Chapter 17
Portfolio Theory, CAPM and Performance Measures

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Abstract This chapter is focused on the "Portfolio Theory" created by Markowitz. This theory has the objective of finding the optimum portfolio for investors; that is, that which gives tangency between an indifference curve and the efficient frontier. In this chapter, the mathematics of this model is developed.

The CAPM, based on this theory, gives the expected return on an asset depending on the systematic risk of the asset. This model detects underpriced and overpriced assets. The critics expressed against the model and their application possibilities are also analyzed.

Finally, the chapter centers on performance measures related to portfolio theory (classic indices, derivative indices and new approaches) and on the performance persistence phenomenon employing the aforementioned indices, including an empirical example.

Keywords CAPM • Performance measures • Penalized internal rate of return index (PIRR) • Penalized internal rate of return for beta index (PIRR for Beta) • Portfolio theory

17.1 Portfolio Theory and CAPM: Foundations and Current Application

17.1.1 Introduction

Portfolio theory has become highly developed and has strong theoretical support, making it essential in the toolbox of any financial expert.

In 1990, three American economists shared the Nobel Prize for Economics: Harry Markowitz, Merton Miller, and William Sharpe. Markowitz is considered the creator of portfolio theory. He began this field of research with his work of 1952, making major additions with his works of 1959 and 1987. Sharpe attempted to simplify the model of his mentor, Markowitz, with his work of 1963, based on which he created the capital asset pricing model (CAPM) in 1964. Miller has also worked on portfolio topics.

Another Nobel Prize winner for Economics, James Tobin, contributed to the advance of portfolio theory with his work in 1958. Merton,[1] Black and Fama are others who have made great advances in the field.

A portfolio is a combination of assets, such as a group of stocks that are diversified to some extent. A portfolio is characterized by the return it generates over a given holding period. Portfolio theory describes the process by which investors seek the best possible portfolio in terms of the tradeoff of risk for return. In this context, risk refers to the situation where the investor is uncertain about which outcomes may eventuate, but can assess the probability with which each outcome may arise. In a risky environment, the choice of an optimal portfolio will depend on the distribution of returns, as well as the preferences of the investor – his or her appetite for risk.

It is clear that all decision makers have their own personal preferences, but it is possible to posit some generalizations. First, let us suppose that all decision makers prefer more wealth to less and, second, that decision makers are risk averse.

We define risk as the variability of the results of an activity; this variability can be measured by standard deviation or by variance. If we assume, as is customary, the marginal utility of an individual as a decreasing function of wealth, then the least risky activity is preferred, given the same average results.

Although the risk aversion behavior of individuals has been questioned, the literature is based on the assumption

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[1] Merton also won a Nobel Prize in 1997 along with Scholes.

C.-F. Lee et al. (eds.), *Handbook of Quantitative Finance and Risk Management*,
DOI 10.1007/978-0-387-77117-5_17,

that risk must be rewarded to be accepted by the individual. This reward is made by awarding "prizes" for risky activity with a higher expected payoff. Therefore, the return premium represents the reward for risk.

17.1.2 The Mean-variance Model

Decision makers will be interested in knowing the probability distribution for the return on their investment. If we assume that this probability distribution is normal, it will be defined by two parameters: the mathematical expectation (mean), and the standard deviation (or its square, the variance). Therefore, investors can be said to make decisions based on mean and variance, hence the "mean-variance" model.

Asset i will produce, in period t, a return r_i, that is a random variable with average μ_i and standard deviation σ_i. Consequently, each asset will be a point on the $\mu-\sigma$ map. The map, shown in Fig. 17.1, represents the group of possible investments (portfolios) from which the investor will be able to choose.

Investors must then choose among the various points on this map of possible opportunities. Each individual has a particular preference, which constitutes the individual's system of indifference curves. An indifference curve is that place of the points on the map that yield equal utility for the individual.

In Fig. 17.2, a system of indifference curves typical for an individual who is averse to risk is shown. Characteristically, the indifference curves are positive-sloped and convex, since the higher the deviation in results, the higher is the mean demanded by individuals to maintain themselves at the same level of indifference. On the other hand, the furthest the curve is from the horizontal axis, the greater the level of utility that it represents.

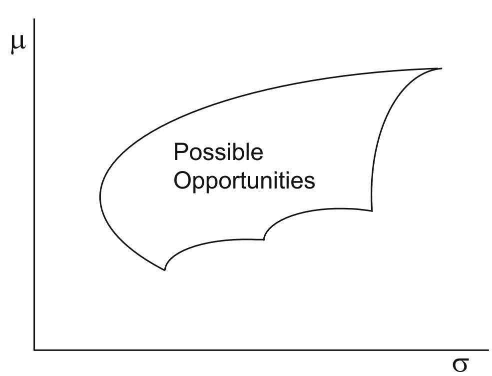

Fig. 17.1 Figure 17.1 represents the group of possible investments (portfolios) from which the investor will be able to choose

17.1.3 The Efficient Frontier

A portfolio is efficient when it yields a higher average return for a given risk, and a lower risk for a determined average return. Such portfolios make up the efficient frontier, which is a subgroup of the minimum-variance frontier, within which are located those portfolios that have a lower risk for a determined average return.

We now show the process for obtaining the minimum-variance frontier, following the work of Merton (1972).[2]

We start with a portfolio made up of n securities (all risky and such that none can be obtained by the linear combination of any others) with r_i returns and in w_i weightings:

$R' = (r_1, r_2, \ldots, r_n)$ $W' = (w_1, w_2, \ldots, w_n)$

We also adopt the following nomenclature:

U is a vector of ones, so $W'U = 1$ (the proportions must add up to 1)

$P = R'W$ is the return on the portfolio and $E(P) = E(R')W$, the average return

Σ is the variance and covariance matrix of returns on n securities

$VAR(P) = W'\Sigma W$, the variance of the portfolio

$COV(R, P) = \Sigma W$, the co-variance vector of the returns on the securities and the portfolio.

Determination of the minimum-variance frontier results in minimization of the variance for each value of the average return; that is, variance is minimized by assuming a return expectation E^*:

$$\begin{aligned} &\text{MIN} : W'\Sigma W \\ &\text{Subject to} : E(P) = E(R')W = E^* \\ &W'U = 1. \end{aligned}$$

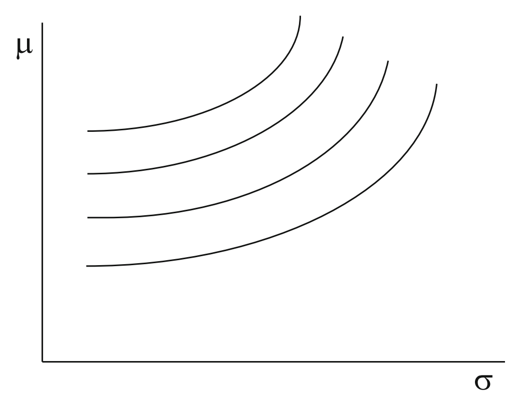

Fig. 17.2 Figure 17.2 shows a system of indifference curves typical for an individual who is averse to risk

[2] Adapted by Gómez-Bezares (2006).

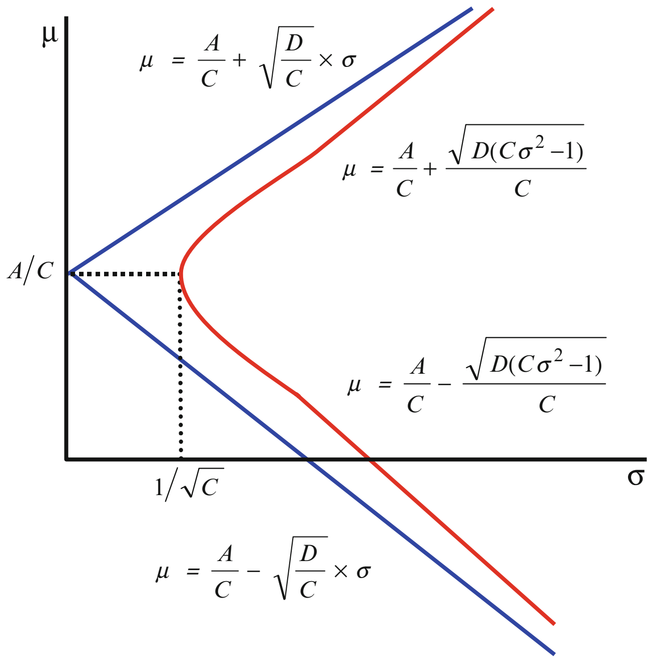

Fig. 17.3 Figure 17.3 shows that the frontier is a hyperbola on the mean-deviation map

Finding the minimum conditioned by Lagrange, we obtain W vector, which provides the proportions, depending on the portfolio expectation, in which appear the securities of the portfolios that compose the frontier:

$$W = \frac{(C \times E(P) - A) \sum^{-1} E(R) + (B - A \times E(P)) \sum^{-1} U}{D}, \tag{17.1}$$

where: $A = U' \sum^{-1} E(R)$; $B = E(R') \sum^{-1} E(R)$; $C = U' \sum^{-1} U$; and $D = BC - A^2$.

It can be demonstrated that the frontier is a hyperbola on the mean-deviation map (see Fig. 17.3), and a parabola on the mean-variance map (Merton 1972)[3] and Gómez-Bezares (2006).

Furthermore, we can rewrite $W = E(P)G + H$, where G and H are vectors, in such a way that, given that each w_i is linearly related with E(P) and knowing the W values of two portfolios that are on the frontier, we obtain G and H and, therefore, we obtain the Ws of all portfolios that make up the frontier.

If we designate λ_1 the Lagrange multiplier corresponding to the first condition, whose formula is $\lambda_1 = 2[C \times E(p) - A]/D$, it can be demonstrated that λ_1 represents the slope – measured by its inverse – of the minimum-variance frontier on the mean-variance map. Depending on how λ_1 varies, we obtain a new portfolio of minimum variance. For positive values of λ_1 we obtain the sub-group called the efficient frontier, that, as well as having minimum variance for a given expectation, has maximum expectation for a given variance.

[3] See also Roll (1977).

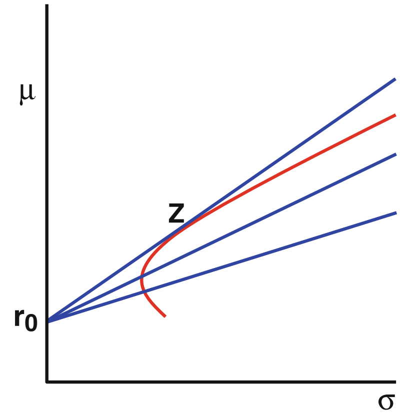

Fig. 17.4 Figure 17.4 shows an efficient frontier with a risk-free asset

17.1.4 Efficient Frontier with a Risk-free Asset

It is possible to construct portfolios as a combination of risky and risk-free assets. On the mean-deviation map, a straight line represents the new portfolios. The highest straight line will be the tangent to the previous efficient frontier on point Z; thus, a new efficient frontier will appear (Fig. 17.4). On the left of Z, we have portfolios composed of a proportion of risk-free assets, and another of portfolio Z; on the right, we invest more than 100% in portfolio Z, accepting debt at the risk-free rate.

We designate W as the vector of the weightings of investments in risky assets, and $(1 - W'U)$ as the proportion invested in the risk-free asset, with r_0 return. In this case, we also minimize variance for a value of the E^* mean:

$$\text{MIN}: W'\Sigma W$$
$$\text{Subject to:} E(P) = E(R')W + (1 - W'U)\, r_0 = E^*$$

Resolving the minimum conditioned by Lagrange, we obtain the vector W:

$$W = \frac{(E(P) - r_0) \sum^{-1} (E(R) - U r_0)}{B - 2Ar_0 + Cr_0^2} = (E(P) - r_0)\, M, \tag{17.2}$$

designating M the corresponding vector. λ continues to be a linear function of E (P):

$$\lambda = \frac{2\,[E(P) - r_0]}{B - 2Ar_0 + Cr_0^2} \tag{17.3}$$

in such a way that the various W vectors are proportional to the M vector. This leads to Tobin's *Separation Theorem* (1958): When there is a risk-free asset (that can be sold short) on the frontier, all W vectors (that contain risky assets) will have the same proportions of assets, with only the proportion between these and the risk-free asset varying.

17.1.5 Efficient Frontier with Inequality Restrictions

Until now the only restriction considered has been that the totality of the proportions invested in securities must add up to the unit. Other equality restrictions could be added and it would still hold that on the frontier, each portfolio is a linear combination of any two portfolios, and that the W values are a linear function of E (P), and therefore of λ_1.

However, the problem becomes more complex when we add inequality restrictions (standard problem) to the basic problem, which contains only equality restrictions.

In the standard problem, there are restrictions such as: $a_i \leq w_i \leq b_i$, so that the relationships stop being linear for any value of λ_1, but are linear within each interval. For example, it could be that up to a value of $\lambda_1 = x$ no inequality restriction appears; that is, there is a basic sub-problem. If, at this point, any restriction stops, it becomes an equality up to $\lambda_1 = y$, meaning that between x and y there is a new basic sub-problem (here the evolution will be linear). Therefore, a standard problem is a group of basic sub-problems. Points x and y are called cornerpoints, meaning that each portfolio on the frontier is a linear combination of two adjacent corner portfolios.

In this way, if there is no risk-free asset, between each two of the adjacent corner portfolios, we will have a piece of hyperbola, but the complete frontier will not yet be a hyperbola (see Fig. 17.1).

If securities cannot be emitted (sold short) except for the risk-free asset, whose short sale means getting into debt, but securities can be bought, the restrictions – that will only affect the risky assets – will take the form $0 \leq w_i \leq \infty$. In this case, since we are able to put ourselves into debt in the risk-free asset, the upper line that is the efficient frontier will be unlimited towards the right. Therefore, the efficient part of the frontier will be a straight line that responds to a basic sub-problem within a standard problem.

17.1.6 Application to the Market

Continuing with the case of inequality restrictions (supposing that there are limitations on short-selling, as in Fig. 17.1),

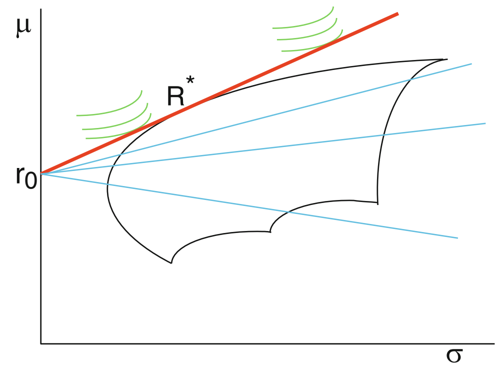

Fig. 17.5 Figure 17.5 represents an optimum portfolio considering that the risk-free asset can be emitted without restriction

individuals will search for their optimum portfolio, which will be that portfolio that reaches the highest possible indifference curve, that is, that in which tangency is produced between one of the indifference curves and the efficient frontier. The problem here is that we do not know where the market portfolio will be located (the sum of all individuals' portfolios), nor whether it will be efficient.

However, if we assume a risk-free asset that can be emitted without restriction, then all individuals will invest a part of their wealth in the risk-free asset – positively or negatively – and another part in the R^* portfolio, which is the market portfolio. This is because all individuals will invest the risk-based portion of their wealth in the proportions of R^* (see Fig. 17.5).

Of course, we assume uniform expectations (agreement) among investors. It is also assumed that there is equilibrium between supply and demand and a uniform interest rate for everyone, in terms of lending and borrowing. In this case, the efficient frontier is the so-called Capital Market Line, CML.

17.1.7 The Capital Asset Pricing Model

The capital asset pricing model (CAPM), or Sharpe-Lintner model, stands out among asset pricing models. This model reflects how the expected return on an asset is a function of the expected returns on the market and the risk-free asset and of the relevant (systematic) risk of that asset. If this model is complied with, at equilibrium, all assets adjust their values so as to offer the return that corresponds to them, thereby allowing investors to determine whether an asset is undervalued or overvalued.

This model was created in the pioneering work of Sharpe (1961, 1964), Lintner (1965), among others, and has been adapted over time to reflect different aspects of the real financial world.

Although the starting hypotheses of the CAPM place us in markets with perfect competition, which creates the potential for lack of realism, the conclusions of the model adapt well to reality.[4] In addition, recent technological advances, new products, and greater knowledge of the markets make markets in general seem ever more like the ideal markets supported by the CAPM.

Portfolio theory is the fundamental theoretical base of the CAPM. This model accepts that there is a risk-free asset with r_0 return (that can be emitted), and claims that the securities that make up a portfolio on the efficient frontier, with P return, must comply with the following equation

$$E(R) = r_0 U + [E(P) - r_0] \frac{COV(R, P)}{VAR(P)}. \quad (17.4)$$

According to this model, the expected return on a security will depend on the portfolio P and the return on the risk-free asset; P represents any portfolio that is located on the efficient frontier. Furthermore, if we suppose there is agreement, all individuals will make the same forecast of the portfolio map, and if the market has reached equilibrium, the market portfolio R^* will be efficient. We return to this concept later.

17.1.8 The Market Model

Sharpe (1963) attempts to reach a simplification of the Markowitz model by proposing the *market model*. The simplifying hypothesis of this model posits that the relations between returns on the various assets stem solely from the relation between returns on all assets and that of a market index. It also assumes the existence of a linear relation between the return on each asset and that of the index.

Using the market portfolio as a proxy for the market index, we have the model

$$R_{it} = \alpha_i + \beta_i R_t^* + \varepsilon_{it}, \quad (17.5)$$

where R_{it} is the return on the asset i in period t, and it is usually considered that ε_{it} is an error term such that:

$E(\varepsilon_{it}) = 0$
$VAR(\varepsilon_{it}) = \sigma^2(\varepsilon_i)$
for all t's, (homoscedasticity)
$COV(\varepsilon_{it}\varepsilon_{it'}) = 0$
for all t's, t' (no autocorrelation)
$COV(\varepsilon_{it,} R_t^*) = 0$ for all t's.

[4] Although we will discuss this matter later.

Hence, we obtain

$$\sigma^2(R_i) = \beta_i^2 \sigma^2(R^*) + \sigma^2(\varepsilon_i) \quad (17.6)$$

in such a way that the first member of this equation represents the risk of the asset measured with its variance, and the second has two components: the first measures the *systematic* risk and the second the *diversifiable* risk.

We commented above that in this model the only relationship between the returns on the assets is the shared relationship with the return on the index, therefore, $COV(\varepsilon_{it}\varepsilon_{jt}) = 0$ if $i \neq j$, but this condition is difficult to accept since the application of factorial analysis demonstrates that there are sectorial factors, which corroborates the idea that, aside from the correlation through the market, there are other sources of correlation. It is precisely based on said condition of zero covariance that it is demonstrated that diversifiable risk really does disappear in the portfolio; however, fortunately, even though there are sector effects, a careful diversification enables us to maintain the distinction between systematic and diversifiable risk.

Systematic risk derives from the general economy and affects all assets to some extent, and diversifiable, or specific risk, is a risk that affects one asset in particular. If an asset has a high diversifiable risk, it will be easy to find other assets that do not correlate with it, forming a diversified portfolio with much less risk.

Systematic risk is measured through the first component of the second member of the formula (17.6), but given that market variance is equal for all the assets, we can measure this risk using the beta parameter. This parameter is the regression slope between the asset return and that of the market (Security Characteristic Line, SCL).

We will call a security an "aggressive security" if it increases market variations ($\beta > 1$) and "defensive" if it decreases them ($\beta < 1$), as shown in Fig. 17.6. The market portfolio has a slope equal to the unit, while the risk-free asset has a null slope. There are also assets with negative slopes that allow the total risk of a well-diversified portfolio to be reduced.

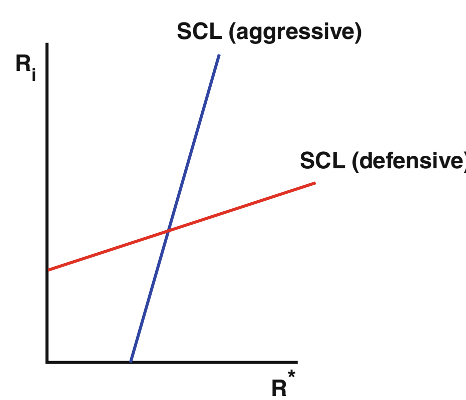

Fig. 17.6 Figure 17.6 shows an aggressive and a defensive security

17.1.9 Relation Between the Market Model and CAPM

The relevant risk from a security is its systematic risk, since it is this risk that each security introduces to the market portfolio, which by definition does not have diversifiable risk. On the other hand, the systematic risk of a portfolio is the linear combination of the systematic risks of the securities that comprise it. Securities have differing levels of systematic risk, depending on how defensive or aggressive they are, and must yield returns accordingly.

The CAPM confirms that securities must produce returns that are dependent entirely on their systematic risk. If we consider formula (17.4) of the CAPM and substitute P for R* (market portfolio) (since this is the only efficient portfolio made up solely of risky assets), we obtain

$$E(R) = r_0 U + \left[E(R^*) - r_0\right] \frac{COV(R, R^*)}{VAR(R^*)} \qquad (17.7)$$

and, given that the regression betas between the asset return and that of the market index are $\beta = \frac{COV(R,R^*)}{VAR(R^*)}$, we arrive at the usual equation from the CAPM

$$E(R) = r_0 U + \left[E(R^*) - r_0\right] \beta, \qquad (17.8)$$

where β is the vector that contains the betas.

Considering CAPM formula (17.8), we note that assets must yield returns, depending on their systematic risk or beta as follows: their expected return is equal to that on the risk-free asset plus a premium for risk, which is the product of multiplying the premium that the market receives by the quantity of systematic risk accepted.

Expected returns on assets, at equilibrium, must be adapted to the previous equation, leading to the Security Market Line, SML (Fig. 17.7). Securities located above the SML will be undervalued, as they produce greater returns than expected and, as they are much in demand, their prices will increase until they reach the SML; those securities located under the SML will be overvalued and will reach the SML by a process inverse to that of those located above the SML.

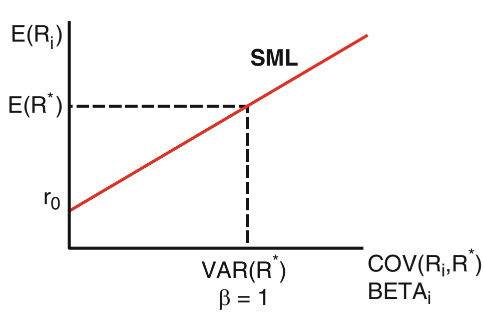

Fig. 17.7 Figure 17.7 shows the Security Market Line

Therefore, intelligent investors will eliminate diversifiable risk from their portfolios because it is not rewarded; they will retain only the systematic risk and *will* be rewarded as a result. The extent of their aversion to risk will lead them to accept a certain quantity of risk along the CML. On the CML are located the efficient portfolios, which do not have diversifiable risk, only systematic – given that the market portfolio has systematic risk only, and that all portfolios on the CML have one portion of the market portfolio and a portion of the risk-free asset.

17.1.10 The Pricing Model

The CAPM is an asset pricing model because it considers that assets are undervalued (overvalued) when their expected return is above (below) that demanded by the model. However, the constraint that a security must produce returns that depend on its systematic risk goes against the idea of business diversification; that is, the investment is not valued by the risk that it brings to the company, rather by the risk brought to the shareholder.

However, and contrary to what is proposed by the CAPM, which does not consider diversifiable risk, there are investments that are difficult to diversify, as is the case for owners of family companies who invest the majority of their savings in their businesses. Also, the fact that few securities are listed on the stock market hinders diversification. These are only two reasons among many of why risk diversification is complex, and why diversifiable risk can be important.

There are other occasions when it is necessary to pay attention to total risk and not just systematic risk – for example, bankruptcy risk, which is dependent on total risk.

All in all, the CAPM produces several interesting results that cannot be ignored. From the CAPM, we deduce an important maxim: *a firm must do only what the shareholder cannot do individually*; that is, the firm should diversify to seek synergies; however, to diversify diversifiable risk, the shareholder alone is sufficient (this is true, above all, in large companies).

17.1.11 Contrasts and Controversy Regarding the CAPM

There are numerous studies that criticize the starting hypotheses of the CAPM and that analyze the effects of their relaxation. In general, we point out two major levels of criticism: for professionals of the financial world, the CAPM is too theoretical, and for the academic world, the model does not adequately reflect a reality that is, in fact, much more complex. However, the most important thing is to determine whether the CAPM can adapt to reality – that is, whether it is sufficiently robust.

At the beginning of the 1970s, the CAPM was viewed optimistically. By the late 1970s, the first criticisms, which grew in the 1980s, arose, and by the 1990s there was a great deal of controversy associated with the CAPM.

To determine whether the CAPM actually works, it seems logical to study whether, using past data, the predicted relationship between an asset's return and its systematic risk is borne out; there are diverse methodologies that enable this task. Equation (17.8) represents an ex ante model that has the drawback that expectations cannot be observed; but if we employ rational expectations, we may test it based on past data (ex post model). To do this, a regression is applied between average returns on assets and their betas (empirical market line)

$$\overline{R_j} = y_0 + y_1\beta_j + u_j, \tag{17.9}$$

where y_0 represents the risk-free interest rate, and y_1 the market premium for risk.

The first contrasts are favorable to the CAPM. In this way for example, Black et al. (1972) achieve a good linear fit between average returns and betas with a positive y_1, although y_0 does not represent the risk-free interest rate. Fama and MacBeth (1973) also obtain results consistent with a linear model, in which beta is the only risk measure and a positive risk premium is observed. In general, at this time, a consensus held that the market correctly prices securities and that the CAPM is a good pricing model. However, the first critics were not long in arriving.

Roll (1977) holds that the only way to test the CAPM is to check whether the market portfolio is efficient ex post; but a problem with the CAPM is that the real market portfolio cannot be observed, as there are no portfolios that include all the assets of the economy.

Another criticism of the CAPM is from Van Horne (1989), who makes clear that betas vary considerably according to the index employed as a proxy for the real market portfolio.

However, the work that generated the most controversy about the CAPM was that of Fama and French (1992), which shows a small power of betas to explain average returns, and which emphasizes the existence of other variables that do explain them, such as size and book-to-market ratio.

Nevertheless, Kothari et al. (1992) examine Fama and French (1992) and find that the tests used are not powerful enough. They also observe, using a different source, that the relationship between average returns and the book-to-market ratio is weak, although they do not make the same observation regarding size. They also find that annual betas make the relationship with returns clearer. That is, in short, they consider that beta continues to be a useful instrument to explain average returns.

Roll and Ross (1994) also analyze the work of Fama and French (1992) and point out that a positive and precise relationship between expected returns and betas will be found if the market index used to find the betas is in the efficient frontier. The problem is that the market portfolio cannot be observed, meaning that proxies must be found, and if the proxy used is not efficient, it should not surprise us that the average returns/betas relationship is not found. Therefore, these authors conclude that although the CAPM cannot be rejected, it is overly dependent on the proxy chosen for the market portfolio, which detracts from the CAPM's robustness.

Later, Fama and French (1993) proposed a multifactor model with five factors to explain return on stocks and bonds (market, size, and book-to-market for stocks, along with two specific factors for bonds). In this way, they reach an approach strongly related to the APT.

Fama and French (1996a) defend their model, insisting that beta, in itself, cannot explain expected returns. Fama and French (1996b) maintain that average returns are related to several variables, which, given that they are not explained by the CAPM, are called anomalies. However, these anomalies disappear when a model with three factors is constructed.

However, the controversy continued. MacKinlay (1995) contends that the multi-factor models do not resolve all deviations of the CAPM. Kim (1995) believes that the problems with using betas to explain expected returns stem from error problems in the variables. Jagannathan and Wang (1996) think that the problem lies in the fact that empirical studies assume that betas remain constant, and they propose a conditional CAPM that is criticized by authors such as Ghysels (1998), and Lewellen and Nagel (2006), who believe that the conditional CAPM does not explain the pricing errors of the traditional model, since the variation in betas and the risk premia would have to be very large to explain such major anomalies in pricing as momentum or value premium.

More recently, Daniel et al. (2001), using the Daniel and Titman test (1997), rejected the model of Fama and French (1993) when evaluating whether the value premium found in the average returns on the largest Japanese stocks represents a compensation for the risk taken. This work was probably undertaken as a way to counter criticism from Davis et al. (2000), who state that a three-factor model better explains the asset premium than does the Daniel and Titman characteristics model (1997).

Many other authors have joined the controversy about the CAPM, for example, Chan and Lakonishok (1993), Grundy and Malkiel (1996), Malkiel and Xu (1997), and Bruner et al. (1998), among others.

Among the most recent studies on portfolio theory, that of Dimson and Mussavian (1999) stands out. These authors conducted a review of financial asset pricing theory and point out that derivative pricing is the latest trend. Another notable study is Jegadeesh and Titman (2001) finding support for the hypothesis of behavioral models, which states that the returns on momentum strategies stem from subsequent reactions to the formation of these strategies. Lastly, Liu (2006) demonstrates that liquidity is an important source of risk, and that liquidity is not considered by the CAPM or by the model of Fama and French. This author believes that a two-factor model (liquidity and market factors) better explains stock returns and justifies the effect of the book-to-market ratio, which cannot be explained by the three-factor model of Fama and French.

In general, there is consensus that investors seek more return when a greater risk is taken, but it is not so clear how they determine either the risk or the reward demanded. Several asset pricing models have attempted to answer these questions; among them, the CAPM is the most often employed model by both financial experts and academics, despite the difficulties inherent in its empirical contrast. The CAPM is not a complete model; rather, it is a simplification of reality. But its competitors do not achieve superior results, and they are more complex.

Furthermore, the models that are created from empirical results and have little theoretical basis, such as that of Fama and French, are susceptible to data mining bias, as Black (1993) shows, making models with rigorous theoretical foundation, such as the CAPM, preferable.

Additionally, and despite all the criticism that it has been subjected to since its appearance, we are able to confirm that the CAPM is very efficacious within companies when it is used to determine investment and finance policies. It is also useful in the markets, as it helps to determine the return that should be demanded from an asset, as well as the level of risk that is warranted. CAPM also enables us to evaluate fund managers, comparing the returns they achieve with the risk they bear.

17.2 Performance Measures Related to Portfolio Theory and the CAPM: Classic Indices, Derivative Indices, and New Approaches

17.2.1 Introduction

When evaluating the behavior of a security, portfolio, or investment fund, there are three characteristics that should be considered: return, risk, and liquidity. This last item can be assumed in markets that are sufficiently liquid. It is the return–risk relationship that must be analyzed, which leads to the issue of performance measures.

According to the classic financial literature, we have two alternative methods to measure performance in portfolio management:

On the one hand, we may estimate average past returns on portfolios during a set time horizon and, later, adjust said returns by the behavior of the reference market in which such portfolios invest, and by a representative measure of the risk borne by such portfolios (if the total risk is considered, this is usually measured through variability in the portfolio return, that is, through the standard deviation[5] of the series of returns; however, if only the systematic risk is considered, it will be measured through the beta coefficient as explained in the previous section of this chapter).

The premise here is that we, as individuals, are averse to risk, and consequently demand a higher expected return for bearing a higher risk; it is precisely the performance measures that, when relating average return to its risk, inform us as to how the analyzed portfolio is rewarding us in terms of return received for the accepted risk.

On the other hand, portfolio management performance can also be measured using a history-based composition of portfolios, real or estimated, and by developing evaluation methodologies for them. The contributions made by Sharpe (1992), Grinblatt and Titman (1993), and Daniel et al. (1997) are important. However, this work focuses on the first alternative for the measurement of performance.

In this section we first examine the classic performance measures usually employed to study stock market portfolios. We will see that the differences between the various measures are attributable to the relevant risk, as well as to the way in which the method of beating the market is measured. Later, we examine other measures that attempt to solve some problems present in the classic measures and, finally, we touch on the latest performance measures that enable better adjustment to the present reality.

17.2.2 Classic Performance Measures

17.2.2.1 The Sharpe Ratio (1966)

$$S = \frac{\mu - r_0}{\sigma}, \tag{17.10}$$

where μ is the average return on a portfolio or asset; r_0 is the risk-free asset return, and σ is the standard deviation of the return on the portfolio or asset.

[5] However, when there is a problem of asymmetry in the distribution of returns, the standard deviation is not a suitable risk measure, as Sortino and Price (1994), Eftekhari et al. (2000), and Pedersen and Satchell (2002) indicate; alternative measures must be considered, such as the standard semi-deviation or the Gini index, among others.

The Sharpe ratio calculates the return premium obtained by the asset or portfolio per unit of total risk accepted. Given that this index responds to total risk, it should be used to evaluate portfolios with a tendency to eliminate diversifiable risk. However, the indices of Treynor and Jensen, as we will see below, are based on the systematic risk, meaning that they should be used to evaluate portfolios that are not very diversified, or individual securities.

17.2.2.2 The Treynor Ratio (1965)

$$T = \frac{\mu - r_0}{\beta}, \tag{17.11}$$

where β is the systematic risk of the portfolio or asset.

This index reflects the return premium obtained by the asset or portfolio per unit of systematic risk borne.

17.2.2.3 The Jensen Index (1968, 1969)

$$J = (\mu - r_0) - \left[\mu^* - r_0\right] \times \beta, \tag{17.12}$$

where μ^* is the average return obtained by the market portfolio. Usually, the market portfolio is approached using a *benchmark* or a stock market index.

This index determines the difference between the excess return on the asset or portfolio over the risk-free asset, and the excess that it should have obtained according to the CAPM. Therefore, an asset or portfolio will be said to have obtained a good performance result when it obtains a positive Jensen index value.

If the Sharpe or Treynor ratios are used as the performance measures to determine whether a portfolio or asset has obtained a good result, it will be necessary to compare this ratio with that obtained by the market portfolio. Also, this comparison will allow us to discover whether active management is superior to passive management.

Furthermore, regardless of which performance index we use, the portfolio or asset that obtains the highest value in such index will always be considered superior.

17.2.3 M^2 and M^2 for Beta

Another indicator of performance based on total risk is Modigliani and Modigliani (1997) M^2 measure. This measure compares portfolio returns, adjusted for risk, with those in the appointed reference portfolio. It is calculated as that portfolio return that has been leveraged up or down with the aim of replicating the risk of the reference portfolio, according to the equation

$$M^2 = \left(\frac{\sigma^*}{\sigma}\right)\mu - \left[\left(\frac{\sigma^*}{\sigma}\right) - 1\right] r_0, \tag{17.13}$$

where σ^* is the standard deviation of the return on the benchmark, and r_0 is the interest rate representing the associated cost of the quantity borrowed, or the interest rate paid for the money loaned.

Therefore, M^2 is a measure directly comparable with market average return, μ^*, meaning that the evaluation of management will be positive when M^2 is higher than μ^*. Also, it can be proven that M^2 generates the same rankings as the Sharpe ratio.

Later, Modigliani (1997) proposed a new version of M^2 that allows the use of systematic risk, rather than total risk: M^2 for beta, which generates the same ranking as the Treynor ratio. The idea behind this measure is that we borrow or lend such that the portfolio has the same beta as the market, later comparing the resulting portfolios returns:

$$\mathrm{M}^2 \text{ for beta} = r_0 + \frac{(\mu - r_0)}{\beta} \tag{17.14}$$

The interest in the use of M^2 and M^2 for beta measures, which technically coincide with the Sharpe and Treynor ratios, respectively, lies in the fact that they are measured as returns, which is much easier to understand by common investors than the Sharpe and Treynor risk premiums. In addition, Modigliani (1997) criticizes the Jensen index, claiming that differences in return cannot be compared when the risks are very different.

17.2.4 Information Ratio and Tracking Error

$$IR = \frac{\mu - \mu^*}{\sigma_{TRACKING-ERROR}} \tag{17.15}$$

Tracking error is defined as the difference between the return obtained within a period by a portfolio and that obtained by the benchmark. The variability of the tracking error is the deviation of said differences ($\sigma_{\text{TRACKING-ERROR}}$).

The information ratio was developed by Sharpe (1994) and, assuming a normal distribution of probability, it reveals the probability that the tracking error will be negative; that is, the probability that the portfolio will yield lower returns than the benchmark. This means that the ratio is very useful for evaluating the manager's ability to beat the market. However, Modigliani and Modigliani (1997) do not consider this a good performance measure as it does not consider total risk.

17.2.5 PIRR Index

The PIRR index came into being as an attempt to resolve the problem of the Sharpe ratio's expression as a quotient. Indeed, if we assume that the distribution of returns on an asset is normal (μ, σ), which is the norm, the numerical value of the Sharpe ratio results from standardizing r_0, meaning that the application of this ratio to the ranking of funds depending on their performance, necessitates classifying them according to the probability that, within any subperiod of time, their returns will be lower than r_0.

This means that this is not a good criterion for classification, because it is not logical that fund X be preferable to fund Y merely because the return on X has a lower probability of falling below the free risk rate. In fact, Y could have an average return much higher than that for X.

All of this reasoning led Gómez-Bezares (1993) to question the risk quotient penalty that the Sharpe ratio imposes, and to study a possible linear penalty; this led him to opt for the penalty of the NPV,[6] ultimately leading to the PPV[7]

$$PPV = \mu_{NPV} - t\sigma_{NPV}, \tag{17.16}$$

where t is the penalty parameter that will vary according to the risk aversion of the decision maker, and the NPV has been calculated at the risk-free interest rate, as we will later penalize it by subtracting t standard deviations. The PPV could be interpreted as the certainty equivalent of a risky investment with a determined average and standard deviation of the NPV.[8] In accordance with this idea, Gómez-Bezares et al. (2004) penalize the IRR,[9] giving rise to the PIRR.[10]

$$PIRR = \mu_{IRR} - t\sigma_{IRR} \tag{17.17}$$

We may interpret the PIRR as the certainty equivalent return on a risky asset with a mean and a standard deviation of IRR; that is, a performance measure adjusted by risk. Returning to the previous nomenclature we have

$$PIRR = \mu - t\sigma. \tag{17.18}$$

The next step consists of assigning a value to t. To do this, we assume an efficient market in which r_0 and μ^* (affected by σ^*) are considered to be equivalent (in the sense that they are similarly desirable for the group of investors): $r_0 = \mu^* - t\sigma^*$. Calculating t, we have $t = (\mu^* - r_0)/\sigma^*$, which we observe is equal to the Sharpe ratio for the market portfolio (S^*). Therefore, we reformulate PIRR

$$PIRR = \mu - S^*\sigma. \tag{17.19}$$

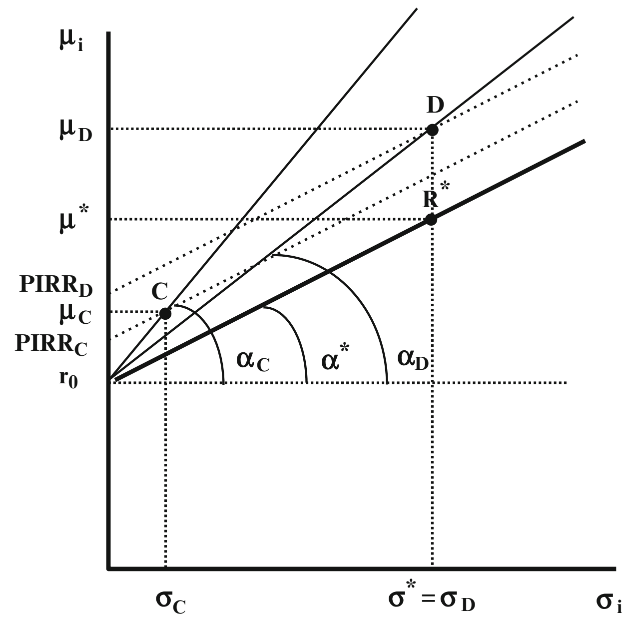

Fig. 17.8 Figure 17.8 shows a comparison between the S and PIRR indices

In Fig. 17.8, we compare the S and PIRR indices. We represent the C and D portfolios and the market portfolio, and observe that the Sharpe ratio coincides with the tangent of the angle that forms the straight line that links r_0 with one of the portfolios with the horizontal line corresponding to r_0; therefore, it is deduced that the Sharpe ratio classifies assets based on a system of indifference straight lines that bundle in such a way that the asset located on the highest line is the best one.

However, we can see that PIRR is equivalent to drawing straight lines parallel to the line that joins r_0 and the market portfolio (PIRR is the point where each line crosses the vertical axis). Thus, we have in this case, a system of parallel indifference lines.

Finally, it is easy to observe that S and PIRR lead to different classifications of portfolios: $S_C > S_D > S^*$; $PIRR_D > PIRR_C > r_0$ (observe that $PIRR^* = r_0$).

17.2.6 PIRR Index for Beta

The Treynor ratio (1965) poses similar problems to the Sharpe ratio; therefore, we will also impose a linear penalty of the risk, in this case

[6] Net present value.

[7] Penalized present value.

[8] Two other characteristics of this system are: if we assume that NPV follows a normal distribution, it is easy to understand t as the standardized value of the PPV; and the system is equivalent to using parallel indifference lines.

[9] Internal rate of return.

[10] Penalized internal rate of return.

$$\text{PIRR for Beta} = \mu - t'\beta, \tag{17.20}$$

where t′ is the penalty parameter, and we again assume that the certainty equivalent of the market portfolio is the risk-free rate: $r_0 = \mu^* - t'\beta^* = \mu^* - t'$ (given that $\beta^* = 1$); calculating the value, we have: $t' = \mu^* - r_0$, therefore

$$\text{PIRR for Beta} = \mu - (\mu^* - r_0)\beta. \tag{17.21}$$

This system can be justified, as can the previous one, by parallel indifference straight lines. It is also apparent that this is equivalent to the Jensen index (1968, 1969).

There are other performance proxies: conditional performance analysis, which considers return and risk expectations to be time-varying; style analysis, which attributes portfolio performance to a group of characteristics that define them; market timing analysis, which breaks down performance into two parts: (1) that which can be attributed to the manager's ability to select securities (stock picking) and (2) that which can be attributed to the manager's ability to anticipate the movements of the market (market timing); among others. However, this chapter does not focus on these analyses.

17.3 Empirical Analysis: Performance Rankings and Performance Persistence

17.3.1 Performance Rankings

In this section, we put into practice what was discussed in the previous section. To do this, we use an example: We take weekly data from the returns on a sample of 198 equity investment funds over 6 years. We also take a stock market index to represent the market portfolio and the risk-free asset is approached by the one-month Treasury bill.[11]

With these data, we calculate the Sharpe and Treynor ratios for each year, as well as the Jensen and PIRR[12] indices. In Table 17.1, we show the percentage of funds that outperform the market portfolio based on each of the performance measures considered, and for each of the 6 years analyzed.[13]

Table 17.1 Table 17.1 reports the percentage of funds that outperform the market portfolio based on different performance measures

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
S	45%	44%	67%	40%	27%	46%
PIRR	45%	44%	67%	40%	27%	46%
J	39%	56%	79%	51%	30%	49%
T	40%	55%	79%	51%	30%	49%

From Table 17.1 it is evident that, based on the S and PIRR indices, exactly the same percentage of investment funds outperform the stock market index. This result is not surprising because, as shown in Fig. 17.8, when an asset beats the market based on the Sharpe ratio, it also beats it based on the PIRR index.

We observe very similar results between the J and T[14] indices. In general, the results obtained are similar based on the four performance measures applied, which leads us to analyze the level of correlation that exists between the classifications of the funds, depending on their performance, derived from the application of the four measures. To do this, we first create a ranking of funds based on each measure, then we apply the Spearman's rho and Kendall's tau-b correlation coefficients to evaluate the level of similarity between these classifications.

17.3.1.1 Spearman's Rho Coefficient

$$r_s = 1 - \frac{6\sum d_i^2}{N(N^2-1)}, \tag{17.22}$$

where N is the number of funds (198, in this case), and d_i is the difference between the place occupied by fund i in two classifications.

The Spearman's rho correlation coefficient agrees with Pearson's coefficient, but the former is an adaptation of the latter for ordinal data.

17.3.1.2 Kendall's Tau-b Coefficient

$$\tau = \frac{2(P-Q)}{N(N-1)}, \tag{17.23}$$

where P is the number of agreements between two classifications, and Q is the number of disagreements.

This coefficient takes values between −1 and $1(-1 \le \tau \le 1)$, and has the disadvantage that, in the case of a draw, the coefficient cannot reach the values −1 and 1, meaning that in such a situation, it would be necessary

[11] The data have been taken from Vargas' PhD Dissertation (2006).

[12] The remainder of the performance measures shown in the previous section are not applied in this example because, as has been stated previously, they give classifications of assets equivalent to those given by one of the measures considered in this example (except IR).

[13] We do not show an individual analysis of the results obtained for each fund, for each performance measure, and for each period because of our purpose of abbreviating.

[14] These should also coincide exactly, but they do not. However, in our example, the first and second years do not coincide and this is due to the special characteristics of one of the analyzed funds that give a negative beta during these 2 years.

Table 17.2 Table 17.2 shows the results for the two correlation coefficients applied on the classifications of the funds considering different performance measures[15]

		J-PIRR	J-S	S-PIRR	S-T	PIRR-T	J-T
1st year	Kendall	0.946	0.835	0.862	0.883	0.790	0.796
	Spearman	0.994	0.960	0.969	0.926	0.900	0.898
2nd year	Kendall	0.830	0.754	0.849	0.858	0.728	0.795
	Spearman	0.926	0.890	0.958	0.952	0.892	0.890
3rd year	Kendall	0.793	0.702	0.832	0.754	0.648	0.801
	Spearman	0.874	0.826	0.946	0.815	0.771	0.918
4th year	Kendall	0.873	0.789	0.879	0.833	0.809	0.869
	Spearman	0.941	0.912	0.972	0.907	0.917	0.971
5th year	Kendall	0.986	0.816	0.821	0.977	0.814	0.813
	Spearman	0.999	0.945	0.947	0.997	0.945	0.943
6th year	Kendall	0.946	0.860	0.882	0.957	0.860	0.868
	Spearman	0.991	0.970	0.978	0.995	0.972	0.971

[15] All of the coefficients in the table are significant at the 1% level.

to correct the expression of the coefficient. However, in the example that we have given, there are no draws.

In Table 17.2, we show the results of the application of the two correlation coefficients on the classifications of the funds derived from the application of the four performance measures analyzed in the example.

Table 17.2 shows a strong similarity between the order of preference determined from the different performance measures, as the coefficients obtained are very close to the unit and are significant at the 1% level.

We can therefore see that the risk linear penalty achieves a methodological improvement of the Sharpe ratio that, in any case, generally maintains the rankings of the funds.

17.3.2 Performance Persistence

In this section, we analyze the possible existence of persistence in the performance results achieved by our sample. To do this, we compare the performance achieved by the funds in two consecutive periods. We apply two methodologies.

First is the *non-parametric methodology*, based on contingency tables and several statistics. It consists of comparing the performance rankings in two consecutive periods, distinguishing, in both periods, two portfolio subgroups ("winners" and "losers") based on the criterion of the median: a fund is a "winner" if its performance is above the median, and a "loser" if it is below. Therefore, we classify the funds as WW if they are winners over two consecutive periods, and WL if they are winners in one period and then losers in a later period, and so on. In addition, the statistical tests of Malkiel (1995), Brown and Goetzmann (1995), and Kahn and Rudd (1995) are applied to determine the significance of the persistence.

17.3.2.1 Malkiel's z-statistic (1995)

$$Z = (Y - np)\,/\,\sqrt{np\,(1-p)}, \tag{17.24}$$

where Z represents a statistic that, approximately, follows the normal distribution (0,1); Y represents the number of winning portfolios in two consecutive periods; n equals WW + WL.

This test shows the proportion of WW compared to WW + WL in such a way that p represents the probability that a winning portfolio over a period will continue to be a winner in the following period. We assign the value of 0.5 to p. If $Z > 1.96$, we reject the null hypothesis of non-persistence at a significance level of 5%.

17.3.2.2 Brown and Goetzmann's Statistic (1995)

$$Z = \frac{\ln(OR)}{\sigma_{\ln(OR)}}, \tag{17.25}$$

where OR is Brown and Goetzmann's Odd's ratio: $OR = \frac{WW\times LL}{WL\times LW}$, and where $\sigma_{\ln(OR)} = \sqrt{1/WW + 1/LL + 1/WL + 1/LW}$. Again, a value of $Z > 1.96$ would confirm a trend toward persistence in performance at the 5% level.

17.3.2.3 Kahn and Rudd's Statistic (1995)

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}, \tag{17.26}$$

where O_{ij} is the real frequency of the i-th row and the j-th column; E_{ij} is the expected frequency of the i-th row and the j-th column.

In the case of a 2×2 contingency table, the distribution shows one degree of freedom. A priori, the four expected frequencies would present the same figure (total number of funds, divided by four), meaning that we could reformulate the χ^2 statistic

$$\chi^2 = \frac{(WW - N/4)^2}{N/4} + \frac{(LL - N/4)^2}{N/4} + \frac{(LW - N/4)^2}{N/4} + \frac{(WL - N/4)^2}{N/4}, \tag{17.27}$$

where N is the total number of portfolios evaluated. If the chi-squared has a critical value above 3.84, it would be indicative of performance persistence at a significance level of 5%.

Following the above example, Tables 17.3 and 17.4 show the performance persistence results – performance is measured by the Sharpe ratio[16] – obtained from application of the non-parametric methodology; in Table 17.3, we show the contingency table, and in Table 17.4, the results of the statistics.

[16] We do not show persistence results obtained from applying other performance measures, due to space considerations and given that the aim of this example is not to analyze a sample, but rather to explain how the different methodologies of persistence analysis work.

We can observe from Tables 17.3 and 17.4 the presence of the phenomenon of performance persistence. In fact, based on Table 17.3, we can confirm that the number of funds that repeat as W or L in two consecutive years is greater than the number of funds that change their category (399 vs. 271). Furthermore, Table 17.4 shows the presence of the phenomenon of persistence given that statistics significant at the level of 1% are obtained for the whole sample.

The other methodology that allows us to analyze the presence of the phenomenon of performance persistence is the *parametric methodology*, based on regression analysis in such a way that it determines, by means of ex post values, whether the relationship between performance in a certain period and the corresponding performance of an earlier period is statistically significant. To do this, the following regression is applied

$$P_{p(t+1)} = \alpha + \beta\ P_{p(t)} + \varepsilon_p, \tag{17.28}$$

where $P_{p(t+1)}$ and $P_{p(t)}$ represent the performance of portfolio p in the periods t + 1 and t, respectively. Positive beta values, together with a significant t statistic, would confirm the existence of performance persistence, while a negative estimation of said coefficient would indicate the existence of the opposite behavior.

Continuing with the above example, in Table 17.5 we show the results of performance persistence–performance

Table 17.3 Table 17.3 reports the contingency table for the Sharpe ratio

	Years 1, 2	Years 2, 3	Years 3, 4	Years 4, 5	Years 5, 6	Total
WW	31	34	35	47	51	198
LL	31	36	36	47	51	201
LW	15	18	20	33	48	134
WL	15	20	21	33	48	137
No. funds	92	108	112	160	198	670

Table 17.4 Table 17.4 shows the results of the statistics that check the persistence in performance measured by Sharpe ratio

	Years 1, 2	Years 2, 3	Years 3, 4	Years 4, 5	Years 5, 6	Total
Z-Malkiel	2.3591	1.9052	1.8708	1.5652	0.3015	3.3328
P	0.0183*	0.0568	0.0614	0.1175	0.7630	0.0009**
OR	4.2711	3.4000	3.0000	2.0285	1.1289	2.1679
Z- B & G	3.2641	3.0335	2.7998	2.2021	0.4263	4.9146
P	0.0011**	0.0024**	0.0051**	0.0277*	0.6699	0.0000**
χ^2- K & R	11.1304	9.6296	8.0714	4.9000	0.1818	24.5075
P	0.0008**	0.0019**	0.0045**	0.0269*	0.6698	0.0000**

*(**) indicates statistical significance at a level of 5% (1%).

Table 17.5 Table 17.5 reports the results of performance persistence with the parametric methodology when performance is measured with the PIRR index

	Years 1, 2	Years 2, 3	Years 3, 4	Years 4, 5	Years 5, 6
No. Funds	92	108	112	160	198
α	−0.000117	0.001	−0.000377	−0.000096	0.000382
T_α	−0.654	11.912**	−2.743**	−1.329	4.171**
β	0.840	0.063	0.602	0.246	0.449
T_β	5.450**	1.036	6.098**	3.725**	4.385**
R^2	0.248	0.010	0.253	0.081	0.089

** indicates statistical significance at the 1% level.

being measured by the PIRR index – obtained through the application of the parametric methodology.

This table confirms the presence of the phenomenon of performance persistence, except between the second and third years (non-significant beta).

17.4 Summary and Conclusions

"Portfolio theory," created by Markowitz, has reached a high level of development thanks to the improvement work carried out on it by several authors.

This theory operates in a risk-based environment (the future return on an asset is known in terms of probability) and has the objective of determining the optimum portfolio for investors. To do this, it places all assets on the $\mu - \sigma$ map (mean and standard deviation of the random return variable, respectively), and to choose among them, it considers the investor's preferences, represented by a system of indifference curves. Therefore, the optimum portfolio is that which gives tangency between an indifference curve and the efficient frontier (a group of portfolios with maximum return and minimum risk).

Furthermore, "portfolio theory" is used as a theoretical foundation for an asset pricing model: the CAPM. This model, created by Sharpe, considers that the expected return on an asset depends on the market expected return and on the risk-free asset return, as well as on the systematic risk (non-diversifiable) of the asset. At equilibrium, according to CAPM, all assets adjust their values to offer the return that corresponds to them, meaning that it is easy to detect if an asset is underpriced or overpriced.

A method of simplification of the relationships between securities also comes from Sharpe, who created the "market model," according to which the relationships between the returns on assets are solely due to their common relationship with the market index return. This model allows us to distinguish between systematic and diversifiable risk (the latter risk is not remunerated, as it can be eliminated by the investor).

The CAPM has been the object of various critiques, especially regarding its starting hypotheses that are alleged to distance this model from reality. However, it has been shown that its conclusions, to a sufficiently significant extent, are fulfilled. Moreover, CAPM has not been overcome by other more complex pricing models, and its usefulness for firms and in markets has been proven.

The second part of this chapter refers to performance measures related to portfolio theory; therefore, classic indices are shown along with other, newer indices that attempt to overcome certain limitations present in the classic indices.

Finally, we show an example in which we analyze the performance of a sample of equity investment funds using several performance measures. We contrast the similarity between the rankings of the funds (depending on their performance) that are obtained from several measures and, finally, we analyze the possible presence of the phenomenon of performance persistence throughout the period analyzed.

Acknowledgement This chapter follows Gómez-Bezares (2006). The authors express their gratitude to Professor Stephen Brown for his helpful comments, which have greatly contributed to the improvement of this chapter.

References

Black, F. 1993. "Beta and return." *Journal of Portfolio Management* 20, 8–18.

Black, F., M. C. Jensen and M. Scholes. 1972. "The capital asset pricing model: some empirical tests." in *Studies in the theory of capital markets*, Jensen (Ed.). Praeger, New York.

Brown, S. J. and W. N. Goetzmann. 1995. "Performance persistence." *Journal of Finance* 50, 679–698.

Bruner, R. F., K. M. Eades, R. S. Harris and R. C. Higgins. 1998. "Best practices in estimating the cost of capital: survey and synthesis." *Financial Practice and Education* 8(1), 13–28.

Chan, L. K. C. and J. Lakonishok. 1993. "Are the reports of beta's death premature?" *Journal of Portfolio Management*, summer 19, 51–62.

Daniel, K., M. Grinblatt, S. Titman and R. Wermers. 1997. "Measuring mutual fund performance with characteristic-based benchmarks." *Journal of Finance* 52, 1035–1058.

Daniel, K. and S. Titman. 1997. "Evidence on the characteristics of cross-sectional variation in stock returns." *Journal of Finance* 52, 1–33.

Daniel, K., S. Titman and K. C. J. Wei. 2001. "Explaining the cross-section of stock returns in Japan: factors or characteristics?" *Journal of Finance* 56, 743–766.

Davis, J. L., E. G. Fama and K. R. French. 2000. "Characteristics, covariances and average returns: 1929 to 1997." *Journal of Finance* 55(1), 389–406.

Dimson, E. and M. Mussavian. 1999. "Three centuries of asset pricing." *Journal of Banking and Finance* 23(12), 1745–1769.

Eftekhari, B., C. S. Pedersen and S. E. Satchell. 2000. "On the volatility of measures of financial risk: an investigation using returns from European markets." *The European Journal of Finance* 6, 18–38.

Fama, E. F. and K. R. French. 1992. "The cross-section of expected stock returns." *Journal of Finance* 47(2), 427–465.

Fama, E. F. and K. R. French. 1993. "Common risk factors in the returns on stocks and bonds." *Journal of Financial Economics* 33, 3–56.

Fama, E. F. and K. R. French. 1996a. "The CAPM is wanted, dead or alive." *Journal of Finance* 51(5), 1947–1958.

Fama, E. F. and K. R. French. 1996b. "Multifactor explanations of asset pricing anomalies." *Journal of Finance* 51(1), 55–84.

Fama, E. F. and J. D. MacBeth. 1973. "Risk, return and equilibrium: empirical tests." *Journal of Political Economy* 81(3), 607–636.

Ghysels, E. 1998. "On stable factor structures in the pricing of risk: Do time-varying betas help or hurt?" *Journal of Finance* 53, 549–573.

Gómez-Bezares, F. 1993. "Penalized present value: net present value penalization with normal and beta distributions." in: *Capital budgeting under uncertainty*, Aggarwal (Ed.). Prentice-Hall, Englewood Cliffs, New Jersey, pp 91–102.

Gómez-Bezares, F. 2006. *Gestión de carteras*, Desclée de Brouwer, Bilbao.
Gómez-Bezares, F., J. A. Madariaga and J. Santibánez. 2004. "Performance ajustada al riesgo." *Análisis Financiero* 93, 6–17.
Grinblatt, M. and S. Titman. 1993. "Performance measurement without benchmarks: an examination of mutual fund returns." *Journal of Business* 66(1), 47–68.
Grundy, K. and B. G. Malkiel. 1996. "Reports of beta's death have been greatly exaggerated." *Journal of Portfolio Management* 22(3), 36–44.
Jagannathan, R. and Z. Wang. 1996. "The conditional CAPM and the cross-section of expected returns." *Journal of Finance* 51(1), 3–53.
Jegadeesh, N. and S. Titman. 2001. "Profitability of momentum strategies: an evaluation of alternative explanations." *Journal of Finance* 56, 699–720
Jensen, M. C. 1968. "The performance of mutual funds in the period 1945–1964." *Journal of Finance* 39, 389–416.
Jensen, M. C. 1969. "Risk, the pricing of capital assets, and the evaluation of investment portfolios." *Journal of Business* 42(2), 167–247.
Kahn, R. N. and A. Rudd. 1995. "Does historical performance predict future performance?" *Financial Analysts Journal* 51, 43–52.
Kim, D. 1995. "The errors in the variables problem in the cross-section of expected stock returns." *Journal of Finance* 50(5) 1605–1634.
Kothari, S. P., J. Shanken and R. G. Sloan. 1992. Another look at the cross-section of expected stock returns, Working Paper, Bradley Policy Research Center, University of Rochester, New York.
Lewellen, J. and S. Nagel. 2006. "The conditional CAPM does not explain asset-pricing anomalies." *Journal of Financial Economics* 82, 289–314.
Lintner, J. 1965. "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets." *Review of Economics and Statistics* 47(1), 13–37.
Liu, W. 2006. "A liquidity-augmented capital asset pricing model." *Journal of Financial Economics* 82, 631–671.
MacKinlay, A. C. 1995. "Multifactor models do not explain deviations from the CAPM." *Journal of Financial Economics* 38, 3–28.
Malkiel, B. G. 1995. "Returns from investing in equity mutual funds 1971 to 1991." *Journal of Finance* 50, 549–572.
Malkiel, B. G. and Y. Xu. 1997. "Risk and return revisited." *Journal of Portfolio Management*, 23, 9–14.
Markowitz, H. 1952. "Portfolio selection." *Journal of Finance* 7(1), 77–91.
Markowitz, H. 1959. *Portfolio selection: efficient diversification of investments*, Wiley, New York.
Markowitz, H. 1987. *Mean-variance analysis in portfolio choice and capital markets*, Basil Blackwell, Oxford.
Merton, R. C. 1972. "An analytic derivation of the efficient portfolio frontier." *Journal of Financial and Quantitative Analysis* 7, 1851–1872.
Modigliani L. 1997. "Don't pick your managers by their alphas," *U.S. Investment Research, U.S. Strategy*, Morgan Stanley Dean Witter, June.
Modigliani, F. and L. Modigliani. 1997. "Risk-adjusted performance." *Journal of Portfolio Management* 23(2), 45–54.
Pedersen, C. S. and S. E. Satchell. 2002. "On the foundations of performance measures under asymmetric returns." *Quantitative Finance* 3, 217–223.
Roll, R. 1977. "A critique of the asset pricing theory's tests." *Journal of Financial Economics* 4(2) 129–176.
Roll, R. and S. A. Ross. 1994. "On the cross-sectional regression between expected returns and betas." *Journal of Finance* 49, pp. 101–121.
Sharpe, W. F. 1961. "Portfolio analysis based on a simplified model of the relationships among securities." Ph.D. Dissertation, University of California, Los Angeles.
Sharpe, W. F. 1963. "A simplified model for portfolio analysis." *Management Science* 9(2), 277–293.
Sharpe, W. F. 1964. "Capital asset prices: a theory of market equilibrium under conditions of risk." *Journal of Finance* 19(3), 425–442.
Sharpe, W. F. 1966. "Mutual fund performance." *Journal of Business* 39, 119–138.
Sharpe, W. F. 1992. "Asset allocation: management style and performance measurement." *Journal of Portfolio Management* 18, 7–19.
Sharpe, W. F. 1994. "The Sharpe ratio." *Journal of Portfolio Management* 21, 49–58.
Sortino, F. and L. Price. 1994. "Performance measurement in a downside risk framework." *Journal of Investing* 3, 59–65.
Tobin, J. 1958. "Liquidity preference as behavior towards risk." *The Review of Economic Studies* 67, 65–86.
Treynor, J. L. 1965. "How to rate management of investment funds." *Harvard Business Review* 43, 63–75.
Van Horne, J. C. 1989. *Financial management and policy*, Prentice-Hall, Englewood Cliffs, New Jersey.
Vargas, M. 2006. "Fondos de inversión españoles: análisis empírico de eficiencia y persistencia en la gestión." PhD Dissertation, University of Zaragoza, Spain.