Chapter 39
Asian Options

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Abstract The payoffs on the expiration dates of Asian options depend on the underlying asset's average price over some prespecified period rather than on its price at expiration. In this chapter we outline the possible applications of these options and describe the different methodologies and techniques that exist for their evaluation as well as their advantages and disadvantages.

Keywords Options • Asian options • Exotic options • Valuation

39.1 Introduction

Asian options are a type of options in the class of exotic options. The payoffs on expiration of Asian options are not dependent on the underlying asset's price at that time as in plain vanilla options, but rather on its average price at some prespecified n periods. The sensitivity of the option's value on expiration date to the underlying asset's price at that time is thus lower for Asian options than for regular options making, Asian options less susceptible to price manipulations.

These options were named Asian options because they were introduced in some Asian markets to discourage price abuse on expiration.[1] Asian options, also called Average Rate Options (AROs), are not traded in any well-known exchange but are quite popular in OTC trading, mainly in energy, oil, and currency markets. In addition to being less sensitive to manipulation they are also effective in hedging cash flows that follow patterns resembling averages over time. For example, many firms periodically make or receive foreign currency payments; Asian options would more closely match such cash flows than options that are defined on the price of the currency on some specific date. Similarly, an airline that continuously buys oil is better off hedging its costs by Asian options rather than by ordinary ones. AROs can also be employed in balance sheet hedging, as firms use average rather than year-end rates to hedge their exposures.

Another feature of the Asian options that makes them popular with some investors (and unpopular with others) is their lower volatility. Since the value of the option at expiration is composed of the average of the price of the underlying asset over n periods, its variance is lower than that of a similar plain vanilla option.

Asian options are sometimes embedded in other securities. In the late 1970s, commodity-linked bond contracts with an average-value settlement price were introduced. These contracts entitle the investors to the average value of the underlying commodity over a certain time interval or the nominal value of the bond, whichever is higher. Hence, the investors are offered a straightforward bond plus an option on the average value of the commodity where the exercise price is equal to the nominal value of the bond. Early examples of such bonds included: Oranje Nassau bonds, 1985,; Mexican Petrobonds 1977; Delaware Gold indexed bond; and BT Gold Limited Notes, 1988.

Some warrant issues include Asian options features as poison pills to preclude hostile takeovers. Firms would issue such warrants to friendly investors. In case of a hostile takeover, the warrants would be exercised by these investors, and because of the averaging of prices, the dilution factor would be low. These types of warrants have been used by such French companies as Compagnie Financiere Indo-Suez and Bouygues.

Asian options are issued either in the European version (Eurasian) in which case they can be exercised only on expiration date or in the American version (Amerasians) where they can be exercised any time prior to expiration. Most of the Asian options appear as Eurasians. Since AROs of the American type can be exercised at any time prior to expiration, the averaging that is designed to protect the investors from manipulation is not as effective as that provided by their European- style counterparts.

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[1] According to legend such options were originally used in 1987 when Bankers Trust's Tokyo office used them for pricing average options on crude oil contracts.

C.-F. Lee et al. (eds.), *Handbook of Quantitative Finance and Risk Management*,
DOI 10.1007/978-0-387-77117-5_39,

Arithmetic averages are usually used in Asian options but geometric averages can be used as well. Asian options can also be broken down into several other categories: those where the average price of the underlying asset is considered or those where different strike prices are averaged. In addition, they can be applied to other exotic options such as basket options (see e.g., Castellacci and Siclari 2003). If the prices are averaged over the entire life of the option they are sometimes called plain vanilla Asian options. If they are averaged over a shorter period closer to expiration they may be called forward starting options.

39.2 Valuation

Most option pricing methods based on Black-Scholes', model, 1973, assume that the underlying asset prices are lognormally distributed. When geometric averaging is used the valuation of Asian options is straightforward because the average price is also lognormally distributed with parameters that can easily be calculated. A straightforward application of the Black Scholes formula is then possible using the known parameters of the distribution (see Turnbull and Wakeman 1991; Curran 1992; Hull 2006). On the other hand, if an arithmetic averaging is used, the distribution of the average is unknown and there are no known closed form formulas for the options valuation. Various methods have been proposed to estimate or approximate the value of the options in this case. These methods include the following: Monte Carlo simulations, diverse mathematical and numerical techniques designed to estimate the distribution of the returns and thereby the value of the options, techniques designed to find lower and upper bounds for the options' values using arbitrage arguments, binomial tree models, and the application of insurance formulas to the evaluation of options. As there is no one method that dominates all others, the choice of the appropriate procedure depends on computational convenience, on the required accuracy, and on the parameters of the options being evaluated. Some models would provide excellent approximations for several sets of values of the parameters but would not perform well for others.[2]

Given the many strong assumptions that underlie most valuation methods (e.g., knowledge of the standard deviation, lognormality, flat yield curve, efficient markets), the inaccuracies involved in most of the extant evaluation techniques do not seem to be too ominous.

Below are some suggested techniques to evaluate average Asian options.

[2] Since most of the options are of the European style, evaluations are provided for the calls, and the puts can be evaluated from the Put-Call parity. For the valuation of American type options, see Ben-Ameur et al. 2002, and Longstaff and Schwartz 2001.

39.2.1 Monte Carlo Simulations

Monte Carlo simulations are usually considered as the most accurate way to evaluate the distribution of the average returns, and hence options. The accuracies of other approximation methods are often measured against the values obtained by the simulations. Unfortunately, simulations are computationally time consuming, especially if the Greeks also need to be calculated (see e.g., Boyle 1977).

39.2.2 Approximations

The simplest approximation for the value of an arithmetic Asian option and the easiest one to calculate is the an equivalent geometric option (see e.g., Kemna and Vorst 1990; Curran 1992; Turnbull and Wakeman 1991; Hull 2006). Consider a newly issued Asian (Eurasian) option on an underlying asset with a volatility of returns σ and a continuous dividend yield (or growth rate) q and suppose the option's value on expiration depends on the average underlying asset's price from issue until that date.[3] The value of the geometric option is obtained by using the usual Black-Scholes formula with a standard deviation, $\sigma/\sqrt{3}$, and a dividend yield, $0.5(r + q + \sigma^2/6)$, where r is the risk free interest rate. Since the geometric average is always lower than or equal to the arithmetic average, the approximations thus obtained underestimate the call options and overestimate the put options. Nevertheless, Turnbull and Wakeman show that estimates based on this method are quite accurate for low standard deviations (20% or lower). Their quality, however, declines when higher standard deviations are considered.

Another relatively easy method for approximating the value of Asian options is to estimate the arithmetic average price by an ad hoc distribution, either lognormal or inverse Gaussian, with the same first and second moment (or higher moments). Such approximations were introduced by Levy (1992), Turnbull and Wakeman (1991), Jacques (1996), and Hull (2006). Turnbull and Wakeman use the Edgeworth series expansion around the log normal distribution to provide an algorithm to compute moments of the arithmetic average. Levy (1992), uses a simpler approach, employing the Wilkinson approximation that utilizes just the first two moments of the distribution.

In the above example, suppose the risk free rate is r, the current price of the underlying asset is S_0, the strike price

[3] If the option is not newly issued and some of the prices that comprise the average have already been observed, then taking that into account, the strike price and the definition of the random variable are adjusted in a straightforward manner.

is K, and that the time to maturity is T. The distribution of the average returns is then approximated by a lognormal distribution with first two moments M_1 and M_2 given by:

$$M_1 = \frac{e^{(r-q)T} - 1}{(r-q)T} S_0$$

and

$$M_2 = \frac{2e^{[2(r-q)+\sigma^2]T} S_0^2}{(r-q+\sigma^2)(2r-2q+\sigma^2)T^2} + \frac{2S_0^2}{(r-q)T^2}\left[\frac{1}{2(r-q)+\sigma^2} - \frac{e^{(r-q)T}}{r-q+\sigma^2}\right]$$

when $q \neq r$. The Asian option can then be regarded as a regular option on a futures contract with current value of F_0 and volatility σ^2 given by:

$$F_0 = M_1$$

and

$$\sigma^2 = \frac{1}{T} \ln\left(\frac{M_2}{M_1^2}\right)$$

The values of the call option, c, and the put option, p, are then given by:

$$c = e^{-rT}\left[F_0 N(d_1) - KN(d_2)\right]$$
$$p = e^{-rT}\left[KN(-d_2) - F_0 N(-d_1)\right]$$
$$d_1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Levy shows that his method provides good approximations for the value of the commonly used Asian options that are usually characterized by low variances or short times to expiration. For options with a standard deviation, σ, and time to expiration, t, satisfying $\sigma\sqrt{t} \leq 0.2$, he shows that the approximations are quite accurate, although their accuracy falls as the standard deviations get larger.

Since the approximations described above have an analytical form, one can explicitly calculate the Greeks that are extremely important for market traders in managing their risks.

39.2.3 Other Mathematical and Numerical Methods

Kemna and Vorst (1990), present a dynamic hedging strategy from which they derive the value of average Asian options using arbitrage arguments. They also show that the price of an ARO is always less than or equal to that of an equivalent plain vanilla option.[4] In addition they also provide some variance reduction methods that are instrumental in calculating confidence intervals and in reducing the computation times. Jacques (1996), extends their method to calculate the values of the replicating portfolios in hedging Asian type options.

Bouaziz et al. (1994), propose a slightly different arbitrage-based model, which works quite well, to approximate the value of Asian options. The advantage of their method is that they also provide upper bounds to the approximation errors. This is an important issue that some of the previous methods have failed to address.

Geman and Yor (1993), have obtained an expression for the Laplace transform of the price of Asian options. However, calculation of the inverse of this transform is quite time consuming. Fu et al. (1998/1999), and Linetsky (2002), also use Laplace transforms to price Asian options and have reached a reasonable degree of accuracy; In addition, Carverhill and Clewlow (1990), make use of Fourier transform techniques.

In general, the price of an Asian option can be found by solving a partial differential equation (PDE) in two space dimensions (see e.g., Ingersoll 1987). Rogers and Shi (1995), have formulated a one-dimensional PDE that can model both floating and fixed strike Asian options. Zhang (1995) presents a semi-analytical approach that is shown to be fast and stable by means of a singularity removal technique and then numerically solving the resultant PDE. Vecer (2001), provides a simple and numerically stable two-term PDE characterizing the price of any type of arithmetically averaged Asian option. The approach includes both continuously and discretely sampled options, which is easily extended to handle continuous or discrete dividend yields.

39.2.4 Binomial Models

As is true for any other option, Asian options can be priced using tree models. This type of method, however, is not used frequently because it could be computationally inconvenient. At any point in time on the tree, the value of the option depends on the average of the price along the path it has taken, and one must then determine a minimum and maximum range at each node. As the number of nodes on a tree grows, so does the number of averages that must be taken,

[4] Turnbull and Wakeman claim however that Kemna and Vorst's result holds true only if the averaging period starts after the initiation (and valuation) of the options. They provide examples showing that when the time to maturity of the option is lower than the averaging period, the value of the Asian option could be higher than that of a standard European option. Geman and Yor 1993, arrive at a similar conclusion.

and this number is exponentially related to the number of possible asset paths. Hull and White (1993), show a way to alleviate this problem by adding a state variable to the tree nodes and approximations are undertaken with interpolation techniques in backward induction.

39.2.5 Applying Insurance Models

The equivalence between insurance and options is well known. Many insurance contracts have payoffs defined in terms of claims accumulations rather than the end-of-period values of the underlying state variables. This property suggests that the analogy between insurance and Asian options is more appropriate in some cases than the analogy between insurance and regular options. The problem of pricing Asian options is then similar to that of calculating stop-loss premiums of a sum of dependent random variables. Results previously used for calculating premiums for such insurance contracts (see e.g., Cummins and Geman 1994; Goovaerts et al. 2000; Jacques 1996; Kaas et al. 2000; Dhaene et al. 2002a, b) can therefore be applied with slight modifications to find analytical lower and upper bounds for the price of Asian options.

39.3 Conclusion

Asian options are popular in markets with thin trading because it is harder to manipulate their values close to expiration than the values on expiration of ordinary options. They are also attractive to some sets of particularly risk averse investors. In addition, many financial instruments may be modeled as Asian options because their values are contingent on the average price of other "underlying" assets. The popularity of Asian options and the absence of a universally accepted formula for their evaluation created the need to rely on approximations. While users of Asian options enjoy the choice of many models for approximations that are quite accurate and easy to use, practitioners still wait for a breakthrough that will make the evaluation more accessible, accurate, and user-friendly.

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