GOAL PROGRAMMING APPROACHES FOR SOLVING FUZZY INTEGER MULTI-CRITERIA DECISION-MAKING PROBLEMS

Omar M. Saad
Department of Mathematics, College of Science, Qatar University, Doha, Qatar

Abstract: Multicriteria decision making can be divided into two parts: multi-attribute decision analysis and multi-criteria optimization. When the number of the feasible alternatives is large, we use multi-criteria optimization. On the other hand, multi-attribute decision analysis is most often applicable to problems with a small number of alternatives in an environment of uncertainty. In this chapter, a goal programming approach was analyzed to solve fuzzy integer multi-criteria decision-making problems.

Key words: Goal programming, integer multi-criteria decision-making problem, iterative goal programming approach, fuzzy integer multi-criteria decision making problem

1. INTRODUCTION

The term "multi-criteria decision making" (MCDM) encompasses a wide variety of problems. Multi-criteria decision making is concerned with the methods and procedures by which multi-criteria can be formally incorporated into the analytical process.

Multi-criteria decision making has, however, two distinct halves: one half, is multi-attribute decision analysis, and the other is multi-criteria optimization (multi-objective mathematical programming).

Multi-attribute decision analysis is most often applicable to problems with a small number of alternatives in an environment of uncertainty. Multi-criteria optimization is often applied to deterministic problems in which the number of feasible alternatives is large.

C. Kahraman (ed.), Fuzzy Multi-Criteria Decision Making.

In recent years research has been carried out in solving multi-criteria integer programming problems, but whereas some has been classified as such, some has appeared in terms such as decision theory.

Treating integer multi-criteria decision-making problems can be classified into three main approaches: vector optimization (multi-objective optimization), goal programming, and interactive approaches. Most of the current research is directed mainly toward the interacting approaches trying to avoid the drawbacks in the other two approaches. Also, the current research includes the stochastic and fuzzy cases.

Most decision problems have multiple objectives that cannot be optimized simultaneously due to the inherent conflict between these objectives. Such problems involve making trade-off decisions to get the "best compromise" solution. Goal programming is a powerful approach that has been proposed for the modeling, solution, and analysis of the multi-criteria decision-making problems. There are a wide variety of goal programming models, including weighted goal programming (Charnes and Cooper, 1961; Ignizio, 1983) lexicographic goal programming, i.e., the use of the so-called "preemptive priority" concept (Ignizio, 1976; 1983), minimax goal programming includes fuzzy programming (Zimmerman, 1978) and interactive goal programming that is used to generate a subset of the nondominated solutions (Ignizio, 1981; Steuer, 1978).

Since goal programming now encompasses any linear, integer, zero-one, or nonlinear multi-objective problem (for which preemptive priorities may be established), the field of applications is wide open. The recent increase in interest in this area has already led to a large number of and a wide variety of actual and proposed applications. For purpose of illustration, we list just a few of these below, and the reader is referred to (Ignizio, 1978):

- Aggregate planning and work force (Dauer and Osman, 1981).
- Qualitative programming for selecting decisions (Zahedi, 1987).
- Curve and response surface fitting (Ignizio, 1977).
- Media planning (Chranes et al., 1968).
- Manpower planning (Chanes and Nilhaus, 1968).
- Program selection (Satterfield and Ignizio, 1974).
- Hospital administration (Lee, 1971).
- Academic resource allocation (Schroeder, 1974).
- Municipal economic planning (Lee and Sevebeck, 1971).
- Transportation problems (Lee and Moore, 1973).
- Energy/water resources (Elchak and Raphael, 1977).
- Radar system design (Ignizio and Satterfield, 1977).
- Sonar system design (Wilson and Ignizio, 1977).

- Planning in wood products (Inoue and Eslick, 1975).
- Portfolio selection (Kumar and Philippatos, 1975)
- Determination of time standards (Mashimo, 1977).
- Development of cost estimating relationship (Ignizio, Inpress).
- Urban renewal planning (Lee and Keown, 1976).
- Merger strategy (Salkin and Jones, 1972).
- Multi-plant/product aggregate production loading (Johnson, 1976).
- BMD systems design (Ignizio and Satterfield, 1977).
- Multi-objective facility location (Harnett and Ignizio, 1972).
- Free flight rockets (Ignizio, 1975a).
- Solar heating and cooling (Ignizio, 1975b).
- Natural gas well sitting (Gochnour, 1976).
- Maintenance level determination (Younis, 1977).
- A Pennsylvania coal model (Kirtland et al., 1977).

All of these applications have one thing in common: they could be forced onto a traditional single-objective model if one so wished. However, those investigating these problems believed that they truly involved multiple, conflicting objectives and were thus most naturally modeled as a goal programming problem (Ignizio, 1978).

2. INTEGER MULTICRITERIA DECISION-MAKING PROBLEM AS A GOAL PROGRAMMING MODEL

The integer multi-criteria decision-making problem (IMCDM) can be formulated mathematically as follows:

(IMCDM):

Maximize $Z(x)$

subject to $x \in X$

where Z: $R^n \rightarrow R^k$, $Z(x) = (z_1(x), z_2(x), ..., z_k(x))$ is a vector-valued criterion with $z_i(x)$, $i = 1, 2, ..., k$ which are real-valued objective functions and X is feasible set. This set might be, for example, of the form:

$$X = \{ x \in R^n \mid Ax \leq b, x \geq 0 \text{ and integer } \}$$

where A is an $(m \times n)$ matrix of constraint function coefficients; x is an $(n \times 1)$ vector of the integer decision variables; b is an $(m \times 1)$ vector of constraint right-hand sides, whose components specify the available resource; and R^n is the set of all ordered n-tuples of real numbers. It is assumed in problem (IMCDM) that the feasible set X is bounded.

The imperative "maximize" in problem (IMCDM) is understood to mean: Find the set of all solutions that have (roughly) the property that increasing the value of one objective $z_k(x)$ decreases the value of at least one other objective function. This set is usually called an *efficient (or nondominated, noninferior, Pareto-optimal, functional-efficient)* set. This set is a surrogate for an optimal solution to a usual optimization problem with a single objective function. The meaning of an efficient solution is given in the following definition.

DEFINITION 1.

A point $x^* \in X$ is said to be an efficient solution of problem (IMCDM) if there exists no other $x \in X$ such that $Z(x) \geq Z(x^*)$ and $Z(x) \neq Z(x^*)$ (see Chankong and Haims, 1983; Cohon, 1978; Geoffrion, 1968, Hwang and Masud, 1979).

Now, let us express the ith objective function in the form: $z_i(x) = c_i^t x$, where the superscript t denotes the transpose and c_i is an n-vector defined as the vector of the coefficients of the ith objective function.

In goal programming, rather than attempting to optimize the objective criteria directly, the decision maker sets to minimize the deviations between goals and levels of achievement within the given set of constraints. Thus, the objective function becomes the minimization of these deviations on the relative importance assigned to them.

Problem (IMCDM) can be transformed into the following integer linear goal programming model (ILGP) consisting of k goals:

(ILGP):

Find x to achieve:

$$z_1(x) = h_1$$

$$z_2(x) = h_2$$

$$\vdots$$

$$z_k(x) = h_k$$

subject to

$$x \in X$$

where $h_1, h_2, \ldots, h_k$ are scalars and represent the desired achievement levels of the objective functions that the decision maker wishes to attain provided that $z_{*_i} \prec h_i \prec z^{*i}, \quad i = 1,2,\ldots,k.$

Note that z^* and z_* provide upper and lower bounds on the objective function values and hence are a great source of information for the decision maker. These bounds can easily be determined by solving:

$$
\begin{aligned}
&\text{Maximize} \quad && z_i(x) \\
&\text{subject to} && \\
& && Ax \leq b, \\
& && x \geq 0 \text{ and integer.}
\end{aligned}
\tag{1}
$$

The solution of problem (1), (x_i^*, z_i^*), is known in the literature as the *ideal solution.* Let $z_{ji} = z_i(x_j);$ then

$$z_{*i} = \min_{\{i\}} z_{ji}, \qquad j = 1,2,\ldots,k. \tag{2}$$

DEFINITION 2.

The goals are ranked as follows: if $i \prec j$ then goal i, $c_i^t x = h_i,$ has a higher priority than goal j, $c_j^t x = h_j$, (see preemptive priorities Charnes and Cooper, 1961; Lee, 1972).

3. AN ITERATIVE GOAL PROGRAMMING APPROACH FOR SOLVING (IMCDM) PROBLEM

In order to solve the integer linear goal program (ILGP) by the iteration algorithm developed in Dauer and Krueger (1977) together with the Gomory's fractional cut shown in Klein and Holm (1978, 1979), we first solve the integer linear optimization problem associated with the first goal viz:

P_1:

minimize $\quad L_1 = d_1^- + d_1^+$

subject to

$$c_1^t x + d_1^- - d_1^+ = h_1,$$
$$Ax \leq b,$$
$$d_1^- \geq 0,\ d_1^+ \geq 0, x \geq 0, \text{ and integer}$$

where d_1^- and d_1^+ are the underattainment and the overattainment, respectively, of the first goal where $d_1^- d_1^+ = 0$.

Suppose this problem has integer optimal value $L_1^* = d_1^{-*} + d_1^{+*}$ with at least one value d_1^{-*} or d_1^{+*} nonzero.

Now, the attainment problem for goal 2 is equivalent to the integer optimization problem P_2, where

P_2:

minimize $\quad L_2 = d_2^- + d_2^+$

subject to

$$c_2^t x + d_2^- - d_2^+ = h_2,$$
$$c_1^t x + d_1^- - d_1^+ = h_1,$$
$$d_1^- + d_1^+ = L_1^*,$$
$$Ax \leq b,$$
$$d_i^- \geq 0,\ d_i^+ \geq 0, x \geq 0, \text{ and integer, } i = 1,2.$$

Letting $L_2^* = d_2^{-*} + d_2^{+*}$ to denote the integer optimal value of problem P_2, we can proceed to goal 3.

The general attainment problem P_j for goal j is written as

P_j :

minimize $\quad L_j = d_j^- + d_j^+$

subject to

$$
\begin{array}{ll}
c_i^t x + d_i^- - d_i^+ = h_i, & 1 \le i \le j \\
d_i^- + d_i^+ = L_i^*, & 1 \le i \le j-1 \\
Ax \le b, & \\
d_i^- \ge 0,\ d_i^+ \ge 0, x \ge 0, \text{ and integer}, & 1 \le i \le j
\end{array}
$$

where d_i^- and d_i^+ are the underattainment and the overattainment, respectively, of the ith goal level and $d_i^- d_i^+ = 0$.

The integer optimal objective value of problem P_j, L_j^*, is the maximum degree of attainment for goal j subject to the maximum attainment of goals 1, 2,…, j−1. Notice that $L_j^* = 0$ if and only if goal j is attained.

Let x^* be the optimal integer solution of the integer attainment problem P_k associated with the minimum L_k^*; then the solution of the ILGP is given by x^*.

The procedure used to solve the ILGP can be summarized as follows.

4. SEQUENTIAL INTEGER GOAL ATTAINMENT ALGORITHM

Step 1. Formulate the ILGP corresponding to the (IMCDM) problem.

Step 2. Solve the integer attainment problem P_1 for goal 1 using Gomory's cutting-plane technique (Klein and Holm, 1978; 1979) and obtain L_1^*.

Step 3. Set $i = 2$.

Step 4. Using $L_1^*, L_2^*, \ldots, L_{i-1}^*$, solve the integer attainment problems P_i using the same cutting-plane technique used in step 2.

Let L_i^* denote the minimum.

Step 5. If $i \ne k$, set $i = i+1$ and go to step 4. Otherwise, go to step 6.

Step 6. Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ denote the integer solution(s) of the attainment problem P_k associated with the minimum L_k^*.

The optimal integer solution(s) of the ILGP is then given by x^*.

5. AN ILLUSTRATIVE EXAMPLE (CRISP CASE)

In this section, we consider the following integer multi-criteria decision-making problem with two objective functions:

(IMCDM):

Maximize $Z(x) = (z_1(x), z_2(x))$

subject to

$$x \in X$$

where

$$X = \left\{ x \in R^2 \;\middle|\; \begin{array}{l} x_1 + x_2 \le 5,\ -x_1 + x_2 \le 0,\ 6x_1 + 2x_2 \le 21,\ x_1, x_2 \ge 0 \\ \text{and integer} \end{array} \right\}$$

and

$$z_1(x) = 2x_1 + x_2$$
$$z_2(x) = x_1 + 2x_2.$$

Suppose that the decision maker specifies the first priority goal to be $z_1(x)$ and the second priority goal to be $z_2(x)$. Consequently, an equivalent integer linear goal program corresponding to the IMCDM problem can be written as follows:

(ILGP):

Goal 1: Achieve $2x_1 + x_2 = h_1$

Goal 2: Achieve $x_1 + 2x_2 = h_2$

subject to

$$x \in X$$

It is easy to see that the aspiration levels of the objectives $z_1(x)$ and $z_2(x)$ are $h_1(x) = 7$ and $h_2(x)$, respectively. The integer linear attainment problem associated with the first goal is written as

P_1:

minimize $\quad L_1 = d_1^{-} + d_1^{+}$

subject to

$$2x_1 + x_2 + d_1^{-} - d_1^{+} = 7$$
$$x_1 + x_2 \le 5$$
$$-x_1 + x_2 \le 0$$
$$6x_1 + 2x_2 \le 21$$
$$d_1^{-},\ d_1^{-},\ x_1,\ x_1 \ge 0 \quad \text{and} \quad \text{integer}$$

This problem can be solved using the following Gomory cuts, (see Klei and Holm, 1978;1979):

$$2x_1 + x_2 \le 7$$
$$x_1 + x_2 \le 4$$
$$x_1 \le 3$$

and the maximum degree of attainment of problem P_1 is $L_1^{*} = 0$, with an optimal integer solution $x^1 = (3,1)$ where $d_1^{-} = 0$ and $d_2^{+} = 0$.

The attainment problem for goal 2 is equivalent to the integer optimization problem P_2, where

P_2:

minimize $\quad L_2 = d_2^{-} + d_2^{+}$

subject to

$$x_1 + 2\bar{x}_2 + d_2^{+} - d_2 = 6$$
$$2x_1 + x_2 + d_1^{-} - d_1^{+} = 7$$
$$d_1^{-} + d_1^{+} = 0$$
$$x_1 + x_2 \le 5$$
$$-x_1 + x_2 \le 0$$
$$6x_1 + 2x_2 \le 21$$
$$d_i^{-},\ d_i^{+},\ x_1,\ x_2 \ge 0 \text{ and integer,}$$
$$i = 1, 2$$

The initial solution $x^1 = (3,1)$, $d_1^{-} = 0$ and $d_2^{+} = 0$ yields a goal 2 value $x_1 + 2x_2 = 5$.

The maximum degree of attainment of goal 2 is $L_2^* = 1$ with an optimal integer solution $x^2 = (3,1)$, where $d_2^- = 1$ and $d_2^+ = 0$. Therefore, the optimal integer solution of the ILGP is given by

$$x^* = (3, 1)$$

$$L_1^* = 0, \quad \text{with} \quad d_1^- = 0, \; d_1^+ = 0$$

$$L_2^* = 1, \quad \text{with} \quad d_2^- = 1, \; d_2^+ = 0$$

6. FUZZY INTEGER MULTI-CRITERIA DECISION-MAKING PROBEM (FIMCDM)

In this section, we begin by introducing the following fuzzy integer multi-criteria decision-making problem with fuzzy parameters in the right-hand side of the constraints as

$$(\text{FIMCDM})_{\tilde{v}}: \qquad \text{Maximize } Z(x)$$

$$\text{subject to } x \in X(\tilde{v})$$

where

$$X(\tilde{v}) = \left\{ x \in R^n / g_r(x) \le \tilde{v}_r, \; (r = 1, 2, \ldots, m), \; x \ge 0 \text{ and integer} \right\}$$

and $Z : R^n \to R^k$, $Z(x) = (z_1(x), z_2(x), \ldots, z_k(x))$ is a vector-valued criterion with $z_i(x)$, $(i=1,2,..,k)$ are real-valued linear objective functions, $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_m)^t$ is a vector of fuzzy parameters, and R^n is the set of all ordered n-tuples of real numbers. Furthermore, the constraints functions $g_r(x)$, $(r = 1, 2, \ldots, m)$ are assumed to be linear.

Now, going back to problem $(\text{FIMCDM})_{\tilde{v}}$, we can write an associated fuzzy integer linear goal programming model $(\text{FILGP})_{\tilde{v}}$ consisting of k goals and having $\tilde{v} \in R^m$ a vector of fuzzy parameters in the right-hand side of the constraints. This model may be expressed as

$(\text{FILGP})_{\tilde{v}}$:

Achieve: $z_1(x) = h_1$,

$z_2(x) = h_2$

•
•
•

$z_k(x) = h_k$

and the constraints are given by

$$g_r(x) \le \tilde{v}_r \,, \; (r = 1, 2, \ldots, m)$$

$$x \ge 0 \text{ and integer}$$

where $h_1, h_2, \ldots, h_k$ are scalars and represent the aspiration levels associated with the objectives $z_1(x), z_2(x), \ldots, z_k(x)$, respectively.

7. FUZZY CONCEPTS

The fuzzy theory has been advanced by L.A. Zadeh at the University of California in 1965. This theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions. The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al. (1974) in the framework of the fuzzy decision of Zadeh and Bellman (Zadeh, 1970).

For the development that follows, we introduce some defintions concerning trapeziodal fuzzy numbers and their membership functions, which come from (Dubois and Prade, 1980) ,and that will be used throughout this part. It should be noted that an equivalent approach can be used in the triangular fuzzy numbers case.

DEFINITION 3.

A real fuzzy number $\tilde{a}$ is a fuzzy subset from the real line R with membership function $\mu_{\tilde{a}}$ (a) that satisfies the following assumptions:

1. $\mu_{\tilde{a}}(a)$ is a continuous mapping from R to the closed interval [0, 1],
2. $\mu_{\tilde{a}}(a) = 0 \qquad \forall\, a \in (-\infty, a_1\,]$,
3. $\mu_{\tilde{a}}(a)$ is strictly increasing and continuous on $[a_1, a_2]$,

4. $\mu_{\tilde{a}}(a) = 1 \qquad \forall\ a \in [a_2, a_3]$,
5. $\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_3, a_4]$,
6. $\mu_{\tilde{a}}(a) = 0 \qquad \forall\ a \in [a_4, +\infty)$.

where a_1, a_2, a_3, a_4 are real numbers and the fuzzy number $\tilde{a}$ is denoted by $\tilde{a} = [a_1, a_2, a_3, a_4]$.

DEFINITION 4.

The fuzzy number $\tilde{a}$ is a trapezoidal number, denoted by $[a_1, a_2, a_3, a_4]$, and its membership function $\mu_{\tilde{a}}(a)$ is given by (see Figure 1).

$$\mu_{\tilde{a}}(a) = \begin{cases} 0, & a \le a_1 \\ 1 - \left(\dfrac{a - a_2}{a_1 - a_2}\right)^2, & a_1 \le a \le a_2 \\ 1, & a_2 \le a \le a_3 \\ 1 - \left(\dfrac{a - a_3}{a_4 - a_3}\right)^2, & a_3 \le a \le a_4 \\ 0, & \text{otherwise.} \end{cases}$$

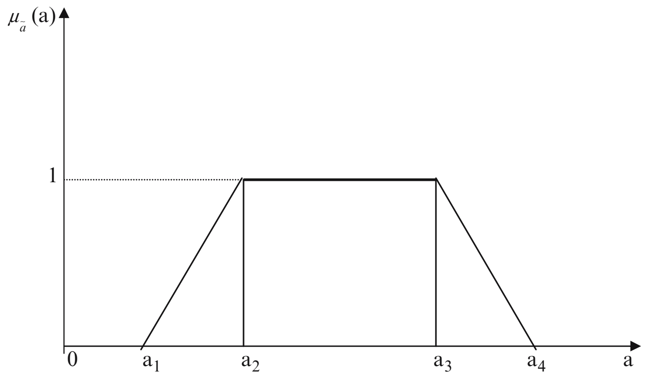

Figure 1. Membership function of a fuzzy number $\tilde{a}$

DEFINITION 5.

The α-level set of the fuzzy number $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership function exceeds the level $\alpha \in [0, 1]$:

$$L_\alpha(\tilde{a}) = \left\{ a \in R \,\middle|\, \mu_{\tilde{a}}(a) \geq \alpha \right\}.$$

For a certain degree $\alpha^* \in [0, 1]$ with the corresponding α-level set of the fuzzy numbers $\tilde{v}_r$, problem $(\text{FILGP})_{\tilde{v}}$ can be understood as the following nonfuzzy integer linear goal programming model written as:

$(\text{FILGP})_v$:

Achieve: $z_1(x) = h_1$

$z_2(x) = h_2$

•
•
•

$z_k(x) = h_k$

subject to

$$g_r(x) \leq v_r, \qquad (r = 1, 2, \ldots, m)$$

$$v_r \in L_\alpha(\tilde{v}_r), \ (r = 1, 2, \ldots, m)$$

$$x \geq 0 \text{ and integer}$$

where $L_\alpha(\tilde{v}_r)$ is the α- level set of the fuzzy parameters $\tilde{v}_r$, (r = 1, 2, …, m).

We now rewrite problem $(\text{FILGP})_v$ above in the following equivalent form:

$(ILGP)_v$:

Achieve: $z_1(x) = h_1$

$z_2(x) = h_2$

•
•
•

$z_k(x) = h_k$

subject to

$$g_r(x) \leq v_r\,, \qquad (r = 1, 2, \ldots, m)$$

$$n_r^{(0)} \leq v_r \leq N_r^{(0)}, \; (r = 1, 2, \ldots, m)$$

$$x \geq 0 \text{ and integer}$$

It should be noted that the constraint $v_r \leq L_\alpha(\tilde{v}_r)$, ($r = 1, 2, \ldots, m$), has been replaced by the equivalent constraint $n_r^{(0)} \leq v_r \leq N_r^{(0)}$, ($r = 1, 2, \ldots, m$), where $n_r^{(0)}$ and $N_r^{(0)}$ are lower and upper bounds on v_r.

Taking into account restrictions $g_r(x) \leq v_r^*$, ($r = 1, 2, \ldots, m$) and for the purpose of solving the integer linear goal program $(ILGP)_v$ at $v_r = v_r^* = N_r^{(0)}$, ($r = 1,2,\ldots,m$) for a certain degree $\alpha = \alpha^* \in [0, 1]$, we use the iterative approach developed in Dauer and Rrueger (1977) together with the Gromory cuts shown in Klein and Holm (1978, 1979). First, we solve the following integer linear optimization problem associated with the first goal, viz:

$P_1(v_r^*)$:

Minimize $\quad L_1 = d_1^- + d_1^+$

subject to

$$z_1(x) + d_1^- - d_1^+ = h_1$$

$$g_r(x) \leq v_r^*, \; (r = 1, 2, \ldots, m)$$

$$d_1^- \geq 0,\; d_1^+ \geq 0,\; x \geq 0 \text{ and integer}$$

where d_1^- and d_1^+ are the underattainment and the overattainment, respectively, of the first goal where $d_1^- \, d_1^+ = 0$.

Suppose this problem has integer optimal value $L_1^* = d_1^{-*} + d_1^{+*}$ with at least one value d_1^{-*} or d_1^{+*} nonzero.

Now, the attainment problem for goal 2 is equivalent to the integer optimization $P_2(v_r^*)$, where

$P_2(v_r^*)$:

Minimize $\quad L_2 = d_2^- + d_2^+$

subject to

$$z_2(x) + d_2^- - d_2^+ = h_2$$

$$z_1(x) + d_1^- - d_1^+ = h_1$$

$$d_1^- + d_1^+ = L_1^*$$

$$g_r(x) \le v_r^*, \; (r = 1, 2, \ldots, m)$$

$$d_i^- \ge 0, \; d_i^+ \ge 0, \; x \ge 0 \text{ and integer}, \; (i = 1,2)$$

Letting $L_2^* = d_2^{-*} + d_2^{+*}$ denotes the integer optimal value of $P_2(v_r^*)$, we can proceed to goal 3.

The general attainment problem $P_j(v_r^*)$ for goal j is written as

$P_j(v_r^*)$:

Minimize $L_j = d_j^- + d_j^+$

subject to

$$z_i(x) + d_i^- - d_i^+ = h_i, \; (1 \le i \le j)$$

$$d_i^- + d_i^+ = L_i^*, \; (1 \le i \le j\text{-}1)$$

$$g_r(x) \le v_r^*, \; (r = 1, 2, \ldots, m)$$

$$d_i^- \ge 0, \; d_i^+ \ge 0, \; x \ge 0 \text{ and integer}, \; (1 \le i \le j)$$

where d_i^- and d_i^+ are the underattainment and the overattainment, respectively, of the ith goal level and $d_i^- \, d_i^+ = 0$.

The integer objective value of $P_j(v_r^*)$, L_j^*, is the maximum degree of attainment for goal j subject to the maximum attainment of goals 1, 2,…, j−1. Notice that $L_j^* = 0$ if and only if goal j is attained.

Let x^* be the optimal integer solution of the integer attainment problem $P_j(v_r^*)$ associated with the minimum L_j^*, then the solution of the integer goal program $(ILGP)_v$ is given by x^* with $\alpha = \alpha^* \in [0, 1]$.

8. AN ITERATIVE GOAL PROGRAMMING APPROACH FOR SOLVING FIMCDM

Now, we develop a solution algorithm to solve the fuzzy integer linear goal program $(FILGP)_{\tilde{\nu}}$ .The outline of this algorithm is as follows (**Alg-II)**:

Step 0. Set $\alpha = {}^{\nu}\alpha^{*} = 0$.

Step 1. Determine the points (a_1, a_2, a_3, a_4) for each fuzzy parameter $\tilde{\nu}_r$, (r = 1, 2, …, m) in program $(FILGP)_{\tilde{\nu}}$ with the corresponding membership function $\mu_{\tilde{a}}(\tilde{\nu}) \geq \alpha^{*}$ for the vector of fuzzy parameters $\tilde{\nu} = (\tilde{\nu}_1, \tilde{\nu}_2, ..., \tilde{\nu}_m)^t$.

Step 2. Convert program $(FILGP)_{\tilde{\nu}}$ into the nonfuzzy integer linear goal program $(ILGP)_{\nu}$.

Step 3. Choose $\nu_r = \nu_r^{*} = N_r^{(0)}$, ($r$ = 1, 2,…, m) and solve problem $P_l(\nu_r^{*})$ using Gomory's cutting- plane method (Klein and Holm, 1978, 1979) and obtain L_1^{*}.

Step 4. Set j =2.

Step 5. Using L_1^{*}, L_2^{*},…, L_{j-1}^{*}, solve $P_j(\nu_r^{*})$ using the same Gomory's cutting-plane method used in step 3.

Let L_j^{*} denotes the minimum.

Step 6. If $j \neq k$, set $j = j + 1$ and go to step 5. Otherwise, go to Step 7.

Step 7. Let x^{*} denotes the optimal integer solution of problem $P_j(\nu_r^{*})$ associated with the minimum L_j^{*}.

Step 8. Set $\alpha = (\alpha^{*} + \text{step}) \in [0, 1]$, and go to Step 1.

Step 9. Repeat again the above procedure until the interval [0, 1] is fully exhausted. Then stop.

9. AN ILLUSTRATIVE EXAMPLE (FUZZY CASE)

Consider the following integer linear goal program involving fuzzy parameters $(\tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3)$ in the right-hand side of the constraints:

$(FILGP)_{\tilde{\nu}}$:

goal 1: Achieve $2x_1 + x_2 = h_1$

goal 2: Achieve $x_1 + 2x_2 = h_2$

subject to

$$x_1 + x_2 \leq \tilde{\nu}_1$$
$$-x_1 + x_2 \leq \tilde{\nu}_2$$

$$6x_1 + 2x_2 \leq \tilde{v}_3$$
$$x_1 \geq 0,\ x_2 \geq 0 \text{ and integers.}$$

Assume that the membership function corresponding to the fuzzy parameters is in the form:

$\mu_{\tilde{a}}(a) =$

$$\mu_{\tilde{a}}(a) = \begin{cases} 0, & a \leq a_1 \\ 1 - \left(\dfrac{a - a_2}{a_1 - a_2}\right)^2, & a_1 \leq a \leq a_2 \\ 1, & a_2 \leq a \leq a_3 \\ 1 - \left(\dfrac{a - a_3}{a_4 - a_3}\right)^2, & a_3 \leq a \leq a_4 \\ 0, & a \geq a_4 \end{cases}$$

where $\tilde{a}$ corresponds to each $\tilde{v}_i$, (i = 1, 2, 3). In addition, we assume also that the fuzzy unmbers are given by the following values:

$$\tilde{v}_1 = (2, 4, 6, 8),\ \tilde{v}_2 = (0, 3, 5, 7),\ \tilde{v}_3 = (18, 20, 22, 24).$$

Setting $\alpha = \alpha^* = 0$, then we get

$$2 \leq \tilde{v}_1 \leq 8,\ 0 \leq \tilde{v}_2 \leq 7,\ 18 \leq \tilde{v}_3 \leq 24.$$

By choosing $v^* = (v_1^*, v_2^*, v_3^*) = (8, 7, 24)$, then the aspiration levels of the goals have been found $h_1 = 10$ and $h_2 = 15$, respectively.

The integer optimization problem associated with the first goal is

$P_1(v_r^*)$:

Minimize $\quad L_1 = d_1^- + d_1^+$

subject to

$$2x_1 + x_2 + d_1^- - d_1^+ = 10$$
$$x_1 + x_2 \leq 8$$
$$-x_1 + x_2 \leq 7$$
$$6x_1 + 2x_2 \leq 24$$
$$d_1^- \geq 0,\ d_1^+ \geq 0,\ x_1 \geq 0,\ x_2 \geq 0 \text{ and integers.}$$

The maximum degree of attainment of problem $P_1(v_r^*)$ is $L_1^* = 0$ with the optimal integer solution:

$$x^1 = (2, 6) \text{ and } d_1^- = 0, \quad d_1^+ = 0.$$

The attainment problem for goal 2 is equivalent to the integer optimization problem $P_2(v_r^*)$ where

$P_2(v_r^*)$:

Minimize $\quad L_2 = d_2^- + d_2^+$

subject to

$$x_1 + 2x_2 + d_2^- - d_2^+ = 15$$
$$2x_1 + x_2 + d_1^- - d_1^+ = 10$$
$$d_1^- + d_1^+ = 0$$
$$x_1 + x_2 \le 8$$
$$-x_1 + x_2 \le 7$$
$$6x_1 + 2x_2 \le 24$$
$$d_i^- \ge 0, \; d_i^+ \ge 0, \; x_1 \ge 0, \; x_2 \ge 0, \text{ and integers } (i = 1, 2).$$

The maximum degree of attainment of goal 2 is $L_2^* = 1$ with the optimal integer solution:

$$x^2 = (2, 6) \text{ and } d_2^- = 1, \; d_2^+ = 0$$

Therefore, the optimal integer solution of the original integer linear goal program is:

$$x^* = (2, 6)$$

$$L_1^* = 0 \quad \text{with} \quad d_1^{-*} = 0, \; d_1^{+*} = 0$$

$$L_2^* = 1 \quad \text{with} \quad d_2^{-*} = 1, \; d_2^{+*} = 0$$

with the corresponding used Gomory cut: $x_2 \le 7$.

On the other hand, setting $\alpha = \alpha^* = 1$, we get:

$$4 \le \tilde{v}_1 \le 6, \; 3 \le \tilde{v}_2 \le 5, \; 20 \le \tilde{v}_3 \le 22.$$

Choosing $v^* = (v_1^*, v_2^*, v_3^*) = (6, 5, 22)$, then the optimal integer solution of the original program has been found:

$$x^* = (2, 4)$$

$$L_1^* = 0 \quad \text{with} \quad d_1^{-*} = 0,\ d_1^{+*} = 0$$

$$L_2^* = 1 \quad \text{with} \quad d_2^{-*} = 1,\ d_2^{+*} = 0$$

with the corresponding used Gomory cut: $3x_1 + x_2 \leq 10$.

Remark. It should be noted that a systematic variation of $\alpha \in [0, 1]$ will yield a new optimal integer solution to the integer linear goal program $(\text{FILGP})_{\tilde{v}}$

10. CONCLUSION

Since goal programming now encompasses any linear, integer, zero–one, or nonlinear multi-objective problem (for which preemptive priorities may be established), the field of applications is wide open. The recent increase in interest in this area has already led to a large number of and wide variety of actual and proposed applications. In this chapter, we have given numerical examples for the IMCDM problem and the FIMCDM. Fuzzy goal programming has many opportunities to develop new approaches to it.

REFERENCES

Chanes, A., and Nilhaus, R.J., 1968, A goal programming model for manpower planning, *Management Science Research Report,* 115, Carnegie-Mellon University, Pittsburgh, PA.

Chankong, V. and Haims, Y.Y., 1983, *Multiobjective Decision Making: (Theory and Methodology)*, Series Vol. 8, North Holland, New York.

Charnes, A., and Cooper, W.W., 1961, *Management Models and Industrial Applications of Linear Programming*, Wiley, New York.

Chranes. A., et al., 1968, A Goal Programming Model for Media Planning, *Management Sciences*, 138−151.

Cohon, J.L., 1978, *Multiobjective Programming and Planning*, Academic Press, New York.

Dauer, J.P. and Osman, M.S.A., 1981, *A Parametric Programming Algorithm for the Solution of Goal Programs with Application to Aggregate Planning of Production and Work Force*, Technical Report, University of Nebraska-Lincoln, Lincoln, NE.

Dauer, J.P., and Krueger, R.J., 1977, An iterative approach to goal programming, *Operational Research Quarterly*, **28:** 671−681.

Dubois, D., and Prade, H., 1980, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York.

Elchak, T., and Raphael, D.L., 1977, An energy planning model for Pennsylvania, *Proceedings of Pittsburgh Conference on Modelling and Simulation*, pp. 77−81.

Geoffrion, A.M., 1968, Proper efficiency and theory of vector maximization, *Journal of Mathematical Analysis and Applications*, **22:** 618−630.

Gochnour, J.R., 1976, A nonlinear goal programming approach to history mapping, Ph.D. Dissertation, Pennsylvania State University.

Harnett, R.M., and Ignizio, J.P., 1972, A heuristic program for the covering problem with multiple objectives, *Proceedings of Seminar on Multiple Criteria Decision Making*, University of South Carolina.

Hwang, C.L., and Masud, A.S., 1979, *Multiple Objective Decision Making—Methods and Applications (A State-of-the-Art Survey)*, Springer-Verlag, Berlin.

Ignizio, J.P., 1975a, The design of a multiple objective systems effectiveness model for the general support rocket system, *Report prepared for Teledyne Brown Engineering*, Huntsville, AL.

Ignizio, J.P., 1975b, The use of goal programming in the design of solar heating and cooling systems, Report prepared for Teledyne Brown Engineering, Huntsville, AL.

Ignizio, J.P., 1976, *Goal Programming and Extensions*, D.C. Health, Lexington Books, Lexington, MA.

Ignizio, J.P., 1977, Curve and Response surface fitting by goal programming, *Proceedings of Pittsburgh Conference on Modelling and Simulation*, pp. 1091−1094.

Ignizio, J.P., 1978, A review of goal programming: a tool for multiobjective analysis, *Journal of the Operational Research Society*, **29**(11)**:** 1109−1119.

Ignizio, J.P., 1981, The determination of a subset of efficient solutions via goal programming, *Computers & Operations Research*, **8:** 9−16.

Ignizio, J.P., 1983, GP-GN: An approach to certain large scale multiobjective integer programming models, *Large Scale Systems*, **4:** 177−188.

Ignizio, J.P., and Satterfield, D.E., 1977, Multicriteria optimization in BMD systems design, *Presented at National ORSA/TIMS Meeting*, Atlanta, GA.

Ignizio, J.P., and Satterfield, D.E., 1977a, Antenna array beam pattern synthesis via goal programming, *Presented at the Military Electronics Defense Exp 77*.

Ignizio, J.P., The Development of Cost Estimating Relationship via Goal Programming, *Engineering. Economy*, 34, In Press.

Inoue, M.S., and Eslick, P.O., 1975, Application of RPMS methodology to a goal programming problem in a wood product industry, *Presented at the AIIE Systems Engineering Conference*, Las Vegas, NY.

Johnson, H.J., 1976, Applying goal programming to multi-plant/product aggregate production loading, *Western Electrical Engineering*, 8−15.

Kirtland, D.A., Taugher, M.F., and Van Konkelenberg, 1977, A linear goal programming approach to the Pennsylvania coal model: utilities demand for non-coking coal, *Research Report for IE 502*, Pennsylvania State University.

Klein, D., and Holm, S., 1978, Discrete right-hand side parameterization for linear integer programs, *European Journal of Operational Research*, **2:** 50−53.

Klein, D., and Holm, S., 1979, Integer programming post-optimal analysis with cutting planes, *Management Sciences*, **25**(1)**:** 64−72.

Kumar, P.C., and Philippatos, G.C., 1975, A goal programming formulation to the selection of portfolios by dual-purpose funds, *Presented at the XXIII TIMS Meeting*, Athens, Greece.

Lee, S.M., 1971, An aggregate resource allocation model for hospital administration, *Presented at Third Annual AIDS Meeting*.

Lee, S.M., 1972, *Goal Programming for Decision Analysis*, Auerbach Publishers, Philadelphia.

Lee, S.M., and Moore, L.J., 1973, Optimizing transportation problem with multiple objectives, *AIIE Transactions,* **5:** 333−338.

Lee, S.M., and Sevebeck, W., 1971, An aggregate model for municipal economic planning, *Policy Science*, **2**(2)**:** 99−115.

Lee, S.M., and Keown, A.J., 1976, *Integer Goal Programming Model for Urban Renewal Planning*, Virginia Polytechnic Institute and State University Paper.

Mashimo, Y., 1977, A goal programming approach to maintenance level determination, *M. E. Research Paper*, Pennsylvania State University.

Salkin, G.R., and Jones, R.C., 1972, A goal programming formulation for merger strategy, In*: Applications of Management Science in Banking & Finance,* EIlon, S. and Fowkes, T. R., (eds.), Gower Press, London.

Satterfield, D.E., and Ignizio, J.P., 1974, The Use of Goal Programming in Program Selection and Resource Allocation, *Presented at the Second International Conference on Systems and Informatics*, Mexico City, Mexico.

Schroeder, R.G., 1974, Resource planning in university management by goal programming, *Operations Research*, **22:** 700−710.

Steuer, R., 1978, *Vector-Maximum Gradient Cone Contraction Techniques*, Multiple Criteria Problem Solving, Zionts, S., (ed.), Springer-Verlag, Berlin.

Wilson, G.L., and Ignizio, J.P., 1977b, The use of computers in the design of sonar arrays, *Presented at 9th International Congress on Acoustics*, Madrid, Spain.

Younis, N.A., 1977, Using goal programming to determine time standards, M. E. Research Report, Pennsylvania State University.

Zadeh, L., and Bellman, R., 1970, Decision Making in a Fuzzy Environment, *Management Sciences*, **17:** 141−164.

Zahedi, F., 1987, Qualitative programming for selecting decisions, *Computers & Operations Research*, **14**(5)**:** 395−407.

Zimmermann, H.J., 1978, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, **1:** 45−55.