Game Theory Models and Their Applications in Inventory Management and Supply Chain

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Abstract Analysis of supply chain politics can benefit from applying game-theory concepts extensively. Game theory tries to enlighten the interactions between individuals or groups of people whose goals are opposed conflicting, or at least partially competing. In this chapter, we review classic game theoretical approaches to modeling and solving certain problems in supply chain management. Both noncooperative and cooperative models are discussed and solution procedures are presented in single-period and multiperiod settings. As used here, a "game" is a metaphor for any interaction among the decision makers in a supply chain.

Key words: inventory, supply chain, noncooperative games, cooperative games, Nash equilibrium, Stackelberg game, transferable utility, core, bargaining, biform games

1 Introduction

Inventory management of physical goods and other products or elements is an integral part of logistic systems common to all sectors of the economy including industry, agriculture, and defense. In a perfectly predictable economy, inventory may be needed in order to take advantage of the economic feature of a particular technology, to synchronize human tasks, or to regulate production process to meet the changing demands. When uncertainty is present, inventories are used as a protection against risk of stock being out.

The existence of inventory in a system generally implies the existence of an organized complex system involving inflow, accumulation, and outflow of some commodities, goods, items, or products. In business, for example, the inflow of goods is generated through procurement, purchase, or production.

The outflow is generated through demand for the goods. Finally, the difference between the rate of outflow and the rate of inflow generates inventory of goods.

The regulation and control of inventory must proceed within the context of this organized system. Rather than being interpreted as idle resources, inventories should be regarded an essential element, the study of which may provide insight in the aggregate operation of the system. The scientific analysis of inventory systems defines the degree of interrelationship between inflow, accumulation, and outflow, and identifies economic control methods for operating such systems.

Traditionally, inventory problems are concerned with a single decision maker, who makes the decisions on the ordered or produced quantity under certain assumptions on the demand, the planning horizon, etc., which the decision maker faces. Although such models capture important aspects of inventory problems, they totally ignore the decisions made by other competitors. In particular, most of such models assume that, if there are two or more products, they cannot be substituted for each other. However, in many real situations this is not true. A customer who cannot find a specific product at one retailer might decide to switch to another retailer who sells the same or a similar product.

It is a fact that, in many production-inventory-transportation problems, one can observe the existence of several decision makers with competitive objectives. In order to have an inventory model, which is able to adequately describe such situations, game theory method should be used. Single-period news-vendor models have typically been used for analyzing such situations [90].

In the current chapter, we are concerned with game theoretic approaches to modeling and solving certain problems in supply chain analysis. The remainder of the paper is organized as follows: Sections 2, 3, and 4 present basic concepts we use throughout the paper. Section 5 presents the application of noncooperative games in inventory management, and in Section 6 their application to supply chain coordination is presented. Section 7 is devoted to cooperative inventory games. Finally, new developments in Game Theory such as bargaining game and biform games, with applications to supply chain, are introduced in Section 8.

2 Basic Concepts in Game Theory

Game theory is a mathematical theory of decision making by participants in conflicting or cooperating situations. Its goal is to explain, or to provide a normative guide for, rational behavior of individuals confronted with strategic decisions or involved in social interaction. The theory is concerned with optimal strategic behavior, equilibrium situations, stable outcomes, bargaining, coalition formation, equitable allocations, and similar concepts related to resolving group differences. Game theory has a profound influence on methodologies of many different branches of sciences, especially those of economics, operations research, and management sciences.

Traditionally, game theory can be divided into two branches: *noncooperative* and *cooperative* game theory. *Noncooperative* game theory uses the notion of a *strategic equilibrium* or simply *equilibrium* to determine rational outcomes of a game. Numerous equilibrium concepts have been proposed in the literature (see [85] for an overview). Some widely used concepts are *dominant strategy*, *Nash equilibrium*, and *subgame perfect equilibrium*.

Nash Equilibrium: Strategies chosen by all players are said to be in Nash equilibrium if no player can benefit by unilaterally changing her strategy. Nash [54, 56] proved that every finite game has at least one Nash equilibrium.

Dominant strategy is one that achieves the highest payoff no matter what the strategies of other players are. In other words, one that is optimal in all circumstances. If strategies are dominant, they also constitute a Nash equilibrium, however, the opposite is not necessarily true.

Subgame perfect equilibrium: Strategies in extensive form are in subgame perfect equilibrium if the strategies constitute a Nash equilibrium at every desicion point.

In *cooperative game theory*, groups of players are taken as primitives and binding agreements can be made between players, which can form coalitions. In such a game, a utility is created when two or more players cooperate and form a coalition. Cooperative game theory can then determine a solution concept that must satisfy a set of assumptions (called axioms). The most important of them are

Pareto optimality: The total utility allocated to the players must be equal to the total utility of the game.

Individual rationality: The utility allocated in each player should be higher than the utility she gains by acting without the coalition.

Kick-back: The utility allocated to a player must always be non-negative.

Monotonicity: If the overall utility increases, the allocation to a player should be higher.

There are several excellent books [5, 42, 51, 62, 71] on the subject, and the reader should turn to them for further details.

3 The Classic Newsboy Problem

The classic newsboy problem is a one-period model in which a firm must choose an inventory level x at a cost c per unit for the perishable product it sells prior to knowing the true level of demand for it. When the demand is realized, the goods are sold at a price r per unit, which is usually assumed to be fixed. Demand is denoted by the random variable w with cumulative distribution $F(W) = P(W \leq w)$, which is assumed to have a continuous density $f(w) = \partial F(W)/\partial w$. Moreover if it is assumed that f is strictly positive on some interval, then F is strictly increasing and therefore it has an inverse function F_w^{-1}. As there is no initial inventory, the quantity ordered by the firm is the total amount available for sale; the firm's sale is the smallest amount between the demand and the inventory level. Excess demand given by $(w-x)^+$ is costly because it results in lost sales. It is therefore penalized by a shortage cost per unit p. Excess inventory, given by $(x-w)^+$, is costly as well because the salvage value s is lower than the cost of procuring inventory. The firm's profit is therefore:

$$\pi = \begin{cases} (r-c)x - (w-x)p \text{ if } x \leq w, \\ (r-c)w + (s-c)x \text{ if } x > w. \end{cases} \tag{1}$$

The firm wants to choose an inventory level x to maximize the expected profit.

$$E(\pi) = rE\min\{w, x\} + sE(x-w)^+ - cx - pE(w-x)^+. \tag{2}$$

Equating marginal revenues with marginal costs yields the optimal inventory x^* as the implicit solution of the equation

$$F(x^*) = \frac{r-c+s}{r+p-s} \Rightarrow x^* = F_w^{-1}\left(\frac{r-c+s}{r+p-s}\right). \tag{3}$$

The assumption that F is strictly increasing implies that x^* is unique. If there are no shortage costs and the salvage value is zero, then

$$F(x^*) = \frac{r-c}{r} \Rightarrow x^* = F_w^{-1}\left(\frac{r-c}{r}\right). \tag{4}$$

For a survey on the news-vendor problem and several of its extensions, see [40].

4 The Competitive Newsboy Model

In the competitive newsboy model, substitution often takes place between different products sold by different retailers when the products have stochastic demands. In such a situation, each retailer's profit depends not only on her own order quantity but also on her competitors' order. In other words, if a customer finds the shelves empty at the first firm she visits, she does not necessarily give up but may travel to another firm in order to satisfy her demand. The actual substitution between any two retailers takes place according to a substitution rate that depends on their products and other factors such as location.

The simplest competitive model has two retailers i and j; each one of them faces a demand w_i and w_j, respectively. Therefore $w = w_i + w_j$ is the industry demand. This allocation of the initial demand to each firm follows some specific splitting rules. If there exists excess demand $(w_i - x_i)^+$ at firm i, then the same proportion of the excess demand should be met by the inventory of firm j. That is, a reallocation of the initial demand at firm i occurs. Hence, the actual demand the firm j faces is

$$R_j = w_j + \beta_i(w_i - x_i)^+, \tag{5}$$

where $\beta_i \in [0, 1]$ is the substitution rate at which i's excess demand is allocated to firm j.

If x_i and x_j denote the firms's inventory levels respectively, then the expected profit for the firm j is

$$E[\pi_j(x_i, x_j)] = rE\min\{x_j, R_j\} + sE(x_j - R_j)^+ - pE(R_j - x_j)^+ - cx_j. \tag{6}$$

Parlar [63] is perhaps the first author to treat an inventory problem using game theory. She examines an extension of the classic newsboy problems in which two retailers (players) sell substitutable products. She modeled the two-product single-period problem as a two-person nonzero-sum game and showed that there exists a unique Nash equilibrium. In her two-player model, substitution occurs with a certain probability.

5 Noncooperative Solution

Noncooperative solution deals with how rational individuals interact with one another in an effort to achieve their own goals. The emphasis is on the strategies of players and the consequences of interaction on payoffs. The purpose is to make predictions on the outcome. The solution concepts that are commonly used are the Nash equilibrium introduced by J.F Nash [54] and the Stackelberg equilibrium introduced by the economist von Stackelberg [89].

5.1 Nash Equilibrium

Single-Period Model Formulation

A *Nash equilibrium* recommends a strategy to each player that the player cannot improve upon unilaterally, that is, given that the other players follow the recommendation. Because the other players are also rational, it is reasonable for each player to expect opponents to follow the recommendation as well. A vector $x^* = (x_i^*)_{i \in N} \in X$ is a Nash equilibrium if and only if for all $i \in N$

$$\pi_i(x_i^*, x_{-i}^*) \geq \pi_i(x_i, x_{-i}^*) \quad \forall x_i \in X. \tag{7}$$

In a Nash equilibrium, each player is doing the best she can do given the strategies of the other players, x_{-i}, i.e., player i has no incentive to deviate from x_i^* when all other players play x_{-i}^*.

Player's i *best response* (function) is the strategy x_i^* that maximizes the player's i payoff. That is

$$x_i^*(x_{-i}) = \arg \max_{x_i} \pi_i(x_i, x_{-i}). \tag{8}$$

The best response function is uniquely defined by the first-order condition if π_i is quasi-concave in x_i. The Nash equilibrium assumes no one of the players has the power to dominate the decision process.

When there is no cooperation between the firms and if both firms are "rational," one of the possible strategies they may adopt is the Nash strategy. A pair of inventory levels (x_i, x_j) $(i, j = 1, 2)$ is a *Nash equilibrium* if neither firm can improve its expected profit by altering its inventory, that is

$$\begin{aligned} E[\pi_i(x_i^*, x_j^*)] &\geq E[\pi_i(x_i, x_j^*)] \;\forall\; x_i \geq 0 \;\; (i, j = 1, 2) \\ E[\pi_i(x_i^*, x_j^*)] &\geq E[\pi_i(x_i^*, x_j)] \;\forall\; x_j \geq 0 \;\; (i, j = 1, 2). \end{aligned} \tag{9}$$

Equation (9) implies that, given the player's j Nash solution x_j^*, player i will not do better if she does not play her Nash solution x_i^*. That is, given the x_j^* , x_i^* maximizes player's objective function, and vice versa. Therefore, the best response for each player will be

$$x_i^*(x_j) = F_{R_i}^{-1}\left(\frac{r_i - c_i + s_i}{r_i + p_i - s_i}\right) \quad (i, j = 1, 2). \tag{10}$$

The best response function can be found by optimizing each player's expected profit function w.r.t the player's own order quantity, provided that $E[\pi_i]$ is continuously differentiable in x_i and it is concave for every x_j. Taking together the best response function of each player, we obtain a best response mapping $R^2 \to R^2$ (see Figure 1). Obviously if x_i^* is a best response to $x_j^*, \forall\; (i, j = 1, 2)$, then the outcome (x_i^*, x_j^*) is a *Nash equilibrium.* Parlar [63, Lemma 1-2, pp. 403–04] has proved that the slope of the best response

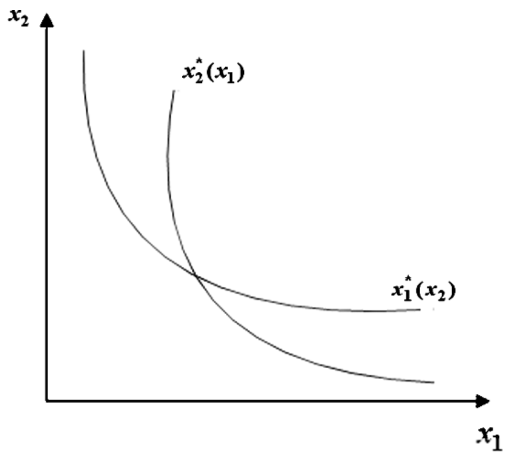

Figure 1. Best response functions in an inventory game

functions is negative, which implies that each player response is monotonically decreasing in the other player's strategy. Indeed, according to [63], the player's Nash solution is a unique point (x_i^*, x_j^*) obtained by solving a system of best responses:

$$x_1^*(x_2^*) = F_{R_1^*}^{-1}\left(\frac{r_1 - c_1 + s_1}{r_1 + p_1 - s_1}\right) \tag{11}$$

$$x_2^*(x_1^*) = F_{R_2^*}^{-1}\left(\frac{r_2 - c_2 + s_2}{r_2 + p_2 - s_2}\right) \tag{12}$$

where $R_i^* = w_i + \beta_i(w_j - x_j^*)^+, i, j = 1, 2$.

Historically, most researchers establish the existence of an equilibrium based on the study of the concavity or quasi-concavity of profit function. Dasgupta and Maskin [27], Parlar [63], Mahajan and van Ryzin [48], Netessine *et al.* [59], among others establish the existence of a Nash equilibrium based on the two above-mentioned properties of the profit function.

However, the existence of a Nash equilibrium for a general case can be established by employing the result of the supermodular game. A function $f(x_1, x_2)$ is supermodular if $f(x_1, x_2) + f(y_1, y_2) \geq f(x_1, y_2) + f(y_1, x_2)$, for all $(x_1, x_2) \geq (y_1, y_2)$. Notice that supermodularity is a weaker condition than concavity, see [95] for a detailed discussion. If the profits are supermodular, then the best response mapping is increased in the other player's strategy. When the best response function has such a monotonicity property, the existence of a Nash equilibrium could be established. The theory of supermodular games is a relatively recent development introduced and advanced by Topkis [84]. See [12, 16, 46, 58, 60] for its application to the competitive news-vendor problem.

As an extension of the model in [63], Wang and Parlar [92] studied the three-product single-period problem. Lippman and McCardle [46] also study an extension of the classic news-vendor problem in which the salvage value of excess inventory and penalty for unmet demand are assumed to be zero. Under this assumption, they examine the equilibrium inventory levels and the rules to reallocate excess demand. They provide conditions under which a Nash equilibrium exists for the case with two or more news-vendors. They examine both the two-firm game and a game with an arbitrary number of players. In their models, initial industry demand is allocated among the players according to a prespecified "splitting rule." This initial allocation may be either deterministic or stochastic. For the two-firm game, they establish the existence of a pure-strategy Nash equilibrium and show that the equilibrium is unique when the initial allocation is deterministic and strictly increasing in the total industry demand for each player. They have proved that competition can lead to higher inventories.

Mahajan and van Ryzin [48] study a model with n retailers that provides substitutable goods, assuming that the demand process is a stochastic sequence of heterogeneous consumers who choose dynamically from the available goods (or choose not to purchase) based on a utility maximization criterion. They demonstrate that an equilibrium exists and show that it is unique for a symmetric game. Their results are similar to [46].

Recent extensions of these models include the work by Rudi and Netessine [75]. They analyze a problem similar to [63] but for an arbitrary number of products. Given mild parametric assumptions, they establish the existence of, and characterize, a unique, globally stable Nash equilibrium. On the other hand, with the substitution structure of their model, they conclude that, under competition, some firms may stock less than under centralization. Under the long-run average payoff criterion, the nonlinear programming formulation developed by Filar *et al.* [33] can be used to compute Nash strategies. If the discounted payoff criterion is considered, then Nonlinear Program (NLP) due to Raghavan and Filar [72] is available. Chand *et al.* [21] and Drezner *et al.* consider the case where the substitution between products take place in a EOQ model.

Multiperiod Model Formulation

Because inventory models used in the literature often involve inventory replenishment decisions that are made over an infinite period of time, multiperiod games should be a logical extension of these inventory models. In the analysis of substitutable product inventory problem over infinite horizon, concepts of *sequential games*, introduced by [38], are used. Two retailers of different products who compete for the substitutable demand of these products are the players of the game. Each player's decision sequence influences the evolution of the process and affects the streams of rewards to all players. A sequential game is said to have a *myopic* solution if its data can be used easily to specify a one-period game such that infinite repetition of a Nash equilibrium of the one-period game comprises an equilibrium for the sequential game.

The mathematical formulation considered is a nonzero-sum game because what is earned (or lost) by one retailer may not be the loss (or earning) of the other retailer although what is earned or lost by each retailer depends on both strategies, not the strategy taken by just that retailer. Demand distributions of the products and the substitution rates are known by both players. So, being aware of all of the parameters and the strategies that can be employed by the opponent, each retailer tries to find out the best strategy as a reply to the opponent. Because the retailers somehow agree (although they do take their actions independently in a strictly competitive environment, they know all the parameters that would affect their decisions) on a pair of strategies, called Nash strategies in the context of nonzero-sum games, this pair is said to be an equilibrium point. Unilateral deviations of either of the players from her Nash strategy do not improve her expected payoff.

Specifically, in a multiple-period setting, we consider two retailers that simultaneously make inventory replenishment decisions at the beginning of each period using a periodic review base-stock policy. If one retailer experiences a stock-out, a portion of the customers who are not satisfied will switch to the other retailer. Leftover inventory at the end of the period is carried over to the next period, incurring inventory holding cost.

At the beginning of each period t, $(t = 1, 2, \ldots, n)$, two retailers review their inventories and simultaneously make replenishment decisions. Let w_i^t denote the exogenously given (random) demand for the product of retailer i in period t. Product i is sold for r_i per unit, $(i = 1, 2)$. Ordering cost is a linear function of the order quantity x_i^t for product i in period t. c_i, which satisfies $0 < c_i < r_i$, is the ordering cost per unit of product i. Let I_i^t be the inventory levels of the retailer's i, at the beginning of period t. Orders are delivered instantaneously so that $z_i^t = I_i^t + x_i^t$ are the inventory levels just after the orders are replenished. p_i is the unit lost sale cost, and h_i is the inventory holding cost per unit of product i per period. Substitution rates are given as the probabilities that a customer switches from one type of product to the other when the product demanded is sold out. β_i is the substitution rate at which i's excess demand is allocate to firm j. Further, the actual demand for retailer i depends on the beginning inventory of retailer j in period t I_j^t as well as on her own beginning inventory level at period t, I_i^t. That is,

$$R_i^t = w_i^t + \beta_j (w_j^t - z_j^t)^+ \quad i, j = 1, 2 \quad t = 1, 2, \ldots, n \tag{13}$$

The inventory balance equations are

$$I_i^{t+1} = [z_i^t - w_i^t - \beta_j (w_j^t - z_j^t)^+]^+ \quad i, j = 1, 2, \quad t = 1, 2, \ldots, n \tag{14}$$

Note that if retailer j cannot satisfy demand w_j^t fully, then the remaining demand $[w_j^t - z_j^t]^+$ switches to retailer i or vice versa. By suppressing subscript t, i.e., considering the order-up-to-levels as $z_i = I_i + x_i$, $i = 1, 2$, when the order-up-to-levels (z_1, z_2) are chosen by the two retailers in a singe period

$$E[\pi_i(z_1, z_2)] = r_i E \min\{R_i, z_i\} - h_i E(z_i - R_i)^+ - p_i E(R_i - z_i)^+ - c_i x_i \tag{15}$$

is the one-period expected profit for retailer i.

Because future payoffs are in general worth less today, it is reasonable to look at discounted payoffs. Suppose that each retailer starts with initial inventories (I_1^1, I_2^2) respectively, the expected discounted profit of retailer i for the remaining period until the end of the planning horizon is given by:

$$E[\pi_i] = E \sum_{t=1}^{\infty} \delta_i^{t-1} \left[r_i \min\{z_i^t, R_i^t\} - h_i (z_i^t - R_i^t)^+ - p_i (R_i^t - z_i^t)^+ - c_i x_i^t \right]. \tag{16}$$

The discount factor is assumed stationary and will be denoted by δ, $0 < \delta < 1$. By using manipulations proposed by Heyman and Sobel in [38], the objective function can be converted to:

$$E[\pi_i] = c_i x_i^1 + \sum_{t=1}^{\infty} \delta_i^{t-1} G_i^t(z_i^t), \quad i = 1, 2 \tag{17}$$

where $G_i^t(z_i^t)$ is the single-period objective function. If we assume that demand is stationary and independently distributed among periods, i.e., $w_i = w_i^t$, we obtain that $G_i^t(z_i^t) = G_i(z_i)$, furthermore if we assume that the inventory policy is stationary as well, i.e., $z_i^t + z_i, t = 1, \dots, n$, then each retailer could solve the problem under consideration as a sequence of the solution to a single-period game, which is

$$z_i^* = F_{R_i^*}^{-1} = \left(\frac{r_i - c_i}{r_i + h_i + p_i - c_i \delta_i} \right) \quad i = 1, 2. \tag{18}$$

For a complete analysis, see Netessine *et al.*[59].

Avsarand and Baykal-Gürsoy [4] analyzed the substitutable product inventory problem using the concepts of stochastic game theory. It is assumed that there are two substitutable products that are sold by different retailers and the demand for each product is random. Game theoretic nature of this problem is the result of substitution between products. Because retailers compete for the substitutable demand, ordering decision of each retailer depends on the ordering decision of the other retailer. Under the discounted payoff criterion, this problem is formulated as a two-person nonzero-sum stochastic game. In the case of linear ordering cost, it is shown that there exists a Nash equilibrium characterized by a pair of stationary base-stock strategies for the infinite horizon problem. This is the unique Nash equilibrium within the class of stationary base-stock strategies.

In addition, more elaborate models capture some effects that are not present in static games. Netessine *et al.* [59] consider the case where when a product is out of stock, the customer often faces a choice of either placing a backorder or turning to a competitor selling a similar product. They consider the four alternative backordering scenarios and formulate each problem as a stochastic dynamic game. They proved that a stationary base-stock inventory policy is a Nash equilibrium of the game and hence it can be found by considering an appropriate static game.

van Mieghem and Dada [86] study a two-period game with capacity choice in the first period and production decision under the capacity constraint in the second period.

5.2 Stackelberg Equilibrium

Stackelberg equilibrium assumes that there is a player who has powerful position and dominates in the desicion process, *the leader*, and the other players, *the followers*, given that they are rational, are free to choose their optimal strategies given their knowledge of the leader's decision. If player i is the leader, she will choose her optimal strategy x_i^*, and the followers's best response x_{-i}^* will be

$$x_{-i}^*(x_i^*) = \{x_{-i}^* | \pi_{-i}(x_i, x_{-i}^*) \geq \pi_{-i}(x_i, x_{-i}\} \tag{19}$$

To find an equilibrium of a Stackelberg game, which is often called the Stackelberg equilibrium, we need to solve a dynamic multiperiod problem via backwards induction.

In a Stackelberg game, one firm, called leader, makes an order first, then the other firm, called follower, makes her order. Because the follower makes her decision after the leader announces hers, the Stackelberg solution will be located on the reaction curve of the follower's defined by equation:

$$x_2^*(x_1) = F_{R_2}^{-1}\left(\frac{r_2 - c_2 + s_2}{r_2 + p_2 - q_2}\right) \tag{20}$$

which means that the follower will always choose her order quantity x_2 to maximize her expected profit for each value of x_1. Intuitively, the leader chooses the best possible point on the follower's best response function; i.e., she tries to solve the following bilevel programming model [63]:

$$\max \quad E[\pi_1(x_1, x_2)] \tag{21}$$

$$\text{where} \quad x_2 \quad \text{solves}$$

$$\frac{\partial E[\pi_2(x_1, x_2)]}{\partial x_2} = 0. \tag{22}$$

Whereas the existence of a Stackelberg equilibrium is easy to demonstrate given the continues payoff function, uniqueness may be considerably harder to demonstrate [18].

Raju and Zhang [69] analyze the Stackelberg game in which one of the retailers is dominant and capable of unilaterally setting a retail price that will be adopted by all other retailers.

Lariviere and Porteus [43] consider a simple supply-chain contract in which a manufacturer sells to a retailer facing a news-vendor problem. The Stackelberg game they set up assumes that first the supplier establishes the wholesale price and then the news-vendor chooses an order quantity, the long contract parameter is the wholesale price. They show that the manufacturer's profit and sales quantity increase with market size, but the resulting wholesale price depends on how the market grows. Anand *et al.* [1] extend the Stackelberg equilibrium concept into multiple periods.

See Netessine and Rudi [57] for a Stackelberg game.

6 Supply Chain Coordination

In another line of research, there exists a large body of research that addresses echelon inventory system with the stationary stochastic demand and fixed lead time. Many of them use the following two-echelon gaming structure: a "manufacturer" wholesales a product to a $n \geq 1$ "retailers," who in turn retail it to the consumer. The literature on competitive supply chain inventory management recognizes that supply chain is usually operated by independent agents with individual preferences and possibly conflicting objectives.

Total expected supply chain profit will be maximized if all decisions are made by a single decision maker with access to all available information. This is referred as the *optimal case* or *first-best case* and is often associated with *centralized control*. Under centralized control, a system manager needs to know how to design a mechanism to optimize the performance of the whole supply chain.

However, in reality, no single agent has control over the entire supply chain, and hence no agent has the power to optimize the supply chain, and each player has his own incentives and state of information. This is referred as a *decentralized control* structure. Under decentralized control, each player needs to know how to behave in order to maximize his profit. In order to increase the total profit of a decentralized supply chain and improve the performance of the players, one strategy is to form contracts among players by modifying their payoffs. The main purpose of a supply chain contract is to overcome an inefficiency known as *double marginalization* [79]. This is because without coordination, the supplier and the retailer only have the incentive to optimize their own profit margin, and their collective decision is always less efficient than what could have been achieved by the system-optimal. Thus, the aim of a coordination contract is to provide the incentive for both players to implement the system-optimal solution, which results in higher total profits for the collective whole. Some contracts provide a means to bring the total profit resulting from decentralized control to the centralized optimal profit. This is referred to as *channel coordination*. Generally speaking, channel coordination may be achieved by three steps: First, determine the optimal solution under centralized control. Next, under decentralized control, apply game theory to determine how the players will behave when they each seek to maximize their own profits, and whether a Nash equilibrium exists. Finally, if the decentralized and centralized solutions differ, investigate how to modify the players' profit so that the new decentralized solution matches the centralized solution.

Consider the case of a supply chain that consists of two echelons: the first echelon is the supplier, usually the manufacturer, and the second echelon consists of two retailers. At the beginning of the period, the retailers place orders x_i and sell them to the customer at a unit price r_i. Supplier produces the product with unit production cost k and supplies x_i units to retailers at a price c_i. It is also assumed that supplier has infinite production capacity. The demand w_i during the period at each retailer is random but distributions are known. Customers encountering a stock-out at retailer i visit retailer j, $(i, j = 1, 2)$ with probability β_{ij} before leaving the system.

Thus, the total demand faced by retailer i is

$$R_i = w_i + \beta_{ij}(w_j - x_j)^+. \tag{23}$$

At the end of the season, the holding cost h_i or shortage cost p_i is incurred depending on whether there is unsold stock or a stock-out.

Assuming the whole supply chain is in centralized control, in order to maximize the total profit, what are optimal orders for both retailers?

In centralized control, a stock-out penalty incurred only when customers leave the system unsatisfied. This includes customers who visit only one retailer and leave unsatisfied and customers who visit both retailers and leave unsatisfied. In the latter case, the amount of the penalty incurred is assumed to be the stock-out penalty cost of the retailer visited first by the customer. The total profit is maximized if supplier should only provide what is needed by the two retailers, i.e., supplier does not face any shortage or holding cost. The expected total profit of the system $E[\pi(x_i, x_j)]$ is

$$E[\pi(x_1, x_2)] = E\left[\sum_{i=1}^{2} r_i \min\{y_i, R_i\} - \sum_{i=1}^{2} h_i(x_i - R_i)^+ - \sum_{i=1}^{2} p_i E(R_i - x_i)^+ - k\sum_{i=1}^{2} x_i\right], \tag{24}$$

which only depends on the retailers's sales quantity. We consider the whole supply chain as an entity, and the money flow within the system is not involved. Therefore, the optimal solution (x_1^*, x_2^*) does not depend on the wholesale prices, c_1 and c_2. Actually, supplier's price decision creates only a transfer payment among firms so it does not influence supply chain's profit.

Because equation (24) is concave, the optimal order quantity of both retailers can be found by solving the system of equations:

$$\begin{aligned} \frac{\partial E[\pi(x_1, x_2)]}{\partial x_1} &= 0 \\ \frac{\partial E[\pi(x_1, x_2)]}{\partial x_2} &= 0 \end{aligned} \tag{25}$$

If the supply chain is under decentralized control, each retailer tries to maximize his own profit. Therefore retailer i $(i = 1, 2)$ profit will be

$$E[\pi_i(x_1, x_2)] = r_i E \min\{x_i, R_j\} - h_i E(x_i - R_i)^+ - p_i E(R_i - x_i)^+ - c_i x_i. \tag{26}$$

Because the decision of one retailer affects the total demand at the other retailer, a game arises as the two retailers make their ordering decisions. Based on the previously presented we know that the Nash equilibrium can be obtained by solving of best responses:

$$x_1^*(x_2^*) = F_{R_1^*}^{-1}\left(\frac{r_1 - c_1}{r_1 + p_1 + h_1}\right) \tag{27}$$

$$x_2^*(x_1^*) = F_{R_2^*}^{-1}\left(\frac{r_2 - c_2}{r_2 + p_2 + h_2}\right) \tag{28}$$

$$\text{where} \quad R_i^* = w_i + \beta_{ij}(w_j - x_j^*)^+, i, j = 1, 2$$

and depends on the wholesale prices c_i. If the chain is not coordinated, each retailer selfishly optimizes its own profit. Hence decentralized decision making may introduce inefficiency in the supply chain as a Nash equilibrium may not be *Pareto optimal*, see [14] for a use of Pareto optimality in the supply chain analysis.

The supply chain coordination can be obtained by determining the wholesale price c_i so as to make the optimal solution (x_1^*, x_2^*) obtained by (25) a Nash equilibrium, that is (x_1^*, x_2^*) must satisfy:

$$\begin{aligned} \frac{\partial E[\pi(x_1, x_2)]}{\partial x_1}|_{x_1=x_1^*, x_2=x_2^*} &= 0 \\ \frac{\partial E[\pi(x_1, x_2)]}{\partial x_2}|_{x_1=x_1^*, x_2=x_2^*} &= 0. \end{aligned} \tag{29}$$

The coordination mechanism modifies each decision maker's objective so that these modified objectives and the total objective of the supply chain yield to the same optimal solution. The mechanism that is mainly used for coordination in the supply chain is a contract. A contract is an argument between two parties. Most supply chain contracts include only two parties usually a supplier and a retailer, but these simplifications allow for studying optimal contracts. Different models of Supply Chain contracts have been developed in the literature. They include the quantity discounts [93], the backup agreements [31], the buy back or return policies [30], the quantity flexibility (QF) contracts [82], the incentive mechanisms [44], and the revenue sharing (RS) contracts [15].

Anupindi and Bassok [2] studied a model with one manufacture and two retailers. They consider two systems: one competitive, where they make independent decisions and stock inventories separately, and one where they cooperate to centralize stocks at a single location. They show that there exits a threshold level for the "market search" above which manufacturer loses, and that for high level of market search, even total supply chain's profit may decrease upon centralization. Market search is measured as the fraction of customers who due to a stock-out at their retailer search for the good at the other. In addition, they show that manufacturer could benefit, in either system, by offering a contract with a holding cost subsidy.

Cachon and Zipkin [19] investigate a two-stage (supplier and retailers) serial supply chain with stationary stochastic demand, fixed transportation time over an infinite horizon, and complete backordering. Both firms incur holding costs and a backorder penalty per unit of time for each unit that is backordered at the retailer; the supplier is not charged for its own backorders, it is only charged when units are backordered at the retailer. That fee reflects the supplier's desire to maintain an adequate stock of its product at the retailer. They compare the base-stock policies chosen under the competitive regime to those selected so to minimize total supply chain costs. Furthermore, they use a linear contract between the supplier and the retailer to modify the payoff of the players and make the total profit close to the global optimum. The model proposed by Cachon [11] is also a two-echelon serial supply chain with stochastic consumer demand. But when a customer arrives at the retailer and the retailer has no stock, a lost sale occurs. As in [19], both firms are concerned about the availability of inventory at the retailer, but in this model stock-outs create opportunity costs rather than backorder penalties.

Wang *et al.* [91] extend Cachon and Zipkin's model to a one-supplier and n-retailers situation. If there exist multiple retailers, the supply from a supplier might not satisfy the demand of multiple retailers. The problem is how to design the distributing scheme of the supplier, and this makes models of supply chain systems more complex. In order to guarantee optimal cooperation in the system, several Nash equilibrium contracts are designed in echelon inventory games and local inventory games.

Coordination technique proposed by Lee and Wang [44] assumes a two stage supply chain with stationary demand, with holding and backorder costs and fixed lead time. Furthermore, assume that supplier cares only about his inventory. The nonlinear transfer payments proposed by Lee and Wang uses the nonlinear transfer payment proposed by Clark and Scarf [24] but this type of payment leads to a Nash equilibrium for the decentralized supply chain. Using similar assumptions, Chen [22] studied a four-stages supply chain where players try to minimize total supply chain costs. The coordination scheme he proposed is linear transfer payments based on accounting inventory and backorders level, where stage is accounting inventory is the actual inventory that could fill its orders at stage $i + 1$ immediately. Porteus [68] proposed a incentive scheme that is a combination of the above two, called responsibility token.

In contrast, Klastorin *et al.* [41], to coordinate a two-echelon distribution system, use price discounts contract. The supplier, in order to influence the buyer's behavior, offers a price discount to any retailer who places an order that coincides with the beginning of retailer's cycle. They show that under specific conditions, this policy can lead to more efficient supply chain management, and present a method for determining the optimal price discount in the decentralized supply chain. For excellent reviews on supply chain coordination and contracts, see Tsay *et al.* [83] and Cachon [13].

6.1 Capacity Allocation in Supply Chain

In many situations, a single supplier provides products to several retailers. If retailers orders are uncertain and capacity is costly, the supplier may not be willing to have capacity that is high enough to cover all orders at any point in time. When the total order from retailers exceeds the supplier's capacity, then he must allocate it among retailers based on some sort of rules. In such a case, the two retailers compete for both supply and demand, and a game called *allocation game* or *shortage game* as is refereed by Lee *et al.* [45] arises.

Three allocation rules are commonly used: *proportional*, where the retailer receives a proportion of the avaliable capacity as a percentages of his order to the total orders; *linear*, where the retailer receives his order minus the difference between total order and capacity divided by the number of retailers; and *uniform*, where the supplier equally divides the avaliable capacity among retailers [17]. However, when the supplier's capacity is finite, the Nash equilibrium exists only under certain conditions.

Assuming that the avaliable capacity of the supplier is κ, suppose that retailer i makes an order $x_i < \kappa, (i = 1, 2)$, then retailer $j, (j = 1, 2)$ can react by making an order either $x_j \leq \kappa - x_i$, where because total orders do not exceed κ, each retailer gets exactly what she orders, or by making an order $x_j > \kappa - x_i$, then the capacity is apportioned by any allocation rule such that $\bar{x}_1 + \bar{x}_2 = \kappa$. Note that what the retailer i gets, $\bar{x}_i (i = 1, 2)$, differs from what he orders. Further assume that a pair of (x_1^*, x_2^*) is the unique Nash equilibrium that solves the news-vendor problem faced by retailers; if $x_1^* + x_2^* \leq \kappa$, then there exists a unique Nash equilibrium allocation as neither retailer has a profitable unileteral deviation.

Now, consider the case where $x_1^* + x_2^* > \kappa$, let $\hat{x}_1 \in x_2^*(x_2)$ for which it holds that $\hat{x}_1 + \bar{x}_2 = \kappa$ and $\hat{x}_2 \in x_2^*(x_1)$ such that $\bar{x}_1 + \hat{x}_2 = \kappa$, then there exists a Nash equilibrium if and only if there exist a pair of allocations $(\bar{x}_1, \bar{x}_2) \in (\hat{x}_1, \hat{x}_2)$ such that $\bar{x}_1$ is the optimal solution to the problem:

$$\begin{aligned} \max \qquad & E[\pi_1(\bar{x}, \kappa - \bar{x})] \\ \text{s.t } \max\{\kappa - \bar{x}_2, 0\} & \leq \bar{x} \leq \kappa \end{aligned} \tag{30}$$

and $\bar{x}_2$ solves the problem:

$$\begin{aligned} \max \qquad & E[\pi_2(\bar{x}, \kappa - \bar{x})] \\ \text{s.t } \max\{\kappa - \bar{x}_1, 0\} & \leq \bar{x} \leq \kappa. \end{aligned} \tag{31}$$

See Dai [26] for a detailed analysis and proofs, and [16, 45] for application of shortages game in the supply chain.

7 Cooperative Games

The subject of cooperative games was first introduced by von Newmann and Mörgestern [88]. Cooperative game theory assumes that binding agreements can be made between players on the advantage of the whole system. One of the main questions is whether the cooperation is stable, i.e., there is an allocation of the total benefit of the system among the players such that no group of players would like to leave the system. Cooperative game theory offers the concept of core as a direct answer to that question. For a long time, cooperative game theory did not enjoy as much attention in inventory management literature as noncooperative game theory. Papers employing cooperative game theory to study inventory problems had been scarce but are becoming more popular. The vast majority of them model the system in a news-vendor setting. In a news-vendor environment, retailers can increase their total profit if they decide to cooperate. The basic cooperation rules that could appear are (1) cooperative players might switch their excess inventory, if any, to anyone who has excess demand so that the latter can save in lost sales penalty cost, and (2) retailers might give a joint order and use this quantity to satisfy the total demand they are faced with. The allocation rules for this cooperation should be based on three criteria, namely nonemptiness of the core, computational ease, and justifiability [36].

If there are $n > 2$ players in the game, then there might be cooperation between some, but not necessarily all, of the players. We can ask which coalitions of players are likely to form and what are the relative bargaining strengths of the coalitions that do form. Label the players $1, 2, \ldots, n$. A coalition of players, S, is then a subset of $N = (1, 2, \ldots, n)$. Let $v(S)$ denote the maximum value $v(S)$ that coalition S can guarantee itself by coordinating the strategies of its members, no matter what the other players do. This is the called the *characteristic function*. By convention, we take $v(\emptyset) = 0$. The worst eventuality is that the rest of the players unite and form a single opposing coalition $T = N - S$. This is then a 2-person noncooperative game and we can calculate the maximum payoff that S can ensure for itself. In such a case, the cooperation is worthwhile, that is, any two groups, which act together, will get no less than that when they act independently. In other words, the property of superadditivity holds, i.e.,

$$v(S \cup T) \geq v(S) + v(T). \tag{32}$$

The distribution of individual rewards will affect whether any coalition is likely to form. Each individual will tend to join the coalition that offers her the greatest reward. Therefore, the game in such a form should provide an indication of how the joint maximum payoff $v(N)$ should be shared among the N players. *An imputation* for an n-person game, with characteristic function v, is defined as a distribution vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ satisfying:

$$\sum_{i}^{n} x_i = v(N) \quad \& \quad x_i \geq v(i) \quad \forall \quad i \in N \tag{33}$$

with x_i being the payoff to player i.

In other words, if x is an indication of how the joint payoff $v(N)$ is distributed among the players and if a player i is rational, then she is willing to join a coalition if and only if she gets no less than the amount she can get by acting independently. The first condition is often referred to as group rationality and the second condition as individual rationality.

Let $E(v)$ be the set of imputations, and $\mathbf{x}, \mathbf{y} \in E(v)$. We say that $\mathbf{y}$ dominates $\mathbf{x}$ over S if

$$y_i > x_i \quad \forall \quad i \in S \quad \& \quad \sum_{i \in S} y_i \leq v(S). \tag{34}$$

In other words, an imputation is dominated if it is dominated via some coalition $S \subseteq N$. Members of the dominating coalition S benefit from forming S and leaving the grand coalition.

The core of a game with characteristic function v is the set, $C(v)$, of all imputations that are not dominated for any coalition. Therefore, an imputation $\mathbf{x}$ is in a core if and only if:

$$\sum_{i}^{n} x_i = v(N) \;\; \& \;\; \sum_{i \in S} x_i \geq v(S) \;\; \forall \;\; S \subseteq N. \tag{35}$$

The core collects undominated imputations. The core is a set-valued solution concept for cooperative games, as it can select multiple payoff vectors. Nonemptiness of the core means that there exists at least one allocation of the joint profits among the players such that no group of players has an incentive to leave. A game is balanced if it has a nonempty core (see Bondareva [9], Shapley [77]), and it is called totally balanced if each subgame $(S, v_{|S})$ is balanced, where $v_{|s}(T) = v(T)$ for all $T \subseteq S$. A subgame is any part of a game that remains to be played after a series of moves and it starts at a point where both players know all the moves that have been made up to that point [25].

Perhaps the first paper employing cooperative games in inventory management is Wang and Parlar [92]. In a model of inventory competition with fixed prices, they use cooperative game theory in one of its original uses: they start with a noncooperative game, then suppose that the players can cooperate on strategy choices with and without Transferable Utility.

What follows is based mainly on Slikker *et al.* [78]. A general cooperative news-vendor situation is characterized by a set of retailers N, the stochastic demand W_i for the good at retailer $i \in N$, furthermore c_i and r_i denote the prices that retailers pay to producer and the customers pay to the retailers, respectively. If several companies cooperate they can, after the realization of demand is known, transship goods. t_{ij} represents the cost of transshipping one unit from i to j, $i, j \in N$ and $t_{ij} \geq 0$. Let X^S be a collection of possible order vector of coalition S retailers defined by:

$$X^S = \left\{ x \in \mathbb{R}^N | x_i^S = 0 \;\; \forall \;\; i \in N \setminus S \;\; \text{and} \;\; x_i^S \geq 0 \;\; \forall \;\; i \in S \right\} \tag{36}$$

and suppose that coalition S has order vector $x^S \in X^S$ and they face demand vector $w^S \in \mathbb{R}^N$ with $w^S = 0$ for all $i \in N \setminus S$. If after the realization of demand, A_{ij}^S is the amount of products that are transshipped from retailer i to retailer j, the amount that is not transshipped is represented by A_{ij}^S for $i = j$. A reallocation matrix of x^S is then

$$A^S = \{A^S \in \mathbb{R}_+^{N\times N} | A_{ij}^S = 0 \text{ if } i \notin S \text{ or } j \notin S$$
$$\sum_{j\in S} A_{ij}^S = x_i^S \ \forall \ i \in S\} \tag{37}$$

The profit of the coalition S is

$$\pi^S(x^s, w^s) = \sum_{j\in S} r_j \min\left\{\sum_{i\in S} A_{ij}^S, x_j^S\right\} - \sum_{i\in S}\sum_{j\in S} A_{ij}^S t_{ij} - \sum_{i\in S} c_i x_i^S \tag{38}$$

The expected profit of coalition S depends on their order quantity vector x^S and the stochastic demand faced by each retailer, W^S, that is

$$\bar{\pi}^S(x^S, W^S) = E\left[\pi^S(x^S, W^S)\right] \tag{39}$$

and the associated game is defined by

$$v(S) = \max_{x\in X^S} \bar{\pi}^S(x, W^S) \ \forall \ S \subseteq N. \tag{40}$$

Slikker *et al.* [78] proved that there exist coalitions and there exist a reallocation matrix A^{*S} and an order quantity x^{*S} that maximizes the expected profit of coalition formation, as well as that the above cooperative news-vendor game has a nonempty core. Hartman *et al.* [37] and Müller *et al.* [49] consider the game in the above-mentioned setting, except that retailers identically single price c and r and in which the value of the group of retailers is their optimal profit if they jointly determine an order size without taking into account the transshipment cost. Both of the above-mentioned papers use the core to show that there is always a cost allocation scheme such that news-vendors will prefer to pool their inventory. The cost game they consider is

$$c(S) = (r - c)E[W^S] - v(S) \ \forall \ S \subseteq N. \tag{41}$$

Hartman *et al.* [37] prove that this game has a nonempty core under certain assumptions about the demand distribution, and Müller*et al.* [49] come up with a more powerful result, namely that the core of the news-vendor games are nonempty regardless of the distribution of the random demands.

7.1 Shapley Value

A solution concept that selects precisely one payoff vector for every cooperative game is the Shapley value. Used on the marginal contributions of all players in the game (N, v), the Shapley value [76], $\Phi(v) = (\Phi_i(v))_{i\in N}$ is defined by:

$$\Phi_i(v) = \sum_{S\subseteq N\setminus\{i\}} \frac{|S|!(|N|-1-|S|)!}{|N|!} \left(v(S\cup\{i\}) - v(S)\right) \tag{42}$$

So far, applications of Shapley value in inventory management are rather scarce, an exception is the paper of Robinson [73], who reexamines the allocation rules proposed by [34] in continuous review single-period inventory model and in terms of Shapley value, and the discussion presented in Granot and Sosic [35].

The Shapley value means that each player should be paid according to how valuable her cooperation is for the other players. In general, the Shapley value need not generate a core element. Hence, it may not be a reasonable prediction of the outcome of a game; because it is not in the core, there exists some subset of players that can deviate and improve their payoffs.

8 Bargaining Theory

In recent years, there is a trend in supply chain literature that considers the use of bargaining theoretic models to expand the view of negotiation and coordination in the supply chain. In this chapter, we present modeling paradigm for supply chain coordination using the notion of bargaining.

Bargaining theory helps to explore the relationship between the expected outcome from direct negotiation. Nash defines a *bargaining problem* as the situation in which two individuals have the opportunity to collaborate for mutual benefits in more than one way [55]. In other words, the bargaining problem arises in situations where there are gains Π from collaboration and is defined as the corresponding attempt to resolve a bargaining situation, i.e., to determine the particular form of cooperation and the corresponding division (π_1, π_2) of the bargaining surplus Π.

In a *bargaining game*, two or more players, who have competitive preferences, negotiate to follow a common mixed strategy in order to conclude with an outcome that is *fair* and *satisfactory* for all of them, that is how to divide bargaining surplus (gains) created from collaboration. It is assumed that both players are rational, self-interested, and risk neutral (expected value maximizers) with complete information. The possible outcome of the agreement depends on the negotiation power of each player.

A bargaining set $\mathcal{B}$ is a set of outcomes that can be jointly achieved by the players, $\mathcal{B} = \{(\pi_1, \pi_2) \in \mathbb{R}^2 : \pi_1 + \pi_2 \leq \Pi \ \text{ and } \ \pi_i \geq 0\}$. The players either reach an agreement $(\pi_1, \pi_2) \in \mathcal{B}$, or fail to reach agreement, in which case the disagreement event $D = (d_1, d_2)$ occurs and each gets nothing. An outcome is Pareto-efficient if it dominated over all possible outcomes, i.e., if no outcome exists that is strictly preferred by one player and not less preferred by any other player.

Given the bargaining set, a solution to the bargaining problem is concerned with the question of which outcome will eventually prevail, i.e., a solution is a rule that picks out one element of the bargaining set. Apparently, two different approaches of solutions to the bargaining problem exist in bargaining theory: (1) *axiomatic (cooperative game) approach*, which requires that the resulting solution should possess a list of axioms, and (2) *strategic (noncooperative game) approach*, in which the outcome is predicted by the notion of subgame perfect equilibrium.

8.1 Axiomatic Solution

The cooperative bargaining process was initiated by J. Nash [55]. In case of cooperative bargaining, the outcomes of negotiation are often described in terms of utilities; the notion of *utility* that satisfies the assumptions von Neumann–Morgenstern is used to quantify individual preferences. Consequently, for each player there is a function, called *utility function* u, which represents and scales her preference over the bargaining set. If $\bar{\pi}_i, \hat{\pi}_i \in \mathcal{B}$, and if for a player $i(i = 1, 2)$, $u_i(\bar{\pi}_i) \geq u_i(\hat{\pi}_i)$, then we can conclude that the outcome $\bar{\pi}_i$ is preferred to outcome $\hat{\pi}_i$ for the player i. Such a utility function is not unique, that is if u_i is a utility function, then the function $v_i = au_i + \beta$ is also a utility function for real numbers a, β and $a > 0$.

He started with a class of problems for which the bargaining set is convex and compact, and for which free disposal is allowed. A bargaining problem, as stated by Nash is concerned with the set of utility pairs, $\mathcal{P} \in \mathbb{R}^2$, that can be derived from the bargaining set $\mathcal{B}$, $\mathcal{P} = \{(p_1, p_2) \in \mathbb{R}^2 : (p_1, p_2) = [u_1(\pi_1), u_2(\pi_2)] \quad (\pi_1, \pi_2) \in \mathcal{B}\}$, where $\mathcal{P}$ is convex, compact, nonempty set, and a pair of utilities $\mathcal{D} = (\delta_1, \delta_2) = (u_1(d_1), u_2(d_2)) \in \mathcal{P}$ a vector on $\mathbb{R}^2$, which is assigned to be the disagreement point. Only if these requirements are satisfied the bargaining problem $\langle \mathcal{P}, \mathcal{D} \rangle$ can properly be called a Nash bargaining problem.

Nash did not build his solution around what the bargainer is doing. He tried to answer the question, "What would a good solution look like?" He came up with a short list of sensible-sounding conditions that a bargaining solution should satisfy. The nice thing about having a set of conditions to start with is that they will limit the set of solutions that you might consider.

> *"Rather than solve the two-person bargaining game by analyzing the bargaining process, one can attack the problem axiomatically by stating general properties that 'any reasonable solution' should possess. By specifying enough such properties one excludes all but one solution" [55].*

Nash proved that a solution to the bargaining problem $\langle \mathcal{P}, \mathcal{D} \rangle$ is a function $\phi(\cdot)$, also known as *arbitration function* that assigns a single outcome $(p_1, p_2) \in \mathcal{P}$ to every bargaining problem $\langle \mathcal{P}, \mathcal{D} \rangle$. Nash proposes that a bargaining solution should satisfy four conditions.

Pareto efficiency. Suppose $(p_1, p_2) = \phi(\mathcal{P}, \mathcal{D})$ is the solution to the bargaining problem $\langle \mathcal{P}, \mathcal{D} \rangle$, and a pair $(\hat{p}_1, \hat{p}_2) \in \mathcal{P}$ then should hold that $(p_1, p_2) > (\hat{p}_1, \hat{p}_2)$. This condition basically says that there is no feasible point $(\hat{p}_1, \hat{p}_2)$ that is Pareto superior to the solution.

Independence of linear transformation. If $v_i = r_i u_i + c_i$, for $i = 1, 2$ and $r_1 > 0$ is a linear transformation of the utility function u_i that generates $\mathcal{P}$, then v_i generates $\mathcal{P}' = \{(r_1 p_1 + c_1, r_2 p_2 + c_2) \in \mathbb{R}^2 : (p_1, p_2) \in \mathcal{P}\}$. Because v_i represents the same preference as u_i if both are applied to the same bargaining set $\mathcal{B}$, the bargaining problem $\langle \mathcal{P}', \mathcal{D}' \rangle$ represents the same bargaining problem with $\langle \mathcal{P}, \mathcal{D} \rangle$ if $\mathcal{D}' = (r_1\delta_1 + c_1, r_2\delta_2 + c_2)$, which is easy to check. Thus a solution to $\phi(\mathcal{P}', \mathcal{D}') = r\phi(\mathcal{P}, \mathcal{D}) + c$. This condition says that if you transform all the elements in $\langle \mathcal{P}, \mathcal{D} \rangle$, you will also transform the solution.

Symmetry. If the bargaining problem $\langle \mathcal{P}, \mathcal{D} \rangle$ is symmetric, and if $\delta_1 = \delta_2$, then $(p_1, p_2) = \phi(\mathcal{S}, \mathcal{D}) \Rightarrow p_1 = p_2$. In symmetric situations, both players get the same.

Independence of irrelevant alternatives. If $\langle \mathcal{P}, \mathcal{D} \rangle$ and $\langle \mathcal{P}', \mathcal{D}' \rangle$ are bargaining problems with $\mathcal{P} \subset \mathcal{P}'$ and $\phi(\mathcal{P}', \mathcal{D}') \in \mathcal{P}$, then $\phi(\mathcal{P}, \mathcal{D}) = \phi(\mathcal{P}', \mathcal{D}')$. This axiom states that the bargain solution does not depend on other available outcomes that the player had the opportunity to choose but did not. See [61] for details and proofs.

The solution that satisfies these four properties is unique and is characterized by the payoff pair $(p_, p_2)$, which maximizes the product of the player's benefits from cooperation, the so-called Nash product.

$$\phi(\mathcal{P}, \mathcal{D}) = \arg \max_{(p_1, p_2) \geq (\delta_1, \delta_2) \in \mathcal{P}} (p_1 - \delta_1)(p_2 - \delta_2). \tag{43}$$

If the symmetric axiom is ignored, the bargaining solution comes to depend on the bargaining powers of the two players, this is the generalized or asymmetric Nash bargaining solution,

$$\phi(\mathcal{P}, \mathcal{D}) = \arg \max_{(p_1, p_2) \geq (\delta_1, \delta_2) \in \mathcal{P}} (p_1 - \delta_1)^{\alpha}(p_2 - \delta_2)^{\beta} \tag{44}$$

where $\alpha, \beta,\ \alpha + \beta = 1$ represents the negotiation power of each player. Among the factors that affect negotiation power are their utility, their risk preference, and their position on the market.

The Nash bargaining solution can be extended to apply in the case with n players, and it can be shown that the unique bargaining solution that satisfies the axioms is the function that satisfies:

$$\phi(\mathcal{P}, \mathcal{D}) = \arg \max_{(p_1, p_2) \geq (\delta_1, \delta_2) \in \mathcal{P}} \prod_{i=1}^{n} (p_i - \delta_i). \tag{45}$$

Kalai and Smorodinsky [39] replace the rather controversial axiom of Independence of irrelevant alternatives with alternative, which they refer to as the *axiom of monotonicity*. Let $p_i^m(\mathcal{P}) = \max\{p_i : pi \in \mathcal{P}\}$ be the maximum that player i could attain (for $i = 1, 2$) in a bargaining situation $\langle \mathcal{P}, \mathcal{D} \rangle$ given that the players are individually rational. The payoff combination defined in this way is called *ideal point.* The Kalai–Smorodinsky solution requires then that if the ideal point belongs to bargaining games $\langle \mathcal{P}, \mathcal{D} \rangle$ and $\langle \mathcal{P}', \mathcal{D}' \rangle \in \mathcal{B}$ and if $\mathcal{P}' \subset \mathcal{P}$, then player i will receive at least as much as in $\langle \mathcal{P}, \mathcal{D} \rangle$ as in $\langle \mathcal{P}', \mathcal{D}' \rangle$. The Kalai–Smorodinsky solution is then a unique function that selects the maximum element in $\mathcal{P}$ on the line that joins the disagreement point (δ_1, δ_2) with the ideal point. For details and proofs, see [53].

To apply the Nash bargaining problem to the supply chain analysis, consider a supply chain with one supplier and one retailer. At the beginning of the period, the retailer places orders x and sells them to the customer at a unit price r. Supplier produces the product with unit production cost k and supplies x units to retailers at a price c. It is also assumed that supplier has infinite production capacity. The demand w faced by the retailer during the period is random but distribution is known. She also faces a unit holding, and shortage costs, denoted by h and p, respectively. Let $\tilde{\pi}_s$ and $\tilde{\pi}_r$ be the supplier's and retailer's profit, respectively, $\pi_s = E[\tilde{\pi}_s]$ and $\pi_r = E[\tilde{\pi}_r]$ their expected profits with:

$$\pi_s = cx - kx = (c - k)x \tag{46}$$

and

$$\pi_r = rE\min\{w, x\} - cx - L(x) \tag{47}$$

where $L(x) = hE(x - w)^+ + pE(w - x)^+$.

In addition, assume that the supply chain makes a positive expected profit Π^C that is greater than the disagreement points and therefore the rational players will always prefer to participate in the game. Furthermore, we assume that disagreement point for supplier is $d_s = kx$ and the retailer's disagreement point is $d_r = L(x)$.

The solution refers to the resulting payoff allocation that each of the players agrees upon. Given that both players are risk neutral, the necessary Pareto efficiency condition ensures the negotiated quantity is always the one that coordinates the whole chain, i.e., $x = x^C$ (where x^C is the coordinating quantity). In other words, this bargaining formulation gives us channel coordination for free [52]. Because the negotiated quantity is always x^C, then $d_s = kx^C$ and the retailers disagreement point is $d_r = L(x^C)$, similar assumptions have been employed in [52]. Suppose that the supplier and retailer negotiate to split the total expected profits of the system. Consequently, the bargaining set can be written as $\mathcal{B} = \{(\pi_s, \pi_r) \in \mathbb{R}^2 : \pi_s + \pi_r \leq \Pi^C, \text{ and } \pi_s, \pi_r \geq 0\}$, which is assumed to be a convex and compact set, in addition the formulation of disagreement points guarantees that it is nonempty. The corresponding Nash bargaining problem is $\mathcal{P} = \{(p_s, p_r) \in \mathbb{R}^2 : (p_s, p_r) = [E(u_s(\pi_s), E(u_r(\pi_r)] \;\; (\pi_s \pi_r) \in \mathcal{B}\}$, where $\mathcal{P}$ is a convex, compact, nonempty set, and $\mathcal{D} = (\delta_s, \delta_r) = (u_s(d_s), u_r(d_r)) \in \mathcal{P}$. Applying the Nash solution concept results in the two players maximizing the following expression

$$\max_{(p_s, p_r) \geq (\delta_s, \delta_r) \in \mathcal{P}} (p_s - \delta_s)(\Pi^C - p_s - \delta_r). \tag{48}$$

Taking the derivative with respect to p_s and p_r and equating to zero, we get respectively:

$$p_s = \frac{\Pi^C - \delta_r + \delta_s}{2} \tag{49}$$

$$p_r = \frac{\Pi^C - \delta_s + \delta_r}{2}. \tag{50}$$

Nagarajan and Bassok [52] consider a cooperative, multilateral bargaining game similar to that where n suppliers are selling complementary components to an assembler. They propose a three-stage game: First (stage 3) the suppliers form coalitions, second (stage 2) the coalitions compete for a position in the bargaining sequence, third (stage 1) the coalitions negotiate with the assembler on the wholesale price and the supply quantity. They show that each player's payoff is a function of the player s negotiation power, the negotiation sequence, and the coalitional structure.

Chae and Heidhues [20] study the effects of integration among downstream local distributors on the entry of upstream producers in a bargaining theoretic framework. They modeled both price formation and the entry of upstream producers in an input market. Using a bargaining solution that generalizes the Nash solution, they showed that a higher degree of concentration among downstream distributors reduces incentives to enter the upstream production industry. The reason is that higher concentration among downstream distributors reduces the bargaining power of upstream producers.

8.2 Strategic Approach

Whereas the cooperative approach is static, in the sense that only the outcome is analyzed without taking into account the bargaining procedure, the strategic approach to bargaining theory, initiated by Ståhl [80] and Rubinstein [74], is more concerned with these situations and analyzes exactly the bargaining procedures, in the attempt to find theoretical predictions of what agreement, if any, will be reached by the bargainers.

To this end, we now present the model developed by Rubinstein [74], in which the procedure is modeled explicitly as a game in real time. Here we think of bargaining as a sequential game. That is, there is a well-defined sequence of moves, and players have preferences over the time of agreement as well as the terms of agreement. There are two players $i, i = 1, 2$, whose task is to divide a single surplus of size 1. Each player is concerned only about the share of the surplus that she receives and prefers to receive more rather than less. Time proceeds without end as $t = 0, 1, 2, \ldots$.

The procedure is as follows. At $t = 0$, one player, say player 1, makes an offer, (π_1, π_2), where π_1 is player 1's share and p_2 is player 2's share where

$\pi_1 + \pi_2 \leq 1$, which is either accepted or rejected. If player 2 accepts the offer, the game ends and the surplus is divided accordingly. If player 2 rejects, she makes a counter offer at period $t+1$, which is either accepted or rejected with counter offer from 1 and so on. If no offer is ever accepted, the payoffs are 0. To simplify matters, we assume that both players have linear utility functions $u_1 = \pi_1$ and $u_2 = \pi_2$. Player i's utility for getting a share π_i of the surplus at time t is equal to $u_i = \pi_i \theta_i^t$, where $\theta \in [0, 1]$ is a fixed discount factor, and it is used to translate expected utility in any given future into present value terms. Rubinstein [74] proved that there is a unique subgame-perfect equilibrium in this game, based on playing the following strategies in every period:

1. Player 1 proposes an offer: $\left(\pi_1 = \frac{1-\theta_2}{1-\theta_1\theta_2}, \pi_2 = \frac{\theta_2(1-\theta_1)}{1-\theta_1\theta_2}\right)$ and accepts player 2's offer if and only if $\pi_1 \geq \frac{\theta_1(1-\theta_2)}{1-\theta_1\theta_2}$.
2. Player 2 proposes an offer: $\left(\pi_1 = \frac{\theta_1(1-\theta_2)}{1-\theta_1\theta_2}, \pi_2 = \frac{1-\theta_1}{1-\theta_1\theta_2}\right)$ and accepts player 1's offer if and only if $\pi_2 \geq \frac{\theta_2(1-\theta_1)}{1-\theta_1\theta_2}$.

Extentions of the Rubistein's model include the case where there is possibility for the negotiation to break down [7] and the influnce of a outside option in the negotiation proposed and implemented in different settings by [8, 50, 67]. The main assumption of these models is that a player can choose to decide to leave a negotiation if there is an outside deal that can optimize her objective.

To apply the noncooperative bargaining game, Ertogal and Wu [32] consider a bargaining situation between a supplier and a retailer (buyer) who negotiate to split certain system surplus, say π . The supplier and the retailer are to make several offers and counter offers before settling on a final agreement. Before entering negotiation, the supplier and retailer each have recallable outside options W_s and W_b, respectively. It is assumed that $\pi \geq W_s + W_r$, otherwise at least one of the players would have no incentive to enter the negotiation.

The sequence of events in our bargaining game is as follows:

1. With equal probability $\frac{p}{2}$, one of the two players proposes an offer that splits the system surplus π into certain amounts.
2. The other player may either:
   (a) accept the offer (the negotiation ends), or
   (b) reject the offer and wait for the next round.
3. With a certain probability $(1 - p)$, the negotiation breaks down and the players take their corresponding outside options.
4. If the negotiation continues, the game restarts from step 1.

They assume that in subgame perfect equilibrium there is an infinite number of solutions leading to gains ranging from m_b to M_b for the buyer, and m_r to M_r for the supplier, where:

$M_s(M_r)$: The maximum share the supplier (the retailer) could receive in a subgame perfect equilibrium for any subgame initiated with the supplier's (the retailer's) offer.

$m_s(m_r)$: The minimum share the supplier (the retailer) could receive in a subgame perfect equilibrium for any subgame initiated with the supplier's (the retailer's) offer.

They formulate and solve the negotiation-sequencing problem as a network flow problem. They proved that the following system of equations defines the subgame perfect equilibrium for the bargaining between the two players:

$$M_s = \pi - \left[\frac{p}{2}\left[(1-p)W_r + \frac{p}{2}(\pi - M_s + m_r) + \pi - M_s\right] + (1-p)W_r\right] \tag{51}$$

$$m_s = \pi - \left[\frac{p}{2}\left[(1-p)W_r + \frac{p}{2}(\pi - m_s + M_r) + \pi - m_s\right] + (1-p)W_r\right] \tag{52}$$

$$M_r = \pi - \left[\frac{p}{2}\left[(1-p)W_s + \frac{p}{2}(\pi - M_r + m_s) + \pi - M_r\right] + (1-p)W_s\right] \tag{53}$$

$$m_r = \pi - \left[\frac{p}{2}\left[(1-p)W_s + \frac{p}{2}(\pi - m_r + M_s) + \pi - m_r\right] + (1-p)W_s\right] \tag{54}$$

and the unique subgame perfect equilibrium strategies of the players are given as follows:

1. If the supplier is the offering party, she will ask for $X_s = \pi - W_r - \frac{p^2}{2(2-p)}(\pi - W_s - W_r)$ share of the surplus and leave $\pi - X_s$ to the retailer.
2. If the retailer is the offering party, she will ask for $X_r = \pi - W_s - \frac{p^2}{2(2-p)}(\pi - W_s - W_r)$ share of the surplus and leave $\pi - X_r$ to the supplier.

This result has important implications in that the bargaining game will end in one iteration when one of the two players initiates the negotiation with the perfect equilibrium offer. They further show that there is a first-mover advantage in this game, but the advantage diminishes as the probability of breakdown approaches zero. Wu [94] expands the model to analyze the trade-off between direct and intermediated exchanges.

Bernstein and Marx [6] address the problem of supply chain performance when one supplier sells to multiple competing retailers and who have bargaining power. They model a retailer's bargaining power through its ability to set a reservation profit level below which it will not participate in the supply chain. They also allow endogenously chosen reservation profit levels for the buyers that may depend on the retailer's opportunities within the supply chain, rather than taking those reservation profit levels as fixed and dependent only on outside opportunities. The retailers may compete in terms of the prices they charge or in terms of the amount of inventory they carry. Their results indicate that supply chain performance is not maximized, or it is maximized conditional on the number of retailers that offer the supplier's product, but some retailers are excluded from trade. They conclude that in equilibrium, retailer's choices of reservation profit levels may induce the supplier to trade only with a strict subset of the retailers, even when all retailers must be included in order for channel profit to be maximized.

de Fontenay and Gans [28] analyze vertical integration in the case of upstream competition in which they demonstrate that vertical integration can alter the joint payoff of integrating parties in ex post bargaining.

Van Mieghem [87] and Chod and Rudi [23] consider settings in which two firms trade capacity after receiving demand information. In [23], the authors consider two independent firms that invest in resources such as capacity or inventory based on imperfect market forecasts. After investment decisions are made, the firms update their forecasts of the market conditions and have the option to trade. Although the negotiation of this trade is formulated in a cooperative fashion, the firms do not cooperate in the investment stage. The problem is formulated as a noncooperative bargaining game, and the existence and uniqueness of an embedded Nash bargaining solution is proved. In Van Mieghem's work [87], after demand revelation, the manufacturer may purchase some of the excess capacity of the subcontractor. She formulates this problem as a noncooperative stochastic investment game. Her results indicate that all decentralization costs are eliminated only when the bargaining parameters depend on demand realization.

8.3 Biform Games

A biform game is a hybrid noncooperative/cooperative game model designed for modeling business interactions. It can be thought of as a noncooperative game with cooperative games as outcomes, and those cooperative games lead to specific payoff. The biform game was first formalized by Brandenburger and Stuart [10]. Hence the noncooperative solution concept of Nash equilibrium extends naturally to the biform game.

To define a biform game, consider a set of players N, indexed by $i = 1, \ldots, n$, and for each player $i \in N$, a finite set X_i of strategies. At the noncooperative stage (first stage), players make decision among their strategies, this game can be analyzed just like any other noncooperative game. Competition is then modeled by a cooperative game (second stage) in which the characteristic value function depends on the chosen actions. The core of this cooperative game is employed to determine the outcome of the game. Even though the core of such game is nonempty, it may yield to a range of outcomes, rather than a unique outcome; as a result, it is not immediately clear what value each player can expect. In such cases, it is necessary to describe each player's preferences over intervals. In a biform game, these preferences are represented by the numbers α_i for each player i. Each player then expects to earn in each possible cooperative game a weighted average of the minimum and maximum values in the core, with α_i being the weight. The parameter α_i can also be interpreted as an index player's i in her bargaining power.

Brandenburger and Stuart [10] proposed the biform game in which players make strategic investments, and then they play a cooperative game determined by their investments. The biform game formulation is employed in Plambeck and Taylor [66] where two independent, price-setting original equipment manufacturers (OEMs) are investing in innovations. They allow OEMs to outsource their productions to an independent contract manufacturer (CM); a bargaining game is employed to model the negotiations among (OEMs) and (CM). They show that the bargaining outcome induces the CM to invest in the system-optimal capacity level and to allocate capacity optimally among the OEMs. A subsequent paper [65] considers the situation where a manufacturer writes quantity flexibility contracts with two buyers. Then, the buyers invest in innovation, and the manufacturer builds capacity. Without renegotiation, quantity flexibility is necessary for the client capacity allocation, but reduces incentives for investment. Typically, allowing renegotiation reduces the flexibility in an optimal contract and increases the total expected profit.

He models the problem as a multivariate, multidimensional, competitive news-vendor problem. He argues that *ex ante* contracts may be too expensive or impossible to enforce, while the supplier's investments (in quality, IT infrastructure, and technology innovation) may be noncontractible.

In the application of the biform game to the news-vendor problem, ordering decisions of different retailers are made competitively whereas allocation decisions take place cooperatively.

In a recent article, Rudi *et al.* [70] consider a two-retailer model with transshipment of stock. They aim to find prices for which the joint decentralization profit achieves the centralized system profit.

Anupindi *et al.* [3] use a hybrid noncooperative/cooperative model to formulate a game where multiple retailers stock at their own locations as well as at several centralized warehouses. In the noncooperative stage, retailers make stocking decisions, for this stage they develop conditions for the existence of a pure Nash equilibrium. In the cooperative stage, retailers use cooperative game theory to characterize possible opportunities for cooperation, similar to Müller *et al.* [49].

Granot and Sosic [35] analyze a similar problem, they consider a network of retailers with stochastic demands: Each chooses its inventory level but in this models retailers are able to hold any inventory left from one period to the other, then demand is realized; and the retailers bargain cooperatively over the transshipment of excess inventory to meet excess demand. Their model has three stages: decision about the order quantity, decision about how much inventory to share with others, and finally the transshipment stage.

Stuart [81] provides a model of the competitive news-vendor problem in which there is price competition following the inventory decisions. The price competition is modeled by considering the core of the induced cooperative game. She shows that with no uncertainty, the inventory decision is equivalent to the capacity decision in Cournot competition. With uncertainty, the analysis again reduces to Cournot competition if the demand uncertainty is characterized by an appropriately constructed, expected demand curve.

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