7. Externalizing Mental Models with Mindtools

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Abstract: Mental models are complex and multi-faceted, so they cannot be adequately represented using any single form of assessment. After reviewing traditional methods for manifesting and representing mental models, we describe how Mindtools can be used by learners to externalize their mental models using different tools that represent different kinds of knowledge.

Keywords: Mental models; knowledge representation; cognitive tools; mindtools; procedural knowledge; structural knowledge; episodic knowledge.

Mental Models

Mental models are the internal constructs that humans construct to represent phenomena in the world. Through interaction with the environment, with others, and with the artifacts of technology, people construct mental models of the world with which they interact (Norman, 1983, p. 7). Based on the mental models, people describe why a system exists and what a system looks like, explain how a system operates and what a current system state is, and predict what a future system state is (Rouse & Morris, 1986). For instance, when driving a car, people construct and manipulate their mental models of how a car operates, what traffic laws are, where they are on a map, and so on. These mental models are different depending on the way people have interacted with the system. Because of a different traffic system, English drivers have a mental model for driving on the left side of the road, whereas the U.S. drivers are uncomfortable applying that model.

Mental models have been researched by cognitive scientists, psychologists, human-computer interaction specialists, and educators, each constructing their own interpretations of mental models. Johnson-Laird (1983), a psychologist, describes mental models as structural analogues of the world as perceived and conceptualized, which enable people to make inferences and predictions, to understand phenomena, to decide and control actions, and to experience events by proxy. Gentner and Stevens (1983) state that mental models are concerned with human knowledge of the world and of how it works. Wilson and Rutherford (1989) reflect a human factors orientation by conceiving of a mental model as a representation formed by a user of a system on the basis of previous experience and current observation. Mental models provide most of the subsequent system understanding and dictate the level of task performance. Vosniadou and Brewer (1994) define mental models as the kinds of mental representations individuals construct when they reason about the physical world. They also assume that mental models are dynamic structures usually created on the spot to meet the demands of specific problem-solving situations. From these explanations, we conclude that a mental model is a representation or a structural analogue of the world that is constructed and manipulated by a person, and it is the basis of cognitive activities such as understanding, predicting, and reasoning which are necessary for specific task performance.

Norman (1983) distinguishes users' mental models from conceptual models of teachers, designers, scientists, and engineers, and he argues that mental models are incomplete, limited, unstable, confused, unscientific, and parsimonious. For instance, Norman (1983) found that people tended to excessively push the clear button of a calculator when they wanted to restart it, whereas they exhibited reluctance to use the clear button during problem solving for the fear of clearing too much. This shows that doubts and superstitions govern users' behavior and enforce extra caution when they operate a machine.

Assessing Mental Models

Just as theorists have differed in their conceptions of mental models, researchers have also differed in the methods they have used to assess mental models. There is no single agreed-upon measurement tool for mental models. Rowe and Cooke (1995) compared several mental model measures in terms of their correlation with troubleshooting performance. They found that laddering interviews, relatedness ratings, and diagramming techniques were predictive of troubleshooting performance, but the think-aloud measure had low correlation with the performance. However, think-aloud protocols are effective in identifying the sequence of states people progress through (Chi, 2006) and in assessing concepts-in-use because participants think aloud while they solve a problem (Jonassen, 2006). Different measurement techniques focus on different aspects of mental models (Royer, Cisero, & Carlo, 1993), so it is hard to assess all aspects of mental models by a single method. Mental models have been measured extensively by five methods: problem solving, verbal report, drawing, categorization, and conceptual pattern representation.

First, problem-solving performance manifests the features of mental models. Mental models are the basis of understanding phenomena and making inferences and predictions (Johnson-Laird, 1983; Norman, 1983), so if people have different mental models, they will understand a problem differently and create different solutions. Gentner and Gentner (1983) showed that the patterns of solving electrical circuit problems were different depending on the mental model the subject had. Based on this assumption, researchers inferred the characteristics of mental models from problem solving outcomes. McCloskey, Caramazza, and Green (1980) asked students to predict the path a metal ball would follow after it came out of a curved tube in order to assess their mental models of physical motion. Thirty six percent of the pathways drawn were curved lines rather than straight lines. They inferred that people who predicted curved pathways had different mental models of physical motion from those who predicted straight pathways. In addition, Azzarello and Wood (2006) recommended unfolding case studies for assessing situational mental models that develop while a person is actively engaged in solving a problem in a specific situation. Unfolding case studies present scenario data in stages, so students' mental model of the case may change with additional information at each stage. The examination of task performance after each stage can reveal how mental models of the situation are evolving. Although problem-solving performance produces only indirect information about mental models, it provides objective evidences and it can be used with other methods effectively.

A second method for assessing mental models, verbal reports, is a direct method for eliciting mental models. Assessment of mental models depends on what people say about their mental models and verbal reports can be done as interviews, explanations or think-aloud protocols (Chi, 2006). This method is based on the notion that individuals had privileged access to their mental models and their report can reveal their cognitive process and thoughts (Ericsson & Simon, 1984). Southerland, Smith, and Cummins (2000) suggested structured interviews as a method of investigating students' conceptual frameworks. Structured interviews have the advantage of allowing students to express what they know and how they can apply the knowledge in their own words. The use of generative questions that require construction and manipulation of mental models are effective in measuring mental models. For example, Vosniadou and Brewer (1992) used such generative questions as "If you were to walk for many days in a straight line, where would you end?" in order to examine children's mental models of the earth. In addition, explanation of observed phenomena is one of the methods measuring mental models (Sabers, Cushing, & Berliner, 1991).

Another form of verbal reports is the think-aloud method (Ericsson & Simon, 1984). For think-aloud, subjects are asked to simply verbalize their thoughts they attend to while performing a task rather than describe or explain what they are doing. Think-aloud protocols have been used for assessing the difference of mental models between experts and novices and the processes in which mental models are constructed and developed for problem solving (Anzai & Yokoyama, 1984; Hong & O'Neil, 1992; Simon & Simon, 1978). Think-aloud protocols are useful data for analyzing mental models because they provide direct information about ongoing thinking processes rather than the outcome of thinking.

Third, drawings can be a complementary method of verbal reports because verbalization of a nonverbal image leads to a biased model. Whitfield and Jackson (1982) found that air traffic controllers had difficulty in verbalizing the image of the system's states and Rouse and Morris (1986) argue that verbal reports have limitation because mental models are frequently pictorial. For this reason, drawings have been used with verbal reports in several mental model research studies. For example, Butcher (2006) asked participants to draw a picture of what they know about the heart and circulatory system and to explain their drawings before and after learning. He categorized drawings according to the mental model of the heart and circulator system and compared students' drawings and verbal explanations in order to examine whether the mental model is improved by learning. In addition, Vosniadou and Brewer (1994) asked children to explain the day and night cycle not only by a verbal response but also by making a drawing. They provided the drawing of a person living on the earth and asked children to draw a picture that made the earth day or night for the person. The drawings represented children's different mental models of the day and night cycle. Drawings can provide information about pictorial aspects of mental models, which are difficult to be measured by verbal reports.

Fourth, categorization of instances reveals how mental models are developed and organized. Categorizing problems based on their similarity has been frequently used for identifying the cognitive difference between experts and novices (Chi, Feltovich, & Glaser, 1981; Hardiman, Dufresne, & Mestre, 1989; Schoenfeld & Herrmann, 1982; Silver, 1979). For example, Chi et al. (1981) asked participants to sort physics problems and to explain the reasons for their categorization. Novices sorted the problems based on the surface features such as the presence of blocks and inclined planes, whereas experts tended to sort them according to the major physics principles that were critical to solutions. This result shows that the mental models of novices are different from those of experts because mental models are the basis of perceiving and sorting problems. Moreover, novices who judged problems based on principles tended to categorize problems similarly to experts and solved problems better than other novices who relied on surface features (Hardiman et al., 1989). Thus, categorization can be used for assessing whether mental models are constructed based on principles or surface features of problems.

Finally, mental models have been represented in the form of concept maps. Concept maps spatially represent concepts and their relationships and they have been used extensively to assess learning outcomes (Jonassen, 2000; 2006). Students can easily create concept maps without statistical analysis and they provide extensive information of conceptual patterns. In addition, multidimensional scaling (MDS, Kruskal, 1964) and Pathfinder (Schvaneveldt, 1990) scaling algorithms have been used for visualizing structural knowledge. The outcomes of MDS and Pathfinder can be assessed both qualitatively and quantitatively. Conceptual patterns of MDS are qualitatively assessed by examining the clusters of concepts and the meaning of dimensions, whereas networks of Pathfinder are qualitatively assessed by analyzing the location and links of concepts and hierarchical features. For instance, Jonassen (1987) used MDS to assess student's conception of Newtonian mechanics. Wilson (1994) used both MDS and Pathfinder to examine the variation of knowledge representation about chemical equilibrium and qualitatively compared conceptual patterns between higher achievers and lower achievers. For quantitative analysis, similarity scores between each person and an expert are frequently used. That is, the higher the similarity score of conceptual patterns is, the closer the mental model of an individual is to that of the expert. The similarity of networks has been reported to predict domain performance highly effectively (Goldsmith, Johnson, & Acton, 1991; Gomez, Hadfield, & Housner, 1996).

Mental Models are Multi-Dimensional

In most of the research on mental models, scholars have attempted to define mental models uni-dimensionally, that is, to identify a single descriptor for mental models. We assume that the construct, mental model, is too complex to be described using any single measure or form of assessment. Mental models are more than structural maps of components. They are dynamic constructions that can be manipulated and tried out. They are multimodal as well as multi-dimensional. Mental models are complex and inherently epistemic, that is, they form the basis for expressing how we know what we know. Because mental models are epistemic, they are not readily known to others and, in fact, not necessarily comprehended by the knower. Jonassen and Henning (1999) showed that troubleshooting performance (a manifestation of procedural knowledge) was positively related to a variety of mental model measures, including structural knowledge, as represented by Pathfinder Networks (Schvaneveldt, 1990), a verbal recollection of their visual image of the system, metaphors that students generated about the system, and retrospective debriefings. That is, students who were better troubleshooters produced better structural knowledge, metaphors, and images of the system they were troubleshooting. That is, the larners had constructed more robust mental models. Each measure of each construct was highly related. They concluded that mental models possess multiple forms of representation. Research to identify all of the relevant components in mental models needs to be conducted. It is likely that mental models also possess executive control or strategic knowledge as well as episodic memories. The latter construct will be described later. In this paper, we briefly describe some of these cognitive dimensions of mental models and also suggest computer-based tools for externalizing representations of those cognitive dimensions. These are tools for externalizing mental models.

Limitation

In this paper, we address only cognitive dimensions (mental models in the head) of mental models. While we accept the existence of social mediated or team mental models derived from the intersection of different individuals' mental models, that discussion is beyond the scope of this chapter.

Modeling Mental Models: Alternatives for Facilitating the Construction and Assessment of Mental Models

The premise of this chapter is that externalizing mental models improves the utility, coherence, and cogency of mental models as wel as providing external representations of those mental models. That is, building external models of internal mental models improves the mental models and the learner's understanding of those models and provides evidence about theor coherence and completeness.

The primary purpose of modeling, from our perspective, is the articulation of mental models. Building explicit models externalizes or reifies mental models. These models are separate fromfrom their referent mental models. Perhaps the most important characteristic is the evaluation of competing alternative models, that is, the comparison of two or more models for their relative fit to the world (Lehrer & Schauble, 2003). Which model better reflects the external world? Comparing and evaluating models require understanding that alternative models are possible and that the activity of modeling can be used for testing rival models.

Modeling is fundamental to human cognition and scientific inquiry (Frederiksen, White, & Gutwill, 1999). Modeling helps learners to express and externalize their thinking; visualize and test components of their theories; and make materials more interesting. Models function as epistemic resources (Morrison & Morgan, 1999). We must first understand what we can demonstrate in the model before we can ask questions about the real system.

If we agree that mental models are multi-faceted and multi-modal, then in order to externally represent mental models, we need to employ multiple representational formalisms. Jonassen (2006) describes a variety of computer-based Mindtools for representing domain knowledge, systems, problems, experiences, and thinking processes. Using computer-based tools, such as concept maps, databases, expert systems, spreadsheets, hypermedia, and teachable agents, to construct models fosters mental model development. That is, there are models in the mind (mental models), and there are external models that represent the models in the mind. The relationship between internal and external models is not well understood. We believe that there is a dynamic and reciprocal relationship between internal mental models and the external models that we construct. The mental models that we construct in the mind provide the material for building external models. The external models in turn regulate the internal models that we build, providing the means for conceptual change (Nersessian, 1999). In this paper, we argue that the construction of models using different computer-based modeling tools (Mindtools) enables learners to tune their internal models.

In the reminder of this chapter, we describe how different Mindtools can be used to construct models of different kinds of knowledge that represent some of the facets of mental models.

Representing Structural Knowledge

Many psychologists equate mental models with concept-map-like representations. Concept maps are representations of structural knowledge (Jonassen, Beissner & Yacci, 1993), knowledge of the semantic relationships among the schemas comprising the model. Structural knowledge is also known as a cognitive structure, the pattern of relationships among concepts in memory (Preece, 1976). For example, Pathfinder nets generated from relatedness data were created to depict mental models (Kraiger & Salas, 1993). Carley and Palmquist (1992) use their own software for constructing interlinked concept circles (maps) based upon text analysis or interviews.

Structural knowledge may be modeled with semantically sensitive software such as concept maps and databases. Figure 1 illustrates a structural model of the molar conversion process. Each concept (node) represents a concept, while each of the lines represents a semantic relationship between the concepts (a proposition). The larger these concept maps are, the more useful they are in supporting mental model construction. A student's concept map for any course, we believe, should probably include more than 2,000 nodes that are inter-connected.

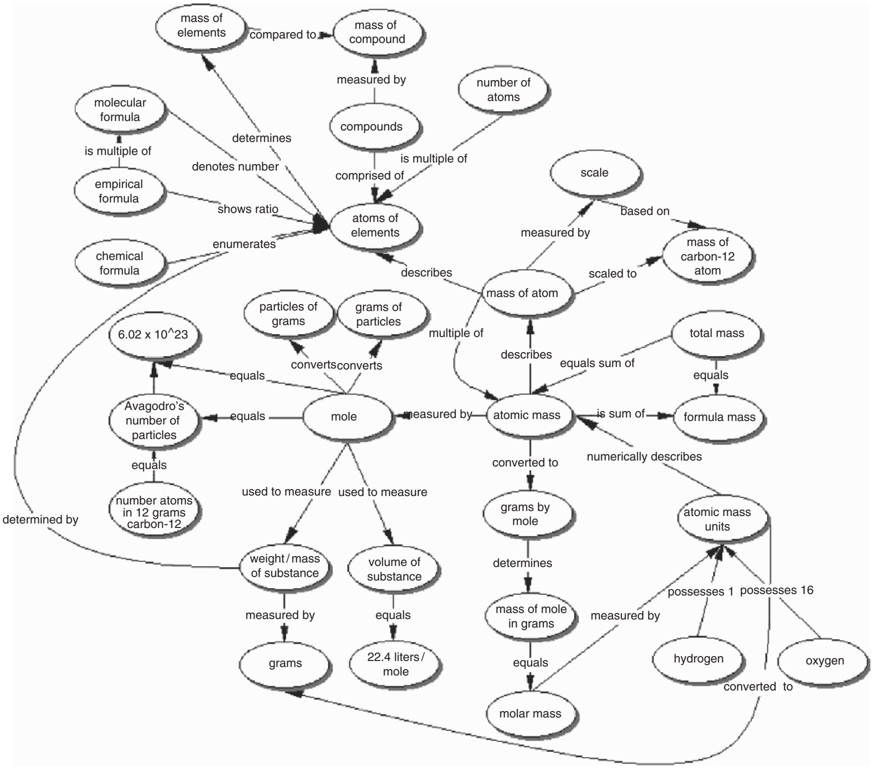

Fig. 1. Concept map representing structural knowledge of molar conversion process.

Databases support more integrated structural models of content. Each cell in a database represents a node. The links are represented by the relationships between records and field. Therefore, databases constrain the kinds of relationships that can be depicted in the model. That characteristic also provides for a more integrated model, with records more tightly yoked to each other. However, both tools enable learners to construct semantic models of the concepts that are integral to a domain or discipline. Too often, teachers attempt to achieve this effect by having students memorize definitions of domain concepts, a process that is much weaker.

Representing Visual Knowledge

Jonassen and Henning (1999) found that mental models also contain a spatial or pictorial representation. As mentioned before, Whitfield and Jackson (1982) and Rouse and Morris (1986) identified pictorial images as an important component of mental models. Images are perhaps the most important dimensional representation of mental models. Wittgenstein (1922) described propositions as imaginal models of reality. Most humans generate mental images of verbal representations. The statement, "The stone gained speed as it rolled down the steep slope" is meaningful only when an image of a mountain with a stone descending along its side is generated. Mental models definitely include mental images of the application of domain knowledge. So, it is important to elicit the learner's mental image of a prototype of the system s/he is constructing.

Generating visual presentations of mental images is very problematic. Mentel images are private and cannot readily be externalized. There are no tools for converting mental images into pixels. Rather, the normal method is to use some sort of graphics program to generate a mental image. That process requires both computer skills and drawing or painting skills. Most of us lack such skills, so our presentations will be inexact models of what we imagine. What humans need are visual prostheses for helping them to visualize ideas and to share those images with others.

Within scientific domains, there are computer-based tools for generating visualizations. In mathematics, tools such as MatLab and Mathematica readily convert mathematical formulas into dynamic visual representations, resulting in better understanding. For example, engineering mechanics students who used Mathematica solved problems requiring calculus more conceptually when compared to traditional students who focused only on the procedures (Roddick, 1995). Being able to interrelate numeric and symbolic representations with their graphical output helps learners to understand mathematics more conceptually.

A number of visualization tools have been developed for the sciences, most especially chemistry. For example, Figure 2 illustrates a molecule of androsterone. Not only does the McSpartan program enable the learners to visualize molecules using five different representations (wire, ball and wire, tube, ball and spoke, and space filling) but it also enables the student to test different bonds and create ions and new molecules. Understanding molecular chemistry is greatly facilitated by visualizing these complex processes. There has been a bit of research on these tools. For example, high school students used eChem to build molecular models and view multiple representations of molecules. Students using the visualization tool were able to generate better mental images of a substance that aided their understanding (Wu, Krajcik, & Soloway, 2001), confirming our belief that there is a reciprocal relationship between mental models and the external models learners construct to represent them.

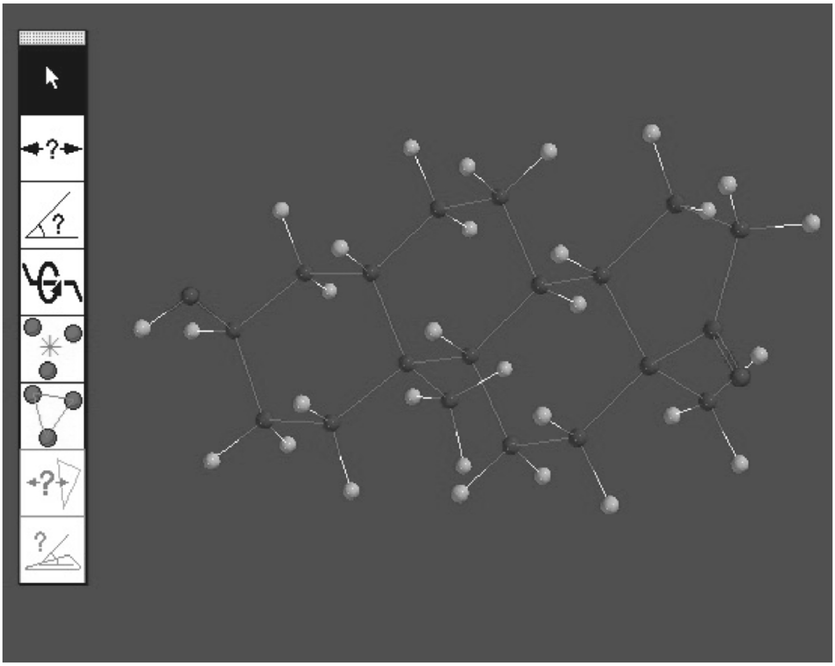

Fig. 2. Visualizing molecules.

Representing Procedural Knowledge

Jonassen and Henning (1999) also showed that the procedural knowledge of effective troubleshooters exceed that of the poorer troubleshooters. Procedural knowledge includes not only a description of the process but also a causal model describing and predicting the performance of the system. The best mental models are runnable, that is, they can be used to model and test how the system functions (Gott, Benett & Gillet, 1988). Assessing procedural knowledge is difficult. Most commonly, researchers ask performers to think aloud while performing a process.

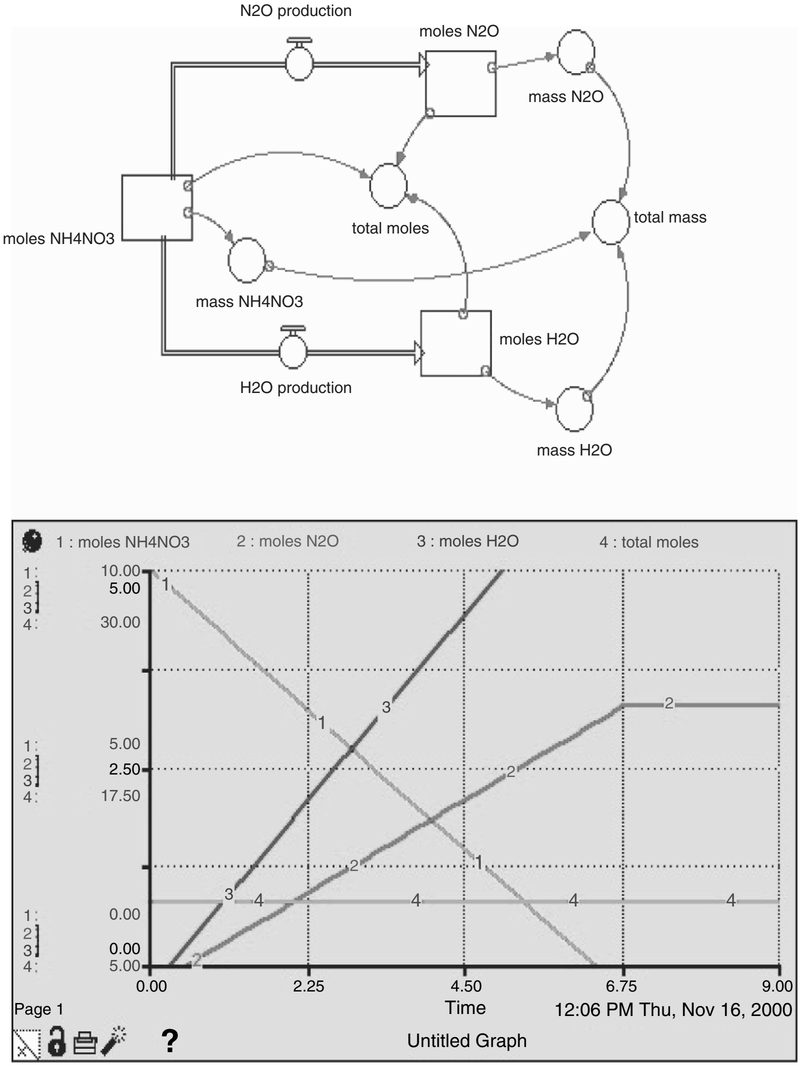

Fig. 3. Stella model of molar conversi on problem

Retrospective debriefing involves asking the performer for explanations of their actions after the performance. These data are difficult to analyze.

A powerful tool for building models of procedural knowledge is the expert system. An expert system is a computer program that attempts to simulate the way human experts solve problems—an artificial decision maker. For example, when you consult an expert (e.g., doctor, lawyer, teacher) about a problem, the expert asks for current information about your condition, searches his or her knowledge base (memory) for existing knowledge to which elements of the current situation can be related, processes the information (thinks), arrives at a decision, and presents a decision or solution. Like a human expert, an expert system is approached by an individual (novice) with a problem. The system queries the individual about the current status of the problem, searches its knowledge base (which contains previously stored expert knowledge) for pertinent facts and rules, processes the information, arrives at a decision, and reports the solution to the user. When used to model procedural knowledge, learners assume the role of an expert and construct a set of IF-THEN rules using an expert system editor. In those rules, they embed the causal and procedural relationships that experts use when making a diagnosis, for instance. The rule base can be tested by running the model and seeing whether the advice that is provided by the rule base, is viable. Building expert systems is technologically easy and intellectually compelling.

Another tool for building external models of procedural knowledge is the systems dynamics tool. The more common kind of tool, the aggregate modeling tool (such as Stella, PowerSim, VenSIm) uses a set of building blocks (stocks, flows, converters, and connectors) to construct a visual model of the components of a system. The systems model in Figure 3 applies the molar conversion process. Students then embed mathematic formulas in the connectors. Students test their models by running them and observing the graphic output in Figure 3. Building systems models is an engaging and powerful process for representing mental models. They probably provide the most complete externalization of mental models that is possible. They can be used to model social, psychological and other processes as well as scientific.

Episodic Knowledge

The strongest kind of memory is episodic. People often remember their experiences with accuracy decades after they occurred. The most common form of external representation of experience is the story. When faced with a problem, the most natural problem-solving process is to first try to recall a similar problem that you have experienced, what you did about it, and how effective that solution was.

Failing that, we tend to communicate with friends or colleagues, tell our problem, and ask if they have experienced a similar one. Frequently they have, and they are usually very willing to share with you a story of their experience along with the lessons learned.

Students can model their own or other people's experiences by collecting stories, indexing them, and entering them into a database to make them accessible. In order to collect stories, it is productive to tell a story to experienced folks about a problem you have. Then ask them if they are reminded of a similar experience. Usually they are. Having collected stories, we must decide what the stories teach us. We tell stories with some point in mind, so the indexing process tries to elucidate what that point is, given a situation. Schank (1990) believes that indexes should include the experience and the themes, goals, plans, results, and lessons from the story. Themes are the subjects that people talk about. Goals motivated the experience. Plans are personal approaches to accomplishing those goals. Results describe the outcome of the experience. The lesson is the moral of the story — the principle that we should take away from the case. Indexing is an engaging analytical process, the primary goal of which is to make the stories accessible.While indexing, we must continually ask ourselves under what situations we would be reminded of this story.

Indexed stories are then entered into a database. The indexes that we construct to describe the stories become the fields of the database. Each field describes the elements of the story on which we may want to retrieve a story. So indexes (fields) may include context, actor, learned lesson, result, or similarity. This process is more fully described by Jonassen and Hernandez-Serrano (2003). It is the process of semantically organizing the story for inclusion in the database that requires the model and conceptual understanding.

Conclusion

The concept of mental models is a powerful construct for describing the meaning that learners make. After describing the various methods that have been used to manifest or assess mental models, we argued that mental models are multidimensional, so no single form of assessment can be used effectively to describe mental models. In order to manifest mental models, learners need to use computer-based modeling tools to externalize their mental models in the form of computer-based models. Because mental models are multi-dimensional, no single modeling tool can manifest the complexity of mental models. So, we suggest that learners use Mindtools to construct models of structural knowledge, procedural knowledge, visual knowledge, and experiential knowledge. Research is needed to identify the most effective combination of modeling tools for representing the underlying complexity of mental models.

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