the above is the following. We have three candidates for president, Red, White and Blue. The latest polling information indicates that the probabilities of each candidate winning is (Red, 0.35), (White, 0.55) and (Blue, 0.1). Our interest here is not on who will be president but on the future interest rates. Based on the campaign statements of the three candidates we are able to conclude that Red will support *low* interest rates and White will support *high* interest rates. For the candidate

Blue we no have information about his attitude toward interest rates. The Dempster-Shafer framework provides an ideal structure for representing this knowledge. Here we let V be the variable corresponding to the future interest rates and let X be the set corresponding to the domain of interest rates, the variable V will assume its value in X. We can now represent our knowledge the value of the future interest rates V using the Dempster-Shafer framework. Here we have three focal sets. The first, A_1, is "low interest rates." The second, A_2 is "high interest rates." The third, A_3, is "unknown interest rate." Furthermore the associated weights are $m(A_1) = 0.35$, $m(A_2) = 0.55$ and $m(A_3) = 0.1$. Each of the A_j are formulated as subsets of X. We note A_3, "unknown interest rate, " is the set X.

Here our interest is in finding the probabilities of events associated with V, that is with arbitrary subsets of X. For example we may be interested in the probability that interest rates will be *less then 4 %*. Because of the imprecision in the information we can't find exact probabilities but we must settle for ranges. Two measures are introduced to capture the relevant information.

Let B be a subset of X the **plausibility** of B, denoted Pl(B), is defined as Pl(B) = $\sum_{i, A_i \cap B \neq \emptyset} m(A_i)$, The **belief** of B, denoted Bel(B), is defined as Bel(B) = $\sum_{i, B \subseteq A_i} m(A_i)$. For any subset B of X Bel(B) $\leq$ Prob(B) $\leq$ Pl(B). The plausibility and belief provide upper and lower bounds on the probability of the subset B.

An important issue in the theory of Dempster-Shafer is the procedure for aggregating multiple belief structures on the same variable. This can be seen as a problem of information fusion. This standard procedure is called Dempster's rule, it is a kind of conjunction (intersection) of the belief structures.

Assume m_1 and m_2 are two independent belief structures on the space X their conjunction is another belief structure m, denoted $m = m_1 \oplus m_2$. The belief structure m is obtained in the following manner. Let m_1 have focal elements A_i, i = 1 to n_1 and let m_2 have focal elements B_j, j = 1 to n_2. The focal elements of m are all the subsets $F_K = A_i \cap B_j \neq \emptyset$ for some i and j. The associated weights are $m(F_K) = \dfrac{1}{1 - T} (m_1(A_i) * m_2(B_j)$ where $T = \sum_{A_i \cap B_j = \emptyset} m_1(A_i) * m_2(B_j)$.

Example: Assume our universe of discourse is X = {1, 2, 3, 4, 5, 6}

$\underline{m_1}$		$\underline{m_2}$
A_1 = {1, 2, 3}	$m_1(A_1) = 0.5$	B_1 = {2, 5, 6}
	$m_2(B_1) = 0.6$	
A_2 = {2, 3, 6}	$m_1(A_2) = 0.3$	B_2 = {1, 4}
	$m_2(B_2) = 0.4$	

$A_3 = \{1, 2, 3, 4, 5, 6\}$ $m_1(A_3) = 0.2$

Taking the conjunction we get: $F_1 = A_1 \cap B_1 = \{2\}$, $F_2 = A_1 \cap B_2 = \{1\}$, $F_3 = A_2 \cap B_1 = \{2, 6\}$, $F_4 = A_3 \cap B_1 = \{2, 5, 6\}$ and $F_5 = A_3 \cap B_1 = \{1, 4\}$.

We note that $A_2 \cap B_2 = \emptyset$. Since only one intersection gives us the null set then $T = m_1(A_2) * m(B_2) = .12$ and $1 - T = 0.88$. Using this we get $m(F_1) = 0.341$, $m(F_2) = 0.227$, $m(F_3) = 0.205$, $m(F_4) = 0.136$ and $m(F_5) = 0.09$.

The above combination of belief structures can be seen to be essentially an intersection, conjunction, of the two belief structures. In [12] Yager provided for an extension of the aggregation of belief structures to any set based operation. Assume ∇ is any binary operation defined on sets, $D = A \nabla B$ where A, B and D are sets. We shall say that ∇ is an "non-null producing" operator if $A \nabla B \neq \emptyset$ when $A \neq \emptyset$ and $B \neq \emptyset$. The union is non-null producing but intersection is not. Assume m_1 and m_2 are two belief structures with focal elements A_i and B_j respectively. Let ∇ be any non-null producing operator. We now define the new belief structure $m = m_1 \nabla m_2$. The belief structure m has focal elements $E_K = A_i \nabla B_j$ with $m(E_K) = m_1(A_i) * m_2(B_j)$. If ∇ is **not** non-null producing we may be forced to do a process called normalization [12]. The process of normalization consists of the following

(1) Calculate $T = \sum\limits_{A_i \nabla B_j = \emptyset} m_1(A_i) * m(B_j)$

(2) For all $E_K = A_i \nabla B_j \neq \emptyset$ calculate $m(E_K) = \dfrac{1}{1-T} m_1(A_i) * m_2(B_j)$

(3) For all $E_K = \emptyset$ set $m(E_K) = 0$.

We can use the Dempster-Shafer structure to represent some very naturally occurring types of information. Assume V is a variable taking its value in the set X. Let A be a subset of X. Assume our knowledge about V is that the probability that V lies in A is "at least α." This information can be represented as the belief structure m which has two focal elements A and X and where $m(A) = \alpha$ and $m(X) = 1$. The information that the probability of A is exactly α can be represented as a belief structure m with focal elements A and $\overline{A}$ where $m(A) = \alpha$ and $m(\overline{A}) = 1 - \alpha$.

An ordinary probability distribution P can also be represented as a belief structure. Assume for each element $x_i \in X$ it is the case P_i is its probability. We can represent this as a belief structure where the focal elements are the individual element $A_i = \{x_i\}$ and $m(A_i) = P_i$. For these types of structures it is the case that for any subset A of X, $Pl(A) = Bel(A)$, thus the probability is uniquely defined as a point rather than interval.

The D-S belief structure can be extended to allow for fuzzy sets [13, 14]. To extend the measures of plausibility and belief we need two ideas from the theory of possibility [15]. Assume A and B are two fuzzy subsets of X, the possibility of B given A is defined as $Poss[B/A] = Max_i[A(x_i) \wedge B(x_i)]$ where $\wedge$ is the min. The certainty of B given A is $Cert[B/A] = 1 - Poss[\overline{B}/A]$. Here $\overline{B}$ is the complement of B, it has

membership grade $\overline{B}(x) = 1 - B(x)$.

Using these we extend the concepts of plausibility and belief. If m is a belief structure on X with focal fuzzy elements A_i and B is any fuzzy subset of X then $Pl(B) = \sum_i Poss[B/A_i] \, m(A_i)$ and $Bel(B) = \sum_i Cert[B/A_i] \, m(A_i)$. The plausibility and belief measures are the expected possibility and certainty of the focal elements.

The combination of belief structures with fuzzy focal elements can be made. If ∇ is some set operation we simply use the fuzzy version of it. For example if m_1 and m_2 are belief structures with fuzzy focal elements then $m = m_1 \cup m_2$ has focal elements $E_K = A_i \cup B_j$ where $E_K(x) = A_i(x) \vee B_j(x)$ ($\vee = max$). Here as in the non-fuzzy case $m(E_K) = m_1(A_i) \, m_2(B_j)$.

Implicit in the formulation for calculating the new weights is an assumption of independence between the belief structures. This independence is reflected in an assumption that the underlying experiments generating the focal elements for each belief structure are independent. This independence manifests itself in the use of the product to calculate the new weights. That is the joint occurrence of the pair of focal elements A_i and B_j is the product of probabilities of each of them $m_1(A_i)$ and $m_2(B_j)$.

In some situations we may have a different relationship between the two belief structures. One very interesting case is called **synonymity**. For two belief structures to be in synonymity they must have their focal elements induced from the same experiment. Thus if m_1 and m_2 are two belief structures on X that are in synonymity they should have the same number of focal elements with the same weights. Thus the focal elements of m_1 are A_i for i = 1 to q, and those of m_2 are are B_j for i = 1 to q then $m_1(A_i) = m_2(B_i)$. In the case of synonymity between m_1 and m_2 if ∇ is any non-null producing set operator then $m = m_1 \nabla m_2$ also has n focal elements $E_i = A_i \nabla B_i$ with $m(E_i) = m(A_i) = m(B_i)$.

4 Probabilistic Uncertainty in the FSM

In the basic FSM, the Mamdani-Zadeh model, the consequent of each rule consists of a fuzzy subset. The consequent of an individual rule is a proposition of the form V *is* D_i. The use of a fuzzy subset implies a kind of uncertainty associated with the output of a rule. The kind of uncertainty is called possibilistic uncertainty and is a reflection of a lack of precision in describing the output. The intent of this proposition if to indicate that the value of the output is constrained by (lies in) the subset D_i.

We now shall add further modeling capacity to the FSM technique by allowing for probabilistic uncertainty in the consequent. A natural extension of the FSM is to consider the consequent to be a fuzzy Dempster-Shafer granule. Thus we shall now consider the output of each rule to be of the form V *is* m_i where m_i is a belief structure

with focal elements D_{ij} which are fuzzy subsets of the universe Y and associated weights $m_i(D_{ij})$. Thus a typical rule is now of the form

When U_1 *is* A_{i1} and U_2 *is* $A_{i2}, \ldots U_r$ *is* A_{ir} then V *is* m_i.

Using a belief structure to model the consequent of a rule is essentially saying that $m_i(D_{ij})$ is the probability that the output of the ith rule lies in the set D_{ij}. So rather than being certain as to the output set of a rule we have some randomness in the rule. We note that with $m_i(D_{ij}) = 1$ for some D_{ij} we get the original FSM.

We emphasize that the use of a fuzzy Dempster-Shafer granule to model the consequent of a rule brings with it two kinds of uncertainty. The first type of uncertainty is the randomness associated with determining which of the focal elements of m_i is in effect if the rule fires. This selection is essentially determined by a random experiment which uses the weights as the appropriate probability. The second type of uncertainty is related to the selection of the outcome element given the fuzzy subset, this is related to the issue of lack of specificity. This uncertainty is essentially resolved by the defuzzification procedure used to pick the crisp singleton output of the system.

We now describe the reasoning process in this situation with belief structure consequents. Assume the input to the system are the values for the antecedent variables, $U_j = x_j$. The process for obtaining the firing levels of the individual based upon these inputs is exactly the same as in the previous situation.

For each rule we obtain the firing level, $\tau_i = \text{Min}[A_{ij}(x_j)]$.

The output of each rule is a belief structure $\widehat{m}_i = \tau_i \wedge m$. The focal elements of $\widehat{m}_i$ are F_{ij} a fuzzy subset of Y where $F_{ij}(y) = \text{Min}[\tau_i, D_{ij}(y)]$, here D_{ij} is a focal element of m_i. The weights associated with these new focal elements are simply $\widehat{m}_i(F_{ij}) = m_i(D_{ij})$.

The overall output of the system *m* is obtained in a manner analogous to that used in the basic FSM, we obtain m by taking a union of the individual rule outputs, $m = \overset{n}{\underset{i=1}{\cup}} \widehat{m}_i$.

Earlier we discussed the process of taking the union of belief structures. For every a collection $<F_{1j_1}, \ldots F_{nj_n}>$ where F_{ij_i} is a focal element of $\widehat{m}_i$ we obtain a focal element of m, $E = \underset{i}{\cup} F_{ij_i}$ and the associated weight is $m(E) = \overset{n}{\underset{i=1}{}} \widehat{m}_i(F_{ij_i})$.

As a result of this third step we obtain a fuzzy D-S belief structure V *is* m as our output of the FSM. We denote the focal elements of m as the fuzzy subsets $E_j, j = 1$ to q, with weights $m(E_j)$. Again we have three choices: present this to a user, try to linguistically summarize the belief structure or to defuzzify to a single value. We shall here discuss the third option.

The procedure used to obtain this defuzzified value $\bar{y}$ is an extension of the previously described defuzzification procedure. For each focal element E_j we calculate

its defuzzified value $\bar{y}_j = \dfrac{\Sigma_i y_i\, E_j(y_i)}{\Sigma_i E_j(y_i)}$. We then obtain as the defuzzified value of m, $\bar{y} =$

$\Sigma_j\, \bar{y}_j\, m(E_j)$. Thus $\bar{y}$ is the expected defuzzified value of the focal elements of m.

The following simple example illustrates the technique just described.

Example: Assume a FSM has two rules

$$\text{If U } is \text{ A}_1 \text{ then V } is \text{ m}_1$$
$$\text{If U } is \text{ A}_2 \text{ then V } is \text{ m}_2.$$

$\mathbf{m_1}$: has focal elements $D_{11} = $ "about two" $= \left\{\dfrac{.6}{1}, \dfrac{1}{2}, \dfrac{.6}{3}\right\}$ and $D_{12} = $ "about five" $=$

$\left\{\dfrac{.5}{4}, \dfrac{1}{5}, \dfrac{.6}{6}\right\}$ with $m_1(D_{11}) = 0.7$ and $m_1(D_{12}) = 0.3$.

$\mathbf{m_2}$: has focal elements $D_{21} = $ "about 10" $= \left\{\dfrac{.7}{9}, \dfrac{1}{10}, \dfrac{.7}{11}\right\}$ and $D_{22} = $ "about 15" $=$

$\left\{\dfrac{.4}{14}, \dfrac{1}{15}, \dfrac{.4}{10}\right\}$ with $m_2(D_{21}) = 0.6$ and $m_2(D_{22}) = 0.4$

Assume the system input is x* and the membership grade of x* in A_1 and A_2 are 0.8 and 0.5 respectively. Thus the firing levels of each rule are $\tau_1 = 0.8$

and $\tau_2 = 0.5$. We now calculate the output each rule $\widehat{m}_1 = \tau_1 \wedge m_1$ and $\widehat{m}_2 = \tau_2 \wedge m_2$.

$\widehat{m}_1$: has focal elements $F_{11} = \tau_1 \wedge D_{11} = \left\{\dfrac{.6}{1}, \dfrac{.8}{2}, \dfrac{.6}{3}\right\}$ and $F_{12} = \tau_1 \wedge D_{12} =$

$\left\{\dfrac{.5}{4}, \dfrac{.8}{5}, \dfrac{.6}{6}\right\}$ with $m(F_{11}) = 0.7$ and $m(F_{12}) = 0.3$

$\widehat{m}_2$: has focal elements $F_{21} = \tau_2 \wedge D_{21} = \left\{\dfrac{.5}{9}, \dfrac{.5}{10}, \dfrac{.5}{11}\right\}$ and $F_{22} = \tau_2 \wedge D_{22} =$

$\left\{\dfrac{.4}{14}, \dfrac{.5}{15}, \dfrac{.4}{10}\right\}$ with $m(F_{21}) = 0.6$ and $m(F_{22}) = 0.4$

We next obtain the union of these two belief structure, $m = m_1 \cup m_2$ with focal elements

$$E_1 = F_{11} \cup F_{21} \qquad m(E_1) = \widehat{m}_1(F_{11}) * \widehat{m}_2(F_{21})$$
$$E_2 = F_{11} \cup F_{22} \qquad m(E_2) = \widehat{m}_1(F_{11}) * \widehat{m}_2(F_{22})$$
$$E_3 = E_{12} \cup F_{21} \qquad m(E_3) = \widehat{m}_1(F_{12}) * \widehat{m}_2(F_{21})$$
$$E_4 = E_{12} \cup F_{22} \qquad m(E_4) = \widehat{m}_1(F_{12}) * \widehat{m}_2(F_{22})$$

Doing the above calculations we get

$$E_1 = \left\{\dfrac{0.6}{1}, \dfrac{0.8}{2}, \dfrac{0.6}{3}, \dfrac{0.5}{9}, \dfrac{0.5}{10}, \dfrac{0.5}{11}\right\} \qquad m(E_1) =$$

0.42

$$E_2 = \left\{\dfrac{0.6}{1}, \dfrac{0.8}{2}, \dfrac{0.6}{3}, \dfrac{0.4}{14}, \dfrac{0.5}{15}, \dfrac{0.4}{10}\right\} \qquad m(E_2) =$$

0.28

$$E_3 = \left\{\dfrac{0.5}{4}, \dfrac{0.8}{5}, \dfrac{0.6}{6}, \dfrac{0.5}{9}, \dfrac{0.5}{10}, \dfrac{0.5}{11}\right\} \qquad m(E_3) =$$

0.18

$$E_4 = \left\{ \frac{0.5}{4}, \frac{0.8}{5}, \frac{0.6}{6}, \frac{0.4}{14}, \frac{0.5}{15}, \frac{0.4}{10} \right\} \qquad m(E_4) = 0.12$$

We now proceed with the defuzzification of the focal elements. Defuzzy(E_1) = $\bar{y}_1$ =5.4, Defuzzy(E_2) = $\bar{y}_2$ = 6.4, Defuzzy(E_3) = $\bar{y}_3$ = 7.23 and Defuzzy(E_4) = $\bar{y}_4$ = 8.34. Finally taking the expected value of these we get

$$\bar{y} = (0.42)(5.4) + (0.28)(6.4) + (0.18) * (7.23) + (0.12)(8.34) = 6.326$$

The development of FSMs with Dempster-Shafer consequents allows for the representation of different kinds of uncertainty associated with the modeling rules.

One situation is where we have a value $\alpha_i \in [0, 1]$ indicating the confidence we have in the ith rule. In this case we have a nominal rule of the form

If U *is* A_i then V *is* B_i

with confidence "at least α_i".

Using the framework developed above we can transform this rule, along with its associated confidence level into a Dempster-Shafer structure

"If U *is* A_i then V *is* m_i."

Here m_i is a belief structure with two focal elements, B_i and Y. We recall Y is the whole output space. The associated weights are $m_i(A_i) = \alpha_i$ and $m(Y) = 1 - \alpha_i$. We see that if $\alpha_i = 1$ then we get the original rule while if $\alpha_i = 0$ we get a rule of the form

If U is A_i then V is Y.

5 Conclusion

We have suggested a framework which can be used for modeling human behavior. The approach suggested has the ability to represent the types of linguistically expressed concepts central to human cognition. It also has a random component which enables the modeling of the unpredictability of human behavior.

References

1. Zadeh, L. A., "A note on web intelligence, world knowledge and fuzzy logic," Data and Knowledge Engineering 50, 291-304, 2004.
2. Yager, R. R., "Using knowledge trees for semantic web querying," in Fuzzy Logic and the Semantic Web, edited by Sanchez, E., Elsevier: Amsterdam, 231-246, 2006.
3. Scott, J., Social Network Analysis, SAGE Publishers: Los Angeles, 2000.
4. Pedrycz, W. and Gomide, F., Fuzzy Systems Engineering: Toward Human-Centric Computing, John Wiley & Sons: New York, 2007.
5. Sugeno, M., Industrial Applications of Fuzzy Control, North-Holland: Amsterdam, 1985.

6. Shafer, G., A Mathematical Theory of Evidence, Princeton University Press: Princeton, N.J., 1976.
7. Yager, R. R. and Liu, L., (A. P. Dempster and G.Shafer, Advisory Editors) Classic Works of the Dempster-Shafer Theory of Belief Functions, Springer: Heidelberg, (To Appear).
8. Lin, T. S., Yao, Y. Y. and Zadeh, L. A., Data Mining, Rough Sets and Granular Computing, Physica-Verlag: Heidelberg, 2002.
9. Bargiela, A. and Pedrycz, W., Granular Computing: An Introduction, Kluwer Academic Publishers: Amsterdam, 2003.
10. Yager, R. R. and Filev, D. P., Essentials of Fuzzy Modeling and Control, John Wiley: New York, 1994.
11. Yager, R. R., "On the retranslation process in Zadeh's paradigm of computing with words," IEEE Transactions on Systems, Man and Cybernetics: Part B 34, 1184-1195, 2004.
12. Yager, R. R., "Arithmetic and other operations on Dempster-Shafer structures," Int. J. of Man-Machine Studies 25, 357-366, 1986.
13. Yager, R. R., "Entropy and specificity in a mathematical theory of evidence," Int. J. of General Systems 9, 249-260, 1983.
14. Yen, J., "Generalizing the Dempster-Shafer theory to fuzzy sets," IEEE Transactions on Systems, Man and Cybernetics 20, 559-570, 1990.
15. Dubois, D. and Prade, H., "Possibility theory as a basis for qualitative decision theory," Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI), Montreal, 1924-1930, 1995.