FUZZY OUTRANKING METHODS: RECENT DEVELOPMENTS

Ahmed Bufardi, Razvan Gheorghe, and Paul Xirouchakis
Institute of Production and Robotics, Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland

Abstract: The main objective of this chapter is to account for the most recent developments related to fuzzy outranking methods with a particular focus on the fuzzy outranking method developed by the authors. The valued outranking methods PROMETHEE and ELECTRE III, which are the outranking methods the most used for application in real-life multi-criteria decision aid problems, are also presented. The description of the general outranking approach is provided.

Key words: Outranking method, fuzzy outranking relation, pair-wise comparison, multicriteria decision aid

1. INTRODUCTION

Outranking methods form one of the main families of methods in multi-criteria decision aid (MCDA). Other important methods are multi-attribute utility theory (MAUT) methods, interactive methods, and the analytic hierarchy process (AHP).

It is worth recalling that the first outranking method called ELECTRE I was developed by Bernard Roy and published in 1968. Since then, a series of outranking methods were developed mainly during the 1970s and 1980s. Among them we can quote ELECTRE II (Roy and Bertier, 1973), ELECTRE III (Roy, 1978), QUALIFLEX (Paelinck, 1978), ORESTE (Roubens, 1982; Pastijn and Leysen, 1989), ELECTRE IV (Roy and Hugonnard, 1982), MELCHIOR (Leclercq, 1984), PROMETHEE I and II

C. Kahraman (ed.), Fuzzy Multi-Criteria Decision Making.

© Springer Science + Business Media, LLC 2008

(Brans and Vincke, 1985), TACTIC (Vansnick, 1986), MAPPACC (Matarazzo, 1986), and PRAGMA (Matarazzo, 1986).

The outranking methods are based on the construction and the exploitation of an outranking relation. The underlying idea consists of accepting a result less rich than the one yielded by multi-attribute utility theory by avoiding the introduction of mathematical hypotheses that are too strong and asking the decision maker some questions that are too intricate (Vincke, 1992a). The concept of an outranking relation is introduced by Bernard Roy who is the founder of outranking methods. According to Roy (1974), an outranking relation is a binary relation S defined on the set of alternatives $\boldsymbol{A}$ such that for any pair of alternatives $(a,b) \in \boldsymbol{A}\times\boldsymbol{A}$: aSb if, given what is known about the preferences of the decision maker, the quality of the evaluations of the alternatives and the nature of the problem under consideration, there are sufficient arguments to state that the alternative a is at least as good as the alternative b, while at the same time no strong reason exists to refuse this statement.

In contrast to the other methods, the outranking methods have the characteristic of allowing incomparability between alternatives. This characteristic is important in situations where some alternatives cannot be compared for one or another reason. According to Siskos (1982), incomparability between two alternatives can occur because of a lack of information, inability of the decision maker to compare the two alternatives, or his refusal to compare them (Siskos, 1982).

In contrast to the valued outranking methods that are well documented in the literature and have been intensively used in practice since 1978 with the publication of ELECTRE III, the fuzzy outranking methods are very recent and are not well documented in the literature, and this is one of the motivations for the redaction of this chapter.

The chapter is structured as follows. The main elements of a general outranking approach are described in Section 2. Section 3 is devoted to the presentation of the PROMETHEE and ELECTRE III, which are the main valued outranking methods considered in both theory and applications. The fuzzy outranking methods are presented in Section 4. Some concluding remarks are given in Section 5.

2. THE OUTRANKING APPROACH

An outranking method is applicable for MCDA problems where the elements of a finite set of alternatives $\boldsymbol{A} = \{a_1, a_2, \ldots, a_n\}$ have to be compared on the basis of the preferences of the decision maker regarding their performances with respect to the elements of a finite set of criteria $F = \{g_1, g_2, \ldots, g_m\}$. It is assumed that each alternative a_i, $i = 1, \ldots, n$ can be evaluated with respect to each criterion g_j, $j = 1, \ldots, m$. The evaluations can be quantitative or qualitative. They can also be deterministic or nondeterministic. In the nondeterministic case, they can be fuzzy or stochastic.

The objective of outranking methods is provide decision aid to decision makers in the form of a subset of "best" alternatives or a partial or complete ranking of alternatives (Pasche, 1991).

According to Roy (1991), the preferences in the outranking concept are determined at two different levels as follows:

- Level of preferences restricted to each criterion. For example, to each criterion g_j, it is possible to associate a restricted outranking relation S_j such that for any two alternatives a and b in $\boldsymbol{A}$:

$$aS_j b \Leftrightarrow a, \text{with respect to } g_j, \text{is at least as good as } b \tag{1}$$

- Level of comprehensive preferences where all criteria are taken into account.

The meaning of an outranking relation is given in Section 1. However, there is a need for a set of conditions to recognize whether a given binary relation can be an outranking relation. The following definition is provided in (Perny and Roy, 1992)

DEFINITION 1.

A fuzzy relation S_j defined on A^2 is said to be a *monocriterion outranking index* for a criterion g_j if a real-valued function t_j, exists defined on A^2, verifying $S_j(a, b) = t_j(a_j, b_j)$ for all a and b in A with a_j and b_j being the crisp scores of a and b on criterion g_j such that:

- $\forall y_0 \in \Re, t_j(x, y_0)$ is a nondecreasing function of x,
- $\forall x_0 \in \Re, t_j(x_0, y)$ is a nonincreasing function of y,
- $\forall z \in \Re, t_j(z, z) = 1$.

It is worth noticing that the three conditions in this definition are also valid for fuzzy outranking relations constructed from fuzzy evaluations on criteria and for global outranking relations.

In the literature, confusion abounds regarding valued and fuzzy outranking relations, and they are often used interchangeably. Even if the valued and fuzzy outranking relations are similar from a mathematical point of view, they represent two different situations:

- The valued outranking relation represents a crisp situation, and the value $S(a, b) \in [0, 1]$ represents the intensity with which the alternative a outranks alternative b and $S(a, b)$ is constructed from crisp evaluations of alternatives a and b.
- The valued outranking relation represents a fuzzy situation, and the value $S(a,b) \in [0, 1]$ represents the degree with which the alternative a is R-related to b and $S(a,b)$ is constructed from fuzzy evaluations of alternatives a and b.

An outranking method is composed of two main phases that are the construction of a global outranking relation and the exploitation of this relation.

The construction phase is composed of two main steps:

- Construction of an outranking relation or related relations such as concordance and discordance indices with respect to each criterion,
- The aggregation of the single outranking relations into a global outranking relation.

The exploitation phase of a valued/fuzzy outranking method can be dealt with in three different ways (Fodor and Roubens, 1994):

- Transformation of the valued/fuzzy outranking relation into another valued/fuzzy relation having particular properties such as transitivity that are interesting for the ranking of alternatives,
- Determination of a crisp relation closed to the valued/fuzzy outranking relation and having specific properties,
- Use of a ranking procedure to obtain a score function as it is the case for PROMETHEE and ELECTRE III methods.

A detailed study of the exploitation phase in the case of crisp relations is provided in (Vincke, 1992b).

3. VALUED OUTRANKING METHODS

The outranking methods that are the most used for application in real-life MCDA problems are ELECTRE III and PROMETHEE, which are valued outranking methods since they are based on the construction and exploitation of a valued "outranking relation." ELECTRE stands for "ELimination Et Choix Traduisant la REalité," and PROMETHEE stands for "Preference Ranking Organization METHod for Enrichment Evaluations."

3.1 ELECTRE III

ELECTRE III is an outranking method proposed by Roy (1978) to deal with multi-criteria decision-making situations in which a finite set of alternatives should be ranked from the best to the worst. It is composed of the following steps:

- The construction of a valued outranking relation;
- The construction of two complete preorders based on descending and ascending distillation chains;
- The comparison of the two complete preorders in order to elaborate a final ranking of the alternatives. This comparison leads to a partial preorder in which it is possible that some alternatives are incomparable.

3.1.1 The Construction Phase of ELECTRE III

Let $\boldsymbol{A} = \{a_1, a_2, \ldots, a_n\}$ be a finite set of n alternatives and $F = \{g_1, g_2, \ldots, g_m\}$ a set of m criteria on which the alternatives in $\boldsymbol{A}$ will be evaluated. Without loss of generality, the criteria can be assumed to be maximizing, i.e., the higher the performance of an alternative on a criterion is, the better the alternative is. ELECTRE III is based on the definition of a valued outranking relation S such that for each ordered pair of alternatives (a,b), $S(a,b) \in [0, 1]$ represents the degree to which alternative a is at least as good as alternative b (the degree to which alternative a is not worse than alternative b).

3.1.1.1 Single Criterion Relations

With each criterion g_j ($j = 1, \ldots, m$) are associated four parameters: a weight w_j, a preference threshold p_j, an indifference threshold q_j, and a veto threshold v_j. It is naturally assumed that for each alternative a: $q_j(g_j(a)) \leq p_j(g_j(a)) \leq v_j(g_j(a))$.

With each criterion g_j ($j = 1, \ldots, m$) are associated a concordance index c_j and a discordance index d_j as follows which are shown in Figures 1 and 2 respectively.

$$c_j(a,b) = \begin{cases} 1 \text{ if } g_j(a) + q_j(g_j(a)) \geq g_j(b), \\ 0 \text{ if } g_j(a) + p_j(g_j(a)) \leq g_j(b), \\ \dfrac{p_j(g_j(a)) + g_j(a) - g_j(b)}{p_j(g_j(a)) - q_j(g_j(a))} \text{ otherwise} * \end{cases} \tag{2}$$

* may occur only in the case when $q_j(g_j(a)) \neq p_j(g_j(a))$.

$$d_j(a,b) = \begin{cases} 0 \;\; \textit{if} \;\; g_j(b) \leq g_j(a) + p_j(g_j(a)), \\ 1 \;\; \textit{if} \;\; g_j(b) \geq g_j(a) + v_j(g_j(a)), \\ \dfrac{g_j(b) - g_j(a) - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))} \text{ otherwise*} \end{cases} \tag{3}$$

* may occur only in the case when $p_j(g_j(a)) \neq v_j(g_j(a))$.

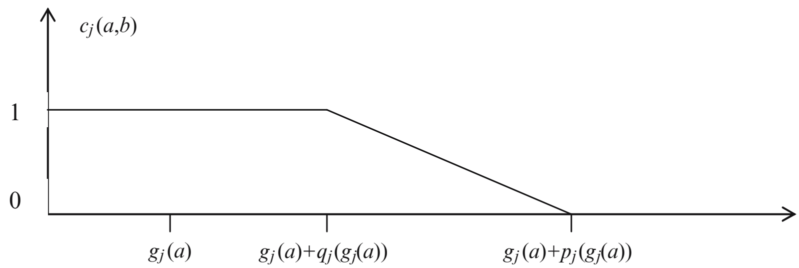


Figure 1. Concordance index of g_j

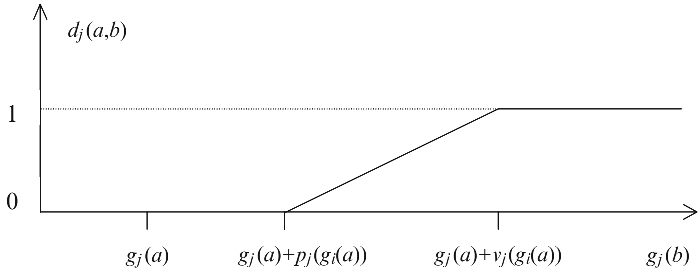


Figure 2. Discordance index of g_j

3.1.1.2 Global Valued Outranking Relation

For each ordered pair of alternatives (a,b), a concordance index $c(a,b)$ is computed in the following way:

$$c(a,b) = \frac{1}{W}\sum_{j=1}^{m} w_j c_j(a,b), \text{ where } W = \sum_{j=1}^{m} w_j \tag{4}$$

It is worth noticing that $c(a,b) = 1$ means that there is no criterion for which alternative b is better than alternative a and $c(a,b) = 0$ means that alternative a is worse than alternative b for all criteria.

The valued outranking relation S is constructed from the concordance and discordance indices. For each ordered pair of alternatives $(a,b) \in \boldsymbol{A}\times\boldsymbol{A}$, $S(a,b)$ is defined in the following way:

$$S(a,b) = \begin{cases} c(a,b) \text{ if } d_j(a,b) \le c(a,b),\ \forall j = 1,\ ...,\ m \\ c(a,b) \times \prod\limits_{j \in J(a,b)} \dfrac{1 - d_j(a,b)}{1 - c(a,b)} \text{ otherwise} \end{cases} \tag{5}$$

where $J = \{j \in \{1, \ldots, m\} / d_j(a, b) > c(a, b)\}$.

The degree of outranking is equal to the concordance index when no criterion is discordant. When at least one criterion is discordant, the degree of outranking is equal to the concordance index multiplied by a factor lowering the concordance index in function of the importance of the discordances. At the extreme, when $d_j(a,b) = 1$ for some criterion g_j, $S(a,b) = 0$. Thus, for each ordered pair of alternatives $(a,b) \in \boldsymbol{A} \times \boldsymbol{A}$, $0 \le S(a,b) \le 1$. S is a valued outranking relation.

3.1.1.3 The Exploitation Phase of ELECTRE III

The second step in ELECTRE III consists in defining two complete preorders from the descending and the ascending distillation chains.

Let $\lambda_0 = \max_{a,b \in A} S(a,b)$. At each iteration of the descending or ascending distillation chain, a discrimination threshold $s(\lambda)$ and a crisp relation D are defined such that:

$$D(a,b) = \begin{cases} 1 \text{ if } S(a,b) > \lambda - S(\lambda) \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

For each alternative a, a qualification score $Q(a)$ is computed as the number of alternatives that are outranked by a (number of alternatives b such that $D(a,b) = 1$) minus the number of alternatives, which outrank a (number of alternatives b such that $D(b,a) = 1$).

ELECTRE III provides the decision makers with two complete preorders. The first preorder is obtained in a descending manner starting with the selection of the alternatives with the best qualification score and finishing with the selection of the alternatives having the worst qualification score. The second preorder is obtained in an ascending manner, first selecting the alternatives with the worst qualification score and finishing with the assignment of the alternatives that have the worst qualification score.

3.1.1.4 Descending Distillation Chain

In the descending procedure, the set of alternatives having the largest qualification score constitutes the first distillate and is denoted as D_1. If D_1 contains only one alternative, the previous procedure is performed in the set $A \backslash D_1$. Otherwise it is applied to D_1 and a distillate D_2 will be obtained. If D_2 is a singleton, then the procedure is applied in $D_1 \backslash D_2$ if it is not empty; otherwise the procedure is applied in D_2. This procedure is repeated until the distillate D_1 is completely explored. Then, the procedure starts exploring $A \backslash D_1$ in order to find a new distillate. The procedure is repeated until a complete preorder of the alternatives is obtained. This procedure is called the descending distillation chain because it starts with the alternatives having the highest qualification and ends with the alternatives having the lowest qualification.

The result of the descending procedure is a set of classes $\overline{C}_1, \overline{C}_2, \ldots, \overline{C}_k$ with $k \leq n$. The alternatives belonging to the same class are considered to be ex-æquo (indifferent), and an alternative belonging to a class outranks all the alternatives belonging to classes with higher indices. Thus, a first complete preorder of the alternatives is obtained.

3.1.1.5 Ascending Distillation Chain

The ascending procedure is the same as the descending procedure except that the criterion of selecting the alternatives is based on the principle of the lowest qualification. The result of this procedure is a set of classes $\underline{C}_1$, $\underline{C}_2$, …, $\underline{C}_h$ with $h \le n$. These classes are written in such a way that two alternatives in the same class are considered to be ex-æquo and an alternative belonging to a class outranks all the alternatives belonging to classes with lower indices. Thus, a second complete preorder of the alternatives is obtained.

3.1.1.6 Partial Preorder of ELECTRE III

The result of ELECTRE III is a partial preorder of the alternatives based on the comparison of the two complete preorders obtained by means of the descending and the ascending distillation chains.

3.1.2 Main Features of ELECTRE III

ELECTRE III has many interesting features among which we can quote:

- Handling imprecise and uncertain information about the evaluation of alternatives on criteria by using indifference and preference thresholds,
- Consideration of incomparability between alternatives; when two alternatives cannot be compared in terms of preference or indifference, they are considered to be incomparable. Indeed, sometimes the information available is insufficient to decide whether two alternatives are indifferent or one is preferred to the other,
- Use of veto thresholds. This is very important for some problems such as those involving environmental and social impacts assessment. According to Rogers and Bruen (1998), within an environmental assessment, it seems appropriate to define a veto as the point at which human reaction to the criterion difference becomes so adverse that it places an “environmental stop” on the option in question. The same can be said about social impact assessment.

ELECTRE III is widely used for different real-world applications such as environmental impact assessment and selection problems in various domains. Examples of these applications can be found in Augusto et al. (2005), Beccali et al. (1998), Bufardi et al. (2004), Cote and Waaub (2000), Hokkanen and Salminen (1994, 1997), Kangas et al. (2001), Karagiannidis and Moussiopoulos (1997), Maystre et al. (1994), Rogers and Bruen (2000), Roy et al. (1986), Teng and Tzeng (1994), and Tzeng and Tsaur (1997). The list is not exhaustive and is given just for illustrative purposes to show the varied and numerous applications of the ELECTRE III method.

3.1.3 Illustrative Example

This illustrative example is taken from Bufardi et al. (2004). The problem considered consists of selecting the best compromise end-of-life (EOL) alternative to treat a vacuum cleaner at its EOL. Theoretically the number of potential EOL alternatives that can be considered is very high. In general only a few EOL alternatives are interesting. Users have their own ways for defining EOL alternatives depending on activity, objectives, experience and constraints from market, legislation, and available technology. In this illustrative example, five EOL alternatives are considered and described as follows. EOL alternative 1 consists of recycling as much as possible and incinerating the rest. EOL alternative 2 consists of recycling only parts with benefits and incinerating the rest. EOL alternative 3 consists of recycling all metals that cannot be incinerated and incinerating all the rest. EOL alternative 4 consists of reusing the motor, recycling metals, and incinerating the rest. EOL alternative 5 consists of landfilling all. The five EOL alternatives are presented in Table 1. The criteria used for the evaluation of EOL alternatives are presented in Table 2. The detailed description of the environmental criteria presented in Table 2 can be found in Goedkoop and Spriensma (2000). Once the EOL alternatives

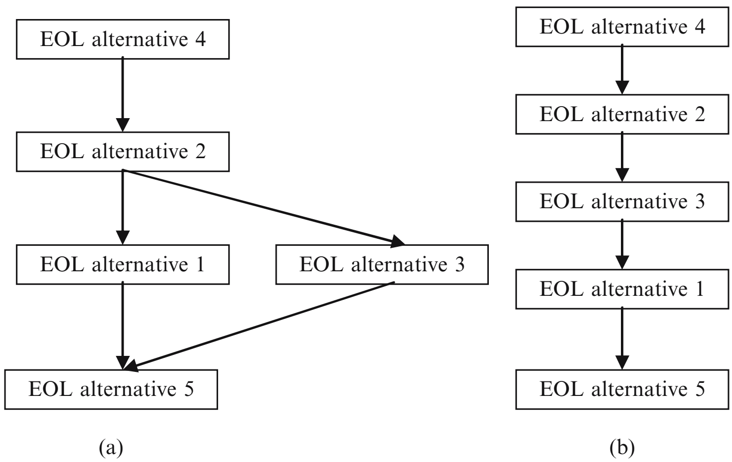


Figure 3. Partial and median preorder

and criteria are selected, each EOL alternative is evaluated with respect to each criterion as shown in Table 3. The results of applying ELECTRE III can be presented in the form of a partial preorder as shown in Figure 3a or a median preorder as shown in Figure 3(b).

Table 1. The EOL Alternatives

No.	Component/subassembly	EOL alternatives				
		1	2	3	4	5
1	Dust bin	*REC*	*INC*	*INC*	*INC*	*LND*
2	2 x Inner Cover	*REC*	*INC*	*INC*	*INC*	*LND*
3	Inner filter asb	*INC*	*INC*	*INC*	*INC*	*LND*
4	Dust bin cover	*INC*	*INC*	*INC*	*INC*	*LND*
5	Lock ring	*REC*	*INC*	*INC*	*INC*	*LND*
6	Spring	*REC*	*INC*	*REC*	*REC*	*LND*
7	Power button cover (+ button)	*REC*	*INC*	*INC*	*INC*	*LND*
8	Spring	*REC*	*INC*	*REC*	*REC*	*LND*
9	Upper VC case	*REC*	*INC*	*INC*	*INC*	*LND*
10	Suction tube	*REC*	*INC*	*INC*	*INC*	*LND*
11	Suction tube sealing	*INC*	*INC*	*INC*	*INC*	*LND*
12	Intermediate tube	*REC*	*INC*	*INC*	*INC*	*LND*
13	Cables	*REC*	*REC*	*INC*	*INC*	*LND*
14	Valve	*INC*	*INC*	*INC*	*INC*	*LND*
15	Intern sealing 1	*INC*	*INC*	*INC*	*INC*	*LND*
16	Intern sealing 2	*INC*	*INC*	*INC*	*INC*	*LND*
17	Spring	*REC*	*INC*	*REC*	*REC*	*LND*
18	Middle	*REC*	*INC*	*INC*	*INC*	*LND*
19	Hepa cover	*REC*	*INC*	*INC*	*INC*	*LND*
20	Hepa filter	*INC*	*INC*	*INC*	*INC*	*LND*
21	Cable coil cover	*REC*	*INC*	*INC*	*INC*	*LND*
22	Cable coil	*INC*	*INC*	*INC*	*INC*	*LND*
23	Cable	*REC*	*REC*	*INC*	*INC*	*LND*
24	Motor Lock ring	*REC*	*INC*	*INC*	*INC*	*LND*
25	Motor bottom seal	*INC*	*INC*	*INC*	*INC*	*LND*
26	Motor sealing	*INC*	*INC*	*INC*	*INC*	*LND*
27	Motor foam	*INC*	*INC*	*INC*	*INC*	*LND*
28	Motor	*REC*	*REC*	*REC*	*REM*	*LND*
29	Motor housing half 2	*REC*	*INC*	*INC*	*INC*	*LND*
30	Motor housing Filter	*INC*	*INC*	*INC*	*INC*	*LND*
31	Motor housing half 1	*REC*	*INC*	*INC*	*INC*	*LND*
32	32 Motor housing seal	*INC*	*INC*	*INC*	*INC*	*LND*
33	Wheels	*INC*	*INC*	*INC*	*INC*	*LND*
34	Lower VC case	*REC*	*INC*	*INC*	*INC*	*LND*
35	Spring	*REC*	*INC*	*REC*	*REC*	*LND*

* remanufacturing/reuse (REM), recycling (REC), incineration with energy recovery (INC), disposal to landfill (LND)

Table 2. List of Criteria

Category	Criterion	Unit	Direction of preferences
Economic	EOL Treatment Cost (C)	[CHF]	Minimization
	Human Health (HH)	[DALY]	Minimization
Environmental	Ecosystem Quality (EQ)	[PDF*m2yr]	Minimization
	Resources (R)	[MJ surplus]	Minimization

Table 3. Evaluation of EOL Alternatives

	Human health (HH) [DALY]	Ecosystem quality (EQ) [PDF*m2yr]	Resources (R) [MJ surplus]	EOL treatment cost (C) [CHF]
EOL alternative 1	−1.08E-05	−0.471	−18.1	0.644125
EOL alternative 2	−0.951E-05	−0.962	−7.49	−0.10601
EOL alternative 3	−0.724E-05	−0.896	−6.76	0.01108
EOL alternative 4	−2.90E-05	−2.02	−36.8	−4.86022
EOL alternative 5	0.0271E-05	0.0103	0.0101	0.38101

3.2 PROMETHEE

PROMETHEE is a MCDA method based on the construction and the exploitation of a valued outranking relation π (Brans and Vincke, 1985). Two complete preorders can be obtained by ranking the alternatives according to their incoming flow and their outgoing flow. The intersection of these two preorders yields the partial preorder of PROMETHEE I where incomparabilities are allowed. The ranking of the alternatives according to their net flow yields the complete preorder of PROMETHEE II.

3.2.1 The Construction Phase of PROMETHEE

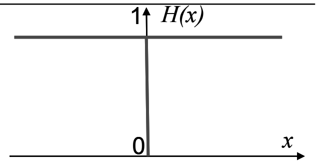
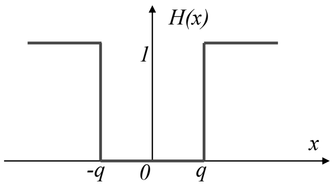
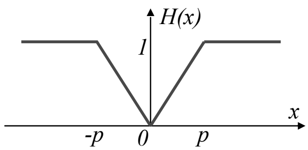
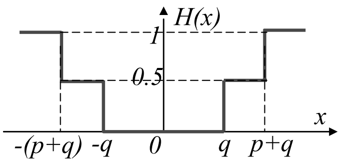
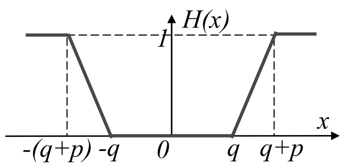
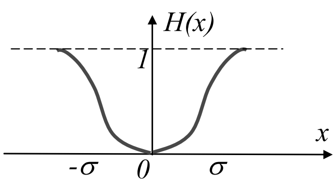
Let $\boldsymbol{A} = \{a_1, a_2, \ldots, a_n\}$ be a finite set of alternatives and $F = \{g_1, g_2, \ldots, g_m\}$ a finite set of criteria on which the alternatives will be evaluated. With each criterion g_j, $j = 1, 2, \ldots, m$, is assigned a weight p_j reflecting its relative importance.

For each pair of alternatives $(a,b) \in \boldsymbol{A} \times \boldsymbol{A}$, an outranking degree $\prod(a,b)$ is computed in the following way:

$$\Pi(a,b) = \frac{1}{P}\sum_{j=1}^{m} p_j H_j(a,b) \tag{7}$$

$P = \sum_{j=1}^{m} p_j$ and $H_j(a,b)$ are numbers between 0 and 1 that are a function of $g_j(a) - g_j(b)$. For the computation of $H_j(a,b)$'s, the decision maker is given six forms of curves described in Table 1. It is worth noticing that in Table 4, the six functions are described for a maximizing criterion where $H(x) = P(a,b)$ if $x \geq 0$ and $H(x) = P(b,a)$ if $x \leq 0$.

Table 4. List of Generalized Criteria

Type of criterion	Analytical definition	Shape
1. Usual	$H(x)=\begin{cases}0 & \text{if } x=0\\ 1 & \text{if } \lvert x\rvert>0\end{cases}$	
2. Quasi	$H(x)=\begin{cases}0 & \text{if } \lvert x\rvert\leq q\\ 1 & \textit{otherwise}\end{cases}$	
3. Linear preference	$H(x)=\begin{cases}\lvert x\rvert/p & \text{if } \lvert x\rvert\leq p\\ 1 & \textit{otherwise}\end{cases}$	
4. Level	$H(x)=\begin{cases}0 & \lvert x\rvert\leq q\\ 0.5 & q<\lvert x\rvert\leq p+q\\ 1 & \lvert x\rvert\geq q+p\end{cases}$	
5. Linear preference and indifference area	$H(x)=\begin{cases}0 & \lvert x\rvert\leq q\\ (\lvert x\rvert-q)/p & q<\lvert x\rvert\leq q+p\\ 1 & \textit{otherwise}\end{cases}$	
6. Gaussian	$H(x)=\begin{cases}0 & x=0\\ 1-e^{-x^2/2\sigma^2} & \lvert x\rvert>0\end{cases}$	

3.2.2 The Exploitation Phase of PROMETHEE

With each alternative are associated two values $\phi^+(a)$ and $\phi^-(a)$.

- $\phi^+(a)$, which is called the *outgoing flow* and is computed in the following way:

$$\phi^+(a) = \sum_{b\in A} \Pi(a,b) \tag{8}$$

- $\phi^-(a)$, which is called the *incoming flow* and is computed in the following way:

$$\phi^-(a) = \sum_{b\in A} \Pi(b,a) \tag{9}$$

It is worth noticing that $\phi^+(a)$ represents the degree by which alternative a outranks the other alternatives and that $\phi^-(a)$ represents the degree by which alternative a is outranked by the other alternatives.

The higher the outgoing flow and the lower the incoming flow, the better the alternative. The two flows induce the following complete preorders (ranking of the alternatives with consideration of indifference) on the alternatives, where P and I are the preference relation and indifference relation, respectively:

- $aP^+b \Leftrightarrow \phi^+(a) > \phi^+(b)$
- $aI^+b \Leftrightarrow \phi^+(a) = \phi^+(b)$
- $aP^-b \Leftrightarrow \phi^-(a) < \phi^-(b)$
- $aI^-b \Leftrightarrow \phi^-(a) = \phi^-(b)$

where P^+, I^+ refer to the outgoing flows while P^-, I^- refer to the incoming flows.

By ranking the alternatives in the decreasing order of the numbers $\phi^+(a)$ and in the increasing order of the numbers $\phi^-(a)$, two complete preorders can be obtained. Their intersection yields the partial order of PROMETHEE I as follows:

$$aSb\,(a \text{ strictly outranks } b) \begin{cases} \text{if } aP^+b \text{ and } aP^-b \\ \text{or } aP^+b \text{ and } aI^-b \\ \text{or } aI^+b \text{ and } aP^-b \end{cases} \tag{10}$$

$$aIb \text{ (} a \text{ is indifferent to } b\text{) if } aI^{+}b \text{ and } aI^{-}b \quad (11)$$

$$aJb \text{ (} a \text{ and } b \text{ are incomparable) otherwise} \quad (12)$$

i.e., $\neg aSb, \neg bSa$ and $\neg aIb$, where "¬" denotes negation.

For each alternative *a*, a *net flow* $\phi(a)$ can be obtained by subtracting the incoming flow $\phi^{-}(a)$ from the outgoing flow $\phi^{+}(a)$; i.e., $\phi(a) = \phi^{+}(a) - \phi^{-}(a)$. By ranking the alternatives in the decreasing order of ϕ, one obtains the unique complete preorder of PROMETHEE II.

3.2.3 Main Features of PROMETHEE

PROMETHEE has many interesting features among which we can quote:

- It is easy to understand. The mathematical background behind PROMETHEE is not complicated and is easy to understand by the users. This is important for the transparency of the results,
- It is easy to use. For each criterion, the decision maker has to fix the weight of this criterion, and at most two parameters of the function are associated with the criterion in order to derive the single-valued outranking relation related to this criterion,
- Consideration of incomparability between alternatives through PROMETHEE I; when two alternatives cannot be compared in terms of preference or indifference, they are considered to be incomparable. Indeed, sometimes the information available is insufficient to decide whether two alternatives are indifferent or one is preferred to the other. PROMETHE is an outranking method easy to understand and to use.

That is why it is widely used for practical MCDA problems in various domains; see, e.g., Al-Rashdan et al. (1999), Anagnostopoulos et al. (2003), Babic and Plazibat (1998), Elevli and Demirci (2004), Geldermann et al. (2000), Gilliams et al. (2005), Goumas and Lygerou (2000), Hababou and Martel (1998), Kalogeras et al. (2005), Le Téno and Mareschal (1998), Mavrotas et al. (2006), and Petras (1997). The list is not exhaustive and is given just for illustrative purposes.

4. FUZZY OUTRANKING METHODS

In these methods, it is assumed that the evaluations of alternatives on criteria are fuzzy.

4.1 Fuzzy Outranking Method of Gheorghe et al.

The fuzzy outranking method presented in this subsection is published in Gheorghe et al. (2004, 2005). Full details can be found in Gheorghe (2005).

4.1.1 Construction of Monocriterion Fuzzy Outranking Relation

The construction of the monocriterion fuzzy outranking relation starts by analyzing the intervals, in our case, the α-cuts of fuzzy performance of two alternatives *a* and *b*.

Let us consider two normalized and convex fuzzy numbers *A* and *B*, representing the performances of alternatives *a* and *b*, respectively (Figure 4). Let μ_A and μ_B be the membership functions of *A* and *B,* respectively. Each α_i-cut is defined by the interval $(a_1^{\alpha_i}, a_2^{\alpha_i})$ for *A* and $(b_1^{\alpha_i}, b_2^{\alpha_i})$ for *B,* respectively, where $i = 1, \ldots, N,$ with *N* denoting the number of α-cuts considered.

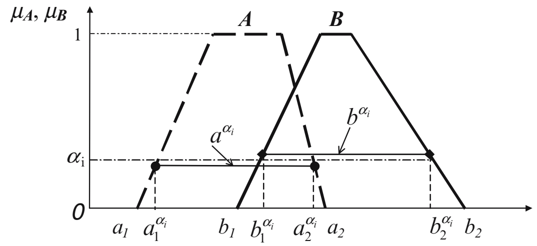


Figure 4. Fuzzy performances of alternatives *a* and *b*

The comparison performances of the alternatives *a* and *b* at the α_i-cut level using the mechanisms shown in Figures 5 and 6 are in accordance with common sense and represent two different view points. When the interval a^{α_i} is entirely on the left of the interval b^{α_i}, there is no doubt that *a* is worse than *b* and that the degree of trueness of the proposition "*a* is not worse than *b*" is 0. When starting to translate a^{α_i} to the right and the two intervals overlap, this degree of trueness increases and reaches the maximum value 1 at the moment when the lower limit (*left*) of a^{α_i} is equal with the lower limit (*left*) of b^{α_i} (Figure 5).

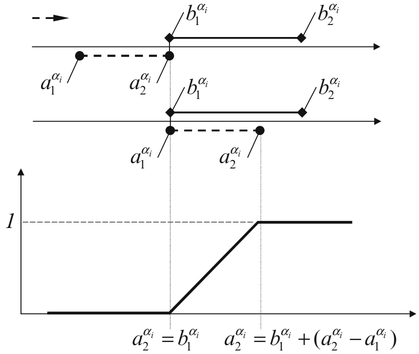


Figure 5. The first case of the achievement of a degree of trueness of 1 of the proposition "*a* is not worse than *b*"

A similar judgment can be performed for the case when the maximum degree of trueness is attained at the moment when the upper limit (*right*) of a^{α_i} is equal with the upper limit (*right*) of b^{α_i} (Figure 6).

Thus the reasoning we have done previously is suitable for the case when a higher value of performance is preferred to a lower value, in other words, for the case when we want to maximize the performance value with respect to a criterion. Similar reasoning can be followed for the case of a minimizing criterion.

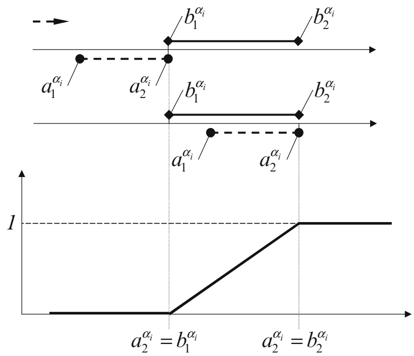


Figure 6. The second case of the achievement of a degree of trueness of 1 of the proposition "*a* is not worse than *b*"

DEFINITION 2.

For each α_i-cut level, two *left* α_i*-cut indices* are defined for, respectively, the case of maximizing and minimizing criteria as the functions $s_{l\_\max}^{\alpha_i}$ and, $s_{l\_\min}^{\alpha_i}$ from $I_{\Re} \times I_{\Re}$ to $[0,1]$, where $I_{\Re}$ is the set of all real intervals:

$$s_{l\_\max}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) = \begin{cases} 0, & a_2^{\alpha_i} < b_1^{\alpha_i} \\ \dfrac{a_2^{\alpha_i} - b_1^{\alpha_i}}{a_2^{\alpha_i} - a_1^{\alpha_i}}, & a_1^{\alpha_i} < b_1^{\alpha_i} \le a_2^{\alpha_i} \\ 1, & a_1^{\alpha_i} \ge b_1^{\alpha_i} \end{cases} \tag{13}$$

$$s_{l\_\min}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) = \begin{cases} 0, & b_2^{\alpha_i} < a_1^{\alpha_i} \\ \dfrac{b_2^{\alpha_i} - a_1^{\alpha_i}}{b_2^{\alpha_i} - b_1^{\alpha_i}}, & b_1^{\alpha_i} < a_1^{\alpha_i} \le b_2^{\alpha_i} \\ 1, & b_1^{\alpha_i} \ge a_1^{\alpha_i} \end{cases} \tag{14}$$

The *right* α_i*-cut indices* can be defined in a similar way as shown in the following definition.

DEFINITION 3.

For each α_i-cut level, two *right* α_i*-cut indices* are defined for, respectively, the case of maximizing and minimizing criteria as the functions $s_{r\_\max}^{\alpha_i}$ and $s_{r\_\min}^{\alpha_i}$ from $I_{\Re} \times I_{\Re}$ to $[0,1]$ such that:

$$s_{r\_\max}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) = \begin{cases} 0, & a_2^{\alpha_i} < b_1^{\alpha_i} \\ \dfrac{a_2^{\alpha_i} - b_1^{\alpha_i}}{b_2^{\alpha_i} - b_1^{\alpha_i}}, & b_1^{\alpha_i} \le a_2^{\alpha_i} < b_2^{\alpha_i} \\ 1, & a_2^{\alpha_i} \ge b_2^{\alpha_i} \end{cases} \tag{15}$$

$$s_{r\_\min}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) = \begin{cases} 0, & b_2^{\alpha_i} < a_1^{\alpha_i} \\ \dfrac{b_2^{\alpha_i} - a_1^{\alpha_i}}{a_2^{\alpha_i} - a_1^{\alpha_i}}, & a_1^{\alpha_i} \le b_2^{\alpha_i} < a_2^{\alpha_i} \\ 1, & b_2^{\alpha_i} \ge a_2^{\alpha_i} \end{cases} \tag{16}$$

DEFINITION 4.

For each α_i-cut level, two *right α_i-cut indices* are defined for, respectively the case of maximizing and minimizing criteria as the functions $s_{\max}^{\alpha_i}$ and, $s_{\min}^{\alpha_i}$ from $I_{\Re} \times I_{\Re}$ to $[0,1]$ such that:

$$s_{\max}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) = (1-\kappa)\cdot s_{l\_\max}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) + \kappa \cdot s_{r\_\max}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}), \forall a, b \in A \quad (17)$$

$$s_{\min}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) = (1-\kappa)\cdot s_{r\_\min}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) + \kappa \cdot s_{l\_\min}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}), \forall a, b \in A \quad (18)$$

The parameter $\kappa \in [0,1]$ represents the degree of optimism of the decision maker (Liou and Wang, 1992). It allows the decision maker to choose which side of the interval is more important. When k increases from 0 to 1, the degree of optimism increases, whereas the degree of pessimism decreases. This type of strategy will be called the horizontal strategy.

In the remaining of this chapter, the notation s or S are used without the index *min* or *max* and refer to the maximization case; however, the related statements are also valid for the minimization case, unless otherwise stated.

PROPOSITION 1.

The α_i-cut indices defined in Definition 4 are fuzzy outranking relations.

The transition from a fuzzy outranking relation defined at the α-cut level to a single criterion fuzzy outranking relation requires an aggregation procedure. Observing the case of fuzzy numbers A and B presented in Figure 7, it follows that the upper α-cut indices favor B, whereas the lower ones favor A. A compensative approach gives a certain discrimination power while still using the biggest amount of information contained in the fuzzy representation of the performances. This idea was exploited in area compensation methods for comparing fuzzy numbers by many authors (Chanas, 1987; Fortemps and Roubens, 1996; Matarazzo and Munda, 2001; Nakamura, 1986). The basic principle is that some nonintersecting areas (i.e., upper left and/or right external areas and lower left and/or right external areas in Figure 7) compensate each other. If we see the previously defined α-cut indices as relative intersections, then their aggregation can be seen as compensation between relative intersections, which is somehow related to the above-mentioned methods. If for linear membership functions the areas considered are relatively simple to be determined, for nonlinear cases, it becomes more difficult. In our α-cut approach besides the fact that we can use inputs stated as a set of α-cut intervals (which avoids possible necessary re-approximations of the original membership function), we prevent the use of integrals for calculating the areas used by an area compensation class of methods.

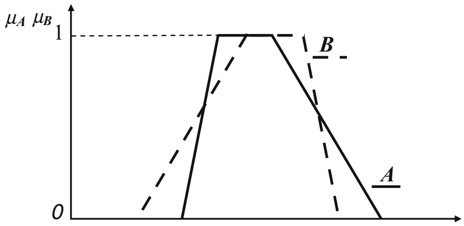


Figure 7. A complex case of comparison of fuzzy numbers

The function used to aggregate the α-cut indices is the *weighted root-power mean* defined for all *x* as follows (Smolíková and Wachowiak, 2002):

$$Fw_{\lambda}(x) = \left(\frac{\sum_{i=1}^{N} \omega_i (x_i)^{\lambda}}{\sum_{i=1}^{N} \omega_i} \right)^{\frac{1}{\lambda}} \tag{19}$$

Using the aggregation function Fw_{λ} to aggregate α_i-cut indices, $i = 1, \ldots, N$, we obtain single criterion fuzzy outranking relation *S* as follows:

$$S(A,B) = \left(\frac{\sum_{i=1}^{N} \omega_i \cdot (s^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}))^{\lambda}}{\sum_{i=1}^{N} \omega_i} \right)^{\frac{1}{\lambda}} \tag{20}$$

PROPOSITION 2.

The single criterion outranking index defined by the relation (20) satisfies the following properties:

- For any convex and normalized fuzzy number B^0, $S(A, B^0)$ is a non-decreasing function of *A*;

- For any convex and normalized fuzzy number A^0, $S(A^0, B)$ is a non-increasing function of B;
- For any fuzzy convex and normalized fuzzy number: $S(C, C) = 1$; hence S is reflexive.

Since the definitions of $s_l^{\alpha_i}$ and $s_r^{\alpha_i}$ allow them to take the value "0," at any α-cut level, some of the particular cases of the relation (20) are excluded:

- Geometrical mean for $\lambda = -1$ due to the possible division by zero;
- Product mean for $\lambda \to 0$, because of the risk of penalty of the result, when an α-cut level of S is 0.

As our intention is to offer the decision maker a flexible decision instrument, cases like *min* or *max* are also excluded. They are dictatorial aggregators, not allowing for compensation between lower and higher values.

Two particular cases are of special interest for the definition of the single criterion fuzzy outranking relation: the weighted arithmetic mean (21) and the weighted square average mean (22).

$$S(A,B) = \frac{\sum_{i=1}^{N} \omega_i \cdot s^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i})}{\sum_{i=1}^{N} \omega_i} \tag{21}$$

$$S(A,B) = \left(\frac{\sum_{i=1}^{N} \omega_i \cdot (s^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}))^2}{\sum_{i=1}^{N} \omega_i} \right)^{\frac{1}{2}} \tag{22}$$

The consideration of weights for α-cut indices makes the final relation more flexible and offers to the decision maker the possibility to decide on the importance of the α-cut levels during the aggregation.

As we have to deal with an enlarged number of weights, equal with the number of α-cuts (which is N), we look to automatically generate the weights. We will search for a method that can give the possibility of changing the weighting vector, such that, for different personalities of the decision maker, we can build different weighting vectors. For example, in the case where the decision maker wants to rely his decisions on α-cuts with less uncertainty, he might be able to slide the highest weights to the highest α-cuts. Alternatively, one might want to give equal importance to all the α-cuts or to assign higher weights to lower α-cuts.

Here we will consider the case when the weights ω_i increase in a linear manner, so the interpolation of these points is a line. As we want to use the information given by all the α-cut indices, this kind of linearity looks convenient, because with two exceptions (the limit functions from this family, which will give 0 for the first α-cut, respectively for the last one), all the weights will be nonzero. The equation of such a line is:

$$\omega_i = \beta \cdot i + c \tag{23}$$

where β is the slope of the line and $c \in \Re$.

Through a series of calculations, using the three particular cases mentioned above and other conditions, the relation (23) becomes

$$\omega_i(\beta) = \beta \cdot \left(i - 1 - \frac{N-1}{2} \right) + \frac{1}{N} = \beta \cdot \left(i - \frac{N+1}{2} \right) + \frac{1}{N} \tag{24}$$

If we consider i as a continuous parameter, then ω_i transforms into a function $\omega(i, \beta) = \beta \times \left(i - 1 - \frac{N-1}{2} \right) + \frac{1}{N}$ of two variables, which can be represented as a surface, as shown in Figure 8.

Therefore, for the case of a maximizing criterion, we obtain the following the single criterion fuzzy outranking relation:

$$S_{\max}(A,B) = \sum_{i=1}^{N} \left(\beta \cdot \left(i - \frac{N+1}{2} \right) + \frac{1}{N} \right)$$

$$[(1-\kappa) \cdot s_{l\_\max}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i}) + \kappa \cdot s_{r\_\max}^{\alpha_i}(a^{\alpha_i}, b^{\alpha_i})]. \tag{25}$$

The expression of the single criterion fuzzy outranking relation for the case of a minimizing criterion can be obtained in a similar way.

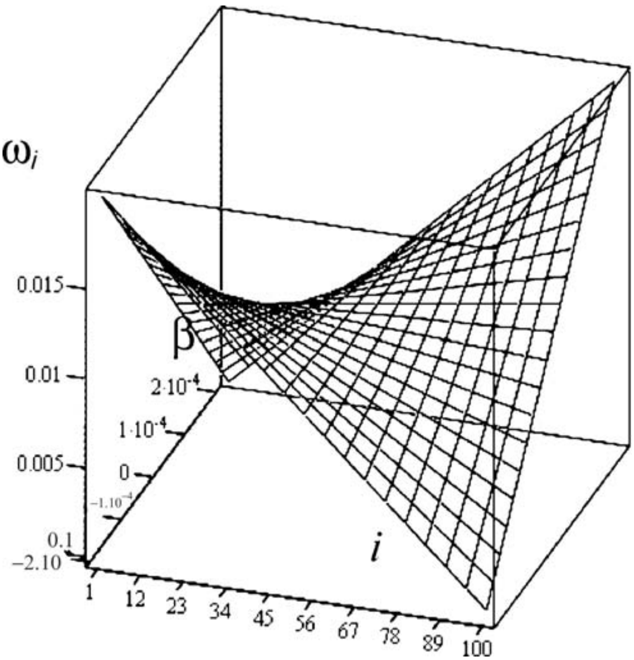


Figure 8. The function *ω(i,β)*

4.1.2 Aggregation of Single Fuzzy Outranking Relations

Here we are interested in aggregating over the set of criteria $g_1, \ldots, g_n$, the single criterion fuzzy outranking relations S_k into a global fuzzy outranking relation S.

Using an aggregation operator M, the global fuzzy outranking relation S is defined for each pair of alternatives (a, b) as follows:

$$S(a,b) = M\left[S_1(a,b),\ \ldots,\ S_n(a,b)\right]. \tag{26}$$

Obviously, S must have the properties of a fuzzy outranking relation, and the following proposition establishes the minimal conditions that an aggregator should fulfill in order to satisfy it.

PROPOSITION 3.

Any aggregator that satisfies the properties of idempotency and monotonicity with respect to the integrand, used to aggregate single criterion fuzzy outranking relations, leads to a global fuzzy relation that is a fuzzy outranking relation.

The Choquet integral (Grabisch, 1999; Marichal, 1999) is an aggregator that satisfies these two properties; consequently, the fuzzy relation obtained by aggregating single criterion fuzzy outranking relations through the use of a Choquet integral is a fuzzy outranking relation.

Considering the Choquet integral as the aggregation operator M, $S(A, B)$ becomes

$$S(A,B) = \sum_{k=1}^{n} S_{(k)}(A,\ B)\ [\mu_{\{(k),\ldots,(n)\}} - \mu_{\{(k+1),\ldots,(n)\}}] \tag{27}$$

4.1.3 Exploitation of the Global Fuzzy Outranking Relation

The type of exploitation to be undergone by the global fuzzy outranking relation depends among others on the type of application for which this exploitation is to be used.

The problem for which this fuzzy outranking method was developed is one in which a large number of decisions has to be taken and for whose solving an automated decision-making procedure has to be put in place (e.g., the selection of the best EOL option for a large number of nodes in a disassembly tree of product with a complex assembly structure, the ranking of design concepts according to their lifecycle performance, including their EOL, etc.; see Gheorghe and Xirouchakis (2006) for a detailed description), but its application goes far beyond this context. It was shown that the α formulation (choice of the best alternative) of the exploitation problem (Roy, 1977) is the most suitable.

Roubens (1989) defined four generalized choice functions C_1, C_2, C_3, and C_4 with all of them being in the authors' opinion, intuitively attractive. As it can be seen from their definition, all these choice functions (and in general all possible choice functions) refer to the strength of the chosen alternative(s) over the rest of alternatives, so they measure somehow the domination of selected alternative(s) over the other or the nondomination of other alternatives on the selected one(s). The superscript "+" is used to denote the choice functions selecting the "best" alternative(s).

The *weak domination* of alternative *a* over all the other alternatives is defined as follows:

$$C_1^+(a) = \underset{b \in \boldsymbol{A}\setminus\{a\}}{T_1} S(a,b) \tag{28}$$

where T_1 is a *t*-norm and $S(a, b)$ is the degree, between 0 and 1, to which *a* is as good as *b*. To be in accordance with Orlovsky's (Orlovsky, 1978) reasoning and terminology, $C_1^+(a)$ can be interpreted as the degree of weak domination of *a* over all the other alternatives in ***A***. The choice set is given by

$$C_1^+(\boldsymbol{A},S) = \left\{ a \in \boldsymbol{A} \mid C_1^+(a) = \max_{b \in \boldsymbol{A}} C_1^+(b) \right\} \tag{29}$$

The *weak nondomination* of *a* by all the other alternatives is defined as follows:

$$C_2^+(a) = \underset{b \in \boldsymbol{A}\setminus\{a\}}{T_1} [1 - S(b,a)] \tag{30}$$

$C_2^+(a)$ is interpreted as the degree of weak nondomination of *a* by all the other alternatives in *A*. The choice set is given by

$$C_2^+(\boldsymbol{A},S) = \left\{ a \in \boldsymbol{A} \mid C_2^+(a) = \max_{b \in \boldsymbol{A}} C_2^+(b) \right\} \tag{31}$$

The *strict domination* of *a* over all the other alternatives is defined as follows:

$$C_3^+(a) = \underset{b \in \boldsymbol{A}\setminus\{a\}}{T_1} P(a,b) \tag{32}$$

$C_3^+(a)$ represents the degree of strict domination of *a* over *b*. *P* is the strict preference relation, and it is defined as $P(a, b)=T_2[(S(a, b), 1 - S(b, a)]$, with T_2 being a *t*-norm. The choice set is given by

$$C_3^+(\boldsymbol{A},S) = \left\{ a \in \boldsymbol{A} \mid C_3^+(a) = \max_{b \in \boldsymbol{A}} C_3^+(b) \right\} \tag{33}$$

The *strict nondomination* of *a* by all other alternatives is defined as follows:

$$C_4^+(a) = \underset{b \in A \setminus \{a\}}{T_1} [1 - P(b,a)] \tag{34}$$

$C_4^+(a)$ represents the degree of strict nondomination of all the other alternatives on *a*. The choice set is given by

$$C_4^+(A,S) = \left\{ a \in A \mid C_4^+(a) = \max_{b \in A} C_4^+(b) \right\} \tag{35}$$

In contrast to the measurement of the strengths of alternatives, it is also interesting to measure their weaknesses. Four weakness-based choice functions C_5^-, C_6^-, C_7^-, and C_8^- are presented in the following.

- The *weak domination* of all alternatives on alternative *a*, representing the degree to which *a* is *weakly dominated* by all the other alternatives, is defined as follows:

$$C_5^-(a) = \underset{b \in A \setminus \{a\}}{T_1} S(b,a) \tag{36}$$

The choice set corresponding to the weak domination (of all alternatives on a given alternative) function C_5^- is given by

$$C_5^-(A,S) = \left\{ a \in A \mid C_5^-(a) = \max_{b \in A} C_5^-(a) \right\} \tag{37}$$

- The *weak nondomination* of an alternative *a* over all other alternatives representing the degree to which *a* doesn't weakly dominate all the other alternatives is defined as follows:

$$C_6^-(a) = \underset{b \in A \setminus \{a\}}{T_1} [1 - S(a,b)] \tag{38}$$

The choice set corresponding to the weak nondomination (of an alternative on all others) function C_6^- is given by

$$C_6^-(A,S) = \left\{ a \in A \mid C_6^-(a) = \max_{b \in A} C_6^-(a) \right\} \tag{39}$$

- The *strict domination* of all alternatives on alternative *a* that gives the degree to which *a* is *strictly dominated* by all the other alternatives is defined as follows:

$$C_7^-(a) = \underset{b\in \boldsymbol{A}\backslash\{a\}}{T_1} P(b,a) \tag{40}$$

The choice set corresponding to the strict domination (of all alternatives on a given alternative) function C_7^- is given by

$$C_7^-(\boldsymbol{A},S) = \left\{ a \in \boldsymbol{A} \mid C_7^-(a) = \max_{b\in \boldsymbol{A}} C_7^-(b) \right\} \tag{41}$$

- The *strict nondomination* of an alternative *a* over all other alternatives representing the degree to which *a doesn't strictly dominate* all the other alternatives is defined as follows:

$$C_8^-(a) = \underset{b\in \boldsymbol{A}\backslash\{a\}}{T_1} [1 - P(a,b)] \tag{42}$$

The choice set corresponding to the strict nondomination (of an alternative on all others) function C_8^- is given by

$$C_8^-(\boldsymbol{A},S) = \left\{ a \in \boldsymbol{A} \mid C_8^-(a) = \max_{b\in \boldsymbol{A}} C_8^-(b) \right\} \tag{43}$$

A ranking method can be obtained using the core concept. Once the set of best alternatives (C_{k+1}) is chosen by the choice function $C(R, \boldsymbol{A}_k)$, which can be any of the choice functions defined above, it is removed from the initial set $\boldsymbol{A}$, and another core set is found between the remaining alternatives ($\boldsymbol{A}_k \backslash C_{k+1}$). This reasoning is applied until the current set ($\boldsymbol{A}_k$) is empty. This algorithm was proposed in (Perny, 1992), and it is described as follows:

Set $k := 0$ and $\boldsymbol{A}_k := \boldsymbol{A}$
  While $\boldsymbol{A}_k \neq \phi$ do
  Begin
    $C_{k+1} := C(R,\boldsymbol{A}_k)$
    $\boldsymbol{A}_{k+1} := \boldsymbol{A}_k \backslash C_{k+1}$
    $k := k+1$
  End

R is one of the relations used to define the first four choice functions (C_1^+ to C_4^+), specifically the weak preference *S* and strict preference *P* relations. The resulting preorder $\succeq_R$ is a complete ranking of sets of single or multiple (indifferent) alternatives from best to worst, where *R* stands for *S* or *P*. Four rankings can be obtained using the *strength* concept.

The same algorithm can be used to obtain a second type of preorder but this time using the last four functions (C_5^- to C_8^-). As they are based on the weakness concept, an ascending preorder from worst to the best will be constructed, denoted by $\preceq_R$. These second type of rankings can be different from the previous one.

The notions of ascending–descending and weak–strict rankings are introduced as follows. Similar concepts were used in methods like ELECTRE II and III, MAPPACC, and PRAGMA. Methods like PROMETHEE I and II use concepts of weakness and strength of alternatives but in a different manner. Four different preorders can be defined as follows:

- *Descending weak preorder* is the complete ranking obtained using the iterated choice functions C_1^+ or C_2^+,
- *Descending strict preorder* is the complete ranking obtained using the iterated choice functions C_3^+ or C_4^+,
- *Ascending weak preorder* is the complete ranking obtained using the iterated choice functions C_5^- or C_6^-,
- *Ascending strict preorder* is the complete ranking obtained using the iterated choice functions C_7^- or C_8^-.
- Looking at the choice functions considered, we see that in fact, C_5^-, C_6^-, C_7^-, and C_8^- are "dual" of the functions of C_1^+, C_2^+, C_3^+ and C_4^+ respectively. So each pair $C_1^+ - C_5^-$, $C_2^+ - C_6^-$, $C_3^+ - C_7^-$ and $C_4^+ - C_8^-$ express the force and the weakness, when used in a ranking procedure. At the same time, pairs like $C_1^+ - C_2^+$ and $C_3^+ - C_4^+$ respectively, $C_5^- - C_6^-$ and $C_7^- - C_8^-$ express another type of "duality" that notions of *"outgoing" domination–non domination* (i.e., of an alternative on all the other alternatives), when talking about strength, respectively *"incoming" domination–non domination* (i.e., of all alternatives on the alternative under consideration), when considering the weakness. And

finally, both "dualities" are present for both weak and strict preference relations.

The eight functions can be used alone to obtain a final ranking (weak or strict preorder). Nevertheless, the rankings obtained from two preorders (one descending and the other ascending), thus allowing incomparability (since an alternative a_i may be preferred over another alternative a_j in one preorder and a_j preferred over a_i in the other preorder), are richer and more interesting, as they take into account concepts that may be opposite, or dual, as shown above. Various ranking procedures based on a pair of choice functions, together with their characterization from the following points of view, can be obtained:

- Type of preference: *weak–strict*,
- Type of the ranking of individual choice functions: *ascending–descending*,
- Concept involved: *strength–weakness*,
- Intuitive meaning of the individual choice functions: *incoming domination*, *incoming nondomination*, *outgoing domination*, and *outgoing nondomination*.

4.1.4 Illustrative Example

The example is adapted from (Wang, 2001). Let us consider the seven valve types (a_1 to a_7), and the criteria are cost, maintenance, criteria sensitivity, leakage, rangibility, and stability (g_1 to g_6). The performance matrix is given in Table 5.

Table 5. Performance Matrix for Seven Valve Types (Trapezoidal Fuzzy Numbers)

Alternatives	Criteria's weights of importance					
	0.217	0.174	0.174	0.217	0.087	0.131
	Performance with respect to criterion g_k					
	g_1	g_2	g_3	g_4	g_5	g_6
A_1	(4, 5, 5, 6)	(5, 6, 7, 8)	(7, 8, 8, 9)	(7, 8, 8, 9)	(7, 8, 8, 9)	(1, 2, 2, 3)
A_2	(7, 8, 8, 9)	(8, 9, 10, 10)	(7, 8, 8, 9)	(2, 3, 4, 5)	(8, 9, 10, 10)	(7, 8, 8, 9)
A_3	(7, 8, 8, 9)	(1, 2, 2, 3)	(7, 8, 8, 9)	(7, 8, 8, 9)	(5, 6, 8, 9)	(5, 6, 7, 8)
A_4	(1, 2, 4, 5)	(4, 5, 5, 6)	(4, 5, 5, 6)	(2, 3, 7, 8)	(4, 5, 8, 9)	(8, 9, 10, 10)
A_5	(7, 8, 8, 9)	(5, 6, 7, 8)	(5, 6, 7, 8)	(8, 9, 10, 10)	(1, 2, 2, 3)	(1, 2, 2, 3)
A_6	(4, 5, 5, 6)	(4, 5, 5, 6)	(2, 3, 4, 5)	(5, 6, 7, 8)	(8, 9, 10, 10)	(8, 9, 10, 10)
A_7	(4, 5, 7, 8)	(8, 9, 10, 10)	(7, 8, 8, 9)	(5, 6, 7, 8)	(8, 9, 10, 10)	(7, 8, 8, 9)

The single criterion fuzzy outranking relations S_k are first calculated for each criterion g_k, $k = 1\ldots6$ using relation (25) for a number of α-cuts $N = 50$. In the second step, S_k are aggregated using the weighted arithmetic (a particular case of the Choquet intergral) mean with the criteria weights of importance given in Table 5. These steps are repeated for the five representative situations given by the pair of parameters (κ,β), representing the decision maker's attitude. Figures 9−13 represent the above-mentioned situations in terms of the global fuzzy outranking relation S.

	1	2	3	4	5	6	7
1	1	0.413	0.652	0.869	0.62	0.804	0.63
2	0.783	1	0.783	0.902	0.783	0.685	0.783
3	0.773	0.638	1	0.695	0.663	0.618	0.638
4	0.241	0.356	0.388	1	0.306	0.605	0.257
5	0.766	0.461	0.635	0.782	1	0.782	0.461
6	0.512	0.435	0.426	0.853	0.262	1	0.652
7	0.817	0.808	0.624	0.902	0.591	0.902	1

Figure 9. $S(a_i,a_j)$ for $\kappa = 0$, $\beta = \beta^c$ (conserv-pessim)

	1	2	3	4	5	6	7
1	1	0.411	0.652	0.869	0.615	0.802	0.495
2	0.783	1	0.783	0.776	0.783	0.682	0.783
3	0.826	0.66	1	0.695	0.658	0.628	0.66
4	0.354	0.368	0.446	1	0.272	0.77	0.392
5	0.783	0.478	0.652	0.782	1	0.782	0.478
6	0.516	0.435	0.446	0.87	0.245	1	0.519
7	0.837	0.837	0.675	0.899	0.62	0.899	1

Figure 10. $S(a_i,a_j)$ for $\kappa = 1$, $\beta = \beta^c$ (conserv-optim)

	1	2	3	4	5	6	7
1	1	0.404	0.652	0.869	0.598	0.795	0.546
2	0.783	1	0.783	0.822	0.783	0.671	0.783
3	0.796	0.633	1	0.695	0.641	0.617	0.633
4	0.252	0.356	0.402	1	0.261	0.662	0.305
5	0.761	0.456	0.63	0.782	1	0.782	0.456
6	0.484	0.435	0.419	0.848	0.24	1	0.577
7	0.81	0.807	0.617	0.888	0.59	0.888	1

Figure 11. $S(a_i, a_j)$ for $\kappa = 0$, $\beta = \beta^m$ (moderate)

	1	2	3	4	5	6	7
1	1	0.396	0.652	0.869	0.578	0.787	0.613
2	0.783	1	0.783	0.876	0.783	0.659	0.783
3	0.759	0.614	1	0.695	0.621	0.61	0.614
4	0.164	0.35	0.371	1	0.238	0.562	0.214
5	0.745	0.44	0.614	0.782	1	0.782	0.44
6	0.452	0.435	0.399	0.832	0.228	1	0.652
7	0.79	0.788	0.578	0.876	0.571	0.876	1

Figure 12. $S(a_i, a_j)$ for $\kappa = 0$, $\beta = \beta^a$ (agress-pessim)

	1	2	3	4	5	6	7
1	1	0.395	0.652	0.869	0.576	0.786	0.444
2	0.783	1	0.783	0.734	0.783	0.658	0.783
3	0.826	0.62	1	0.695	0.619	0.612	0.62
4	0.248	0.352	0.404	1	0.23	0.712	0.357
5	0.749	0.444	0.618	0.782	1	0.782	0.444
6	0.453	0.435	0.404	0.836	0.224	1	0.484
7	0.795	0.795	0.591	0.875	0.578	0.875	1

Figure 13. $S(a_i, a_j)$ for $\kappa = 1$, $\beta = \beta^a$(agress-optim)

The complete preorders given by the functions $C_1^+ (\equiv C_4^+)$, $C_2^+ (\equiv C_3^+)$, $C_5^- (\equiv C_8^-)$, and $C_6^- (\equiv C_7^-)$ were determined for each of the five representative decision attitudes. Because the fuzzy numbers expressing the performances of the considered alternatives interfere very little and in a trivial manner, we observed an influence that is not strong enough to change the partial preorders when sliding from a conservative to an aggressive attitude. Some changes are noticed when varying the other parameter (κ). Table 6 shows the complete preorders:

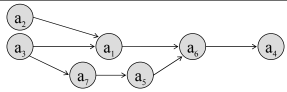
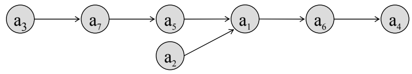
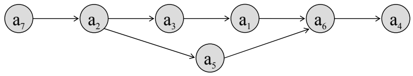
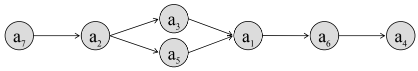
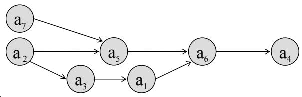
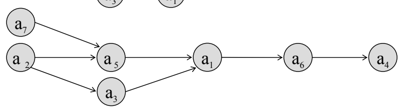
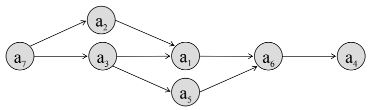
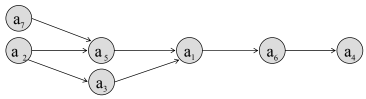
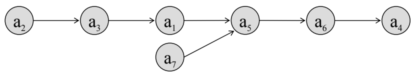
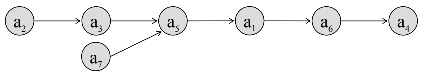
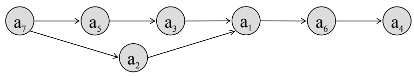
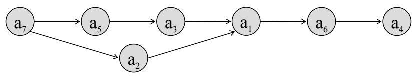
Table 6. Complete Preorder for the Seven Types of Valves

No.	Choice functions	Decision strategy (κ, β)	Preference
1	$C_1^+ \equiv C_4^+$	(0, βa), (0, βc)	2 > 3 > 1 > 7 > 5 > 6 > 4
		(1,βa), (0.5,βm), (1,βc)	2 > 3 > 7 > 5 > 1 > 6 > 4
2	$C_2^+ \equiv C_3^+$	(0, βa), (0, βc)	3, 5, 7 > 2 > 1 > 6 > 4
		(1,βa), (0.5,βm), (1,βc)	3, 5, 7 > 2 > 1 > 6 > 4
3	$C_5^- \equiv C_8^-$	(0, βa), (0, βc)	7 > 2 > 5 > 3 > 1 > 6 > 4
		(1,βa), (0.5,βm), (1,βc)	7 > 2 > 5 > 3 > 1 > 6 > 4
4	$C_6^- \equiv C_7^-$	(0, βa), (0, βc)	7 > 2 > 3 > 1 > 5 > 6 > 4
		(1,βa), (0.5,βm), (1,βc)	7 > 2 > 3 > 5 > 1 > 6 > 4

For each decision strategy, six partial preorders can be derived from the above table by intersecting pairs of choice functions. They are shown in Table 7.

Besides the theoretical foundations of this method, its advantages are related to the practical aspects, namely the format of the input data that can be used (general, nonanalytical representations of fuzzy numbers) where the preference function is described as a vector of α-cuts. Six rankings are proposed. One, several, or all of them can be used to reinforce the choice or the ranking. They enclose different choice ideas, all together offering a large "palette" of concepts. It is up to the decision maker which of them is to be used in the concrete problem. The concepts that are proposed are easy to understand, and they give transparency to the decision process.

Table 7. Partial Ranking of the Eight Valve Types

No.	Choice functions	Decision strategy (κ, β)	Preference
1	$(C_1^+ \equiv C_4^+)$ – $(C_2^+ \equiv C_3^+)$	$(0, \beta^a)$, $(0, \beta^c)$	
		$(1,\beta^a)$, $(0.5,\beta^m)$, $(1,\beta^c)$	
2	$(C_1^+ \equiv C_4^+)$ – $(C_5^- \equiv C_8^-)$	$(0, \beta^a)$, $(0, \beta^c)$	
		$(1,\beta^a)$, $(0.5,\beta^m)$, $(1,\beta^c)$	
3	$(C_5^- \equiv C_8^-)$ – $(C_6^- \equiv C_7^-)$	$(0, \beta^a)$, $(0, \beta^c)$	
		$(1,\beta^a)$, $(0.5,\beta^m)$, $(1,\beta^c)$	
4	$(C_2^+ \equiv C_3^+)$ – $(C_6^- \equiv C_7^-)$	$(0, \beta^a)$, $(0, \beta^c)$	
		$(1,\beta^a)$, $(0.5,\beta^m)$, $(1,\beta^c)$	
5	$(C_1^+ \equiv C_4^+)$ – $(C_6^- \equiv C_7^-)$	$(0, \beta^a)$, $(0, \beta^c)$	
		$(1,\beta^a)$, $(0.5,\beta^m)$, $(1,\beta^c)$	
6	$(C_2^+ \equiv C_3^+)$ – $(C_5^- \equiv C_8^-)$	$(0, \beta^a)$, $(0, \beta^c)$	
		$(1,\beta^a)$, $(0.5,\beta^m)$, $(1,\beta^c)$	

4.2 Other Fuzzy Outranking Methods

All outranking methods briefly described in this subsection consider fuzzy evaluations of alternatives on criteria; therefore, they are fuzzy outranking methods.

4.2.1 Method of Czyżak and Słowiński (1996)

This method is an adaptation of ELECTRE III to the case where the concordance and discordance indices are determined from the fuzzy evaluations of alternatives on criteria through the use of four different measures using possibility and necessity concepts from possibility theory developed in Dubois and Prade (1988). The aggregation of the possibility and necessity measures to drive the concordance and discordance indices is realized through the use of a *weighted root-power mean*. Apart from an adjustment of the monocriterion concordance and discordance indices through some transformation, the rest of the method is similar to ELECTRE III. The method is illustrated through its application to the ground water management problem considered in Duckstein et al. (1994).

4.2.2 Method of Wang (1997)

This method is based on the consideration of a fuzzy preference relation P defined each pair of alternatives (a, b) whose respective fuzzy scores on a given criterion are A and B as follows:

$$P(a,b) = \frac{D(A,B) + D(A \cap B,0)}{D(A,o) + D(B,0)} \tag{44}$$

where $D(A,B)$ represents the areas where A dominates B, $D(A \cap B,0)$ represents the intersection areas of A and B, $D(A,0)$ represents the area of A, and $D(B, 0)$ represents the area of B. It is worth recalling that this fuzzy relation is considered by Tseng and Klein (1989) for the problem of ranking fuzzy numbers.

The outranking relation is defined for:

- The case of a pseudo-order preference model where a preference and indifference thresholds are associated with criteria,
- The case of a semi-order preference model where only indifference thresholds are associated with criteria,

- The case of a complete-preorder preference model where the preference and indifference thresholds are null for each criterion.

The exploitation phase is based on the consideration of the concepts of dominance and non-dominance sets.

The method is illustrated through its application to the problem of evaluating and comparing design concepts in conceptual design.

Güngör and Arikan (2000) applied a similar method to the problem of energy policy planning.

4.2.3 Method of Wang (1999)

In this method, the concordance and discordance indices are determined from the fuzzy evaluations of alternatives on criteria through the use of possibility and necessity measures. More specifically, for two design requirements r_i and r_j, the concordance and discordance indices of criterion C_k with the assertion "r_i is at least as good as r_j" are defined as follows:

$$CI_k(r_i, r_j) = \theta POSS_k(r_i \geq r_j) + (1-\theta) NESS_k(r_i \geq r_j) \tag{45}$$

$$DI_k(r_i, r_j) = NESS_k(r_j \geq r_i) \tag{46}$$

where θ is a preference ratio such that $0 \leq \theta \leq 1$.

The global outranking relation is obtained from monocriterion concordance and discordance indices through the use of the aggregation method developed by Siskos et al. (1984).

The method is illustrated through its application to the problem of prioritizing design requirements in quality function deployment in the case of a car design.

4.2.4 Method of Wang (2001)

In this method, the construction of the fuzzy outranking relation is similar to that of Czyżak and Słowiński (1996) since the concordance and discordance indices are determined from the fuzzy evaluations of alternatives on criteria through the use of four different measures using possibility and necessity concepts. However, the exploitation of the global fuzzy outranking relation is different from ELECTRE III since it is based on the determination of the set of nondominated alternatives as it is considered in Orlovsky (1978). The method is illustrated through its application to the problem of ranking engineering design concepts in conceptual design.

5. CONCLUSION

In this chapter we made a clear distinction between outranking methods based on the construction and exploitation of a valued outranking relation and outranking methods based on the construction and exploitation of a fuzzy outranking relation since they are applicable to two different situations. Indeed, the outranking methods with a valued outranking relation are applicable to the situation where the evaluations of alternatives on criteria are crisp, whereas the outranking methods with a fuzzy outranking relation are applicable to the situation where the evaluations of alternatives on criteria are fuzzy. The outranking methods with a valued outranking relation are called valued outranking methods and the outranking methods with a fuzzy outranking relation are called fuzzy outranking methods. In the literature the fuzzy and valued outranking methods are often confused and the clarification made in this chapter allows avoiding this confusion.

All fuzzy outranking methods deal with the problem of comparing fuzzy numbers; however, they consider different approaches:

- Gheorghe et al. (2004) consider an approach based on α-cuts;
- Czyżak and Słowiński (1996) and Wang (1999, 2001) consider an approach based on possibility and necessity measures;
- Wang (1997) and Güngör and Arikan (2000) consider an approach based on the comparison of areas of fuzzy numbers.

The valued outranking methods ELECTRE III and PROMETHEE are widely applied to real-world problems; however, they are not suitable to the problems where the evaluations of alternatives on criteria are fuzzy. The fuzzy outranking methods presented in this chapter are quite recent compared with the valued outranking methods, and even if they were applied to specific problems, they can be adapted to any MCDA problem where the evaluations of alternatives on criteria are fuzzy.

In this chapter, we provided two illustrative examples, one for a valued outranking method, namely ELECTRE III, and one for a fuzzy outranking method, namely the method developed by the authors. The objective is to show that outranking methods can be applied to various problems with the mention that valued outranking methods are suitable to problems with crisp evaluations (i.e., the case of the treatment of products at their EOL) and fuzzy outranking methods are suitable to problems with fuzzy evaluations (i.e., the case of design concept selection in conceptual design).

REFERENCES

Al-Rashdan, D., Al-Kloub, B., Dean, A., and Al-Shemeri, T., 1999, Environmental impact assessment and ranking the environmental projects in Jordan, *European Journal of Operational Research*, **118:** 30−45.

Anagnostopoulos, K., Giannoupoulo, M., and Roukounis, Y., 2003, Multicriteria evaluation of transportation infrastructure projects: An application of PROMETHEE and GAIA methods, *Advanced Transportation*, **14:** 599−608.

Augusto, M., Figueira, J., Lisboa, J., and Yasin, M., 2005, An application of a multi-criteria approach to assessing the performance of Portugal's economic sectors: Methodology, analysis and implications, *European Business Review*, **17:** 113−132.

Babic, Z., and Plazibat, N., 1998, Ranking of enterprises based on multicriterial analysis, *International Journal of Production Economics*, **56−57:** 29−35.

Beccali, M., Cellura, M., and Ardente, D., 1998, Decision making in energy planning: the ELECTRE multicriteria analysis approach compared to a fuzzy sets methodology, *Energy Conversion and Management,* **39:** 1869−1881.

Brans, J.-P., and Vincke, Ph., 1985, A preference ranking organization method, *Management Sciences*, **31:** 647−656.

Bufardi, A., Gheorghe, R., Kiritsis, D., and Xirouchakis, P., 2004, A multicriteria decision−aid approach for product end−of−life alternative selection, *International Journal of Production Research*, **42:** 3139−3157.

Chanas, S., 1987, Fuzzy optimization in networks, in: *Optimization Models Using Fuzzy Sets and Possibility Theory*, Kacprzyk, J., and Orlovski, S.A., (eds.), pp. 308−327, Reidel, Dordrecht.

Cote, G., and Waaub, J.-P., 2000, Evaluation of road project impacts: Using the multicriteria decision aid, *Cahiers de Geographie du Quebec*, **44:** 43−64.

Czyżak, P., and Słowiński, R., 1996, Possibilistic construction of fuzzy outranking relation for multiple-criteria ranking, *Fuzzy Sets Systems*, **81:** 123−131.

Dubois, D., and Prade, H., 1988, *Possibility Theory: An Approach to Computerised Processing of Uncertaint*y, Plenum Press, New York.

Duckstein, L., Treichel, W., and El Magnouni, A., 1994, Ranking ground−water management alternatives by multicriterion analysis, ASCE, *Journal of Water Resources Planning And Management*, **120:** 546–565.

Elevli, B., and Demirci, A., 2004, Multicriteria choice of ore transport system for an underground mine: Application of PROMETHEE methods, *Journal of the South African Institute of Mining and Metallurgy*, **104:** 251−256.

Fodor, J., and Roubens, M., 1994, *Fuzzy Preference Modeling and Multicriteria Decision Support*, Kluwer, Dordrecht.

Fortemps, Ph., and Roubens, M., 1996, Ranking and defuzzification methods based on area compensation, *Fuzzy Sets Systems*, **82:** 319−330.

Geldermann, J., Spengler, T., and Rentz, O., 2000, Fuzzy outranking for environmental assessment. Case study: iron and steel making industry, *Fuzzy Sets Systems***, 115:** 45−65.

Gheorghe, R., 2005, A new fuzzy multicriteria decision aid method for conceptual design, PhD thesis, EPF Lausanne.

Gheorghe, R., Bufardi, A., and Xirouchakis, P., 2004, Construction of a two-parameters outranking relation from fuzzy evaluations, *Fuzzy Sets Systems*, **143:** 391−412.

Gheorghe, R., Bufardi, A., and Xirouchakis, P., 2005, Construction of global fuzzy preference structures from two-parameter single-criterion fuzzy outranking relations, *Fuzzy Sets Systems*, **153:** 303−330.

Gheorghe, R., and Xirouchakis, P., 2006, Decision-based methods for early phase sustainable product design, *International Journal of Engineering Education*, in press.

Gilliams, S., Raymaekers, D., Muys, B., and Van Orshoven, J., 2005, Comparing multiple criteria decision methods to extend a geographical information system on afforestation, *Computers and Electronics in Agriculture,* **49:** 142−158.

Goedkoop, M., and Spriensma, R., 2000, The Eco-indicator 99—A damage oriented method for Life Cycle Impact Assessment, Methodology Report, 2nd edition, Pre Consultantsbu, Amersfoort, The Netherlands.

Goumas, M., and Lygerou, V., 2000, An extension of the PROMETHEE method for decision making in fuzzy environment: Ranking of alternative energy exploitation projects, *European Journal of Operational Research*, **123:** 606−613.

Grabisch, M., 1999, *Fuzzy Measures and Integrals: Theory and Applications*, Physica-Verlag, Heidelberg.

Güngör, Z., and Arikan, F., 2000, A fuzzy outranking method in energy policy planning, *Fuzzy Sets Systems*, **114:** 115−122.

Hababou, M., and Martel, J.−M., 1998, Multicriteria approach for selecting portfolio manager*, INFOR Journal*, **36:** 161−176.

Hokkanen, J., and Salminen, P., 1994, Choice of a solid waste management system by using the ELECTRE III method, in *Applying MCDA for Environmental Management*, Paruccini, M., (ed.), Kluwer Academic Publishers, Dordrecht.

Hokkanen, J., and Salminen, P., 1997, ELECTRE III and IV decision aids in an environmental problem, *Journal of Multi-Criteria Decision Analysis,* **6:** 215−226.

Kalogeras, N., Baourakis, G., Zopounidis, C., and van Dijk, G., 2005, Evaluating the financial performance of agri−food firms: a multicriteria decision−aid approach, *Journal of Food Engineering,* **70:** 365−371.

Kangas, A., Kangas, J., and Pykäläinen, J., 2001, Outranking methods as tools in strategic natural resources planning, *Silva Fennica*, **35:** 215−227.

Karagiannidis, A., and Moussiopoulos, N., 1997, Application of ELECTRE III for the integrated management of municipal solid wastes in the Greater Athens Area, *European Journal of Operational Research*, **97:** 439−449.

Le Téno, J.F., and Mareschal, B., 1998, An interval version of PROMETHEE for the comparison of building product's design with ill−defined data on environmental quality, *European Journal of Operational Research***, 109:** 522−529.

Leclercq, J.-P., 1984, Propositions d'extension de la notion de dominance en presence de relations deordre sur les pseudo-criteres: Melchior, *Revue Belge de Recherche Operationnelle de Statistique et de lnformatique*, **24**(1)**:** 32–46.

Liou, T.S., and Wang, M.J., 1992, Ranking fuzzy numbers with integral value, *Fuzzy Sets Systems***, 50:** 247−255.

Marichal, J.-L., 1999, Aggregation operators for multicriteria decision aid, PhD thesis, Université de Liège.

Matarazzo, B., 1986, Multicriterion analysis of preferences by means of pairwise actions and criterion comparisons (MAPPACC), *Applied Mathematics and Computation*, **18**(2)**:** 119–141.

Matarazzo, B., and Munda, G., 2001, New approaches for the comparison of L−R fuzzy numbers: A theoretical and operational analysis, *Fuzzy Sets Systems*, **118:** 407−418.

Mavrotas, G., Diakoulaki, D., and Caloghirou, Y., 2006, Project prioritization under policy restrictions: A combination of MCDA with 0–1 programming, *European Journal of Operational Research*, **171:** 296−308.

Maystre, L., Pictet, J., and Simos, J., 1994, *Méthodes Multicritères ELECTRE. Description, Conseils Pratiques Et Cas 'Application A Gestion Environementale*, Presses Polytechniques et universitaires Romandes, Lausanne.

Nakamura, K., 1986, Preference relations on a set of fuzzy utilities as a basis for decision making, *Fuzzy Sets Systems*, **20:** 147−162.

Orlovsky, S.A., 1978, Decision-making with a fuzzy preference relation, *Fuzzy Sets Systems*, **1:** 155−167.

Paelinck, J., 1978, Qualiflex, a flexible multiple-criteria method, *Economic letters*, **3:** 193–197

Pasche, C., 1991, EXTRA: An expert system for multicriteria decision making, *European Journal of Operational Research*, **52:** 224−234.

Pastijn, H., and Leysen, J., 1989, Constructing an outranking relation with ORESTE, *Mathematical and Computer Modelling*, **12**(10–11)**:** 1255–1268

Perny, P., 1992, Modélisation, agrégation et exploitation de préférences floues dans une problématique de rangement, PhD thesis, Université Paris−Dauphine.

Perny, P., and Roy, B., 1992, Fuzzy outranking relations in preference modeling, *Fuzzy Sets Systems*, **49:** 33−53.

Petras, J.C.E., 1997, Ranking the sites for low- and intermediate-level radioactive waste disposal facilities in Croatia, *International Transactions in Operational Research*, **4:** 135−159.

Rogers, M., and Bruen, M., 1998, Choosing realistic values of indifference, preference and veto thresholds for use with environmental criteria within ELECTRE, *European Journal of Operational Research*, **107:** 542−551.

Rogers, M., and Bruen, M., 2000, Using ELECTRE III to choose route for Dublin Port Motorway, *Journal of Transportation Engineering*, **126:** 313−323.

Roubens, M., 1982, Preference Relations on Actions and Criteria in Multicriteria Decision Making, *European Journal of Operational Research*, **10:** 51–55.

Roubens, M., 1989, Some properties of choice functions based on valued binary relations, *European Journal of Operational Research*, **40:** 309−321.

Roy, B., 1968, Classement et choix en présence de points de vue multiples (la méthode ELECTRE), R.I.R.O., **8:** 57–75.

Roy, B., and Bertier, P., 1973, La méthode ELECTRE II – *Une application au media-planning*, M. Ross (Ed.), OR 72, 291-302, North Holland, Amsterdam.

Roy, B., 1974, Critères multiples et modélisation des préférences : l'apport des relations de surclassement, *Revue d'Economie Politique*, **1:** 1–44.

Roy, B., 1977, Partial preference analysis and decision−aid: The fuzzy outranking relation concept, in *Conflicting Objectives in Decisions*, Bell, D.E., Keeney, R. L., and Raiffa, H. (eds.), pp. 40−75, John Wiley and Sons, New York.

Roy, B., 1978, ELECTRE III: algorithme de classement basé sur une représentation floue des préférences en présence des critères multiples, *Cahiers du CERO*, **20:** 3−24.

Roy, B., and Hugonnard, J.C., 1982, Ranking of suburban line extension projects on the Paris metro system by a multicriteria method, *Transportation Research Part A: General*, **16**(4)**:** 301–312

Roy, B., 1991, The outranking approach and the foundations of ELECTRE methods, *Theory and Decisions*, **31:** 49–73.

Roy, B., Présent, M., and Silhol, D., 1986, A programming method for determining which Paris Metro stations should be renovated, *European Journal of Operational Research*, **24:** 318−334.

Siskos, J., 1982, A way to deal with fuzzy preferences in multi-criteria decision problems, *European Journal of Operational Research*, **10:** 314−324.

Siskos, J., Lochard, J., and Lombard, J., 1984, A multicriteria decision making methodology under fuzziness: application to the evaluation of radiological protection in nuclear power plant, in: *TIMS/Studies in the Management Sciences*, H. J. Zimmermann, ed., pp. 261−283, North-Holland, Amsterdam.

Smolíková, R., and Wachowiak, M. P., 2002, Aggregation operators for selection problems, *Fuzzy Sets Systems*, **131:** 23−34.

Teng, G.-Y., and Tzeng, G.-H., 1994, Multicriteria evaluation for strategies of improving and controlling air quality in the super city: A case study of Taipei city, *Journal of Environmental Management*, **40:** 213−229.

Tseng, T.Y., and Klein, C.M., 1989, New algorithm for the ranking procedure in fuzzy decision making, *IEEE Transactions On Systems, Man, And Cybernetics*, **19:** 1289−1296.

Tzeng, G.-H., and Tsaur, S.-H., 1997, Application of multiple criteria decision making for network improvement, *Journal of Advanced Transportation*, **31:** 49−74.

Vincke, Ph., 1992a, *Multicriteria Decision−Aid*, John Wiley and Sons, Chichester.

Vincke, Ph., 1992b, Exploitation of a crisp relation in a ranking problem, *Theory and Decision*, **32:** 221−240.

Wang, J., 1997, A fuzzy outranking method for conceptual design evaluation, *International Journal of Production Research*, **35:** 995−1010.

Wang, J., 1999, Fuzzy outranking approach to prioritize design requirements in quality function deployment, *International Journal of Production Research*, **37:** 899−916.

Wang, J., 2001, Ranking engineering design concepts using a fuzzy outranking preference model, *Fuzzy Sets Systems*, **119:** 161−170.

Vansnick, J.-C., 1986, On the problem of weights in multiple criteria decision making (the noncompensatory approach), *European Journal of Operational Research*, **24**(2)**:** 288–294.